① The power, input to the rotor at 440V, 50Hz 3ϕ 6 pole induction motor is 50kW. The rotor emf makes 120 cycles per minutes. Friction and windage losses are 2kW. Nr

① Calculate

(i) Slip (ii) rotor speed (iii) rotor Cu-loss (iv) mech power developed (v) output power (vi) output torque

Soln:

$$\boxed{S = \frac{N_s - N_r}{N_s} \qquad N_s = \frac{120f}{P}}$$

$$\boxed{P_{in} \text{ to rotor} = \sqrt{3}\, V_L I_L \cos\phi = 50kW}$$

$$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$N_r = 120 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s} = \frac{1000 - 120}{1000} = 0.88$$

$S < 1$ always

$$\boxed{\text{Rotor Cu-loss} = S \times \text{input power to rotor}}$$

Rotor Cu-loss = 0.88 × 50 = 44 kW

$$\boxed{\eta_r = \frac{\text{Power delivered by rotor}}{\text{i/p power to rotor}}}$$

$$\boxed{\eta_r = \frac{N_r}{N_s}} = \frac{120}{1000}$$

$$\frac{120}{1000} = \frac{\text{Power delivered by rotor}}{50\,kW}$$

∴ P^n delivered by rotor = 6 kW → mech power developed

$$\boxed{\text{Output power} = \text{mech power developed} - \text{friction loss}}$$

P_{out} = 6 kW − 2 kW

= 4 kW

$$\boxed{\text{Power} = \text{Torque} \times \frac{2\pi N_r}{60}}$$

$$T = \frac{P \times 60}{2\pi N_r} \qquad T_{out} = \frac{60 \times P_{out}}{2\pi N_r} = \frac{60 \times 4 \times 10^3}{2\pi \times 120}$$

$$= 318.30 \text{ N-m}$$

② A 4-pole, 50 Hz 3ϕ slip-ring induction motor has star connected strator and rotor windings. The rotor windings has resistance of 0.8Ω and reactance of 4Ω per phase at stand still. The emf induced between the slip rings at stand still is 400V. The starter to rotor turn ratio is 4. The motor runs at 1490 rpm at no-load and 1300 rpm at full load.

E_2 (line)

Calc

(i) Starting current, $N = 0$, I_1 (starting) = ?

(ii) no-load current, $N = 1490$ rpm, I_1 (starting) = ?

(iii) full-load current $N = 1300$ rpm, I_1 (starting) = ?

