(MARR)

→ A minimum return the company will accept on the money it invest is called MARR.

Factors influence the determination of MARR

(a) amount of fund available for investment
(b) the nature of investment alternatives
(c) Types of organization involved.

1. Equivalent worth method
   (i) Present worth method
   (ii) Future worth method
   (iii) Annual worth method

PW > 0 accept
PW = 0 remain indifferent
PW < 0 reject

same for annual and FW method

2. Rate of return method
   (a) Internal rate of return (IRR)
   (b) External rate of return (ERR)

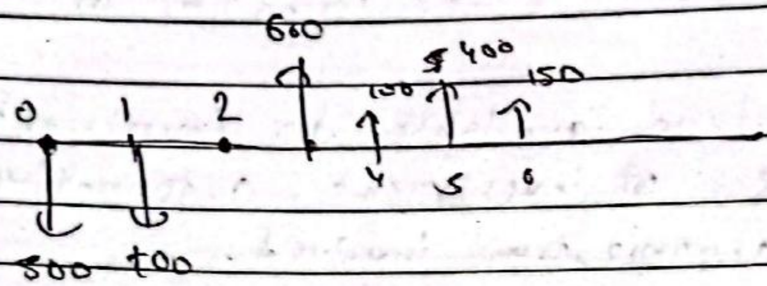
3. Payback period method
   (a) Simple period method
   (b) Discounted period method

4. Benifit cost ratio method
   (a) conventional
   (b) modified

(Basic method of engineering economics analysis)

Theory

Find the NPV of the cash as shown below effective interest rate is 10% compounded annually.

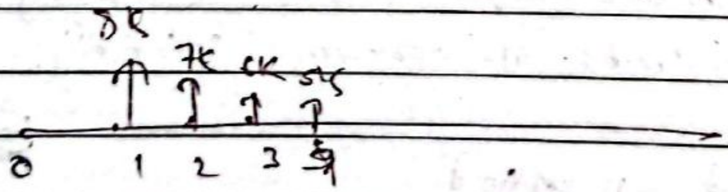


$$PW = -500 - 100(1+0.1)^{-1} + 600(1+0.1)^{-3}$$

$$+ 100(1+0.1)^{-4} + 400(1+0.1)^{-5} + 150(1+0.1)^{-6}$$

$$PW = 261.22 > 0 \quad \text{(accept)}$$

#



MARR = 10%

$$FW = 8000(1+0.1)^{3} + 7000(1+0.1)^{2} + 6000(1+0.1)^{1} + 5000$$

$$= 30718$$

$$FW > 0 \quad \text{(accept)}$$

$$AW = FW\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= 38748\left[\frac{0.15}{(1.15)^4 - 1}\right]$$

AW matra ma huncha

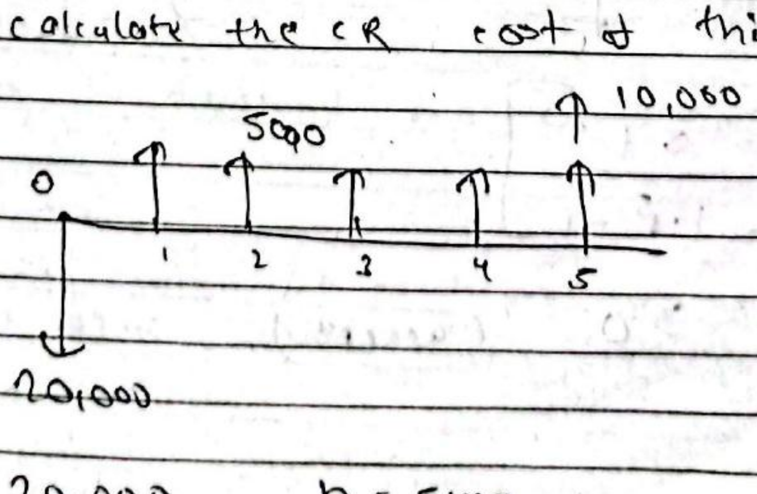
$$= 6151.75 > 0 \quad \text{accept}$$

Capital Recovery

Investment ... future value

$$CR = I\,(A/P,\ i\%,\ n) - SV\,(A/F,\ i\%,\ n)$$

\# Consider a machine cost Rs 20000 and expected to save Rs 5000 per year from the operation for the period of 5 years, The SV at the end of 5 years is Rs 10000, is it worthwhile to purchase this machine. Justify by using annual worth formulation take MARR = 10%. Also calculate the CR cost of this machine.



$I = 20{,}000 \qquad n = 5\text{ yrs}$

$SV = 10{,}000 \qquad i = 10\% \qquad AW = ? \qquad CR = ?$

4

$$FW = -20,000(1+0.1)^5 + 5000\left[\frac{(1+0.1)^5 - 1}{0.1}\right] + 10,000$$

$$FW = 8315.3$$

now,

$$AW = FW\left[\frac{0.1}{1.1^5 - 1}\right]$$

changes everything to AW

$$= 1362.02 > 0. \text{ accept}$$

$$AW = -20,000\left[\frac{(1+0.1)^5 \times 0.1}{(1+0.1)^5 - 1}\right] + 5000 + 10,000\left[\frac{0.1}{1.1^5 - 1}\right]$$

$$= 1362 > 0 \text{ (accept)}$$

(5)

$$CR = I\,(A/P,\ i\%,\ n) - SV\,(A/F,\ i\%,\ n)$$

$$= 20,000\left[\frac{1.1^5 \times 0.1}{1.1^5 - 1}\right] - 10,000\left[\frac{0.1}{1.1^5 - 1}\right]$$

$$= 3637.97$$

Rate of Return method

→ ROR is defined as the interest rate earned on the unpaid ~~balance~~ balance of an installment loan. IRR is that interest that makes the NPW = 0

(a) IRR

→ It is the rate of interest earned on the unrecovered balanced of the investment.

