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25 YEARS ANNIVERSARY SOICT

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Parallel in tree-related problems

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10.1 Prefix and Suffix

Prefix, Suffix Problems

- Concept: Let A[1..n] be a sequence of n integer elements
  - P[i] is called i_th prefix sum of array A, if : $P[i] = \sum A[j]$ with $j \in 1..i$
  - S[i] is called i_th suffix sum of array A, if : $S[i] = \sum A[j]$ with $j \in i..n$
- Problem: Build an algorithm on PRAM:
  - Input: A[1..n];
  - Output: P[1..n] (or S[1..n])
- These two problems (prefix or sufix) are the same.

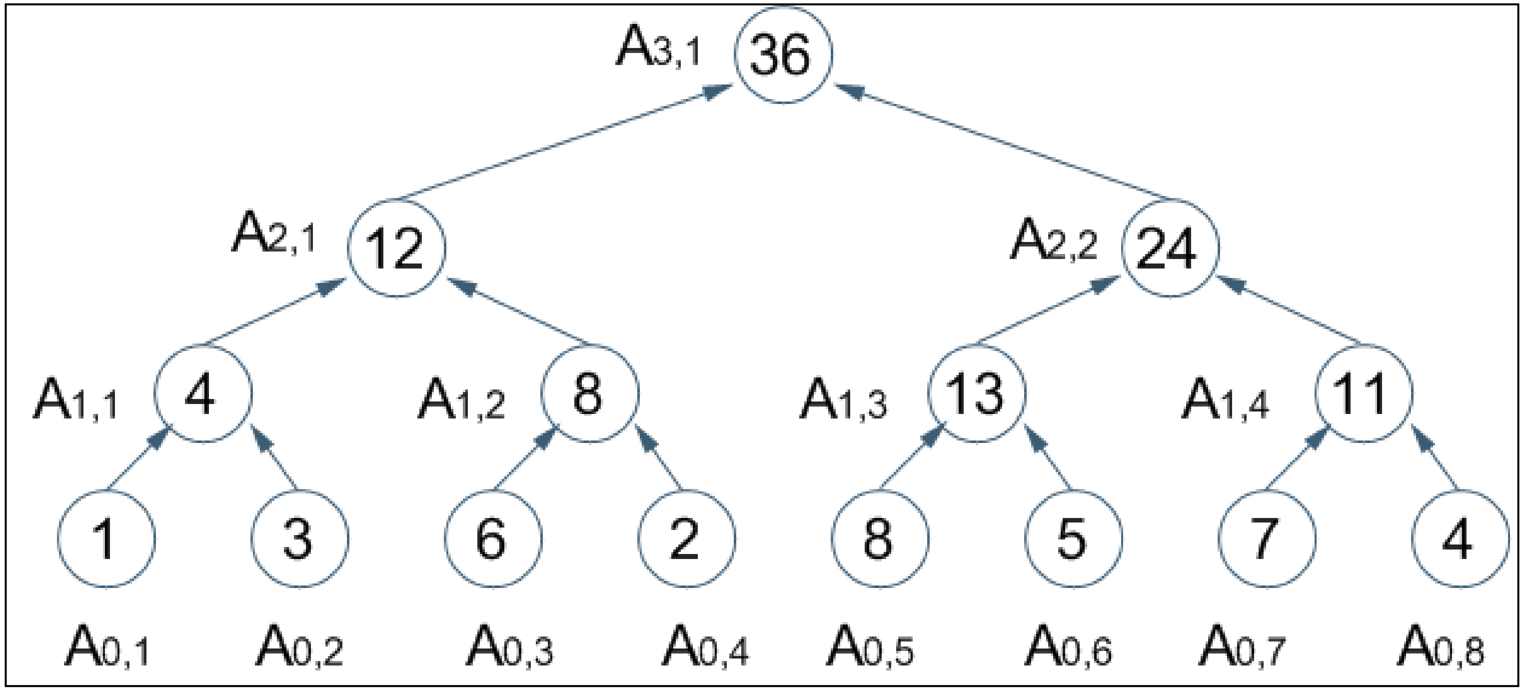
Prefix Sum

- Approaches:
  - Method 1: Using Balanced Tree + Growing by Doubling
  - Method 2: Recursive.
  - Method 3: Using Jumping Pointer

Using Balanced Tree + Growing by Doubling

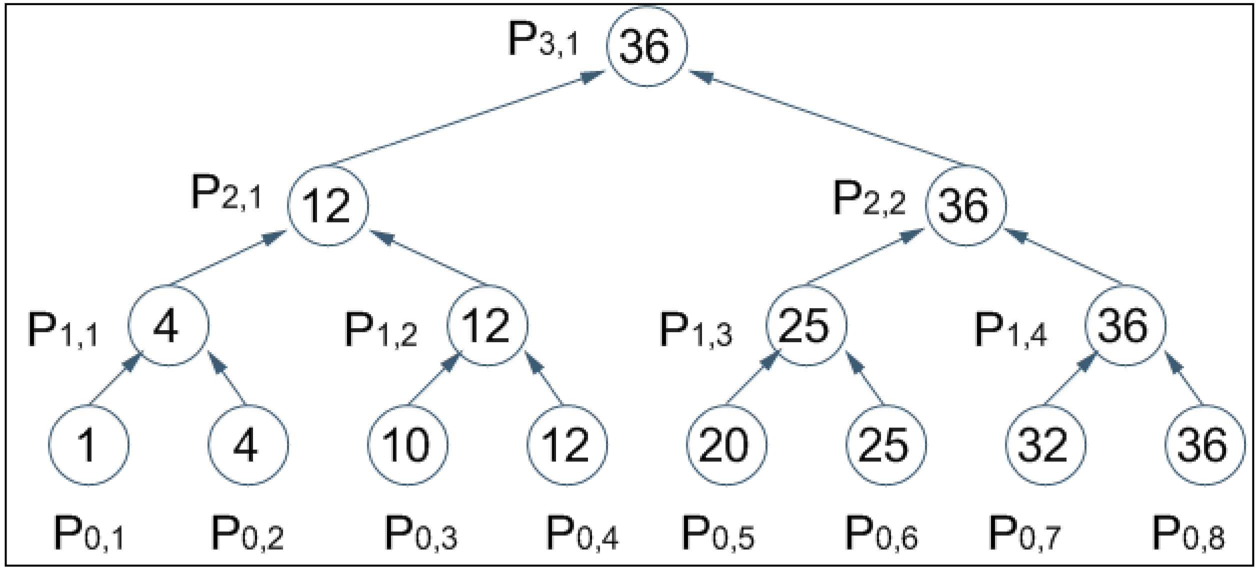
- Comment:
  - Balanced Tree Method returns a result at the top of the tree.
  - In order to obtain the sequence of prefix sum → using other nodes in the balanced tree created above.
- Idea:
  - Build a new tree P, each node is called: $P_{i,j}$ with i is the level index, j is the processor index.
  - Assuming that $P_{i,j}$ is root of a sub-tree, has leftmost leaf that is $P_{0,k}$, hence $P_{i,j} = \sum A[t]$ where $t \in 1..k$

Step1. Create a balanced tree



- Each node in the tree is represented by $A_{i,j}$ where:
  - i is the level index.
  - j is the processor index.
- And: $A_{i+1,j} = A_{i,2j-1} + A_{i,2j}$

Step 2. Build a new tree P



- Comments:
  - P tree is built from top to bottom with the top : $P_{k,1} = A_{k,1} = \sum A[i]$ where $i = 1..n$, $k = \log_2 n$.
  - $P_{i,1} = A_{i,1}$ where $i = k-1 .. 0$.
  - $P_{i,j} = P_{i+1,j/2}$ with j is even; $P_{i,j} = P_{i+1,[j/2]} + A_{i,j}$ with j is odd ($i = k-1..0$)

```
input       : A[1..n]; n = 2^k
output      : P[1..n] | P[i] = Prefix_sum[i]
begin
    for i = 1 to n do in parallel
        A[0,i]      =     A[i];
    end parallel
    for i = 1 to k do
        for j = 1 to n/2^i do in parallel
            A[i,j]=     A[i-1,2j-1] + A[i-1,2j];
        end parallel
    end for
    P[k,1]      =     A[k,1];
    for i = k -1 downto 0 do
        for j = 1 to 2^(k-i) do in parallel
            if j = 1 then P[i,1] = A[i,1];
            else if j chẵn then P[i,j] = P[i+1,j/2]
            else P[i,j] = P[i+1,[j/2]] + A[i,j]
        end parallel
    end for
    for i = 1 to n do in parallel
        P[i]  =     P[0,i];
    end parallel
end.
```

Complexity Evaluation

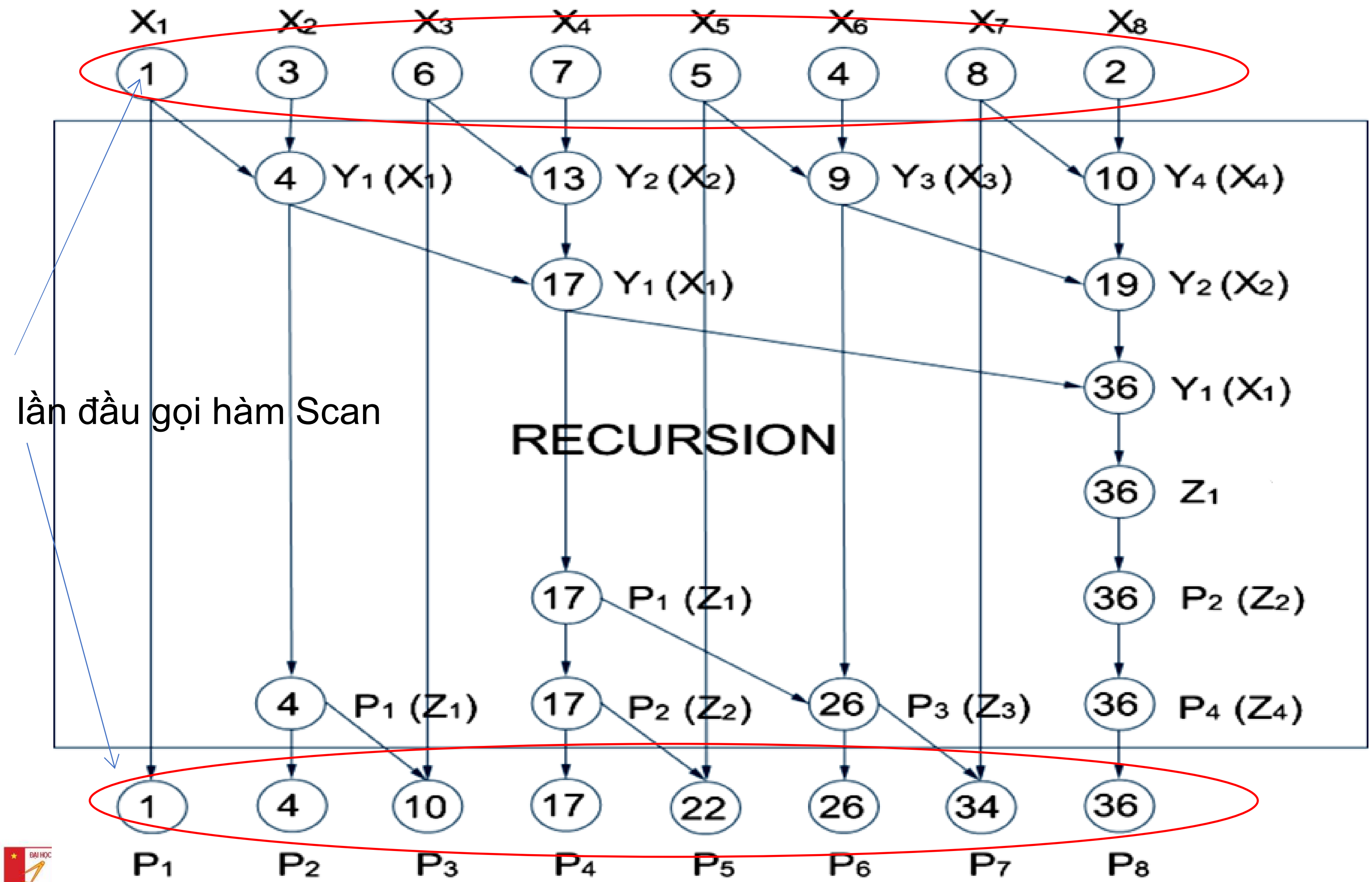
- The algorithm is divided into 2 parts:
  - Part 1: create the balanced tree : $O(\log_2 n)$ in PRAM EREW.
  - Part 2: build P tree with $O(\log_2 n)$ serial steps.
    - Node $P_{i,j}$ needs $P_{i+1,[j/2]}$: this value can be read in serial mode in PRAM EREW.
    - When j is even: $P_{i,j}$ needs $P_{i+1,j/2}$
    - When j is odd: $P_{i,j}$ needs $A_{i,j}$ first, then $P_{i+1,[j/2]}$

Recursive Method

- Idea:
  - Using recursive method to build A, P trees.
  - For example, we can build a balanced tree thanks to a recursive method as follows:

```
function S = Reduce(A[1..n])
begin
        if n = 1 then
                S = A[1];
                return S;
        end if
        for i = 1 to n/2 do in parallel
                A[i] = A[2i-1] + A[2i]
        end parallel
        S = Reduce(A[1..n/2];
end
```

Recursive Method's Illustration



Recursive Method

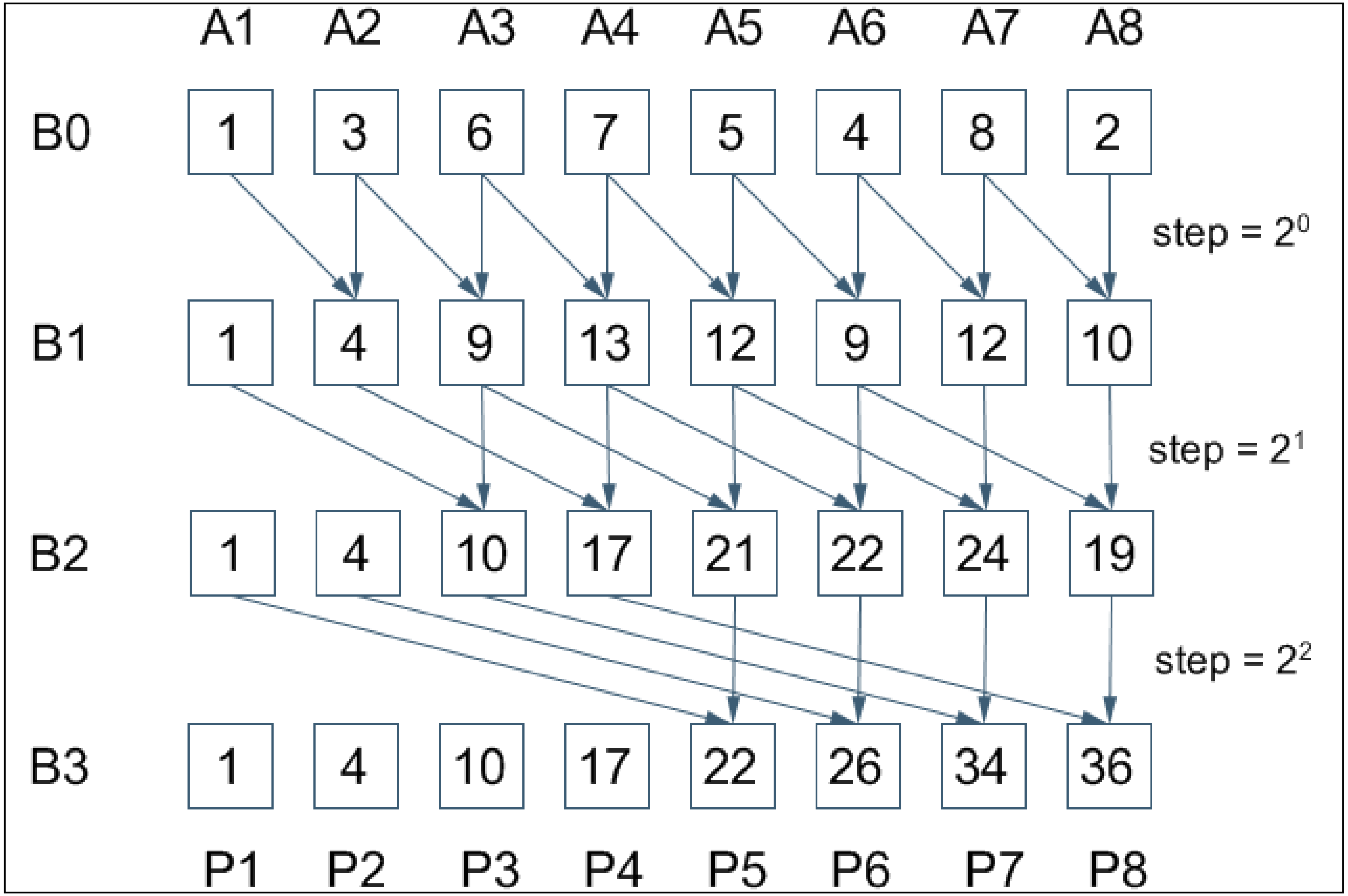
- The duration of the algorithm is equal to the number of recursive function calls.
- In a recursive call: O(1) time unit
- Total duration time: $O(\log_2 n)$.

```
function P[1..n] = Scan(X[1..n])
begin
        if n = 1 then
                P[1] = X[1];
                return P;
        end if
        for i = 1 to n/2 do in parallel
                Y[i] = X[2i-1] + X[2i]
        end parallel

        Z[1..n/2] = Scan(Y[1..n/2]);

        for i = 1 to n do in parallel
                if i chẵn then  P[i] = Z[i/2];
                elseif i = 1 then P[1] = X[1];
                else P[i] = Z[(i-1)/2] + X[i];

        end parallel
        return P;
end
```

Jumping Pointer's Algorithm

- Idea:
  - Initial step: $P_0[i] = A[i]$.
  - At step k:
    - step = 2^{k-1}.
    - $P_k[i] = P_{k-1}[i] + P_{k-1}[i\text{-step}]$ with $\forall$ i | step < i $\leq$ n.
  - We can see that:
    - At step k: $P_k[i] = \sum A[j]$ with $j \in 1..i$; $i \in 1..2^k$.
    - So to calculate P[1..n] requires $\log_2 n$ repeating steps.