Soln ÷

[4-pole, 50 Hz, 3 phase Y → stator / Y → rotor]
Induction motor

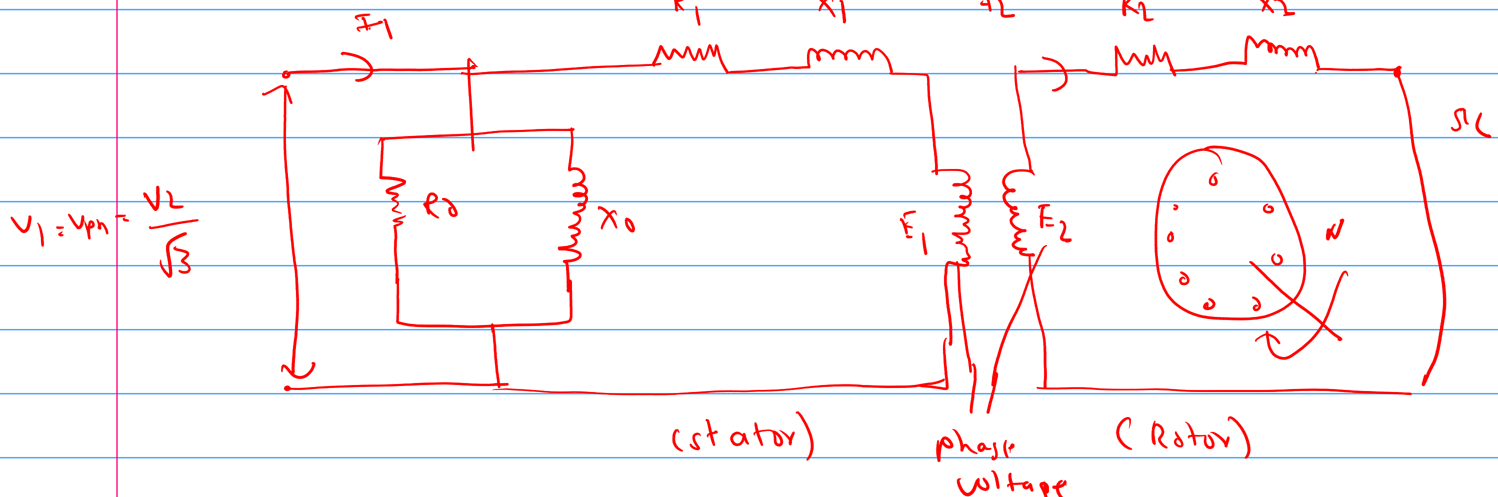
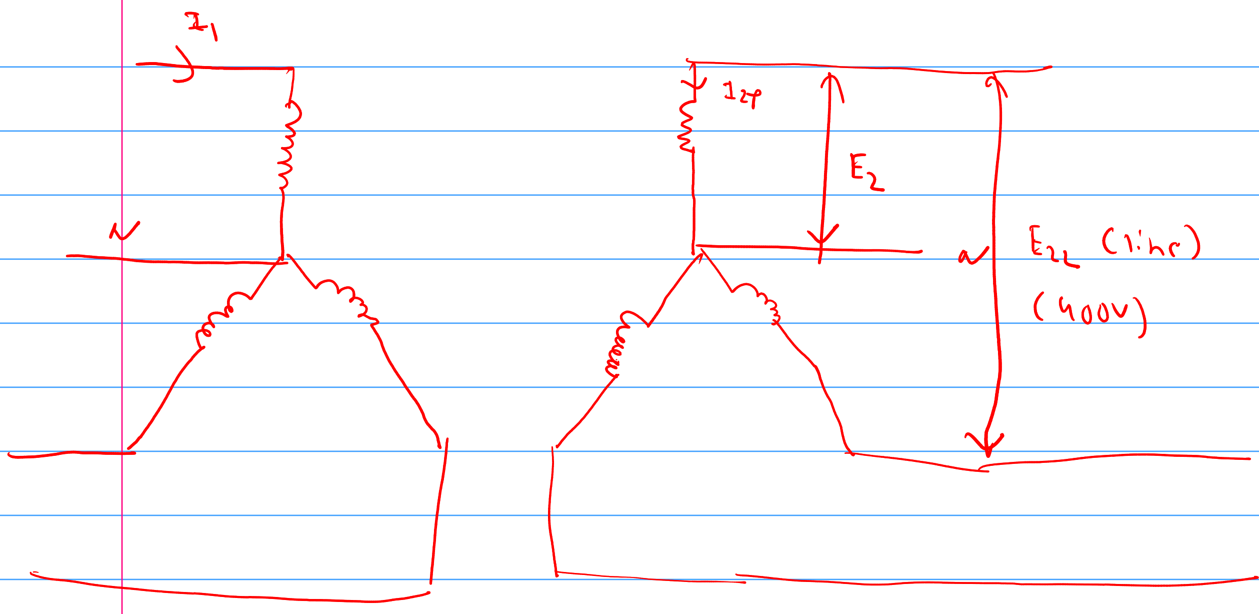


fig: per-phase equivalent ckt

Stand → E_2 / X_2

Running → SE_2 / SX_2



(stator) (Rotor)

phase voltage (E_2) $= \dfrac{E_{2L}}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94\,V$