Advantages

→ Easy to understand and communicate

Disadvantage

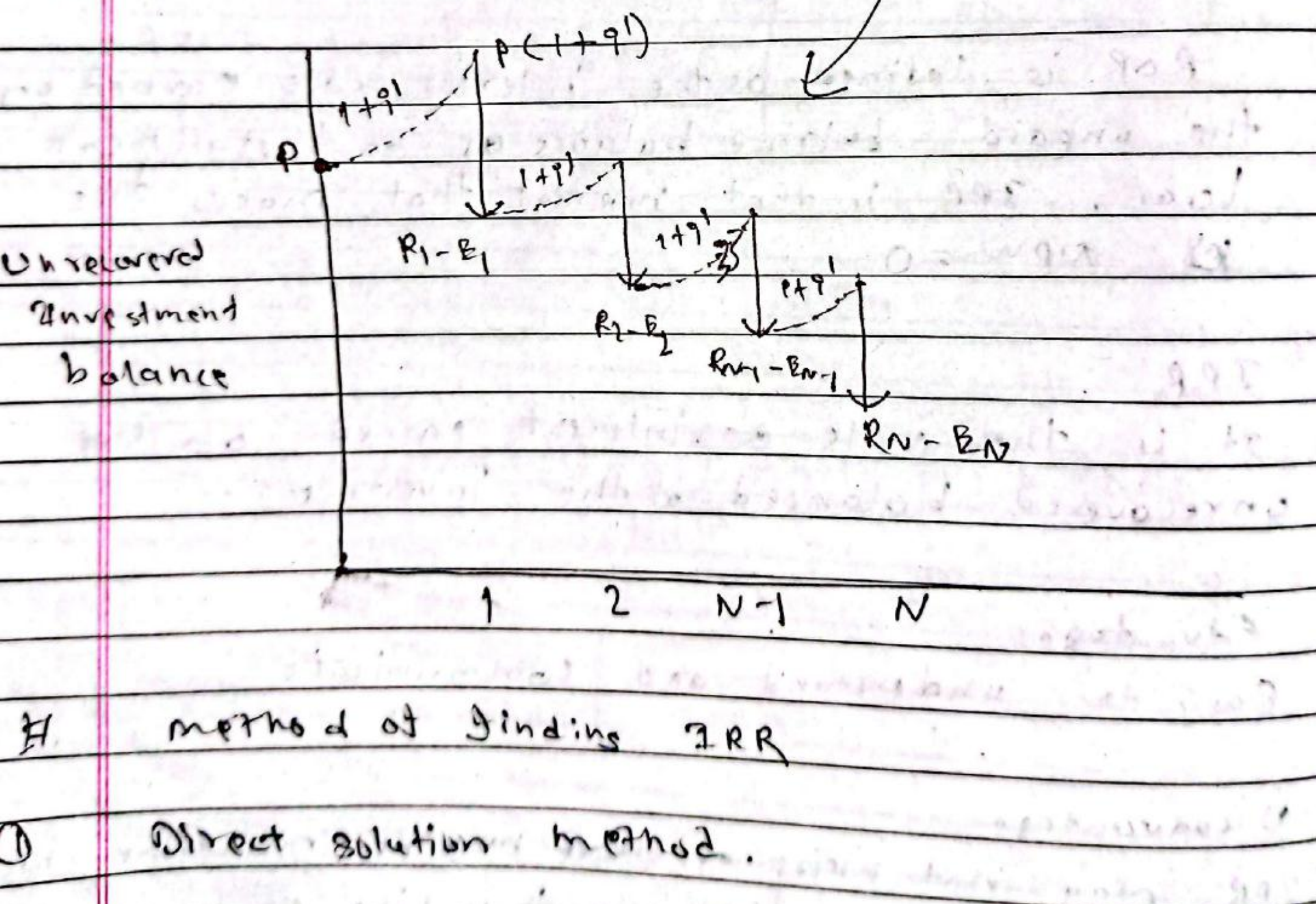
→ IRR may not exist or there may be multiple IRR

→ Does not distinguish between investing or borrowing

Decision Rule.

If IRR > MARR (accept the project)
IRR = MARR (Remain indifferent)
IRR < MARR (Reject the project)

Representation by investment balance diagram



method of finding IRR

(1) Direct solution method.

Period	0	1	2	3	4
1	-1000	0	0	0	1500
2	-2000	1300	1500	0	0

(7)

Soln project 1

$NPW = -1000 + 1500(1+i)^{-4}$

we know

$NPW = 0$

$0 = -1000 + 1500(1+i)^{-4}$

$1000 = 1500(1+i)^{-4}$

So $i = 0.1066$

$i = 10.66\%$

Project 2

$NPW = -2000 + 1300(1+i)^{-1} + 1500(1+i)^{-2}$

$0 = -2000 + 1300(1+i)^{-1} + 1500(1+i)^{-2}$

$2000 = \frac{1300}{(1+i)} + \frac{1500}{(1+i)^2}$

$i = 1.25 - 1$
$= 0.25$

$2000 = \frac{1300(1+i)+1500}{(1+i)^2}$

$i = 25\%$

$2000(1+i)^2 - 1300(1+i) = 1500$

$2000(1+i)^2 - 1300(1+i) - 1500 = 0$

$1+i = x$

$i = 1.25, -0.6$

$i = 2.25, 0.4$

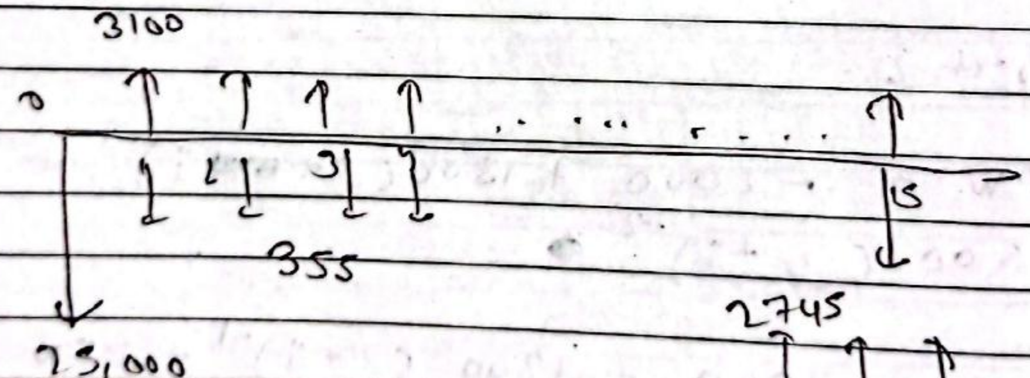
$x = 1.25$ or -0.6 (rejected) negative

$x = 1.25$

(8)

(7) Trial and error method

→ A company purchase a piece of construction equipment for rental purposes. The expected income is Rs 3100 annually for its useful life of 15 yrs. Expenses are estimated to be Rs 355 annually. If the purchase price is Rs 25,000 ~~athe~~ and there is no salvage value, what is the prospective rate of return neglecting taxes. Prepare UAB both in table and diagram.



$$NPW = -25{,}000 + 2745\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

$$0 = -25{,}000 + 2745\left[\frac{(1+i)^{15} - 1}{(1+i)^{15} \times i}\right]$$

Let,

9

+ve ra -ve niskini gare PW i ko select garne

$i = 6\%$, PW = Rs 1660.12

$i = 8\%$, PW = Rs −1504.23

$i = ?$, PW = 0

Now,

$\cancel{\frac{x_2 - x_1}{y_2 - y_1} = \frac{x - x_1}{y - y_1}}$

The formula for linear interpolation btw two point (x_1, y_1) and (x_2, y_2)

