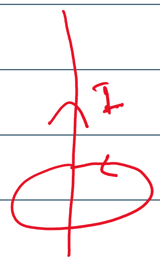
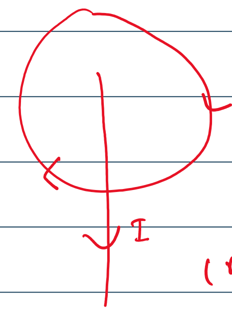
The steady (time invariant magnetic field)

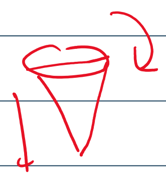


If current is upward magnetic field produced direction is anticlockwise

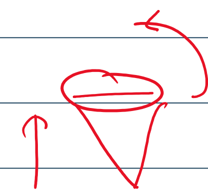


downwards (mf direction clockwise)

(easy to remember right hand rules)



Screw rotate clockwise moves downward



opposite

Magnetic materials → Materials that show magnetic properties

→ non-magnetic :→ not show magnetic properties

(i) Dimagnetic materials → materials weakly repelled by a magnetic field
es : copper, gold

(ii) paramagnetic materials → weakly attracted by a magnetic field
es : Aluminium, platinum

(iii) Ferromagnetic materials → strongly attracted by a magnetic field and can retain their magnetization even after the external field is removed. es : Iron, cobalt, nickel

The magnetic fields that are constant in time are called steady magnetic fields. The steady (time invariant) magnetic fields are produced by steady currents and the theory of steady magnetic fields is called magnetostatics.

Biot - Savart law

definition

→ "The magnitude of the magnetic field intensity at any point P produced by the differential element is directly proportional to the product of the current, the magnitude of the differential length, the sine of angle lying between the filament and a line connecting the filament to the point P and is inversely proportional to the square of the distance between the filament and point P."

The direction of the magnetic field intensity is normal to the plane containing the differential filament and the line drawn from the filament to the point P.

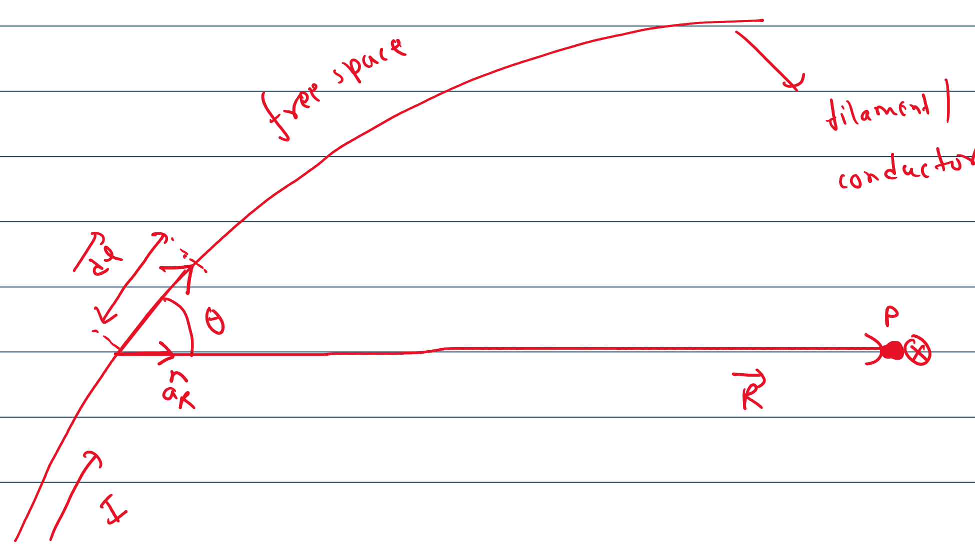


Fig: For calculating magnetic field intensity at P due to a differential current element using Biot-Savart law.

