Formulas

(i) Cylindrical to rect

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

(ii) Rect to cylindrical

$$\rho = \sqrt{x^2+y^2}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

Spherical to rect

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

(ii) Rect to spherical

$$r = \sqrt{x^2+y^2+z^2}$$
$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos^{-1}\frac{z}{r}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow \text{angle measurement}$$

Cylindrical to spherical

$$\phi = \phi$$
$$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2}$$
$$\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\frac{z}{\sqrt{\rho^2+z^2}}$$

(ii) Spherical to cylindrical

$$\phi = \phi$$
$$\rho = r\sin\theta$$
$$z = r\cos\theta$$

#

	$\hat{a}_r$	$\hat{a}_\theta$	$\hat{a}_\phi$
$\hat{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{a}_z$	$\cos\theta$	$-\sin\theta$	0

	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1

	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_r$	$\sin\theta$	0	$\cos\theta$
$\hat{a}_\theta$	$\cos\theta$	0	$-\sin\theta$
$\hat{a}_\phi$	0	1	0

①

Chapter 1 (5 marks)

① Co-ordinate System

→ Any point in the space with a particular reference

Scalars and vectors

→ Quantites having only magnitude
eg: Mass, Distance, density, pressure, volume

Vectors

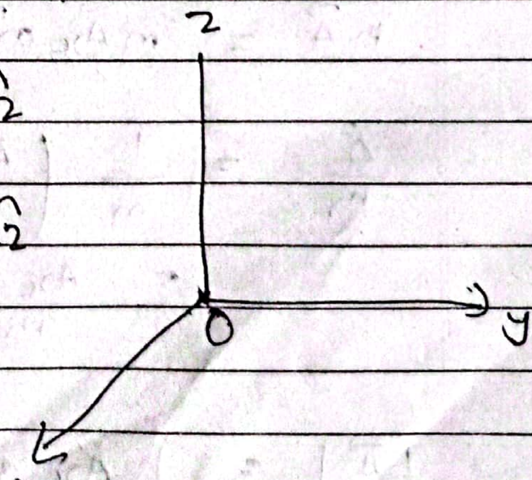
→ Those quantities having both a magnitude and a direction in space.
eg: Force, velocity, acceleration, magnetic ~~fat~~ flux density, current density

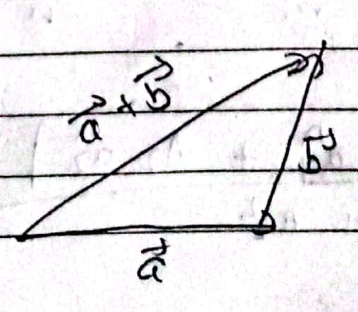
Algebraic Operation Involving Vectors :-

① Vector addition and subtraction :-

$\vec{A} = A_x \cdot \hat{a_x} + A_y \cdot \hat{a_y} + A_z \cdot \hat{a_z}$

$\vec{B} = B_x \cdot \hat{a_x} + B_y \hat{a_y} + B_z \hat{a_z}$





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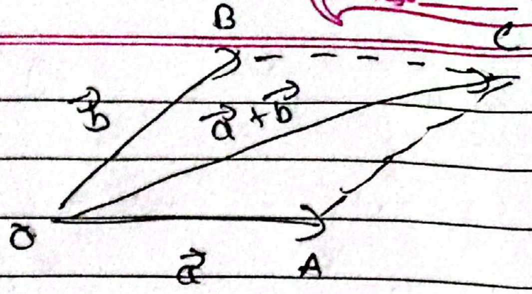


$$\vec{a} + \vec{b} = \vec{OA} + \vec{AC}$$
$$= \vec{OA} + \vec{AC}$$

$$\vec{A} + \vec{B} = (A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z})$$
$$+ (B_x \hat{a_x} + B_y \hat{a_y} + B_z \hat{a_z})$$

$$= (A_x + B_x)\hat{a_x} + (A_y + B_y)\hat{a_y} + (A_z + B_z)\hat{a_z}$$

$$\therefore \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$= (A_x - B_x)\hat{a_x} + (A_y - B_y)\hat{a_y} + (A_z - B_z)\hat{a_z}$$

(iii) multiplication and division by a scalar

$$m\vec{A} = mA_x \hat{a_x} + mA_y \hat{a_y} + mA_z \hat{a_z}$$

$$\frac{\vec{A}}{m} = \frac{1}{m}\left(A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}\right)$$
$$= \frac{A_x}{m}\hat{a_x} + \frac{A_y}{m}\hat{a_y} + \frac{A_z}{m}\hat{a_z}$$

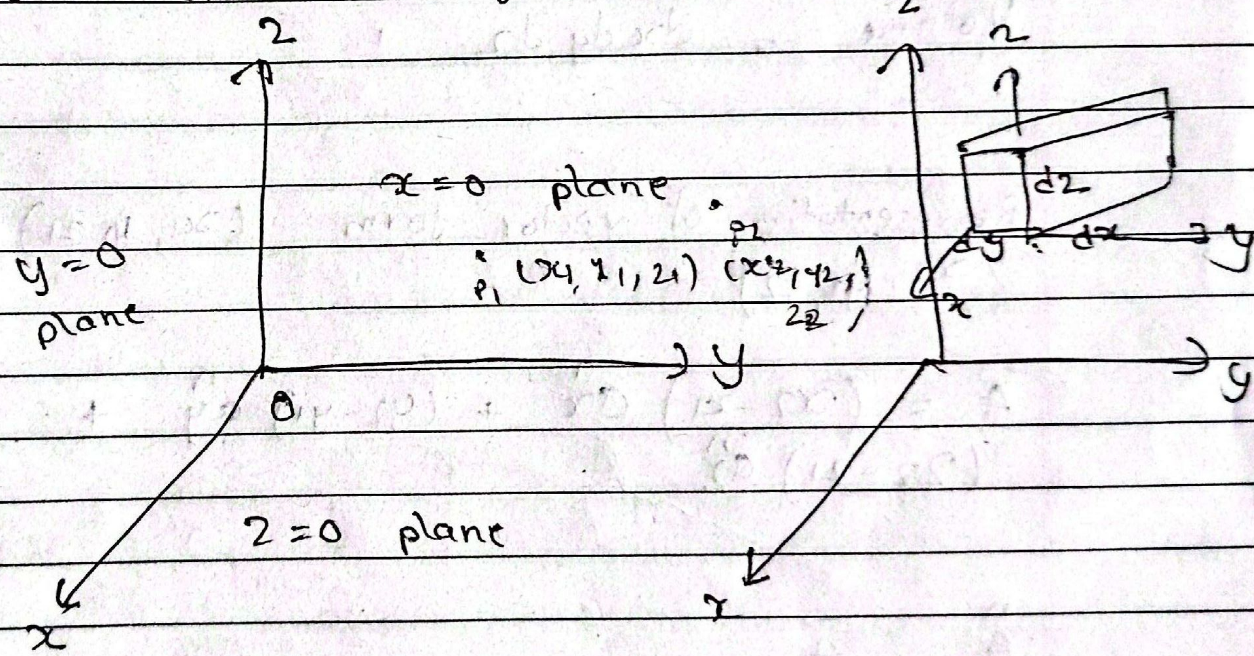
$$|\vec{A}| = |A_x \hat{a_x} + A_y \cdot \hat{a_y} + A_z \hat{a_z}|$$
$$= \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Unit vector along A $(\hat{a}_A) = \frac{\vec{A}}{|\vec{A}|}$

Classification of co-ordinate System

1. Rectangular co-ordinate system
2. Cylindrical / circular co-ordinate system
3. Spherical co-ordinate system

Rectangular co-ordinate system :



length $(\vec{dl}) = dx \cdot \hat{ax} + dy\,\hat{ay} + dz\,\hat{az}$

$x \rightarrow dx$
$y \rightarrow dy$ ie length change
$z \rightarrow dz$

(6)

Area ds in x - direction / x - plane
= dy dz

Area ds in y - direction / y - plane
= dx dz

Area ds in z - direction / z - plane
= dx dy

Volume = dx dy dz

Representation of vector form (x_1, y_1, z_1) to (x_2, y_2, z_2)

$$\vec{A} = (x_2 - x_1)\,\hat{a_x} + (y_2 - y_1)\,\hat{a_y} + (z_2 - z_1)\,\hat{a_z}$$

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$$= \frac{(x_2 - x_1)\,\hat{a_x} + (y_2 - y_1)\,\hat{a_y} + (z_2 - z_1)\,\hat{a_z}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