$x_1 = 6\%$, $y_1 = 1660.12$

$x_2 = 8\%$, $y_2 = -1504.23$

$x = ?$, $y = 0$

$$x = x_1 + \frac{(y - y_1)(x_2 - x_1)}{(y_2 - y_1)}$$

~~i = 0.06 + (0 −~~

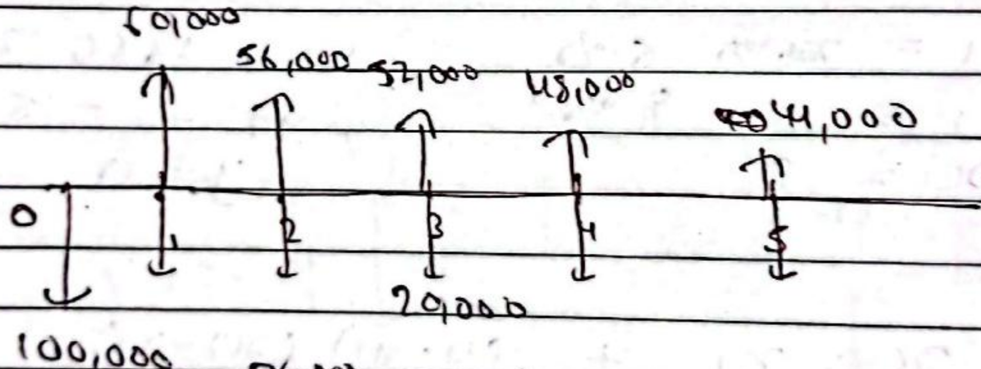
$$i = 0.06 + \frac{(0 - 1660.12)(0.08 - 0.06)}{(-1504.23 - 1660.12)}$$

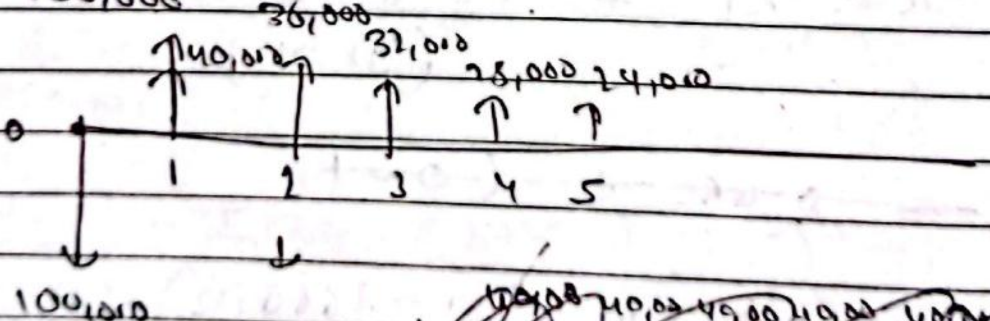
$= 0.0704$

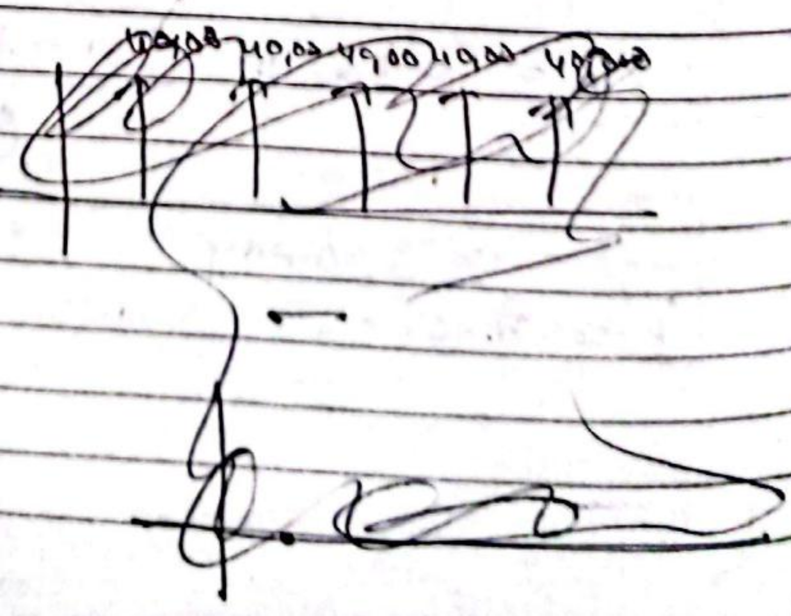
$i = 7.04\%$.

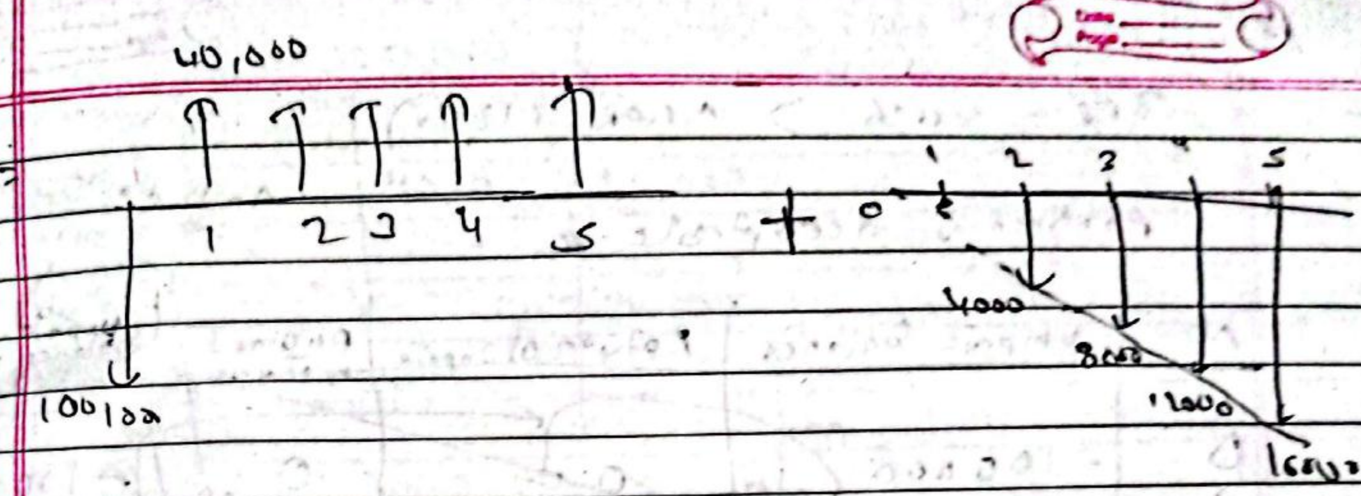
Initial investment = Rs 100 000
Salvage value = 0
Annual O and M cost = Rs 20,000
Useful life = 5 yrs
Annual benefit = 60,000 at the end of 1st year and decrease by 4000 each year for the remaining years. MARR = 12%.
Draw CF diagram.