We know,

$$dH \propto \frac{I\, dl \sin\theta}{R^2} \propto \frac{I\ |\vec{dl}|\,|\hat{a}_R|\,\sin\theta}{R^2}$$

mag

In vector form

$|\vec{dl}|\,|\hat{a}_R|\,\sin\theta$

$$\vec{dH} \propto \frac{I\,\vec{dl} \times \hat{a}_R}{R^2}$$

Or $\vec{dH} = K\, \dfrac{I\, \vec{dl} \times \hat{a}_R}{R^2}$, where $K = \dfrac{1}{4\pi}$ = proportionality constant

Or, $\vec{dH} = \dfrac{I}{4\pi R^2}\, \vec{dl} \times \hat{a}_R$

Or, $\vec{dH} = \dfrac{I\, \vec{dl} \times \vec{R}}{4\pi R^3}$; $\hat{a}_R = \dfrac{\vec{R}}{|\vec{R}|}$

$\therefore$ $\boxed{\vec{dH} = I\, \dfrac{\vec{dl} \times \vec{R}}{4\pi R^3}}$

This is the differential expression of Biot-Savart law. The direction of $\vec{dH}$ at point P is inward the page $\otimes$ / $\odot \rightarrow$ outward

Direction → Thumb → current

curl fingers → mag field

Direction → Thumb → current
curl fingers → mag field

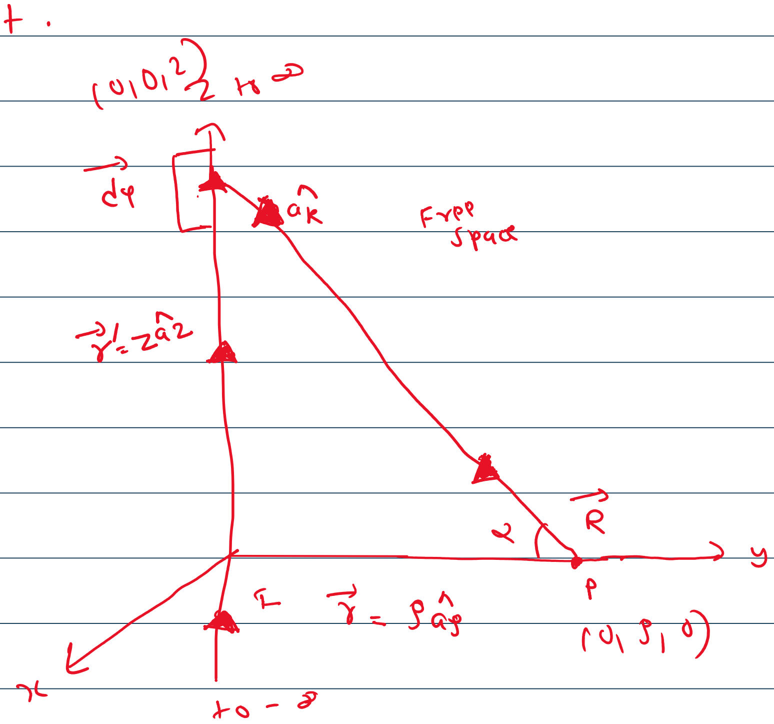
Total magnetic field intensity is :-

$$\boxed{\vec{H} = \oint I \frac{\vec{dl} \times \vec{R}}{4\pi R^3}}$$

Imp

Application of Biot - Savart Law (numerical lai more imp sometimes derivation)

① Magnetic field intensity due to an infinitely long straight filament carrying a dc current .



Consider an infinitely long straight filament carrying a dc current I

Consider an infinitely long straight filament carrying a dc current I is placed on z-axis as shown in the figure in cylindrical co-ordinate system. let P be the point where the value of magnetic field intensity is to be determined.

$$\vec{r'} + \vec{R} = \vec{r} \quad \text{(triangle law)}$$

$$\text{or } \vec{R} = \vec{r} - \vec{r'}$$
$$= \rho \hat{a}_\rho - z\hat{a}_z$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\rho \hat{a}_\rho - z\hat{a}_z}{\sqrt{\rho^2+z^2}}$$

