Machine Learning
(IT3190E)

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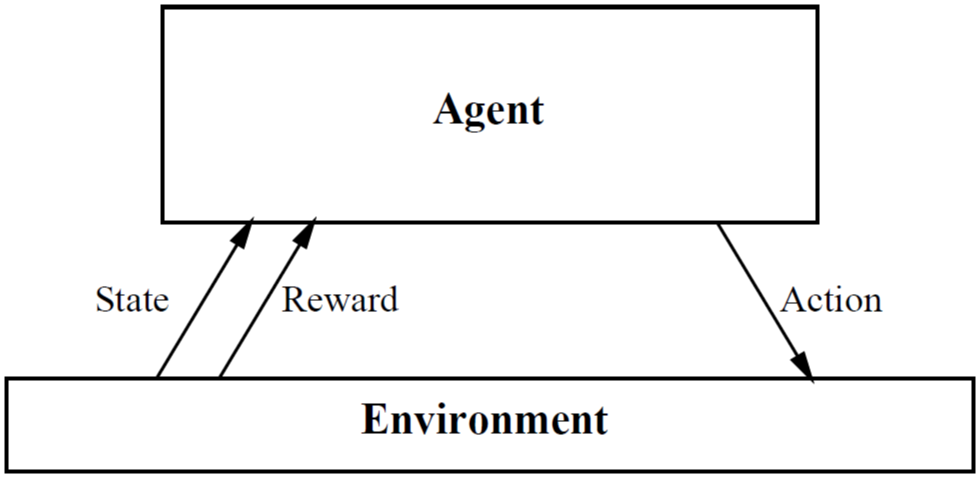
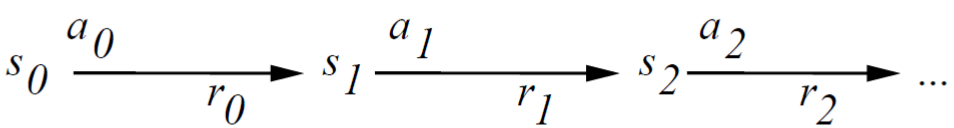
The course's content:

- Introduction
- Performance evaluation of ML system
- Supervised learning
- Unsupervised learning
- Ensemble learning
- **Reinforcement learning**
  - **Introduction of Reinforcement learning**
  - ***Q*-Learning**

Reinforcement Learning problem





(T. M. Mitchell. Machine Learning. McGraw-Hill, 1997)

- Goal: Learn to choose actions that maximize

  $r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$, where $0 \le \gamma < 1$

  (γ is the discount factor for future rewards)

Characteristics of Reinforcement learning

- What makes Reinforcement Learning (RL) different from other machine learning paradigms?
  - There is no supervisor, only a **reward** signal
  - Training examples are of form ((*S*, *A*), *R*)
  - Feedback is delayed, not instantaneous
  - Time really matters (sequential, not independent data)
  - Agent's actions affect the subsequent data it receives
- Examples of RL
  - Play games better than humans
  - Manage an investment portfolio
  - Make a humanoid robot walk
  - …

Reward

- A **reward** R_t is a scalar feedback signal
- Indicates how well agent is doing at step t
- The agent's job is to maximize cumulative reward
- Reinforcement learning is based on the **reward hypothesis**:
  - All goals can be described by the maximization of expected cumulative reward