Soln :









$$NPV = -10,000 + 40,000\left[\frac{(1+i)^5 - 1}{i \times (1+i)^5}\right]$$

$$+ \frac{-4000}{i^2}\left[\frac{(1+i)^5 - 1 - 5i}{(1+i)^5}\right]$$

$i = 0.18$ $\quad NPV = 4161.84$

$i = 0.22$ $\quad NPV = -3884.29$

$i = ?\ (x)$ $\quad NPV = 0\ (y)$

$$x = x_1 + \frac{(y-y_1)(x_2-x_1)}{(y_2-y_1)}$$ (linear interpolation)

$i = 0.20066$

$i = 20\ \%$

(17)

IRR = 20% > MARR (12%)

project is acceptable #

cash flow

NO	unpaid balance	Return of unpaid	payment received	unpaid balance at EOY
0	-100000	0	0	-100000
1	-100000	-20000	40,000	-80000
2	-80000	-16,000	36,000	-

add

No	unpaid balance	Return of unpaid	Received amount	Unpaid balance at EOY
0	-100000	0 (20%)	0 (cash flow)	-100000
1	-100000	-20,000	40,000	-80,000
2	-80,000	-16,000	36,000	-60,000
3	-60,000	-12,000	32,000	-40,000
4	-40,000	-8,000	28,000	-20,000
5	-20,000	-4,000	24,000	0

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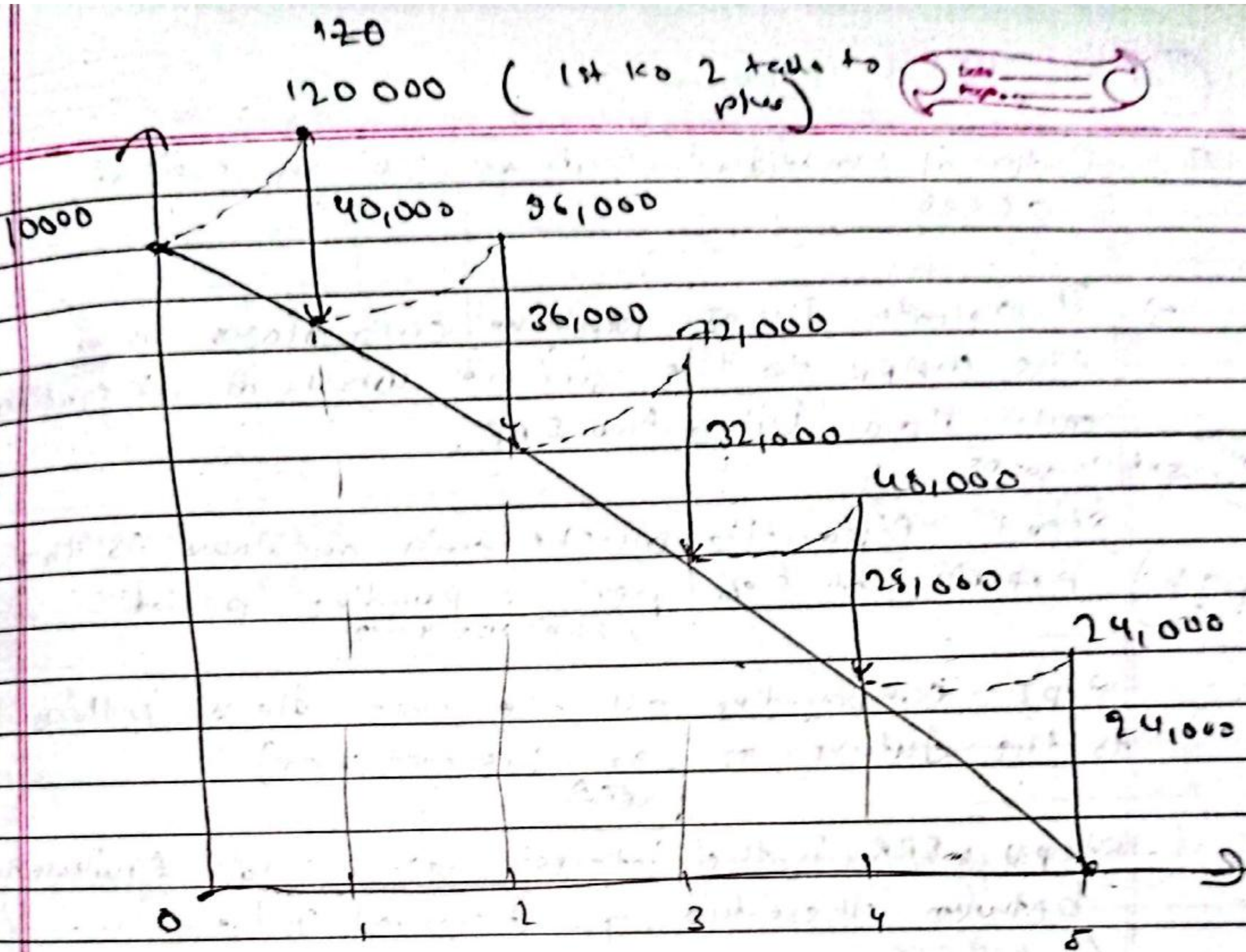


(Jailo 0 na meet huncha)

(unpaid amount balanced diagram)

Date
Page

① External / modified rate of return method (ERR)

→ It equates FW of positive cash flows using the MARR to the future worth of negative cash flow using the ERR.

provide hota ERR use garne

(P/F) Step 1: Discounting all the cash outflows to the preset at ε% per compounding period (ERR) use garera

Step 2: Compounding all the cash ~~flows~~ inflows to the future at ε% (reinvest rate) (MARR)

Step 3: ERR is the interest rate that equivalence between these two quantities at FW.

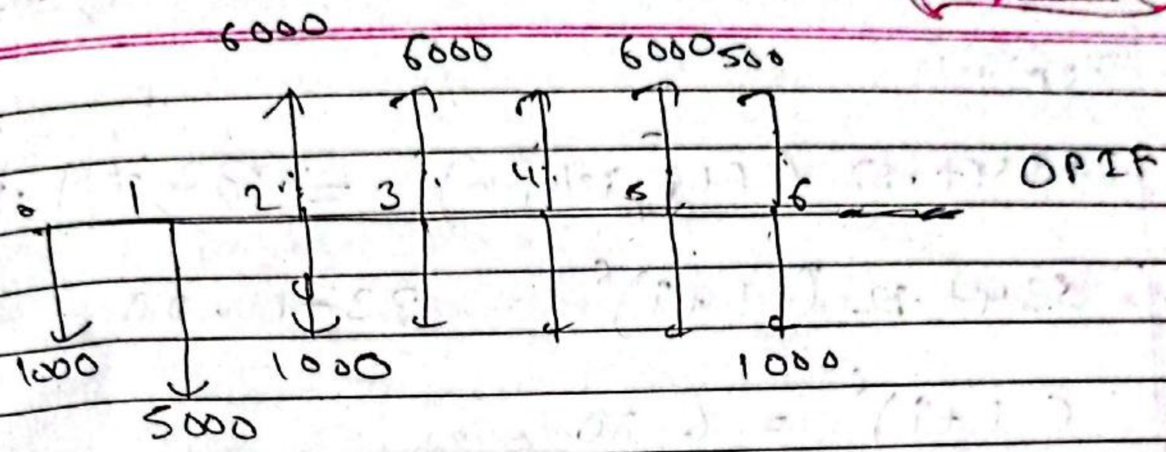
$$\sum_{t=0}^{n} R_t (1+r)^{n-t} = \sum_{t=0}^{n} C_t (1+i)^{n-t}$$

ε nikalne yaha bata

Step: convert step 1 to future and evaluate both and find i which give ERR.
Evalute step 1 as future worth with step 2 and find i.