Using Bio-savart law, magnetic field intensity at point P due to dl is : (imp) #

$$d\vec{H} = \frac{I\, d\vec{l} \times \vec{R}}{4\pi R^3} = \frac{I}{4\pi(\rho^2+z^2)^{3/2}}\left[dz\,\hat{a}_z \times (\rho\hat{a}_\rho - z\hat{a}_z)\right]$$

$$= \frac{I}{4\pi(\rho^2+z^2)^{3/2}}\rho\, dz\, \hat{a}_\phi \quad \left(\because \hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi\right)$$

$\hat{a}_z \times \hat{a}_z = 0$

cross product

z, ρ, φ

Total value of magnetic field intensity is calculated by integrating from z = -∞ to z = +∞

$$\vec{H} = \int d\vec{H} = \int_{z=-\infty}^{z=+\infty} \frac{I\rho\, dz\, \hat{a}_\phi}{4\pi(\rho^2+z^2)^{3/2}} = \frac{I\rho\, \hat{a}_\phi}{4\pi}\int_{-\infty}^{\infty}\frac{dz}{(\rho^2+z^2)^{3/2}} \quad \text{—(i)}$$

$$\text{let } I = \int_{-\infty}^{\infty}\frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho \tan\alpha$ | $z = -\infty \Rightarrow \alpha = -\pi/2$

$dz = \rho \sec^2\alpha \, d\alpha$ | $z = +\infty \Rightarrow \alpha = \pi/2$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{(\rho^2 + \rho^2 \tan^2\alpha)^{3/2}} = \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{[\rho^2(1+\tan^2\alpha)]^{3/2}}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{\rho^3 (\sec^2\alpha)^{3/2}} = \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{\rho^3 \sec^3\alpha} = \frac{1}{\rho^2}\int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha$$

$$= \frac{1}{\rho^2}[\sin\alpha]_{-\pi/2}^{\pi/2} = \frac{2}{\rho^2}$$

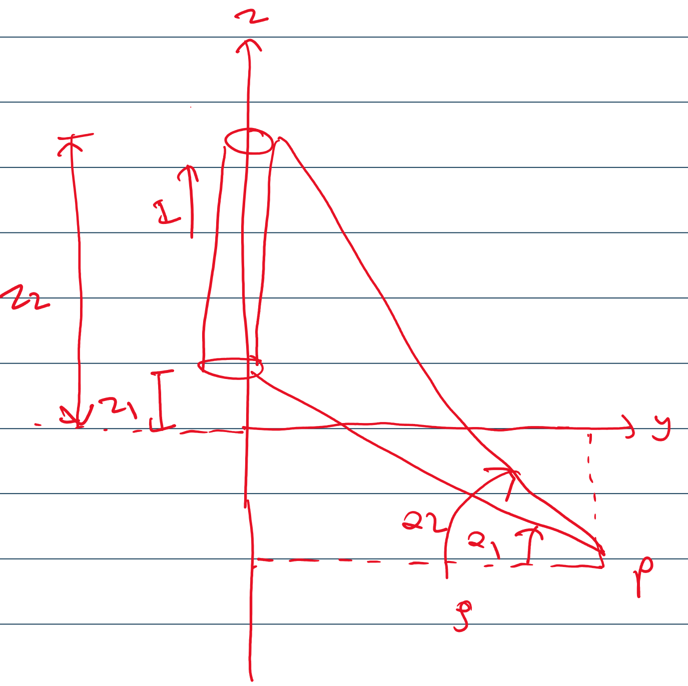
Now from (i)

$$\vec{H} = \frac{I \rho \, \hat{a}_\phi}{4\pi} \times \frac{2}{\rho^2}$$

$$\boxed{\therefore \vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

This shows that the magnitude of the field is not a function of ϕ or z, and varies inversely with ρ. The direction of $\vec{H}$ is along the unit vector $\hat{a}_\phi$.

case II : If the length of filament is finite



$$\vec{H} = \frac{I\rho}{4\pi}\hat{a}_\phi \int_{z=z_1}^{z=z_2} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

$$\text{let } I = \int_{z_1}^{z_2} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho\tan\alpha$ | $z = z_1 \Rightarrow \alpha = \alpha_1$ (say)