$\left(s = \dfrac{N_s - \overset{\to 0}{N_r}}{N_s} \right) \quad \boxed{S = 1}$

Now,

$$I_2 = \frac{E_2}{\sqrt{(R_2)^2 + (X_2)^2}} = \frac{230.94}{\sqrt{0.8^2 + 4^2}} = 56.61\,A$$

now

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Given,

$$\frac{N_1}{N_2} = 4$$

or, $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$ | $\dfrac{N_1}{N_2} = \dfrac{I_2}{I_1}$ | $4 = \dfrac{I_2}{I_1}$

or $\boxed{I_1 = \dfrac{I_2}{4} = \dfrac{56.61}{4} = 14.1525\,A}$

→ Starting current at stand still condition

(ii) No-load current, $N_r = 1490$ rpm, I_1 (starting)

$N_r = 1490$ rpm, $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$ rpm

$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1490}{1500} = 6.67 \times 10^{-3}$ $S < 1$

Now $I_2 = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} = 1.9214$ A

$\frac{I_1}{I_2} = \frac{N_2}{N_1}$ | $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ | $4 = \frac{1.9214}{I_1}$

or

$\therefore$ $\boxed{I_1 = 0.48035 \text{ A}}$ → no load

(iii) Full-load current

$N_s = \frac{120f}{P} = 1500$ rpm $N_r = 1300$ rpm

$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1300}{1500} = 0.133$

$S = 0.133$, $SE_2 = 0.133 \times 230.94 = 30.7843$ V

$I_2 = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} = \frac{30.7843}{\sqrt{0.8^2 + (0.133 \times 4)^2}}$

$= 32.0202$ A

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

or $$4 = \frac{32 \cdot 0202}{I_1}$$

$\therefore$ I_1 (full-load) $= 8.005A$ #

\# A 3ϕ, 400V (line), 50Hz 2 pole induction motor has a rotor circuit resistance of 2Ω and rotor circuit reactance of 8Ω at stand still. It develops a starting torque of 10 N-m. At which speed, the motor developed maximum torque and calculate the value of max torque.

Soln:-

(3ϕ, 400V, 50Hz, 2 pole)

$$V_l = \frac{V_L}{\sqrt{3}}$$

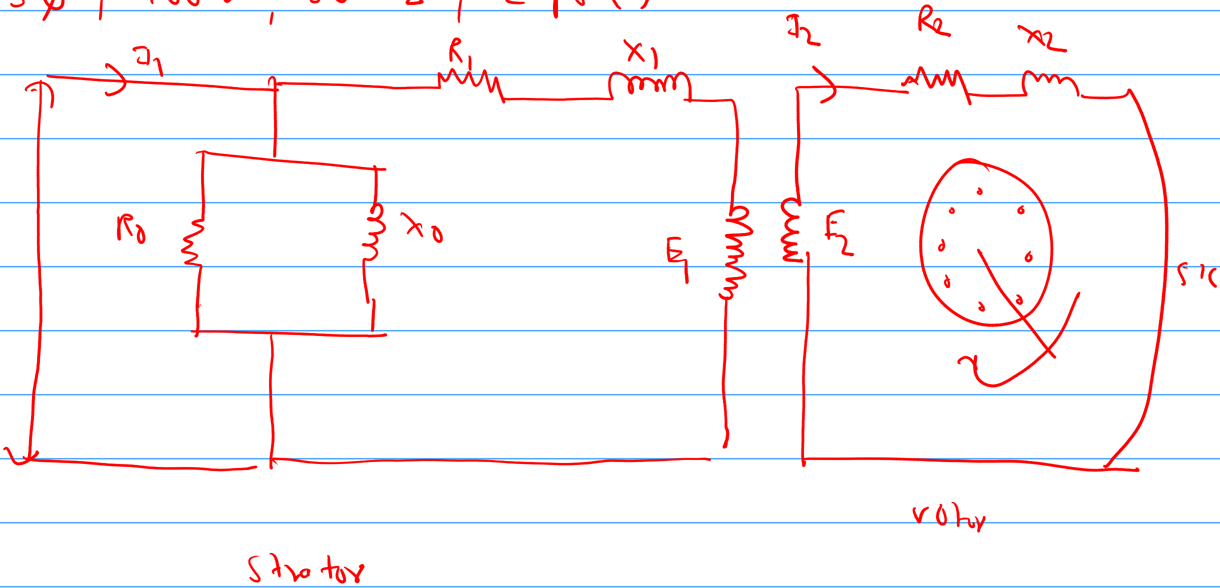


fig: per-phase equivalent circuit

$\boxed{\begin{array}{l}\phi n \text{ ma not mention so} \\ \text{strator} \rightarrow \text{Star connected}\end{array}}$

