ĐẠI HỌC BÁCH KHOA HÀ NỘI
VIỆN CÔNG NGHỆ THÔNG TIN VÀ TRUYỀN THÔNG

2. Language Models

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Language Models

- Compute the probability of a sentence or sequence of words in a language
- A good language model will consider: grammatically correct sentences are more fluently than words with a random order.
- E.g.: P(“hôm nay trời đẹp”) > P(“trời đẹp nay hôm”)

N-gram Language Models

- Goal: compute the probability of a sentence or sequence of words:

  $P(W) = P(w_1,w_2,w_3,w_4,w_{5,}\ldots,w_m)$

- Recall the definition of conditional probabilities

  $p(B|A) = P(A,B)/P(A)$ Rewriting: **$P(A,B) = P(A)P(B|A)$**

- More variables:

  $P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

- The Chain Rule in General

  $P(w_1,w_2,w_3,w_4,w_{5,}\ldots,w_m) = P(w1)*P(w2|w1)*P(w3|w1w2)*\ldots*P(wm|w1w2w3\ldots wm{-}1)$

  P("hôm nay trời đẹp") =

  P(hôm) * P(nay|hôm) * P(trời|hôm nay) * P(đẹp |hôm nay trời)

How to estimate these probabilities?

$$\text{P(đẹp } |hôm\ nay\ trời) = \frac{P(hôm\ nay\ trời\ \text{đẹp})}{P(hôm\ nay\ trời)}$$

- It is impossible to store all such probabilities, especially with m being the length of the natural language text

➢ Use Markov chain assuming that a word depends only on n-1 words before it (n-gram model)

$$P(w_m|w_1w_2w_3..w_{m-1}) = P(w_m|w_1,w_2,w_3,..,w_{m-1})$$

$$= P(w_m|w_{m-n}w_{m-n+1}w_{m-n+2}...w_{m-1})$$

N-gram Language Models

- Unigram model: $$P(w_1 w_2 \ldots wn) \sim \prod_i P(w_i)$$
- Bigram model: $$P(w_1 w_2 \ldots wn) \sim \prod_i P(w_i | wi\ _{-\ 1})$$
- Trigram model: $$P(w_1 w_2 \ldots wn) \sim \prod_i P(w_i | wi\ _{-\ 1}\, w_{i\ -\ 2})$$

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Example 1

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

Example 2
Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i’m looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i’m looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

- $P(english|want) = .0011$
- $P(chinese|want) = .0065$
- $P(to|want) = .66$
- $P(eat | to) = .28$
- $P(food | to) = 0$
- $P(want | spend) = 0$
- $P (i | \text{<s>}) = .25$

Practical Issues

- We do everything in log space
  - Avoid underflow
  - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

- Google Book N-grams
  - http://ngrams.googlelabs.com/
- KenLM
  - https://kheafield.com/code/kenlm/

Google N-Gram Release

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensible 40
- serve as the individual 234

http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to “real” or “frequently observed” sentences
    - Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

Extrinsic evaluation of N-gram models

- Best evaluation for comparing models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
  - Compare accuracy for A and B

Intrinsic evaluation

- **Intrinsic** evaluation using **perplexity (độ phức tạp)**
    - Bad approximation
        - unless the test data looks **just** like the training data
        - So **generally only useful in pilot experiments**

Intuition of Perplexity

- The **Shannon Game**:
  - How well can we predict the next word?

    I always order pizza with cheese and ____

    The 33rd President of the US was ____

    I saw a ____
  - Unigrams are terrible at this game. (Why?)
- A better model of a text
  - is one which assigns a higher probability to the word that actually occurs

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= (\frac{1}{10}^{N})^{-\frac{1}{N}} \\
&= \frac{1}{10}^{-1} \\
&= 10
\end{aligned}
$$

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - We need to train robust models that generalize!
  - One kind of generalization: Zeros!
    - Things that don’t ever occur in the training set but occur in the test set

Zeros

- Training set:
  … denied the allegations
  … denied the reports
  … denied the claims
  … denied the request

