Ex 1.

a, All people in homes in the city of Hanoi

b, All possible p tosses use the same coin

c, All pairs of the new type of tennis shoe.

d, All drives from her suburban home to her midtown office

Ex 2.

a, Mean : 8.6

b, Median : 9.5

c, Mode : 5, 10

Ex 3.

Mean : 2.6583

Median : 2.7

Variance : 0.3371

Standard deviation : 0.5806

Ex 4. The distribution of lengths of life that light bulbs have is N(800, 40)

The random variable $Z = \dfrac{\hat{X} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}}$ has a standard normal distribution

$$P(\hat{X} < 775) = P\left(\frac{\hat{X} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}} < \frac{775 - \mu_x}{\frac{\sigma_x}{\sqrt{n}}}\right)$$

$$= P(Z < -2.5)$$

$$= 0.0062$$

E5.

$$\sigma_{\hat{x}} = \frac{\sigma_x}{\sqrt{n}}$$

$$n = 36 - \sigma_{\hat{x}} = 2 \Rightarrow \sigma_x = 12$$

$$\frac{\sigma_x}{\sqrt{n}} = 1.2 \Rightarrow \sqrt{n} = 10 \Rightarrow n = 100$$

E6.

a, $\mu_x = 174.5$, $\sigma_x = 6.9$

$$\Rightarrow \mu_{\hat{x}} = 174.5 \quad , \quad \sigma_{\hat{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{6.9}{\sqrt{25}} = 1.38$$

b, The random variable $Z = \dfrac{\hat{X} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}}$ has a standard normal distribution

$$\Rightarrow P(172.5 < \hat{X} < 175.8) = P\left(\frac{172.5 - \mu_x}{\frac{\sigma_x}{\sqrt{n}}} < Z < \frac{175.8 - \mu_x}{\frac{\sigma_x}{\sqrt{n}}}\right)$$

$$= P(-1.45 < Z < 0.94)$$

$$= P(Z < 0.94) - P(Z < -1.45)$$

$$= 0.8264 - 0.00735 = 0.7529$$

$\Rightarrow$ The number of sample means in the interval is

$$200 \times 0.7529 \approx 151$$

c, Similarly

$$P(\hat{X} < 172.0) = P(Z < -1.81)$$

$$= 0.0351$$

$\Rightarrow$ The number of sample means in the interval is

$$200 \times 0.0351 \approx 7$$

E7

$$f(x \mid (x-10)^2<4) = \frac{f(x)}{P((x-10)^2<4)} = \frac{f(x)}{P(8<x<12)}$$

$$= \frac{f(x)}{P(\frac{8-\eta}{\sigma}<z<\frac{12-\eta}{\sigma})}$$

$$X \text{ is } N(10,1) \Rightarrow f(x \mid (x-10)^2<4) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-10)^2}}{0.9545}$$

E8

$$P(x>43) = P(z>1.5)$$
$$= 1-P(z<1.5)$$
$$= 1-0.93319$$
$$= 0.06681$$

E9

$$P(1 \leq x \leq 2) = P(0.5 \leq z \leq 1)$$
$$= P(z \leq 1) - P(z \leq 0.5)$$
$$= 0.84134 - 0.69146$$
$$= 0.14988$$

$$P(1 \leq x \leq 2 \mid x \geq 1) = \frac{P(1 \leq x \leq 2)}{1-P(x \leq 1)} = \frac{0.14988}{1-P(z \leq 0.5)}$$

$$= 0.48577$$

E10

$$P(x<1024) = P(z<1.2) = 0.88493$$

$$P(x<1024 \mid x>961) = \frac{P(-1.95<x<1.2)}{1-P(z<-1.95)} = \frac{0.88493-0.02559}{1-0.02559}$$

$$= 0.88191$$

$$P(31<\sqrt{x}<32) = P(961<x<1024) = 0.85934$$

E11

Let A be the lowest possible grade to get an A

$P(z \geq A) = 0.12$

$\Rightarrow P\left(z \geq \frac{A-74}{7}\right) = 0.12$

$\Rightarrow P\left(z \leq \frac{A-74}{7}\right) = 0.88$

$\Rightarrow \frac{A-74}{7} = 1.175 \Rightarrow A = 82.225$

$\Rightarrow$ The lowest possible A is 83, the highest possible B is 82

E12

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = P(-3 < z < 3)$$
$$= 1 - 2P(z < -3)$$
$$= 0.9973 > 8.9$$

