

Computer Vision

Chapter 4: Feature detection and Image matching

1

Plan

- Edges
 - Detection
 - linking
- Feature extraction
 - Global features
 - Local features
- Image Matching and Applications

2

Edge detection

Role of edges & general approach

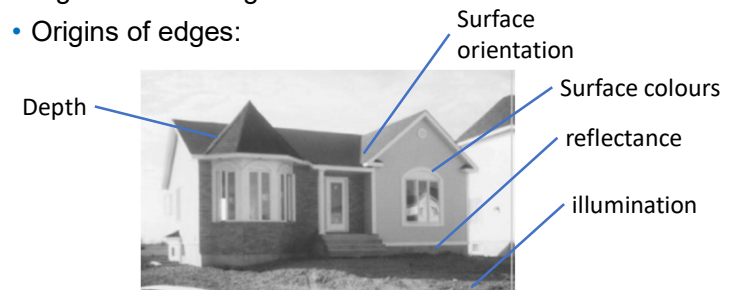
Image gradient

Edges detectors: sobel, prewitt, Canny detector

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What are edges/contours ?

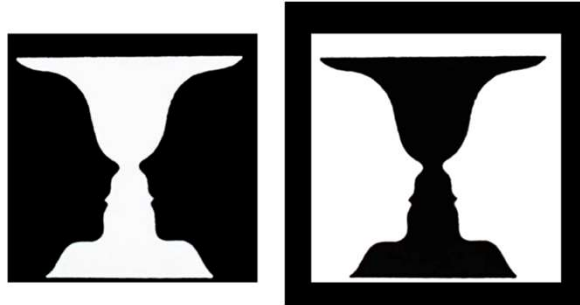
- Local changes in the images
- Typically occur on the boundary between different regions in an image
- Origins of edges:



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Edge is important?

- What do you see ?



Edge is important?

- (A) Cave painting at Chauvet, France, about 30,000 B.C.;
- (B) Aerial photograph of the picture of a monkey as part of the Nazca Lines geoglyphs, Peru, about 700 – 200 B.C.;
- (C) Shen Zhou (1427-1509 A.D.): Poet on a mountain top, ink on paper, China;
- (D) Line drawing by 7-year old I. Lleras (2010 A.D.).

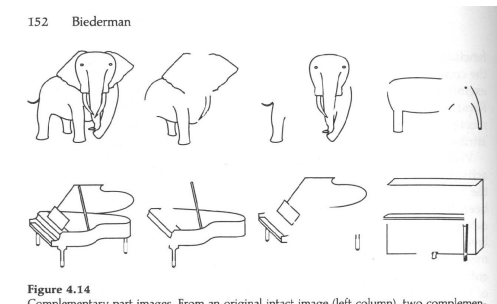
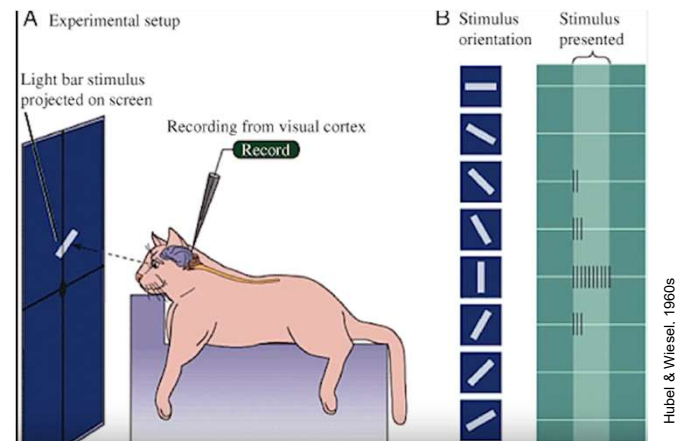
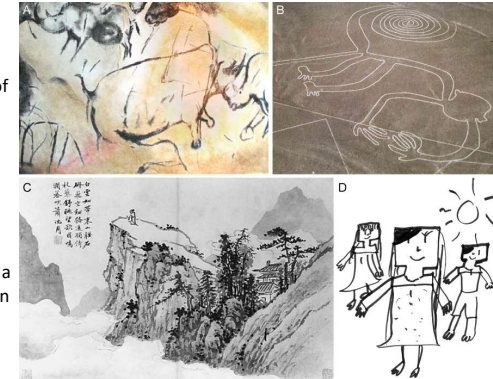
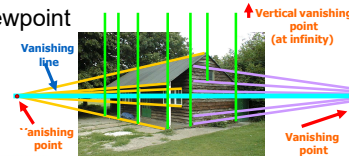


Figure 4.14
Complementary-part images. From an original intact image (left column), two complemen-

Can we recognize these objects?

Edge detection

- Goal: Identify sudden changes (discontinuities) in an image
 - Intuitively, most semantic and shape information from the image can be encoded in the edges
 - More compact than pixels
- Why?
 - Extract information, recognize objects
 - Recover geometry and viewpoint



Source: J. Hayes

How to find edges?