Illustration



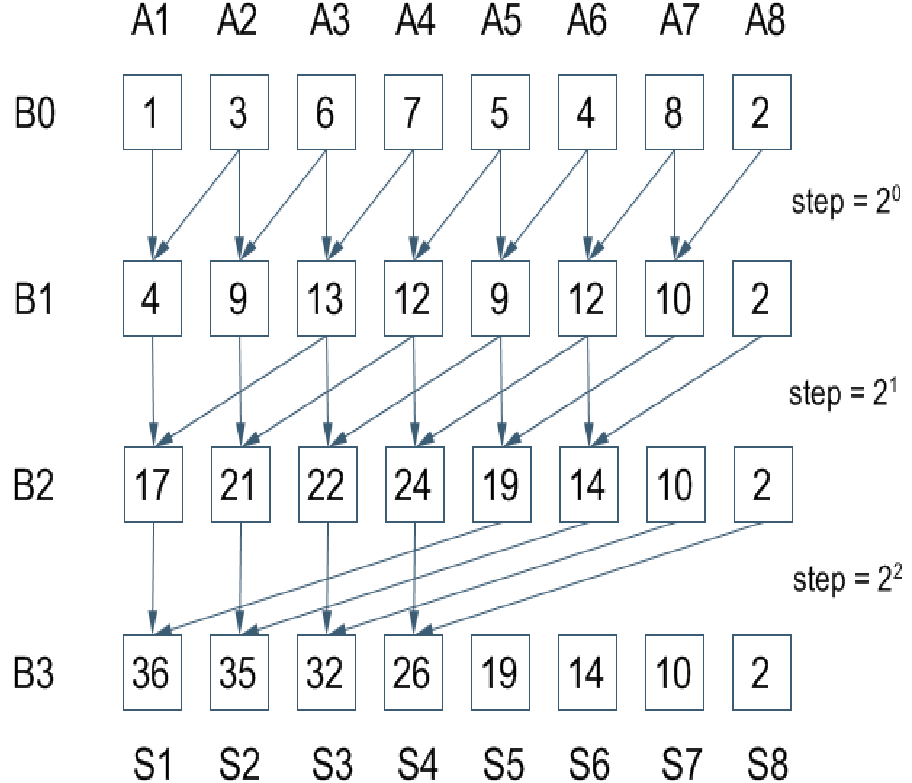
The algorithm's illustration

- Execution time: Serial iteration's number: O(k) = O($\log_2$n).
- Assignments can be performed by reading/writing separately → PRAM EREW.

```
input    : A[1..n]; n = 2^k
output   : P[1..n] | P[i] = Prefix_sum[i]
begin
    for i = 1 to n do in parallel
        P[i] = A[i];
    end parallel
    for i = 1 to k do
        for j = 1 to n do in parallel
            if j > 2^(i-1) then
                P[j] = P[j] + P[j-2^(i-1)];
            end if
        end parallel
    end for
end
```

Suffix sum

- Same as Prefix sum problem.
- For example, Jumping Pointer method for Suffix sum as follows:




```
input     : A[1..n]; n = 2^k
output    : S[1..n] | S[i] = Suffix_sum[i]
begin
    for i = 1 to n do in parallel
        S[i] = A[i];
    end parallel
    for i = 1 to k do
        for j = 1 to n do in parallel
            if j ≤ n-2^(i-1) then
                S[j] = S[j] + S[j+2^(i-1)];
            end if
        end parallel
    end for
end
```

10.2 Tree-related Problems

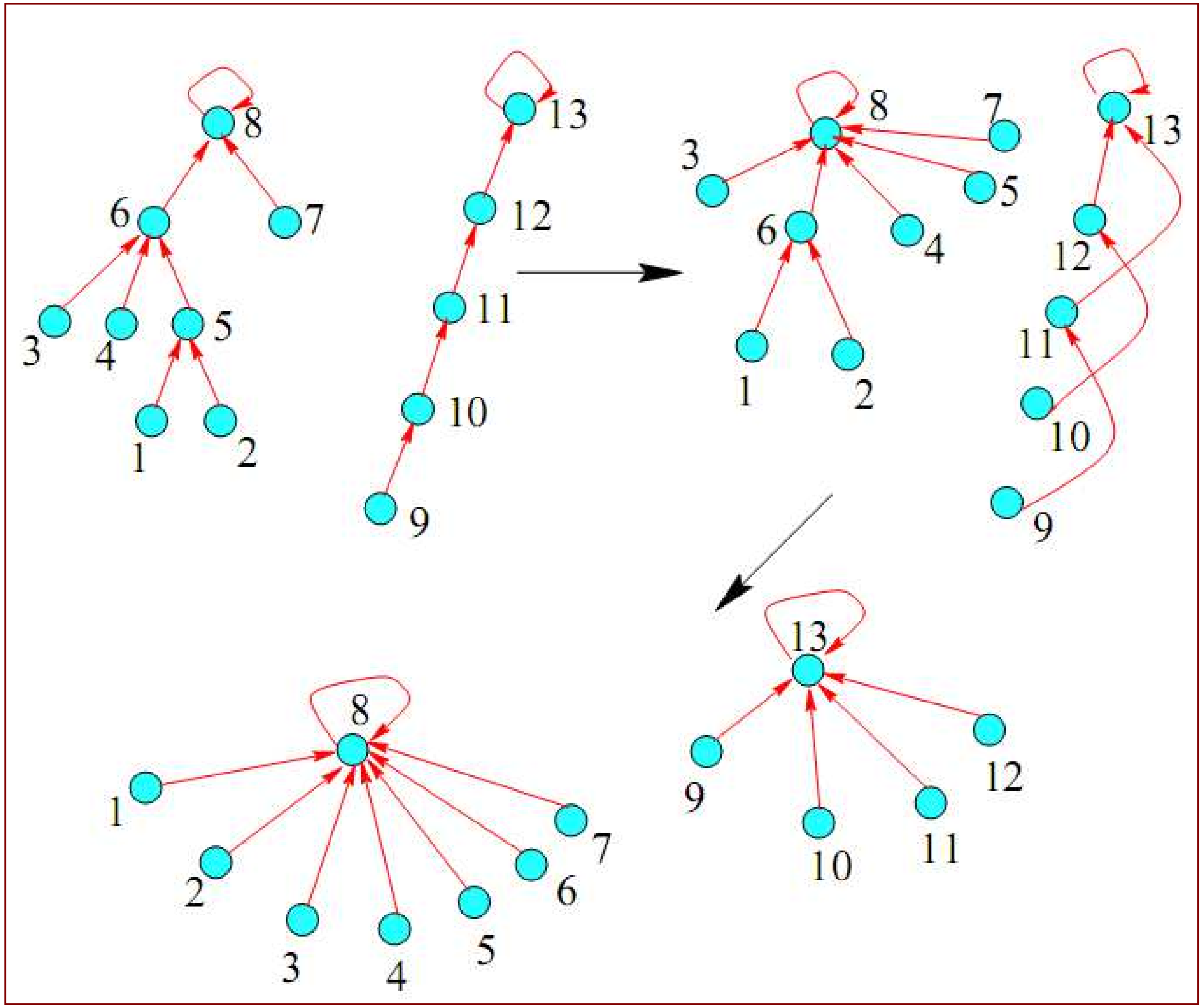
10.2.1 Rooted-directed Tree

- Definition: Rooted-directed Tree T is a directional graph with a special node r satisfying:
  - $\forall$ node v $\in$ V - {r} : have an out-of-degree outdegree(v) = 1 while the node r has outdegree(r) = 0.
  - $\forall$ node v $\in$ V - {r}: $\exists$ 1 path from v to r.
  - => The node r is called the root of tree.
- Tree T is presented by an array P[1..n],
  - P[i] = j if j is the father of i in the tree.
  - Root is the node that points to itself: P[r] = r.

Identifying tree root in the forest

- Problem speaking:
  - Let F be a forest of root-oriented trees.
  - F is presented by an array P[1..n].
  - For each node i in the forest, identifying the root of the trees that contains the node i, which is called S[i].
- Approach:
  - Use the jumping pointer technique.

Identifying tree root in the forest



Identifying tree root in the forest

```
input   : rừng F xác định bởi P[1..n]
output : S[1..n], S[i] -- gốc của cây con chứa nút i
begin
      for i = 1 to n do in parallel
            S[i] = P[i];
            while S[i] <> S[S[i]] do          pointer
                  S[i]  = S[S[i]];            jumping
            end while.
      end for.
end.
```

- Complexity: if h is the height of the highest tree in the forest.
  - Execution time: $O(\log_2 h)$.
  - Cost: $O(n.\log_2 h)$. →

Tree’s Suffix-sum problem

- Problem speaking:
  - Forest F is presented by an arrayP[1..n].
  - The nodes on the tree are weighted W[1..n].
  - Tree root’s weight equals 0.
  - Let's determine the total weight from any node (for example node v) to the root node r of a sub-tree containing node v.
- Approach:
  - Jumping pointer technique.

Tree’s Suffix-sum problem

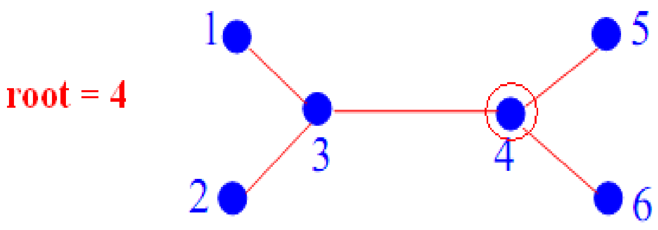
```
input   : rừng F xác định bởi P[1..n], W[1..n]
output : R[1..n], R[i] -- trọng số đi từ i tới S[i]
begin
      for i = 1 to n do in parallel
            S[i] = P[i];
            while S[i] <> S[S[i]] do
                  W[i] = W[i] + W[S[i]];
                  S[i]  = S[S[i]];
            end while.
      end for.
end.
```

- Complexity evaluation as before.
- PRAM model: CREW for 1 parent node can have multiple child nodes

Rooted Tree

- Let T = (V,E) be a tree defined by the list of adjacent vertexes and 1 node r $\in$V. Let's build the T tree with the root is r by defining p(v), with each node v $\neq$ r, as the parent node of v.
- Approach:
  - Set up a Euler cycle on T tree.
  - Let's assume u is the last node in the adjacent list of r. Set s(<u,r>) = 0.
  - Setting the weight for the <x,y> = 1 on T tree and performing suffix_sum in the tree.
  - For each <x,y>, defining x = p(y) if suffix_sum (<x,y>) is greater than suffix_sum(<y,x>).

Rooted Tree problem (root: 4)

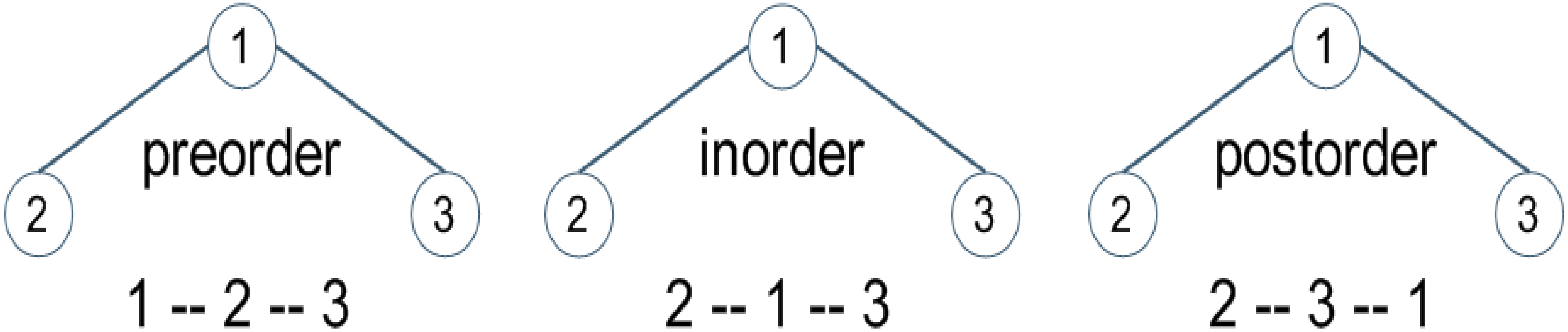


v	adj(v)
1	3
2	3
3	2, 1, 4
4	3, 5, 6
5	4
6	4

edge	successor
<3,1>	<1,3>
<3,2>	<2,3>
<2,3>	<3,1>
<1,3>	<3,4>
<4,3>	<3,2>
<3,4>	<4,5>
<5,4>	<4,6>
<6,4>	null
<4,5>	<5,4>
<4,6>	<6,4>

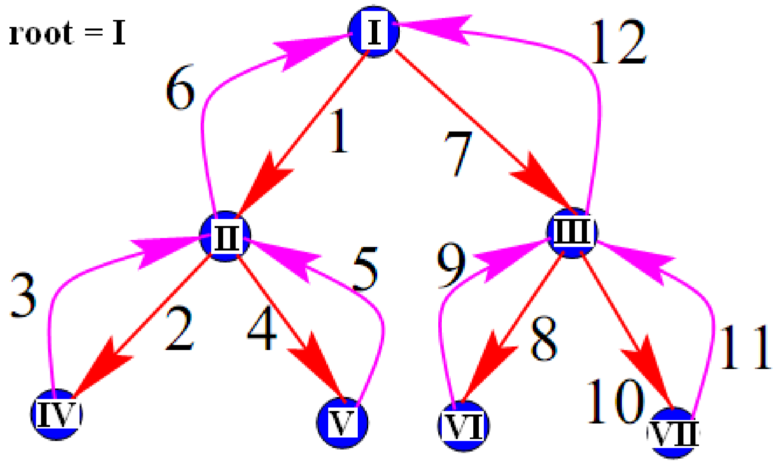
Euler Tour				Result	
Thứ tự	Cạnh	Giá trị	Suffix_sum	v	p(v)
1	<4,3>	1	10	1	3
2	<3,2>	1	9	2	3
3	<2,3>	1	8	3	4
4	<3,1>	1	7	4	4
5	<1,3>	1	6	5	4
6	<3,4>	1	5	6	4
7	<4,5>	1	4		
8	<5,4>	1	3		
9	<4,6>	1	2		
10	<6,4>	1	1		

10.2.2 Tree traversing problem



- Let T = (V,E) be a tree with root node r.
- 3 ways to traverse a binary tree:
  - Pre-Order.
  - In-Order.
  - Post-Order

Approach



Euler Tour		Tree	
Thứ tự	Cạnh	v	p(v)
1	<1,2>	1	1
2	<2,4>	2	1
3	<4,2>	3	1
4	<2,5>	4	2
5	<5,2>	5	2
6	<2,1>	6	3
7	<1,3>	7	3
8	<3,6>		
9	<6,3>		
10	<3,7>		
11	<7,3>		
12	<3,1>		

Post-Order

- Set up the Euler cycle on the T tree.
- With root r, identifying the rooted-directed tree (with $\forall$ v identify p(v) as the father of v).
- Set the weight to the edges:
- $w(<v,p(v)>) = 1$ & $w(<p(v),v>) = 0$.
- For each $<u,v>$, determine suffix_sum for $<u,v>$. It is called $S(<u,v>)$
- Traversing position of node v is: $|V| - S(<v,p(v)>)$.
- Finally traversing the root node r.

Post-Order

Euler Tour				Tree	
Thứ tự	Cạnh	Giá trị	Suffix_sum	v	p(v)
1	<1,2>	0	6	1	1
2	<2,4>	0	6	2	1
3	<4,2>	1	6	3	1
4	<2,5>	0	5	4	2
5	<5,2>	1	5	5	2
6	<2,1>	1	4	6	3
7	<1,3>	0	3	7	3
8	<3,6>	0	3		
9	<6,3>	1	3		
10	<3,7>	0	2		
11	<7,3>	1	2		
12	<3,1>	1	1		

Post-Order

- Position of vertexes :
  - S(<2,1>) = 4 → Position(2) = 7 - 4 = 3.
  - S(<3,1>) = 1 → Position(3) = 7 - 1 = 6.
  - S(<4,2>) = 6 → Position(4) = 7 - 6 = 1.
  - S(<5,2>) = 5 → Position(5) = 7 - 5 = 2.
  - S(<6,3>) = 3 → Position(6) = 7 - 3 = 4.
  - S(<7,3>) = 2 → Position(7) = 7 - 2 = 5.
- The traversing order is:
  - [4 → 5 → 2 → 6 → 7 → 3 → 1]

Pre-Order

- Set up the Euler cycle on the T tree.
- With root r, identifying the rooted-directed tree (with ∀ v, identifying p(v) as father of v).
- Set the weight to the edges:
- w(<v,p(v)>) = 0 & w(<p(v),v>) = 1.
- For each <u,v>, determining suffix_sum for <u,v>. It is called S(<u,v>)
- Traversing position v is: |V| - S(<p(v),v>).
- We traverse root node first.

Pre-Order

Euler Tour				Tree	
Thứ tự	Cạnh	Giá trị	Suffix_sum	v	p(v)
1	<1,2>	1	6	1	1
2	<2,4>	1	5	2	1
3	<4,2>	0	4	3	1
4	<2,5>	1	4	4	2
5	<5,2>	0	3	5	2
6	<2,1>	0	3	6	3
7	<1,3>	1	3	7	3
8	<3,6>	1	2		
9	<6,3>	0	1		
10	<3,7>	1	1		
11	<7,3>	0	0		
12	<3,1>	0	0		

Pre-Order

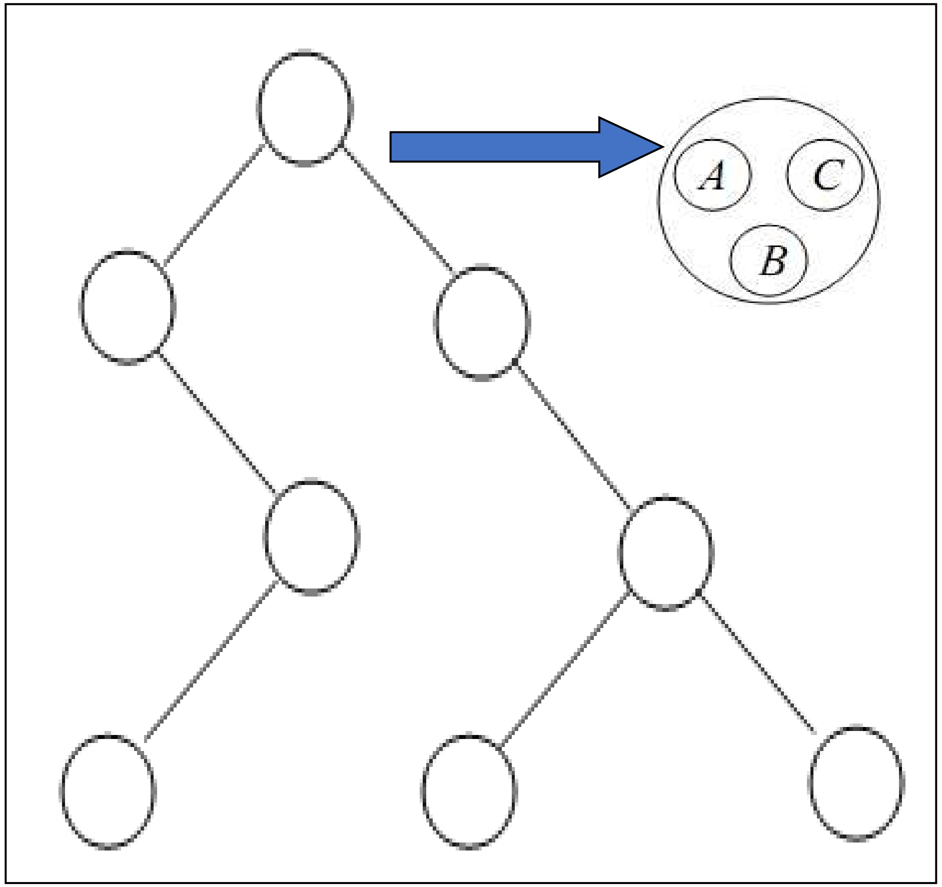
- Position of vertexes as follows:
  - S(<1,2>) = 6 → Position(2) = 7 - 6 = 1.
  - S(<1,3>) = 3 → Position(3) = 7 - 3 = 4.
  - S(<2,4>) = 5 → Position(4) = 7 - 5 = 2.
  - S(<2,5>) = 4 → Position(5) = 7 - 4 = 3.
  - S(<3,6>) = 2 → Position(6) = 7 - 2 = 5.
  - S(<3,7>) = 1 → Position(7) = 7 - 1 = 6.
- The traversing order is:
  - [1 → 2 → 4 → 5 → 3 → 6 → 7]