$$T_R = \frac{s R_2 k E_2^2}{R_2^2 + (s X_2)^2}$$

$T_s = 10 \text{ N-m}$

$R_2 = 2\Omega$

$X_2 = 8\Omega$

at starting $N_r = 0$, $s = \frac{N_s - N_r}{N_s} = 1$

$$T_s = \frac{k E_2^2 R_2}{R_2^2 + X_2^2}$$

or $$10 = \frac{k \cdot E_2^2 \times 2}{(2)^2 + (8)^2}$$

or $k E_2^2 = 340$ —— (i)

$V_1 = \frac{V_1 \text{ (line)}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$ (star connected)

$V_1 = E_1 = 230.94V$

Now

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

let turn ratio $\frac{N_2}{N_1} = 1$ (if not given)

$E_2 = E_1 = 230.94\,V$

put E_2 in (i)

$340 = K \times (230.94)^2$

$\therefore K = 6.375 \times 10^{-3}$

For max torque, $S_m = \frac{R_2}{X_2} = \frac{2}{8} = 0.25$

$$T_{max} = \frac{S_m R_2 K E_2^2}{R_2^2 + (S_m X_2)^2} = \frac{0.25 \times 2 \times 6.375 \times 10^{-3} \times 230.94^2}{2^2 + (0.25 \times 8)^2}$$

$= 21.2498$ N-m

Speed at which it develop max torque (N_r) = ?

$S_m = \frac{N_s - N_r}{N_s}$, $N_s = \frac{120f}{P} = \frac{120 \times 50}{2}$

$= 3000$ rpm

or $0.25 = \frac{3000 - N_r}{3000}$

$\therefore$ $\boxed{N_r = 2250 \text{ rpm}}$

$V_{1,L} \rightarrow$ line voltage

\# A 3 phase 400V, 50 Hz, 4-pole induction motor has rotor circuit resistance 2Ω and reactance of 8Ω at standstill. It develops a starting torque at 5N-m. The stator to rotor turn ratio is unity. Calculate the torque developed by the motor when it runs at 1400 rpm.