- Intensity profile of an image is the set of intensity values taken from regularly spaced points along a line segment or multi-line path in an image

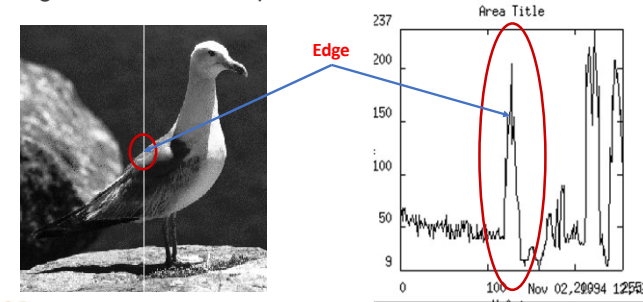
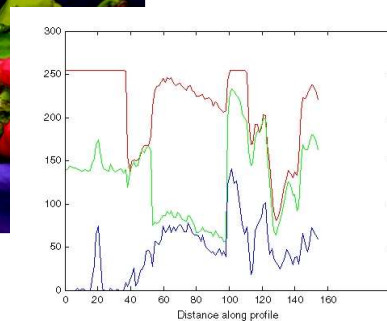


Image profile

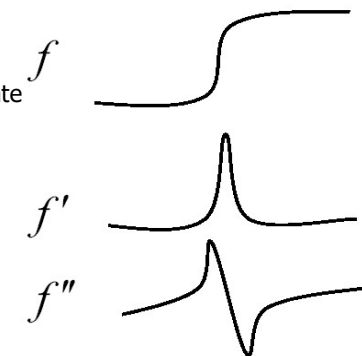


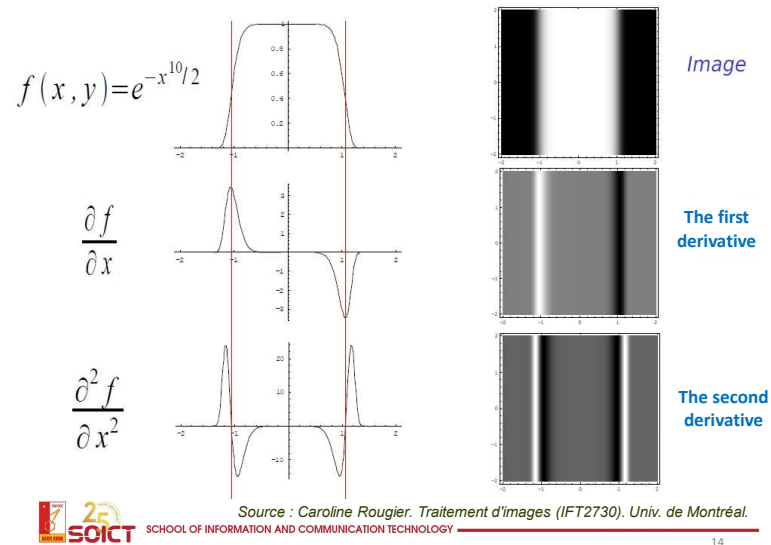
<https://www.mathworks.com/help/images/intensity-profile-of-images.html>

How to find edges?

- An edge is a place of rapid change in the image intensity function

- Extrema of 1st derivative
- Zero-crossing of 2nd derivative





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Image gradients

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Derivatives

- Derivative in 1D:

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

- Discrete derivative in 1D

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

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Types of Discrete derivative in 1D

Backward $\frac{df}{dx} = f(x) - f(x-1) = f'(x)$

Forward $\frac{df}{dx} = f(x) - f(x+1) = f'(x)$

Central $\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$

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1D discrete derivate filters

- Backward filter:

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x) \quad [0 \quad 1 \quad -1]$$

- Forward:

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x) \quad [-1 \quad 1 \quad 0]$$

- Central:

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x) \quad [1 \quad 0 \quad -1]$$

1D discrete derivate example

- Backward filter: $[0 \quad 1 \quad -1]$

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

$$f'(x) = 0 \quad 5 \quad -5 \quad 0 \quad 15 \quad -5 \quad 0 \quad 0$$

$$\begin{array}{l} f(x) : \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 50 \quad 50 \quad 50 \quad 50 \quad 50 \\ f'(x) : \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 50 \quad 0 \quad 0 \quad 0 \quad 0 \end{array}$$

Discrete derivate in 2D

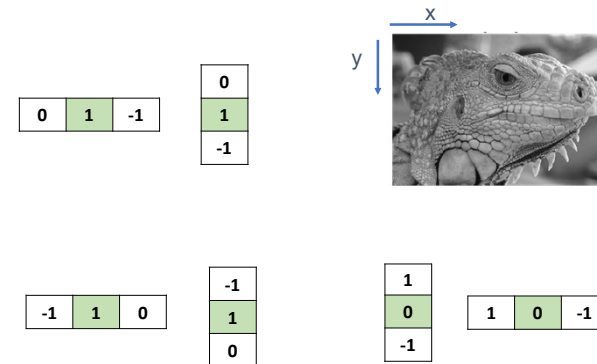
Given function $f(x, y)$

Gradient vector $\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$

Gradient magnitude $|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$

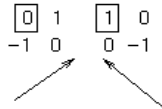
Gradient direction $\theta = \tan^{-1} \frac{f_x}{f_y}$

Discrete derivate filters wrt. x and y



Discrete derivate filters wrt. x and y

- Robert filter (the first approximation filter for image derivative - 1965)



- Prewitt filter

$$\frac{1}{3} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & -0 & -1 \end{bmatrix}$$

$$\frac{1}{3} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

- Sobel filter

$$\frac{1}{4} \times \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & -0 & -1 \end{bmatrix}$$

$$\frac{1}{4} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$



Image I (9 x 8)

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100

 M_x : Derivative filter wrt x

$$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

 M_y : Derivative filter wrt y $I_x = I * M_x = \text{the 1st derivative wrt x}$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	100	100	0	0	0	0	0	0
0	100	100	0	0	0	0	0	0
0	100	100	0	0	0	0	0	0
0	100	100	0	0	0	0	0	0
0	100	100	0	0	0	0	0	0



Image I (9 x 8)

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100
0	0	0	100	100	100	100	100	100