A different approach

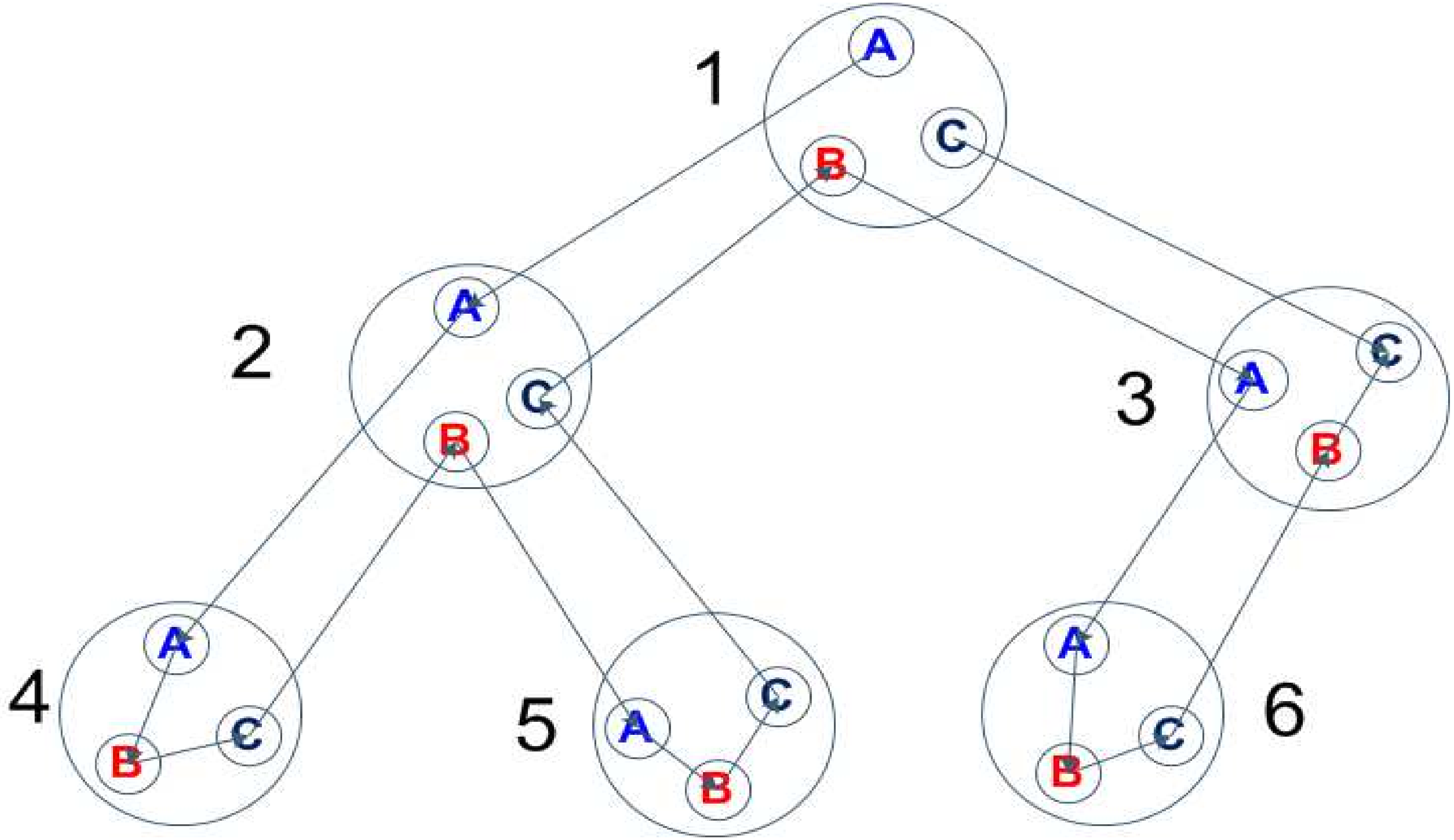
- For the binary tree:
  - Each v node is considered to be 3 child nodes: v[a], v[b], v[c].
  - Rules of the node [a]:
    - If v has a left child that is u, then: v[a] → u[a].
    - If v does not have left child: v[a] → v[b].
  - Rules of the node [b]:
    - If v have a right child that is u, then: v[b] → u[a].
    - If v does not have right child: v[b] → v[c].
  - Rules of the node [c]:
    - If v is u's left child, then: v[c] → u[b].
    - If v is u's right child,: v[c] → u[c].
    - If v is the root node: v[c] → NULL.

Illustration

- Look at the tree as pictured on the side. Each node is presented by a set of 3 child nodes A, B, C
- Assigning the appropriate values A,B,C to problems:
  - Traversing trees
  - Calculate height, number of child nodes, …

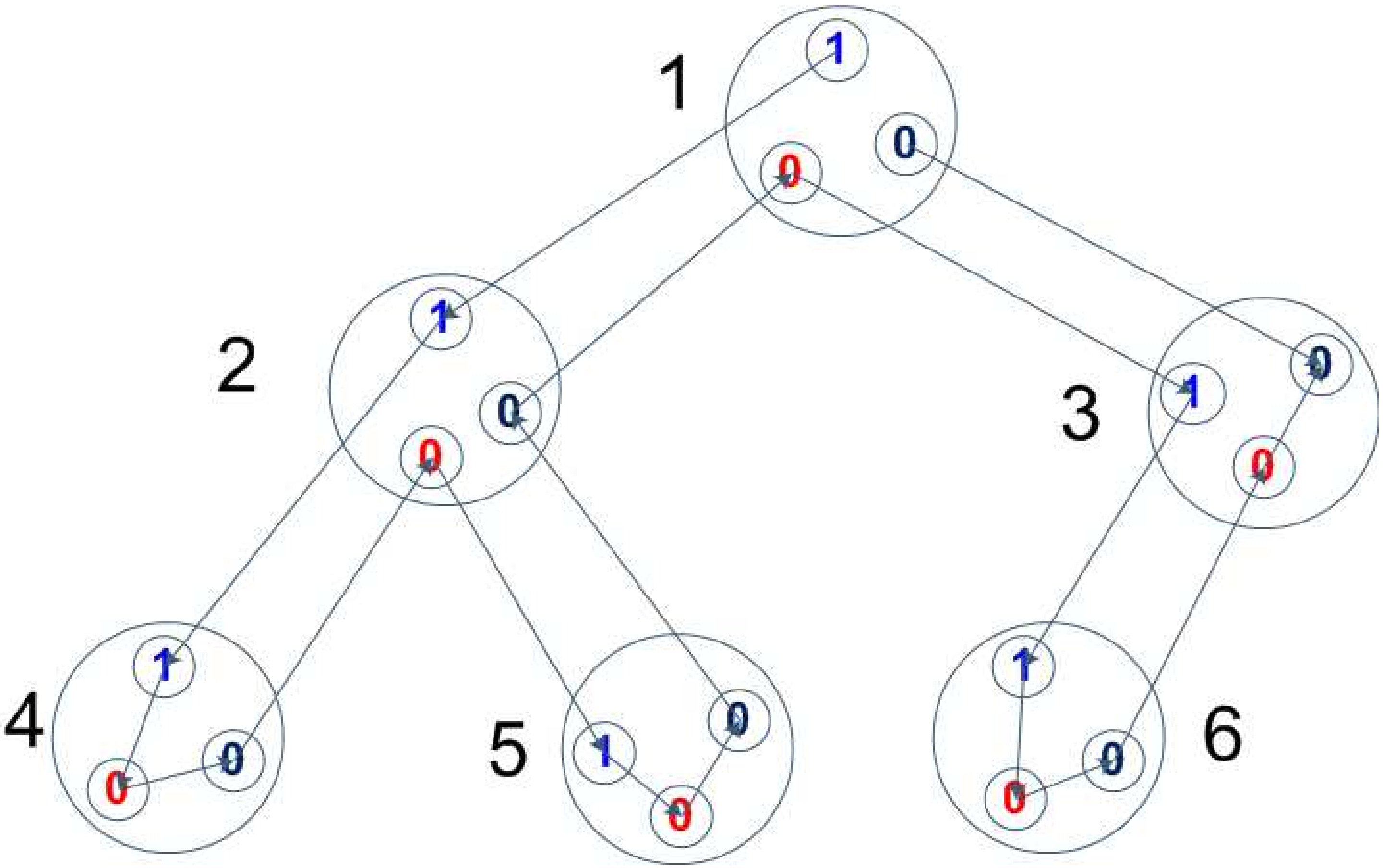


Euler cycle and Linked List

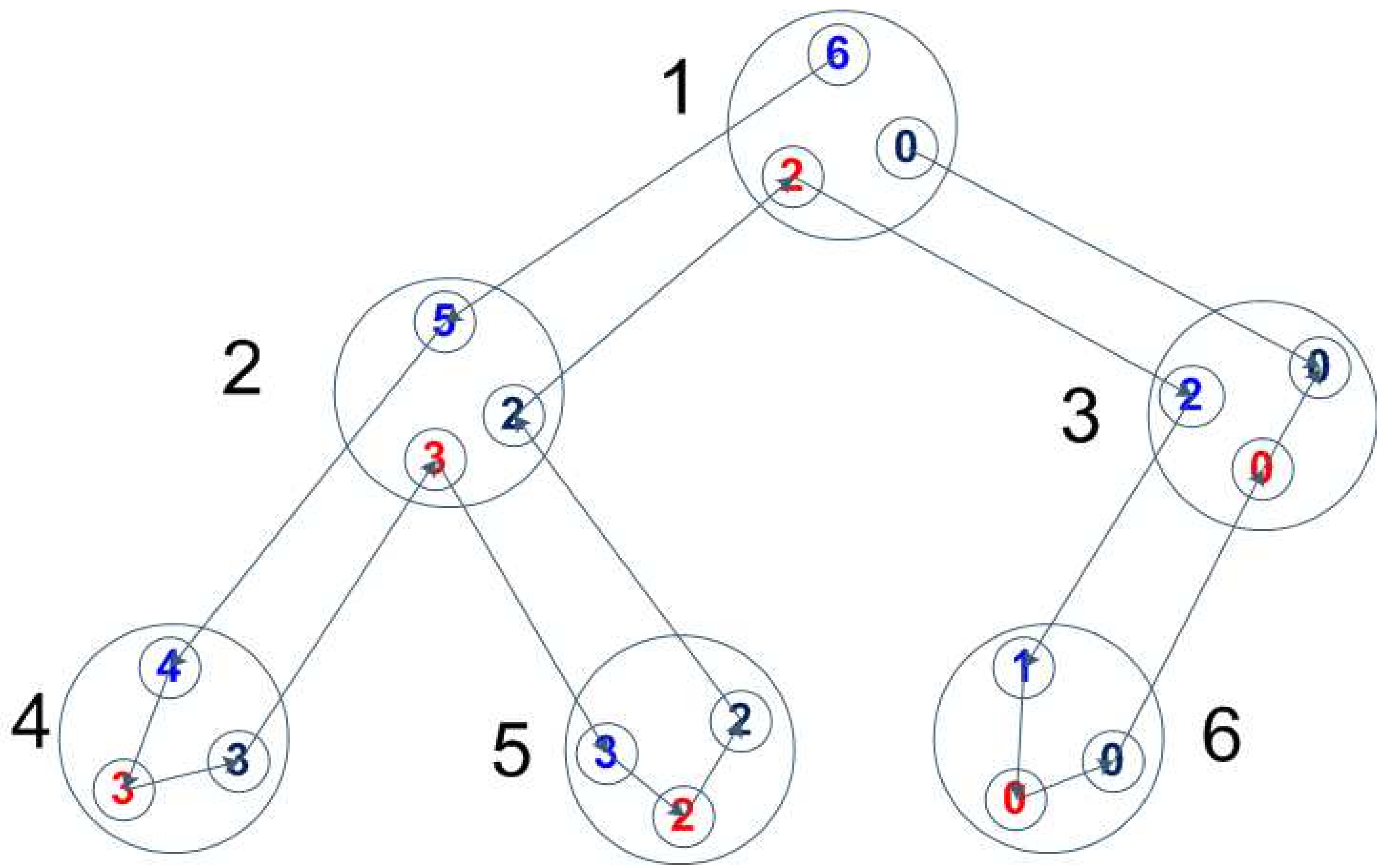


1[A] → 2[A] → 4[A] → 4[B] → 4[C] → 2[B] → 5[A] → 5[B] →5[C] → 2[C] → 1[B] → 3[A] → 6[A] → 6[B] → 6[C] → 3[B] → 3[C] → 1[C] → ●(NULL)

PreOrder: A = 1, B = 0, C = 0.



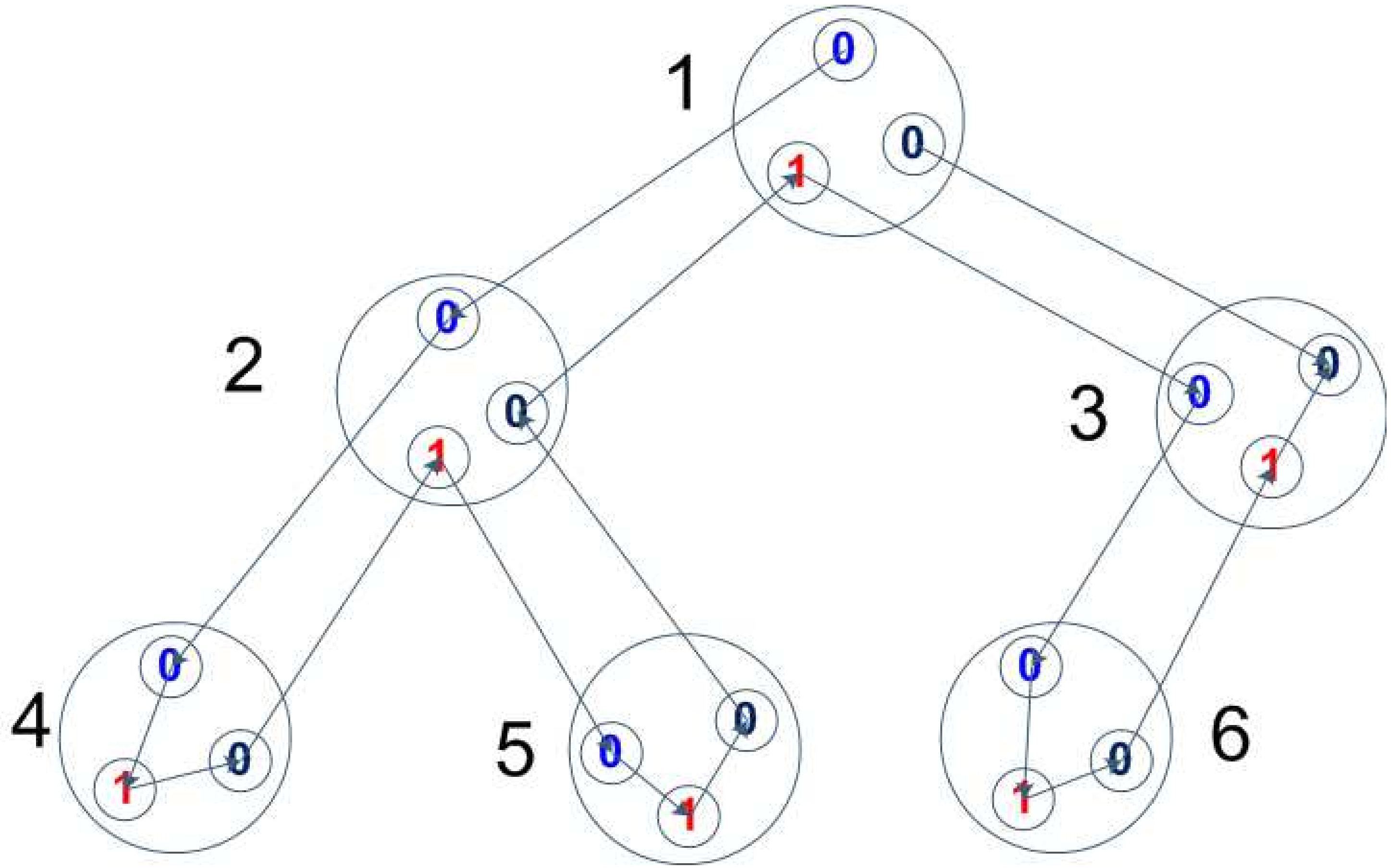
Calculate the Suffix-Sum of the List.



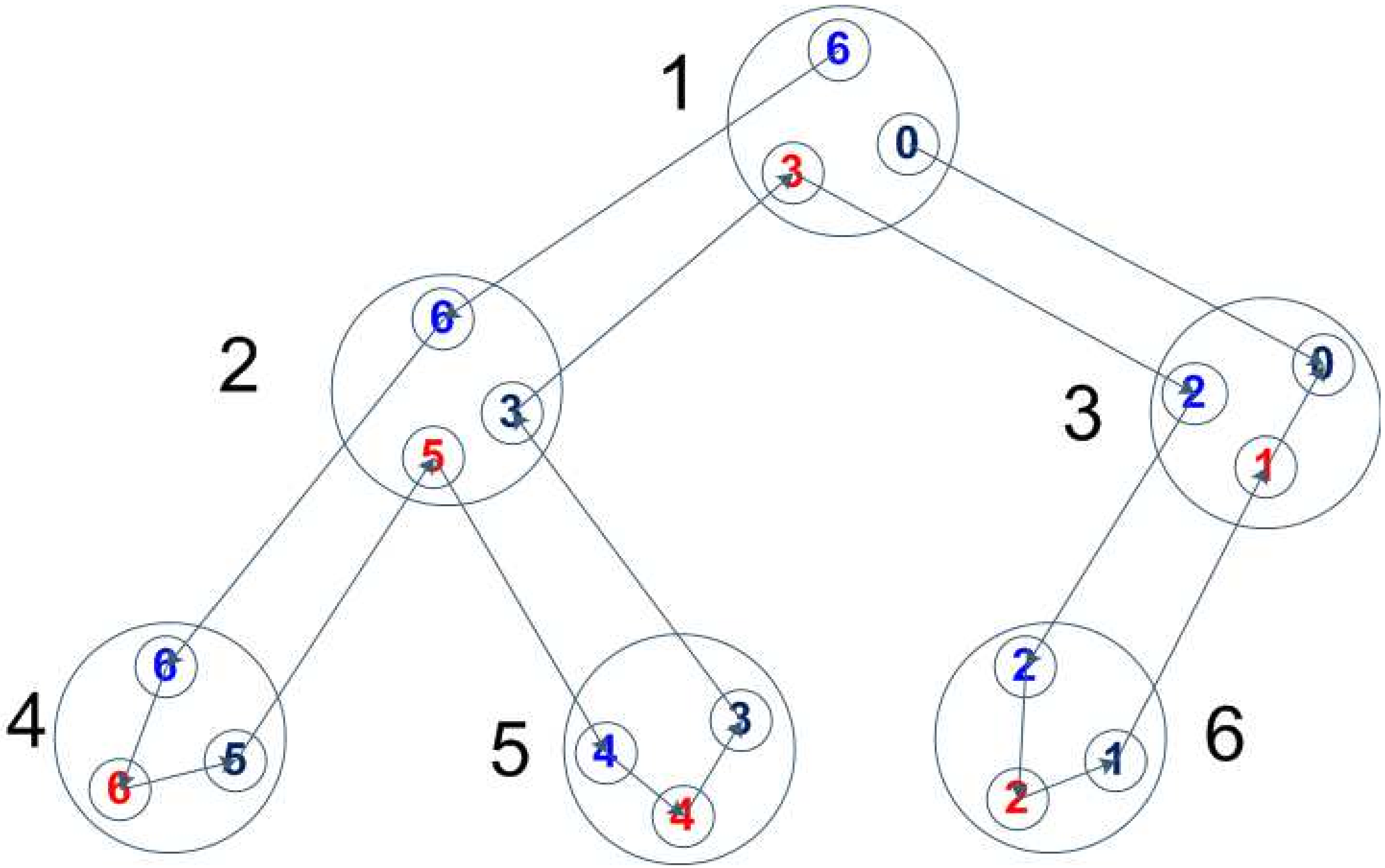
PreOrder's order

- Node's traversing order v: |V| - v[A] + 1.
  - 1[A] = 6 → Position(1) = 6 – 6 + 1 = 1.
  - 2[A] = 5 → Position(2) = 6 – 5 + 1 = 2.
  - 3[A] = 2 → Position(3) = 6 – 2 + 1 = 5.
  - 4[A] = 4 → Position(4) = 6 – 4 + 1 = 3.
  - 5[A] = 3 → Position(5) = 6 – 3 + 1 = 4.
  - 6[A] = 1 → Position(6) = 6 – 1 + 1 = 6.
- The traversing order is: [1 → 2 → 4 → 5 → 3 → 6]

InOrder: A = 0, B = 1, C = 0.