(15)

#



MARR = 20% , reinvestment rate ε% = 15%
calc ERR = ? . Is the project accepted . ?

Step 1 :
Outflows → present using ε%

(↑ → cash inflow ↓ → cash outflow)
(money received money spend)

$$1000 + 5000\,(1+0.15)^{-1}$$
$$= 5347.82$$

Step 2 :

$$= 5000 \left[\cancel{\frac{(1.15)^5 - 1}{(1.15)^5 \cdot 0.15}} \quad \frac{(1.15)^5 - 1}{0.15} \right]$$

$$= 33711.90$$

Step 3 :

$$5347.82\,(F/P, i\%, n) = 33711.00$$

$$5347.82\,(1+i)^6 = 33711.00$$

$$(1+i)^6 = 6.30$$

$$1+i = (6.30)^{1/6}$$

$$1+i = 1.36$$

$$i = 0.35592$$

$$\therefore i = 35.92\% = ERR$$

Here,

ERR > MARR (accept the project)

Payback period method

→ It is the length of time in which an investment pays back its original cost

Rule

→ If the calculated payback period is less than or equal to some pre-specified payback period, then accept the project otherwise reject it.

(P)

Types

simple payback period

→ It does not consider the time value of money

→ It is easy to calculate

→ It is interpreted in years

$$\text{simple payback period} = \frac{\text{Initial investment}}{\text{Annual saving}}$$

Calculate the simple payback period for the given cash flow:

period	0	1	2	3	4	5
net cash flow	-25000	8000	8000	8000	8000	13000

period	net cash flow	cummulitative C.F (cs)
0	-25000	-25,000
1	8000	-17,000
2	8000	-9,000
3	8000	-1000
4	8000	7000
5	13000	20,000

Payback period lies between 3 and 4

$$\text{Payback period} = 3 + \frac{1000}{8000} = 3.125 \text{ yrs}$$

Discounted payback period

→ It is the time period that the project takes to pay back its initial investment, taking the time value of money into account.

→ By this time you have discounted the cash flows, you might as well calculate the NPV.

Example:

① A company has decided to purchase a new machine cost of 10,000 and is likely to bring in after tax cash inflows of Rs 4000 in the 1st year Rs 4500 in the 2nd year Rs 10000 in the third year 8000 in the 4th year. The company has a policy of buying equipment only if the payback period is 3 years or less.

~~Soln~~ Calculate the discounted payback period of the machine by using a discount rate of 10%.

Soln:

(16)

present manus in 10%.

year	net cash flow.	NPW at discounted C.F	cummulative C.F
	10% PW		
0	-10000	-10000	-10000
1	4000	3636	-6364
2	4500	3719	-2645
3	10000	7513	4865
4	8000	5464	

from qn

Payback period is in between 2 and 3

$$\text{Payback period} = 2 + \frac{2645}{7513} = 2.35 \text{ yrs}$$

Payback period < 3
(so accepted) #

Benefit Cost Ratio method

→ Benefit ratio is defined as the ratio of equivalent worth of benefit to the equivalent worth of cost
→ The main objective of this method is used to find out desirability of public projects as far as the expected benefits on the capital investment are concerned.

Types of B/C ratio

(20)

① Type Conventional B/C ratio
② Modified B/C ratio

AW ra PW ma ni huncha

B/C ≥ 1 (accept)

(1) Convention B/C Ration with PW

$$B/C = \frac{PW(B)}{I + PW(O \text{ and } m) - PW(SV)} \quad \left(\frac{B}{I + Om - SV} \right.$$

B → benefit I → initial investment O and m = operation and maintanance cost , SV = salvage value

(2) Modified B/C ratio with PW

$$B/C = \frac{PW(B) - PW(O \text{ and } m)}{I - PW(SV)} \quad \left(\frac{B - Om}{I - SV} \right.$$

Determine both types of B/C ratio from the following cashflow. Initial investment = 3 00 000
Annual revenue = 85000
Annual cost = 15000
SV = 20% of initial investment
Useful life = 6 yrs MARR = 10%
By using PW approach.

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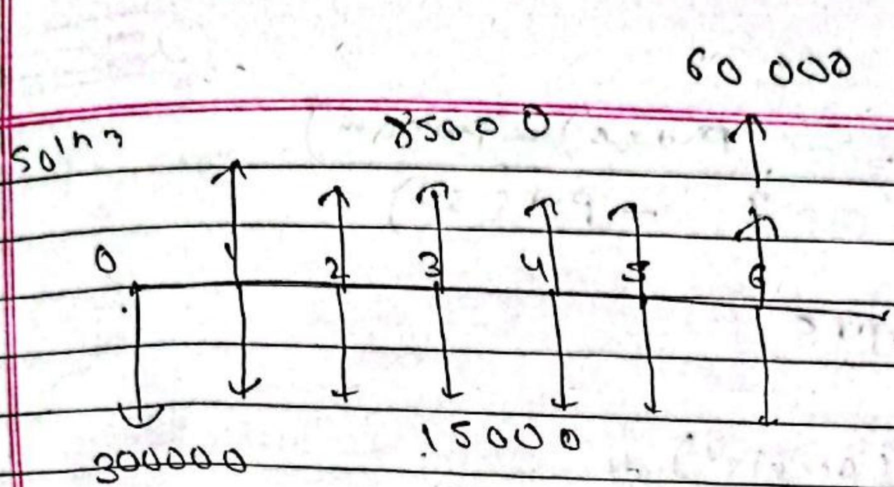
Sol^n



$I = 300\,000$, $SV = 60\,000$, $AR = 8500$ $AC = 1500$

$PW(B) = 8500\,(1+0.1)^{6}$

$$= 85000 \left[\frac{1.1^{6} - 1}{1.1^{6} \times 0.1} \right] = 370197.15 \quad \cancel{37019.71}$$

$$PW(O \text{ and } M) = 15000 \left[\frac{1.1^{6} - 1}{1.1^{6} \times 0.1} \right] = \cancel{6532.89} \quad 65328.91$$

$PW(SV) = 60{,}000\,(1+0.1)^{-6} = 33868.4358$

(1) Conventional $B/C = \dfrac{PW(B)}{I + P(OM) - P(SV)}$

$$= \frac{370197.15}{300000 + 65328.91 - 33868.4358}$$

$= 1.116$

$B/C \geq 1$ so accepted

$$\text{modified } B/C = \frac{PW(B) - PW(OM)}{I - PW(SV)}$$

$= 1.145$

$B/C \geq 1$ (accepted) #

~~Conv~~

$$B/C_{conv} = \frac{FW(B)}{FW(I) - SV + FW(OM)}$$

$$B/C \text{ modified} = \frac{FW(B) - FW(OM)}{FW(I) - SV}$$

example:

