Chapter-8 Numericals

Wednesday, March 12, 2025 7:15 PM

(1) A current filament of 5A in the $\hat{a}_y$ direction is parallel to y axis at $x = 2m$, $z = -2m$. Find $\vec{H}$ at origin.

Soln →

For filament of infinite length

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

$$\vec{\rho} = \underbrace{(0,0,0)}_{f.p} - \underbrace{(2,0,-2)}_{i.p} = -2\hat{a}_x + 2\hat{a}_z$$

$$\rho = \sqrt{(-2)^2 + (2)^2} = 2\sqrt{2}$$

$$\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho$$

$\hat{a}_I$ ↓ line element current direction in question $\hat{a}_y$

So,

$$\hat{a}_\phi = \hat{a}_y \times \hat{a}_\rho$$

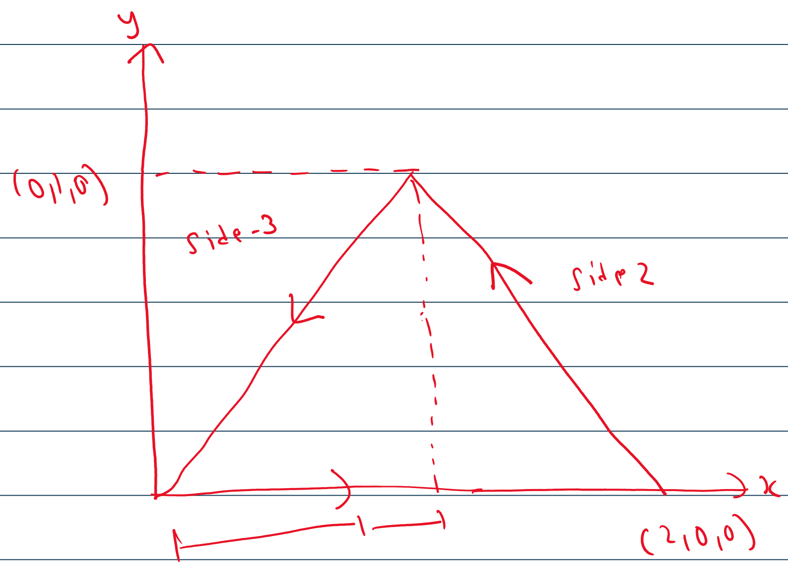
$$= \hat{a}_y \times \frac{\vec{\rho}}{|\vec{\rho}|} = \hat{a}_y \times \left[\frac{-2\hat{a}_x + 2\hat{a}_z}{2\sqrt{2}}\right]$$

$$= \frac{\hat{a}_y \times -\hat{a}_x}{\sqrt{2}} + \frac{\hat{a}_y \times \hat{a}_z}{\sqrt{2}} = \frac{\hat{a}_z}{\sqrt{2}} + \frac{\hat{a}_x}{\sqrt{2}}$$

$$= \frac{\hat{a}_x + \hat{a}_z}{\sqrt{2}}$$

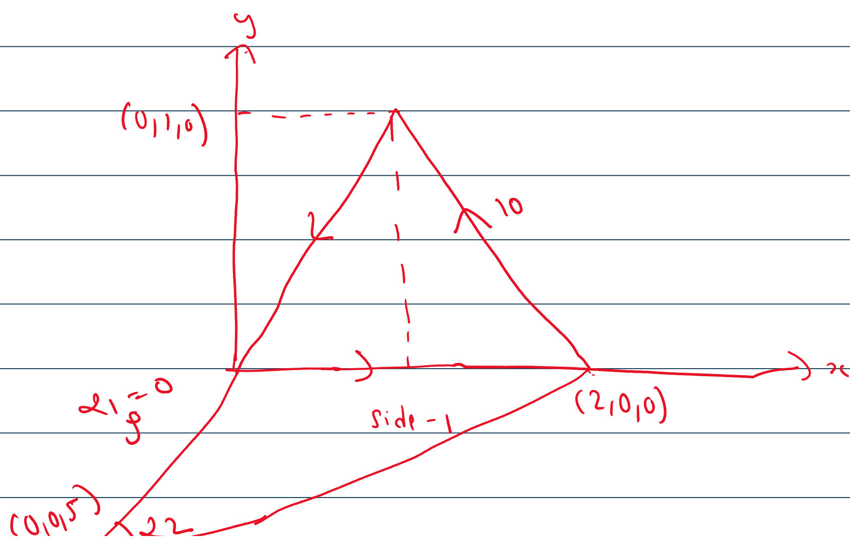
$$\therefore \quad \vec{H} = \frac{5}{2\pi \times 2\sqrt{2}} \left(\frac{\hat{a}_x + \hat{a}_z}{\sqrt{2}} \right) = \frac{5}{8\pi} \left(\hat{a}_x + \hat{a}_z \right) \text{ A/m}$$

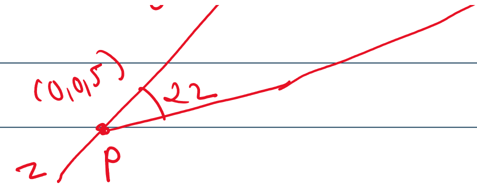
The conducting triangular loop (in figure below) carries a current of 10A. Find $\vec{H}$ at P (0,0,5) due to side 1 of the loop.



Side 1

Soln÷





we have $\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi$

both path above P so α_2, α_1 is +ve #

$\alpha_1 = 0°$, $\alpha_2 = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$ $I = 10\,A$ (given)

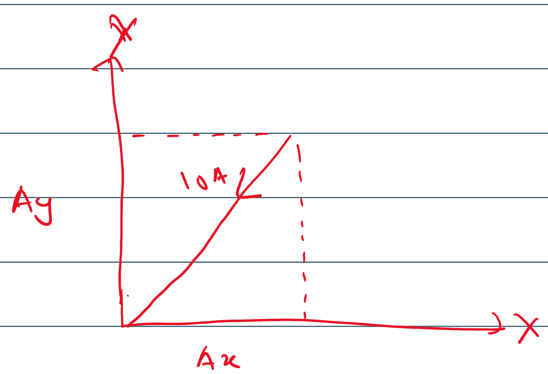
$\rho = 5$ $\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho$

$= \hat{a}_x \times \hat{a}_z = -\hat{a}_y$

$\vec{H} = \frac{40}{4\pi \times 5}[\sin 21.8° - \sin 0°](-\hat{a}_y)$

$= -0.059\,\hat{a}_y$ A/m

Due to side -3



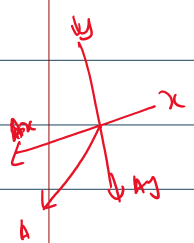
P(0, 0, 5) along x-axis

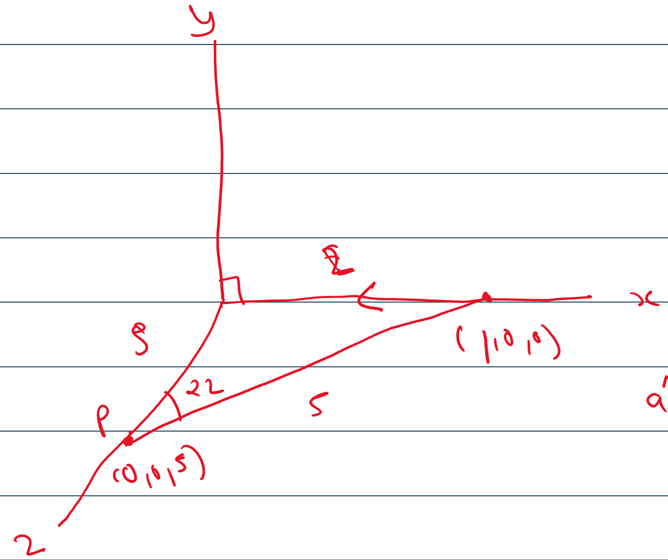
P(0, 0, 5) along y-axis

P(0, 0, 5) along x-axis

P(0, 0, 5) along y-axis

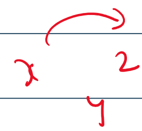
$$\vec{H} = \vec{H}_x + \vec{H}_y$$





$$\alpha_2 = \tan^{-1}\left(\frac{1}{5}\right)$$

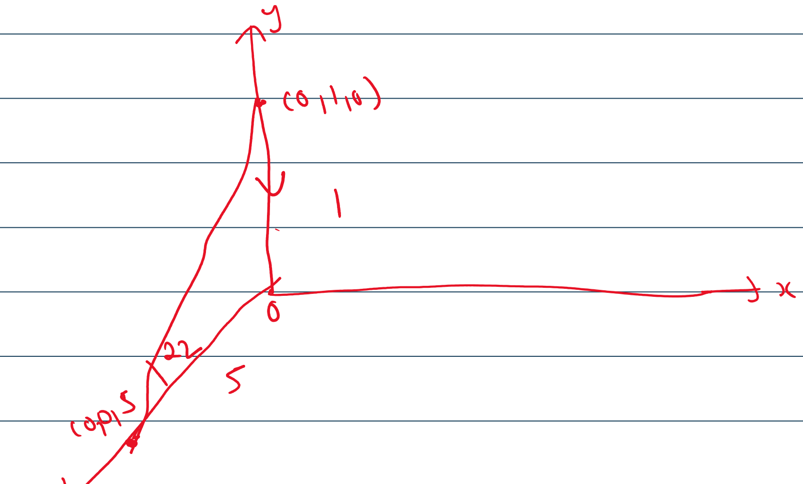
$$= 11.30^\circ$$



$$\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho$$
$$= -\hat{a}_x \times \hat{a}_z$$
$$= \hat{a}_y$$

