2021 Junes - Statistics

Example 1:

a) All people in homes in Hanoi
b) All possible tosses of such coin
c) All pairs of the new type of tennis shoes.
d) Every drives from the lawyer home to the office.

Exercise 2:

a) $\mu = 8.6$
b) Rearrangement: 5, 5, 5, 6, 9, 10, 10, 10, 11, 15
=> median = $(9+10)/2 = 9.5$
c) mode = $\{5; 10\}$

Exercise 3:

$\mu = 2.66$
mediance = 2.7
$$\sigma^2 = \frac{1}{n-1} \sum_{x \in S} (x-\mu)^2 = 0.34$$
$\Rightarrow \sigma = 0.59$

Exercise 4:

$X \sim N(800, 40^2)$
The sample taken from X, size = 16
$$\begin{cases} \mu_{\hat{x}} = \mu_X = 800 \\ \sigma_{\hat{x}} = \dfrac{\sigma_X}{\sqrt{n}} = \dfrac{40}{\sqrt{16}} = 10 \end{cases}$$
(Central limit theorem)

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$\Rightarrow \hat{X} \sim N(800, 10^2)$

Let $Z = \dfrac{\hat{X} - 800}{10} \sim N(0,1)$

$P(\hat{X} < 775) = P\left(\dfrac{\hat{X}-800}{10} < \dfrac{775-800}{10}\right)$

$= P(Z < -2.5)$

$= \Phi(-2.5)$

$= 1 - \Phi(2.5)$

$= 1 - 0.9938$

$= 0.0062$

<u>Exercise 5:</u>

Let $\hat{X} \sim$ sample with size $= 36$ $(n = 36)$

σ_X : population standard deviation

$\sigma_{\hat{X}} = \dfrac{\sigma_X}{\sqrt{n}} \Rightarrow 2 = \dfrac{\sigma_X}{\sqrt{36}} \Rightarrow \sigma_X = 12$

$\sigma_{\hat{X}} = 1.2 \Leftrightarrow 1.2 = \dfrac{12}{\sqrt{n}} \Leftrightarrow n = 100$

<u>Exercise 6:</u>

a)

X : population of 1000 students $\sim N(174.5; 6.9)$

$\hat{X}$: sample of 25 students $\sim N\left(174.5; \dfrac{6.9}{5}\right)$

$\sim N(174.5; 1.38)$

$Z = \dfrac{\hat{X} - 174.5}{1.38} \sim N(0; 1)$

b) $P(172.5 < \hat{X} < 175.8)$

$= P\left(\dfrac{172.5 - 174.5}{1.38} < \dfrac{\hat{X} - 174.5}{1.38} < \dfrac{175.8 - 174.5}{1.38}\right)$

$= P(-1.45 < Z < 0.94)$
$= \Phi(0.94) - \Phi(-1.45)$
$= \Phi(0.94) - 1 + \Phi(1.45)$
$= 0.7529$
=> Number sample means fall <...> $= 200 \times 0.7529$
$= 151$

c)
$P(\hat{X} < 1720)$
$= 0.0351$
=> The numberof sample means falling below 172 cm is $200 \times 0.0351 = 7$

Exercise 7:

$n = 36$
$\mu_{\bar{x}} = 2.6$
$\sigma_{x} = 0.3$

+) 95% confidence:

$$\mu_{\bar{x}} - z_{0.025} \frac{\sigma_x}{\sqrt{n}} < \mu < \hat{\mu} + z_{0.025} \frac{\sigma_x}{\sqrt{n}}$$

$$\Rightarrow 2.6 - 1.96 \times \frac{0.3}{\sqrt{36}} < \mu < 2.6 + 1.96 \times \frac{0.3}{\sqrt{36}}$$

$$\Rightarrow 2.5 < \mu < 2.7$$

+) 99% confidence:

$$2.47 < \mu < 2.73$$

Exercercises 8:

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$n = 100$
$\mu_{\hat{X}} = 23500$
$\sigma_{\hat{X}} = 3900$
$H_0 : \mu \leq 20000$
$H_1 : \mu > 20000$
Test statistic
$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{23500 - 20000}{3900/10} = 8.97$$
$P(z > 8.97) = 0$
$\Rightarrow$ Reject H_1

Exercise 9:

$Y = X(8) - X(5)$
Second moment of Y:
$E(Y^2) = E[(X(8) - X(5))^2]$
$= E[X(8)^2] - 2E[X(8)X(5)] + E[X(5)^2]$
$= R(0) + -2R(3) + R(0)$
$= 2R(0) - 2R(3)$
$= 2A(1 - e^{-3\alpha})$

Exercise 10:
$X(t)$ is stationary process
$R_{XX}(\tau) = A\delta(\tau)$
$h(t) = e^{-ct}u(t)$ is a linear system
$\Rightarrow Y(t)$ is stationary process
$R_{YY}(t_1, t_2) = R_{XX}(\tau) * h(t_2) * h(t_1)$
$= A\delta(t) * h(t_2) * h(t_1)$
$= A\, h(\tau) * h(t_1)$
$= A\, h(t_2 - t_1) * h(t_1)$