Lecture 8: Classification

Classification: Definition

- Given a collection of records (training set).
  - Each record is characterized by a tuple (x, y) where x is the attribute set and y is the class label.
- Task: Learn a model that maps each attribute set x into one of the predefined class labels y.
- Example:

Task	Attribute set, $\boldsymbol{x}$	Class label, y
Categorizing email messages	Features extracted from email message header and content	spam or non-spam
Identifying tumor cells	Features extracted from x-rays or MRI scans	malignant or benign cells
Cataloging galaxies	Features extracted from telescope images	Elliptical, spiral, or irregular-shaped galaxies

General Approach for Building Classification Model

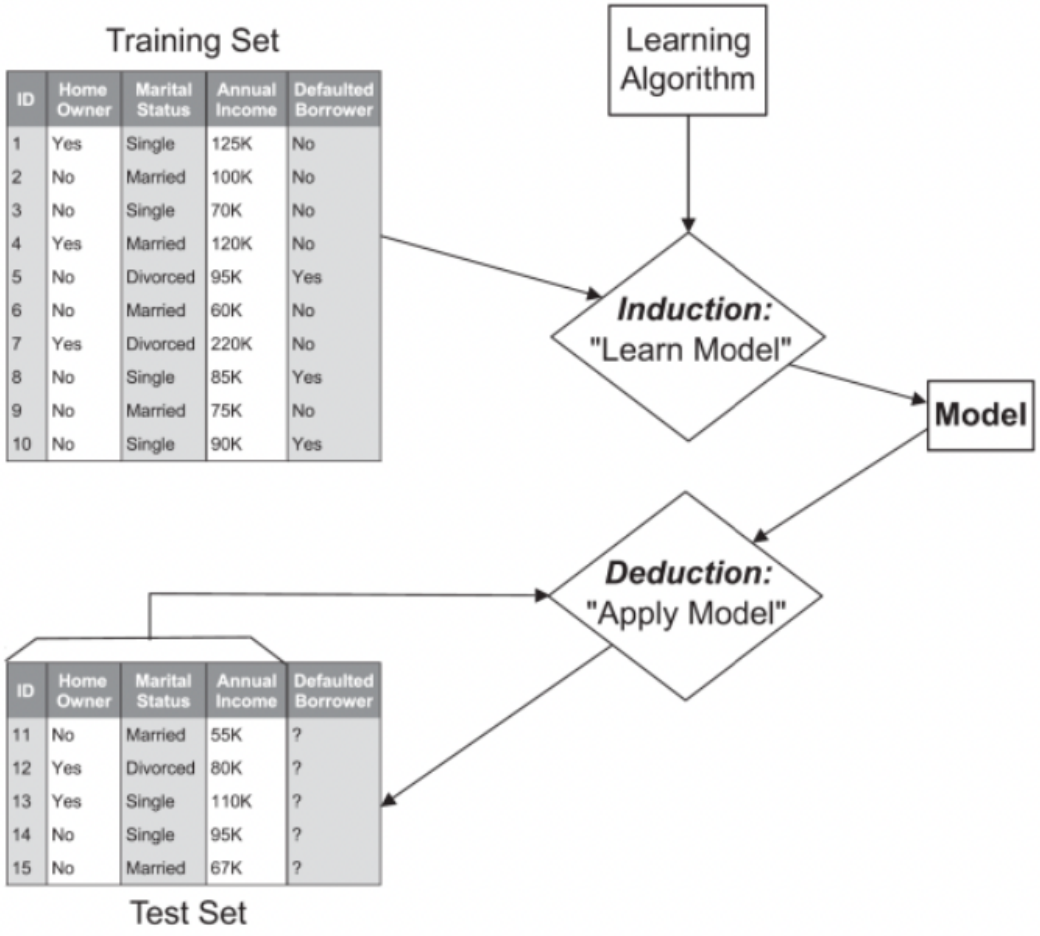


Figure 3.3. General framework for building a classification model.

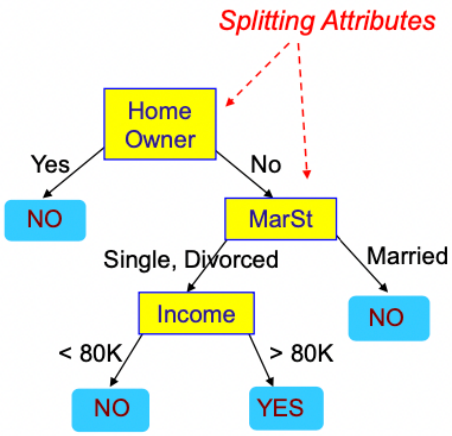
Classification Techniques

- Base Classifiers:
  - Decision Tree based Methods.
  - Rule-based Methods.
  - Nearest-neighbor.
  - Naïve Bayes and Bayesian Belief Networks.
  - Support Vector Machines.
- Ensemble Classifiers:
  - Booting.
  - Bagging.
  - Random Forests.