$$\vec{H}_x = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi = \frac{10}{4\pi 5}(\sin 11.30^\circ - \sin 0^\circ)\,\hat{a}_y$$

$$= 0.0314\,\hat{a}_y$$



$$\rho = 5, \quad \alpha_1 = 0, \quad \alpha_2 = \tan^{-1}\left(\frac{1}{5}\right) = 11.30^\circ$$

$$\vec{H}_y = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi$$

$$(\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho)$$
$$= -\hat{a}_y \times \hat{a}_z$$
$$= -\hat{a}_x$$

$$\vec{H}_y = \frac{10}{4\pi\times 5}(\sin 11.30^\circ - \sin 0)\cdot -\hat{a}_x = -0.0314\,\hat{a}_x$$

$$\boxed{\vec{H} = -0.0314\,\hat{a}_x + 0.0314\,\hat{a}_y}$$

Given $\vec{H} = (10r^2/\sin\theta)\,\hat{a}_\theta + 180\,r\cos\theta\,\hat{a}_\phi$ A/m in free space. Find the current in the $\hat{a}_\theta$ direction through the conical surface $\theta = 30^\circ$, $0 \le \phi \le 2\pi$, $0 \le r \le 2$ by using either side of stoke's theorem.

Soln ÷

(θ = const, ϕ vary, r vary)

$$\vec{H} = \frac{10r^2}{\sin\theta}\,\hat{a}_\theta + 180\,r\cos\theta\,\hat{a}_\phi \text{ A/m}$$

$\hat{a}_\theta$ direction

Stoke's theorem

$$\oint \vec{H}\cdot d\vec{l} = \int_S (\nabla \times \vec{H})\, ds$$

Now,

For spherical co-ordinate system

$$\vec{dl} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$$

for taking path

initial

$$\begin{tikzcd}
A\,(0,\underset{\text{const}}{30^\circ},0) \arrow[r, "d\phi \text{ change}"] & B\,(0,30^\circ,2\pi) \arrow[r, "dr \text{ change}"] & C\,(2,30^\circ,2\pi) \arrow[d, "d\phi \text{ change}"] \\
 & & D\,(2,30^\circ,0) \arrow[llu, "dr \text{ change}"]
\end{tikzcd}$$

cyclic
must form a closed path like this #

$$L.H.S = \int_{abcda} \vec{H}\cdot\vec{dl}$$

$$= \int_a^b \vec{H}\cdot\vec{dl} + \int_b^c + \int_c^d + \int_d^a \vec{H}\cdot\vec{dl}$$

= for a-b and c-d only ϕ changes

so $\vec{dl} = r\sin\theta\,d\phi\,\hat{a_\phi}$

$$\vec{H}\cdot\vec{dl} = 180\,r\cos\theta \cdot r\sin\theta\,d\phi$$

$$= 180\,r^2\sin\theta\cos\theta\,d\phi$$

$$= 90\,r^2\sin 2\theta\,d\phi$$

similarly

from b to c and D to a, only r changes

$$\vec{dl} = dr\,\hat{a}_r$$

$$\vec{H}\cdot\vec{dl} = 0$$

Now,

$$\oint_c \vec{H}\cdot\vec{dl} = \int_a^b \vec{H}\cdot\vec{dl} + \int_c^d \vec{H}\cdot\vec{dl}$$

$$= \int_{\phi=0}^{2\pi} 90r^2\sin 2\theta\, d\phi \;(\theta=30^\circ, r=0) + \int_{\phi=2\pi}^{0} 90r^2 \sin 2\theta\, d\phi \;(r=2,\ \theta=30^\circ)$$

$$= 0 + 90\times(2)^2\times\sin 60^\circ \times \int_{2\pi}^{0} d\phi$$

$$= -90\times 2^2 \times \sin 60^\circ \times 2\pi$$

$$= -1957.9103$$

Now, R.H.S

For spherical co-ordinate system

$$\nabla\times\vec{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right]\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\hat{a}_\theta$$

$$+\frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right)\hat{a}_\theta$$

Here, $\theta = 30^\circ \rightarrow$ const other vary (surface ma 2 vary chayo one const)

Here, $\theta = 30^\circ \rightarrow$ const [illegible] (2 vary chayo one const)

$$dS = r \sin\theta \, dr \, d\theta \, \hat{a}_\theta$$

$(\hat{a}_\theta \cdot \hat{a}_\theta = 1)$ other $= 0$

Now,

$$(\nabla \times \vec{H}) \, d\vec{S} = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial (r H_\phi)}{\partial r}\right] \cdot r \sin\theta \, dr \, d\phi$$

$$= \frac{1}{r}\left[\frac{1}{\sin\theta} \cdot 0 - \frac{\partial}{\partial r}\left(r \cdot 180 \, r \cos\theta\right)\right] \cdot r \sin\theta \, dr \, d\phi$$

$$= \frac{1}{r}\left[-180 \cos\theta \cdot 2r\right] \cdot r \sin\theta \, dr \, d\phi$$

$$= -360 \cos\theta \sin\theta \, r \, dr \, d\phi$$

$$= -180 \, r \sin 2\theta \, dr \, d\phi$$

Now,

$$\int_S (\nabla \times \vec{H}) \, ds = -\int_S 180 r \sin 2\theta \, dr \, d\phi$$

$$\theta = 30^\circ$$

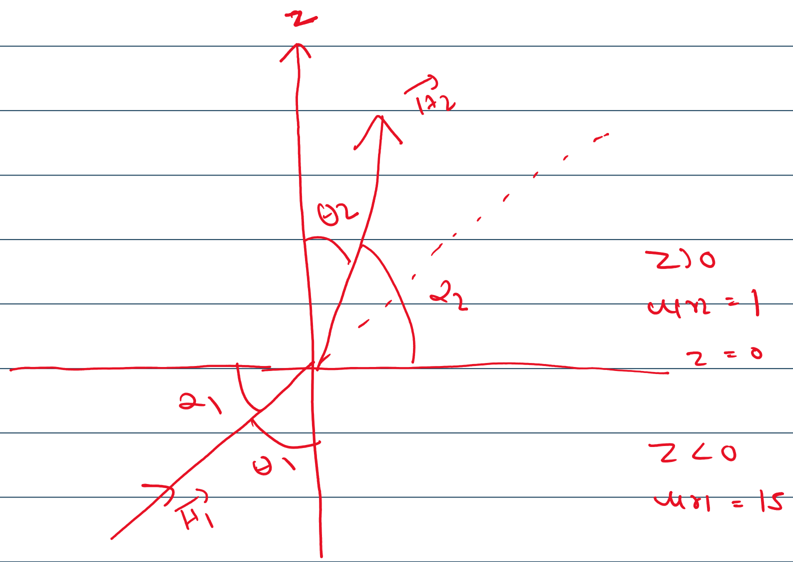
$$= -180 \times \sin 60^\circ \int_0^2 r \, dr \int_{\phi = 0}^{2\pi} d\phi$$

$$= -1957.9103$$

$\therefore$ L.H.S = R.H.S stokes theorem is verified #

Q. In region I ($z = 0^-$), $\vec{B}_1 = 1.2\hat{a}_x + 0.8\hat{a}_y + 0.4\hat{a}_z$ (T). Find H_2 (at $z = 0^+$) and the angles between the field vectors and a tangent to the interface $\mu_{r1} = 15$, $\mu_{r2} = 1$.

Soln:



$$\vec{H}_2 = \vec{H}_{N2} + \vec{H}_{t2} \quad \text{—— (i)}$$

Given, $\vec{B}_1 = 1.2\hat{a}_x + 0.8\hat{a}_y + 0.4\hat{a}_z$ T

Since $z = 0$ sepperates two regions, we must have

$$B_{N1} = \vec{B}_1 \cdot \hat{a}_z = (1.2\hat{a}_x + 0.8\hat{a}_y + 0.4\hat{a}_z) \cdot \hat{a}_z$$

$$= 0.4 \text{ T}$$

$$\vec{B}_{N1} = B_{N1}\hat{a}_z = 0.4\hat{a}_z \text{ T}$$

$$\vec{B}_1 = \vec{B}_{N1} + \vec{B}_{t1}$$

or, $\vec{B}_{t1} = \vec{B}_1 - \vec{B}_{N1}$

$$= 1.2\hat{a}_x + 0.8\hat{a}_y \text{ T}$$

$$\vec{H}_{t1} = \frac{\vec{B}_{t1}}{\mu_1} = \frac{\vec{B}_{t1}}{\mu_0\mu_{r1}} = \frac{(1.2\hat{a}_x + 0.8\hat{a}_y)}{4\pi \times 10^{-7} \times 15}$$

$$\vec{H}_{t1} = \frac{\vec{B}_{t1}}{\mu_1} = \frac{\vec{B}_{t1}}{\mu_0\mu_{r1}} = \frac{(1.2\hat{a}_x + 0.8\hat{a}_y)}{4\pi\times10^{-7}\times15}$$

$$= 63.66\times10^3\,\hat{a}_x + 42.44\times10^3\,\hat{a}_y \text{ A/m}$$

Using boundary conditions, we calculate $\vec{H}_{t2}$ as :-

$$\vec{H}_{t1} = \vec{H}_{t2}$$

$$\therefore \vec{H}_{t2} = 63.66\times10^3\,\hat{a}_x + 42.44\times10^3\,\hat{a}_y \text{ A/m}$$