$I = 6000$

$B = 3000$

$OM = 1000$

$SV = 1500$

$MARR = 10\%$, $n = 10$

$FW(I) = 6000(1+0.1)^{10} = 15562.45$

$FW(B) = 3000\left[\frac{1.1^{10}-1}{0.1}\right] = 47812.27$

$FW(OM) = 1000\left[\frac{1.1^{10}-1}{0.1}\right] = 15937.42$

$SV = 1500$

$$B/C_{conv} = \frac{\cancel{15562.45}\quad 47812.27}{15562.45 + 15937.42 - 1500}$$

$$= 1.593 \geq 1 \text{ (justified)}$$

$$B/C_{modified} = \frac{47812.27 - 15937.42}{15562.45 - 1500}$$

$$= 2.26 \geq 1 \text{ (justified)}$$

B/C ratio conv $= \dfrac{AW(B)}{\cancel{CR + AW(OM)}}$ $\quad \dfrac{AW(B)}{CR + AW(OM)}$

A/P annual, annual A/F

$CR = I - SV$

$$B/C \text{ ratio modified} = \frac{AW(B) - AW(OM)}{CR}$$

B/C using AW method

I = Rs 90,000

AR = Rs 50,000

AC = Rs 2000

SV = 20 000

MARR = 12%, n = 10 yrs

$CR = I(A/P, i\%, n) - SV(A/F, i\%, n)$

$$= 90{,}000\left[\frac{0.12 \times 1.12^{10}}{1.12^{10}-1}\right] - 20{,}000\left[\frac{0.12}{1.12^{10}-1}\right]$$

$= 14788.89148$

$$B/C_{conv} = \frac{50{,}000}{14788.89 + 2000}$$

$= 2.97 \geq 1$ (justified)

$$B/C_{modified} = \frac{50{,}000 - 2000}{14788.89}$$

$= 3.24 \geq 1$ (justified)

Economic analysis	Financial analysis
① It is done by government project	private project
② determines the best use of resources over project life.	determines financial feasibility of the project
③ consumer oriented	investor oriented
④ profit is never a goal	profit is the goal
⑤ B/C ratio analysis is used	ROR and PW are most used.

Cost = 250000

SV = 50000

n = 5 yr

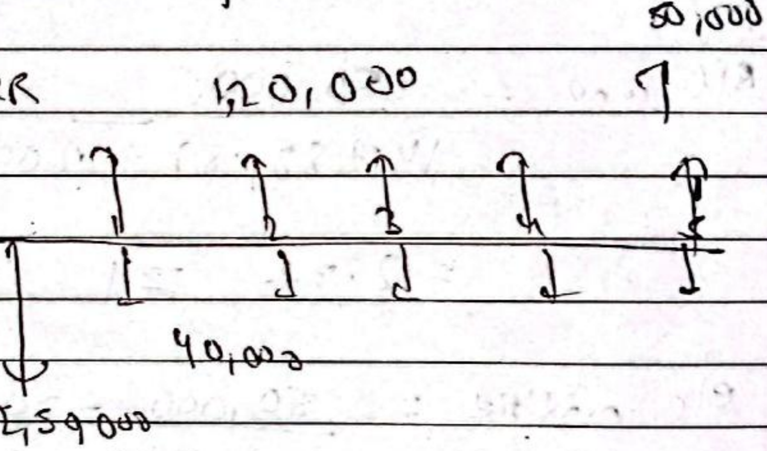
AE = 40000

AR = 120000

MARR = 20%

(i) IRR using AW formulation

Let i% be IRR



$$AW = -2,50,000 \left[\frac{i \times (1+i)^5}{(1+i)^5 - 1} \right] + 120000 - 40000$$

$$+ 50,000 \left[\frac{i}{(1+i)^5 - 1} \right] = 0$$

$\therefore x = 21.37\%$ (Direct method)

By interpolation

$i = \cancel{20\%}\ 0.20$ $\quad$ AW = 3124.0593

$i = \cancel{-841.18}\ 0.22$ $\quad$ AW = −841.1869

$i = ?$ $\quad$ AW = 0

$$x = x_1 + \frac{(y-y_1)(x_2-x_1)}{(y_2-y_1)}$$

$$i = 0.20 + \frac{(0 - 3124.0593)(0.22 - 0.20)}{(-841.1869 - 3124.0593)}$$

$= 0.2157$

$= 21.57\%$

IRR > MARR (justified)

BCR using FW :

$FV(I) = 2,50,000\,(1+0.2)^5 = \cancel{6228}\ 622080$

$FW(OM) = 40,000 \left[\frac{1.2^5 - 1}{0.2}\right] = 297664$

$FV(B) = 120000 \left[\frac{1.2^5 - 1}{0.2}\right] = 892992$

$B/C_{conv} = \dfrac{892992}{622080 + 297664 - 50000}$

$= 1.026 \geq 1$ (justified)

$B/C_{modified} = \dfrac{892992 - 297664}{622080 + -50000} = 1.04 \geq 1$ (justified) e find ERR

Step:

Cash outflow to present (d)

$$= 250000 + 40,000 \left[\frac{1.2^5 - 1}{1.2^5 \times 0.2} \right]$$

$$= 369624.4856$$

Step 2:

Cash inflow to future (A)

$$50,000 + 120\,000 \left[\frac{1.2^5 - 1}{0.2} \right]$$

$$= 942992$$

Step 3:

$$369624.4856\,(1+i)^5 = 942992$$

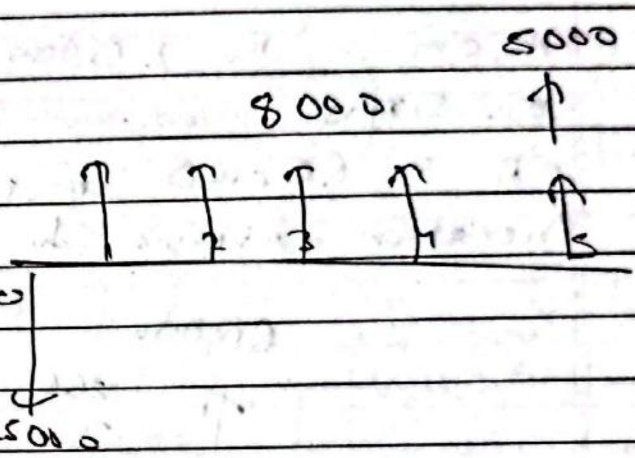
$i = 0.2060$

$i = 20.60\,\% = ERR$

ERR > MARR (accept) #

Determine IRR by trial and error process

I = 25000
AR = 8000
SV = 5000
n = 5 year
MARR = 20%



Using PW method

$PW(i\%) = 0$

$$PW(i\%) = -25000 + 8000\left[\frac{(1+i)^5 - 1}{(1+i)^5 \times i}\right] + 5000(1+i)^{-5}$$

— (i)