Example of a Decision Tree

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
	categorical	categorical	continuous	class
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

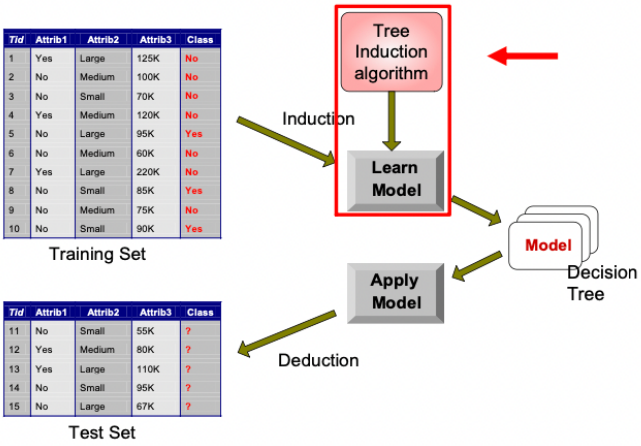
Training Data



Model: Decision Tree

- Apply Model to Test Data?
- There could be more than one tree that fits the same data.

Decision Tree Classification Task



Decision Tree Induction

- Many Algorithms:
  - Hunt's Algorithm (one of the earliest).
  - CART.
  - ID3, C4.5.
  - SLIQ, SPRINT.

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
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3	No	Single	70K	No
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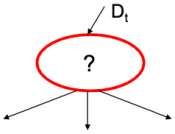
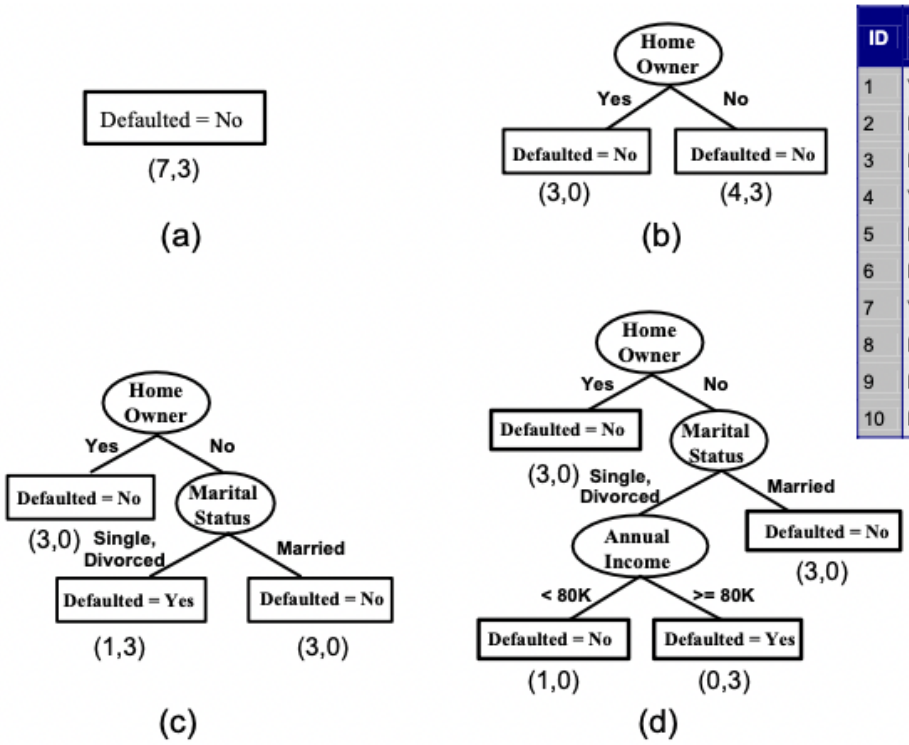


General Structure of Hunt's Algorithm

- Let D_t be the set of training records that reach a node t.
- General Procedure:
  - If D_t contains records that belong the same class y_t then t is a leaf node labeled as y_t.
  - If D_t contains records that belong to more than one class, use an attribute test to split the data into smaller subsets.
  - Recursively apply the procedure to each subset.

Hunt's Algorithm



ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
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6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Design Issues of Decision Tree Induction

- How should training records be split?
  - Method for expressing test condition.
    - Depending on attribute types.
  - Measure for evaluating the goodness of a test condition.
- How should the splitting procedure stop?
  - Stop splitting if all the records belong to the same class or have identical attribute values.
  - Early termination.

Methods for Expressing Test Conditions

- Depends on attribute types.
  - Binary.
  - Nominal.
  - Ordinal.
  - Continuous.