Now

$$\vec{B}_{N1} = \vec{B}_{N2}$$

or $0.4\,\hat{a}_z = \mu_0\mu_{r2}\vec{H}_{N2}$

or $$\vec{H}_{N2} = \frac{0.4\,\hat{a}_z}{\mu_0\mu_{r2}} = \frac{0.4\,\hat{a}_z}{4\pi\times10^{-7}\times1} = 318.309\times10^3\,\hat{a}_z \text{ A/m}$$

$$\therefore \vec{H}_2 = \vec{H}_{t2} + \vec{H}_{N2}$$

$$= 63.66\times10^3\,\hat{a}_x + 42.44\times10^3\,\hat{a}_y + 318.309\times10^3\,\hat{a}_z \text{ A/m}$$

Calculation at θ_1 and θ_2

$$\cos\theta_1 = \frac{|\vec{H}_{N1}|}{|\vec{H}_1|}$$

$$\vec{H}_{N1} = \frac{1}{\mu_1}\vec{B}_{N1} = \frac{\vec{B}_{N1}}{\mu_0\mu_{r1}} = \frac{0.4\,\hat{a}_z}{4\pi\times10^{-7}\times15} = 21.22\times10^3\,\hat{a}_z \text{ A/m}$$

$$H_{N1} = \frac{1}{\mu_1} B_{N1} = \frac{B_{N1}}{\mu_0 \mu_{r1}} = \frac{0.4\,\hat{a}_z}{4\pi\times10^{-7}\times15} = 21.22\times10\,\hat{a}_z \text{ A/m}$$

$$|\vec{H}_{N1}| = 21.22\times10^3 \text{ A/m}$$

$$\vec{H}_1 = \vec{H}_{N1} + \vec{H}_{t}$$

$$= 63.66\times10^3\,\hat{a}_x + 42.44\times10^3\,\hat{a}_y + 21.22\times10^3\,\hat{a}_z$$

$$|\vec{H}_1| = 79.39\times10^3 \text{ A/m}$$

$$\cos\theta_1 = \frac{21.22\times10^3}{79.38\times10^3} = 0.2672 \Rightarrow \theta_1 = 74.5^\circ$$

now, $\alpha_1 = 90^\circ - \theta_1 = 90^\circ - 74.5^\circ = 15.5^\circ$

$$\cos\theta_2 = \frac{|\vec{H}_{N2}|}{|\vec{H}_2|} = \frac{318.309\times10^3}{327.37\times10^3} = 0.9723 \Rightarrow \theta_2 = 13.515^\circ$$

$$\alpha_2 = 90^\circ - \theta_2 = 90^\circ - 13.515 = 76.484^\circ$$

$$\alpha_1 = 15.5^\circ,\ \alpha_2 = 76.484^\circ$$

The magnetic field intensity is given in a certain region of free space as $\vec{H} = \frac{x+2y}{z^2}\,\hat{a}_y + \frac{2}{z}\,\hat{a}_z$ A/m. Find the total current passing through the surface $z=4$ $1<x<2$, $3<y<5$ in the $\hat{a}_z$ direction.

$$I = \int_S \vec{J} \cdot \vec{ds}$$

$$= \int_S (\nabla \times \vec{H}) \vec{ds}$$

Now,

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \hat{a}_y$$

$$+ \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z$$

$$\vec{ds} = dx\,dy\,\hat{a}_z$$

$$(\nabla \times \vec{H}) \vec{ds} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) dx\,dy$$

$$= \left[\frac{\partial}{\partial x} \left(\frac{x+2y}{z^2} \right) - \frac{\partial}{\partial y} \left(0 \right) \right] dx\,dy$$

$$= \frac{1}{z^2} \left(1 + 0 \right) dx\,dy$$

$$= \frac{1}{z^2} dx\,dy$$

$$I = \int_S \frac{1}{z^2} dx\,dy = \int_S \frac{1}{4^2} dx\,dy = \frac{1}{4^2} \int_1^2 dx \int_3^5 dy$$

$$I = \int_S \frac{1}{z^2} dx\,dy = \int_S \frac{1}{4^2} dx\,dy = \frac{1}{4^2} \int_1 dx \int_3 dy$$

$(z=4)$

$$= \frac{1}{8} A$$

\# Calculate the value of the vector current density in a cylindrical co-ordinate at P_1 ($\rho = 1.5$, $\phi = 90^\circ$, $z = 0.5$) if $\vec{H} = \frac{2}{\rho} (\cos 0.2\phi)\, \hat{a}_\rho$.

Now,

$$\vec{J} = \nabla \times \vec{H}$$

$$= \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \hat{a}_\phi + \frac{1}{\rho} \left(\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) \hat{a}_z$$

$$= \left(\frac{1}{\rho} \cdot 0 - 0 \right) \hat{a}_\rho + \left[\frac{\partial}{\partial z} \left(\frac{2}{\rho} \cos 0.2\phi \right) - 0 \right] \hat{a}_\phi + \frac{1}{\rho} \left(0 - \frac{\partial}{\partial \phi} \left(\frac{2}{\rho} \cos 0.2\phi \right) \right) \hat{a}_z$$

$$= 0 + 0 + \frac{1}{\rho} \left(0 - \frac{2}{\rho} (-\sin 0.2\phi) \times 0.2 \right) \hat{a}_z$$

$$= 2 \times 0.2 \times \frac{1}{\rho^2} \times \sin 0.2\phi \;\hat{a}_z$$

At P ($\rho = 1.5$, $\phi = 90^\circ$, $z = 0.5$)

$$= 2 \times 0.2 \times \frac{1}{1.5^2} \times \sin 0.2 \times 90 \;\hat{a}_z$$

$= \quad 0.054\,\hat{a}_z$

$\vec{J} = 0.054\,\hat{a}_z \text{ A/m}^2$

The magnetic field intensity in a certain region of space is given by $\vec{H} = (2\rho + z)\,\hat{a}_\rho + \frac{2}{z}\,\hat{a}_z$ A/m. Find the total current passing through the surface $\rho = 2$, $\frac{\pi}{4} < \phi < \frac{\pi}{2}$, $3 < z < 5$ in $\hat{a}_\rho$ direction.

Soln:

$$I = \int_S \vec{J} \cdot \vec{ds}$$

$$= \int_S (\nabla \times \vec{H})\, \vec{ds}$$

$$\vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$(\nabla \times \vec{H})\, \vec{ds} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right) \cdot \rho\, d\phi\, dz$$

$$= \left(\frac{1}{\rho}\frac{\partial}{\partial \phi}\left(\frac{2}{z}\right) - \frac{\partial}{\partial z}(0)\right) \cdot \rho\, d\phi\, dz$$

$$= 0$$

$$\therefore I = 0 \text{ A}$$

A current distribution give rise to the vector magnetic potential $\vec{A} = x^2y\,\hat{a}_x + y^2x\,\hat{a}_y - 4xyz\,\hat{a}_z$ wb/m. Calculate the following

$\psi = \int_s \vec{B}\cdot\vec{ds}$

(a) $\vec{B}$ at $(-1,2,5)$

(b) flux through the surface defined by $z=1$, $0 \le x \le 1$, $-1 \le y \le 4$

soln ÷

vector mag potential

(i) $\vec{B} = \nabla \times \vec{A}$

H ko sata A ho yeha

$$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$$

$$= \left(\frac{\partial}{\partial y}(-4xyz) - \frac{\partial}{\partial z}(y^2x)\right)\hat{a}_x + \left(\frac{\partial}{\partial z}(x^2y) - \frac{\partial}{\partial x}(-4xyz)\right)\hat{a}_y$$

$$+ \left(\frac{\partial}{\partial x}(y^2x) - \frac{\partial}{\partial y}(x^2y)\right)\hat{a}_z$$

$$= -4xz\,\hat{a}_x + 4yz\,\hat{a}_y + (y^2 - x^2)\,\hat{a}_z$$

at $(-1,2,5)$

$$\vec{B} = 20\,\hat{a}_x + 40\,\hat{a}_y + 3\,\hat{a}_z \quad \text{wb/m}^2 / T$$

(b) $\psi = \oint_s \vec{B}\cdot\vec{ds}$

(b) $\psi = \oint_S \vec{B} \cdot \vec{ds}$

so in z direction

$\vec{ds} = dx\,dy\,\hat{a}_z$ (z = const)

$\psi = \oint_S (y^2 - x^2)\, dx\, dy$

$= \oint_S y^2\, dx\, dy - \oint_S x^2\, dx\, dy$

$= \int_{-1}^{4} y^2 dy \int_0^1 dx - \int_0^1 x^2 dx \int_{-1}^{4} dy$

$= \frac{65}{3} - \frac{5}{3} = \frac{60}{3} = 20$ wb

Evaluate both side of stoke's theorem for the field $\vec{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y$ A/m. Rectangular path around the region $2 \le x \le 5$, $-1 \le y \le 1$, $z = 0$. ds is $\hat{a}_z$.