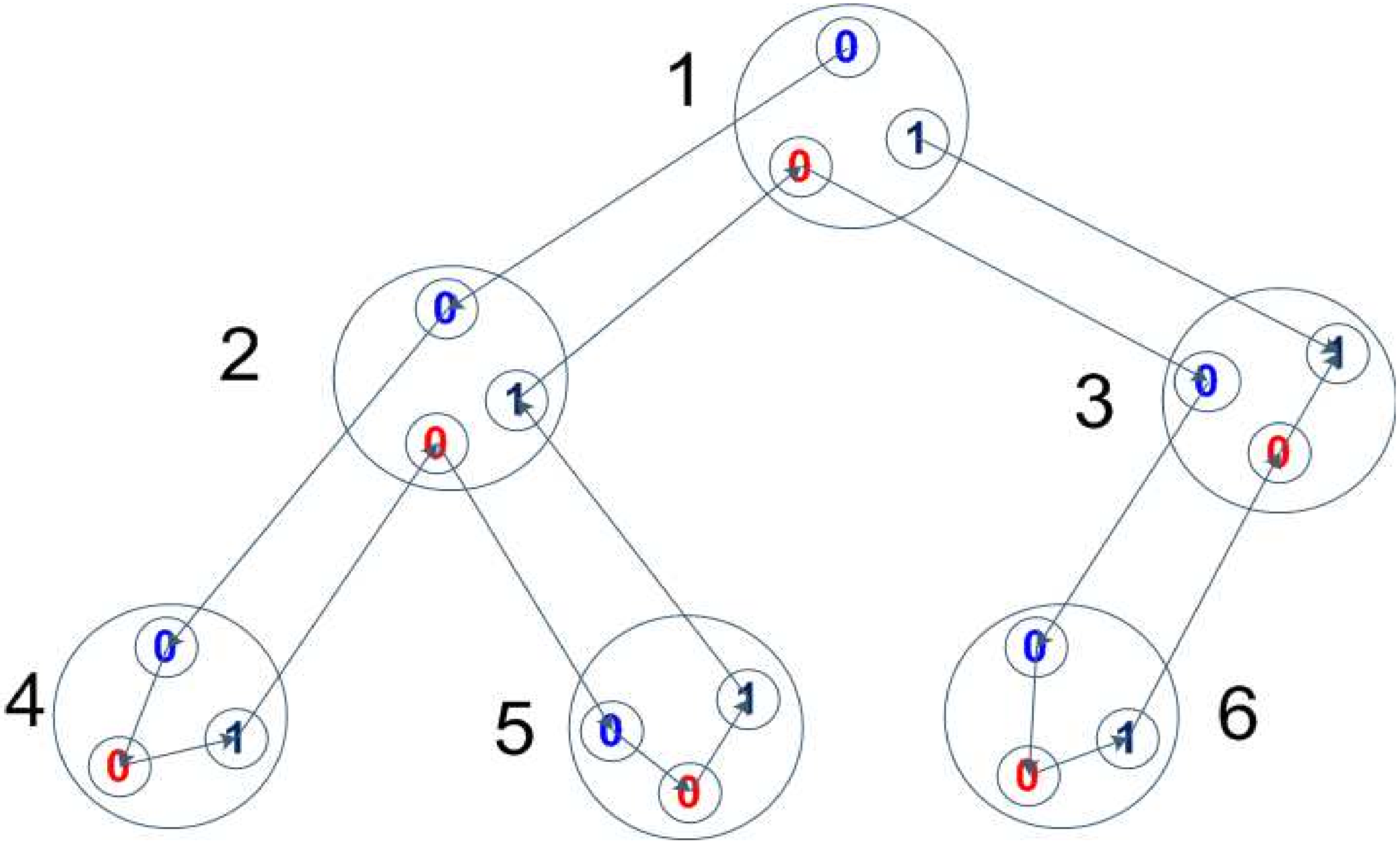
Calculate the Suffix-Sum of the List



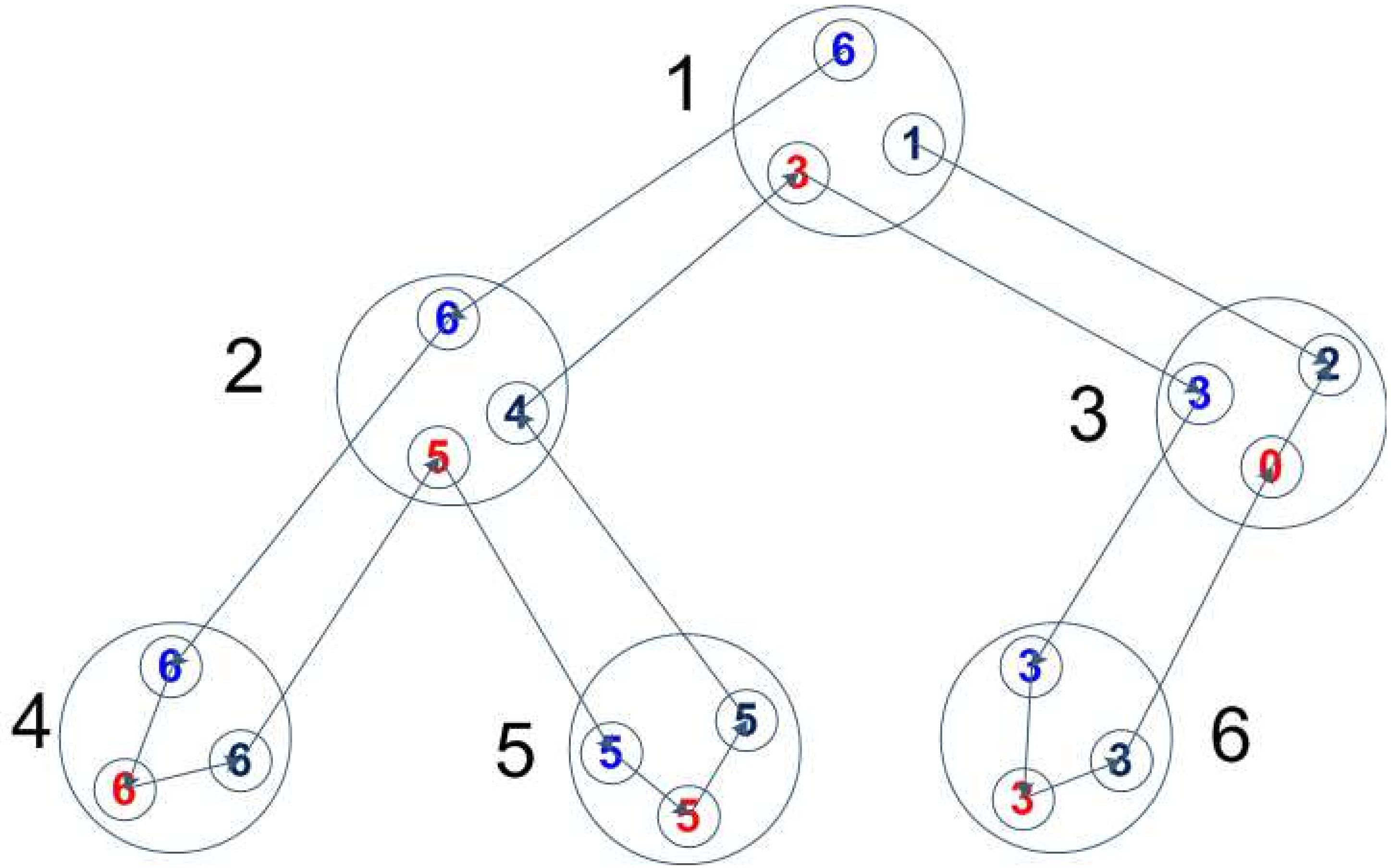
InOrder's order

- Node's traversing order v: |V| - v[B] + 1.
  - 1[B] = 3 → Position(1) = 6 – 3 + 1 = 4.
  - 2[B] = 5 → Position(2) = 6 – 5 + 1 = 2.
  - 3[B] = 1 → Position(3) = 6 – 1 + 1 = 6.
  - 4[B] = 6 → Position(4) = 6 – 6 + 1 = 1.
  - 5[B] = 4 → Position(5) = 6 – 4 + 1 = 3.
  - 6[B] = 2 → Position(6) = 6 – 2 + 1 = 5.
- The traversing order is: [4 → 2 → 5 → 1 → 6 → 3]

PostOrder: A = 0, B = 0, C = 1.



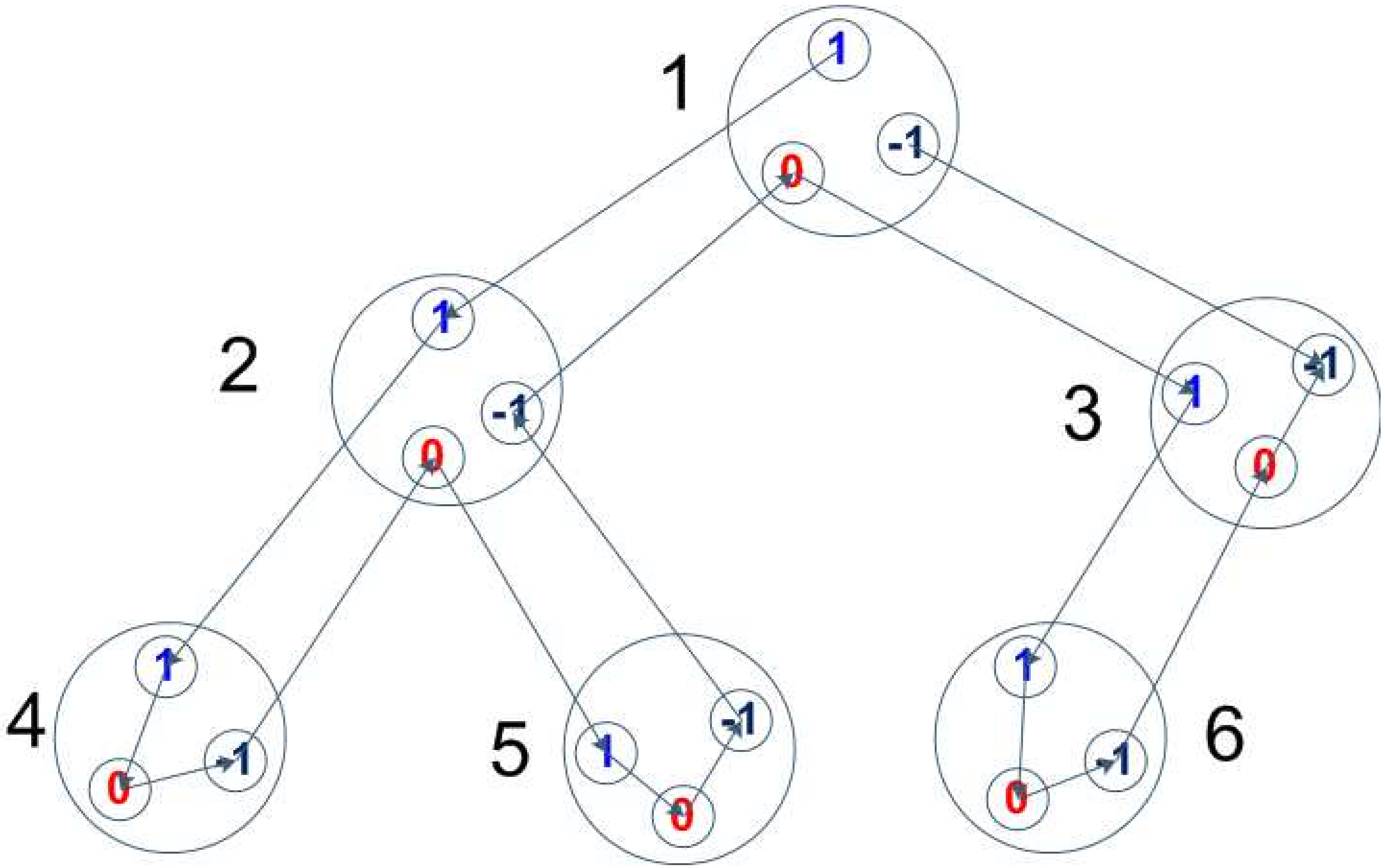
Calculate the Suffix-Sum of the List



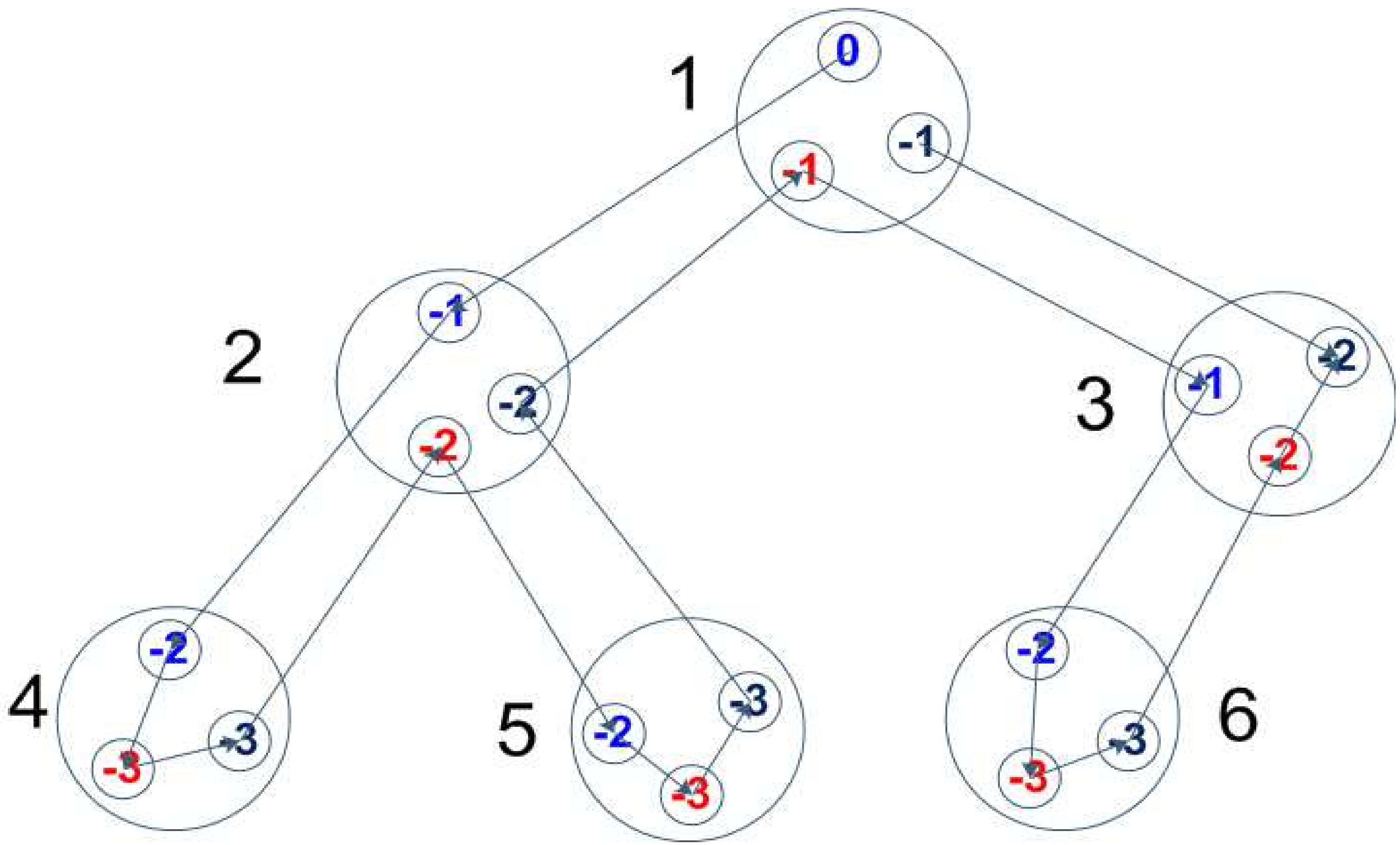
PostOrder's order

- Node's traversing order v: |V| - v[C] + 1.
  - 1[C] = 1 → Position(1) = 6 – 1 + 1 = 6.
  - 2[C] = 4 → Position(2) = 6 – 4 + 1 = 3.
  - 3[C] = 2 → Position(3) = 6 – 2 + 1 = 5.
  - 4[C] = 6 → Position(4) = 6 – 6 + 1 = 1.
  - 5[C] = 5 → Position(5) = 6 – 5 + 1 = 2.
  - 6[C] = 3 → Position(6) = 6 – 3 + 1 = 4.
- The traversing order is: [4 → 5 → 2 → 6 → 3 → 1]

Depth(v) : A = 1, B = 0, C = -1



Calculate the Suffix-Sum of the List

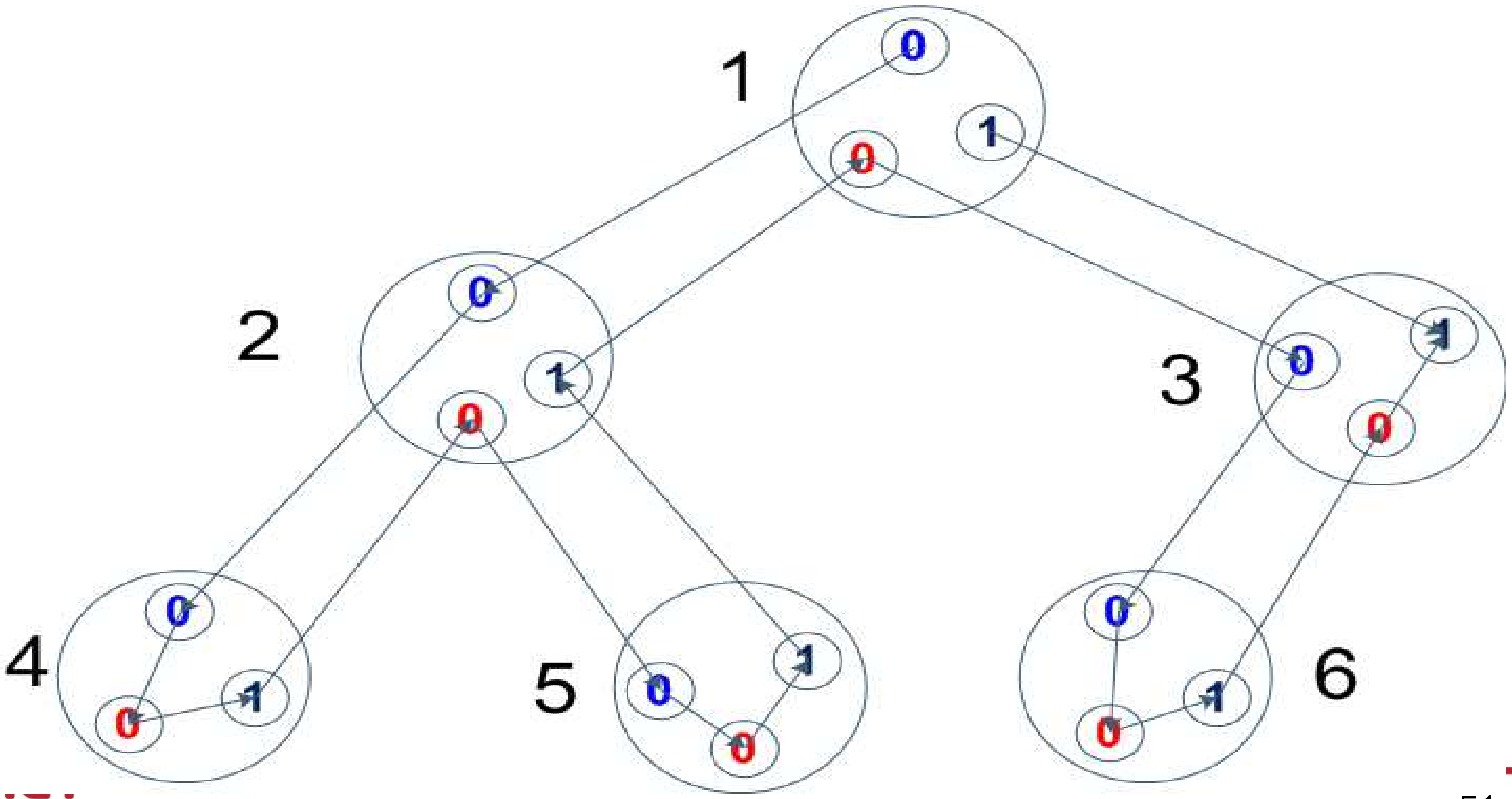


Specify the depth of nodes

- Depth of node v: abs(v[A])
  - 1[A] = 0 → Depth(1) = 0.
  - 2[A] = -1 → Depth (2) = 1.
  - 3[A] = -1 → Depth (3) = 1.
  - 4[A] = -2 → Depth (4) = 2.
  - 5[A] = -2 → Depth (5) = 2.
  - 6[A] = -2 → Depth (6) = 2.
- Node's Height: Height(v) = H - Depth(v) where H = max{ Depth(v)}.