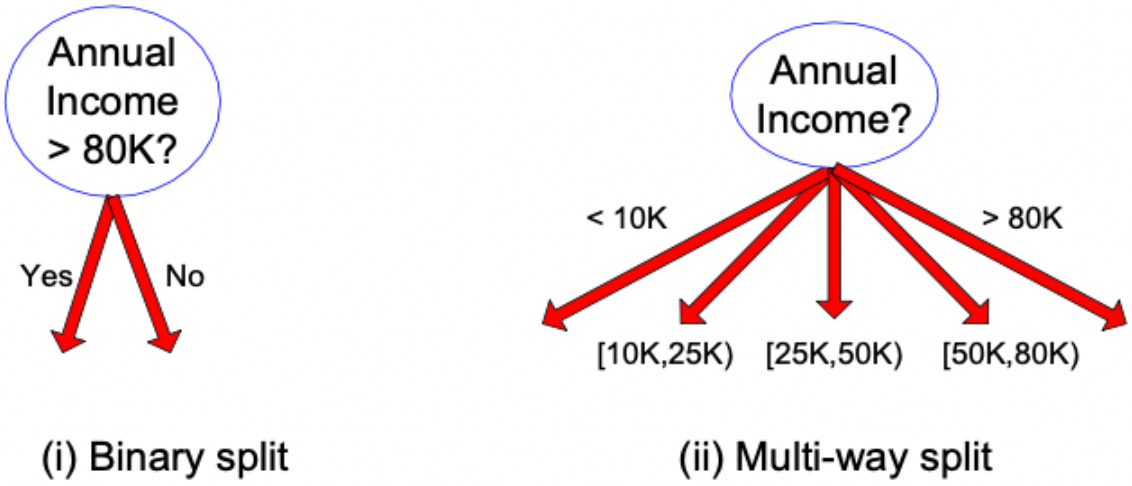
Test Condition for Nominal Attributes

- Multi-way split:
  - Use as many partitions as distinct values.
- Binary split:
  - Divides values into 2 subsets.

Test Condition for Ordinal Attributes

- Multi-way split:
  - Use as many partitions as distinct values.
- Binary split:
  - Divides values into 2 subsets.
  - Preserve order property among attribute values.

Test Condition for Continuous Attributes



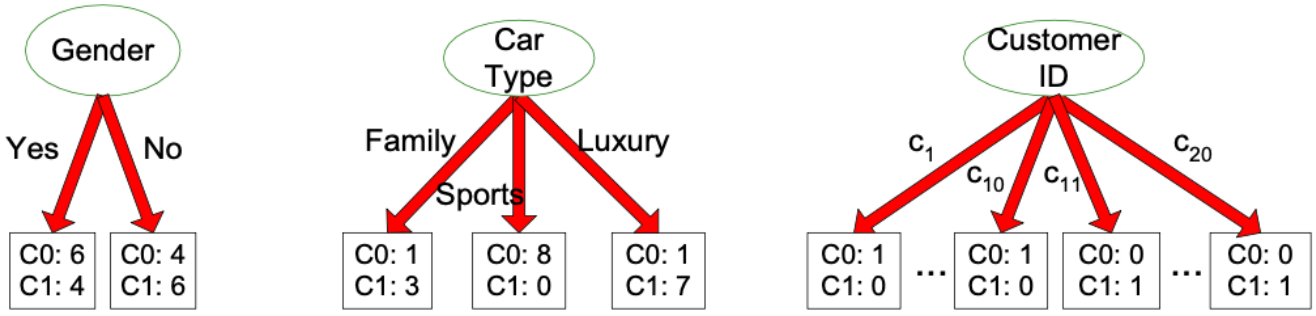
Splitting Based on Continuous – Attributes

- Different ways of handling:
  - Discretization to form an ordinal categorical attribute.
    - Ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
      - Statistic: discretize once at beginning.
      - Dynamic: repeat at each node.
  - Binary Decision: $(A < v)$ or $(A \geq v)$.
    - Consider all possible splits and finds the best cut.
    - Can be more compute intensive.

How to determine the Best Split

Customer Id	Gender	Car Type	Shirt Size	Class
1	M	Family	Small	C0
2	M	Sports	Medium	C0
3	M	Sports	Medium	C0
4	M	Sports	Large	C0
5	M	Sports	Extra Large	C0
6	M	Sports	Extra Large	C0
7	F	Sports	Small	C0
8	F	Sports	Small	C0
9	F	Sports	Medium	C0
10	F	Luxury	Large	C0
11	M	Family	Large	C1
12	M	Family	Extra Large	C1
13	M	Family	Medium	C1
14	M	Luxury	Extra Large	C1
15	F	Luxury	Small	C1
16	F	Luxury	Small	C1
17	F	Luxury	Medium	C1
18	F	Luxury	Medium	C1
19	F	Luxury	Medium	C1
20	F	Luxury	Large	C1

- Before Splitting: 10 records of class 0, 10 records class of 1.