 M_x : Derivative filter wrt x

$$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

 M_y : Derivative filter wrt y $I_y = I * M_y = \text{the 1st derivative wrt y}$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	100	100	100	100	100	100	100
0	0	100	100	100	100	100	100	100
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0



3x3 image gradient filters

$$\frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

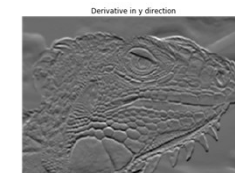
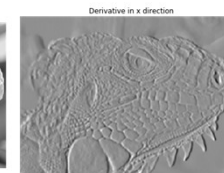


Image gradient

- Derivative of image wrt. x + Derivative wrt. y
→ Image gradient: magnitude and direction

- Gradient magnitude: gradient intensity for each pixel (*mostly used*)

$$|G| = \sqrt{(G_x^2 + G_y^2)} \approx |G_x| + |G_y|$$

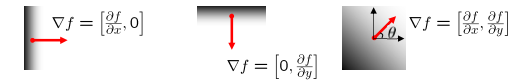
- Gradient Direction: main direction of each pixel

$$\theta = \tan^{-1} G_y / G_x$$



Image gradient

- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$



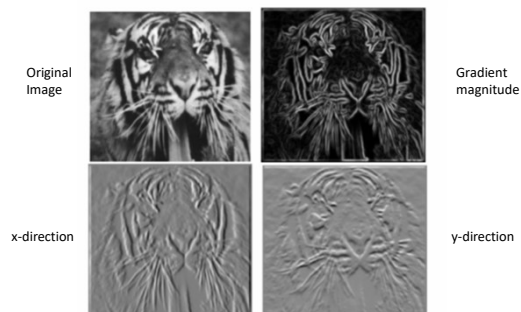
- The **gradient vector** points in the direction of most rapid increase in intensity
- The **gradient direction** is given by $\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$
 - how does this relate to the direction of the edge?
- The *edge strength* is given by the **gradient magnitude**

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seltz



Example



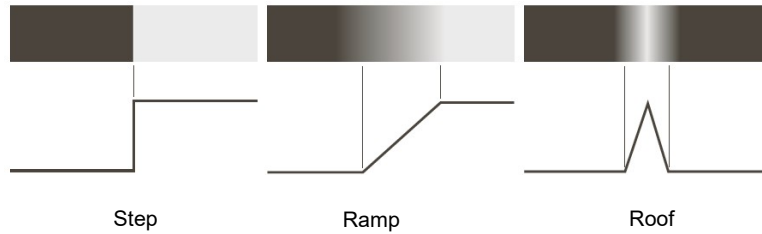
- Which one is the gradient in the x-direction? How about y-direction?



Edge detector

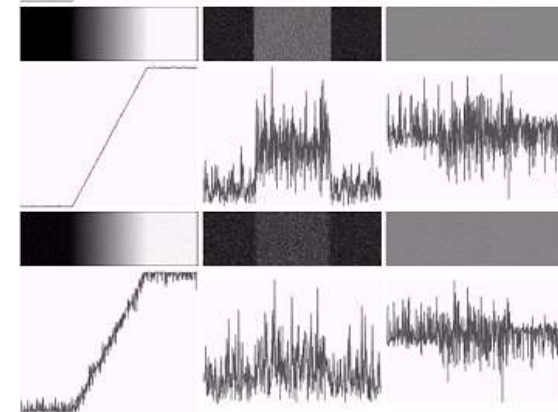


Type of edges



Source : Gonzalez and Woods. Digital Image Processing 3ed. Prentice-Hall, 2008.

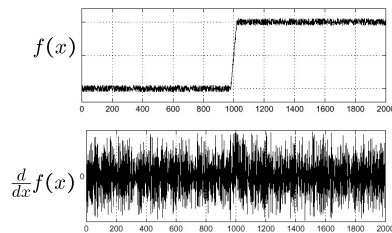
Effects of noise



Source : Gonzalez and Woods. Digital Image Processing 3ed. Prentice-Hall, 2008.

Effects of noise

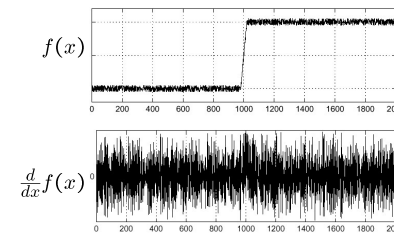
- Consider a single row or column of the image
 - Plotting intensity as a function of position gives a signal



Where is the edge?

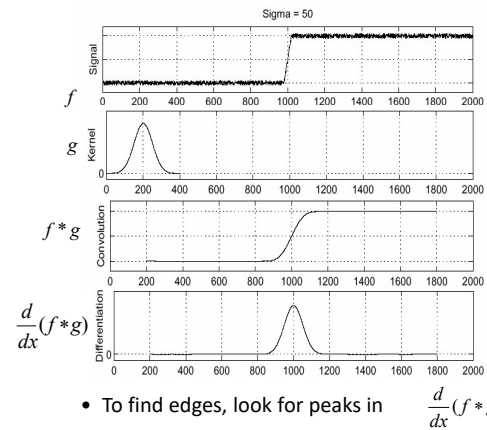
Effects of noise

- Solution: smoothing the image



Where is the edge?

