QUESTIONS

Question 1 In an experiment, A, B, and C are events with probabilities $P[A] = P[B] = P[C] = 0.3$, $P[ABC] = 0$, and each pair of events is independent.

(a) Find $P[ABC^c]$.

(b) Find $P[A^cB^cC^c]$.

Question 2 Jack plays a game, which assigns a number that is uniformly distributed between 0 and 5. If the game assigns a number less than or equal to k, then he loses 1 dollar, on the other hand, if the game assigns a number larger than k, then he will gain 1 dollar.

(a) Find the expected profit of the game.

(b) What is the k value, which minimizes the variance of the profit obtained in this game?

(c) If you were to play this game 10 times, what is the probability that you gain 2 dollars.

Question 3 Suppose that in a certain drug, the concentration of a particular chemical is a random variable with a continuous distribution for which the PDF g is as follows:

$$g_X(x) = \begin{cases} \dfrac{3}{8}x^2, & \text{if} \quad 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Suppose the concentrations X and Y of the chemical in two separate batches of the drug are independent random variables for each of the PDF is g. Determine

(a) The joint PDF of X and Y;

(b) $P(X > Y)$;

(c) CDF $F_Z(z)$, where $Z = 2X + Y$.

Question 4 Independent random samples of $n_1 = 35$ and $n_2 = 46$ observations were selected from two normal populations, $X_1 \sim \mathcal{N}(\mu_1; \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2; \sigma_2^2)$:

	Population 1	Population 2
Sample mean	$\overline{x}_1 = 32.2$	$\overline{x}_2 = 34.6$
Sample variance	$s_1^2 = 5.9$	$s_2^2 = 4.8$

(a) Find a 95% confidence interval for $E(X_2) = \mu_2$.

(b) How large a sample is needed in **Question 4**(a) if we wish to be 98% confident that our sample mean will be within 1.2 of the true mean?

(c) Suppose you wish to detect a difference between the population means. Conduct the test and state your conclusions. Use a 0.01 level of significance.

(d) Find $P[X_1 > 37.055]$.