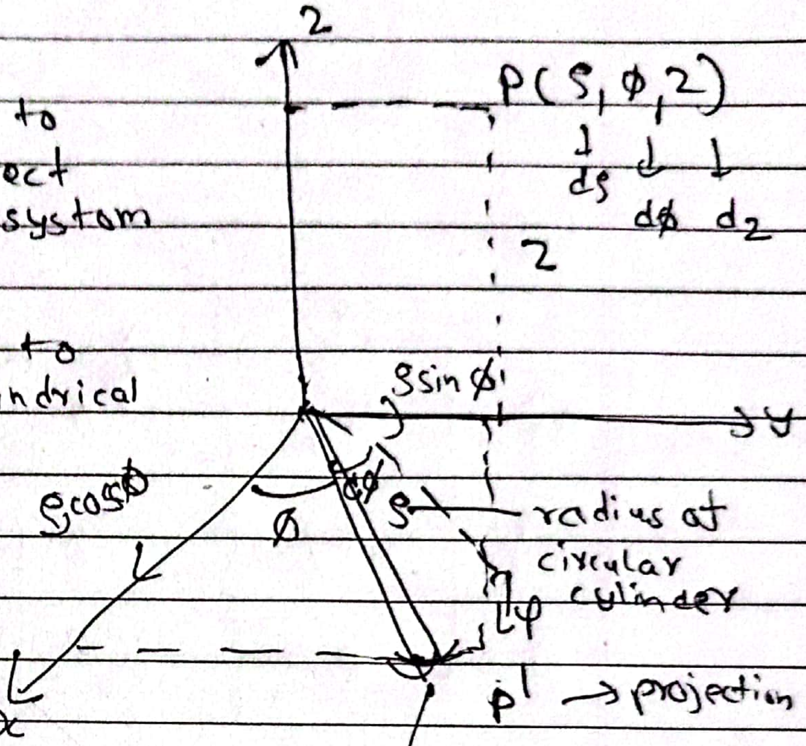
(5)

$\boxed{P(x, y, z)}$

(11) Cylindrical co-ordinate system $P(\rho, \phi, z)$

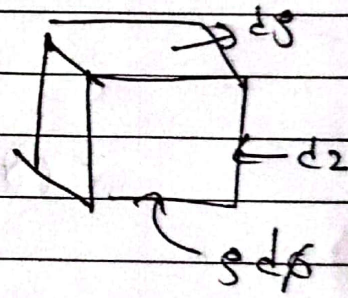
$$\left.\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z \end{aligned}\right\}$$ cyln to rect system

$$\left.\begin{aligned} \rho &= \sqrt{x^2+y^2} \\ \phi &= \tan^{-1}\left(\frac{y}{x}\right) \\ z &= z \end{aligned}\right\}$$ rect to cylindrical



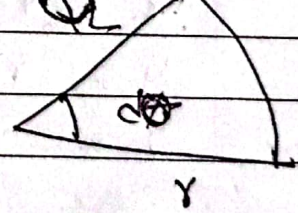
$$d\phi = \frac{\varphi}{\rho}$$

$$\varphi = \rho\, d\phi$$



$z \to dz$
$\rho \to d\rho$
$\phi = \rho\, d\phi$

ρ, ϕ, z
$\downarrow$
$\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$



$$\vec{d\varphi} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

~~(length vector comp)~~

$$d\theta = \frac{\varphi}{r}$$

$$\varphi = r\, d\theta$$

except z small change

Area at z-plane $= \rho\, d\rho\, d\phi$

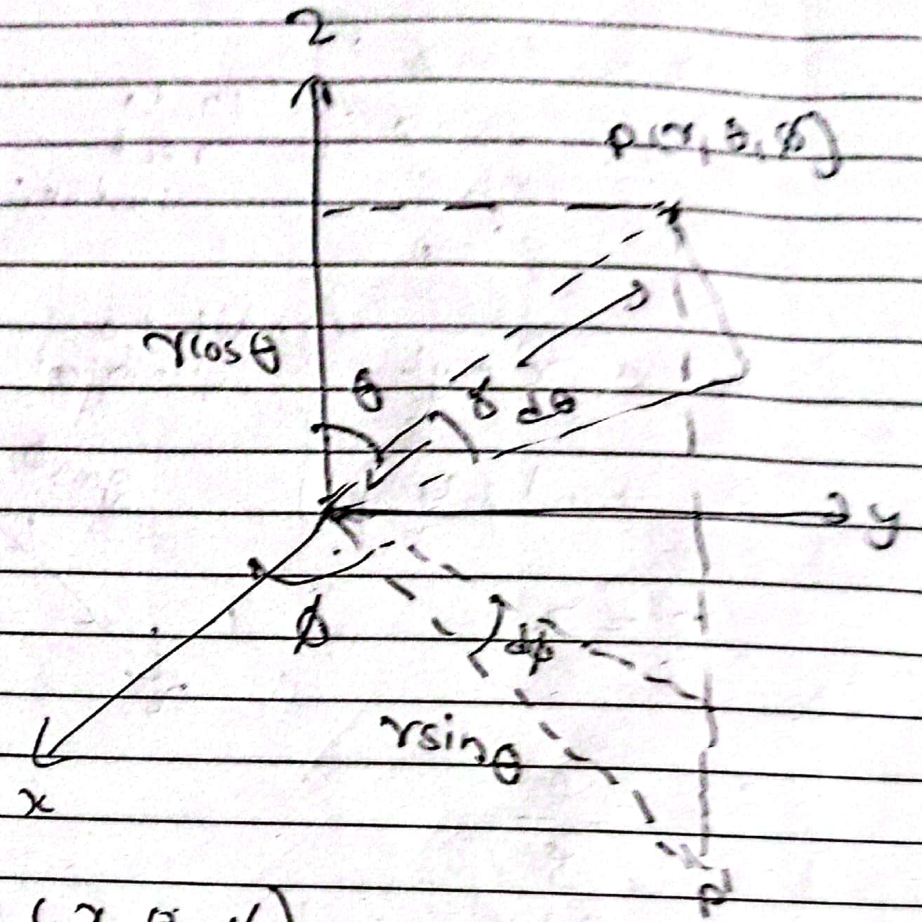
Area at ρ-plane $= \rho\, d\phi\, dz$

" " ϕ-plane $= d\rho\, dz$

volume $= \rho\, d\rho\, d\phi\, dz$

6

Spherical Co-ordinate System



P (x, y, z)   P (r, θ, φ)

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

Sphe. to Rect

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

rect to spherical

Elementory change in length $(r \to dr$

$(\phi \to d\phi$

$(\theta \to d\theta)$

$r \to dr$

$\phi \to r\sin\theta \, d\phi$

$\theta \to r\, d\theta$

length $(d\vec{\varphi}) = dr\, \hat{a_r} + r\,d\theta\, \hat{a_\theta} + r\sin\theta\, d\phi\, \hat{a_\phi}$

Area

r-plane $= r^2 \sin\theta\, d\phi\, d\theta$

θ-plane $= r\sin\theta\, d\phi\, dr = r\,dr \sin\theta\, d\phi$

ϕ-plane $= r.\, dr\, d\theta$

volume $= r\,dr\,d\theta\, r\sin\theta\, d\phi$

$= r^2 \sin\theta\, dr\, d\theta\, d\phi$

Rectangular to Cylindrical co-ordinate System

$\vec{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$ (Rect)

$\vec{A_c} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$ (cylindrical)

where,

$A_\rho = \vec{A} \cdot \hat{a_\rho}$

$= (A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}) \cdot \hat{a_\rho}$

$= A_x \hat{a_x} \cdot \hat{a_\rho} + A_y \hat{a_y} \cdot \hat{a_\rho} + A_z \hat{a_z} \cdot \hat{a_\rho}$

8

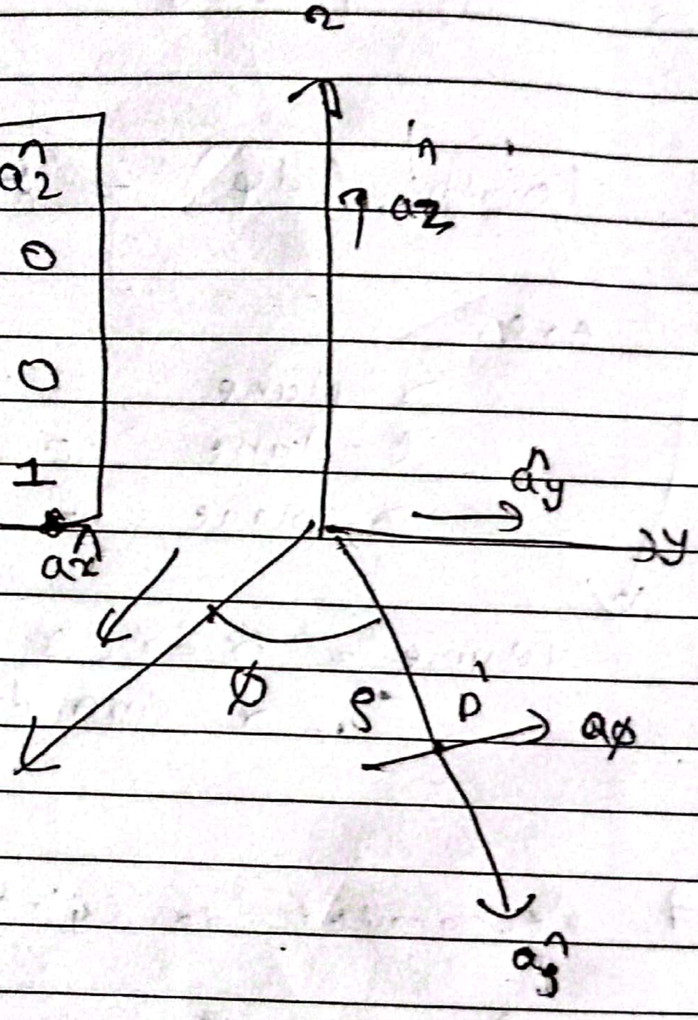
$A_\phi = \vec{A} \cdot \hat{a_\phi}$

$= A_x \, \hat{a_x} \cdot \hat{a_\phi} + A_y \, \hat{a_y} \cdot \hat{a_\phi} + A_z \hat{a_z} \cdot \hat{a_\phi}$