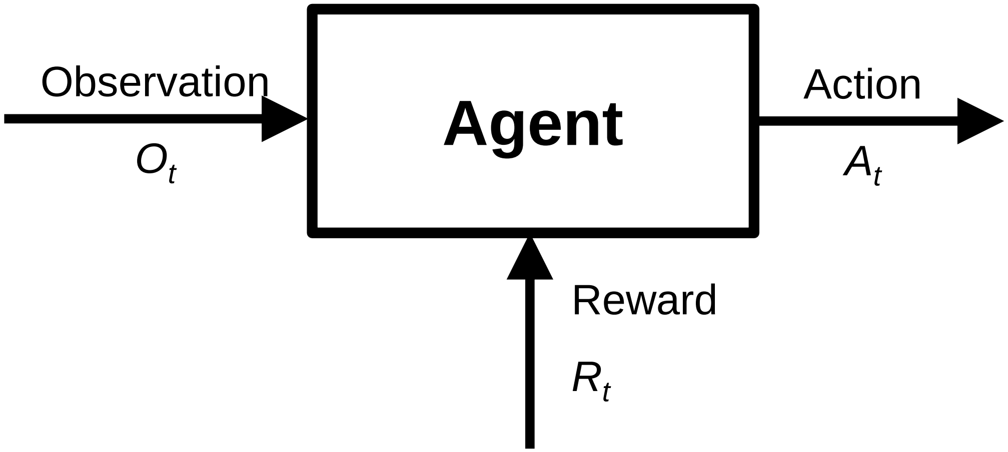
Examples of reward

- Play games better than humans
  - + reward for increasing score
  - - reward for decreasing score
- Manage an investment portfolio
  - + reward for each $ in bank
- Make a humanoid robot walk
  - + reward for forward motion
  - - reward for falling over

Sequential decision making

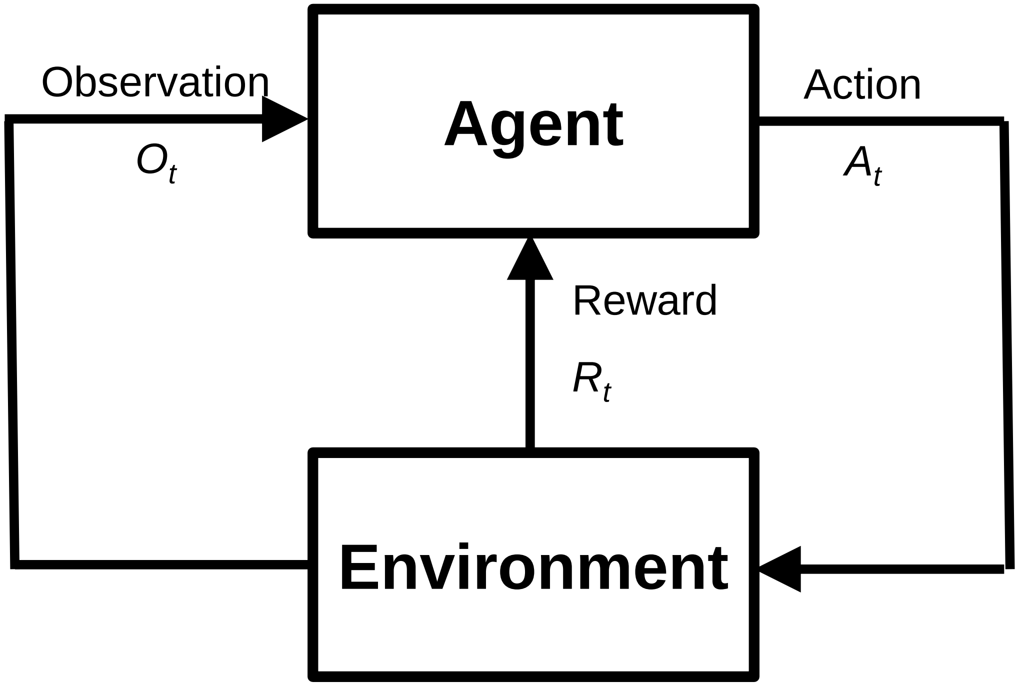
- Goal: **Select actions to maximize total future reward**
- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward
- Examples:
  - A financial investment (may take months to mature)
  - Blocking opponent moves (might help winning chances many moves from now)

Agent and Environment (1)



- At each step t the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t

Agent and Environment (2)



- At each step t the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t
- At each step t the environment:
  - Receives action A_t
  - Emits observation O_{t+1}
  - Emits scalar reward R_{t+1}
- t increments at environment step