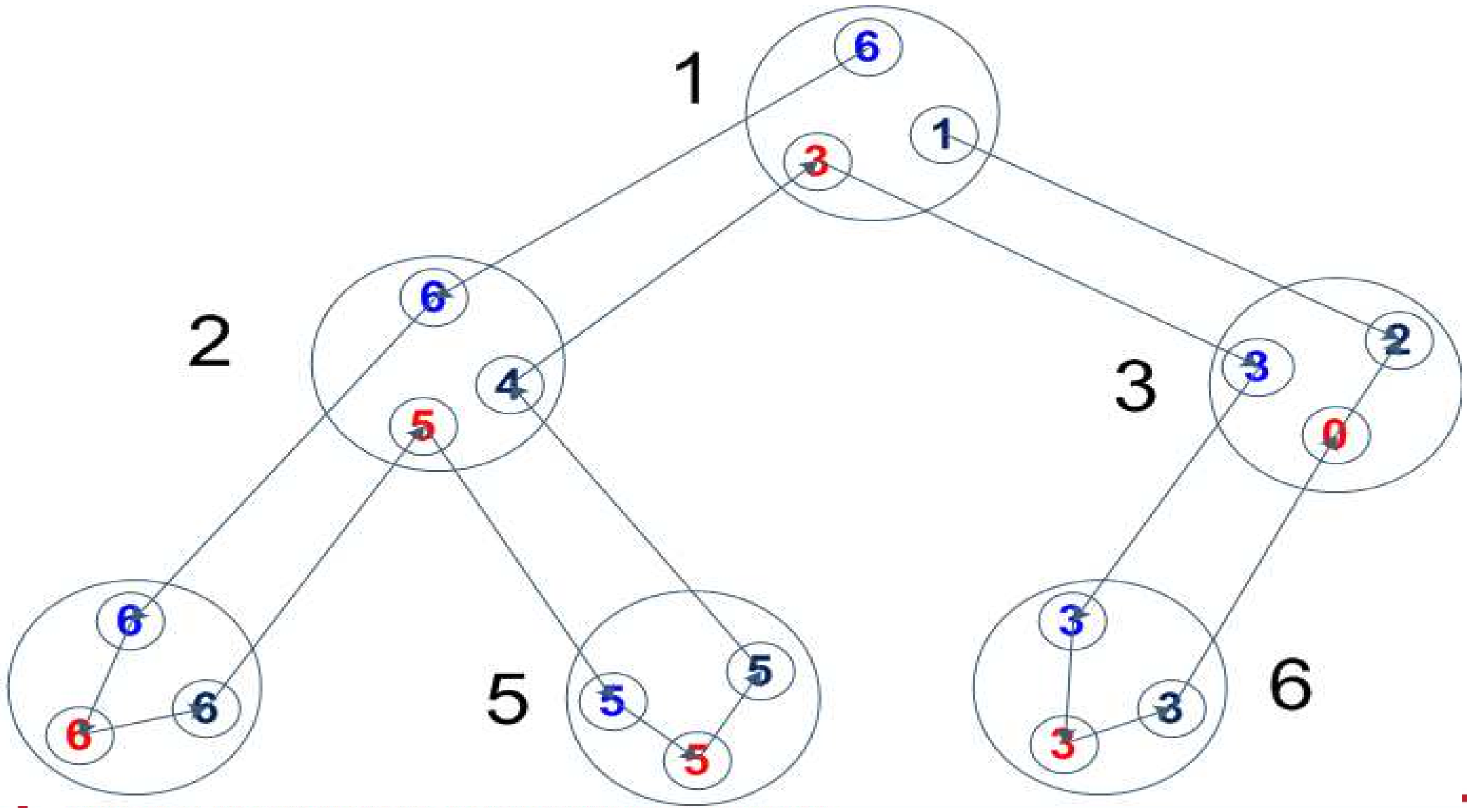
Determining the size of the tree with the root v

- For all v, specifying the number of nodes in the sub-tree that consider v as the root. Set A = 0, B = 0, C = 1.



Determining the size of the tree with the root v

- Calculate the Suffix-sum of the list based on the Euler cycle



Determine the size of the tree with the root v

- Size(v) = v[A] – V[C] + 1.
  - Size(1) = 6 – 1 + 1 = 6.
  - Size(2) = 6 – 4 + 1 = 3.
  - Size(3) = 3 – 2 + 1 = 2.
  - Size(4) = 6 – 6 + 1 = 1.
  - Size(5) = 5 – 5 + 1 = 1.
  - Size(6) = 3 – 3 + 1 = 1.

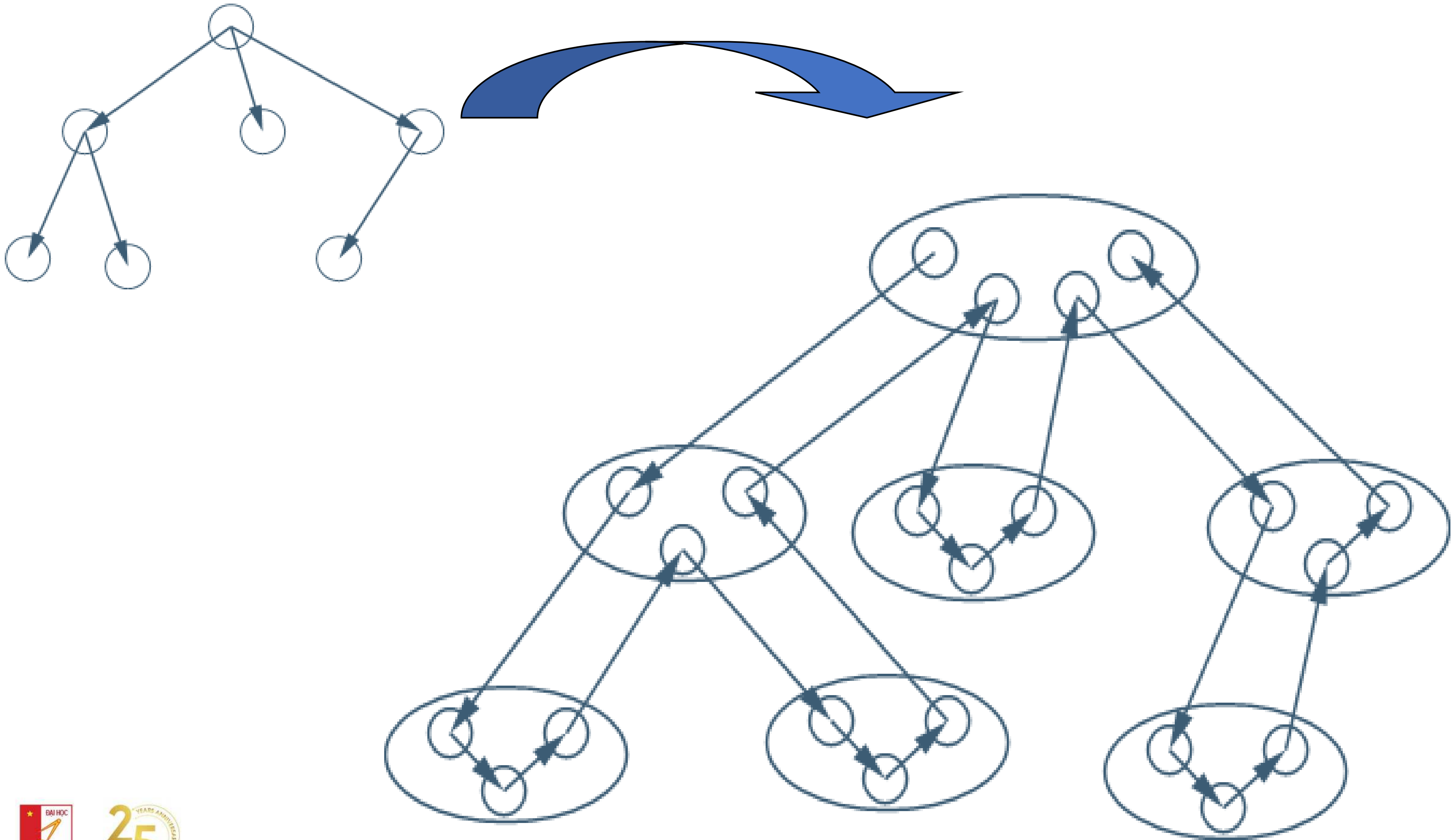
Euler cycle for general tree

- Consider node v. Supposing {v1, v2, .., vm} are the children of v from left to right.
- Node v is presented by m+1 child nodes:
  - v[A] : the entrance point of v in the Euler cycle.
  - v[C] : point out of v in euler cycle.
  - v[B_k]: connect to the child nodes of v_{k+1}. (k = 1..m-1)
- If v is a leaf node or has only 1 child, v is still presented by v[A], v[B], v[C].

Node's connecting rules

- Rules for A:
  - If v has an outer-left child v_1 then v[A] connected to v_1[A].
  - If v does not have children, then v[A] connects to v[B].
- Rules for B:
  - If v is the leaf node, then v[B] connects to v[C].
  - If v has child nodes $\{v_1, v_2, .., v_m\}$ then v[B_k] connected to v_{k+1}[A] with k = 1..m-1.
- Rules for C:
  - If v is the outer-right child of u then v[C] connects to u[C].
  - If v is the k-child of u then v[C] connects to u [B_k].
  - If v is the root, then v[C] connects to NULL.

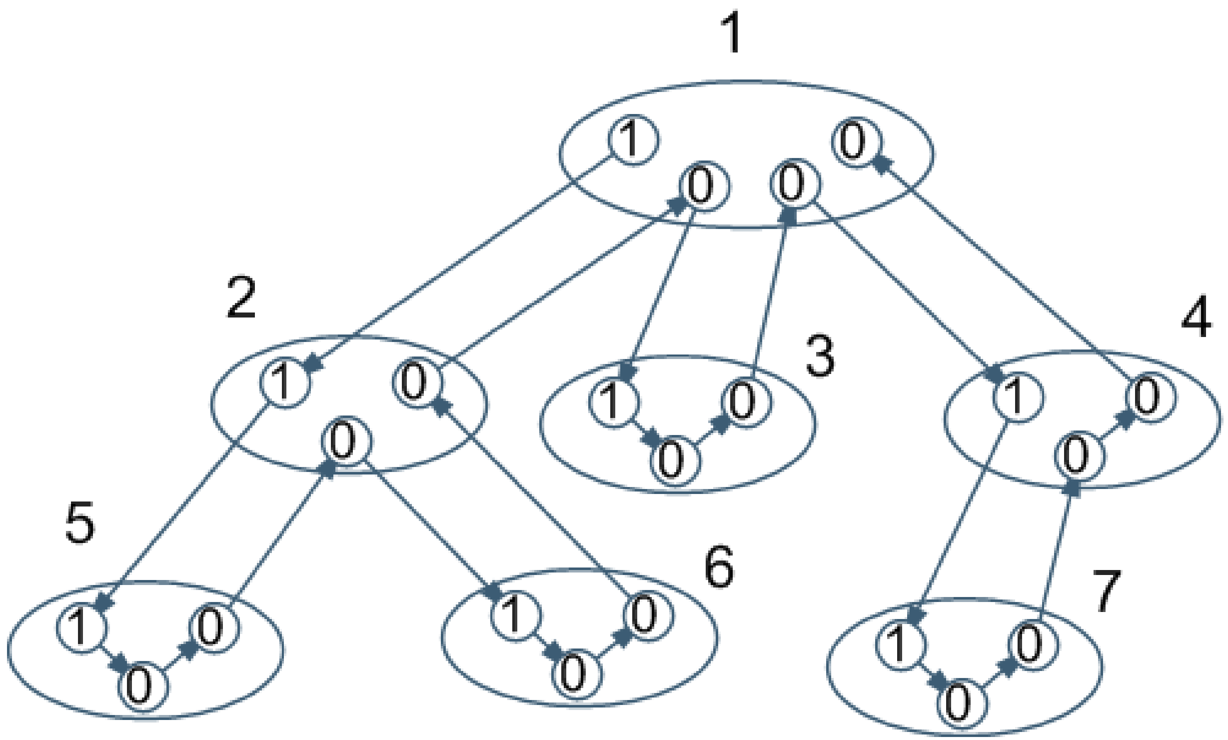
Illustration



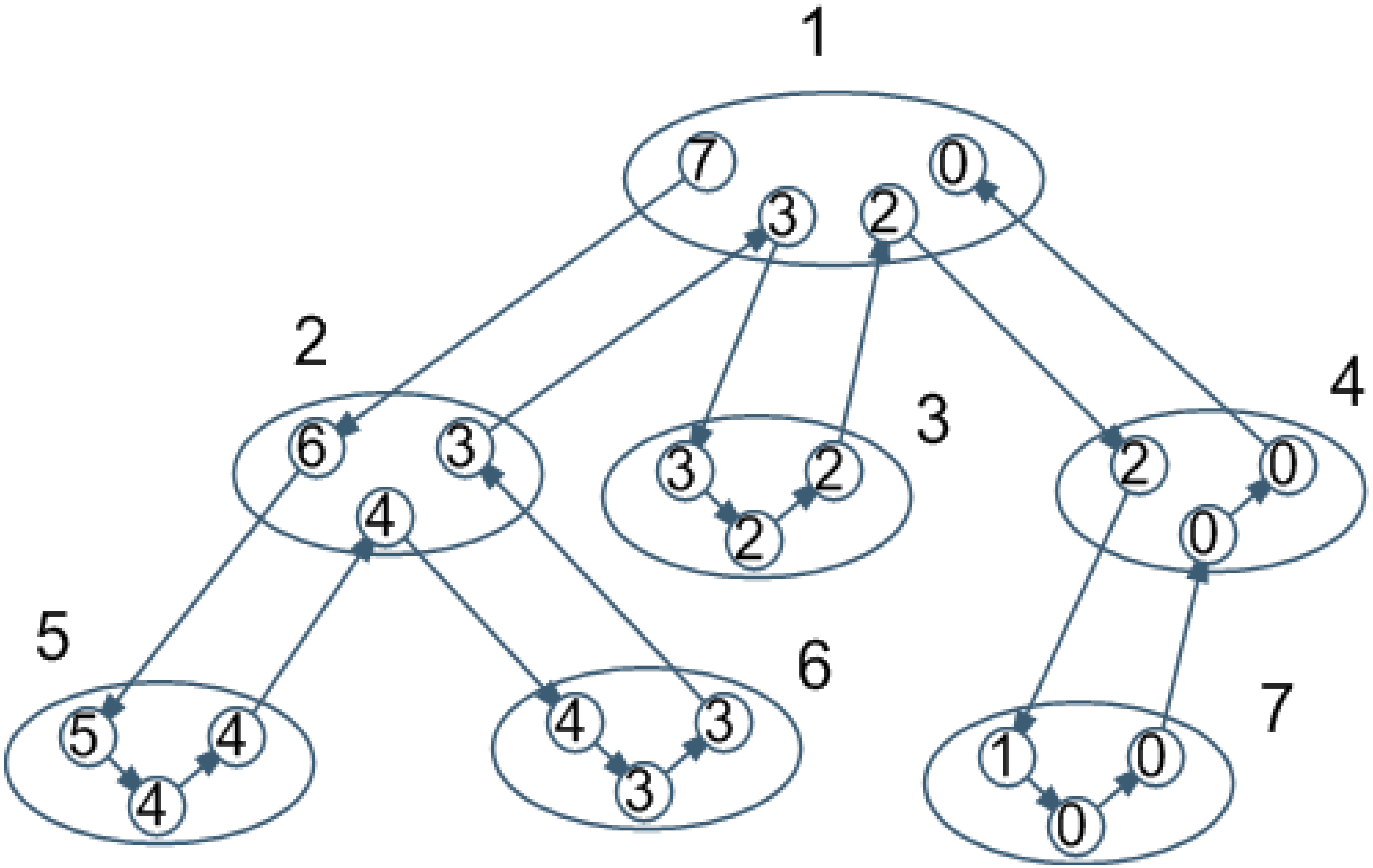
Problems with the general tree

- Traversing problem:
  - No Inorder concept.
  - PreOrder and PostOrder's traversing are based on the A input node A and output node C. Values $B_k = 0$.
    - Preorder : A = 1; C = 0.
    - Postorder: A = 0; C = 1.
- Depth problem: A = 1; C = -1.
- Sub-tree size problem: A = 0; C = 1.