$A_z = A_z$

Conversion table

	$\hat{a_\rho}$	$\hat{a_\phi}$	$\hat{a_z}$
$\hat{a_x}$	$\cos\phi$	$-\sin\phi$	0
$\hat{a_y}$	$\sin\phi$	$\cos\phi$	0
$\hat{a_z}$	0	0	1



Rectangular to spherical co-ordinate system

$\vec{A} = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

$\vec{A}_R = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$

$A_r = \vec{A} \cdot \hat{a_r}$

$= (A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}) \cdot \hat{a_r}$

$= A_x \hat{a_x} \cdot \hat{a_r} + A_y \hat{a_y} \cdot \hat{a_r} + A_z \hat{a_z} \cdot \hat{a_r}$

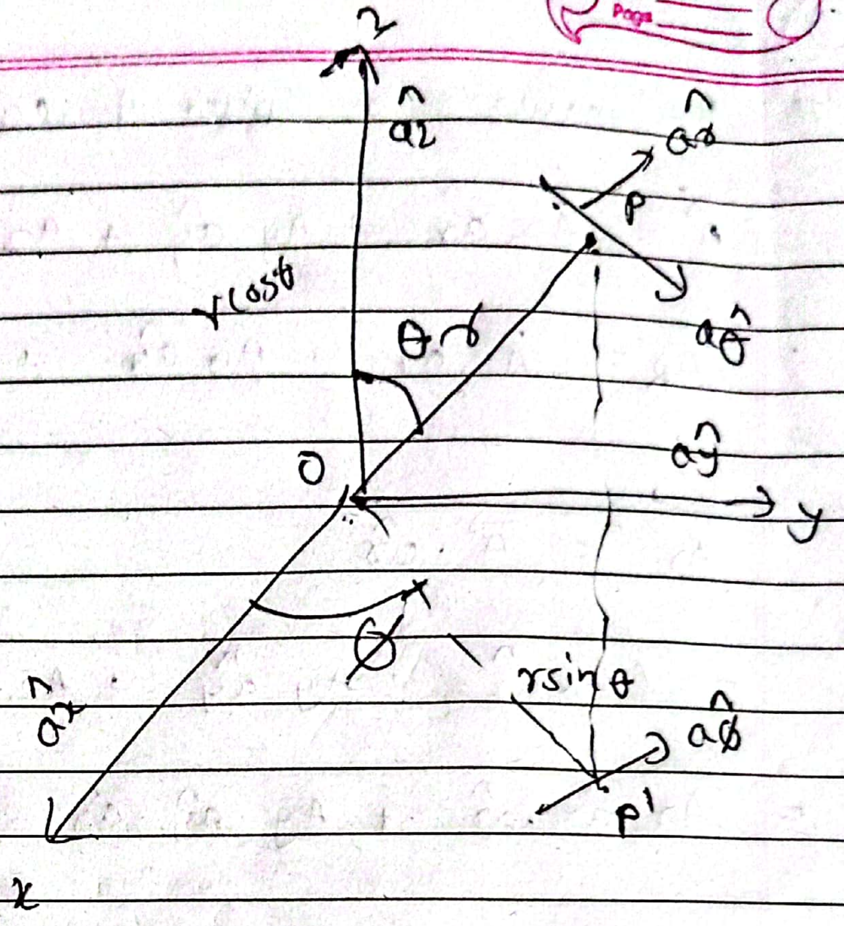
$A_\theta = \vec{A} \cdot \hat{a_\theta}$

$= A_x \hat{a_x} \cdot \hat{a_\theta} + A_y \hat{a_y} \cdot \hat{a_\theta} + A_z \hat{a_z} \cdot \hat{a_\theta}$

$A_\phi = \vec{A} \cdot \hat{a_\phi}$

$= A_x \hat{a_x} \cdot \hat{a_\phi} + A_y \hat{a_y} \cdot \hat{a_\phi} + A_z \hat{a_z} \cdot \hat{a_\phi}$

(11)



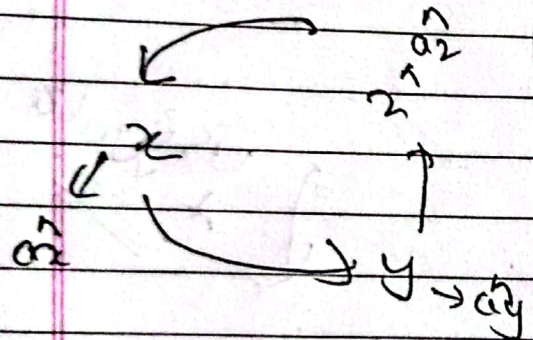
Conversion table

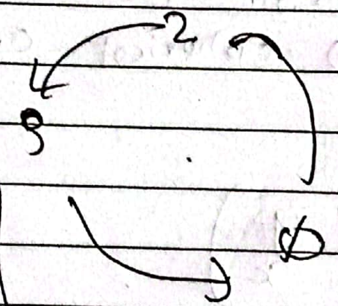
	$\hat{a_r}$	$\hat{a_\theta}$	$\hat{a_\phi}$
$\hat{a_x}$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{a_y}$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{a_z}$	$\cos\theta$	$-\sin\theta$	0

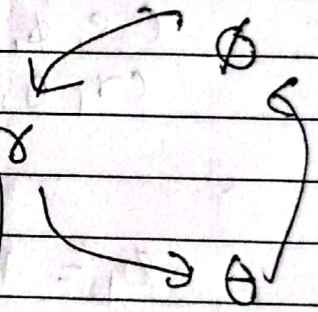
Dot product of unit vector in cylindrical and spherical co-ordinate system

	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_r$	$\sin\theta$	0	$\cos\theta$
$\hat{a}_\theta$	$\cos\theta$	0	$-\sin\theta$
$\hat{a}_\phi$	0	1	0

Cross Product







→ $\hat{a}_x \times \hat{a}_y = \hat{a}_z$
→ $\hat{a}_y \times \hat{a}_z = \hat{a}_x$
→ $\hat{a}_z \times \hat{a}_x = \hat{a}_y$

→ $\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z$
→ $\hat{a}_\phi \times \hat{a}_z = \hat{a}_\rho$
→ $\hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$

$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$
$\hat{a}_\theta \times \hat{a}_\phi = \hat{a}_r$
$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$

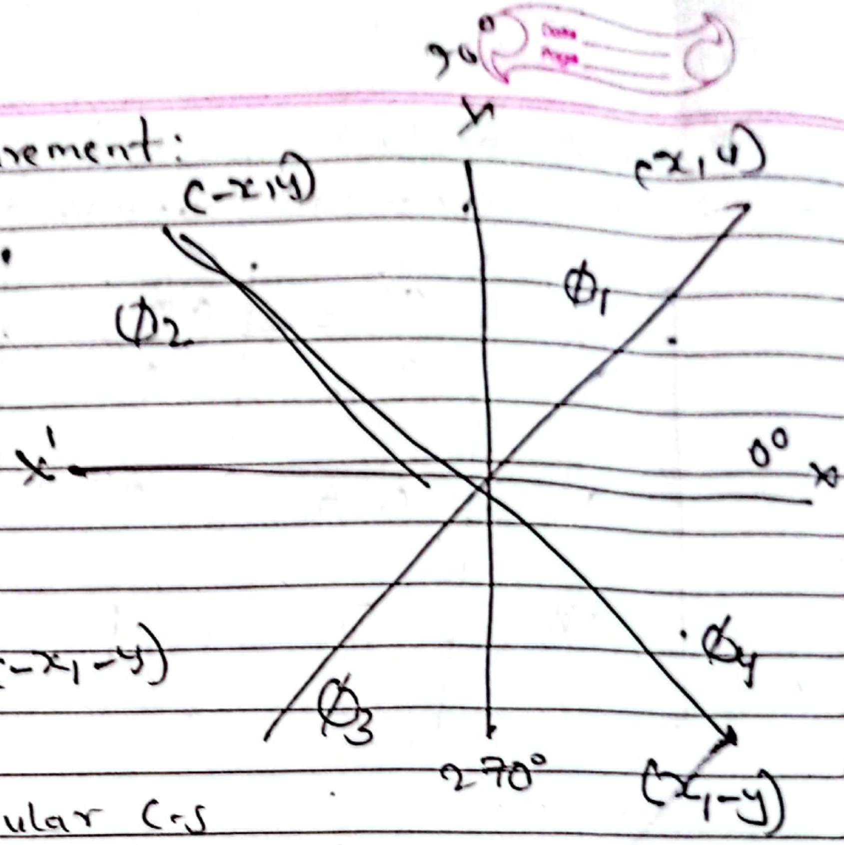
→ $\hat{a}_y \times \hat{a}_x = -\hat{a}_z$

