Code : 103202

B.Tech 2nd Semester Exam., 2021

(New Course)

MATHEMATICS—II

(Linear Algebra, Transform Calculus and Numerical Methods)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are **NINE** *questions in this paper.*

(iii) Attempt **FIVE** *questions in all.*

(iv) Question No. **1** *is compulsory.*

1. Answer any *seven* of the following : 2×7=14

(a) The eigenvalues of a matrix A are 2, 3, 1, then find the eigenvalues of $A^{-1} + A^2$.

(b) If A and B are symmetric matrices, then prove that $AB - BA$ is a skew-symmetric matrix.

$(AB-BA)^T = AB - BA$

$(AB)^T - (BA)^T$

(c) Prove that the matrix

$A^{-1} = A^T$

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

is orthogonal.

(d) Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+(\delta^2/4)}$.

(e) Find the missing values in the table :

x :	45	50	55	60	65
y :	3	____	2	____	−2·4

(f) Use Euler's method to obtain an approximate value of $y(0\cdot4)$ for the equation $y' = x + y$, $y(0) = 1$ with $h = 0\cdot1$.

(g) Obtain the approximate value of $y(1\cdot2)$ for the initial value problem $y' = -2xy^2$, $y(1) = 1$ using Taylor series second-order method with step size $h = 0\cdot1$.

$L(t\,f(t)) = -\frac{d}{ds}(F(s))$

(h) Find the Laplace transforms of $te^{-t}\sin 2t$.

(i) Find the inverse Laplace transforms of

$$L^{-1}\left(\frac{s^3}{s^4 - a^4}\right)$$

(j) Evaluate :

$$\int_0^\infty e^{-x^2}\,dx$$

2. (a) Investigate, for what values of λ and μ do the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have *(i)* no solution, *(ii)* unique solution and *(iii)* infinite solutions. 7

(b) Find the rank of the matrix

$$\begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{bmatrix}$$

by reducing it to normal form. 7

3. (a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Hence compute A^{-1}. 4 − 1 = 3 — 7

(b) Reduce the matrix

$$P = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

to diagonal form. — 7

4. (a) Find a real root of the equation $x \log_{10} x = 1 \cdot 2$ by Regula-Falsi method, correct to four decimal places. — 7

(b) The following table gives the population of a town during the last six censuses. Estimate the population in 1913 by Newton's forward difference formula : — 7

Years	:	1911	1921	1931	1941	1951	1961
Population (in thousands)	:	12	15	20	27	39	52

(Continued)

5. (a) Find

$$\int_0^6 \frac{e^x}{1+x}\,dx$$

approximately by using Simpson's 3/8 rule on integration. — 7

(b) Evaluate

$$\int_0^8 x \sec x \, dx$$

using eight intervals by Trapezoidal rule. — 7

6. (a) State convolution theorem of Laplace transform and using it find

$$L^{-1}\left\{\frac{1}{(s^2+4)(s+2)}\right\}$$ — 7

(b) Use Laplace Transform to solve :

$$\frac{dx}{dt} + y = \sin t, \quad \frac{dy}{dt} + x = \cos t$$

given that $x = 2$, $y = 0$ at $t = 0$. — 7

(Turn Over)

7. (a) Find the Fourier transform of $f(x)$, defined by

$$f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$$

and hence evaluate

$$\int_{-\infty}^{\infty}\frac{\sin as\cos sx}{s}\,ds$$ 10

(b) Evaluate : 4

$$\int_0^{\infty} e^{-st}\,t^3\sin t\,dt$$

8. (a) Solve the initial value problem $yy'=x$, $y(0)=1$, using the Euler method in $0\le x\le 0\cdot 8$, with $h=0\cdot 2$ and $h=0\cdot 1$. Compare the results with the exact solution at $x=0\cdot 8$. Extrapolate the result. 6

(b) Given the initial value problem :

$$\frac{dy}{dx}=\frac{y-x}{y+x},\quad y(0)=1$$

Find $y(1)$ by Runge-Kutta fourth-order method taking $h=0\cdot 5$. 8

9. Obtain the approximate value of $y(0\cdot 2)$ for the initial value problem $y'=x^2+y^2$, $y(0)=1$. Using the methods $y_{n+1}=y_n+hf(x_n, y_n)$, as predictor and

$$y_{n+1}=y_n+\frac{h}{2}[f(x_n, y_n)+f(x_{n+1}, y_{n+1})],$$

as corrector, with $h=0\cdot 1$. Perform two corrector iterations per step. 14

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