PreOrder: $A = 1, B_k = 0, C = 0$



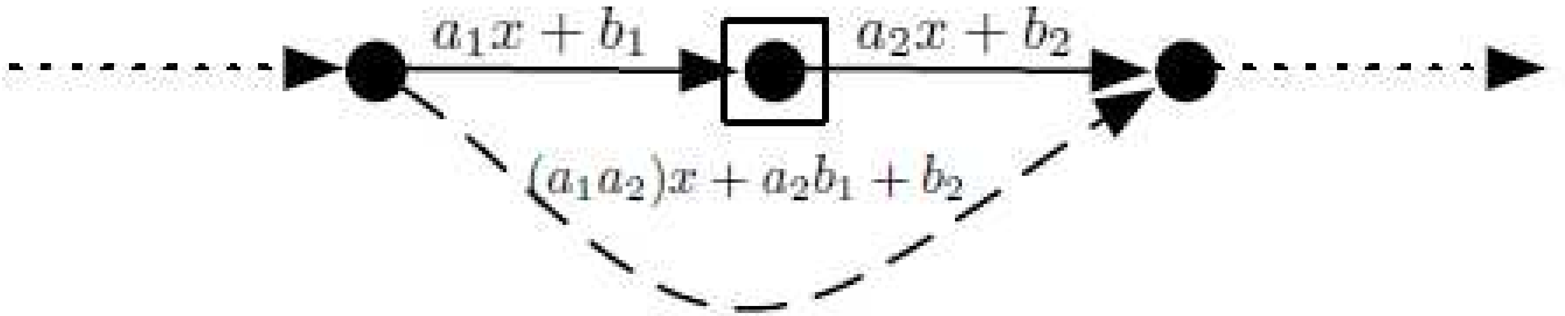
PreOrder: $A = 1, B_k = 0, C = 0$



[1 → 2 → 5 → 6 → 3 → 4 → 7]

10.2.3 Tree Contraction

- For a binary tree. Let's reduce a main tree to a smaller tree consisting of 1 root and 2 child nodes.
- For example, the example shortens 3 nodes to 2 nodes:

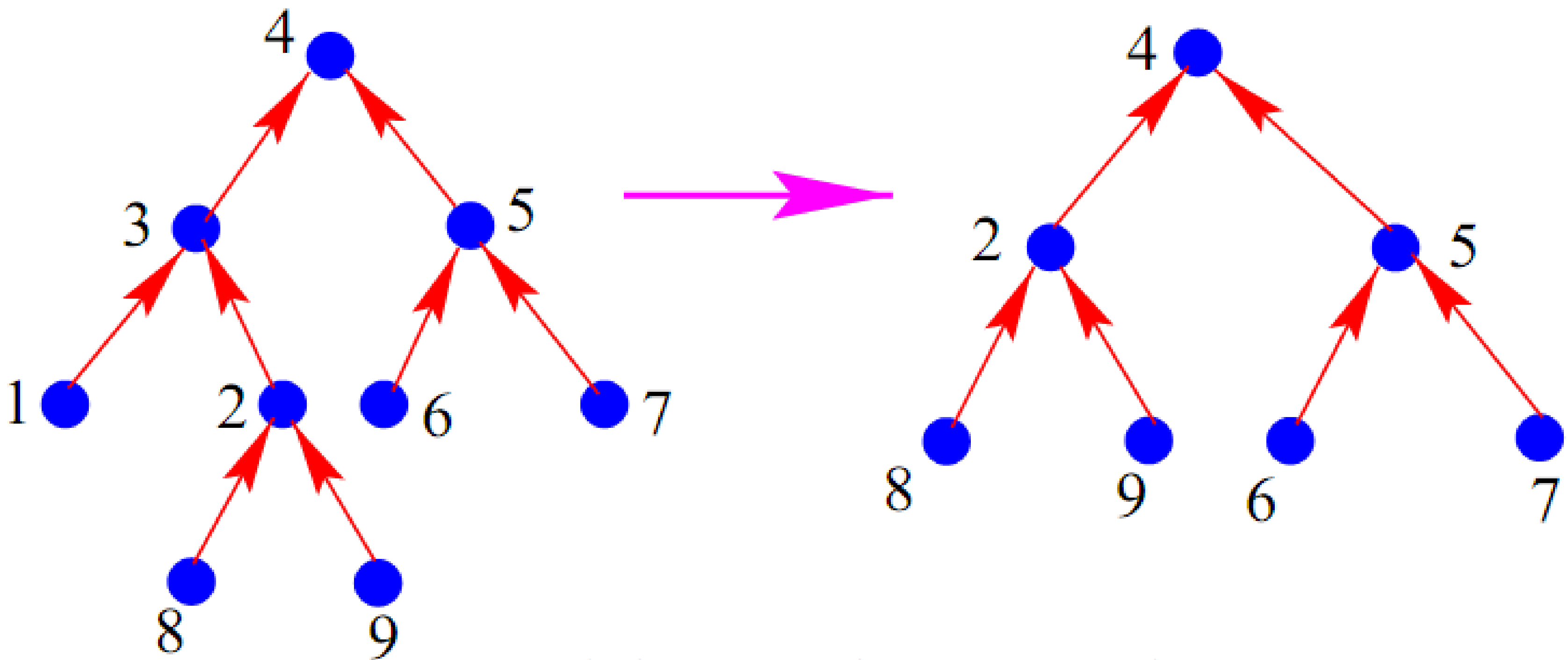


Approach

- Tree T = (V,E) is a binary tree with root r:
  - p(v) is the parent node of v on the T tree.
  - sib(v) is the brother node of v: being the child of the same parent node. (sib = sibling).
- RAKE operation for leaf node v: p(v) ≠ r
  - Delete nodes v, p(v) on the T tree.
  - Connect sib(v) to p(p(v)) on the T tree.

Approach

- RAKE operation - reduce the leaf nodes:



Applying Rake to node 1

Approach

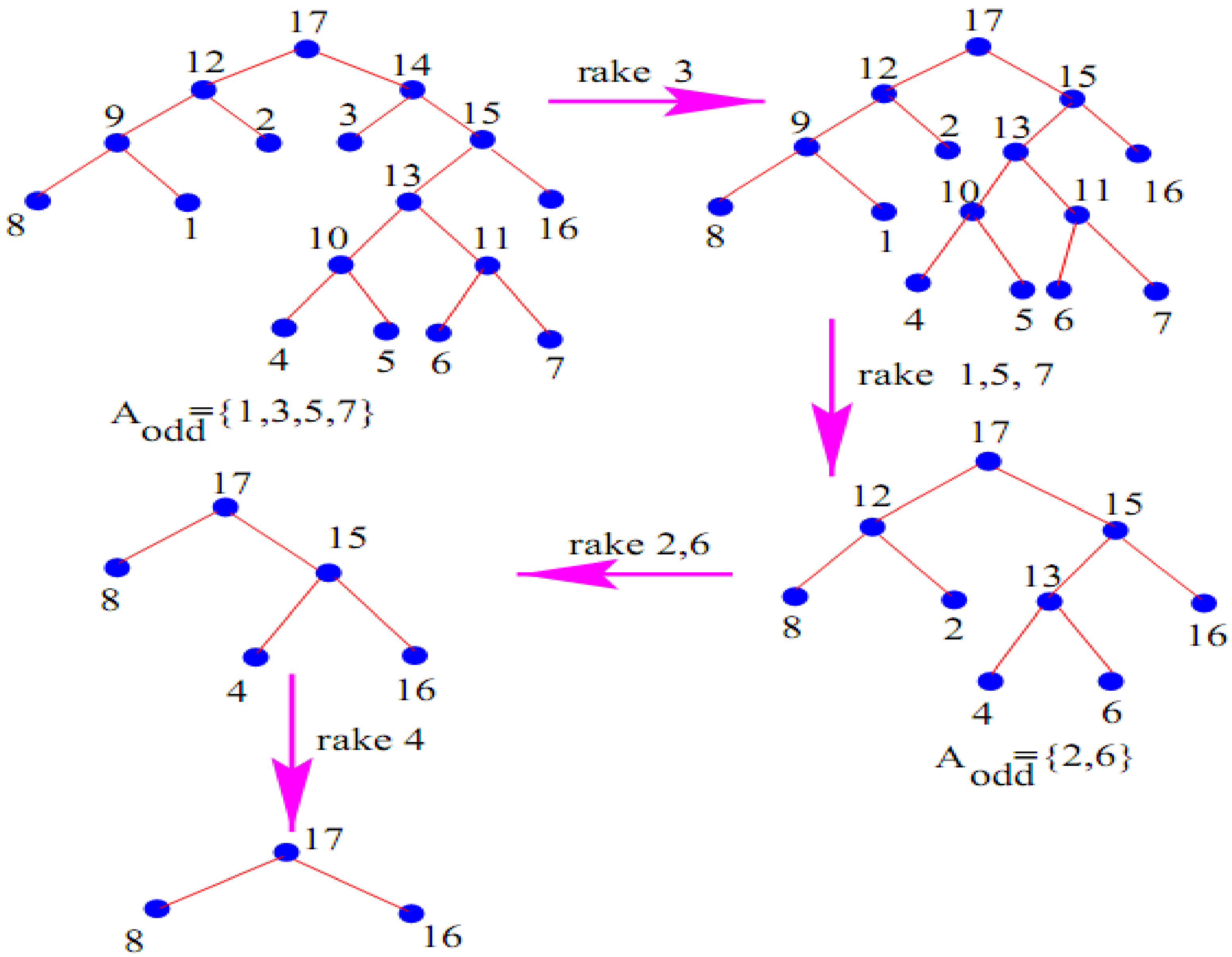
- Problems arises:
  - The RAKE can't be performed with the leaf node connected to the root.
  - All leaves cannot be excluded by a RAKE operation in parallel?
  - → it works only on the leaves that their fathers do not adjacent to each other.
  - For example, nodes of 1, 8, 9 cannot be RAKE together.

Approach

- Solution:
  - Each parent node must store information about its left child and its right child nodes.
  - Highlight leaf nodes in the order from 1..n
  - Consider nodes with odd index:
    - The nodes are the left child, and their fathers won't be together. It is called as Group 1.
    - The same with the right-child nodes. It is called as Group 2.
  - → implemented in parallel on each group in turn will ensure that RAKE operation’s condition is not violated.

Algorithm steps

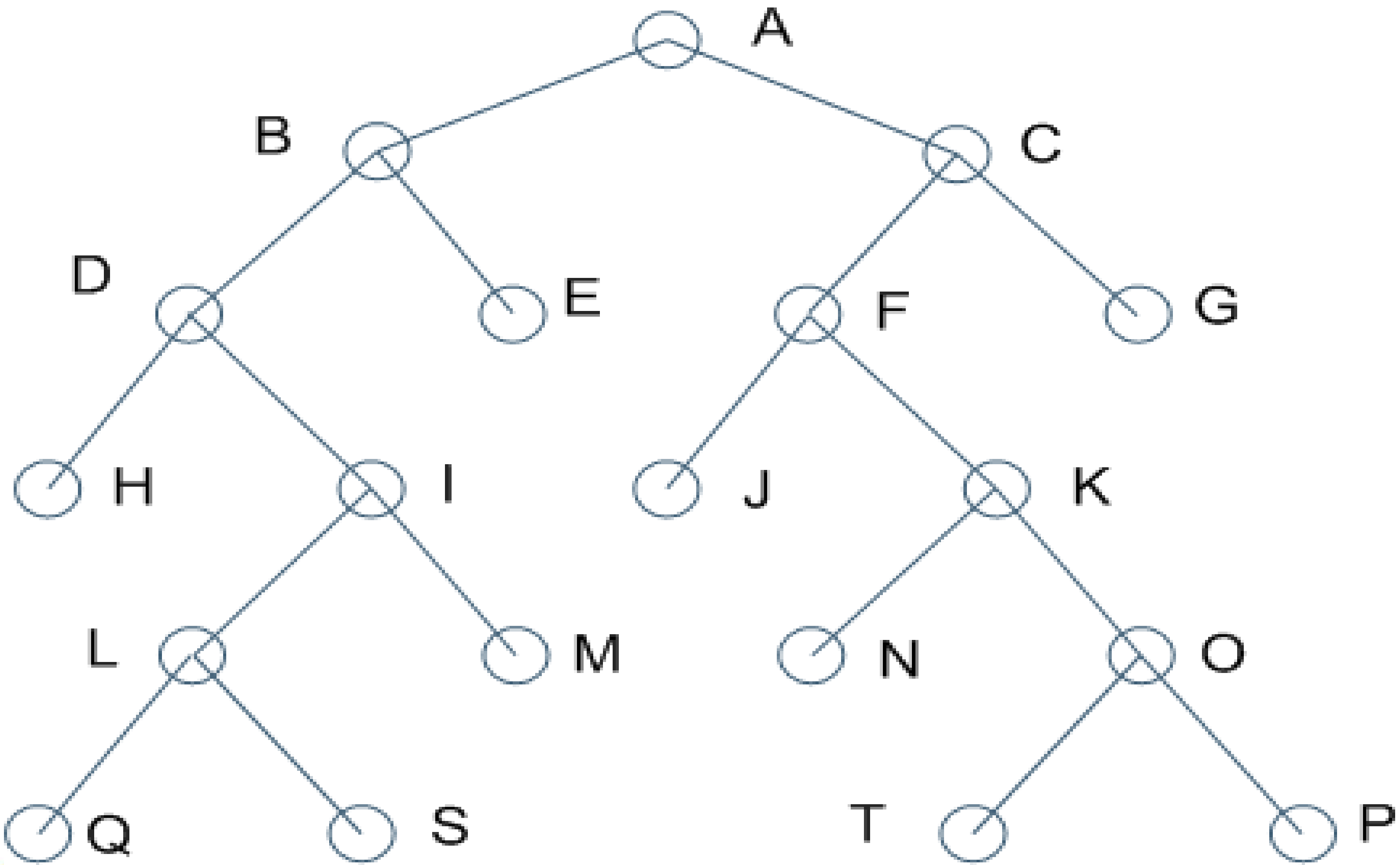
- S1. Marking the leaf nodes in order from 1..n to save to array Z, except for 2 leaf nodes located on the left, on the right end.
- Repeat:
  - S2. Performing RAKE with Z[k] nodes if k is odd and the node must be left child.
  - S3. Performing RAKE with Z[k] nodes with remaining odd-values k.
  - S4. Assign Z = set of Z[k] if k is even.
- Until there are 3 nodes left, then the algorithm stops.

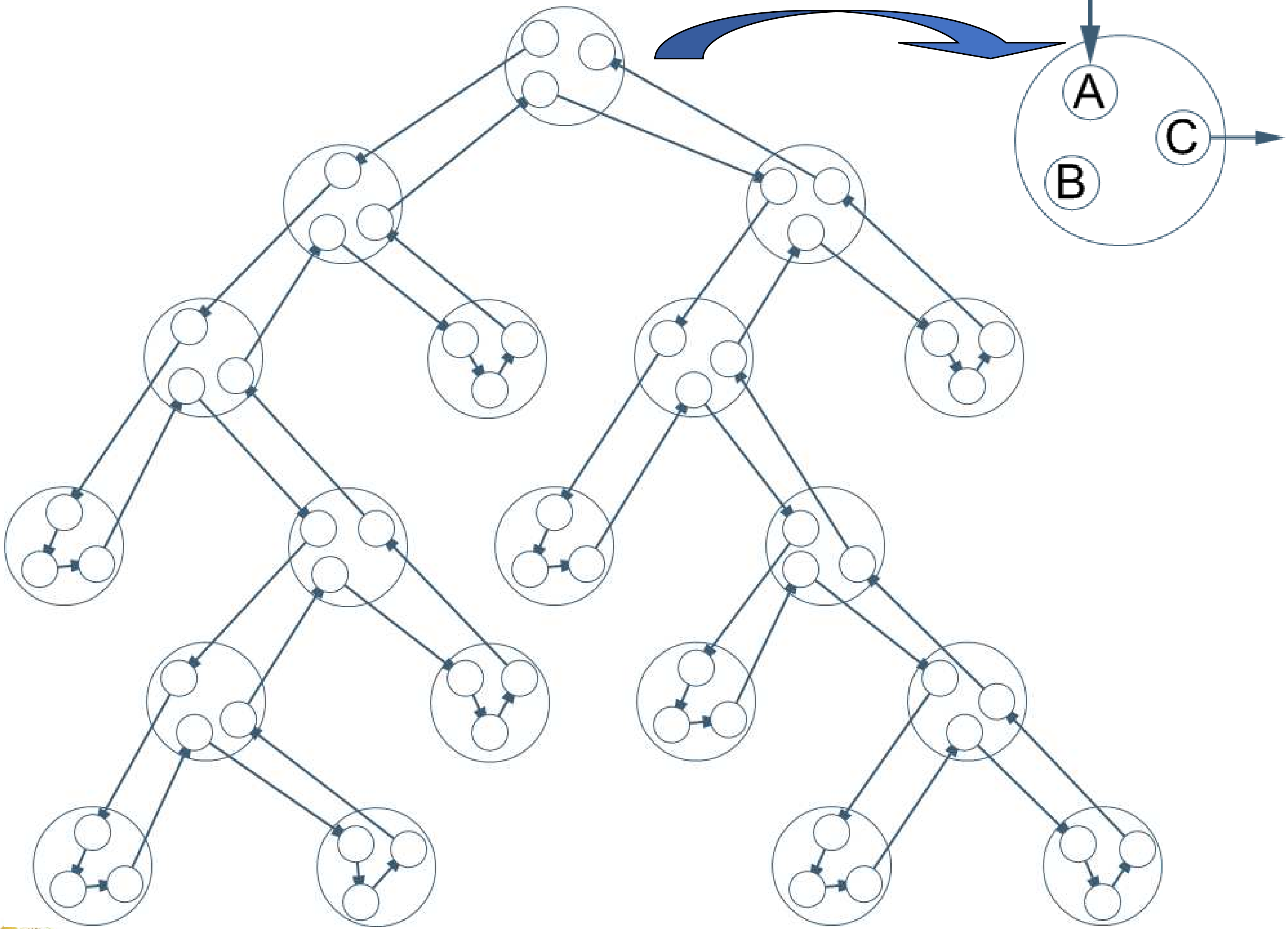

17
12
14
9
2
3
15
13
8
1
10
11
16
4
5
6
7
A odd ={1,3,5,7}
rake 3
17
12
15
9
2
13
8
1
10
11
16
4
5
6
7
rake 1,5, 7
17
12
15
8
2
13
16
4
6
A odd ={2,6}
rake 2,6
17
15
8
4
16
rake 4
17
8
16

Detailed steps

- Solving step 1 of the algorithm:
  - Given tree T = (V,E).
  - Numbering the leaves from left to right (except for left-end/right-end nodes) in order from 1...n.
- Solution:
  - Using Euler cycle.
- Illustration with binary tree (each node has exactly 2 sub-nodes).