$\hat{a}_\phi \times \hat{a}_\rho = -\hat{a}_z$

$\hat{a}_\theta \times \hat{a}_r = -\hat{a}_\phi$

(13)

~~Agl~~ Angle measurement:



$(x, y, z) \rightarrow$ rectangular C.S

$(\rho, \phi, z) \rightarrow$ cylindrical C.S

$(r, \theta, \phi) \rightarrow$ spherical C.S

usual

$\phi_1 = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi_2 = 180° - \tan^{-1}\left(\frac{y}{x}\right) \qquad \left(90 + \tan^{-1}\left(\frac{y}{x}\right)\right)$

$\phi_3 = 270 - \tan^{-1}\left(\frac{y}{x}\right) \qquad \left(180 + \tan^{-1}\left(\frac{y}{x}\right)\right)$

$\phi_4 = 360 - \tan^{-1}\left(\frac{y}{x}\right) \qquad \left(270° + \tan^{-1}\left(\frac{y}{x}\right)\right)$

(14)

~~① Expression the~~

Numerical

① Express the unit vector $\hat{a_x}$ in spherical components at $\rho = 2.5$, $\phi = 0.7$ rad, $z = 1.5$

soln ÷

$\boxed{\text{rect to spherical}}$

$\vec{F} = \hat{a_x}$

$\vec{F_s} = F_r \hat{a_r} + F_\theta \hat{a_\theta} + F_\phi \hat{a_\phi}$

$F_r = \vec{F} \cdot \hat{a_r}$
$= \hat{a_x} \cdot \hat{a_r}$
$= \sin\theta \cos\phi$

$F_\theta = \vec{F} \cdot \hat{a_\theta}$
$= \hat{a_x} \cdot \hat{a_\theta}$
$= \cos\theta \cos\phi$

$F_\phi = \vec{F} \cdot \hat{a_\phi}$
$= \hat{a_x} \cdot \hat{a_\phi}$
$= -\sin\phi$

$\rho = 2.5$
$\phi = 0.7$ rad
$z = 1.5$

$\phi = \phi = 0.7 \text{ rad} = 40.107^\circ$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{z}{\sqrt{\rho^2 + r^2}}\right)$

$= \cos^{-1}\left(\frac{1.5}{\sqrt{(2.5)^2 + (1.5)^2}}\right)$

$= 59.036^\circ$

Now,

$F_r = \sin\theta \cos\phi$

$= \sin 59.036^\circ \cdot \cos 40.107^\circ$

$= 0.6558$

$F_\theta = \cos\theta \cos\phi$

$= 0.3935$

$F_\phi = -\sin\phi = -0.6442$

$$\boxed{\vec{F_S} = 0.6558\,\hat{a_r} + 0.3935\,\hat{a_\theta} - 0.6442\,\hat{a_\phi}}$$

(2) Transform the vector $\vec{A} = y\hat{a_x} + (x+y)\hat{a_z}$ at $P(-2,6,3)$ in cylindrical system.

Soln:-

$\vec{A} = y\hat{a_x} + (x+y)\hat{a_z}$

$P(-2,6,3)$ at (x,y,z)

$\vec{A} = 6\hat{a_x} + 4\hat{a_z}$

(14)

$\vec{A}_{\rho c} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$

$\rho = \sqrt{x^2+y^2}$ $\qquad \rho = \sqrt{(-2)^2+(6)^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$ $\qquad = \sqrt{40}$

$z = 2$ $\qquad \phi = \tan^{-1}\left(\frac{6}{-2}\right)$

$= 180^\circ - \tan^{-1}\left(\frac{6}{2}\right)$

$= 108.435^\circ$

$\hat{A_\rho} = \vec{A} \cdot \hat{a_\rho}$

$= (6\hat{a_x} + 4\hat{a_z})\,\hat{a_\rho}$

$= 6\,\hat{a_x}\cdot\hat{a_\rho} + 4\,\hat{a_z}\cdot\hat{a_\rho}$

$= 6\cos\phi + 0$

$= 6\cos\phi = 6\cos(180.435^\circ) = -1.897$

$\hat{A_\phi} = \vec{A} \cdot \hat{a_\phi}$

$= (6\hat{a_x} + 4\hat{a_z}) \cdot \hat{a_\phi}$

$= 6\,\hat{a_x}\cdot\hat{a_\phi} + 4\,\hat{a_z}\cdot\hat{a_\phi}$

$= -6\sin\phi + 4\times 0$

$= -6\sin(180.435^\circ) = -5.69$

$\hat{A_z} =$ Here

$A_z = 4$ (cylindrical and rect z same)

$\vec{A}(\rho, \phi, z) = -1.897\,\hat{a_\rho} - 5.69\,\hat{a_\phi} + 4\,\hat{a_z}$ #

(17)

$x \quad y \quad z$

Given two fixed point $C(-3, 2, 1)$ and $D(r=5, \theta=20^\circ, \phi=-70^\circ)$ find

(a) The spherical co-ordinate of C
(b) cartesian co-ordinate of D
(c) the distance from C and D.

Soln:-

(a)

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{(-3)^2+(2)^2+(1)^2}$$
$$= 3.7416$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{1}{3.7416}\right) = 74.498^\circ$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-3}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{2}{3}\right)$$

$$= 146.30^\circ$$

$$(r, \theta, \phi) = (3.74, 74.498^\circ, 146.30^\circ)$$

(18)

Date ______
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(b)

Sol^n:- $5 \quad 20^\circ \quad -70^\circ$

$(r, \theta, \phi) \rightarrow (x, y, z)$

$x = r\sin\theta\cos\phi = 5\sin 20^\circ \cdot \cos(-70^\circ) = 0.585$

$y = r\sin\theta\sin\phi = -1.607$

$z = r\cos\theta = 5 \times \cos 20^\circ = 4.698$

(c) Distance form (C, D)

$C(-3, 2, 1) \qquad D(0.585, -1.607, 4.698)$

$$d_{CD} = \sqrt{(0.585+3)^2 + (-1.607-2)^2 + (4.698-1)^2}$$

$= 6.287$

7. Convert the vector: $\vec{F} = F_x\hat{a}_x + F_y\hat{a}_y + F_z\hat{a}_z$ to spherical co-ordinate system

Sol^n:-

$\vec{F} = F_x\hat{a}_x + F_y\hat{a}_y + F_z\hat{a}_z \qquad (r, \theta, \phi)$

Let,

$\vec{F} = F_r\hat{a}_r + F_\theta\hat{a}_\theta + F_\phi\hat{a}_\phi$ —— (i)

(15)

$$F_r = \vec{F} \cdot \hat{a_r} = (F_x \hat{a_x} + F_y \hat{a_y} + F_z \hat{a_z}) \hat{a_r}$$

$$= F_x \sin\theta \cos\phi + F_y \sin\theta \sin\phi + F_z \cos\theta$$

$$F_\theta = \vec{F} \cdot \hat{a_\theta} = (F_x \hat{a_x} + F_y \hat{a_y} + F_z \hat{a_z}) \hat{a_\theta}$$

$$= F_x \cos\theta \cos\phi + F_y \cos\theta \sin\phi + F_z (-\sin\theta)$$

$$F_\phi = \vec{F} \cdot \hat{a_\phi} = (F_x \hat{a_x} + F_y \hat{a_y} + F_z \hat{a_z}) \cdot \hat{a_\phi}$$

$$= F_x (-\sin\phi) + F_y \cos\phi + F_z \times 0$$

$$= -\sin\phi F_x + F_y \cos\phi$$

From (i)

$$\vec{F} = (F_x \sin\theta \cos\phi + F_y \sin\theta \sin\phi + F_z \cos\theta) \hat{a_r}$$

$$+ (F_x \cos\theta \cos\phi + F_y \cos\theta \sin\phi - F_z \sin\theta) \hat{a_\theta}$$

$$+ (-F_x \sin\phi + F_y \cos\phi) \hat{a_\phi}$$

(V) (Insights)

