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Ngày
No

Homework 1

Ex 1

1.1) $$F(x) = \begin{cases} \int_{-\infty}^{x} f(t)\,dt & \text{if } x \le -1 \\ \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{x} f(t)\,dt & \text{if } -1 < x \le 2 \\ \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{x} f(t)\,dt & \text{if } x > 2 \end{cases}$$

$$= \begin{cases} 0, & x \le -1 \\ x^3/9 + 1/9 & \text{if } -1 < x \le 2 \\ 1, & x > 2 \end{cases}$$

1.2) $$P(0 < X \le 1) = \int_0^1 \frac{x^2}{3}\,dx = \frac{1}{9}$$

Ex 2

2.1)

$$f_A(x) = \begin{cases} 0.31, & \text{if } x = 1 \\ 0.38, & \text{if } x = 2 \\ 0.10, & \text{if } x = 3 \\ 0.01, & \text{if } x = 4 \\ 0.09, & \text{if } x = 5 \\ 0.11, & \text{if } x = 6 \\ 0 & \text{otherwise} \end{cases}$$

$$f_B(x) = \begin{cases} 0.20 & \text{if } x = 0 \\ 0.10 & \text{if } x = 1 \\ 0.30 & \text{if } x = 2 \\ 0.10 & \text{if } x = 3 \\ 0.10 & \text{if } x = 4 \\ 0.05 & \text{if } x = 5 \\ 0.15 & \text{if } x = 6 \end{cases}$$

$$F_A(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.31 & \text{if } 1 \le x < 2 \\ 0.69 & \text{if } 2 \le x < 3 \\ 0.79 & \text{if } 3 \le x < 4 \\ 0.80 & \text{if } 4 \le x < 5 \\ 0.89 & \text{if } 5 \le x < 6 \\ 1 & \text{if } x \ge 6 \end{cases}$$

$$F_B(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.20 & \text{if } 0 \le x < 1 \\ 0.30 & \text{if } 1 \le x < 2 \\ 0.60 & \text{if } 2 \le x < 3 \\ 0.70 & \text{if } 3 \le x < 4 \\ 0.80 & \text{if } 4 \le x < 5 \\ 0.85 & \text{if } 5 \le x < 6 \\ 1 & \text{if } x \ge 6 \end{cases}$$

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2.2)

Ex3 A: 1ST line B: 2nd line C: 3rd line

H: defective

$P(H) = P(A)\,P(H|A) + P(B).\,P(H|B) + P(C).\,P(H|C)$

$= 2.45\%$

$$P(A|H) = \frac{P(H|A)P(A)}{P(H)} = 24.49\%$$

$$P(B|H) = \frac{P(H|B)P(B)}{P(H)} = 55.1\%$$

$$P(C|H) = \frac{P(H|C)P(C)}{P(H)} = 20.41\%$$

Ex4 $F(x) = (1 - e^{-\alpha x})\,U(x-c)$

$f(x) = \alpha e^{-\alpha x} U(x-c) + (1 - e^{-\alpha x}) + \delta(x-c)$

Ex5

$$D^T = \begin{pmatrix} 1 & 2 & 1 & 3 & 4 & 3 \\ 1 & 1 & 2 & 1 & 3 & 3 \\ 3 & 4 & 4 & 5 & 2 & 3 \end{pmatrix}$$

$\bar{D} = (7/3 \quad 11/6 \quad 7/2)$

$$Cov = \frac{1}{n-1} \sum_{i=1}^{6} (D_i - \bar{D})(D_i - \bar{D})^T$$

$$= \begin{pmatrix} 22115 & 213 & -215 \\ 213 & 29130 & -7110 \\ -215 & -7110 & 11110 \end{pmatrix}$$