History and State

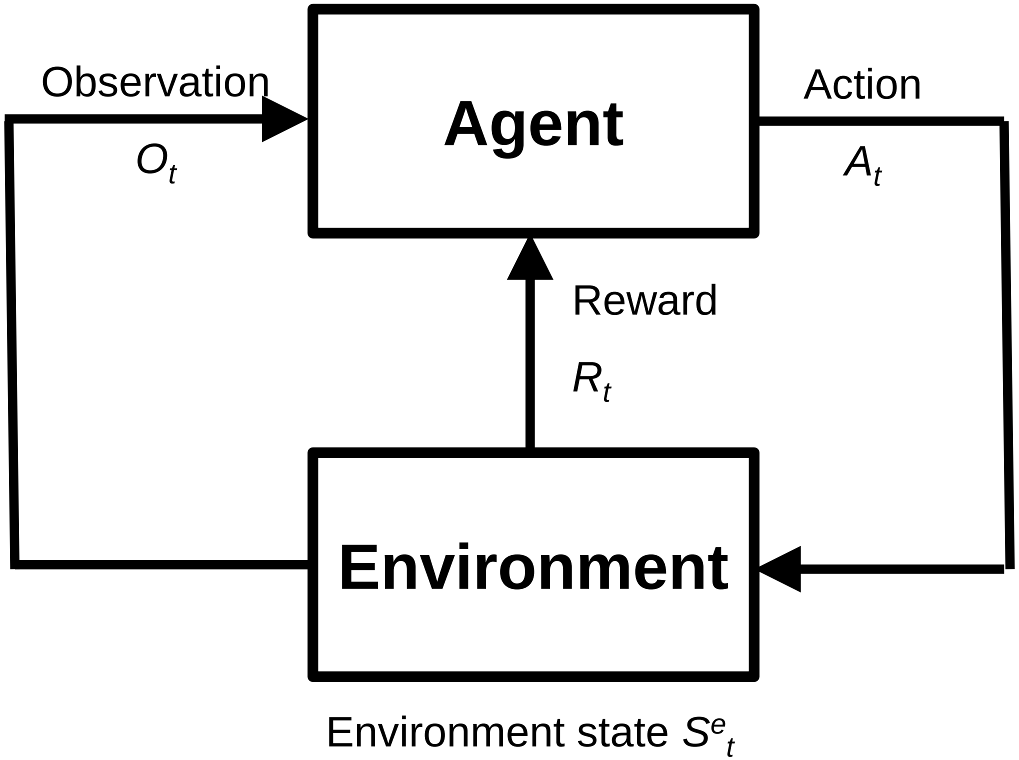
- The **history** is the sequence of observations, actions, rewards:

$$H_t = O_1, R_1, A_1, \ldots , A_{t-1}, O_t, R_t$$

  - All observable variables up to time t
  - The sensorimotor stream of the agent
- What happens next depends on the history:
  - The agent selects actions
  - The environment selects observations/rewards
- **State** is the information used to determine what happens next
- Formally, state is a function of the history:

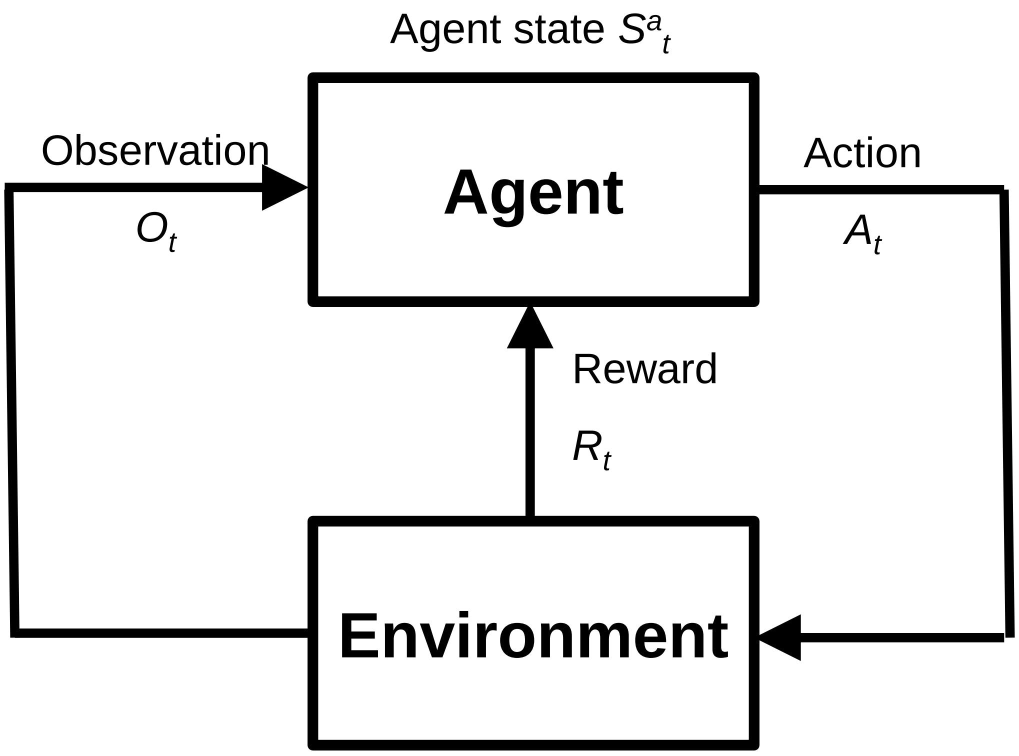
$$S_t = f(H_t)$$

Environment state



- The **environment state** S^e_t is the environment's private representation
  - The information the environment uses to pick the next observation/reward
- The environment state is not usually visible to the agent