Order leaves from left to right

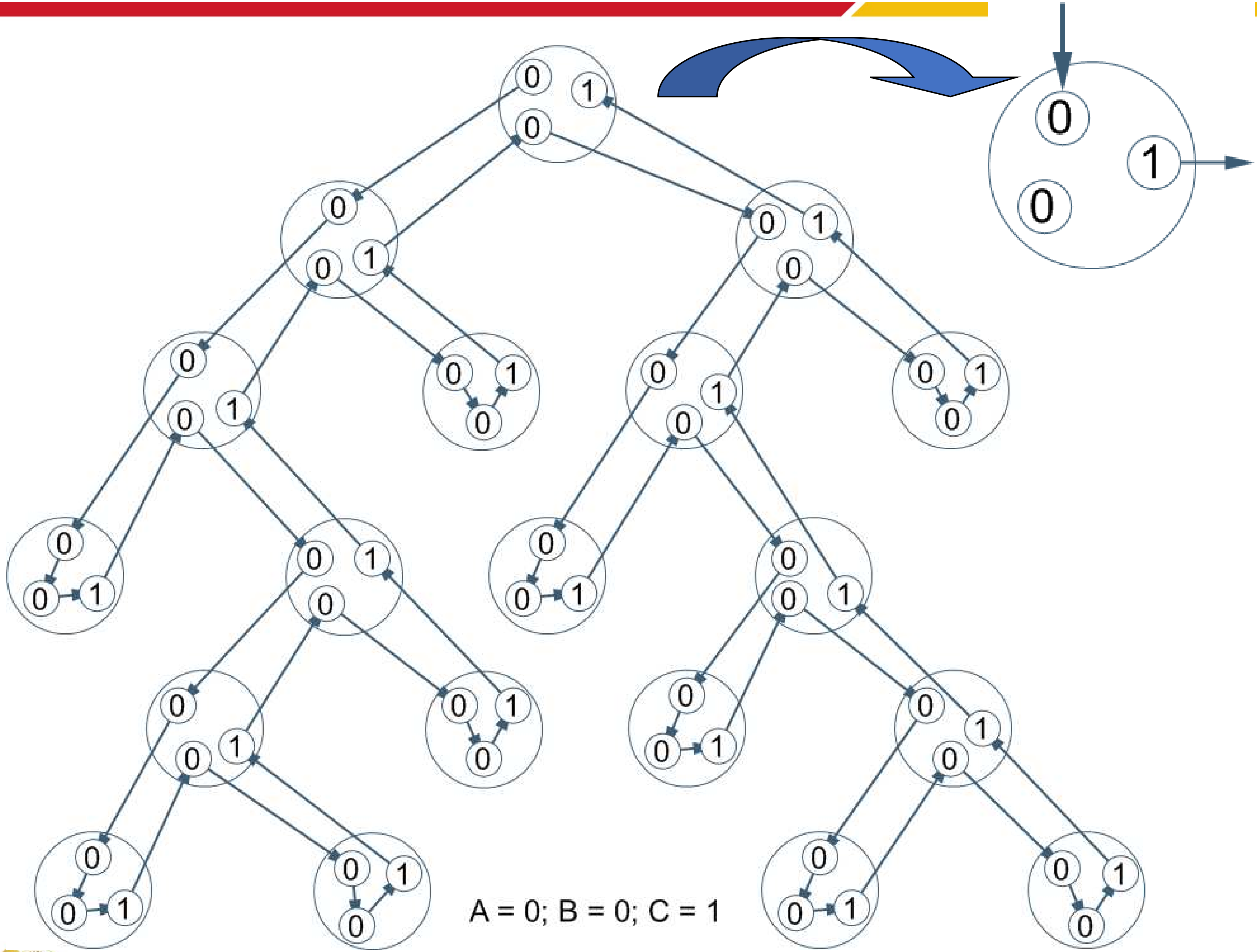


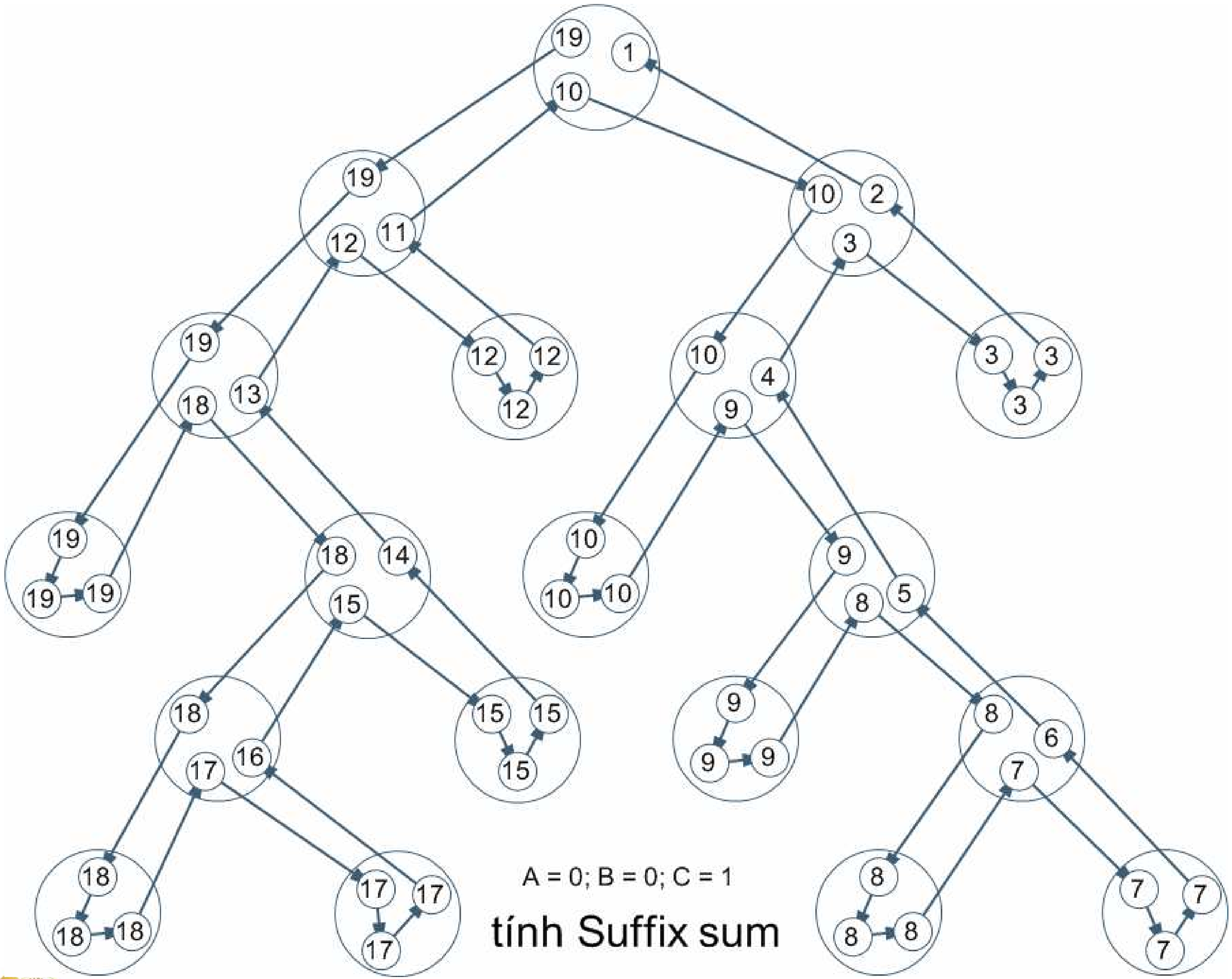

A
B
C

Defining leaf nodes

- Building the Euler cycle on tree.
- At each node v : v[A] = 0; v[B] = 0; v[C] = 1.
- Calculate the Suffix-Sum for nodes on the list generated from the Euler cycle.
- Leaf node has following characteristics: suffix-sum values at its child nodes are equal: v[A] = v[B] = v[C].
- From the picture, we have the leaves as follows:

  [H Q S M E J N T P G]


A = 0; B = 0; C = 1

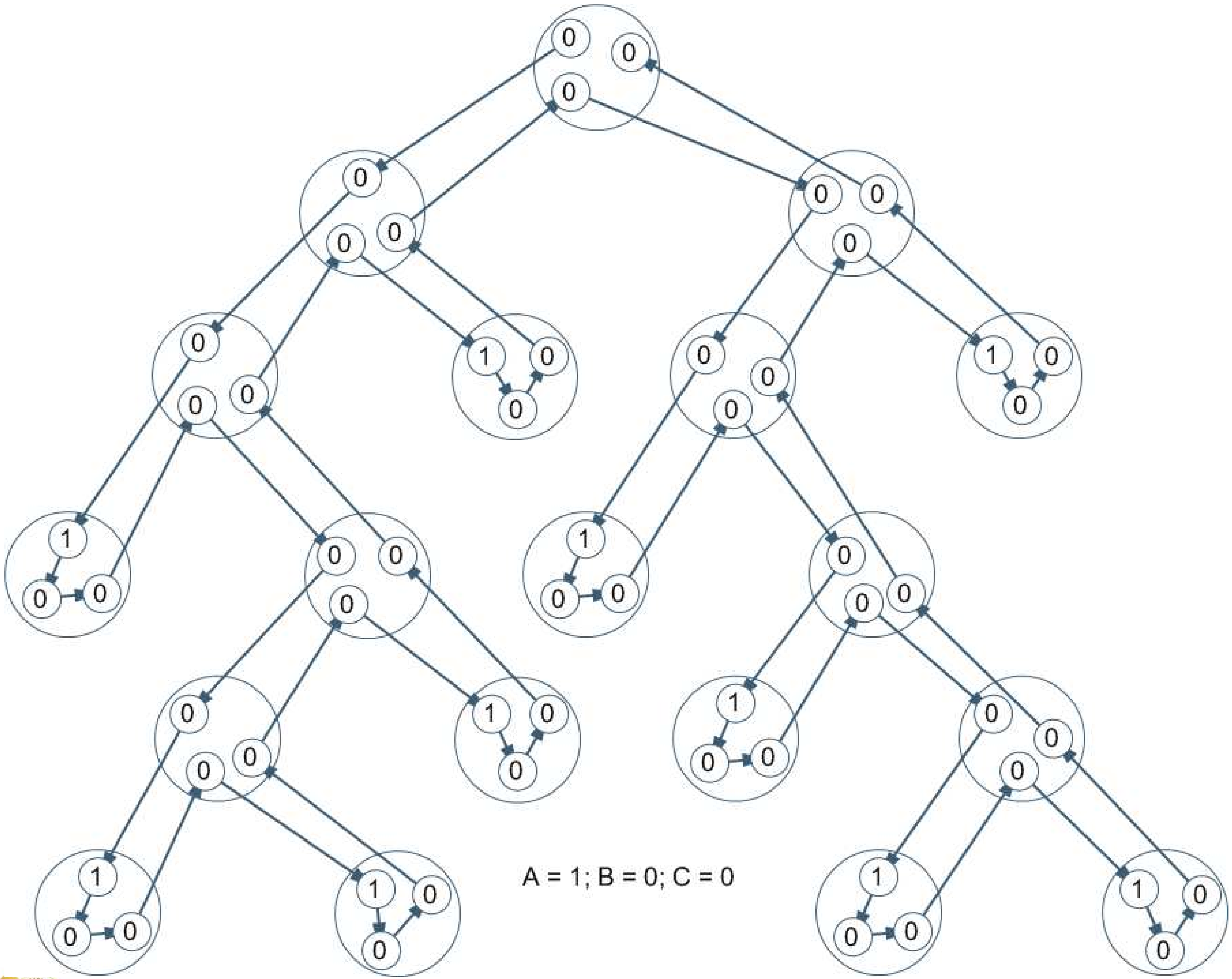


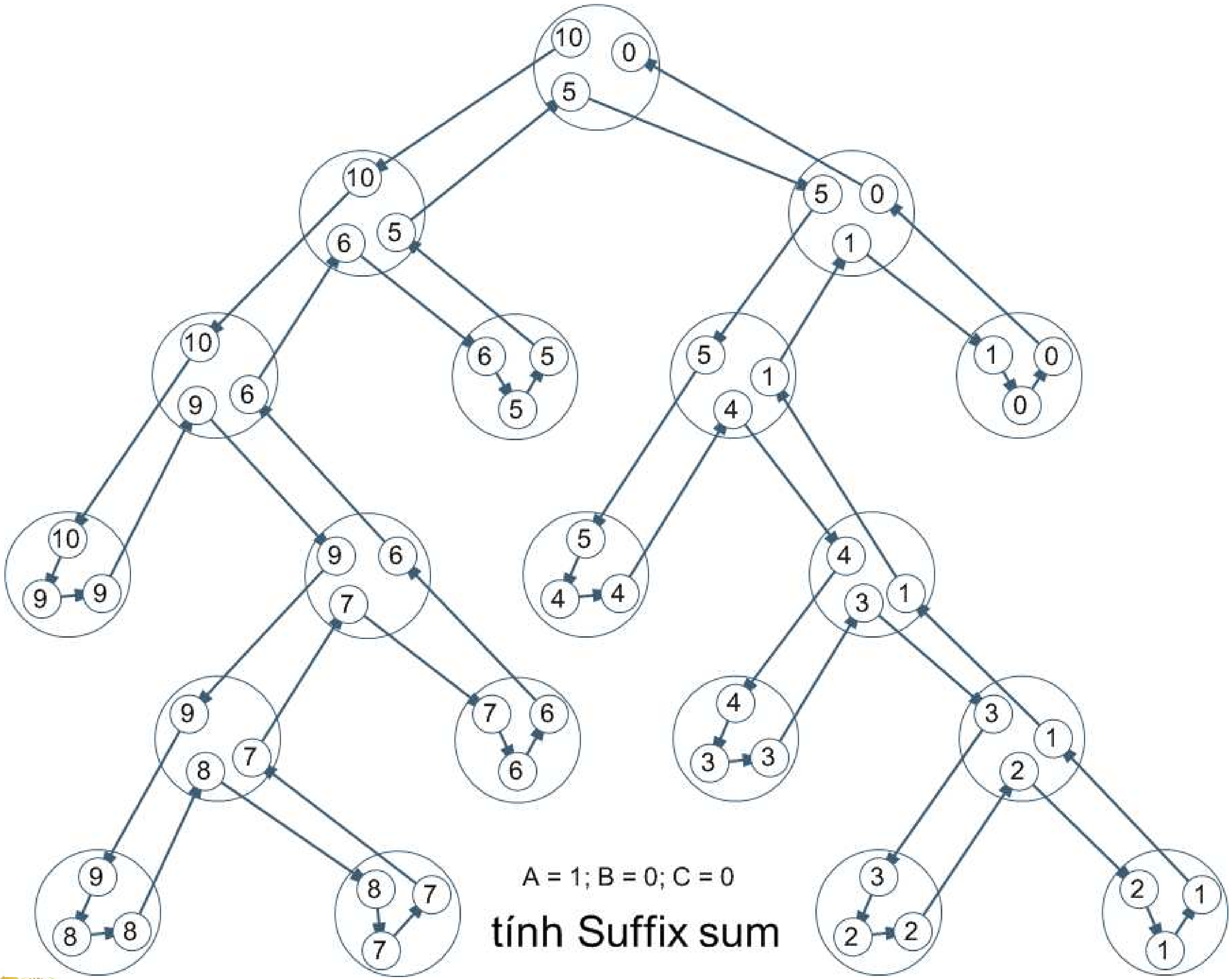
A = 0; B = 0; C = 1

tính Suffix sum

Set the order for the leaves

- Set A = 1, B = 0, C = 0.
- Calculate the Suffix-sum for nodes on the list generated from the Euler cycle.
- Order of the leaves sorted from right to left through value v[A]
- Node can be numbered left-to-right using formula: |number of leaves| - v[A] + 1.
- Store the leaves except for the leaf on the left end and for the leaf on the right end in array Z[1..n].