  P("offer" | denied the) = 0

- Test set
  … denied the offer
  … denied the loan

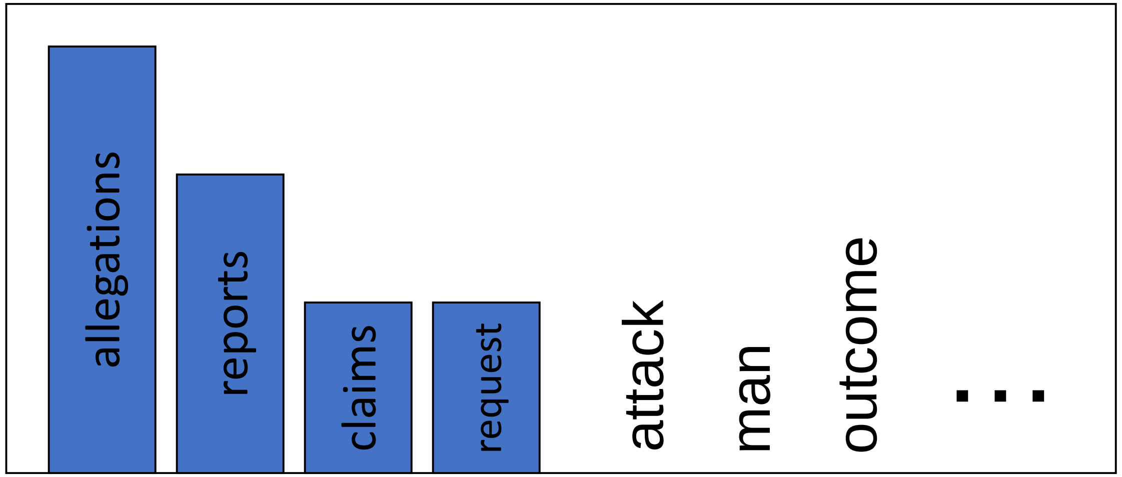
→ Probability of a sentence or a sequence of words =0

➢ Using smoothing

The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

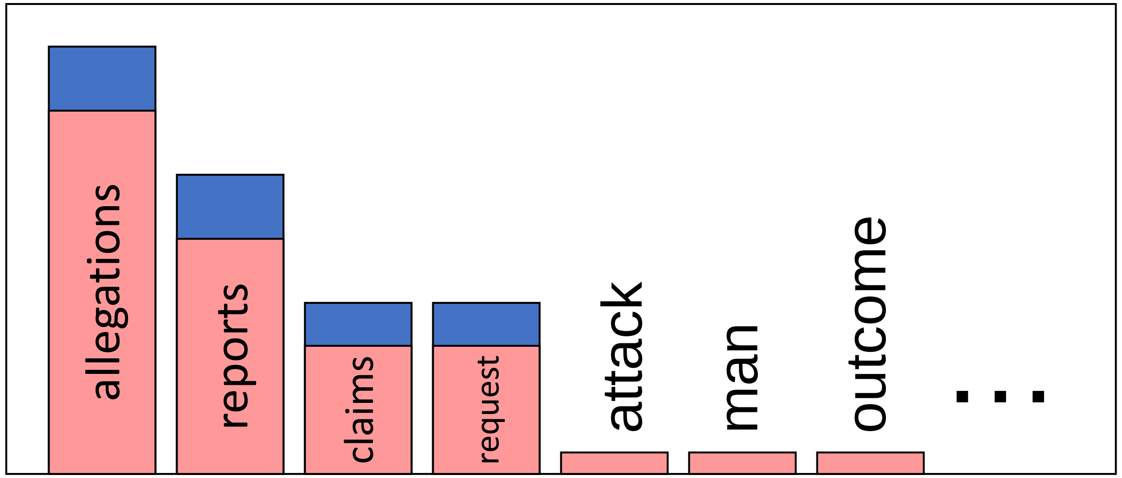
P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total



- Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other

7 total



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!

- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$