If $\vec{A} = 3\hat{a}_r + 2\hat{a}_\theta - 6\hat{a}_\phi$ and $\vec{B} = 4\hat{a}_r + 3\hat{a}_\phi$ determine

(a) $\vec{A}\cdot\vec{B}$ (b) $|\vec{A}\times\vec{B}|$ (c) The vector comp of $\vec{A}$ along $\hat{a}_z$ at $(1, \frac{\pi}{3}, \frac{5\pi}{4})$

soln:-

(a) $\vec{A}\cdot\vec{B} = (3\hat{a}_r + 2\hat{a}_\theta - 6\hat{a}_\phi)\cdot(4\hat{a}_r + 3\hat{a}_\phi)$

$= \cancel{3\times 4\,\hat{a}_r \; 2\times 0\,\hat{a}_\theta}$

$= \cancel{3\times 4\,\hat{a}_r\cdot\hat{a}_r}\;(3\times 4 - 6\times 3) = (12-18)$

$= -6$

(b) $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ 3 & 2 & -6 \\ 4 & 0 & 3 \end{vmatrix}$$

$= \hat{a}_r(3\times 2 - (0\times -6)) - \hat{a}_\theta(3\times 3 - (-6\times 4))$

$+ (0-8)\hat{a}_\phi$

$= 6\hat{a}_r - 33\hat{a}_\theta - 8\hat{a}_\phi$

$\therefore |\vec{A}\times\vec{B}| = \sqrt{(6)^2 + (-33)^2 + (-8)^2}$

$= 34.48$

(2)
confusing so imp

(c) Vector comp of $\vec{A}$ along $\hat{a_z}$ at $\left(1, \frac{\pi}{3}, \frac{5\pi}{4}\right)$

$\vec{A} \cdot \vec{a_z}$ mag $\quad (\vec{A} \cdot \hat{a_z})\, \hat{a_z}$

magnitude comp

$$= (3\hat{a_r} + 2\hat{a_\theta} - 6\hat{a_\phi}) \cdot \hat{a_z}$$

$$= 3\,\hat{a_r} \cdot \hat{a_z} + 2\,\hat{a_\theta} \cdot \hat{a_z} - 6\,\hat{a_\phi} \cdot \hat{a_z}$$

$$= 3\cos\theta - 2\sin\theta - 0$$

$$= 3\cos\theta - 2\sin\theta$$

now,

$\hat{a_z}$ in terms of spherical system

$$\vec{F} = \hat{a_z}$$

$$\vec{F} = F_r\hat{a_r} + F_\theta\hat{a_\theta} + F_\phi\hat{a_\phi}$$

where

$$F_r = \vec{F} \cdot \hat{a_r} = \hat{a_z} \cdot \hat{a_r} = \cos\theta$$

$$F_\theta = \vec{F} \cdot \hat{a_\theta} = \hat{a_z} \cdot \hat{a_\theta} = -\sin\theta$$

$$F_\phi = \vec{F} \cdot \hat{a_\phi} = \hat{a_z} \cdot \hat{a_\phi} = 0$$

$$\vec{F} = \cos\theta\, \hat{a_r} - \sin\theta\, \hat{a_\theta}$$

Thus,

$$\hat{a_z} = \cos\theta\, \hat{a_r} - \sin\theta\, \hat{a_\theta}$$

The vector component

(iii)

$\cancel{(3\cos 60^\circ - 2\sin 60^\circ)(\cos 60^\circ \hat{a_r} - \sin 60^\circ \hat{a_\theta})}$

$= \cancel{-0.323}$

$\Rightarrow (\vec{A} \cdot \hat{a_2})\, \hat{a_2}$

$= (3\cos\theta - 2\sin\theta)(\cos\theta\, \hat{a_r} - \sin\theta\, \hat{a_\theta})$

at $\left(1, \frac{\pi}{3}, \frac{8\pi}{4}\right)$

$= (3\cos 60^\circ - 2\sin 60^\circ)(\cos 60^\circ \hat{a_r} - \sin 60^\circ \hat{a_\theta})$

$= -0.116\, \hat{a_r} + 0.2009\, \hat{a_\theta}$

(2) → done already

(3) Transform $\vec{A} = 3\hat{a_\rho} - \hat{a_\phi}$ at $P(3, 4, -1)$ (x, y, z)
in cartesian co-ordinate system

(a) Soln:- → insigned ma wrong soln

$\vec{A} = 3\hat{a_\rho} - \hat{a_\phi}$

$A_R = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$

$A_x = \vec{A} \cdot \hat{a_x}$
$= (3\hat{a_\rho} - \hat{a_\phi})\, \hat{a_x}$
$= 3\cos\phi + \sin\phi$

$A_y = \vec{A} \cdot \hat{a_y}$
$= 3\sin\phi - \cos\phi$

$A_z = -1 = z$

3)

$\rho = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = 5$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$

$z = -1$

~~$A_R =$~~ $\rho\phi$

$A_x = 8\cos(53.13^\circ) + \sin(53.13)$
$= 2.6$

$A_y = 3\sin(53.13) - \cos(53.13)$
$= 1.8$

$A_z = 0$

$\vec{A_R} = 2.6\,\hat{a_x} + 1.8\,\hat{a_y}$

4) Transform the vector field $\vec{B} = y\hat{a_x} - x\hat{a_y} + z\hat{a_z}$ into cylindrical coordinates

soln $\vec{B} = y\hat{a_x} - x\hat{a_y} + z\hat{a_z}$

$\vec{B_c} = B_\rho \hat{a_\rho} + B_\phi \hat{a_\phi} + B_z \hat{a_z}$

$x = \rho\cos\phi$
$y = \rho\sin\phi$
$z = z$

$B_\rho = \vec{B}\cdot\hat{a_\rho}$

$= y\,\hat{a_x}\cdot\hat{a_\rho} - x\,\hat{a_y}\cdot\hat{a_\rho} + z\,\hat{a_z}\cdot\hat{a_\rho}$

$= y\cos\phi - x\sin\phi + 0$

$= \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi$

$= 0$

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$B_\phi = \vec{B} \cdot \hat{a_\phi}$

$= y(\hat{a_x} \cdot \hat{a_\phi}) - x(\hat{a_y} \cdot \hat{a_\phi}) + z(\hat{a_z} \cdot \hat{a_\phi})$

$= -y \sin\phi - x \cos\phi + 0$

$= -\rho \sin^2\phi - \rho \cos^2\phi$

$= -\rho(\sin^2\phi + \cos^2\phi)$

$= -\rho$

$B_z = z$

$\therefore \vec{B} = -\rho \hat{a_\phi} + z \hat{a_z}$

5 (a) Transform to cylindrical coordinates $\vec{G} = (2x+y)\hat{a_x} - (y-4x)\hat{a_y}$ at point $P(\rho, \phi, z)$

(b) Convert to cartesian comp $\vec{H} = 20\hat{a_\rho} - 10\hat{a_\phi} + 3\hat{a_z}$ at point $P(x=5, y=2, z=-1)$

Soln:-

(a) $\vec{G} = (2x+y)\hat{a_x} - (y-4x)\hat{a_y}$ at $P(\rho, \phi, z)$

$\vec{G_c} = G_\rho \hat{a_\rho} + G_\phi \hat{a_\phi} + G_z \hat{a_z}$

$G_\rho = \vec{G} \cdot \hat{a_\rho}$

$= (2x+y)\hat{a_x} \cdot \hat{a_\rho} - (y-4x)\hat{a_y} \cdot \hat{a_\rho}$

$= (2x+y)\cos\phi - (y-4x)\sin\phi$

$= (2\rho\cos\phi + \rho\sin\phi)\cos\phi - (\rho\sin\phi - 4\rho\cos\phi)\sin\phi$

$= 2\rho\cos^2\phi + \rho\sin\phi\cos\phi - \rho\sin^2\phi + 4\rho\sin\phi\cos\phi$

$= \boxed{2\rho\cos^2\phi + 5\rho\sin\phi\cos\phi - \rho\sin^2\phi}$

$G_\phi = \vec{G} \cdot \hat{a_\phi} = \boxed{4\rho\cos^2\phi - 3\rho\sin\phi\cos\phi - \rho\sin^2\phi}$

$\vec{G_z} = \vec{G} \cdot \hat{a_z} = \boxed{0}$

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$\therefore$ $\vec{G} = (2\rho\cos^2\phi - \rho\sin^2\phi + 5\rho\sin\phi\cos\phi)\hat{a_\rho}$
$+ (4\rho\cos^2\phi - \rho\sin^2\phi - 3\rho\sin\phi\cos\phi)\hat{a_\phi}$