Soln:

Now,

$\oint \vec{H} \cdot \vec{dl} = \oint_S (\nabla \cdot \vec{H})\, \vec{ds}$

R.H.S $= \oint \vec{H} \cdot \vec{dl}$

$$R.H.S = \oint_{z=\text{const}} \vec{H} \cdot \vec{dl}$$

path formation

$$A(2,-1,0) \xrightarrow{x\text{-change}} B(5,-1,0) \xrightarrow{y\text{ change}} C(5,1,0)$$

$$C(5,1,0) \xrightarrow{x\text{-change}} D(2,1,0) \xrightarrow{y\text{-change}} A(2,-1,0)$$

y-change can also happen again result is same

anyway make it cyclic rectangular path :

$$L.H.S = \int_a^b \vec{H} \cdot \vec{dl} + \int_b^c \vec{H} \cdot \vec{dl} + \int_c^d \vec{H} \cdot \vec{dl} + \int_d^a \vec{H} \cdot \vec{dl}$$

For a-b and c-d x only change

$$\boxed{\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z \quad \text{for rectangular}}$$

$$\vec{H} \cdot \vec{dl} = (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\,dx\,\hat{a}_x$$

$$= 6xy\,dx$$

For b-c and d-a , y-only change

$$\vec{H} \cdot \vec{dl} = (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\,dy\,\hat{a}_y$$

$$= -3y^2\,dy$$

Now,

Now,

$$\oint \vec{H}\cdot d\vec{l} = \underset{(a-b)}{\int_{\substack{x=2\\(y=-1)}}^{5} 6xy\,dx} + \underset{(b-c)}{\int_{y=-1}^{1} -3y^2\,dy} + \underset{(c-d)}{\int_{x=5\ (y=1)}^{2} 6xy\,dx}$$

$$+ \underset{(d-a)}{\int_{y=1}^{-1} -3y^2\,dy}$$

$$= -63 - 2 - 63 + 2$$

$$= -126\ A$$

$$R.H.S = \int_s (\nabla \times \vec{H})\, d\vec{s} \qquad d\vec{s} = dx\,dy\,\hat{a}_z$$

$$(\nabla \times \vec{H})\cdot d\vec{s} = \underbrace{\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)}_{(\hat{a}_z \cdot \hat{a}_z)} dx\,dy$$

$$= \left(0 - \frac{\partial(6xy)}{\partial y}\right) dx\,dy$$

$$= -6x\,dx\,dy$$

$$\text{Now } R.H.S = \int_s -6x\,dx\,dy$$

$$= -6\int_2^5 x\,dx \int_{-1}^{1} dy$$

$= -126\ A$

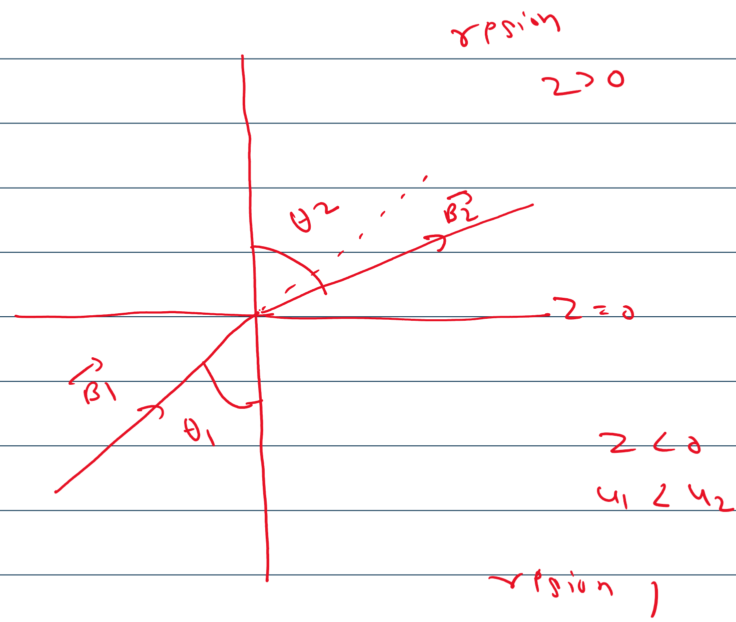
$\therefore$ L.H.S = R.H.S Stoke theorem verified #

not
R.H.S of stoke theorem $= \int_S (\nabla \times \vec{H})\, \vec{ds}$ (current)

$= \int_S J \cdot \vec{ds} = \vec{I}$

stoke to current dinchg

let $\mu_1 = 4\mu H/m$ in region 1 where $z > 0$ while $\mu_2 = 7\mu H/m$ in region 2 wherever $z < 0$. moreover let $\vec{K} = 80\hat{a}_x$ A/m on the surface $z = 0$. If $\vec{B_1} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$ mT in region 1, find $\vec{B_2}$.



$B_{N1} = \vec{B_1} \cdot \hat{a}_z = 10^{-3}$ T

$\vec{B_{N1}} = B_N \hat{a}_z = 10^{-3}\hat{a}_z$ T

Using boundary condition

$$B_{N2} = B_{N1}$$

$$\vec{B}_{N2} = 10^{-3}\,\hat{a}_z \text{ T}$$

$$\vec{B}_1 = \vec{B}_{N1} + \vec{B}_{t1}$$

or $(2\hat{a}_x - 3\hat{a}_y + \hat{a}_z) \times 10^{-3} = 10^{-3}\hat{a}_z + \vec{B}_{t1}$

$\therefore\ \vec{B}_{t1} = (2\hat{a}_x - 3\hat{a}_y) \times 10^{-3}$ T

Using boundary condition

$$\vec{H}_{t1} - \vec{H}_{t2} = \hat{a}_{N12} \times \vec{K}$$

$1-2 \rightarrow \hat{a}_z$ T

$$\vec{H}_{t1} - \vec{H}_{t2} = \hat{a}_z \times 80\,\hat{a}_x$$

x, y, z

or $\dfrac{\vec{B}_{t1}}{\mu_0\mu_{r1}} - \dfrac{\vec{B}_{t2}}{\mu_0\mu_{r2}} = 80\,\hat{a}_y$

or $\dfrac{\vec{B}_{t1}}{\mu_{r1}} - \dfrac{\vec{B}_{t2}}{\mu_{r2}} = \mu_0\, 80\,\hat{a}_y$

or $\dfrac{(2\hat{a}_x - 3\hat{a}_y) \times 10^{-3}}{4 \times 10^{-6}} - \dfrac{\vec{B}_{t2}}{7 \times 10^{-6}} = 4\pi \times 10^{-7} \times 80\,\hat{a}_y$

or $\left(\dfrac{2\times 10^{-3}\,\hat{a}_x - 3\times 10^{-3}\,\hat{a}_y}{4\times 10^{-6}} - 4\pi\times 10^{-7} \times 80\,\hat{a}_y\right) 7\times 10^{-6} = \vec{B}_{t2}$

$\therefore\ \vec{B}_{t2} = 3.5\times 10^{-3}\,\hat{a}_x - 5.25\times 10^{-3}\,\hat{a}_y$

$$\therefore \vec{B_2} = \vec{B_{N2}} + \vec{B_{t2}}$$

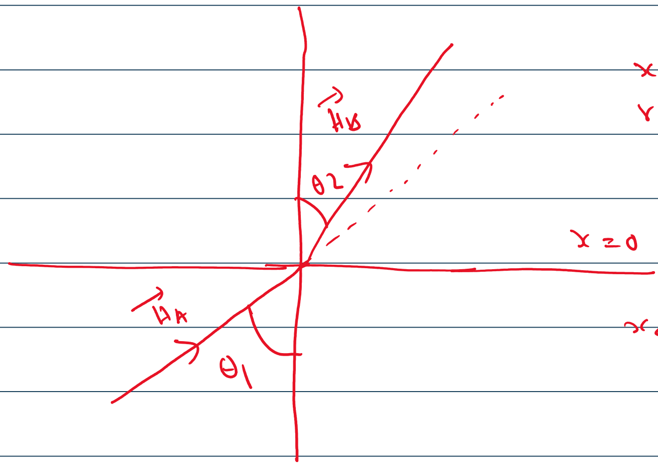
$$= 3.5\times10^{-3}\hat{a}_z - 5.25\times10^{-3}\hat{a}_y + 10^{-3}\hat{a}_z \text{ T}$$

\# let the permeability be 5μ H/m in region A where $x<0$ and 20μ H/m in region B where $x>0$. If there is a surface current density

$$K = 150\hat{a}_y - 200\hat{a}_z \text{ A/m} \quad \text{at } x=0 \quad \text{and if } \vec{H_A} = 300\hat{a}_x - 400\hat{a}_y + 500\hat{a}_z \text{ A/m}$$

Find (a) $|\vec{H_{tA}}|$ (b) $|\vec{H_{NA}}|$ (c) $|\vec{H_{tB}}|$ (d) $|\vec{H_{NB}}|$

soln:-



$$\text{Now} \quad \vec{H_A} = 300\hat{a}_x - 400\hat{a}_y + 500\hat{a}_z \text{ A/m}$$

$$H_{NA} = \vec{H_A}\cdot\hat{a}_x = 300$$

$$\vec{H_{NA}} = 300\hat{a}_x$$

$$\vec{H_A} = \vec{H_{NA}} + \vec{H_{TA}}$$

$$\text{or} \quad 300\hat{a}_x - 400\hat{a}_y + 500\hat{a}_z = 300\hat{a}_x + \vec{H_{TA}}$$