Agent state



- The **agent state** S^a_t is the agent's internal representation
  - The information the agent uses to pick the next action
  - It is the information used by reinforcement learning algorithms
- It can be a function of history:

$$S^a_t = f(H_t)$$

Information state

- An **information state** (a.k.a. Markov state) contains all useful information from the history
- A state S_t is **Markov** if and only if:

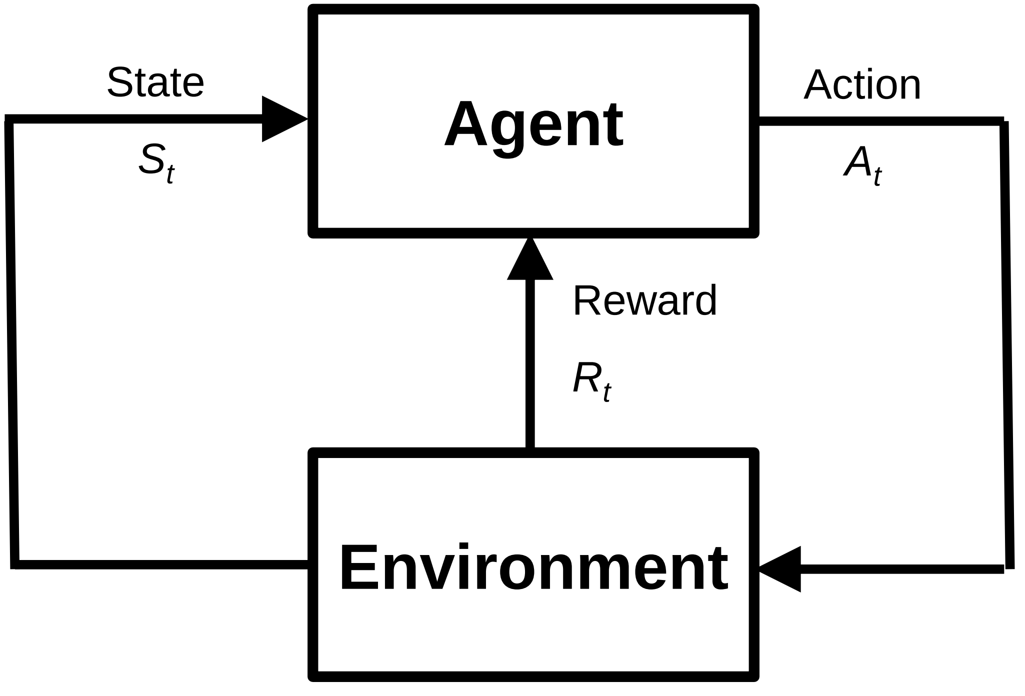
$$P(S_{t+1} \mid S_t) = P[S_{t+1} \mid S_1, \ldots, S_t)$$

  - The future is independent of the past given the present

$$H_{1:t} \rightarrow S_t \rightarrow H_{t+1:\infty}$$

  - Once the state is known, the history may be thrown away
  - The state is a sufficient statistic of the future
  - The environment state S^e_t is Markov
  - The history H_t is Markov

Fully observable environments



- **Full observability**: Agent **directly** observes environment state

$$O_t = S^a_t = S^e_t$$

- Agent state = Environment state = Information state
- Formally, this is a Markov decision process (MDP)

Partially observable environments

- **Partial observability**: Agent **indirectly** observes environment:
  - E.g., A robot with camera vision isn't told its absolute location
  - E.g., A trading agent only observes current prices
  - E.g., A poker playing agent only observes public cards
- Now, Agent state ≠ Environment state
- Formally this is a **partially observable Markov decision process (POMDP)**
- Agent must construct its own state representation S^a_t :
  - E.g., By using complete history: $S^a_t = H_t$
  - E.g., By using a recurrent neural network: $S^a_t = \sigma(S^a_{t-1}W_s + O_tW_o)$