soln ÷

$R_2 = 2\Omega \quad X_2 = 8\Omega$

$\frac{N_1}{N_2} = 1$ standstill

sX_2, sE_2 running

$N_r = 1400$ rpm

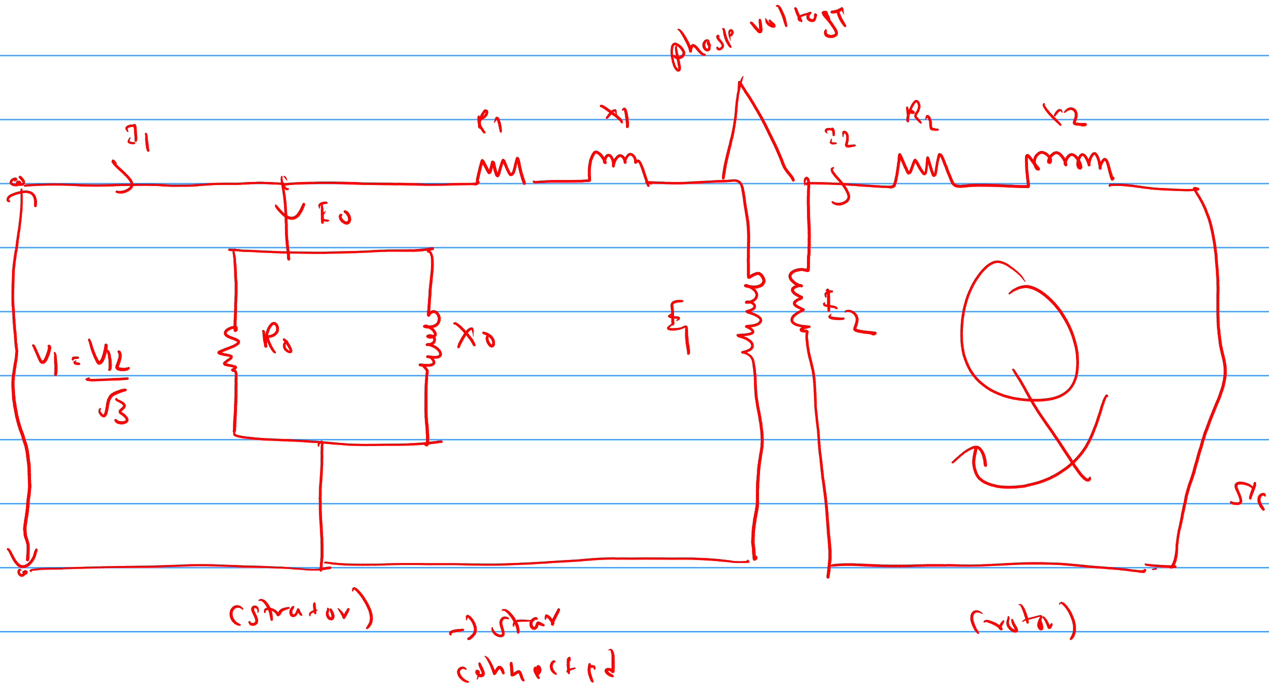


fig: per-phase equivalent ckt

$T_s = SN\text{-}m \quad , N = 0 \quad , S = 1$

$R_2 = 2\Omega \quad | \quad X_2 = 8\Omega$

at starting

$$T_s = \frac{K E_2^2 R_2}{(R_2)^2 + (X_2)^2}$$

or $$5 = \frac{K \cdot (E_2)^2 \times 2}{(2)^2 + (8)^2}$$

or $$170 = K(E_2)^2 \quad \text{—— (i)}$$

$$V_1 = \frac{V_{1,2}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\,V$$

$V_1 = E_1 = 230.94\,V$

now,

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{E_2}{E_1}$$

$\left(\frac{N_1}{N_2} = 1\right)$

or $$1 = \frac{E_2}{E_1} \qquad \therefore E_2 = 230.94\,V$$

$$K = \frac{170}{(230.94)^2} = 3.1875 \times 10^{-3}$$

$T_R = ?$ at $N_r = 1400$ rpm

$$S = \frac{N_S - N_r}{N_S} \qquad N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = \frac{1500 - 1400}{1500} = 0.0667$$

$$T_R = \frac{K_S E_2^2 R_2}{R_2^2 + (SE_2)^2} = \frac{3.1875 \times 10^3 \times 0.0667 \times 230.94^2 \times 2}{(2)^2 + (0.0667 \times 8)^2}$$

$$= 5.29067 \text{ N-m} \quad , N_r = 1400 \text{ rpm}$$

\# A 6-pole, 50 Hz 3ϕ induction motor has rotor resistance of 0.4Ω/phase, max torque is 200 Nm at 850 rpm.

Find (i) torque at 4% slip

(ii) additional rotor resistance to get $\left(\frac{2}{3}\right)^{rd}$ of max torque.