$\therefore \quad \vec{H}_{TA} = -400\,\hat{a}_y + 500\,\hat{a}_z$

$$|\vec{H}_{TA}| = \sqrt{(-400)^2 + 500^2}$$

$$= 640.31 \text{ A/m}$$

Using boundary condition

1-2 $\uparrow \hat{a}_x$

$$\vec{H}_{TA} - \vec{H}_{TB} = \hat{a}_{N_{12}} \times \vec{K}$$

or $\quad -400\,\hat{a}_y + 500\,\hat{a}_z - (\hat{a}_x)(150\,\hat{a}_y - 200\,\hat{a}_z) = \vec{H}_{TB}$

or $\quad \vec{H}_{TB} = -400\,\hat{a}_y + 500\,\hat{a}_z - 150\,\hat{a}_z + 200(-\hat{a}_y)$

or, $\quad \vec{H}_{TB} = -400\,\hat{a}_y - 200\,\hat{a}_y + 500\,\hat{a}_z - 150\,\hat{a}_z$

or $\quad \vec{H}_{TB} = -600\,\hat{a}_y + 350\,\hat{a}_z$ A/m

Again,

Using boundary condition

$$\vec{B}_{NB} = \vec{B}_{NA}$$

or, $\quad \cancel{\mu_0}\,\mu_{r2}\,\vec{H}_{NB} = \cancel{\mu_0}\,\mu_{r1}\,\vec{H}_{NA}$

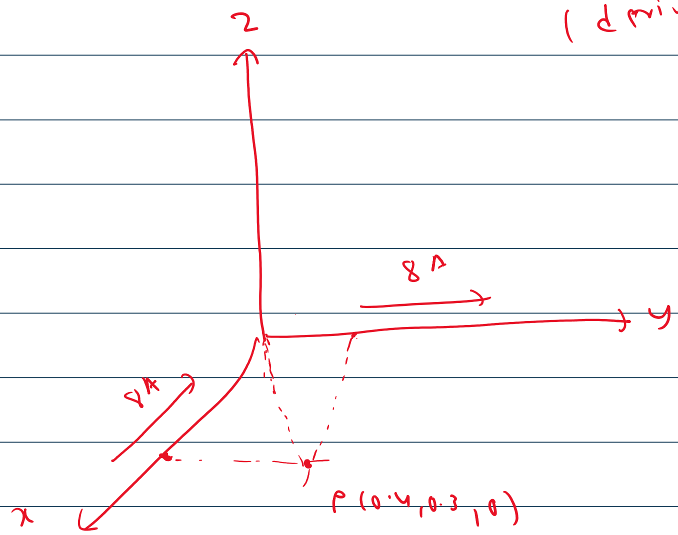
or $\quad \vec{H}_{NB} = \dfrac{\mu_{r1}}{\mu_{r2}}\,\vec{H}_{NA}$

$$= \frac{5\mu}{20\mu} \times 300\,\hat{a}_x$$

$= 750\,\hat{a}_x$

\# Determine $\vec{H}$ at $P(0.4, 0.3, 0)$ in the field from an 8A filament current directed inward from infinity to the origin on the positive x-axis, and outward to infinity along the y-axis as shown in figure below:

(derivation ma yo axes used)

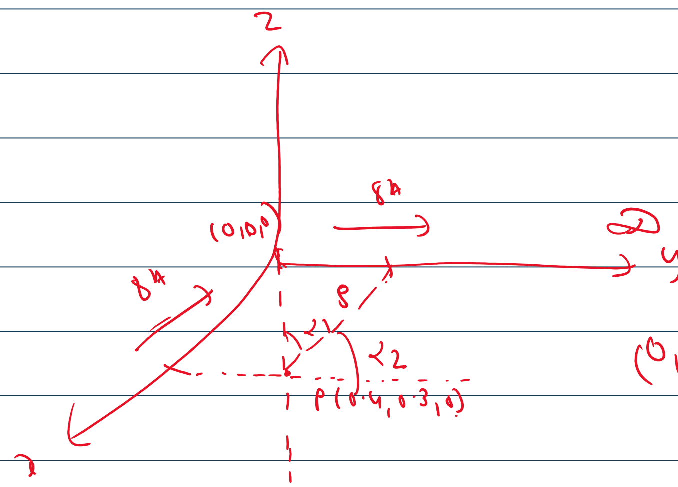


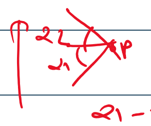
Soln ÷

For a finite length of conductor carrying a current

$$\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi$$

(Due to y-axis)





α_1 -ve $\quad \alpha_2 = +ve$

(0,0,0) behind P(0.4, 0.3, 0)

$$\vec{H}_y = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi$$

$$\vec{\rho} = 0.4\hat{a}_x \qquad |\vec{\rho}| = 0.4$$

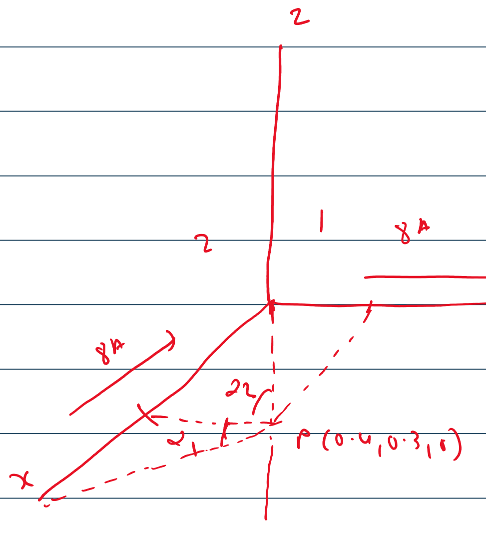
$$\hat{a}_\phi = \hat{a}_i \times \hat{a}_\rho = \hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$\alpha_1 = -\tan^{-1}\left(\frac{0.3}{0.4}\right) = -36.869^\circ, \quad \alpha_2 = 90^\circ \quad (\text{at infinity})$$

$$\vec{H}_y = \frac{8}{4\pi\, 0.4}[\sin 90^\circ + \sin 36.87^\circ]\,(-\hat{a}_z)$$

$$= -2.546\,\hat{a}_z$$

For x-axis



muni ko -ve

$$-\tan^{-1}\left(\frac{\infty}{0.3}\right)$$
$$-90^\circ$$

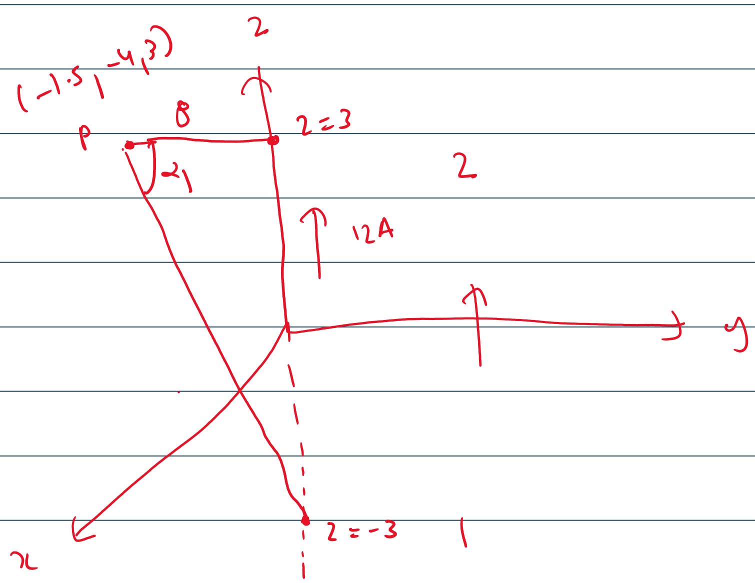
$$\alpha_1 = -90^\circ, \quad \alpha_2 = \tan^{-1}\left(\frac{0.4}{0.3}\right) = 53.13^\circ$$

$$\rho_\phi = 0.3$$
$$a_\phi = -\hat{a}_x \times \hat{a}_y = -\hat{a}_z$$

$$\vec{H} = \frac{\rho}{4\pi(0.3)}\left[\sin 53.1^\circ + \sin 90^\circ\right](-\hat{a}_z) \quad = -\frac{12}{\pi}\,\hat{a}_z \text{ A/m}$$

$$\vec{H} = \vec{H}_x + \vec{H}_y = -6.37\,\hat{a}_z \text{ A/m}$$

\# Find the vector magnetic field $\vec{H}$ in at $P(-1.5, -4, 3)$ caused by a current filament of 12A in the $\hat{a}_z$ direction on the z-axis and extending from $z=-3$ to $z=3$



$\alpha_2 = 0$

$\rho = \text{F.P} - \text{I.P}$

$= (-1.5, -4, 3) - (0, 0, 3)$

$= -1.5\,\hat{a}_x - 4\,\hat{a}_y$

$|\rho| = \sqrt{(-1.5)^2 + (-4)^2} = 4.272$

$$\alpha_1 = \tan^{-1}\left(\frac{6}{4.272}\right) = 54.55^\circ$$

$x \to y$
$\uparrow \quad \downarrow$
z

$$\hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho$$

$$= \hat{a}_z \times \frac{\vec{\rho}}{|\rho|} = \hat{a}_z \times \left(\frac{-1.5\hat{a}_x - 4\hat{a}_y}{4.272}\right)$$

$$= \frac{-1.5\hat{a}_y + 4\hat{a}_x}{4.272}$$

$$= -0.351\hat{a}_y + 0.936\hat{a}_x$$

Now,

$$\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi$$

$$= \frac{12}{4\pi(4.272)}(\sin 0 - \sin 54.55^\circ)(0.936\hat{a}_x - 0.351\hat{a}_y)$$

$$= -0.182(0.936\hat{a}_x - 0.351\hat{a}_y)$$

$$= (-0.17\hat{a}_x + 0.064\hat{a}_y) \text{ A/m}$$

\# Given the magnetic vector potential $\vec{A} = -\frac{\rho^2}{4}\hat{a}_z \frac{Wb}{m}$, calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2m$, $0 \le z \le 5m$.