Solution: smooth first



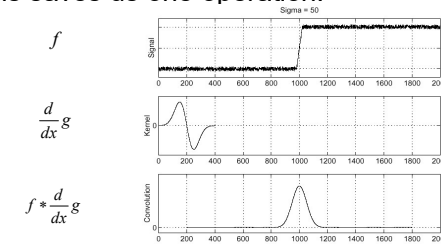
Source: S. Seitz

Derivative theorem of convolution

- This theorem gives us a very useful property:

$$\frac{d}{dx}(f * g) = \frac{df}{dx} * g = f * \frac{dg}{dx}$$

- This saves us one operation:



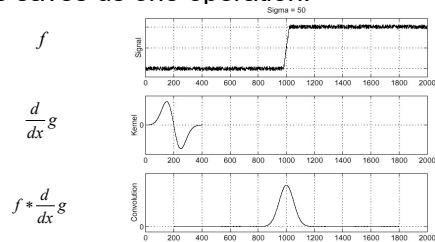
Source: S. Seitz

Derivative theorem of convolution

- This theorem gives us a very useful property:

$$\frac{d}{dx}(f * g) = \frac{df}{dx} * g = f * \frac{dg}{dx}$$

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Source: S. Seitz

Sobel Operator

- Gaussian Smoothing + differentiation

$$G_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Gaussian smoothing
differentiation

$$G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

➔ Less sensible to noise

Prewitt Operator

- Mean smoothing + differentiation

$$Gx = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1 \quad 0 \quad -1]$$

$$Gy = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} [1 \quad 1 \quad 1]$$

→ Less sensible to noise

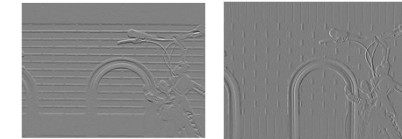


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Simple edge detector with 1st derivative

- Convolve the original image with 2 kernels to calculate approximations of the derivatives



- Compute the gradient magnitude



- Thresholding: choose edges to be the pixel above a threshold T



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Simple edge detector with 1st derivative

Original image



Gradient magnitude using Sobel operator



Threshold T = 25



Threshold T = 60



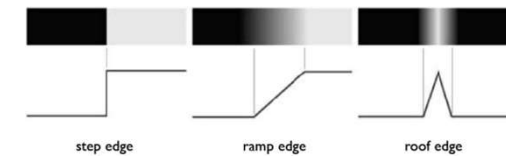
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Source : Caroline Rougier. Traitement d'images (IFT2730). Univ. de Montréal.

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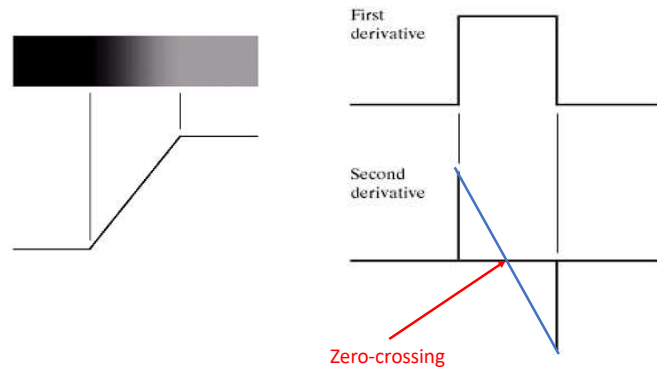
Problems

- Poor localization (multiple adjacent pixels)
- Thresholding value favors certain directions over others
 - Can miss oblique edges more than horizontal or vertical edges → False negatives



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Edge detector with 2nd derivative



Edge detector with 2nd derivative

- 2nd derivative with Laplacian filter:

– convolution the image with one of 2 filters

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Edge detection

– Compute the 2nd derivative of the images

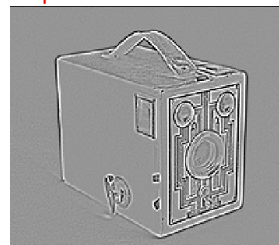
– Find the zero-crossing pixels → edges

Edge detector with 2nd derivative

Image



Laplacian

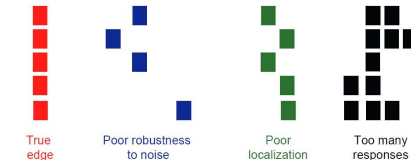


- Single response
- Sensible to noise

“Optimal” edge detector

- Criteria:

- **Good detection:** the optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges)
- **Good localization:** the edges detected must be as close as possible to the true edges
- **Single response:** the detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge



Canny detector

- This is probably the **most widely used** edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that **optimizes the product of signal-to-noise ratio and localization**



J. Canny, *A Computational Approach To Edge Detection*, IEEE Trans. PAMI, 8:679-714, 1986.

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Canny detector

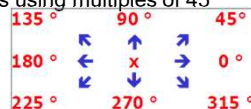
- Optimal:
 - Detection: **weak edges detected**
 - Good location: **close to the real edges**
 - Unique response: **edge thickness = 1**



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Canny detector: Steps

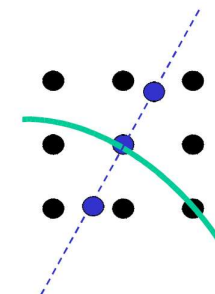
- 1) Apply a gaussian filter on the image
 - Lowpass filter to **remove noise**
- 2) Compute the **gradient intensity** in the image
 - Sobel filter in X and Y
 - Compute the magnitude $|G| = |G_x| + |G_y|$
- 3) Compute image **gradient direction**
 - Gradient direction $\theta = \arctan(G_y / G_x)$
 - Round directions using multiples of 45°



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Canny detector: Steps

- 4) Non-maxima suppression
 - If the gradient magnitude of a pixel (x,y) is **inferior to the one of its 2 neighbors** along the gradient direction
- set this magnitude for (x,y) to zero

