Chapter 1

Probability

1.1 Experiments

Problem 1.1 A fax transmission can take place at any of three speeds depending on the condition of the phone connection between the two fax machines. The speeds are high (h) at 14400 b/s, medium (m) at 9600 b/s, and low (l) at 4800 b/s. In response to requests for information, a company sends either short faxes of two (t) pages, or long faxes of four (f) pages. Consider the experiment of monitoring a fax transmission and observing the transmission speed and length. An observation is a two-letter word, for example, a high-speed, two-page fax is ht.

(a) What is the sample space of the experiment?

(b) Let A_1 be the event "medium-speed fax." What are the outcomes in A_1?

(c) Let A_2 be the event "short (two-page) fax." What are the outcomes in A_2?

(d) Let A_3 be the event "high-speedfax or low-speed fax." What are the outcomes in A_3?

(e) Are A_1, A_2, and A_3 mutually exclusive?

(f) Are A_1, A_2, and A_3 collectively exhaustive?

Problem 1.2 An integrated circuit factory has three machines X, Y, and Z. Test one integrated circuit produced by each machine. Either a circuit is acceptable (a) or it fails (f). An observation is a sequence of three test results corresponding to the circuits from machines X, Y, and Z, respectively. For example, aaf is the observation that the circuits from X and Y pass the test and the circuit from Z fails the test.

(a) What are the elements of the sample space of this experiment?

(b) What are the elements of the sets

$$\begin{aligned} Z_F &= \{\text{circuit from } Z \text{ fails}\}, \\ X_A &= \{\text{circuit from } X \text{ is acceptable}\}. \end{aligned}$$

(c) Are Z_F and X_A mutually exclusive?

(d) Are Z_F and X_A collectively exhaustive?

(e) What are the elements of the sets

$$C = \{\text{more than one circuit acceptable}\},$$
$$D = \{\text{at least two circuits fail}\}.$$

(f) Are C and D mutually exclusive?

(g) Are C and D collectively exhaustive?

Problem 1.3 Find out the birthday (month and day but not year) of a randomly chosen person. What is the sample space of the experiment. How many outcomes are in the event that the person is born in July?

Problem 1.4 Let the sample space of the experiment consist of the measured resistances of two resistors. Give four examples of event spaces.

1.2 Counting Methods

Problem 1.5 Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. How many different code words are there? How many code words have exactly three 0's?

Problem 1.6 Consider a language containing four letters: A, B, C, D. How many three-letter words can you form in this language? How many four-letter words can you form if each letter appears only once in each word?

Problem 1.7 On an American League baseball team with 15 field players and 10 pitchers, the manager must select for the starting lineup, 8 field players, 1 pitcher, and 1 designated hitter. A starting lineup specifies the players for these positions and the positions in a batting order for the 8 field players and designated hitter. If the designated hittermust be chosen among all the field players, howmany possible starting lineups are there?

Problem 1.8 A basketball team has three pure centers, four pure forwards, four pure guards, and one swingman who can play either guard or forward. A pure position player can play only the designated position. If the coach must start a lineup with one center, two forwards, and two guards, how many possible lineups can the coach choose?

1.3 Probability

Problem 1.9 In a certain city, three newspapers A, B, and C are published. Suppose that 60 percent of the families in the city subscribe to newspaper A, 40 percent of the families subscribe to newspaper B, and 30 percent of the families subscribe to newspaper C. Suppose also that 20 percent of the families subscribe to both A and B, 10 percent subscribe to both A and C, 20 percent subscribe to both B and C, and 5 percent subscribe to all three newspaper A, B, and C. What percentage of the families in the city subscribe to at least one of the three newspapers?

Problem 1.10 From a group of 3 freshmen, 4 sophomores, 4 juniors and 3 seniors a committee of size 4 is randomly selected. Find the probability that the committee will consist of a) 1 from each class; b) 2 sophomores and 2 juniors; c) Only sophomores and juniors.

Problem 1.11 A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all 4 defective bulbs will be obtained by the same person?

Problem 1.12 Suppose that three runners from team A and three runners from team B participate in a race. If all six runners have equal ability and there are no ties, what is the probability that three runners from team A will finish first, second, and third, and three runners from team B will finish fourth, fifth, and sixth?

Problem 1.13 Suppose that a school band contains 10 students from the freshman class, 20 students from the sophomore class, 30 students from the junior class, and 40 students from the senior class. If 15 students are selected at random from the band, what is the probability that at least one students from each of the four classes?

Problem 1.14 Suppose that 10 cards, of which 5 are red and 5 are green, are placed at random in 10 envelopes, of which 5 are red and 5 are green. Determine the probability that exactly x envelopes will contain a card with a matching color $(x = 0, 1, 2, ., 10)$.

Problem 1.15 Consider two events A and B with $P(A) = 0.4$ and $P(B) = 0.7$. Determine the maximum and minimum possible values of $P(A \cap B)$ and the conditions under which each of these values is attained.

Problem 1.16 Suppose that four guests check their hats when they arrive at a restaurant, and that these hats are returned to them in a random order when they leave. Determine the probability that no guest will receive the proper hat.

