E1

1.1 $$F(x) = \begin{cases} 0, & x \leq -1 \\ \int_{-1}^{x} \frac{x^2}{3}\,dx = \frac{x^3}{9} + \frac{1}{9}, & -1 < x < 2 \\ 1, & x \geq 2 \end{cases}$$

1.2

$$P\{0 < X \leq 1\} = F(1) - F(0) = \frac{1}{9}$$

$$P\{0 < X \leq 1\} = \int_0^1 f(x)\,dx = \int_0^1 \frac{x^2}{3}\,dx = \frac{1}{9}$$

E3

3.1 Let A, B, C be the events that the product was made by line 1, 2, 3 respectively

Let S be the event that the product is ~~diffe~~ defective

$$P(S) = P(S|A)P(A) + P(S|B)P(B) + P(S|C)P(C)$$
$$= 0.02 \times 0.30 + 0.03 \times 0.45 + 0.02 \times 0.25$$
$$= 0.0245$$

3.2 $$P(A|S) = \frac{P(S|A)P(A)}{P(S)} = 0.24$$

$$P(B|S) = \frac{P(S|B)P(B)}{P(S)} = 0.55$$

$$P(C|S) = \frac{P(S|C)P(C)}{P(S)} = 0.21$$

E4 $$F(x) = (1 - e^{-\alpha x})\,U(x - c)$$

$$= \begin{cases} 0, & x \leq c \\ 1 - e^{-\alpha x}, & x > c \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 0, & x \leq c \\ \alpha e^{-\alpha x}, & x > c \end{cases}$$

E5

$\bar{x} = \frac{7}{3}$ $\bar{y} = \frac{11}{6}$ $\bar{z} = \frac{7}{2}$

$\Rightarrow$ The covariance matrix :
$$\begin{pmatrix} \frac{11}{9} & \frac{5}{9} & -\frac{1}{3} \\ \frac{5}{9} & \frac{29}{36} & -\frac{7}{12} \\ -\frac{1}{3} & -\frac{7}{12} & \frac{11}{12} \end{pmatrix}$$

E6

a,
$$P_k = P(\eta - k\sigma < x < \eta + k\sigma)$$
$$= P(-k < z < k)$$
$$= P(z < k) - P(z < -k)$$
$$= 1 - 2P(z < -k)$$

$$P_1 = 1 - 2 \times 0.15866 = 0.68268$$
$$P_2 = 1 - 2 \times 0.02275 = 0.9545$$
$$P_3 = 1 - 2 \times 0.00135 = 0.9976$$

b, $P_k = 0.9 \Rightarrow P(z < -k) = 0.05$
$\Rightarrow k = 1.65$

$P_k = 0.99 \Rightarrow P(z < -k) = 0.005$
$k = 2.57$

$P_k = 0.999 \Rightarrow P(z < -k) = 0.0005$
$k = 3.29$

c, $P\{\eta - z_u\sigma < x < \eta + z_u\sigma\} = \gamma$

$\Leftrightarrow 1 - 2P(z < -z_u) = \gamma$

$\Leftrightarrow P(z < -z_u) = \frac{1-\gamma}{2}$

E7

+) $\gamma = 0.95 \Rightarrow \delta = 0.05 \Rightarrow \frac{\delta}{2} = 0.025 \Rightarrow z_{1-\frac{\delta}{2}} = 1.96$

$\Rightarrow$ The 95% confidence interval:

$$\bar{x} - z_{1-\delta/2}\frac{\sigma_x}{\sqrt{n}} < \mu_x < \bar{x} + z_{1-\frac{\delta}{2}}\frac{\sigma_x}{\sqrt{n}}$$

$$2.6 - 1.96\frac{2.6}{\sqrt{36}} < \mu_x < 2.6 + 1.96\frac{2.6}{\sqrt{36}}$$

$$1.75 < \mu_x < 3.45$$

+) $\gamma = 0.99 \Rightarrow \delta = 0.01 \Rightarrow \frac{\delta}{2} = 0.005 \Rightarrow z_{1-\frac{\delta}{2}} = 2.575$

The 99% confidence interval

$$2.6 - 2.575\frac{2.6}{\sqrt{36}} < \mu_x < 2.6 + 2575\frac{2.6}{\sqrt{36}}$$

$$1.48 < \mu_x < 3.72$$

E8 The null hypothesis

$H_0: \mu = 20000$

The alternative hypothesis

$H_1: \mu > 20000$

$\bar{X} = 23500, \sigma = 3900$

$$z = \frac{23500 - 20000}{\frac{3900}{\sqrt{100}}} = 8.97$$

$$P(z > 8.97) = 1 - P(z < 8.97) \simeq 0$$

$\Rightarrow$ We reject the null hypothesis

$\Rightarrow$ Automobile is driven on the average more than 20000km per year

E9

The second momentum of r.v y:

$$E[y^2] = E[x^2(8) + x^2(5) - 2x(8)x(5)]$$
$$= E[x^2(8)] + E[x^2(5)] - 2E[x(8)x(5)]$$
$$= R(0) + R(0) - 2R(3)$$
$$= 2A - 2Ae^{-3\alpha}$$

E10

$$R_{xy}(t_1, t_2) = h(t_2) * R_{xx}(t_1, t_2)$$
$$= \int_{-\infty}^{\infty} h(\alpha) R_{xx}(t_1, t_2 - \alpha)\, d\alpha$$
$$= \int_{-\infty}^{\infty} e^{-c\alpha} u(\alpha) A\delta(T - \alpha)\, d\alpha$$
$$= Ae^{-\alpha c T} u(T)$$

$$R_{yy}(t_1, t_2) = h(t_1) * R_{xy}(t_1, t_2)$$
$$= \int_{-\infty}^{\infty} h(\alpha) R_{xy}(t_1 - \alpha, t_2)\, d\alpha$$
$$= \int_{-\infty}^{\infty} e^{-c\alpha} u(\alpha) A e^{-c(\tau + \alpha)} u(\tau + \alpha)\, d\alpha$$

$$t > 0 \Rightarrow \int_0^{\infty} e^{-c\alpha} A e^{-c(T + \alpha)}\, d\alpha$$
$$= A \int_0^{\infty} e^{-cT - 2c\alpha}\, d\alpha$$
$$= \frac{Ae^{-cT}}{2c} = y(T)$$