- What test condition is the best?
- Greedy approach:
  - Nodes with purer class distribution are preferred.
- Need a measure of node impurity.

C0: 5
C1: 5

High degree of impurity

C0: 9
C1: 1

Low degree of impurity

Measures of Node Impurity

- Gini Index

$$Gini\ Index = 1 - \sum_{i=0}^{c-1} p_i(t)^2$$

Where $\boldsymbol{p_i(t)}$ is the frequency of class $\boldsymbol{i}$ at node t, and $\boldsymbol{c}$ is the total number of classes

- Entropy

$$Entropy = -\sum_{i=0}^{c-1} p_i(t) log_2 p_i(t)$$

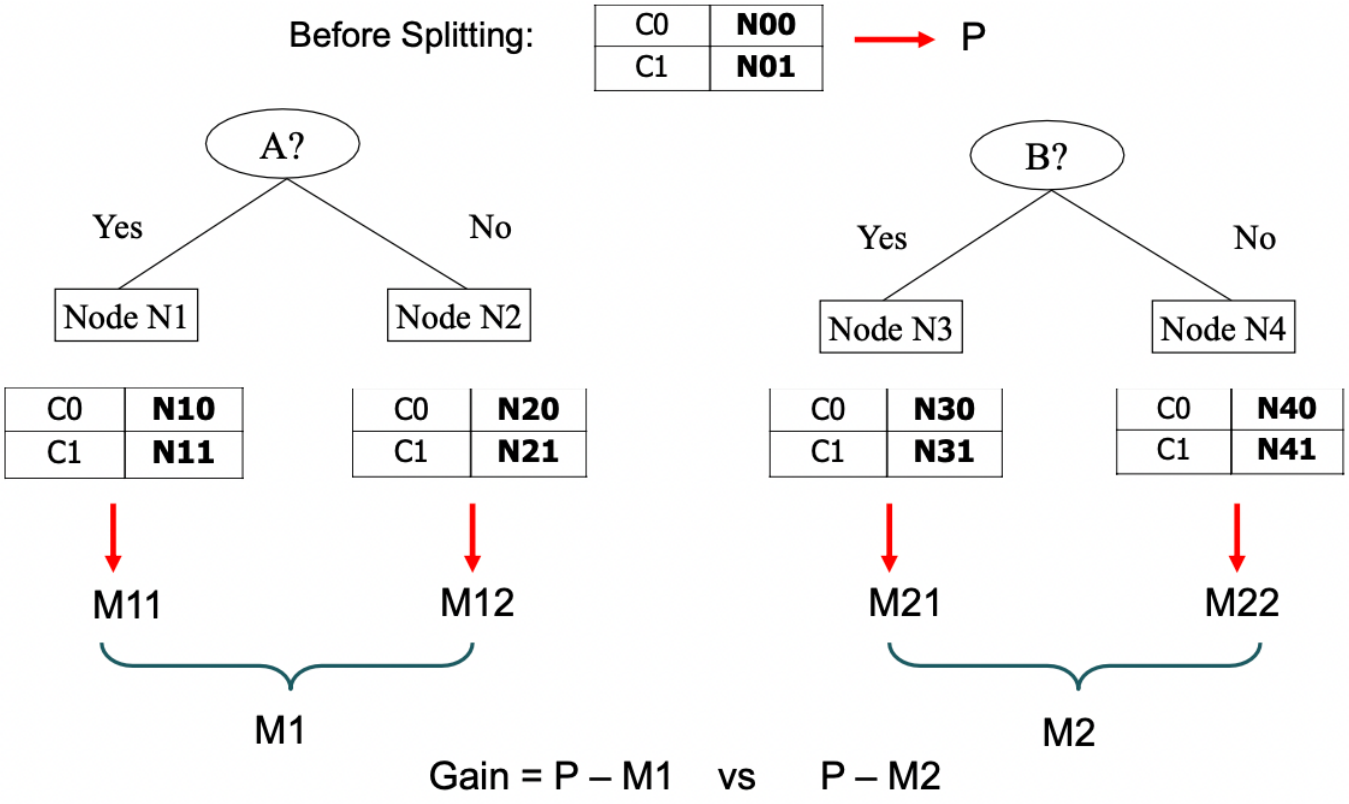
- Misclassification error

$$Classification\ error = 1 - \max[p_i(t)]$$

Finding the Best Split

1. Compute impurity measure (P) before splitting.
2. Compute impurity measure (M) after splitting.
   - Compute impurity measure of each child node.
   - M is the weighted impurity of child nodes.
3. Choose the attribute test condition that produces the highest gain.
   - Gain = P – M

   Or equivalently, lowest impurity measure after splitting (M).