(b) $\vec{H} = 20\hat{a_\rho} - 10\hat{a_\phi} + 3\hat{a_z}$ $\quad P(x=5, y=2, z=-1)$

soln:-

$\vec{H_R} = H_x\hat{a_x} + H_y\hat{a_y} + H_z\hat{a_z}$ $\quad (\phi = \tan^{-1}(\frac{y}{x})$

$= \tan^{-1}(\frac{2}{5})$

$= 21.80^\circ$

$H_x = \vec{H}\cdot\hat{a_x}$

$= 20\hat{a_\rho}\cdot\hat{a_x} - 10\hat{a_\phi}\cdot\hat{a_x} + 3\hat{a_z}\cdot\hat{a_x}$
$= 20\cos\phi + 10\sin\phi + 0$
$= 20\cos 21.80^\circ + 10\sin 21.80^\circ$
$= 22.28$

$H_y = \vec{H}\cdot\hat{a_y}$
$= 20\hat{a_\rho}\cdot\hat{a_y} - 10\hat{a_\phi}\cdot\hat{a_y} + 3\hat{a_z}\cdot\hat{a_y}$
$= 20\sin\phi - 10\cos\phi$
$= 20\sin 21.80^\circ - 10\cos 21.80^\circ$
$= -1.857$

$H_z = \vec{H}\cdot\hat{a_z}$
$= 20\hat{a_\rho}\cdot\hat{a_z} - 10\hat{a_\phi}\cdot\hat{a_z} + 3\hat{a_z}\cdot\hat{a_z}$
$= 0 - 0 + 3$
$= 3$

~~Fro~~
Hence, $\vec{H_R} = 22.28\hat{a_x} - 1.857\hat{a_y} + 3\hat{a_z}$

(5)

(1) → done

(2) Transform $\vec{A_c} = x\hat{a_x} + xy\hat{a_z}$ at $(1, 2, 3)$ in cartesian to $\vec{A_{cy}}$ in cylindrical.

Soln:

$\vec{A_c} = \hat{a_x} + 2\hat{a_z}$

$\vec{A_{cy}} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$

$\vec{A_\rho} = A_c \cdot \hat{a_\rho}$

$= (\hat{a_x} + 2\hat{a_z}) \cdot \hat{a_\rho}$

$= \cos\phi + 0$

$= \cos\phi$

$\phi = \tan^{-1}\frac{2}{1} = 63.43°$

$A_\rho = \cos 63.43° = 0.447$

$A_\phi = \vec{A_c} \cdot \hat{a_\phi}$

$= (\hat{a_x} + 2\hat{a_z}) \cdot \hat{a_\phi}$

$= \hat{a_x} \cdot \hat{a_\phi} + 2\,\hat{a_z} \cdot \hat{a_\phi}$

$= -\sin\phi + 2\times 0 = -\sin\phi = -\sin 63.43 = -0.89$

$A_z = \vec{A_c} \cdot \hat{a_z}$

$= (\hat{a_x} + 2\hat{a_z}) \cdot \hat{a_z} = \hat{a_x} \cdot \hat{a_z} + 2\hat{a_z} \cdot \hat{a_z}$

$= 0 + 2 = 2$

From eqn (i)

$\vec{A_{cy}} = 0.447\,\hat{a_\rho} - 0.894\,\hat{a_\phi} + 2\hat{a_z}$ #

Q Express the vector field $\vec{W} = (x-y)\hat{a_y}$ in cylindrical and spherical co-ordinates.

Soln÷

(a) For cylindrical coordinate system,

$$\vec{W} = W_\rho \hat{a_\rho} + W_\phi \hat{a_\phi} + W_z \hat{a_z}$$

Where,

$$W_\rho = \vec{W} \cdot \hat{a_\rho} = (x-y)\hat{a_y} \cdot \hat{a_\rho}$$

$$= (\rho\cos\phi - \rho\sin\phi) \cdot \sin\phi$$

$$= \rho\cos\phi\sin\phi - \rho\sin^2\phi$$

$$W_\phi = \vec{W} \cdot \hat{a_\phi} = (\rho\cos\phi - \rho\sin\phi)\cos\phi$$

$$= \rho\cos^2\phi - \rho\sin\phi\cos\phi$$

$$W_z = \vec{W} \cdot \hat{a_z} = (x-y)\hat{a_y} \cdot \hat{a_z} = 0$$

$\therefore$ $\vec{W} = \rho(\cos\phi - \sin\phi)\sin\phi\,\hat{a_\rho} + \rho(\cos\phi - \sin\phi)\cos\phi\,\hat{a_\phi}$

(b) For spherical co-ordinate system

$$\vec{W} = W_r \hat{a_r} + W_\theta \hat{a_\theta} + W_\phi \hat{a_\phi}$$

$$W_r = \vec{W} \cdot \hat{a_r}$$

$$= (x-y)\hat{a_y} \cdot \hat{a_r}$$

(a) 28

$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\sin\theta\sin\phi$

$W_\theta = \vec{W}\cdot\hat{a}_\theta = (x-y)\,\hat{a}_y\cdot\hat{a}_\theta$

$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\cos\theta\sin\phi$

$W_\phi = \vec{W}\cdot\hat{a}_\phi = (x-y)\,\hat{a}_y\cdot\hat{a}_\phi$

$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\cos\phi$

$\therefore \vec{W} = r\sin\theta(\cos\phi - \sin\phi)\sin\theta\sin\phi\,\hat{a}_r +$

$r\sin\theta(\cos\phi - \sin\phi)\cos\theta\sin\phi\,\hat{a}_\theta + r\sin\theta$
$(\cos\phi - \sin\phi)\cos\phi\,\hat{a}_\phi$

(3) Transform vector $\vec{A} = (\rho\cos\phi\,\hat{a}_\rho + 2\hat{a}_z)$ at point $(1, 30^\circ, 2)$ in cylindrical co-ordinate system to a vector in spherical co-ordinate system.

Soln:

$P(1, 30^\circ, 2) \rightarrow P(\rho, \phi, z)$

In spherical coordinate system

$\vec{A} = A_R\,\hat{a}_R + A_\theta\,\hat{a}_\theta + A_\phi\,\hat{a}_\phi$

where,

$A_R = \vec{A}\cdot\hat{a}_R = (\rho\cos\phi\,\hat{a}_\rho + 2\hat{a}_z)\cdot\hat{a}_R$

$= \rho\cos\phi\,\hat{a}_\rho\cdot\hat{a}_R + 2\,\hat{a}_z\cdot\hat{a}_R$

$=$

In spherical co-ordinate system,

$$\vec{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$$

where,

$$\cancel{\vec{A_\rho} = A_\rho \cdot \hat{a_\rho}}$$
$$\cancel{= (r\cos\phi \, \hat{a_r} + 2\hat{a_z}) \cdot \hat{a_\rho}}$$
$$\cancel{= r\cos\phi \, \hat{a_r} \cdot \hat{a_\rho} + 2 \, \hat{a_z} \cdot \hat{a_\rho}}$$
$$\cancel{= r\cos\phi \sin\theta + 0}$$

$$\vec{A_r} = A_r \cdot \hat{a_r}$$
$$= (\rho\cos\phi \, \hat{a_\rho} + 2\hat{a_z}) \cdot \hat{a_r}$$
$$= \rho\cos\phi \sin\theta + 2\cos\theta$$

$$A_\theta = \vec{A} \cdot \hat{a_\theta}$$
$$= (\rho\cos\phi \, \hat{a_\rho} + 2\hat{a_z}) \cdot \hat{a_\theta}$$
$$= \rho\cos\theta\cos\phi - 2\sin\theta$$

$$A_\phi = \vec{A} \cdot \hat{a_\phi}$$
$$= (\rho\cos\phi \, \hat{a_\rho} + 2\hat{a_z}) \, \hat{a_\phi}$$
$$= 0 + 0 = 0$$

For point $(1, 30^\circ, 2)$

$$\phi = \phi = 30^\circ$$

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$r = \sqrt{\rho^2 + z^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\cancel{\phi = co}$ $\theta = \cos^{-1} \frac{z}{R} = \cos^{-1} \frac{2}{\sqrt{5}} = 26.565^\circ$

$\therefore A_\rho = 2.17614, \; A_\theta = -0.11982, \; A_\phi = 0$

$\therefore \vec{A} = 2.17614 \, \hat{a}_\rho - 0.11982 \, \hat{a}_\theta$

(10) Express a scalar potential field $V = x^2 + 2y^2 + 3z^2$ in spherical co-ordinates. Find the value of V at point $P(2, 60^\circ, 90^\circ)$.