Soln:

$$\vec{B} = \nabla \times \vec{A}$$

$$= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\hat{a}_z$$

Given, $\vec{A} = -\frac{\rho^2}{4}\hat{a}_z$, $A_\rho = 0$, $A_\phi = 0$, $A_z = -\frac{\rho^2}{4}$

$$\vec{B} = -\frac{\partial}{\partial \rho}\left(-\frac{\rho^2}{4}\right)\hat{a}_\phi$$

$$= \frac{1}{4} \times 2\rho\, \hat{a}_\phi = \frac{1}{2}\rho\, \hat{a}_\phi$$

$$\text{Total flux } (\psi) = \int_S \vec{B}\cdot\vec{ds}$$

$$= \int_S \frac{1}{2}\rho\, \hat{a}_\phi\, d\rho\, dz\, \hat{a}_\phi$$

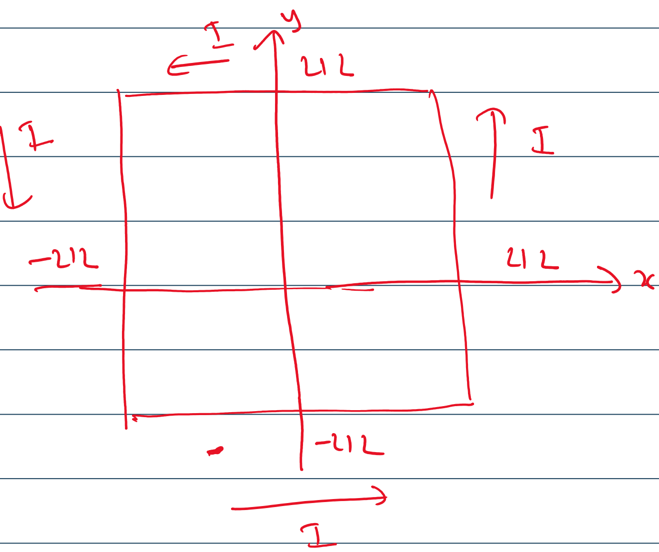
$$= \int_S \frac{1}{2}\rho\, d\rho\, dz$$

$$= \frac{1}{2}\int_1^2 \rho\, d\rho \int_0^5 dz$$

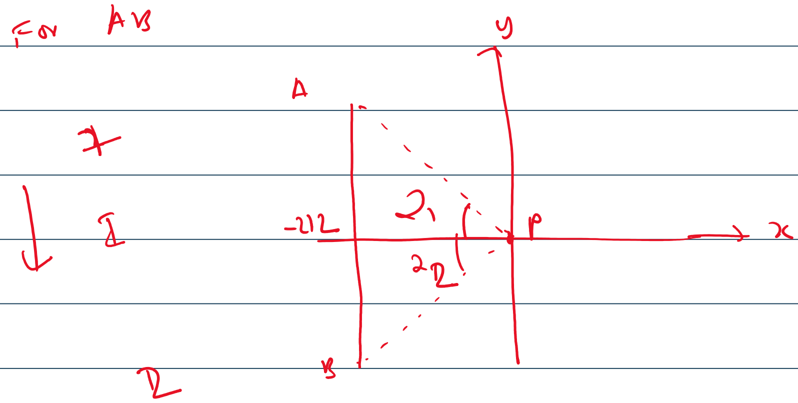
$$= 3.75 \text{ wb}$$

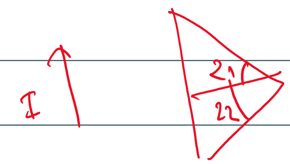
Find $\vec{H}$ at the centre of a square current loop at side L

I $\uparrow y$



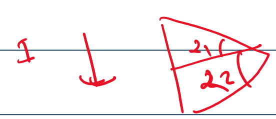
For AB





α_2 −ve

but



$$\rho = \frac{L}{2}, \quad \alpha_1 = -45^\circ, \quad \alpha_2 = 45^\circ$$

$$\vec{H}_y = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi$$

$\alpha_1 \rightarrow$ −ve

$$\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho = -\hat{a}_y \times a_x = \hat{a}_z$$

$$\vec{H}_y = \frac{I}{2\pi L}\left[\sin 45^\circ - \sin(-45^\circ)\right]\hat{a}_\phi$$

$I \quad 2\sin 45^\circ \, \hat{a}_\phi$

$$= \frac{I}{\cancel{2}\pi 2} \cancel{2} \sin 45^\circ \, \hat{a}_\phi$$

$$= \frac{I}{\pi 2\sqrt{2}} \hat{a}_\phi$$

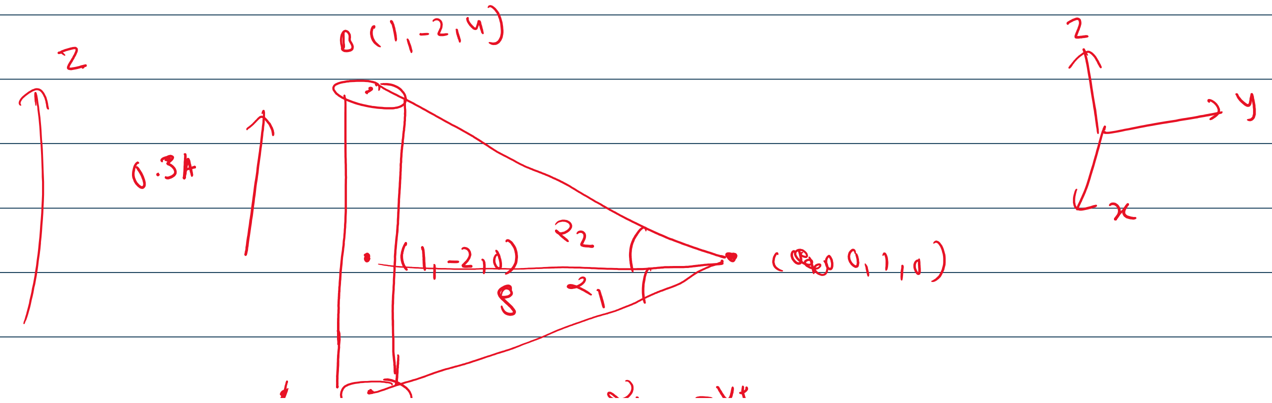
$$= \frac{I}{\pi 2\sqrt{2}} \hat{a}_z$$

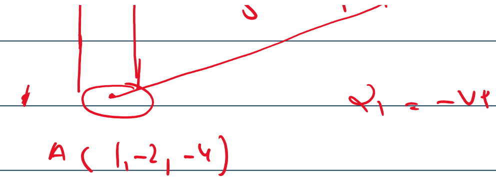
Because of symmetry along both x-axis and y-axis

$$\boxed{\begin{aligned} \vec{H} &= 4\vec{H_y} \\ &= \frac{4I}{\pi 2\sqrt{2}} \hat{a}_z = \frac{4I}{\pi 2\sqrt{2}} \hat{a}_N \end{aligned}}$$

\# A current of 0.3A in the $\hat{a}_z$ direction in free space is in a filament parallel to the z-axis and passing through the point (1,-2,0). Find the magnetic field intensity $\vec{H}$ at (0,1,0) if the filament lies in the interval $-4 < z < 4$.

Soln ÷





Now,

$$\vec{H} = \frac{I}{4\pi\rho}\left(\sin\alpha_2 - \sin\alpha_1\right)\hat{a}_\phi$$

$$\vec{\rho} = F.P - I.P = (0,1,0) - (1,-2,0)$$

$$= -\hat{a}_x + 3\hat{a}_y$$

$$|\vec{\rho}| = \sqrt{1^2+3^2} = \sqrt{10}$$

$$\alpha_2 = \tan^{-1}\left(\frac{4}{\sqrt{10}}\right) = 51.671^\circ$$

$$\alpha_1 = -\tan^{-1}\left(\frac{4}{\sqrt{10}}\right) = -51.671^\circ$$

$$\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho = \hat{a}_z \times \frac{\vec{\rho}}{|\vec{\rho}|} = \hat{a}_z \times \frac{-\hat{a}_x + 3\hat{a}_y}{\sqrt{10}}$$

x y z

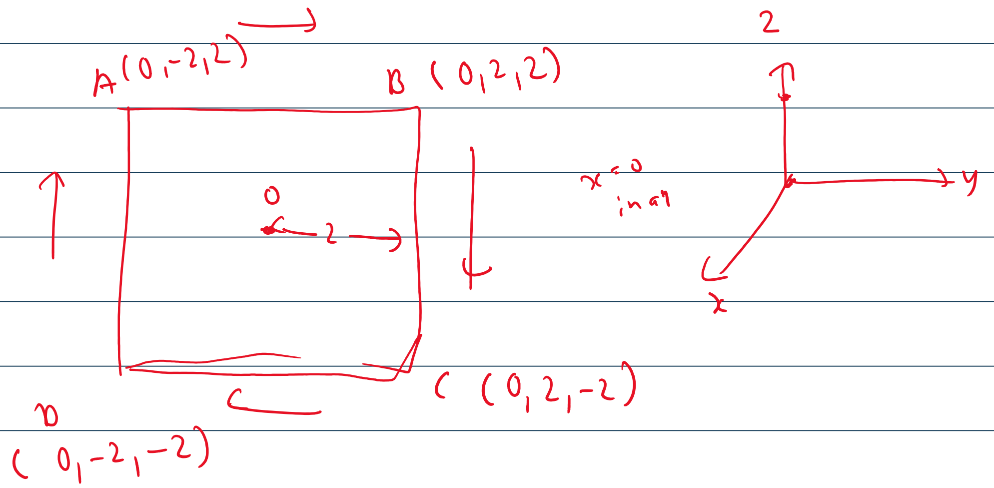
$$= \frac{-\hat{a}_y - 3\hat{a}_x}{\sqrt{10}}$$

$$\therefore \vec{H} = \frac{0.3}{4\pi\sqrt{10}}\left[\sin 51.67^\circ - \sin(-51.67^\circ)\right]\frac{-\hat{a}_y - 3\hat{a}_x}{\sqrt{10}}$$