E13

Probability of a correct answer is 0.25

$\Rightarrow$ Mean $\eta = 80 \times 0.25 = 20$

Variance $\sigma^2 = np(1-p) = 20 \times 0.75 = 15$

$\Rightarrow \sigma = 3.87$

$P(25 \leq x \leq 30) \approx P(24.5 < y < 30.5)$

with y is a continuos r.v. $N(20, 3.87)$

$$\Rightarrow P(25 \leq x \leq 30) \approx P(1.16 < z < 2.71)$$
$$= P(z < 2.71) - P(z < 1.16)$$
$$= 0.99664 - 0.87698$$
$$= 0.1196$$

$= 0.9545$

E15

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 6x^2(1-x)\, dx$$

$$= 0.5$$

$$\sigma^2 = E(X^2) - E^2(X) = \int_0^1 6x^3(1-x)\, dx - 0.25$$

$$= 0.3 - 0.25 = 0.05$$

$$\sigma = 0.224$$

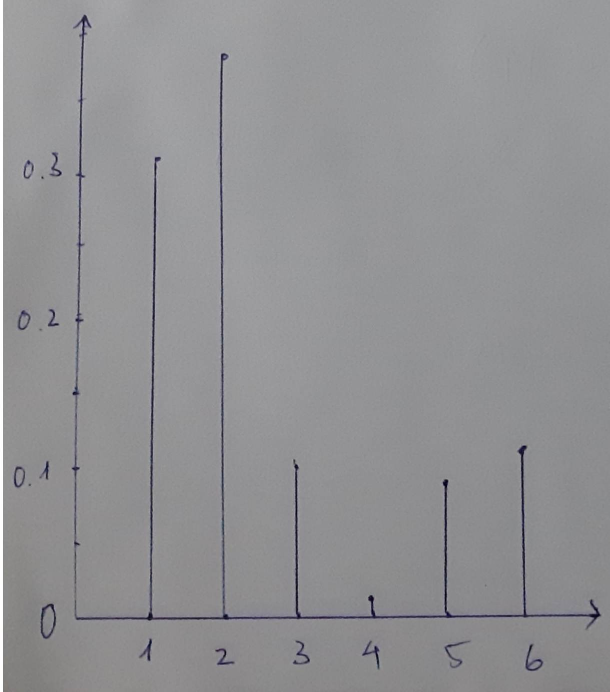
$$P(0.052 < x < 0.948) = \int_{0.052}^{0.948} 6x(1-x)\, dx$$