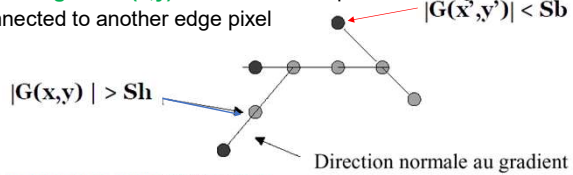


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Canny detector: Steps

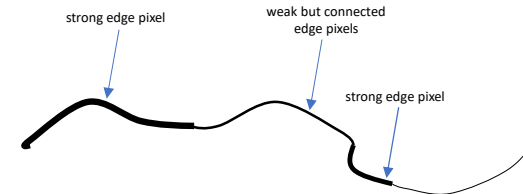
5) Edge thresholding (hysteresis)

- Using two thresholds: a threshold high (S_h) and a threshold low (S_b)
- For each pixel in the gradient magnitude:
 - IF $\text{magnitude}(x,y) < S_b$, THEN set the pixel to zero (non-edge)
 - IF $\text{magnitude}(x,y) > S_h$, THEN the edge is an edge
 - IF $S_b \leq \text{magnitude}(x,y) \leq S_h$, THEN the pixel is an edge IF it is connected to another edge pixel



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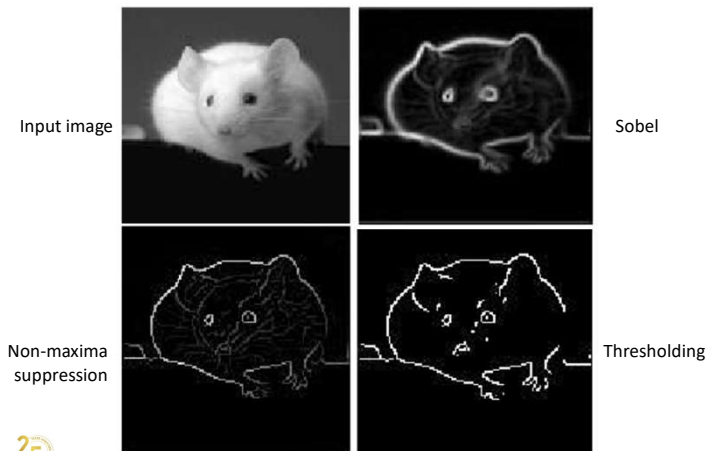
Hysteresis thresholding



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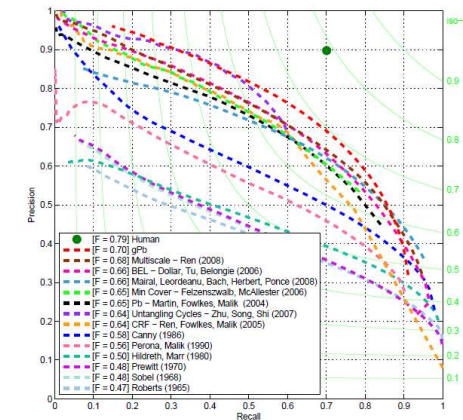
Source: S. Seitz

Canny detector



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45 years of boundary detection



Source: Arbelaez, Maire, Fowlkes, and Malik. TPAMI 2011 (pdf)



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Edge linking

Hough Transform
RANSAC



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Hough transform



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Hough transform

- The Hough transform (HT)
 - can be used to detect lines
 - was introduced in 1962 (Hough 1962) and first used to find lines in images a decade later (Duda 1972)
 - Goal: to find the location of lines in images.
- **Caveat:** Hough transform can *detect lines, circles* and other *structures* ONLY if their *parametric equation is known*
- It can give **robust detection** under *noise* and *partial occlusion*



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Hough transform

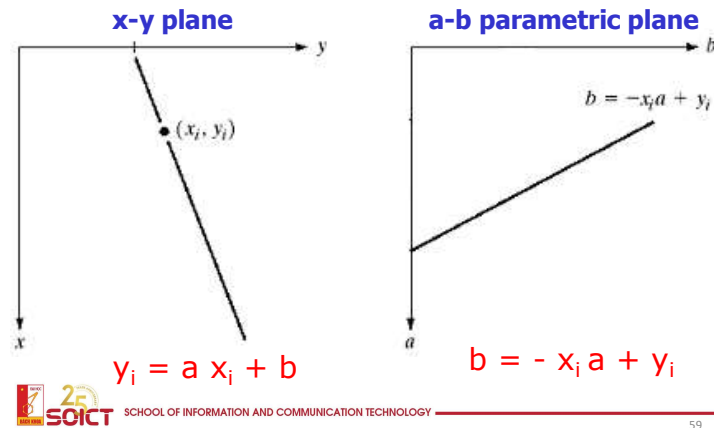
- **Global** approach to detect continuous edges
 - *From the x-y plane to the parametric plane a-b*
- **x-y plane**
 - $y_i = a x_i + b$
 - an infinity of lines going through one (x_i, y_i) pair
 - one sole line for the (a,b) pair
- **a-b parametric plane**
 - $b = -x_i a + y_i$
 - one sole line for the (x_i, y_i) pair
 - an infinity of lines going through one (a,b) pair



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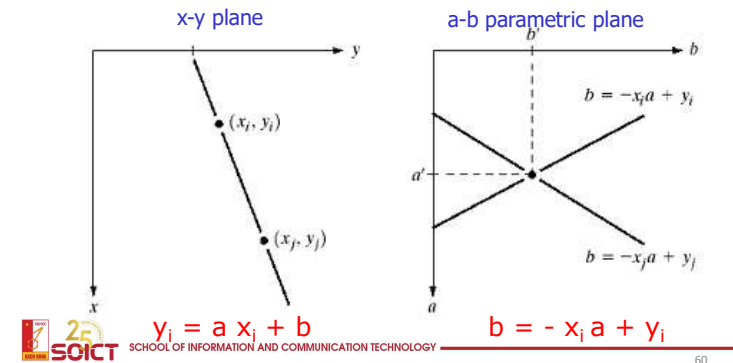
x-y plane vs a-b plane