Soln ÷

(6-pole, 50Hz, 3ϕ) $R_2 = 0.4\Omega$ $T_{max} = 200$ Nm

$N_r = 850$ rpm

(1) Torque at 4% slip, $s = 0.04$

$$T_R = \frac{s R_2 K E_2^2}{R_2^2 + (sX_2)^2}$$

we know

$T_{max} = 200$ Nm

$N_r = 850$ rpm

$$S_m = \frac{R_2}{X_2} = \frac{0.4}{X_2} \quad \text{— (i)}$$

$$S = \frac{N_s - N_r}{N_s} \quad \Big| \quad N_s = \frac{120 f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$S = \frac{1000 - 850}{1000} = 0.15 = S_m$$

so, $\frac{0.4}{X_2} = 0.15$ $\therefore X_2 = 2.6667\ \Omega/\text{phase}$

$$T_{max} = \frac{S_m K E_2^2 R_2}{(R_2)^2 + (S_m X_2)^2} = \frac{0.15 K E_2^2 \times 0.4}{(0.4)^2 + (2.6667 \times 0.15)^2}$$

$$200 = 0.187 K E_2^2$$

$$\therefore 1066.68 = K E_2^2 \quad \text{— (ii)}$$

from (i)

T_R at $s = 0.04$

$$T_R = \frac{sR_2 \, k E_2^2}{(R_2)^2 + (sX_2)^2} = \frac{0.04 \times 0.4 \times 1066.68}{(0.4)^2 + (0.04 \times 2.667)^2}$$

$$= 99.5861 \text{ N-m}, \quad s = 0.04$$

(ii) addition rotor resistance to get $\frac{2}{3}$ of max torque

$R_2' = R_2 + R_{add}$

$T_s = \frac{2}{3} T_{max}$

or $\frac{R_2' \, k E_2^2}{(R_2')^2 + (X_2)^2} = \frac{2}{3} \times 200$ — from (i)

or $\frac{(R_2') \times 1066.68}{(R_2')^2 + (2.6667)^2} = \frac{2}{3} \times 200$

$\therefore \quad R_2' = 1.0188$

$R_2' = R_2 + R_{add}$

$1.0188 = 0.4 + R_{add}$

$\therefore \quad R_{add} = 0.58\,\Omega$

A 380 V 4-pole 50 Hz, 3 phase slip ring induction motor has a star connected stator winding and a star connected rotor winding. At standstill the voltage between two slip rings is 180 V. The stator impedance is 0.5 + j2.5 Ω. The rotor resistance and reactance at standstill are 0.06 Ω and 0.3 Ω respectively. The stator to rotor turn ratio is 1:1. The motor consumes 500 W at no-load. It develops a max torque of 150 N-m calc.

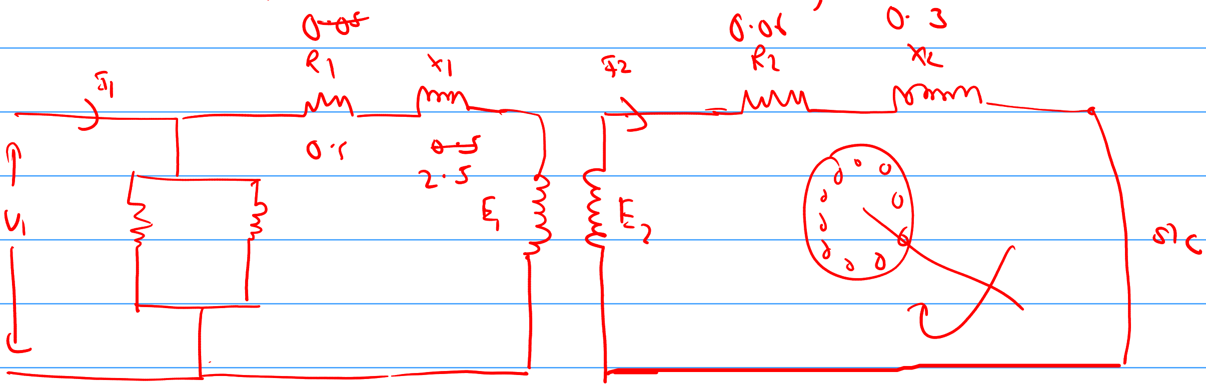
$V_2 = E_2$ (phase)