Measure of Impurity

- Gini Index for a given node t

$$Gini\ Index = 1 - \sum_{i=0}^{c-1} p_i(t)^2$$

Where $p_i(t)$ is the frequency of class i at node t, and c is the total number of classes

- Maximum of 1-1/c when records are equally distributed among all classes, implying the least beneficial situation for classification.
- Minimum of 0 when all records belong to one class, implying the most beneficial situation for classification.
- Gini index is used in decision tree algorithms such as CART, SLIQ, SPRINT.
- For 2-class problem (p, 1-p):
  - GINI = 1 - p^2 - $(1-p)^2$ = 2p(1-p)

C1	0
C2	6
Gini=0.000	

C1	1
C2	5
Gini=0.278	

C1	2
C2	4
Gini=0.444	

C1	3
C2	3
Gini=0.500	

Computing GINI Index of a Single Node

$$Gini\ Index = 1 - \sum_{i=0}^{c-1} p_i(t)^2$$

C1	0
C2	6

P(C1) = 0/6 = 0 P(C2) = 6/6 = 1

Gini = 1 – $P(C1)^2$ – $P(C2)^2$ = 1 – 0 – 1 = 0

C1	1
C2	5

P(C1) = 1/6 P(C2) = 5/6

Gini = 1 – $(1/6)^2$ – $(5/6)^2$ = 0.278

C1	2
C2	4

P(C1) = 2/6 P(C2) = 4/6

Gini = 1 – $(2/6)^2$ – $(4/6)^2$ = 0.444

Computing GINI Index for a Collection of Nodes

When a node p is split into k partitions (children)

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

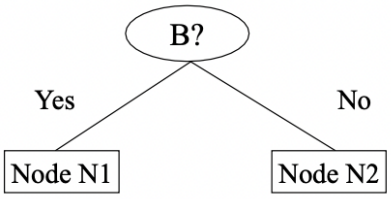
where, n_i = number of records at child i,

n = number of records at parent node p.

Binary Attributes: Computing GINI Index

- Splits into 2 partitions (child nodes).
- Effect of Weight partitions:
  - Larger and purer partitions are sought.
- Gini(N1) = $1 - (5/6)^2 - (1/6)^2 = 0.278$
- Gini(N2) = $1 - (2/6)^2 - (4/6)^2 = 0.444$

	Parent
C1	7
C2	5
Gini = 0.486	



	N1	N2
C1	5	2
C2	1	4
Gini=0.361		

Weighted Gini of N1 N2
= 6/12 * 0.278 +
6/12 * 0.444
= 0.361

Gain = 0.486 – 0.361 = 0.125

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset.
- Use the count matrix to make decisions.

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	8	1
C2	3	0	7
Gini	0.163		

Two-way split
(find best partition of values)

	CarType	
	{Sports, Luxury}	{Family}
C1	9	1
C2	7	3
Gini	0.468	

	CarType	
	{Sports}	{Family, Luxury}
C1	8	2
C2	0	10
Gini	0.167	

- Which of these is the best?

Continuous Attributes: Computing Gini Index

- Use Binary Decision based on one value.
- Several Choices for the splitting value.
  - Number of possible splitting values = Number of distinct values.
- Each Splitting value has a count matrix associate with it.
  - Class counts in each of the partitions, A ≤ v and A > v.
- Simple method to choose the best v:
  - For each v, scan the database to gather count matrix and compute its Gini index.
  - Computationally inefficient! Repetition of work.
- For efficient computation: For each attribute,
  - Sort the attribute on values.
  - Linearly scan these values, each time updating the count matrix and computing GINI index.
  - Choose the split position that has the least GINI index.

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Cheat	No		No		No		Yes		Yes		Yes		No		No		No		No	
Annual Income																				
Sorted Values →	60		70		75		85		90		95		100		120		125		220	
Split Positions →	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		***0.300***		0.343		0.375		0.400		0.420	

Measure of Impurity: Entropy

- At each node t:

$$Entropy = -\sum_{i=0}^{c-1} p_i(t) log_2 p_i(t)$$

- Where $p_i(t)$ is the frequency of class i at node t, and c is total number of classes.
  - Maximum of $log_2 c$ when records are equally distributed among all classes, implying the least beneficial situation for classification.
  - Minimum of 0 when all records belong to one class, implying most beneficial situation of classification.
- Entropy based computations are quite like the GINI index computations.