Soln ÷

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

$V = (r \sin\theta \cos\phi)^2 + 2(r \sin\theta \sin\phi)^2 + 3(r\cos\theta)^2$

$= r^2 \sin^2\theta \cos^2\phi + 2r^2 \sin^2\theta \sin^2\phi + 3r^2 \cos^2\theta$

$V(2, 60^\circ, 90^\circ)$

$= (2)^2 \sin^2 60^\circ \cos^2 90^\circ + 2 \times (2 \sin 60^\circ \sin 90^\circ)^2$

$+ 3 \times (2)^2 \times \cos^2 60^\circ$

$= 9V$ #

(31)

(11) Express in cartesian components (a) the vector at A ($\rho = 4$, $\phi = 40°$, $z = -2$) that extends to B ($\rho = 5$, $\phi = -110°$, $z = 2$) (b) a unit vector at B directed towards A.

Soln :-

(a) A ($\rho = 4$, $\phi = 40°$, $z = -2$)

B ($\rho = 5$, $\phi = -110°$, $z = 2$)

For A

$x = \rho\cos\phi = 4\cos 40° = 3.064$

$y = \rho\sin\phi = 4\sin 40° = 2.571$

$A(x, y, z) = (3.064, 2.571, -2)$

For B

$x = \rho\cos\phi = 5 \times \cos(-110) = -1.71$

$y = \rho\sin\phi = 5 \times \sin(-110) = -4.698$

$B(x, y, z) = (-1.71, -4.698, 2)$

$\vec{AB} = \vec{B} - \vec{A}$

$= (-4.774, -7.269, 4)$

$= -4.774\hat{a_x} - 7.269\hat{a_y} + 4\hat{a_z}$

(b) $\vec{BA} = 4.77\hat{a_x} + 7.269\hat{a_y} - 4\hat{a_z}$

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$\hat{a}_{BA} = \frac{\vec{BA}}{|\vec{BA}|}$

$= \frac{4.774\hat{a_x} + 7.289\hat{a_y} - 4\hat{a_z}}{\sqrt{(4.774)^2 + (7.289)^2 + (-4)^2}}$

$= 0.498\hat{a_x} + 0.758\hat{a_y} - 0.417\hat{a_z}$

(12) Give the vector in Cartesian coordinates that ~~exten~~ extends from $P(r=4, \theta=20^\circ, \phi=10^\circ)$ to $Q(r=7, \theta=120^\circ, \phi=75^\circ)$

Soln÷

For $P(4, 20^\circ, 10^\circ)$

$x = r\sin\theta\cos\phi = 1.35$

$y = r\sin\theta\sin\phi = 0.2376$

$z = r\cos\theta = 3.768$

For $Q(7, 120^\circ, 75^\circ)$

$x = 1.57, \; y = 5.86, \; z = -3.5$

$\vec{PQ} = \vec{Q} - \vec{P}$

$= (1.57 - 1.35)\hat{a_x} + (5.86 - 0.2376)\hat{a_y} + (-3.5 - 3.768)\hat{a_z}$

$= 0.22\hat{a_x} + 5.6224\hat{a_y} - 7.26\hat{a_z}$

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(13) Given the vector in spherical coordinates at $M (x=5, y=1, z=2)$ that extends to $N (2,4,6)$

Soln:

$$\vec{MN} = \overset{N}{(2,4,6)} - \overset{M}{(5,1,2)}$$
$$= (2-5)\hat{a_x} + (4-1)\hat{a_y} + (6-2)\hat{a_z}$$
$$= -3\hat{a_x} + 3\hat{a_y} + 4\hat{a_z}$$

let $\vec{MN} = -3\hat{a_x} + 3\hat{a_y} + 4\hat{a_z} = \vec{A}$

In spherical co-ordinate system

$$\vec{A_s} = A_r\hat{a_r} + A_\theta\hat{a_\theta} + A_\phi\hat{a_\phi}$$

$$A_r = \vec{A} \cdot \hat{a_r}$$
$$= (-3\hat{a_x} + 3\hat{a_y} + 4\hat{a_z}) \cdot \hat{a_r}$$
$$= -3\sin\theta\cos\phi + 3\sin\theta\sin\phi + 4\cos\theta$$

We have,

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1}{5} = 11.30^\circ$$

$$\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\frac{2}{\sqrt{5^2+1^2+2^2}} = 68.58^\circ$$

So, $A_r = -0.731$

$A_\theta = \vec{A} \cdot \hat{a_\theta} = -4.583$

$A_\phi = \vec{A} \cdot \hat{a_\phi} = 3.529$

$$\therefore \boxed{\vec{A_s} = -0.731\hat{a_r} - 4.583\hat{a_\theta} + 3.529\hat{a_\phi}}$$

(14) How far it is form A ($r = 110$, $\theta = 30°$, $\phi = 60°$) to B ($r = 30$, $\theta = 75°$, $\phi = 123°$).

Soln:

For (110, 30°, 60°)

$x = r \sin\theta \cos\phi = 110 \sin 30 \cos 60° = 27.5$

$y = r \sin\theta \sin\phi = 110 \sin 30 \sin 60 = 47.63$

$z = r \cos\theta = 110 \cos 60 = 95.262$

For (30°, 75°, 125°)

$x = -16.62$

$y = 23.73$

$z = 7.764$

Now,

A (27.5, 47.63, 95.262) and B (-16.62, 23.73, 7.764)

$$d_{AB} = \sqrt{(-16.62 - 27.5)^2 + (23.73 - 47.63)^2 + (7.764 - 95.262)^2}$$

$= 100.86$ units.

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Q. Given points A ($\rho = 5$, $\phi = 70^\circ$, $z = -3$) and B ($\rho = 2$, $\phi = -30^\circ$, $z = 1$) find:

(a) ~~find~~ a unit vector in cartesian coordinates at A directed towards B.

(b) a unit vector in cylindrical coordinates at A directed towards B.

Soln:-

(a) For A $(5, 70^\circ, -3)$

$x = 5\cos 70^\circ = 1.71 \quad y = 5\sin 70^\circ = 4.698$

$z = -3$

For B $(2, -30^\circ, 1)$

$x = 2\cos(-30^\circ) = 1.732$

$y = 2\sin(-30^\circ) = -1$

$z = 1$

$\vec{AB} = \vec{B} - \vec{A}$

$= 0.022\,\hat{a}_x - 5.698\,\hat{a}_y + 4\,\hat{a}_z$

$$\hat{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{0.022\,\hat{a}_x - 5.698\,\hat{a}_y + 4\,\hat{a}_z}{\sqrt{(0.022)^2 + (-5.698)^2 + 4^2}}$$

$= 0.00316\,\hat{a}_x - 0.818\,\hat{a}_y + 0.574\,\hat{a}_z$

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(b) In cylindrical

$$\vec{F} = F_\rho \hat{a}_\rho + F_\phi \hat{a}_\phi + F_z \hat{a}_z$$

$$F_\rho = \vec{F} \cdot \hat{a}_\rho = (0.022\hat{a}_x - 5.698\hat{a}_y + 4\hat{a}_z) \cdot \hat{a}_\rho$$
$$= 0.022\cos\phi - 5.698\sin\phi + 0$$
$$= 0.022\cos 70 - 5.698\sin 70$$
$$= -5.346$$

$$F_\phi = \vec{F} \cdot \hat{a}_\phi = (0.022\hat{a}_x - 5.698\hat{a}_y + 4\hat{a}_z) \cdot \hat{a}_\phi$$
$$= -0.022\sin\phi - 5.698\cos\phi$$
$$= -0.022\sin 70^\circ - 5.698\cos 70^\circ$$
$$= -1.969$$

$$F_z = \vec{F} \cdot \hat{a}_z = (0.022\hat{a}_x - 5.698\hat{a}_y + 4\hat{a}_z) \cdot \hat{a}_z$$
$$= 4$$

From equation (i)

$$\vec{F} = -5.346\hat{a}_\rho - 1.969\hat{a}_\phi + 4\hat{a}_z$$

$$\hat{a}_F = \frac{\vec{F}}{|\vec{F}|} = \frac{-5.346\hat{a}_\rho - 1.969\hat{a}_\phi + 4\hat{a}_z}{\sqrt{(-5.346)^2 + (-1.969)^2 + 4^2}}$$
$$= -0.767\hat{a}_\rho - 0.282\hat{a}_\phi + 0.574\hat{a}_z$$

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(16) Transform the vector $\vec{A} = 4\hat{a_x} - 2\hat{a_y} - 4\hat{a_z}$ into spherical co-ordinate at a point P (x = -2, y = -3, z = 4)

Soln:

In spherical co-ordinate system

$\vec{A} = A_r \hat{a_r} + A_\theta \hat{a_\theta} + A_\phi \hat{a_\phi}$.

where,

$A_r = \vec{A} \cdot \hat{a_r}$

$= (4\hat{a_x} - 2\hat{a_y} - 4\hat{a_z}) \cdot \hat{a_r}$

$= 4\sin\theta\cos\phi - 2\sin\theta\sin\phi - 4\cos\theta$

At P (x = -2, y = -3, z = 4)