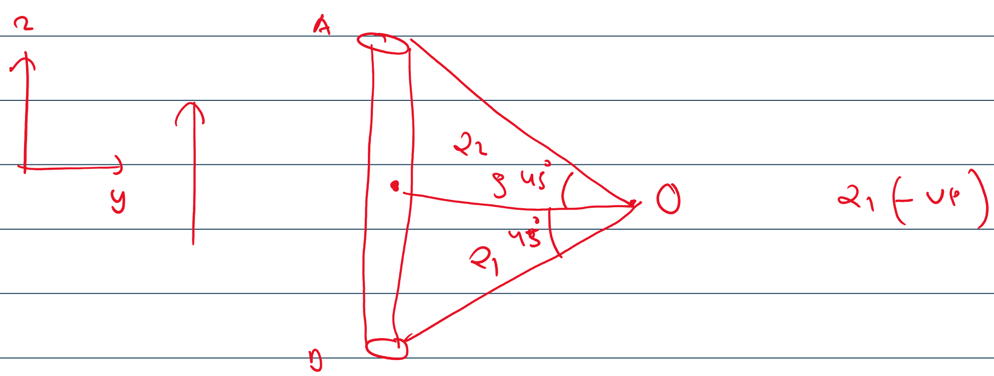
$$= 3.7455\times10^{-3}\left(-\hat{a}_y - 3\hat{a}_x\right) \text{ A/m}$$

\# A current carrying square loop with vertices A (0, -2, 2), B (0, 2, 2), C (0, 2, -2), D (0, -2, -2) is carrying a dc current of 20 A in direction along A - B - C - D - A. Find magnetic field intensity $\vec{H}$ at the center of current carrying loop.

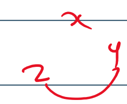
Soln :-



For AD



$$\vec{H}_{AD} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi$$



$\rho = 2$, $\alpha_2 = 45°$, $\alpha_1 = -45°$, $\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho$

$= \hat{a}_z \times \hat{a}_y$

$= -\hat{a}_x$

$$\vec{H}_{AD} = \frac{20}{4\pi(2)}\left[\sin 45^\circ - \sin(-45^\circ)\right](-\hat{a}_x)$$

$$= \frac{20}{8\pi} 2\sin 45^\circ (-\hat{a}_x) = -1.125\,\hat{a}_x \text{ A/m}$$

Due to symmetry, field at center is :

$$\vec{H} = 4\vec{H}_{AD} = -4.501\,\hat{a}_x \text{ A/m}$$

\# (a) Evaluate the closed line integral of $\vec{H}$ from $P_1(5,4,1)$ to $P_2(5,6,1)$ to $P_3(0,6,1)$ to $P_4(0,4,1)$ to P_1 using straight line segments, if $\vec{H} = 0.1y^3\hat{a}_x + 0.4x\hat{a}_z$ A/m (b) Determine the quotient of the closed line integral and the area enclosed by the path as an approximation to $(\nabla\times\vec{H})_z$. (c) Determine $(\nabla\times\vec{H})_z$ at the center of the area.

Soln :

(a) $$\oint \vec{H}\cdot d\vec{l} = \int_{P_1}^{P_2}\vec{H}\cdot d\vec{l} + \int_{P_2}^{P_3}\vec{H}\cdot d\vec{l} + \int_{P_3}^{P_4}\vec{H}\cdot d\vec{l} + \int_{P_4}^{P_5}\vec{H}\cdot d\vec{l}$$

$$\int_{P_1}^{P_2}\vec{H}\cdot d\vec{l} = \int_{y=4}^{6}(0.1y^3\hat{a}_x + 0.4x\hat{a}_z)(dy\,\hat{a}_y) \quad (x=5, z=1)$$

$$= 0$$

$$\int^{P_3}\vec{H}\cdot d\vec{l} = \int^{0}(0.1y^3\hat{a}_x + 0.4x\hat{a}_z)(dx\,\hat{a}_x)$$

$$\int_{P2}^{P3} \vec{H}\cdot\vec{dl} = \int_{x=5}^{0} (0.1y^3 \hat{a}_x + 0.4x\hat{a}_z)(dx\,\hat{a}_x)$$

$$= \int_{x=5}^{0} 0.1y^3\,dx \qquad (y=6)$$

$$= 6^3 \times 0.1 \int_{x=5}^{0} dx = -108$$

$$\int_{P3}^{P4} \vec{H}\cdot\vec{dl} = \int_{y=6}^{y=4} \vec{H}\cdot\vec{dl} = 0$$

$$\int_{P4}^{P1} \vec{H}\cdot\vec{dl} = \int_{x=0}^{5} 0.1y^3\,dx \quad (y=4) = 0.1\times 4^3 \times \int_{x=0}^{5} dx$$

$$= 32$$

$$\therefore \oint \vec{H}\,\vec{dl} = -76\text{ A}$$

(b) $$(\nabla\times\vec{H})_z = \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta S_N}$$ (first part of derivation of Stokes theorem)

$$\Delta S_N = (5-0)\times(6-4) = 10\text{ m}^2$$

(x-changes from 0 to 5
y-changes from 4 to 6
z is const)

$$(\nabla\times\vec{H})_z = \frac{-76}{10} = -7.6\text{ A/m}^2$$

① $(\nabla \times \vec{H})_z = 0.3y^2\,\hat{a}_z$ (only $\hat{a}_z$ part)

(unit)

center $\left(\frac{5+0}{2}, \frac{4+6}{2}, \frac{1+1}{2}\right) = (2.5, 5, 1)$

$$(\nabla \times \vec{H})_z \Big|_{(2.5,5,1)} = 0.3 \times 5^2\,\hat{a}_z = 7.5\,\hat{a}_z \text{ A/m}^2$$

\# Evaluate both sides of stoke's theorem for the field $\vec{G} = 10\sin\theta\,\hat{a}_\phi$ and the surface $r = 3$, $0 \le \phi \le 90°$ — , $0 \le \theta \le 90°$. let the surface have the $\hat{a}_r$ direction.

Soln ÷

Stoke's theorem is ÷

$$\oint \vec{G} \cdot d\vec{l} = \int_s (\nabla \times \vec{G})\,d\vec{s}$$

R.H.S $= \int_s (\nabla \times \vec{G})\,\vec{ds}$ | $\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$

$$(\nabla \times \vec{G})\cdot\vec{ds} = \frac{1}{r\cancel{\sin\theta}}\left[\frac{\partial(G_\phi \sin\theta)}{\partial\theta} - \frac{\partial G_\theta}{\partial\phi}\right] r^2 \cancel{\sin\theta}\, d\theta\, d\phi$$

$$= r\left[\frac{\partial(10\sin^2\theta)}{\partial\theta} - 0\right] d\theta\, d\phi$$

$$= r \times 10 \times 2 \sin\theta \cdot \cos\theta \, d\theta \, d\phi$$

$$= 10 r \sin 2\theta \, d\theta \, d\phi$$

Now,

$$\int_S 10 r \sin 2\theta \, d\theta \, d\phi \qquad (r = 3)$$

$$\text{or} \quad 10 \times 3 \int_0^{\pi/2} \sin 2\theta \, d\theta \int_0^{\pi/2} d\phi$$

$$\int_S (\nabla \times \vec{h}) \cdot \vec{ds} = 15\pi$$

Now,

$$L.H.S = \oint \vec{h} \cdot \vec{dl}$$

The closed path which encloses the surface is :-

$$\begin{tikzcd}
A(3,0,0) \arrow[r, "\theta\text{-change}"] & B(3,90^\circ,0) \arrow[d, "\phi\text{-change}"] \\
D(3,0,90^\circ) \arrow[u, "\phi\text{ change}"] & C(3,90^\circ,90^\circ) \arrow[l, "\theta\text{-change}"']
\end{tikzcd}$$

$$L.H.S = \oint \vec{h} \cdot \vec{dl} = \int_A^B \vec{h} \cdot \vec{dl} + \int_B^C \vec{h} \cdot \vec{dl} + \int_C^D \vec{h} \cdot \vec{dl} + \int_D^A \vec{h} \cdot \vec{dl}$$