Computing Entropy of a Single Node

$$Entropy = -\sum_{i=0}^{c-1} p_i(t) log_2 p_i(t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0 P(C2) = 6/6 = 1

Entropy = – 0 log 0 – 1 log 1 = – 0 – 0 = 0

C1	**1**
C2	**5**

P(C1) = 1/6 P(C2) = 5/6

Entropy = – (1/6) $\log_2$ (1/6) – (5/6) $\log_2$ (1/6) = 0.65

C1	**2**
C2	**4**

P(C1) = 2/6 P(C2) = 4/6

Entropy = – (2/6) $\log_2$ (2/6) – (4/6) $\log_2$ (4/6) = 0.92

Computing Information Gain After Splitting

- Information Gain:

$$Gain_{split} = Entropy(p) - \sum_{i=1}^{k} \frac{n_i}{n} Entropy(i)$$

  - Parent Node, p is split into k partitions (children).
  - n_i is number of records in child node i.
- Choose the split that achieve most reduction (maximizes GAIN).
- Used in the ID3 and C4.5 decision tree algorithm.
- Information gain is the mutual information between the class variable and the splitting variable.

Problem with large number of partitions

- Node impurity measures tend to prefer splits that result in large number of partitions, each being small but pure.
- Customer ID has highest information gain because entropy for all the children is zero.

Gain Ratio

$$Gain\ Ratio = \frac{Gain_{split}}{Split\ Info} \qquad Split\ Info = -\sum_{i=1}^{k} \frac{n_i}{n} log_2 \frac{n_i}{n}$$

  - Parent Node, p is split into k partitions (children).
  - n_i is the number of records in child node i.
- Adjusts Information Gain by the entropy of the partitioning (Split Info).

- Higher entropy partitioning (large number of small partitions) is penalized.
- Used in C4.5 algorithm.
- Designed to overcome the disadvantage of Information Gain.

	CarType		
	Family	Sports	Luxury
C1	1	8	1
C2	3	0	7
Gini	0.163		

SplitINFO = 1.52

	CarType	
	{Sports, Luxury}	{Family}
C1	9	1
C2	7	3
Gini	0.468	

SplitINFO = 0.72

	CarType	
	{Sports}	{Family, Luxury}
C1	8	2
C2	0	10
Gini	0.167	

SplitINFO = 0.97

Measure of Impurity: Classification Error

- Classification error at a node t:

$$Error(t) = 1 - \max_i [p_i(t)]$$

- Maximum of 1 – 1/c when records are equally distributed among all classes, implying the least interesting situation.
- Minimum of 0 when all records belong to one class, implying the most interesting situation.

Computing Error of a Single Node

$$Error(t) = 1 - \max_i [p_i(t)]$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0 P(C2) = 6/6 = 1

Error = 1 – max (0, 1) = 1 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6 P(C2) = 5/6

Error = 1 – max (1/6, 5/6) = 1 – 5/6 = 1/6

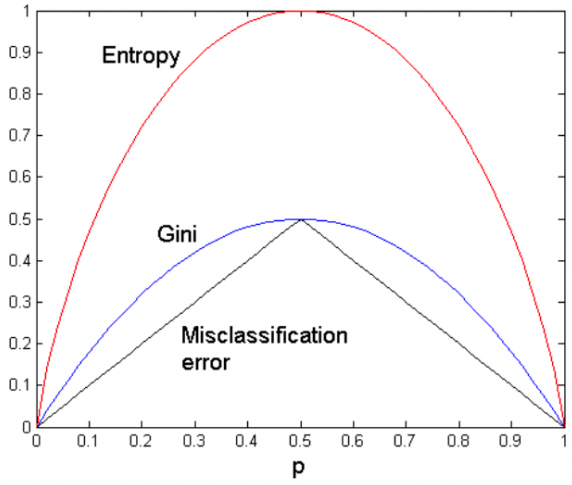
C1	**2**
C2	**4**

P(C1) = 2/6 P(C2) = 4/6

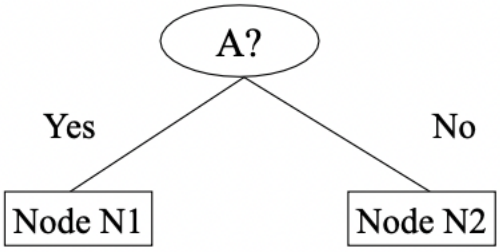
Error = 1 – max (2/6, 4/6) = 1 – 4/6 = 1/3

Comparison among Impurity Measures

For a 2-class problem:



Misclassification Error vs. Gini Index



	Parent
C1	7
C2	3
Gini = 0.42	

	N1	N2
C1	3	4
C2	0	3
Gini=0.342		

- Gini improves but error maintains the same.