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Line vs Points

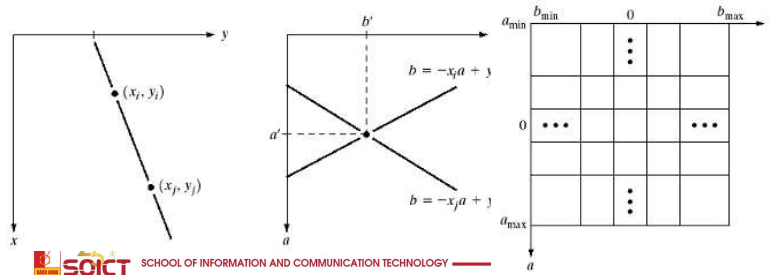
All the points (x, y) on a line in the **x-y plane** are going through one sole point (a', b') in the **a-b parametric plane**



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Main idea for the Hough transform

- Accumulation cells - Matrix (a, b)
- Build an **voting image**
 - each point is voting for a particular line
- The lines receiving more votes are kept



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Computing the Hough transform

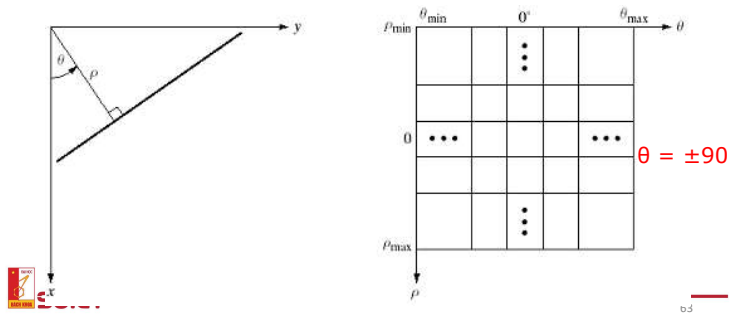
- We compute the **contour points** of the input image
 - *Sobel, Prewitt, Canny, ...*
- For each contour point, we compute a line (a, b)
 - *Result is one line in the a-b plane for each pixel (x, y)*
- The maximum peaks in the a-b parametric plane show the lines with the maximum of points in the x-y plane
 - *The points indicating line crossing in the a-b plane show the real lines existing in the x-y plane*



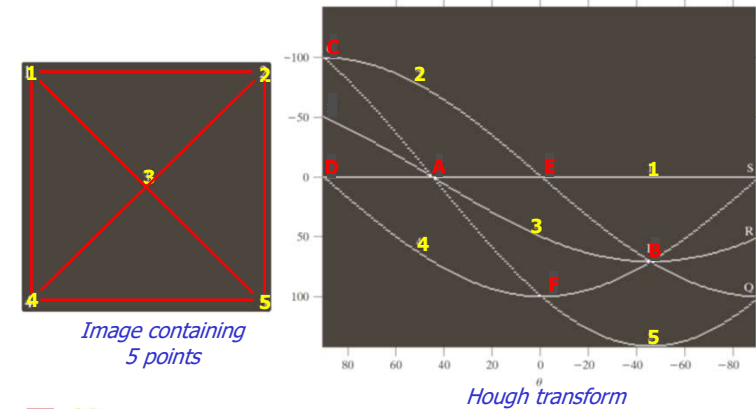
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Problem with the (a,b) space

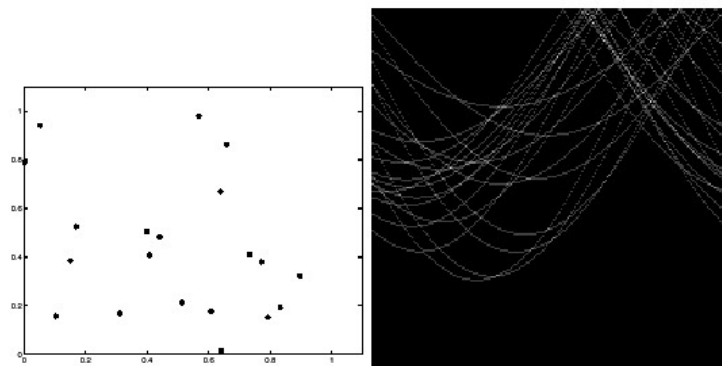
- **Problem:** for a vertical line, $a = \infty$!
 - **Solution:** representing using polar coordinates (ρ, θ)
- $\rho = x \cos \theta + y \sin \theta$



Example with 5 points



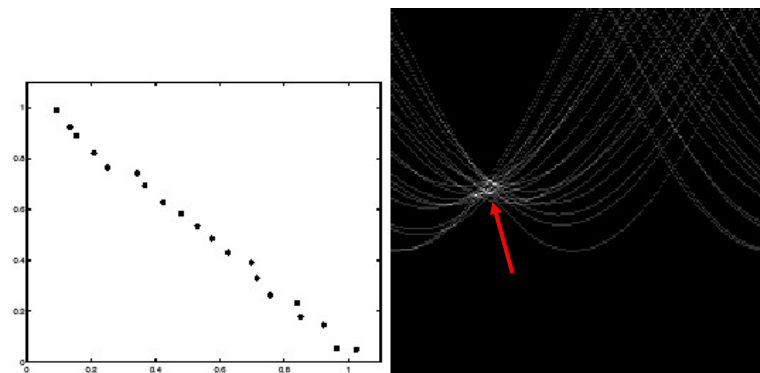
Hough transform (random points)



