Inspire the love of learning

(II) 2021 June - Statistics

(1) The population contains all individuals / subjects about which we want to collect information.

a = All people in homes in the city of Hanoi.

b = All possible tosses of the used coin.

c = All pairs of the new type of tennis shoe.

d = Every drive from the lawyer's suburban home to her midtown office.

(2) a = The mean is:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i = 8,6$$

b = Firstly, we sort the records in ascending order:

5, 5, 5, 6, 9, 10, 10, 10, 11, 15.

The median is:

$$x_{med} = \frac{9+10}{2} = 9,5.$$

c = The modes are: 5, 10.

(3) The mean is 2,658

The median is 2,7

The variance is: $\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 = 0,3428$

The standard deviation is 0,585

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(4) ⊕ $\mu_{\bar{x}} = 800$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{16}} = 10$

⊕ $P(X \leq 775) = P\left(\frac{X - 800}{10} < \frac{775 - 800}{10}\right)$

$= P(Z \leq -2,5)$

$= 1 - P(Z \leq 2,5)$

$= \boxed{0,00621}$

(5) ⊕ $\sigma_1 = 2$, $n_1 = 36$

⊕ $\sigma_1 = \frac{6}{\sqrt{n_1}} \Rightarrow 6 = 12$

⊕ $\sigma_2 = 1,2 \Rightarrow \sqrt{n_2} = 10 \Rightarrow \boxed{n_2 = 100}$

(6) a $\mu_{\bar{x}} = M = \boxed{174,5}$

$\sigma_{\bar{x}} = \frac{6}{\sqrt{n}} = \frac{6,9}{\sqrt{25}} = \boxed{1,38}$

b $P(172,5 < X < 175,8) = P(-1,45 < Z < 0,94)$

$= P(Z < 0,94) - P(Z < -1,45)$

$= P(Z < 0,94) - 1 + P(Z < 1,45)$

$= 0,7529$

Hence, the number of sample means that fall between 172,5 and 175,8 cm is $200 \times 0,7529 \simeq \boxed{151}$

c $P(X \leq 172,0) = P(Z < -1,81)$

$= 1 - P(Z < 1,81)$

$= 0,03593$

Hence, the number of sample means falling below 172cm is $200 \times 0,03593 = \boxed{7}$

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(7)

⊕ $\mu_{\bar{x}} = 2,6$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0,3}{\sqrt{36}} = 0,05.$

⊕ $95\% \Rightarrow \alpha/2 = 0,025 \Rightarrow z_{\alpha/2} = 1,96.$

95% confidence interval is:

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$\Rightarrow 2,6 - 1,96 \times 0,05 < \mu < 2,6 + 1,96 \times 0,05.$

$\Leftrightarrow \quad 2,502 < \mu < 2,698.$

⊕ $99\% \Rightarrow \alpha/2 = 0,005 \Rightarrow z_{\alpha/2} = 2,575$

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$\Rightarrow 2,6 - 2,575 \times 0,05 < \mu < 2,6 + 2,575 \times 0,05$

$\Leftrightarrow \quad 2,4713 < \mu < 2,7288.$

(8)

⊕ $\mu_{\bar{x}} = 23500$

$\sigma = 3900$

$n = 100$

⊕ $H_0 : \mu = 20000$

$H_1 : \mu > 20000$

⊕ $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{23800 - 20000}{\sigma/\sqrt{n}} = 8,97.$

⊕ $P(Z > 8,97) = 1 - P(Z < 8,97) \simeq 1 - 1 = 0.$

⇒ The P-value is ≈0 ⇒ Reject the null hypothesis.

Hence, automobile is driven on the average more than 20000 kilometers per year

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(9) The second moment of y:

$$E(Y^2) = E((x(8) - x(5))^2)$$
$$= E(x^2(8) + x^2(5) - 2x(8)x(5))$$
$$= E(x^2(8)) + E(x^2(5)) - 2E(x(8).x(5))$$
$$= R(0) + R(0) - 2R(3)$$
$$= 2A - 2Ae^{-3\alpha}$$

(10) ⓐ $R_{yy}(t_1, t_2) = R_{xx}(t_1, t_2) * h(t_2) * h(t_1)$

$$= R_{xy}(t_1, t_2) * h(t_1)$$

ⓑ $R_{xy}(t_1, t_2) * h(t_1) = \int_{-\infty}^{+\infty} h(\alpha) R_{xx}(t_1, t_2 - \alpha)\, d\alpha$

$$= \int_{-\infty}^{+\infty} e^{-c\alpha} u(\alpha) . A . \delta(T - \alpha)\, d\alpha$$