	N1	N2
C1	3	4
C2	0	3
Gini=0.342		

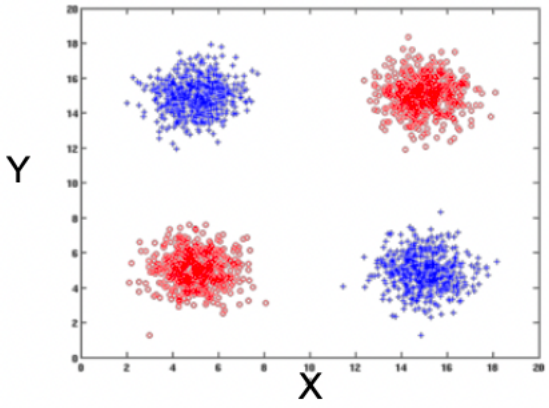
	N1	N2
C1	3	4
C2	1	2
Gini=0.416		

- Misclassification error for all three cases = 0.3!

Decision Tree Based Classification

- Advantages:
  - Relatively inexpensive to construct.
  - Extremely fast at classifying unknown records.
  - Easy to interpret for small-sized trees.
  - Robust to noise (especially when methods to avoid overfitting are employed).
  - Can easily handle redundant attributes.
  - Can easily handle irrelevant attributes (unless the attributes are interacting).
- Disadvantages:
  - Due to the greedy nature of splitting criterion, interacting attributes (that can distinguish between classes together but not individually) may be passed over in favor of other attributed that are less discriminating.
  - Each decision boundary involves only a single attribute.