Major components of an RL agent

An RL agent may include one or more of these components:

- **Policy**: Agent's behavior function
- **Value function**: How good is each state and/or action
- **Model**: Agent's representation of the environment

Policy

- A **policy** is the agent's behavior
- It is a map from state to action
- *Deterministic* policy: $a = \pi(s)$
- *Stochastic* policy: $\pi(a|s) = P(A_t{=}a \mid S_t{=}s)$

Value function

- **Value function** is a prediction of future reward
- Used to evaluate the goodness/badness of states
- And therefore to select between actions

$$v_\pi(s) = E_\pi(R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s)$$

where R_{t+1}, R_{t+2}, … are generated by following policy π starting at state s

Model

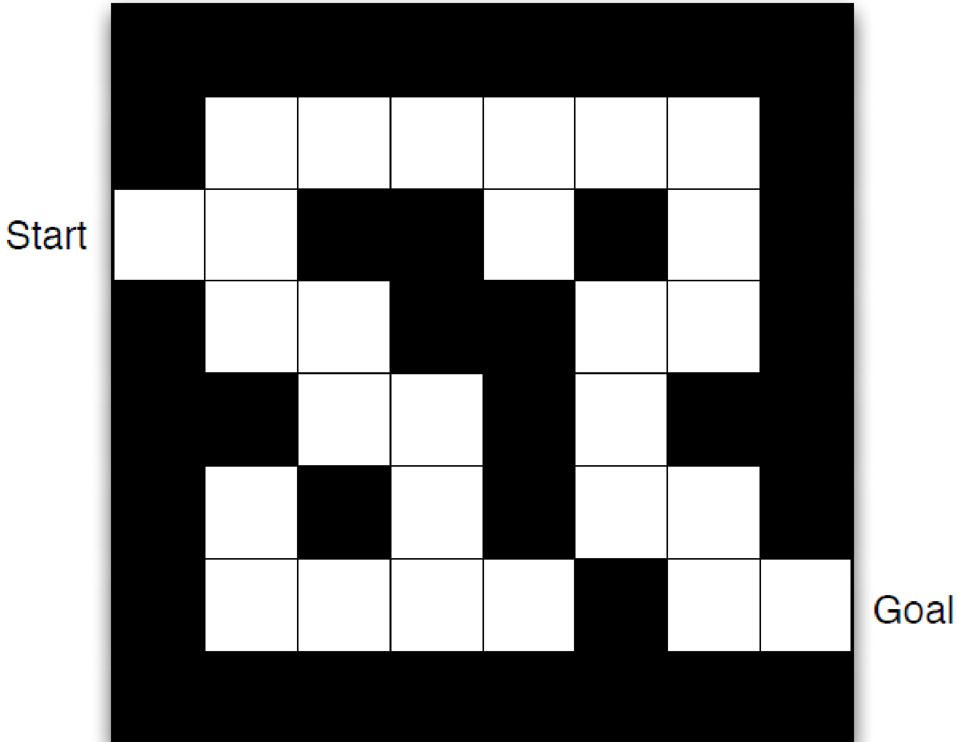
- A **model** predicts what the environment will do next
- P predicts the next state