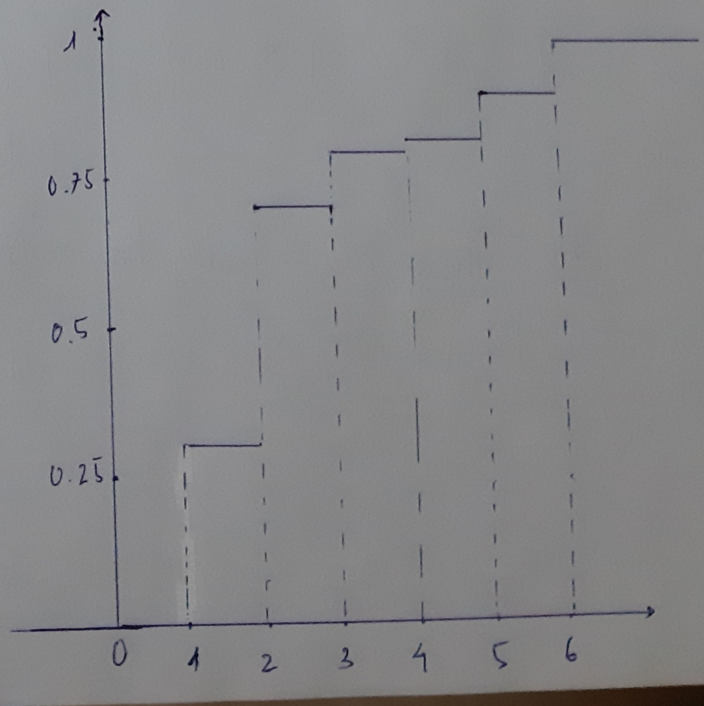
$$= 0.984$$

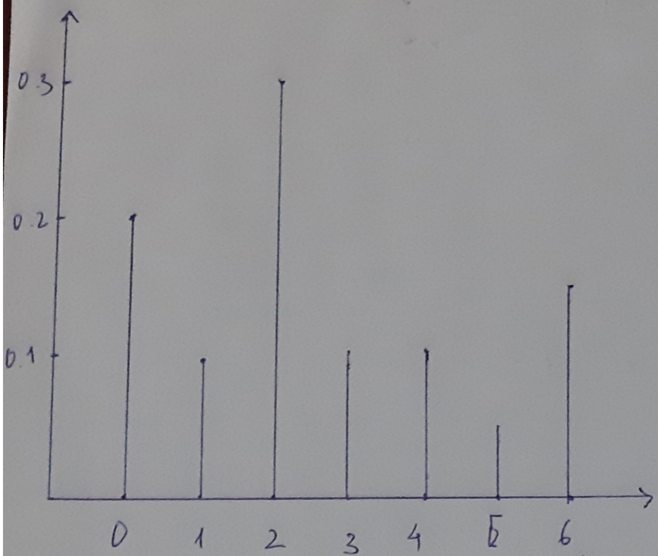
By Chebyshev's theorem: $P > 1 - \frac{1}{2^2} = 0.75$

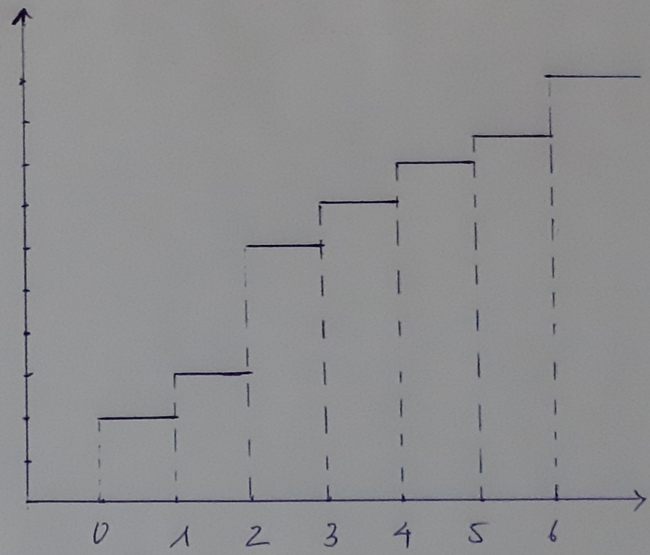
$\Rightarrow$ The result satisfies Chebyshev's theorem.

E2 Probability density function and cumulative distribution func. of A









Probability density & distribution functions of B

2.2

$E(A) = 2.52$ $\qquad$ $E(B) = 2.55$

$\sigma_A^2 = 2.7496$ $\qquad$ $\sigma_B^2 = 3.9475$

$\Rightarrow$ B is more disperse than A

E14

a, $P(-4 < X < 20) = P(8 - 4 \times 3 < X < 8 + 4 \times 3)$

According to Chebyshev's theorem,

$$P(-4 < X < 20) \geq 1 - \frac{1}{4^2} = \frac{15}{16}$$

b, $P(|X - 8| \geq 6) = P(2 \leq X \leq 14)$

$= P(8 - 2 \times 3 \leq X \leq 8 + 2 \times 3)$

According to Chebyshev's Theorem,

$$P(|X - 8| \geq 6) \geq 1 - \frac{1}{2^2} = 0.75$$