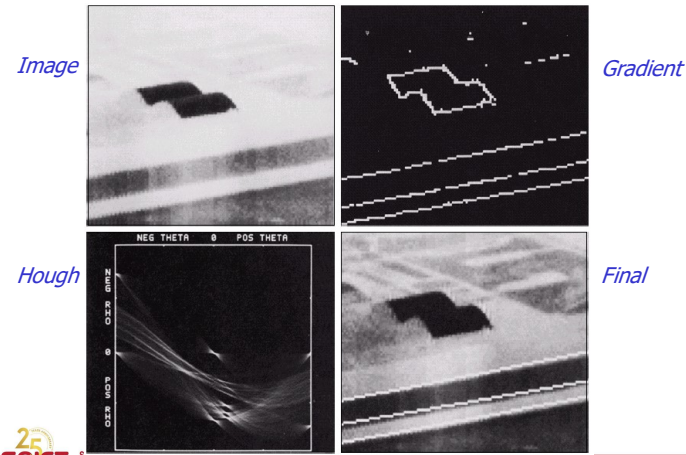
The transform of random points does not give any precise results

Hough transform (straight line)

The transform for aligned points result in a line detection



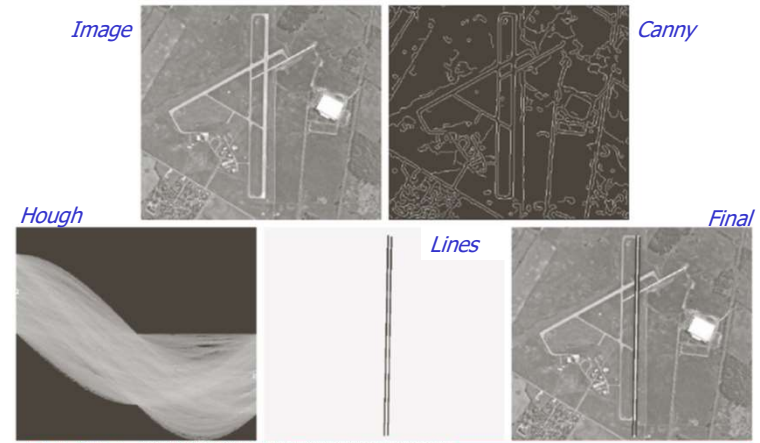
Example



Source : Gonzalez and Woods. Digital Image Processing. Prentice-Hall, 2002.

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Other example



Source : Gonzalez and Woods. Digital Image Processing. Prentice-Hall 3ed, 2008.

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Ransac

CS231: Ransac, Juan Carlos Niebles and Ranjay Krishna, Stanford Vision and Learning Lab



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Ransac

- A model fitting method:
 - A learning technique to estimate parameters of a model by random sampling of observed data
 - Used for :
 - Line detection
 - Correspondance problem (matching between 2 sets of features)
 - ...

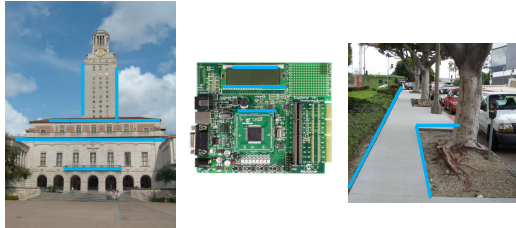


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Example: Line Fitting

- Why fit lines?
 - Many objects characterized by presence of straight lines



Slide credit: Kristen Grauman

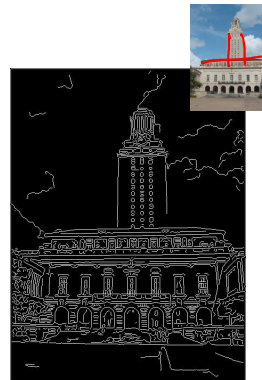
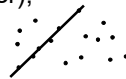
Fitting as search in parametric space

- Choose a **parametric model** to represent a set of features
- Membership criterion is **not local**
 - Can't tell whether a point belongs to a given model just by looking at that point.
- Three main questions:
 - **What model** represents this set of features best?
 - **Which of several model instances** gets which feature?
 - How **many model instances** are there?
- Computational **complexity** is important
 - It is infeasible to examine every possible set of parameters and every possible combination of features

Source: L. Lazebnik

Difficulty of Line Fitting

- Extra edge points (clutter), multiple models:
 - Which points go with which line, if any?
- Only some parts of each line detected, and some parts are missing:
 - How to find a line that bridges missing evidence?
- Noise in measured edge points, orientations:
 - How to detect true underlying parameters?



Slide credit: Kristen Grauman

Voting

- It's **not feasible to check all combinations** of features by fitting a model to each possible subset.
- Voting is a general technique where we let **the features vote for all models** that are compatible with it.
 - Cycle through features, cast votes for model parameters.
 - Look for model parameters that receive a lot of votes.
- **Noise & clutter** features will cast votes too, *but* typically their votes should be inconsistent with **the majority of "good" features**.
- Ok if some features not observed, as model can span multiple fragments.

Slide credit: Kristen Grauman

RANSAC [Fischler & Bolles 1981]

- **R**ANdom **S**Ample **C**onsensus
- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those
- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.