Ex6

6.1) Put $z = \frac{X-\eta}{\sigma}$

$$\Rightarrow p_k = P(-k < z < k) = \Phi(k) - \Phi(-k)$$
$$= 2\Phi(k) - 1$$

$p_1 = 0.6826$
$p_2 = 0.9544$
$p_3 = 0.9974$

6.2) $p = 0.9 \Leftrightarrow \Phi = 0.95 \Leftrightarrow k = 1.65$
$p = 0.99 \Leftrightarrow I = 0.995 \Leftrightarrow k = 2.575$
$p = 0.999 \Leftrightarrow I = 0.9995 \Leftrightarrow k = 3.05$

6.3) $\gamma = 2\Phi(z_u) - 1 \Rightarrow z_u = \Phi^{-1}\left(\frac{\gamma+1}{2}\right)$

Ex7 Put $z = X - 10 \Rightarrow z \sim N(0,1)$

$$f(x \mid (x-10)^2 < 1) = \frac{f(x)}{P((x-10)^2 < 4)} = \frac{\varphi(x)}{P(-2 < z < 2)} = \frac{f(x)^{25}}{2\Phi(2) - 1}$$

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Ex8 X: resistance of resistor

$X \sim N(40, 2^2)$

$Z = \frac{X-40}{2} \sim N(0,1)$

$P(X>43) = P(Z > \frac{3}{2}) \simeq 1 - \Phi(312) = 6.68\%$

Ex9 $X \sim N(0, 2^2) \Rightarrow Z = \frac{X}{2} \sim N(0,1)$

a) $P(1 \leq x \leq 2) = P\left(\frac{1}{2} \leq Z < 1\right) = 0.15$

b) $P(1 \leq x \leq 2 \mid x > 1) = \frac{P(1 \leq x \leq 2)}{P(X>1)} = \frac{P(\frac{1}{2} \leq Z \leq 1)}{1 - P(Z \leq \frac{1}{2})} = 0.22$

Ex10 $X \sim N(1000, 20)$

$Z = \frac{X-1000}{20} \sim N(0,1)$

f) $P(X < 1024) = P(Z < 1.2) = 0.885$

$P(X > 961) = 1 - P(Z < -1.95) = 0.975$

$P(X < 1024 \mid X > 961) = \frac{P(961 < X < 1024)}{P(X > 961)} = 0.892$

f) $P(31 < \sqrt{X} \leq 32) = P(961 < X \leq 1024)$

$= P(-1.95 < Z \leq 1.2) = 0.8593$

EX11: X: grade of student $\sim N(74, 7^2)$

$\Rightarrow Z = \frac{X-74}{7} \sim N(0,1)$

$P(X > a) = 12\% \Leftrightarrow P\left(z > \frac{a-74}{7}\right) = 0.12$

$\Leftrightarrow P\left(z < \frac{a-74}{7}\right) = 0.88$

$\Leftrightarrow \frac{a-74}{7} = 1.175$

$\Leftrightarrow a = 82.225$

$\Rightarrow$ Lowest A: 83
Highest B: 82

Ex12: $P(\mu - 3\sigma < X < \mu + 3\sigma)$
$= \Phi(3) - \Phi(-3) = 0.9974$

Ex13 $\sum_{k=25}^{30} C_{80}^{k} \left(\frac{1}{4}\right)^{k} \left(\frac{3}{4}\right)^{80-k} = 0.1193$

Ex14 a) $P(-4 < X < 20)$
$= P(8 - 3.4 < X < 8 + 3.4) \geq 1 - \frac{1}{4^2} = \frac{15}{16}$

(Chebysev Theorem)

b) $P(|X-8| \geq 6) = 1 - P(|X-8| < 6) \leq 1 - \left(1 - \frac{1}{2^2}\right) = \frac{1}{4}$

Ex15 $\mu = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{0}^{1} 6x^2(1-x)\,dx = \frac{1}{2}$

$\sigma^2 = E(X^2) - E^2(X) = \int_{0}^{1} 6x^3(1-x)\,dx - \frac{1}{4} = \frac{1}{20}$

$\Rightarrow P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9839$

Chebysev theorem: $P(\mu - 2\sigma < X < \mu + 2\sigma) \geq \frac{3}{4}$