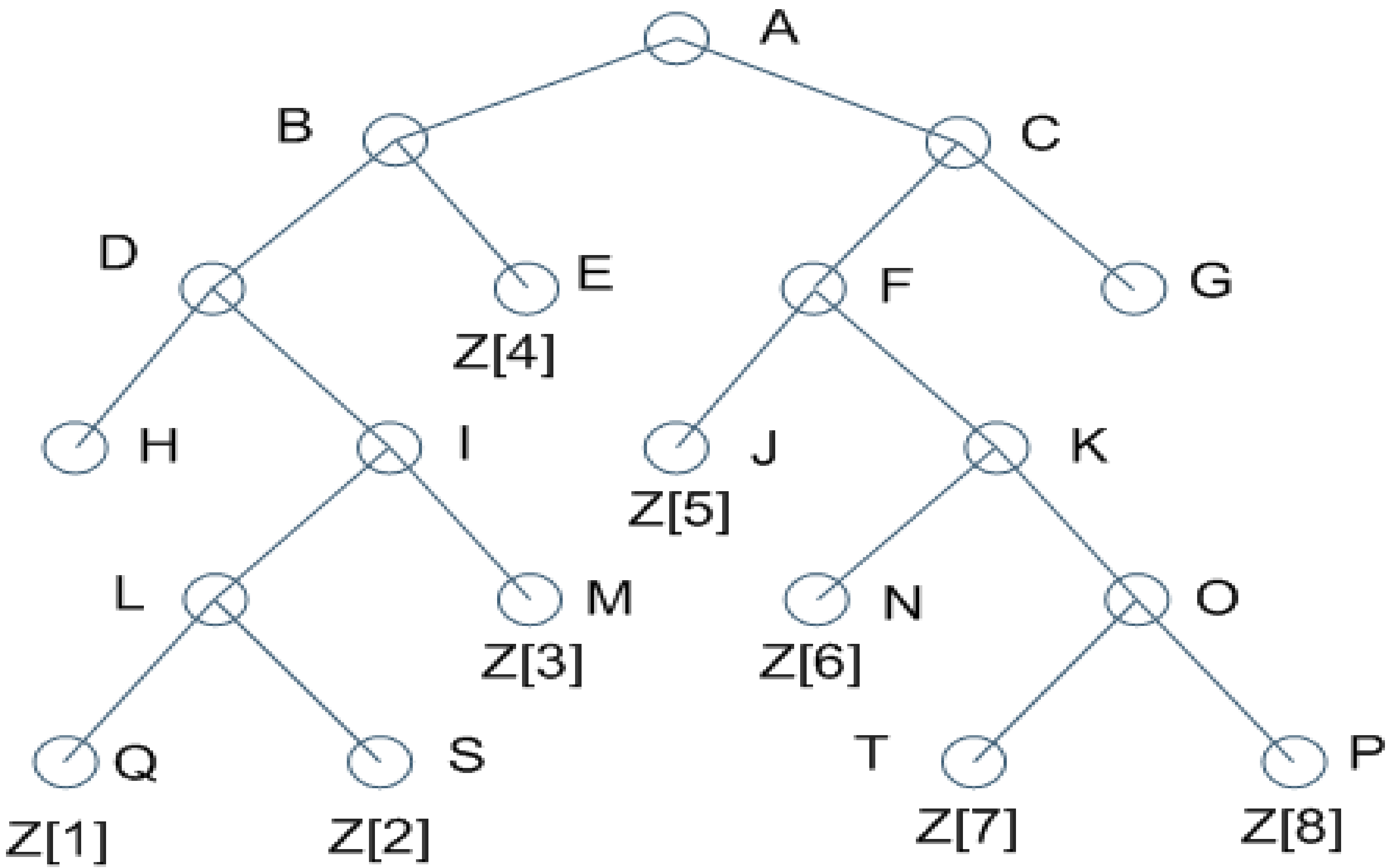

A = 1; B = 0; C = 0



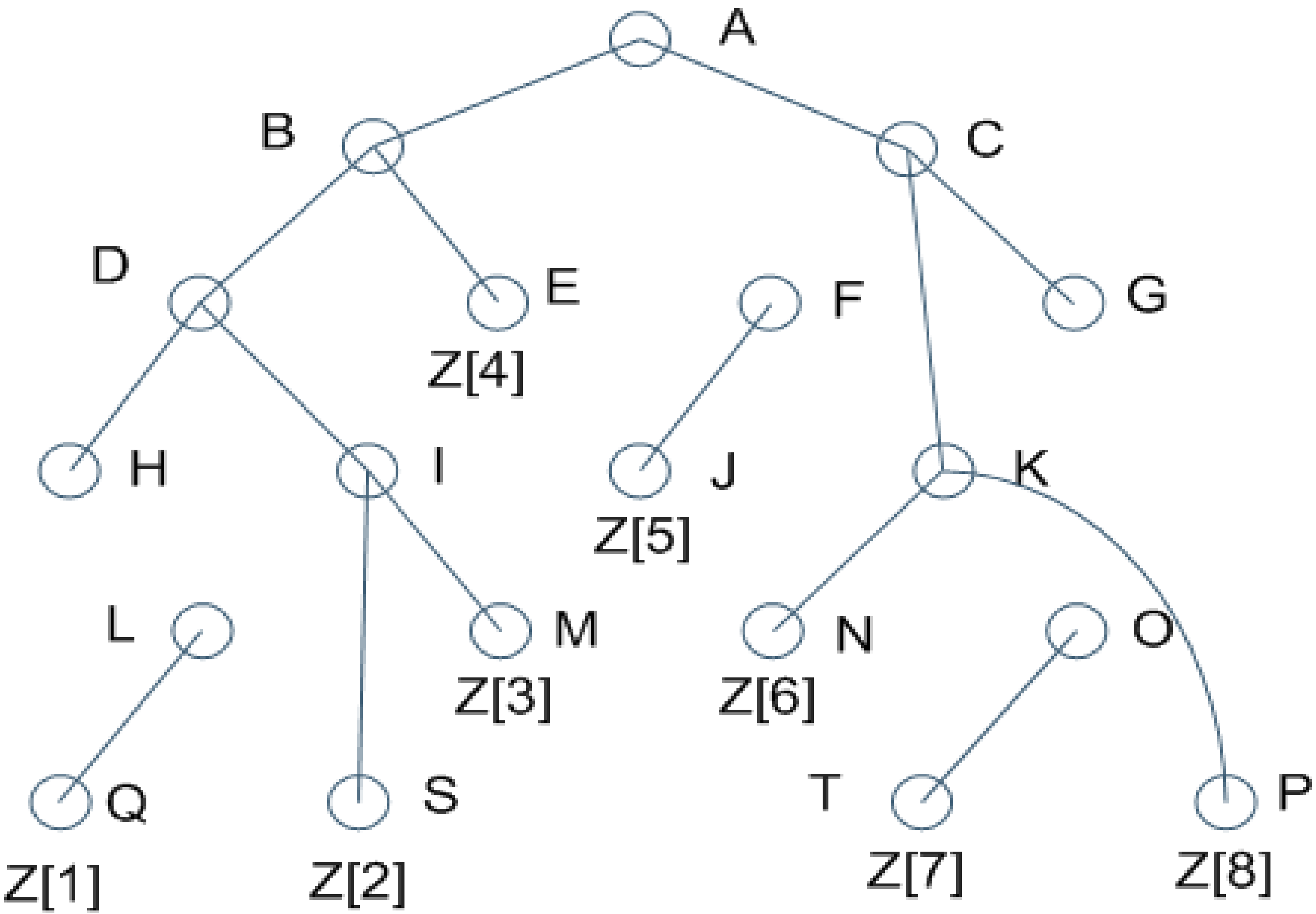
A = 1; B = 0; C = 0

tính Suffix sum

Assigning the order of leaves from left to right

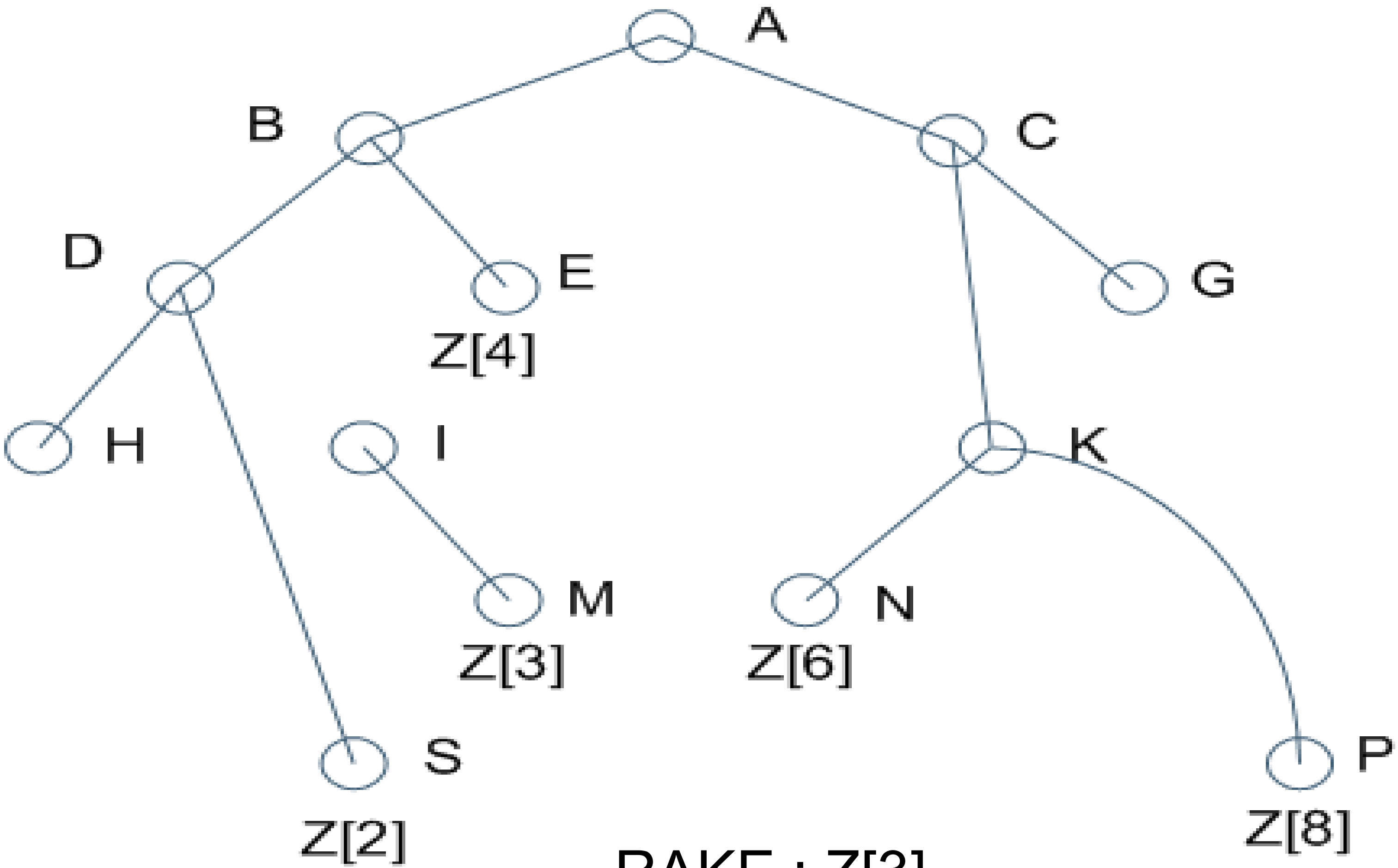


Tree Contraction (Step 1.1)



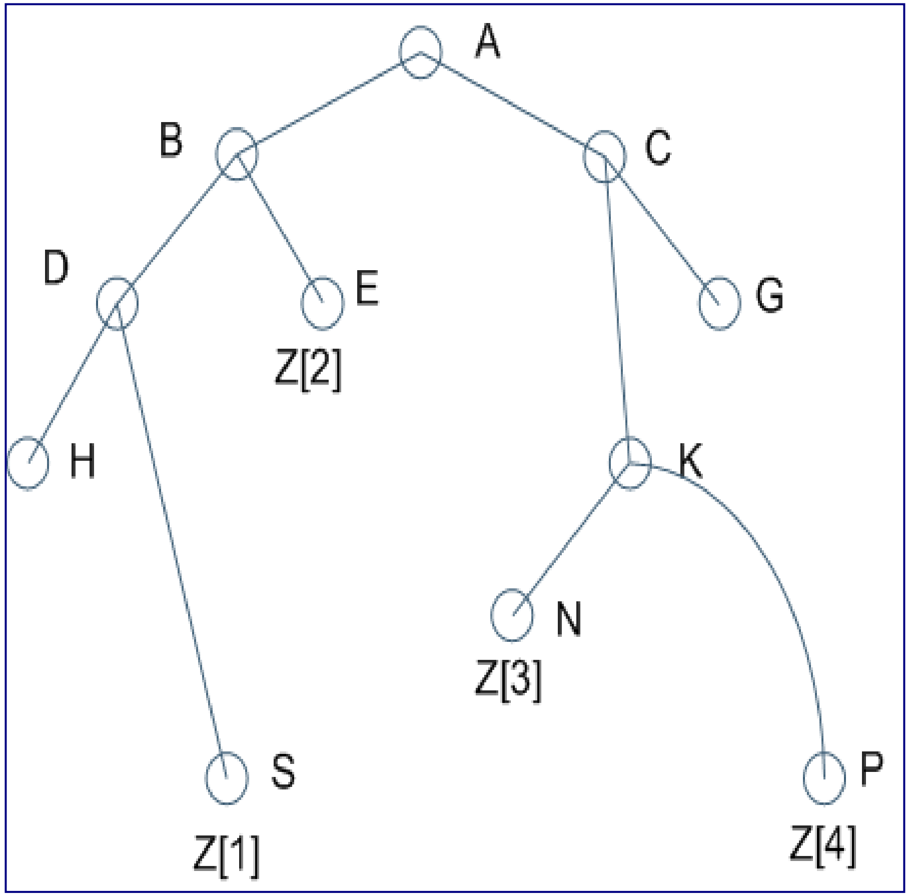
RAKE : Z[1], Z[5], Z[7]

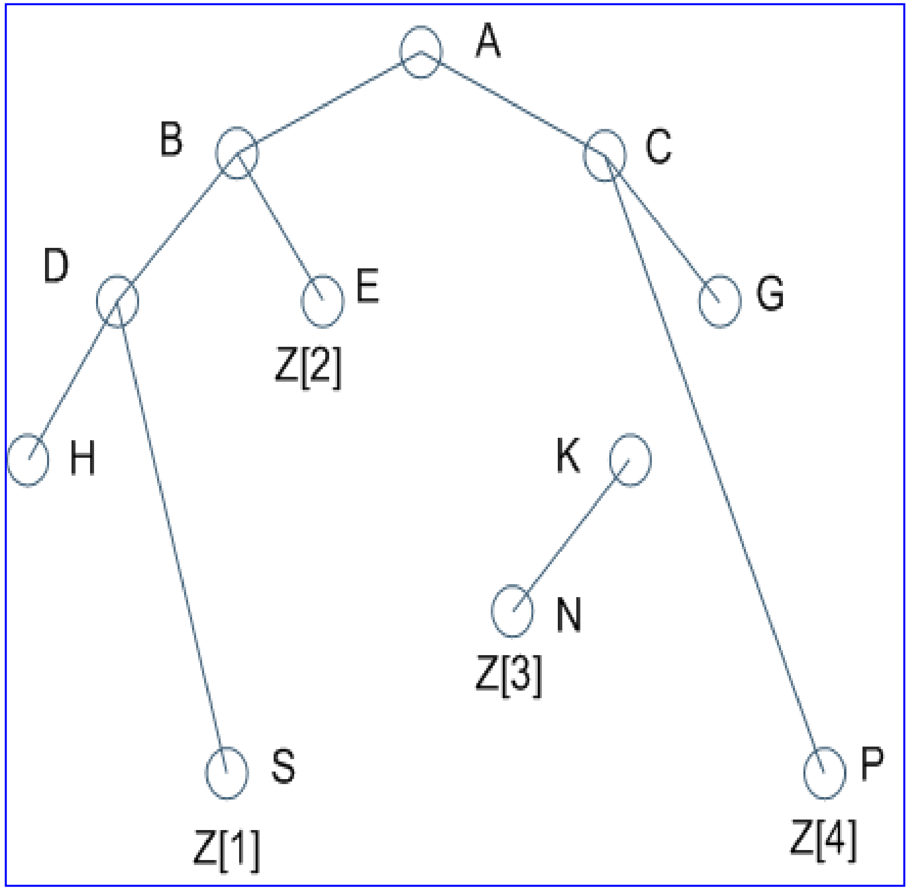
Tree Contraction (Step 1.2)



RAKE : Z[3]

Tree Contraction (Steps 1.3-2.1)

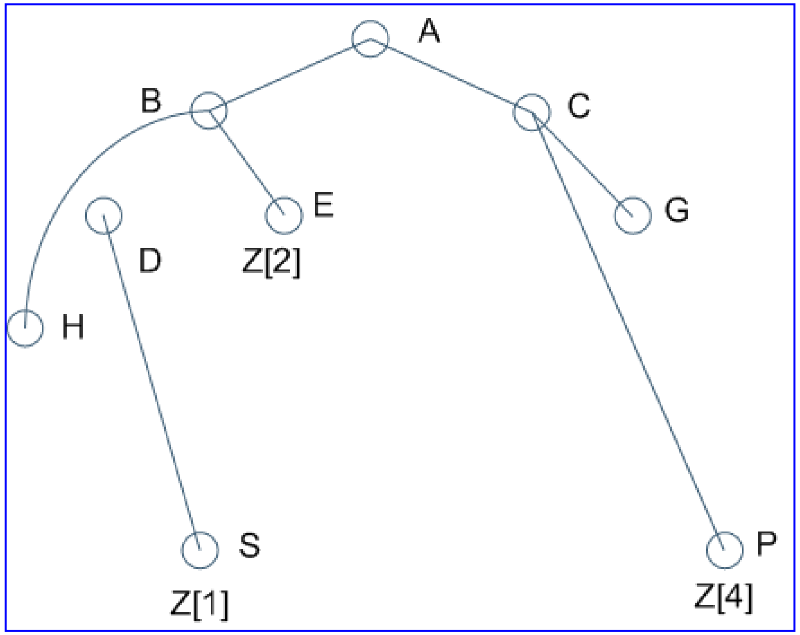




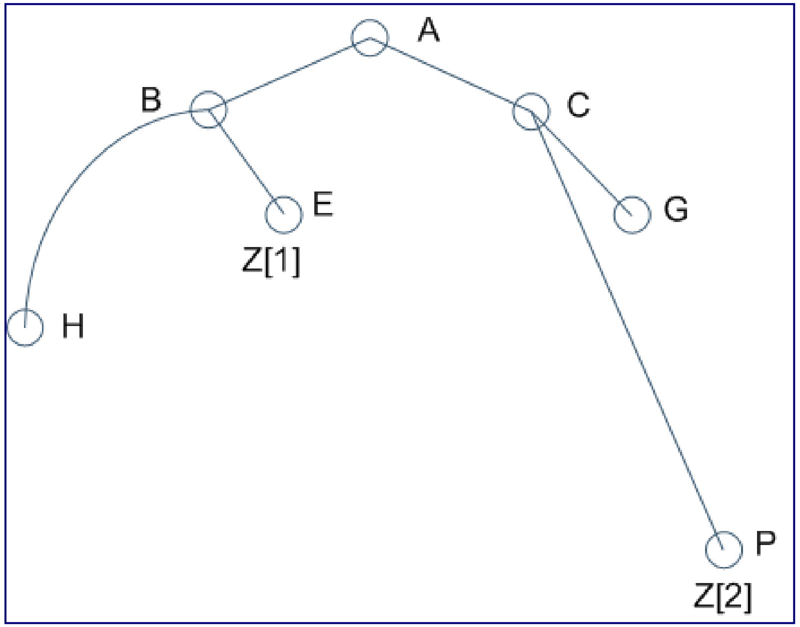
Z = Z[k] with k is even

RAKE: Z[3]

Tree Contraction (Steps 2.2-2.3)

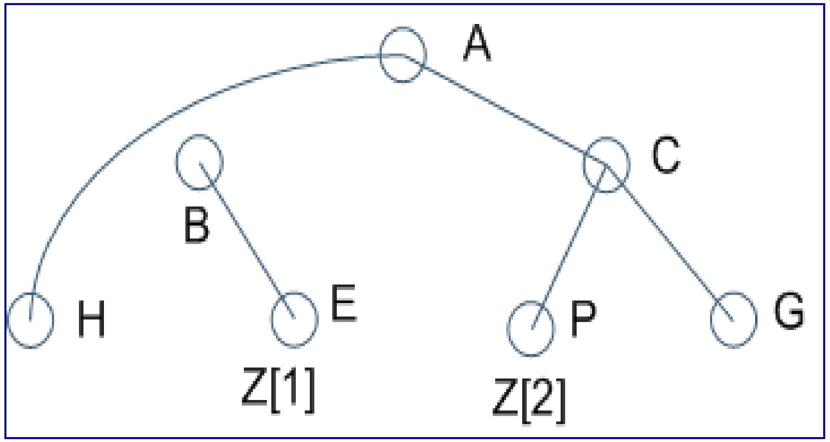


RAKE : Z[1]

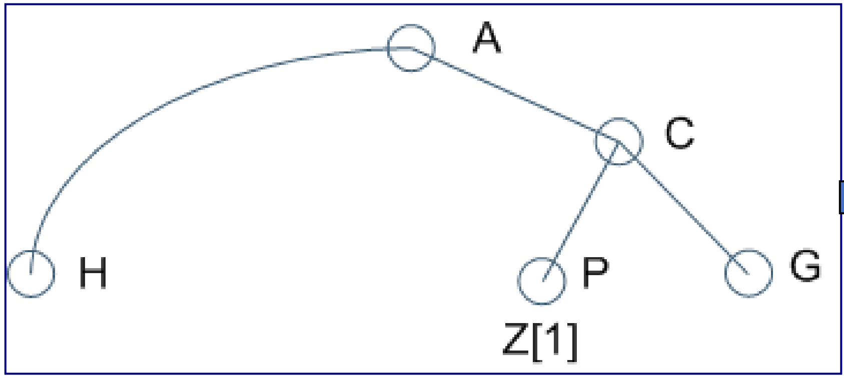


Z = Z[k] with k is even

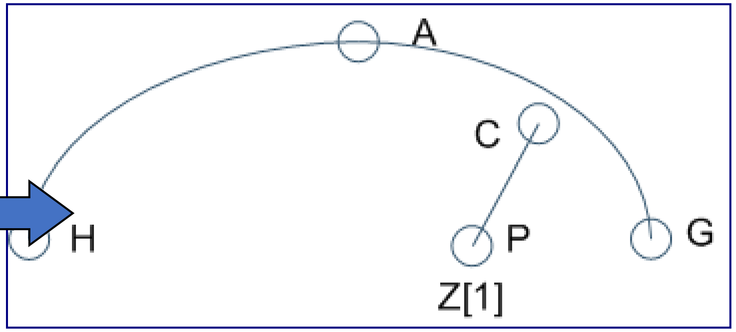
Tree Contraction (steps 3.1-3.2)



RAKE: Z[1]

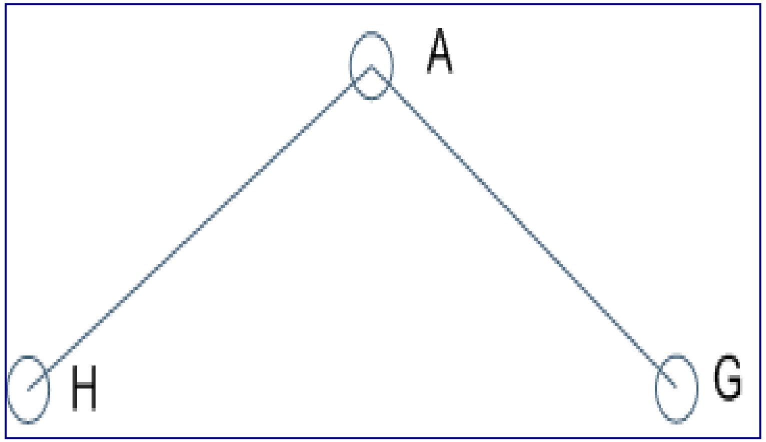


Assign: Z = Z[k] with even



RAKE : Z[1]

End



10.3 Write parallel tree-based programs

Write parallel tree-based programs

- Choose a tree-based algorithm
- Write a program implemented the chosen algorithm
- Run the program in a cluster consisting at least 2 connected linux-based computers.
- Evaluating the performance of the algorithm

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