  $$P^a_{ss'} = P(S_{t+1}=s' \mid S_t=s, A_t=a)$$

- R predicts the next (*immediate*) reward

  $$R^a_s = E(R_{t+1} \mid S_t=s; A_t=a)$$

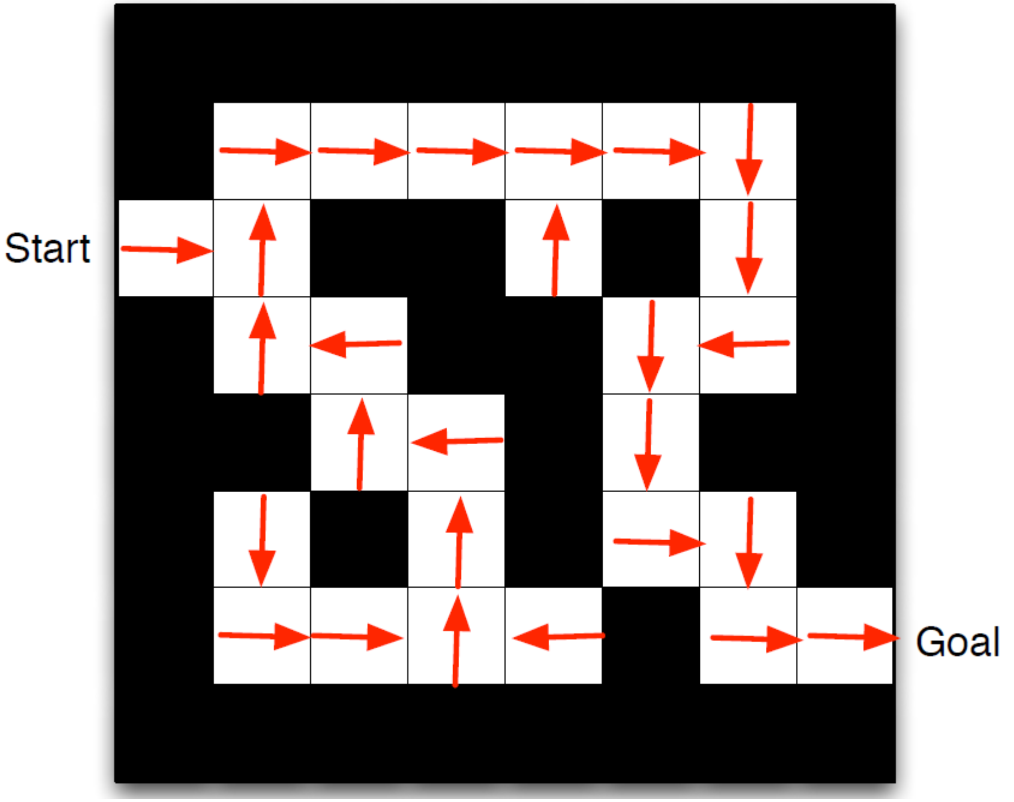
Maze example



(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

- Rewards: -1 per time-step
- Actions: N, E, S, W
- States: Agent's location

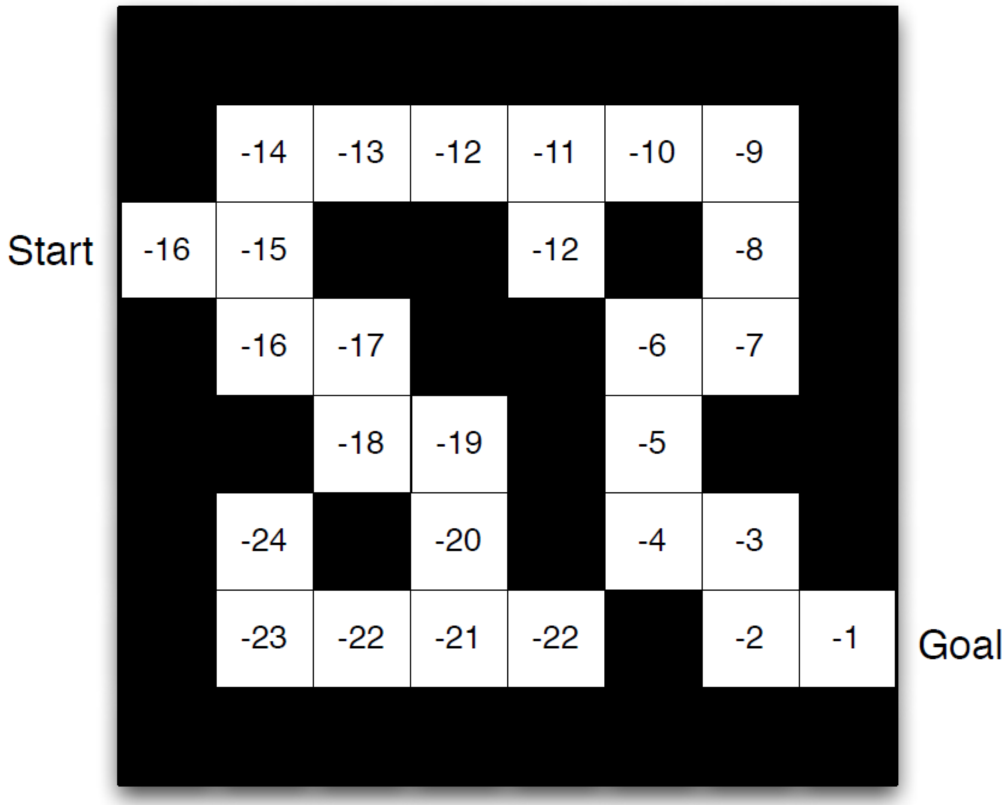
Maze example: Policy



(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

- Arrows represent policy $\pi(s)$ for each state s

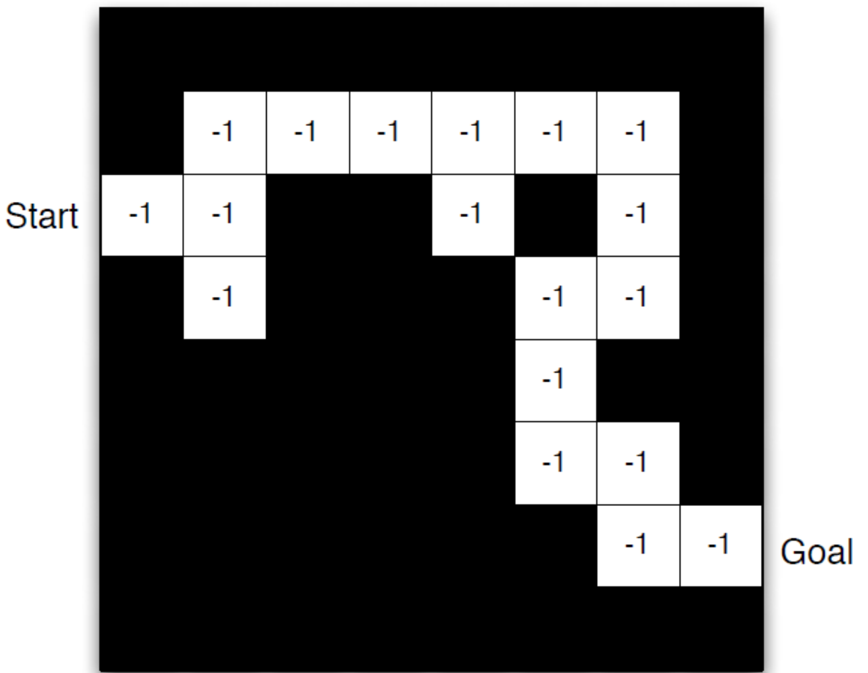
Maze example: Value function



- Numbers represent value $v_\pi(s)$ of each state s

(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

Maze example: Model



(https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf)

- Agent may have an internal model of the environment
- Dynamics: How actions change the state
- Rewards: How much reward from each state
- The model may be imperfect
- Grid layout represents transition model $P^{a}_{ss'}$
- Numbers represent immediate reward R^{a}_{s} from each state *s* (same for all actions *a*)

Categorizing RL agents (1)

- Value-based
  - No policy
  - Value function
- Policy-based
  - Policy
  - No value function
- Actor critic
  - Policy
  - Value function

Categorizing RL agents (2)

- Model-free
  - Policy and/or Value function
  - No model
- Model-based
  - Policy and/or Value function
  - Model

Exploration and Exploitation (1)

- Reinforcement learning is like *trial-and-error learning*
- The agent should discover a good policy
- From its experiences of the environment
- Without losing too much reward along the way

Exploration and Exploitation (2)

- **Exploration** finds more information about the environment
- **Exploitation** exploits known information to maximize reward
- It is usually important to **both** *explore* and *exploit*

Exploration and Exploitation: Examples

- Restaurant selection
  - Exploitation: Go to your favorite restaurant
  - Exploration: Try a new restaurant
- Online banner advertisements
  - Exploitation: Show the most successful advertisement
  - Exploration: Show a different advertisement
- Game playing
  - Exploitation: Play the move you believe is best
  - Exploration: Play an experimental move

Q-Learning: What to learn

- We might try to have agent learn the value function v_π
- It could then do a lookahead search to choose best action from any state s because

$$\pi(s) = \underset{a}{argmax} \left(r(s,a) + \gamma v_\pi(\delta(s,a))\right)$$

- A problem:
  - This works well if agent knows δ: $S \times A \rightarrow S$, and r: $S \times A \rightarrow R$
  - But when it doesn't, it can't choose actions this way