$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-3}{-2} = 180 + \tan^{-1}\left(\frac{3}{2}\right)$

$= 236.3^\circ$

$\theta = \cos^{-1}\frac{z}{r} = \frac{2}{\sqrt{x^2+y^2+z^2}} = 42.03^\circ$

$A_r = -3.343$

$A_\theta = \vec{A} \cdot \hat{a_\theta} = (4\hat{a_x} - 2\hat{a_y} - 4\hat{a_z}) \cdot \hat{a_\theta}$

$= 4\hat{a_x} \cdot \hat{a_\theta} - 2\hat{a_y} \cdot \hat{a_\theta} - 4\hat{a_z} \cdot \hat{a_\theta}$

$= 4\cos\theta\cos\phi - 2\cos\theta\sin\phi - 4(-\sin\theta)$

$= 2.266$

$A_\phi = \vec{A} \cdot \hat{a_\phi}$

$= (4\hat{a_x} - 2\hat{a_y} - 4\hat{a_z}) \cdot \hat{a_\phi}$

$= 4\hat{a_x} \cdot \hat{a_\phi} - 2\hat{a_y} \cdot \hat{a_\phi} - 4\hat{a_z} \cdot \hat{a_\phi}$

$= 4(-\sin\phi) - 2\cos\phi - 0$

$= 4.438$

From eqn (i)

$\vec{A} = \cancel{-3.4} \; -3.343\hat{a_r} + 2.266\hat{a_\theta} - 4.438\hat{a_\phi}$

Next box #

Transform $\vec{G} = (2x+y)\hat{a_x} - (y-4x)\hat{a_y}$ at point $Q(\rho, \phi, 2)$

Soln:

Given,

$\vec{G} = (2x+y)\hat{a_x} - (y-4x)\hat{a_y}$

now,

$\vec{G} = (2\rho\cos\phi + \rho\sin\phi)\hat{a_x} - (\rho\sin\phi - 4\rho\cos\phi)\hat{a_y}$

$\vec{G}_{cy} = G_\rho\hat{a_\rho} + G_\phi\hat{a_\phi} + G_z\hat{a_z}$

$\vec{G} = 2\rho\cos\phi\,\hat{a_x} + \rho\sin\phi\,\hat{a_x} - \rho\sin\phi\,\hat{a_y} + 4\rho\cos\phi\,\hat{a_y}$

$G_\rho = \vec{G} \cdot \hat{a_\rho}$

$= 2\rho\cos^2\phi + \rho\sin\phi\cos\phi \cancel{- \rho\sin^2\phi + 4\rho\cos\phi\sin\phi}$

$= 2\rho\cos^2\phi + 5\rho\sin\phi\cos\phi - \rho\sin^2\phi$

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$$a_\phi = \vec{a} \cdot \hat{a}_\phi$$

$$= -2\rho\cos\phi\sin\phi - \rho\sin^2\phi - \rho\sin\phi\cos\phi$$
$$+ 4\rho\cos^2\phi$$

$$= 4\rho\cos^2\phi - 3\rho\cos\phi\sin\phi - \rho\sin^2\phi$$

$$a_z = \vec{a} \cdot \hat{a}_z$$
$$= 0$$

$$\therefore \vec{a} = (2\rho\cos^2\phi + 5\rho\sin\phi\cos\phi - \rho\sin^2\phi)\,\hat{a}_\rho$$
$$+ (4\rho\cos^2\phi - 3\rho\cos\phi\sin\phi - \rho\sin^2\phi)\,\hat{a}_\phi$$

Transform $\vec{W} = (x-y)\,\hat{a}_y$ in spherical system

$$\vec{W} = (x-y)\,\hat{a}_y$$
$$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\,\hat{a}_y$$

$$\vec{W}_{sp} = W_r\hat{a}_r + W_\theta\hat{a}_\theta + W_\phi\hat{a}_\phi$$

$$W_r = \vec{W} \cdot \hat{a}_r$$
$$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\sin\theta\sin\phi$$
$$= r\sin^2\theta\sin\phi\cos\phi - r\sin^2\theta\sin^2\phi$$

$$W_\theta = \vec{W} \cdot \hat{a}_\theta$$
$$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\cos\theta\sin\phi$$
$$= r\sin\theta\cos\theta\sin\phi\cos\phi - r\sin\theta\cos\theta\sin^2\phi$$

$W_\phi = \vec{W} \cdot \hat{a_\phi}$

$= (r\sin\theta\cos\phi - r\sin\theta\sin\phi)\cos\phi$

$= r\sin\theta\cos^2\phi - r\sin\theta\sin\phi\cos\phi$

replace

$\vec{W_{sp}} = W_r\hat{a_r} + W_\theta\hat{a_\theta} + W_\phi\hat{a_\phi}$

Express the vector field $\vec{D} = \dfrac{x\hat{a_x} + y\hat{a_y}}{x^2+y^2}$ in cylindrical components and cylindrical variable. $(\rho = 2, \phi = 36, z = 5)$

Soln:

Now,

$$\vec{D} = \frac{x\hat{a_x} + y\hat{a_y}}{x^2+y^2}$$

$x = \rho\cos\phi = 1.619$ $\quad y = \rho\sin\phi = 1.175$

$\vec{D} = 0.4045\hat{a_x} + 0.294\hat{a_y}$

$D_\rho = D_\rho\hat{a_\rho} + D_\phi\hat{a_\phi} + D_z\hat{a_z}$

$D_\rho = \vec{D}\cdot\hat{\rho} = 0.5$

$D_\phi = 0$

$D_z = 0$

$\therefore \vec{D_{cy}} = 0.5\hat{a_\rho}$

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At a point P(-3, -4, 5) transform the vector to spherical co-ordinate system that extends from P to Q (2, 0, -1).

soln:

$\vec{PQ} = (2,0,-1) - (-3,-4,-5)$
$= 5\hat{a_x} + 4\hat{a_y} - 6\hat{a_z}$

$P_{sp} = P_r \hat{a_r} + P_\theta \hat{a_\theta} + P_\phi \hat{a_\phi}$

$P_r = \vec{PQ} \cdot \hat{a_r}$
$= 5\sin\theta\cos\phi + 4\sin\theta\sin\phi - 6\cos\theta$

$P_\theta = \vec{PQ} \cdot \hat{a_\theta}$
$= 5\cos\theta\cos\phi + 4\cos\theta\sin\phi + 6\sin\theta$

$P_\phi = \vec{PQ} \cdot \hat{a_\phi}$
$= -5\sin\phi + 4\cos\phi$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{(-3)^2+(-4)^2+(5)^2}$
$= 5\sqrt{2}$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{5}{5\sqrt{2}}\right) = 45^\circ$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-4}{-3}\right) = 180^\circ + \tan^{-1}\left(\frac{4}{3}\right)$
$= 233.13^\circ$

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or Putting value of θ and ϕ we get

$$\vec{P_{SP}} = -8.63\,\hat{a_r} - 0.141\,\hat{a_\theta} + 1.6\,\hat{a_\phi}$$

Old Questions

Define vector field. A field vector is given by an expression $\vec{A} = \frac{1}{\sqrt{x^2+y^2+z^2}}(x\hat{a_x} + y\hat{a_y} + z\hat{a_z})$ at point $(2, 30^\circ, 1)$

→ A vector field is a mathematical construct where each point in a given space is assigned a vector. These vectors represent both a direction and magnitude at each point.

Soln:

$$\vec{A} = \frac{1}{\sqrt{x^2+y^2+z^2}}(x\hat{a_x} + y\hat{a_y} + z\hat{a_z})$$

$$\vec{A_c} = A_\rho\hat{a_\rho} + A_\phi\hat{a_\phi} + A_z\hat{a_z}$$

$$A_\rho = \vec{A}\cdot\hat{a_\rho}$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2}}(x\cos\phi + y\sin\phi + 0)$$

$$= \cancel{(x\cos 30^\circ + \sin 30^\circ + 0)}$$

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$$\frac{\sqrt{6}}{20}\left(2\cos 30^\circ \cdot \cos 30^\circ + 2\sin^2 30^\circ\right)$$

$$\frac{\sqrt{10}}{20}\left(2 \times 1\right)$$

$$= \frac{\sqrt{10}}{10}$$

$$= 0.316$$

$$A_\phi = \vec{A} \cdot \hat{a_\phi}$$

$$= \frac{\sqrt{6}}{20}\left(-2\cos 30^\circ \sin 30^\circ + 2\sin 30^\circ \cos 30^\circ + 0\right)$$

$$= 0$$

A

$$A_z = \vec{A} \cdot \hat{a_z}$$

$$= \frac{\sqrt{10}}{20} \times 6$$

$$= 0.94$$

$$\vec{A}_{cy} = 0.316\, \hat{a_\rho} + 0.94\, \hat{a_z}$$ #