$$\int_A^B \vec{H} \cdot \vec{dl} = \int 10\sin\theta \, \hat{a_\phi} \cdot r\,d\theta \, \hat{a_\theta} = 0$$

$$\int_B^C \vec{H} \cdot \vec{dl} = \int_{\phi=0}^{\pi/2} 10\sin\theta \, \hat{a_\phi} \cdot r\sin\theta \, d\phi \, \hat{a_\phi}$$

$$= 10\, r \sin^2\theta \int_0^{\pi/2} d\phi$$

$$(r = 3, \ \theta = \pi/2)$$

$$= 15\pi$$

$$\int_C^D \vec{H} \cdot \vec{dl} = \int_{\theta=\pi/2}^{0} (10\sin\theta \, \hat{a_\phi})(r\,d\theta \, \hat{a_\theta}) = 0$$

$$\int_D^A \vec{H} \cdot \vec{dl} = \int_{\phi=\pi/2}^{0} (10\sin\theta \, \hat{a_\phi})(r\sin\theta \, d\phi \, \hat{a_\phi})$$

$$= \quad r = 3, \ \theta = 0$$

$$= 0 \quad \#$$

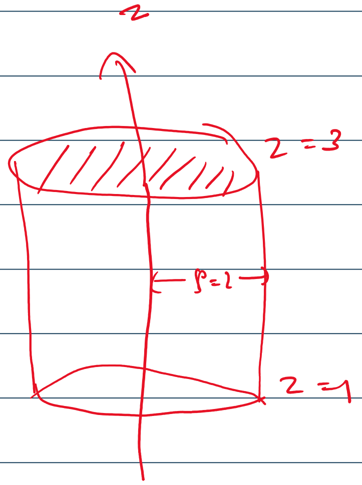
L.H.S $= 15\pi$

$\therefore$ L.H.S = R.H.S stoke theorem is verified #

Different type #

\# Given the field $\vec{H} = \rho^2 \sin^2\phi \, \hat{a_\rho} + \rho^2 \cos^2\phi \, \hat{a_\phi} + 2z^2 \hat{a_z}$, evaluate both sides

Given the field $\vec{A} = \rho^2 \sin^2\phi\, \hat{a}_\rho + \rho^2 \cos^2\phi\, \hat{a}_\phi + 2z^2 \hat{a}_z$, evaluate both sides of stoke's theorem for the path formed by the intersection of the cylinder $\rho = 2$, and the plane $z = 1$, and for the surface defined by $\rho = 2$, $1 \le z \le 3$, and $z = 3$, $0 \le \rho \le 2$.



soln ÷

Stoke's theorem is ÷

$$\oint \vec{A} \cdot \vec{dl} = \int_s (\nabla \times \vec{A})\, \vec{ds}$$

The given cylinder is open at $z = 1$ and has its top at $z = 3$ i.e, it is closed at the upper end.

The circle formed by the intersection of the cylinder $\rho = 2$, $z = 1$ encloses the given surface.

$\oint \vec{A} \cdot \vec{dl}$ has to be evaluated for the circle represented by $\rho = 2$, $z = 1$.

For this circle, $\vec{dl} = \rho\, d\phi\, \hat{a}_\phi$

$$L.H.S = \oint \vec{A} \cdot \vec{dl} = \oint (\rho^2 \sin^2\phi\, \hat{a}_\rho + \rho^2 \cos^2\phi\, \hat{a}_\phi + 2z^2 \hat{a}_z)\,(\rho\, d\phi\, \hat{a}_\phi)$$
$$(\rho = 2, z = 1)$$

$$= \int_{\phi=0}^{2\pi} \rho^3 \cos^2\phi\, d\phi$$

$$= 2^3 \int_0^{2\pi} \cos^2\phi\, d\phi$$

$$= 8\pi$$

$$R.H.S = \int (\nabla \cdot \vec{A})\, \vec{ds}$$

$$R.H.S = \int_S (\nabla \cdot \vec{h})\, \vec{ds}$$

$$\nabla \times \vec{h} = (3\rho \cos^2\phi - \rho \sin 2\phi)\, \hat{a}_z$$

$$R.H.S = \int_{\substack{\text{cylindrical} \\ \text{surfac}}} (\nabla \times \vec{h})\, \vec{ds} + \int_{\substack{\text{top} \\ \text{surface}}} (\nabla \times \vec{h})\, \vec{ds}$$

$$= \int_{S_1} (\nabla \times \vec{h})\, \vec{ds} \quad (\rho = 2) + \int_{S_2} (\nabla \times \vec{h})\, \vec{ds} \quad (z = 3)$$

$$= \int_{S_1} (3\rho \cos^2\phi - \rho \sin 2\phi)\, \hat{a}_z \cdot (\rho\, d\phi\, dz\, \hat{a}_\rho) +$$

$$\int_{S_2} (3\rho \cos^2\phi - \rho \sin 2\phi)\, \hat{a}_z \cdot (\rho\, d\rho\, d\phi\, \hat{a}_z)$$

$$= 0 + \int_{S_2} (3\rho^2 \cos^2\phi - \rho^2 \sin 2\phi)\, d\rho\, d\phi$$

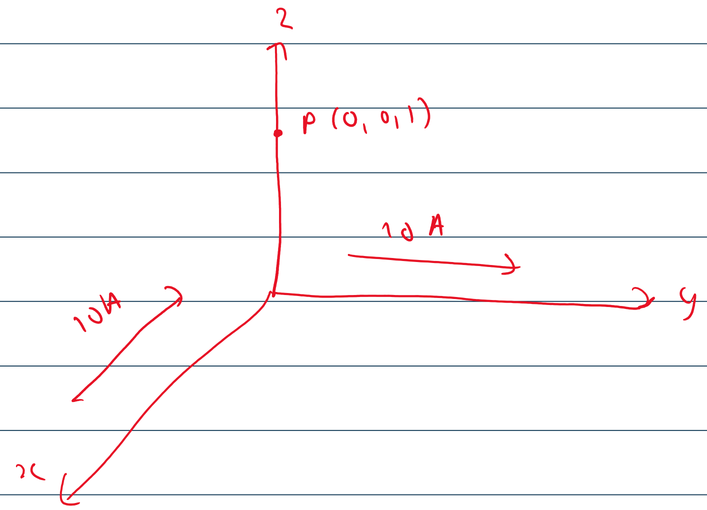
$$= \int_{S_2} 3\rho^2 \cos^2\phi\, d\rho\, d\phi - \int_{S_2} \rho^2 \sin 2\phi\, d\rho\, d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\rho=0}^{2} 3\rho^2 \cos^2\phi\, d\rho\, d\phi - \int_{\phi=0}^{2\pi} \int_{\rho=0}^{2} \rho^2 \sin 2\phi\, d\rho\, d\phi$$

$$= 8\pi$$

L.H.S = R.H.S proved #

A filamentary current of 10A is directed in from ∞ to origin on the +ve x-axis and then back out to infinity along the positive y axis. Use the Biot-Savart law to find $\vec{H}$ at P(0,0,1)



$\longrightarrow +\hat{a}_y$

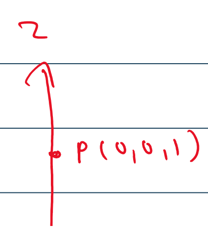
Due to y-axis

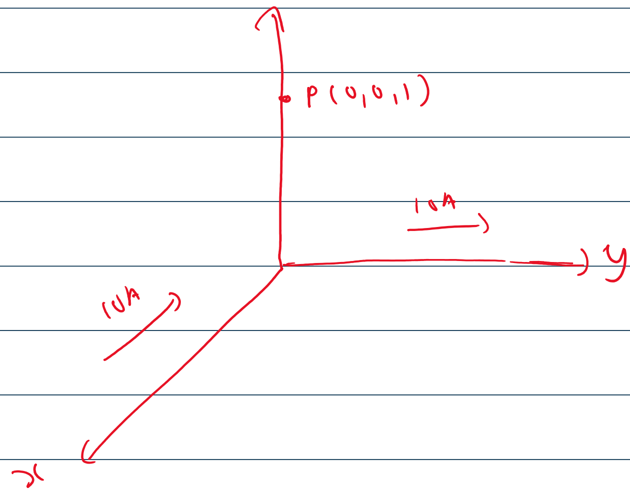
$\rho = 1 \ , \ \alpha_1 = 0^\circ \ , \ \alpha_2 = 90^\circ \qquad (\tan^{-1}\infty = 90^\circ)$

$\hat{a}_\phi = \hat{a}_I \times \hat{a}_\rho = \hat{a}_y \times \hat{a}_z = \hat{a}_x$

$$\vec{H}_y = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi$$

$$= \frac{10}{4\pi 1}(\sin 90^\circ - \sin 0)\hat{a}_x = \frac{10}{4\pi}\hat{a}_x \ A/m$$





$\rho = 1$, $\hat{a}_\phi = -\hat{a}_x \times \hat{a}_z = \hat{a}_y$

$\overrightarrow{-\hat{a}_x}$

$\alpha_1 = -90^\circ$, $\alpha_2 = 0$

$$\vec{H}_x = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi$$

$$= \frac{10}{4\pi}(\sin 0 + \sin 90^\circ)\hat{a}_y$$

$$= \frac{10}{4\pi}\hat{a}_y$$

$$\vec{H} = \vec{H}_x + \vec{H}_y$$

$$= \frac{10}{4\pi}\hat{a}_x + \frac{10}{4\pi}\hat{a}_y$$