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Slide credit: Kristen Grauman

RANSAC [Fischler & Bolles 1981]

RANSAC loop:

1. Randomly **select a seed group** of points on which to base transformation estimate (e.g., a group of matches)
 2. **Compute transformation** from seed group
 3. Find **inliers** to this transformation
 4. If the number of inliers is **sufficiently large**, **re-compute least-squares estimate** of transformation on all of the inliers
- Keep the transformation with the largest number of inliers

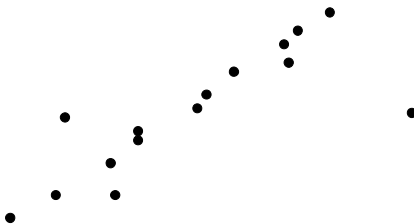


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line
 - *How many points do we need to estimate the line?*

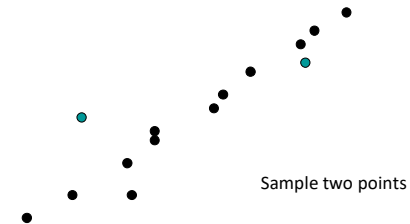


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

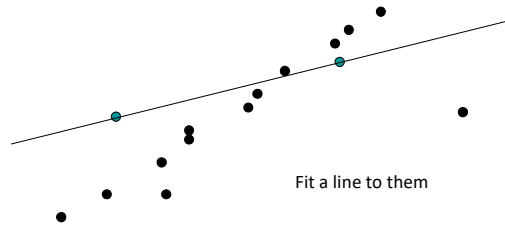


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

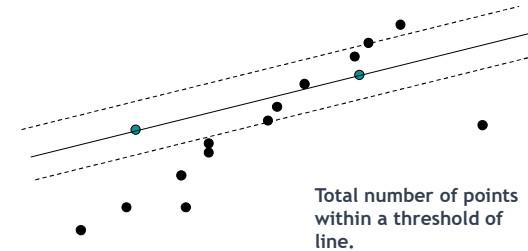


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

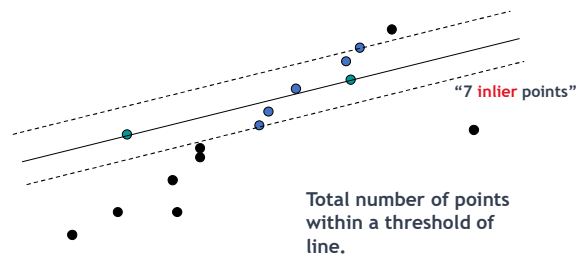


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

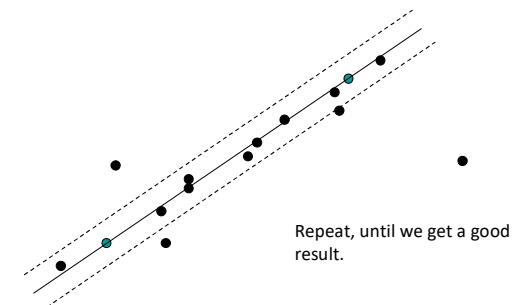


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

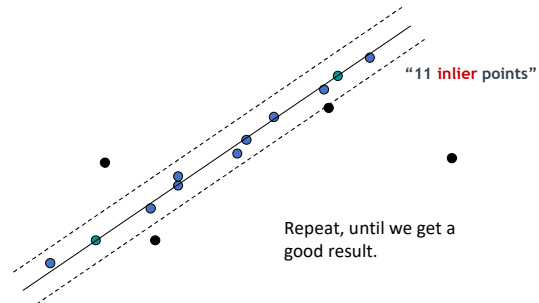


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Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line



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Slide credit: Kristen Grauman

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:

n — the smallest number of points required

k — the number of iterations required

t — the threshold used to identify a point that fits well

d — the number of nearby points required

to assert a model fits well

Until k iterations have occurred

Draw a sample of n points from the data

uniformly and at random

Fit to that set of n points

For each data point outside the sample

Test the distance from the point to the line

against t ; if the distance from the point to the line

is less than t , the point is close

end

If there are d or more points close to the line

then there is a good fit. Refit the line using all

these points.

end

Use the best fit from this collection, using the

fitting error as a criterion



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RANSAC: How many samples?

- How many samples are needed?
 - Suppose w is fraction of inliers (points from line)
 - n points needed to define hypothesis (2 for lines)
 - k samples chosen
 - p : desired probability that we get a good sample

Prob. that a single sample of n points is correct: w^n

Prob. that all k samples fail is: $(1 - w^n)^k$

⇒ Choose k high enough to keep this below desired failure rate $(1-p)$.



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Slide credit: David Lowe

RANSAC: Computed k ($p=0.99$)

Sample size n	Proportion of outliers						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

p : desired probability that we get a good sample

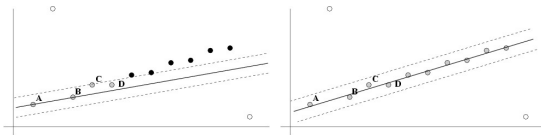


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Slide credit: David Lowe

After RANSAC

- RANSAC divides data into **inliers** and **outliers** and yields estimate computed from minimal set of inliers.
- Improve **this initial estimate with estimation over all inliers** (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



RANSAC: Pros and Cons

- **Pros:**
 - General **method suited for a wide range of model fitting** problems
 - **Easy** to implement and easy to calculate its failure rate
- **Cons:**
 - **Only handles a moderate percentage of outliers** without cost blowing up
 - Many real problems have **high rate of outliers** (but sometimes selective choice of random subsets can help)
- A voting strategy, the Hough transform can handle high percentage of outliers

Plan

- Edges
 - Detection
 - linking
- Feature extraction (**next lesson**)
 - Global features
 - Local features
- Image Matching and Applications (**next lesson**)

References

- Lecture 3: CS231 - Juan Carlos Niebles and Ranjay Krishna, Stanford Vision and Learning Lab
- Vision par Ordinateur, Alain Boucher, IFI