Problem 1.17 Suppose that A, B and C are three independent events such that P(A) = 1/4, P(B) = 1/3 and P(C) = 1/2. a) What is the probability that none of these three events will occur? b) Determine the probability that exactly one of these three events will occur.

Problem 1.18 Three players A, B and C take turns tossing a fair coin. Suppose that A tosses the coin first, B tosses the second and C tosses third and cycle is repeated indefinitely until someone wins by being the first player to obtain a head. Determine the probability that each of the three players will win.

Problem 1.19 Computer programs are classified by the length of the source code and by the execution time. Programs with more than 150 lines in the source code are big (B). Programs with ≤ 150 lines are little (L). Fast programs (F) run in less than 0.1 seconds. Slow programs (W) require at least 0.1 seconds. Monitor a program executed by a computer. Observe the length of the source code and the run time. The probability model for this experiment contains the following information: $P[LF] = 0.5$, $P[BF] = 0.2$, and $P[BW] = 0.2$. What is the sample space of the experiment? Calculate the following probabilities:
(a) $P[W]$; (b) $P[B]$; (c) $P[W \cup B]$.

Problem 1.20 Mobile telephones perform handoffs as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than three minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

What is the probability $P[H_0]$ that a phone makes no handoffs? What is the probability a call is brief? What is the probability a call is long or there are at least two handoffs?

Problem 1.21 Proving the following facts: (a) $P[A \cup B] \geq P[A]$; (b) $P[A \cup B] \geq P[B]$; (c) $P[A \cap B] \leq P[A]$; (d) $P[A \cap B] \leq P[B]$.

1.4 Law of Total Probability

Problem 1.22 Given the model of handoffs and call lengths in Problem 1.20,

(a) What is the probability that a brief call will have no handoffs?

(b) What is the probability that a call with one handoff will be long?

(c) What is the probability that a long call will have one or more handoffs?

Problem 1.23 You have a six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even-numbered.

(a) What is $P[R_3|G_1]$, the conditional probability that 3 is rolled given that the roll is greater than 1?

(b) What is the conditional probability that 6 is rolled given that the roll is greater than 3?

(c) What is $P[G3|E]$, the conditional probability that the roll is greater than 3 given that the roll is even?

(d) Given that the roll is greater than 3, what is the conditional probability that the roll is even?

Problem 1.24 You have a shuffled deck of three cards: 2, 3, and 4. You draw one card. Let C_i denote the event that card i is picked. Let E denote the event that card chosen is a even-numbered card.

(a) What is $P[C_2|E]$, the probability that the 2 is picked given that an even-numbered card is chosen?

(b) What is the conditional probability that an evennumbered card is picked given that the 2 is picked?

Problem 1.25 Two different suppliers, A and B, provide a manufacturer with the same part. All suppliers of this part are kept in a large bin. In the past, 5 percent of the parts supplied by A and 9 percent of the parts supplied by B have been defective. A supplies four times as many parts as B. Suppose you reach into the bin and select a part and find it is non-defective. What is the probability that it was supplied by A?

Problem 1.26 Suppose that traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.75 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green.
a) What is the probability that the second light is green?
b) What is the probability that you wait for at least one light?

Problem 1.27 A factory has three machines A, B, and C. Past records show that the machine A produced 40% of the items of output, the machine B produced 35% of the items of output, and machine B produced 25% of the items. Further 2% of the items produced by machine A were defective, 1.5% produced by machine B were defective, and 1% produced by machine B were defective.
a) If an item is drawn at random, what is the probability that it is defective?
b) An item is acceptable if it is not defective. What is the probability that an acceptable item comes from machine A?

1.5 Independent

Problem 1.28 Is it possible for A and B to be independent events yet satisfy $A = B$?

Problem 1.29 In an experiment, A, B, C, and D are events with probabilities $P[A] = 1/4$, $P[B] = 1/8$, $P[C] = 5/8$, and $P[D] = 3/8$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find $P[A \cap B]$, $P[A \cup B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

(c) Find $P[C \cap D]$, $P[C \cap D^c]$, and $P[C^c \cap D^c]$.

(d) Are C^c and D^c independent?

Problem 1.30 In an experiment, A, B, C, and D are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

(c) Find $P[D]$, $P[C \cap D^c]$, $P[C^c \cap D^c]$, and $P[C|D]$.

(d) Find $P[C \cup D]$ and $P[C \cup D^c]$.

(e) Are C and D^c independent?

1.6 Bernoulli Trials

Problem 1.31 Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit.

Problem 1.32 Suppose each day that you drive to work a traffic light that you encounter is either green with probability $7/16$, red with probability $7/16$, or yellow with probability $1/8$, independent of the status of the light on any other day. If over the course of five days, G, Y, and R denote the number of times the light is found to be green, yellow, or red, respectively, what is the probability that $P[G = 2, Y = 1, R = 2]$? Also, what is the probability $P[G = R]$?

Problem 1.33 An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.

Problem 1.34 A midterm test has 4 multiple choice questions with four choices with one correct answer each. If you just randomly guess on each of the 4 questions, what is the probability that you get exactly 2 questions correct? Assume that you answer all and you will get (+5) points for 1 question correct, (-2) points for 1 question wrong. Let X is number of points that you get. Find $P(X = 13)$.