Exercises

E1: Given continuous r.v *X* with probability density function *f(x)*:

$$f(x) = \begin{cases} x^2/3, & -1 < x < 2 \\ 0, & x \leq -1; x \geq 2 \end{cases}$$

1.1 Determine its probability distribution function *F(x)*.

1.2 Calculate probability $P\{0<X\leq 1\}$ using *F(x)* and using *f(x)*.

E2: Suppose we have two experiment A and B with experimental results that are discrete r.vs with following probability density functions respectively:

A \ x	1	2	3	4	5	6
$f_a(x)$	0.3 1	0.38	0.10	0.01	0.09	0.11

B \ x	0	1	2	3	4	5	6
$f_b(x)$	0.20	0.10	0.30	0.10	0.10	0.05	0.15

2.1 Sketch probability distribution functions and density functions of these r.vs.

2.2 Compare dispersion of experimental results of two experiments.

E3: In a plant, three assembly lines make 30% , 45% and 25% of products respectively. It is known that 2%, 3%, 2% of products made by each line are defective respectively.

3.1 Calculate probability that a randomly selected product is defective ?

3.2 From which assembly line a defective product comes from ?

E4: find f(x) if $F(x) = (1 - e^{-\alpha x})U(x - c)$

E5: Experimental results are samples of 3D vector r.v (x, y, z) and have a set of values: (1, 1, 3), (2, 1, 4), (1, 2, 4,), (3, 1, 5), (4, 3, 2), (3, 3, 3).

Calculate covariance matrix of this r.v.

E6: The r.v X is $N(\eta, \sigma)$ and $P\{ \eta - k\sigma < x < \eta + k\sigma\} = p_k$,

a) Find p_k if k = 1, 2, 3
b) Find k for p_k = 0.9, 0.99, 0.999.
c) If $P\{ \eta - z_u\sigma < x < \eta + z_u\sigma\} = \gamma$, express z_u in term of γ

E7: the r.v X is N(10, 1). Find $f(x|(x-10)^2<4)$.

E8: A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms'?

E9: if x is N(0, 2), find: $P\{1\leq x\leq 2\}$ and $P\{1\leq x\leq 2|x\geq 1\}$

E10: If x is N(1000, 20), find: P{x<1024}; P{x<1024|x>961}; P{31<√x≤32}

E11: The average grade for an exam is 74, and the standard deviation is 7. If 12%; of the class is given A's, a nd the grades are curved to follow a normal distribution,what is the lowest possible A and the highest possible B?

E12: According to Chebyshev's theorem ($P((\mu - k\sigma < X < \mu + k\sigma) \geq 1\text{-}1/k^2$), the probability that any random variable assumes a value within 3 standard deviations of the mean is at least 8/9. If it is known that the probability distribution of a random variable X is normal with mean μ, and variance σ^2, what is the exact value of $P(\mu - 3\sigma < X < \mu + 3\sigma)$?

E13: A multiple-choice quiz has 200 questions each with 4 possible answers of which only 1 is the correct answer. What is the probability that sheer guesswork yields from 25 to 30 correct answers for 80 of the 200 problems about which the student has no knowledge?

E14: A random variable X has a mean $\mu = 8$, a variance $\sigma^2 = 9$, and an unknown probability distribution. Find

(a) $P(-4 < X < 20)$, (b) $P(|X - 8| \geq 6)$.

E15: Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$, where X has the density function

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0 \quad , & elsewhere \end{cases}$$

and compare with the result given in Chebyshev's theorem.