(1) Speed at which the motor develops max torque

(2) Power developed by motor when it is running at 1350 rpm.

soln:

(380V, 4 pole, 50 Hz, 3Ø induction motor)



(1) $T_{max} = ;$

$$T_{max} = \frac{s_m k E_2^2 R_2}{R_2^2 + (s_m X_2)^2} \quad , \quad s_m = \frac{R_2}{X_2} = \frac{0.06}{0.3} = 0.2$$

$$N_r = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S_m = \frac{N_s - N_r}{N_s}$$

or, $$0.2 = \frac{1500 - N_r}{1500}$$

$\therefore N_r = 1200\ rpm \rightarrow$ speed at which torque is max :

(ii) Power developed by motor when running at 1350 rpm

$$P_{out} = \frac{2\pi N}{60} \times T_{out}$$

$$= \frac{2\pi N}{60} \times \frac{sR_2 k E_2^2}{(R_2)^2 + (sX_2)^2} \quad \text{——— (*)}$$

$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1350}{1500} = 0.1$$

Given, $T_{max} = 150\ Nm$

$$T_{max} = \frac{kE_2^2}{2X_2}$$

or, $$150 = \frac{kE_2^2}{2 \times 0.3} \qquad (E_2\ (\text{phase}) = 90V$$

or, $$90 = kE_2^2 \quad \text{——— (ii)}$$

$$P_{out} = \frac{2\pi N_r}{60} \times \frac{sR_2 k E_2^2}{(R_2)^2 + (sX_2)^2}$$

$$= \frac{2\pi \times 1350}{60} \times \left[\frac{0.1 \times 0.06 \times 90}{(0.06)^2 + (0.1 \times 0.3)^2} \right]$$

$$= 16964.600\ W = 16.964\ kW$$ #

(star $V_p = \frac{V_L}{\sqrt{3}}$ ~~delta~~ delta $V_p = V_L$)

The data obtained from the test of 3ϕ star connected 400V induction motor are ÷

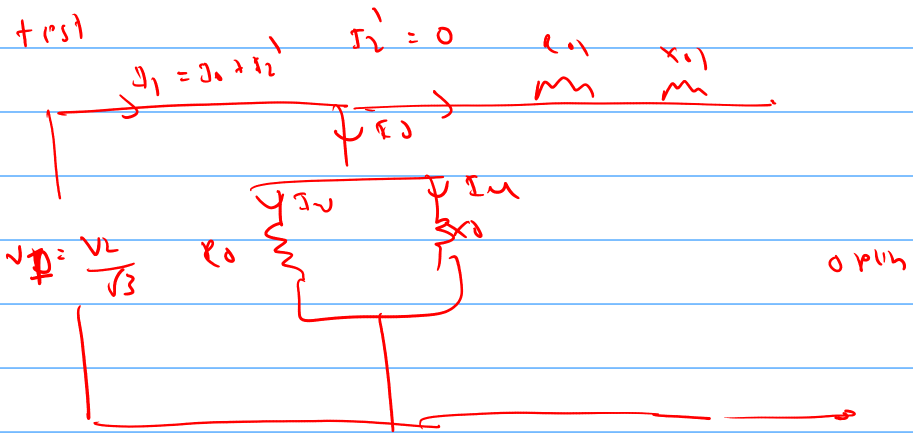
R_0, X_0

line voltage

→ no-load test : $V_1 = 400V$, $I_0 = 20A$, $W_1 = 5000W$, $W_2 = 3200W$

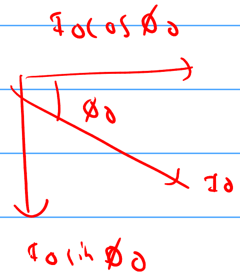
blocked rotor test : $V_{sc} = 50V$, $I_{sc} = 60A$, $W_1 = 2300W$, $W_2 = 750W$