Now,
when

$i = 0.2$ PW = 934.28
$i = 0.22$ PW = −240.88
$i = ?$ PW = 0

$$x = x_1 + \frac{(y-y_1)(x_2-x_1)}{(y_2-y_1)}$$

$$= 0.2 + \frac{(0 - 934.28)(0.22-0.2)}{(-240.88 - 934.28)}$$

$= 0.2159$

$i = 21.59\%$

IRR = 21.59% > MARR (20%)
accepted

(x)

t) $P = 100000$

$SV = 0$

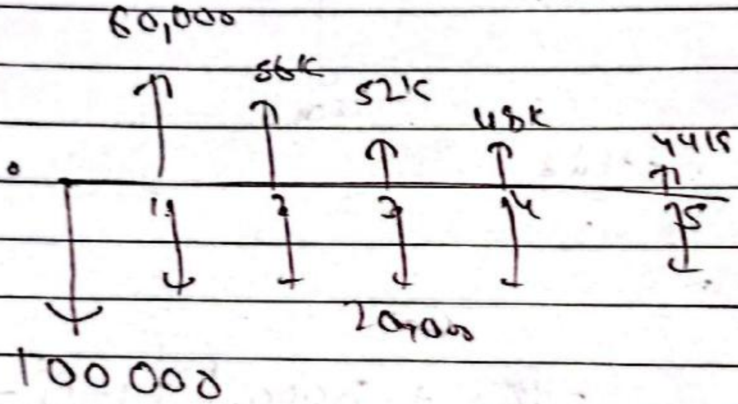
A OM = Rs 20000

$n = 5$ yrs

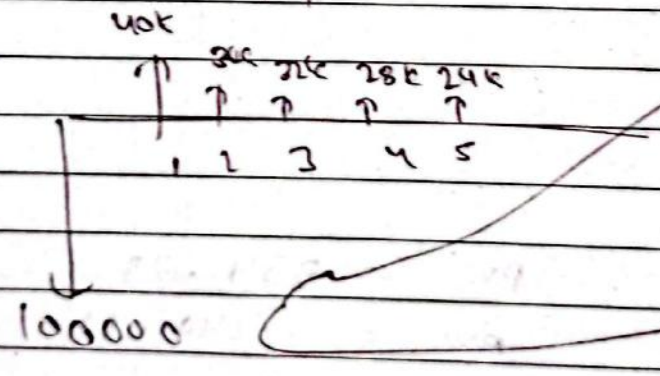
AB = 60000 at end of 1st year

thereafter decreases by 4000 each year

MARR deko chaina bhane nikalo



(ii)



$PW(i\%) = 0$

or, $-100000 + 40,000(1+i)^{-1} + 36000(1+i)^{-2} + 32000(1+i)^{-3} + 28000(1+i)^{-4} + 24000(1+i)^{-5}$

on solving

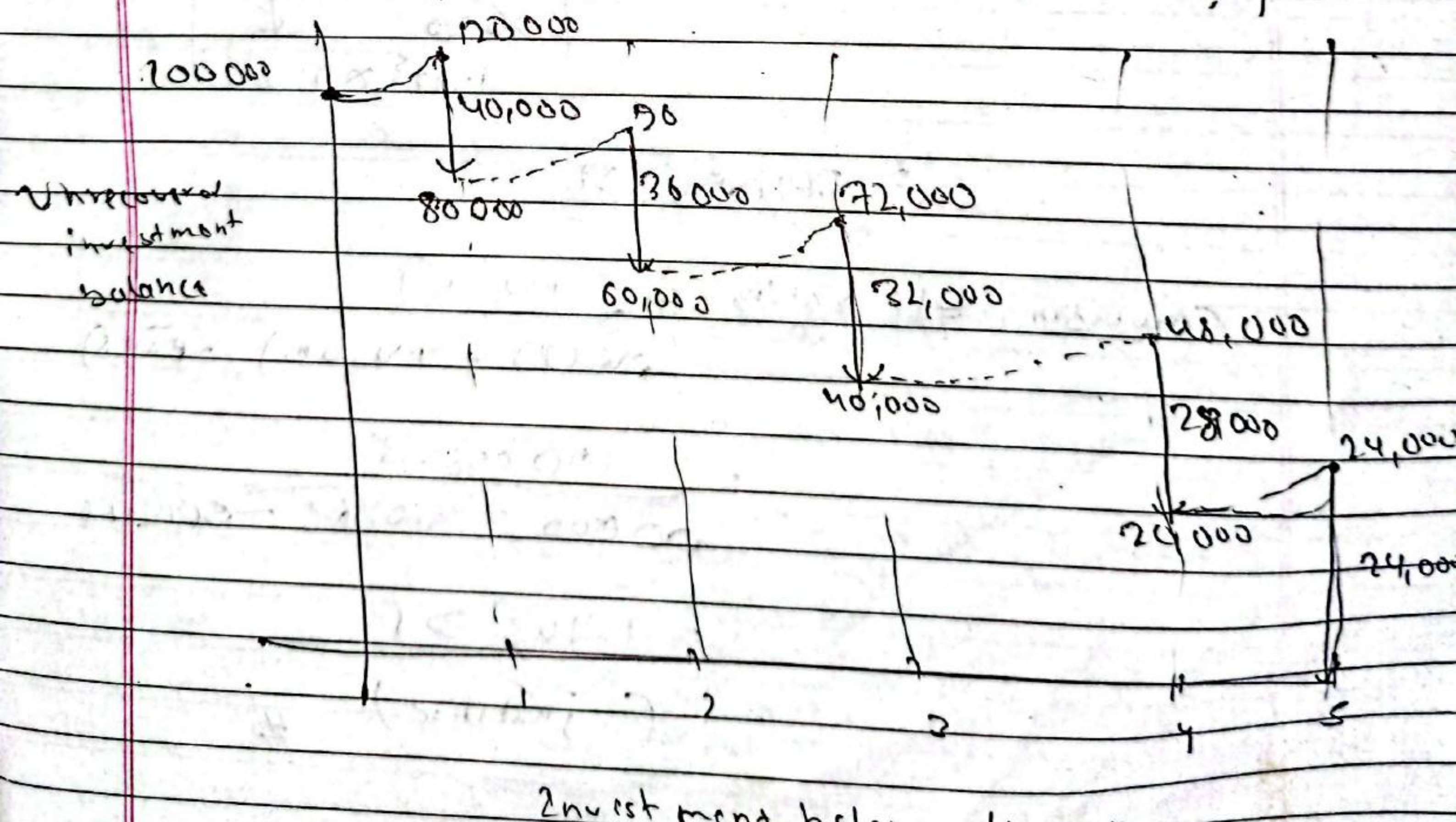
$i = 0.2$

IRR = 20% = MARR (remain indifferent)

(a)

i) UBD

year	Unpaid balance at beginning year	Return on Unpaid balance	Payment received	Unpaid balance at EOY
0	−100000	0	0 (cash flow)	−100000
1	−100000	−20000	40000	−80000
2	−80000	−16000	36000	−60000
3	−60000	−12000	32000	−40000
4	−400000	−8000	28000	−20000
5	−20000	−4000	24000	0



Investment balance diagram

(a)

(b) B/C ration using PW formulation, MARR = 12%

(SV) = 10000

$$PW(B) = \frac{60,000}{1.12} + \frac{56000}{1.12^2} + \frac{52000}{1.12^3} + \frac{48000}{1.12^4} + \frac{44000}{1.12^5}$$

$= 190628.5$

$$PW(S) = \frac{10\,000}{1.12^5} = 5674.26$$

$$PW(O\,and\,M) = 20,000 \times \left[\frac{1.12^5 - 1}{1.12^5 \times 0.12}\right]$$