Q-Function

- Define new function very similar to v_π

  $$Q(s,a) = r(s,a) + \gamma . v_\pi(\delta(s,a))$$

- If agent learns Q, it can choose optimal action

  $$\pi(s) = \underset{a}{argmax} \left(r(s,a) + \gamma v_\pi(\delta(s,a)) \right)$$

  $$\pi(s) = \underset{a}{argmax}\ Q(s,a)$$

- Q is the value function the agent will learn

Training rule to learn Q

- Note that Q and v_π are closely related
$$v_\pi(s) = \max_{a'} Q(s,a')$$
- Which allows us to write Q recursively as
$$Q(s_t,a_t) = r(s_t,a_t) + \gamma.v_\pi(\delta(s_t,a_t))$$
$$= r(s_t,a_t) + \gamma.(\max_{a'} Q(s_{t+1},a'))$$
- Let's Q^* denote leaner (agent)'s current approximation to Q, consider the training rule:
$$Q^*(s,a) \leftarrow r(s,a) + \gamma.(\max_{a'} Q^*(s',a'))$$
where s' is the state resulting from applying action a in state s

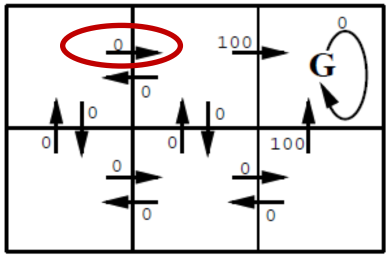
Q-Learning for deterministic worlds

- For each s,a initialize table entry $Q^*(s,a) \leftarrow 0$
- Observe current state s
- Do forever:
  - Select an action a and execute it
  - Receive immediate reward r
  - Observe the new state s'
  - Update the table entry for Q*(s,a) as follows:

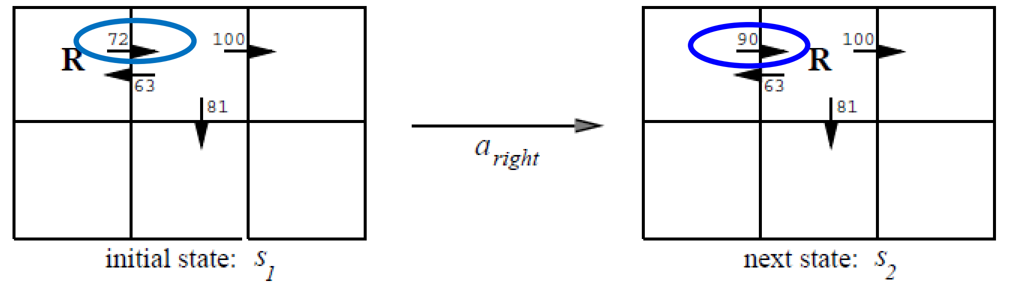
$$Q^*(s,a) \leftarrow r + \gamma.\left(\max_{a'} Q^*(s',a')\right)$$

  - $s \leftarrow s'$

Updating Q^*



$r(s, a)$ (immediate reward) values



(T. M. Mitchell. Machine Learning. McGraw-Hill, 1997)

- $$\begin{aligned} Q^*(s_1, a_{right}) &\leftarrow r + \gamma . (\max_{a'} Q^*(s_2, a')) \\ &\leftarrow \color{red}{\mathbf{0}} + 0.9 \,.\, \max(63, 81, 100) \\ &\leftarrow \color{blue}{\mathbf{90}} \end{aligned}$$
- Note that if rewards are non-negative, then

  $$\forall s, a, n\text{: } Q^*_{n+1}(s,a) \geq Q^*_n(s,a),$$

  $$\forall s, a, n\text{: } 0 \leq Q^*_n(s,a) \leq Q(s,a)$$

Q-Learning for non-deterministic worlds

- What if reward and next state are non-deterministic?
- We redefine v_π and Q by taking expected values

$$v_\pi(s) = E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

$$Q(s,a) = E[r(s,a) + \gamma v_\pi(\delta(s,a))]$$

- Q-Learning generalizes to non-deterministic worlds
  - Alter the training rule to:

$$Q^*_n(s,a) \leftarrow (1-\alpha_n).Q^*_{n-1}(s,a) + \alpha_n.[r + \max_{a'} Q^*_{n-1}(s',a')]$$

where $\alpha_n = \frac{1}{1+visits_n(s,a)}$

References

- D. Silver. *Lecture 1: Introduction to Reinforcement Learning* (https://www.davidsilver.uk/wp-content/uploads/2020/03/intro_RL.pdf).
- T. M. Mitchell. *Machine Learning*. McGraw-Hill, 1997.