(i) no-load test



line voltage

$$W_1 + W_2 = \sqrt{3}\, V_L I_0 \cos\phi_0$$



$$\cos\phi_0 = \frac{5000 + 3200}{\sqrt{3} \times 400 \times 20}$$

$$= 0.59178$$

$$\phi_0 = 53.71^\circ$$

$I_w = 11.8356\,A$, $I_\mu = 16.12192\,A$

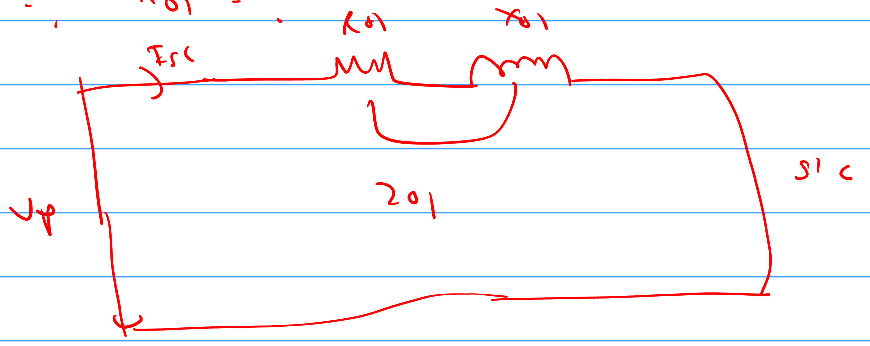
$$R_0 = \frac{V_p}{I_w} = \frac{V_L}{\sqrt{3} I_w} = \frac{400}{\sqrt{3} \times 11.8356} = 19.51\,\Omega$$

$$X_0 = \frac{V_p}{I_\mu} = \frac{V_L}{\sqrt{3} I_\mu} = 14.3246\,\Omega$$

(iii) Block rotor test

$V_{sc} = 50V$ (line), $I_{sc} = 60A$, $W_1 = 2300W$, $W_2 = 750W$

$R_{01} = ?$ $X_{01} = ?$



$$W_1 + W_2 = 3 I_{sc}^2 \times R_{01}$$

or, $2300 + 750 = 3 \times (60)^2 \times R_{01}$

$\therefore R_{01} = 0.2824 \Omega$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

$$Z_{01} = \frac{V_p}{I_{sc}} = \frac{V_L}{\sqrt{3} I_{sc}}$$

$$= \frac{V_{sc}}{\sqrt{3} I_{sc}}$$

$$= \frac{50}{\sqrt{3} \times 60} = 0.4811 \Omega$$

$$X_{01} = \sqrt{(0.4811)^2 - (0.2824)^2}$$

$$= 0.3895 \Omega$$

The power input to a 3-phase induction motor is 50 kw & the corresponding stator losses are 2 kw. Calculate (a) power developed by rotor & rotor copper loss when the slip is 3% (b) output horse power of the motor if the friction & windage losses are 1 kw (c) efficiency of the motor.

$P_{out} = P_{in}$ - strator loss - rotor cu-loss - friction and windage loss

Soln :-

$P_{in} = 50\,kW$ strator loss $= 2\,kW$

Pin to the roto = Pin - strator loss

$= 50 - 2$

$= 48\,kW$

$\boxed{\text{cu-loss at rotor} = s \times \text{i/p power to rotor}}$

$= 0.03 \times 48$

$= 1.44\,kW$

Power developed by rotor = power i/p to rotor - cu-loss at rotor

$= 48 - 1.44$

$= 46.56\,kW$

Friction and windage loss = 1 kW

O/p power = power developed by rotor - friction and windage loss

$= 46.56 - 1$

$= 45.56\,kW$

$\eta = \frac{O/P}{i/p} \times 100\% = \frac{45.56}{50} \times 100\%$

$= 91.12\%$

Slip (S) is always < 1 #

O/p power induction motor = P i/p - stator loss - rotor cu-loss - friction loss

$$Pin = \frac{Pout}{pin} \times 100\%$$

frequency of rotor at different value of S

$$\boxed{f_r = sf}$$