$= 72095.52$

$$\text{Convention B/C ration} = \frac{PW(B)}{PW(I) + PW(OM) - PW(S)}$$

$$= \frac{190698.5}{100000 + 72096.5 - 5674.26}$$

$= 1.145 > 1$

(justified)

(3)

(1) Disounted payback period

Year	cash flow	PW of CF	cum CF
0	-100000	-100000	-100000
1	40000	35714	-64286
2	36000	28698.8	-35586.2
3	~~34000~~ 32000	22776.9	-12809.25
4	28000	17794.5	4985.50
5	24000	13618.24	18603.44

3	-12809.25
4	4985.50
?	0

$$x = x_1 + \frac{(y - y_1)(x_2 - x_1)}{(y_2 - y_1)}$$

$$= 3 + \frac{(0 + 12809.25)(4-3)}{(4985.50 + 12809.25)}$$

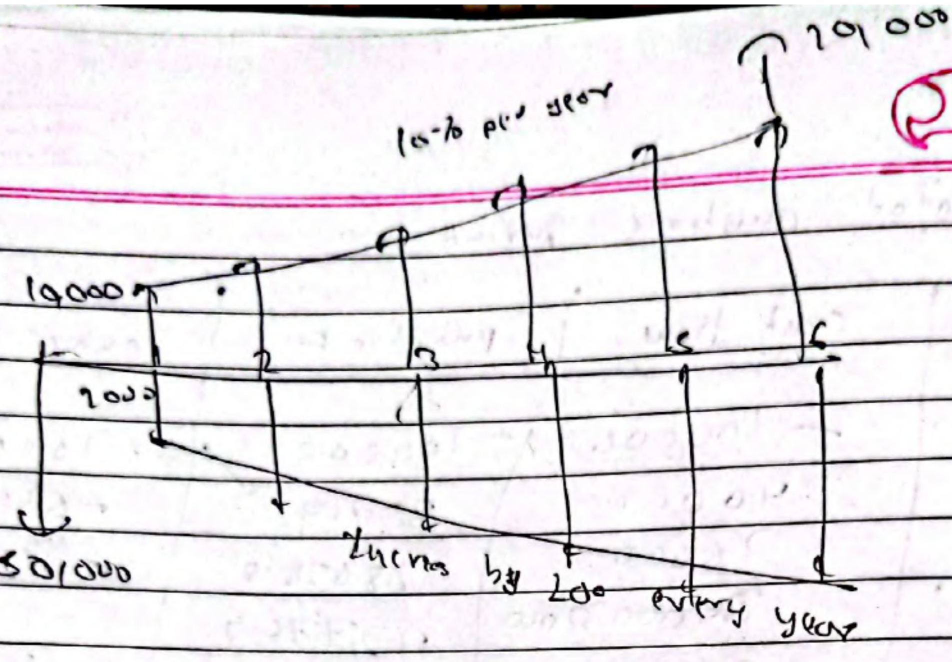
$= 3.7198$

$= 3.72 > 3$ yrs → Given in Q1n

(so accepted)

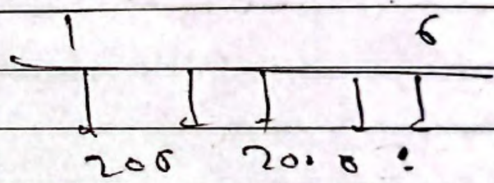
(38)

#



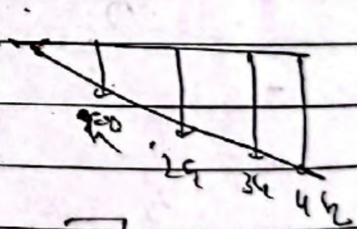
SV = 20,000

FW (0 and m) =



$2000\,(F/A,\ i\%,\ n)$

$+\ \dfrac{G}{i}\left[\dfrac{(1+i)^n-1}{i}\right] - \dfrac{nG}{i}$ +



$= 2000\left[\dfrac{1.15^6-1}{\cancel{0.15}}\ \dfrac{1.15^6-1}{0.15}\right] + \dfrac{200}{0.15}\left[\dfrac{1.15^6-1}{0.15}\right] - \dfrac{6\times 200}{0.15}$

$= 21178.128$

1+S

$$FW(B) = 10{,}000 \left\{ \frac{(1+0.15)^6 - (1+0.1)^6}{0.15 - 0.1} \right\}$$

$$= 108299.9531$$

$$B/C_{con} = \frac{FW(B)}{FW(I) - FW(S) + FW(OM)}$$

$$= \frac{108299.9531}{115653.0383 - \cancel{0} + 21179.128}$$

20,000

$$= 0.926 < 1$$

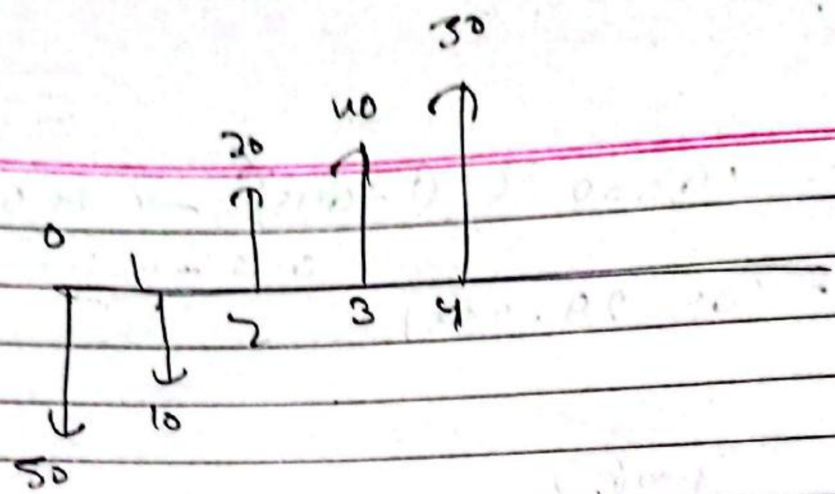
(not justified)

$$B/C \text{ modified} = \frac{FW(B) - FW(OM)}{FW(I) - FW(S)}$$

$$= 0.910 < 1$$

(not justified) #

3c



$i = 15\%$

For IRR

$$PW = -50 - 10(1+i)^{-1} + 30(1+i)^{-2} + 40(1+i)^{-3} + 50(1+i)^{-4}$$

at PW = 0

$$0 = -50 - 10(1+i)^{-1} + 30(1+i)^{-2} + 40(1+i)^{-3} + 50(1+i)^{-4}$$

$i = 0.2658$

IRR = 26.58 %

B/C ration → MARR

(~~if not given~~ assume 10% MARR)

7

\# Payback period nikalda = 0 ako nai payback period #

\# when calculating ERR using reinvestment rate given

MARR = 12%, reinvestment rate = 14% → use this

this for comparision

~~∴ reinvestment rate not given~~
~~use MARR~~
~~to B/C ratio~~

\# B/C ratio ma alway use MARR #