SOME QUESTIONS

Probability and Statistics

1. A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all 4 defective bulbs will be obtained by the same person?

2. Suppose that a school band contains 20 students from the freshman class, 25 students from the sophomore class, and 35 students from the junior class. If 10 students are selected at random from the band, what is the probability that at least one student from each of the three classes?

3. If 10 percent of the balls in a certain box are red, and if 20 balls are selected from the box at random, with replacement, what is the probability that more than three red balls will be obtained?

4. (a) A four-engine plane can fly if at least two engines work. If the engines operate independently and each malfunctions with probability q, what is the probability that the plane will fly safely?
   (b) A-two engine plane can fly if at least one engine works and if an engine malfunctions with probability q, what is the probability that the plane will fly safely?
   (c) Which plane is the safest?

5. A rat maze consists of a straight corridor, at the end of which is a branch; at the branching point, the rat must either turn right or left. Assume 10 rats are placed in the maze, one at a time.
   (a) If each is choosing one of the two branches at random, what is the distribution of the number that turn right?
   (b) What is the probability at least 9 will turn the same way?

6. In a certain assembly plant, three machines, C_1, C_2, and C_3, make 35%, 30%, and 35%, respectively, of the products. It is known from past experience that 2%, 1.5%, and 1.5% of the products made by each machine, respectively, are defective.
   (a) Now, suppose that a finished product is randomly selected. What is the probability that it is defective?
   (b) If a product was chosen randomly and found to be non-defective, what is the probability that it was made by machine C_1?

7. Continuous random variable X has probability density function

$$f_X(x) = \begin{cases} kx(3-x), & 0 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find k?

   (b) Find the cumulative distribution function of X.

   (c) Define the random variable Y by $Y = 2X - 5$. Find the cumulative distribution function of Y.

8. Let X be a random variable denoting the proportion of students in a class who get a grade lower than C. Suppose X is a random variable with the following probability density function:

$$f_X(x) = \begin{cases} kx(4-x), & \text{if } x \in (0,4), \\ 0, & \text{if } x \notin (0,4). \end{cases}$$

   (a) Find k such that $f_X(x)$ is a probability density function.

   (b) A class is assumed to be unsuccessful if the proportion of students with a grade lower than C is greater than 0.5. Assume 6 classes take the course and their successes are independent of each other. What is the probability that at least two of the classes will be unsuccessful?

9. A midterm test has 4 multiple choice questions with four choices with one correct answer each. If you just randomly guess on each of the 4 questions, what is the probability that you get exactly 2 questions correct? Assume that you answer all and you will get (+5) points for 1 question correct, (-2) points for 1 question wrong. Let X is the number of points that you get. Find the probability mass function of X and the expected value of X.

10. Suppose the number of customers, arriving at a market is modeled as a Poisson Process with a mean arrival rate $\lambda = 4$ customers/hour.

    (a) If exactly 5 customers arrive during 9:30-10:30 what is the probability that at least 9 customers arrive during 9:30-11:00?

    (b) If less than 8 customers arrive during 9:30-13:30, the market is assumed to be nonprofitable. What is the probability that exactly 1 day will be profitable in a week? (Assume that the market is open 6 days a week).