Handling Interactions

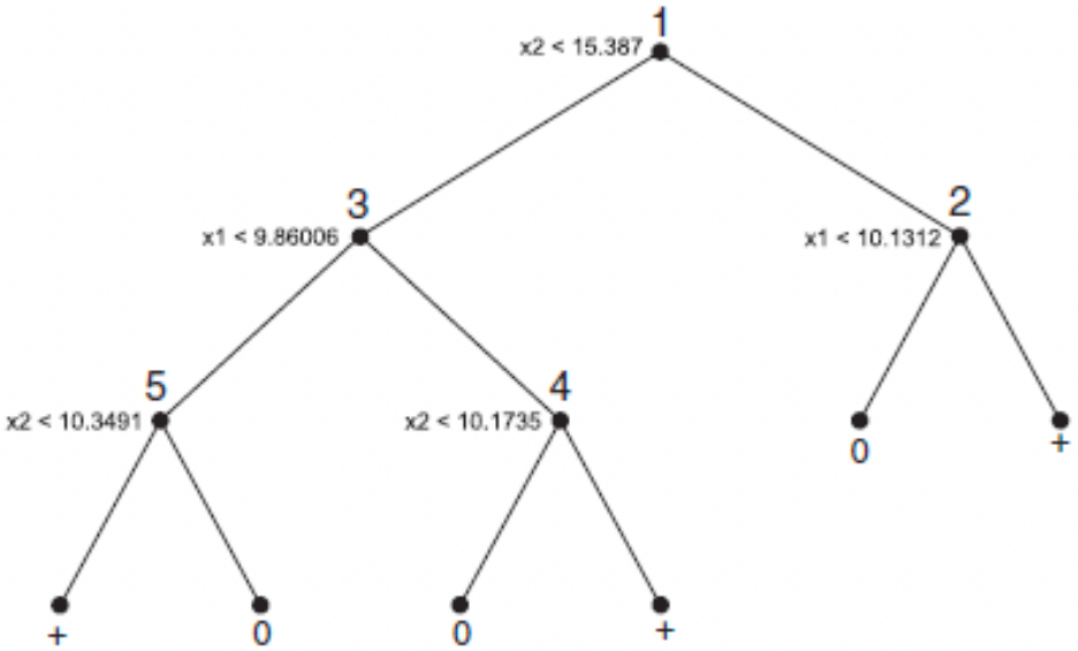


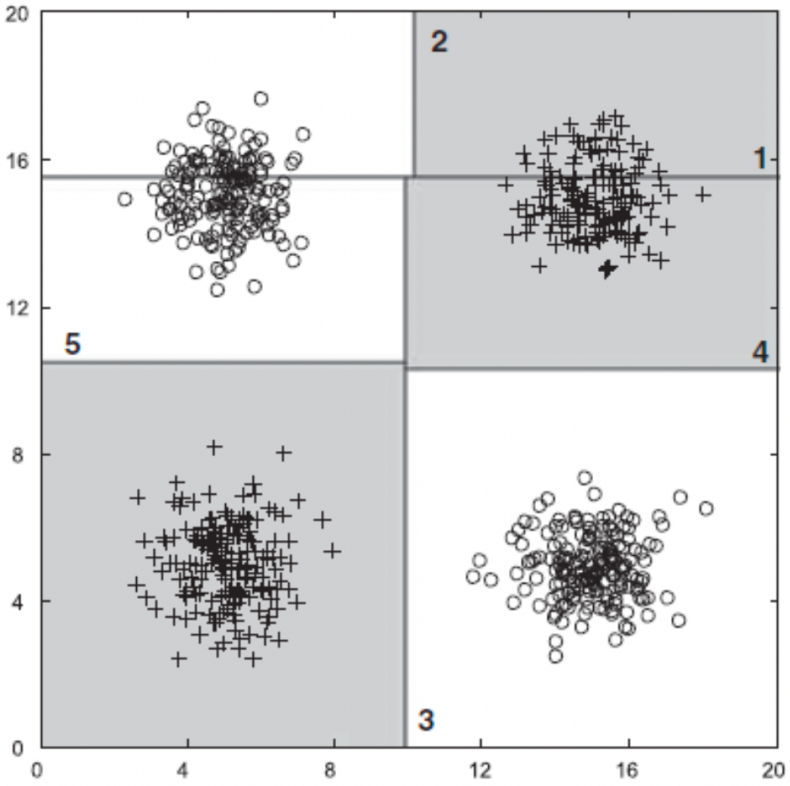
+ : 1000 instances

o : 1000 instances

Entropy (X) : 0.99
Entropy (Y) : 0.99

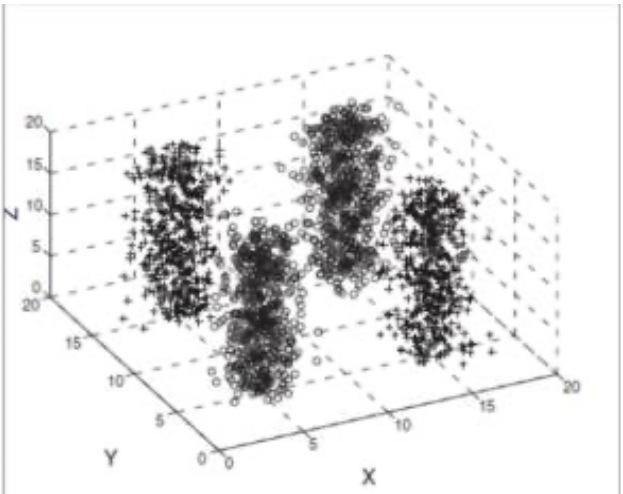
Handling Interactions – 6 leaf nodes





Handling Interactions Given Irrelevant Attributes

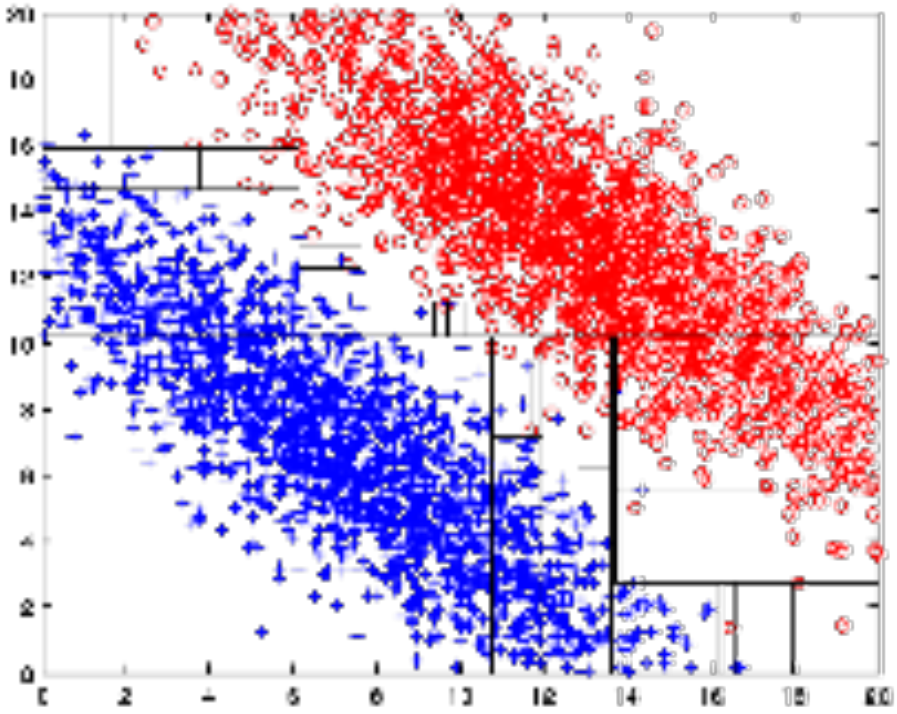
- Adding Z as a noisy attribute generated from a uniform distribution.
- Attribute Z will be chosen for splitting.



(a) Three-dimensional data with attributes X, Y, and Z.

Entropy (X) : 0.99
Entropy (Y) : 0.99
Entropy (Z) : 0.98

- Both positive (+) and negative (o) classes generated from skewed Gaussians with centers at (8, 8) and (12, 12) respectively.



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