$dz = \rho\sec^2\alpha\, d\alpha$ | $z = z_2 \Rightarrow \alpha = \alpha_2$ (say)

$$\vec{H} = \frac{I\rho}{4\pi}\hat{a}_\phi \int_{\alpha=\alpha_1}^{\alpha_2} \frac{\rho\sec^2\alpha\, d\alpha}{(\rho^2+\rho^2\tan^2\alpha)^{3/2}} = \frac{I\rho\hat{a}_\phi}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho\sec^2\alpha\, d\alpha}{\rho^3\sec^3\alpha}$$

$$= \frac{I\rho}{4\pi}\hat{a}_\phi \int_{\alpha_1}^{\alpha_2} \frac{1}{\rho^2}\cos\alpha\, d\alpha$$

$$= \frac{I\rho}{4\pi}\, a_\phi \int_{\alpha_1}^{\alpha_2} \frac{1}{\rho^2} \cos\alpha \, d\alpha$$

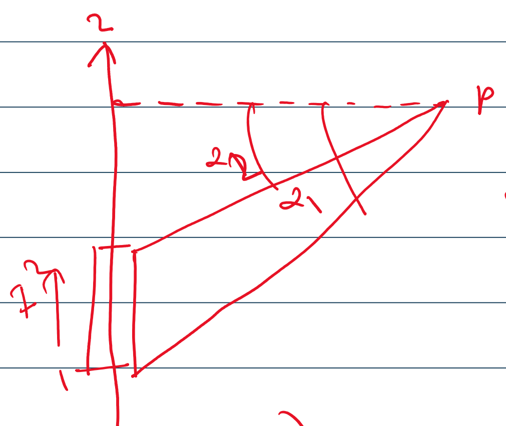
$$= \frac{I}{4\pi\rho}\, \hat{a}_\phi \left[\sin\alpha \right]_{\alpha_1}^{\alpha_2}$$

$$= \frac{I}{4\pi\rho} \left[\sin\alpha_2 - \sin\alpha_1 \right] \hat{a}_\phi$$

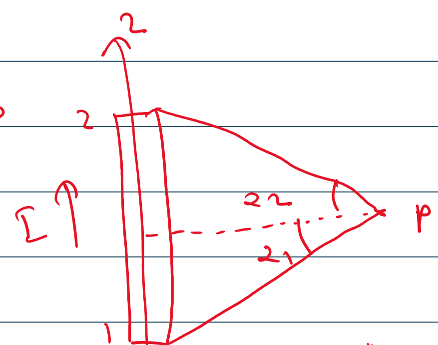
$$\therefore \boxed{\vec{H} = \frac{I}{4\pi\rho} \left[\sin\alpha_2 - \sin\alpha_1 \right] \hat{a}_\phi}$$

---

* Imp for numerical figures

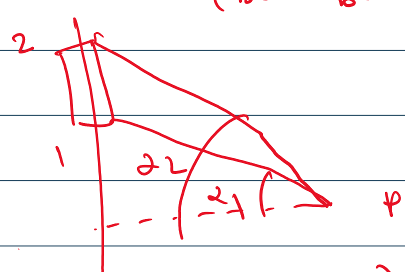


α_1, α_2 (−ve)
(both ends below P)



$\alpha_1 \to -ve$
$\alpha_2 \to +ve$

(one end below P
one end above)



(+ve)

α_1, α_2 (+ve)

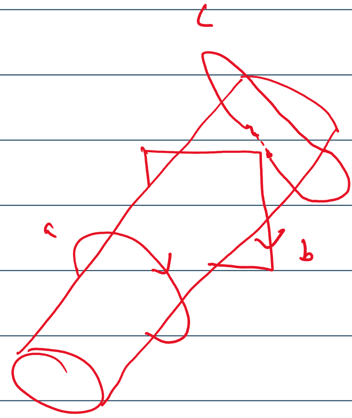
(both above P)

Ampere's circuital law (V. IMP)

→ Ampere circuital law state that the line integral of $\vec{H}$ about any closed path is exactly equal to the direct current enclosed by that path.

Mathematically,

$$\oint \vec{H} \cdot \vec{dl} = I$$



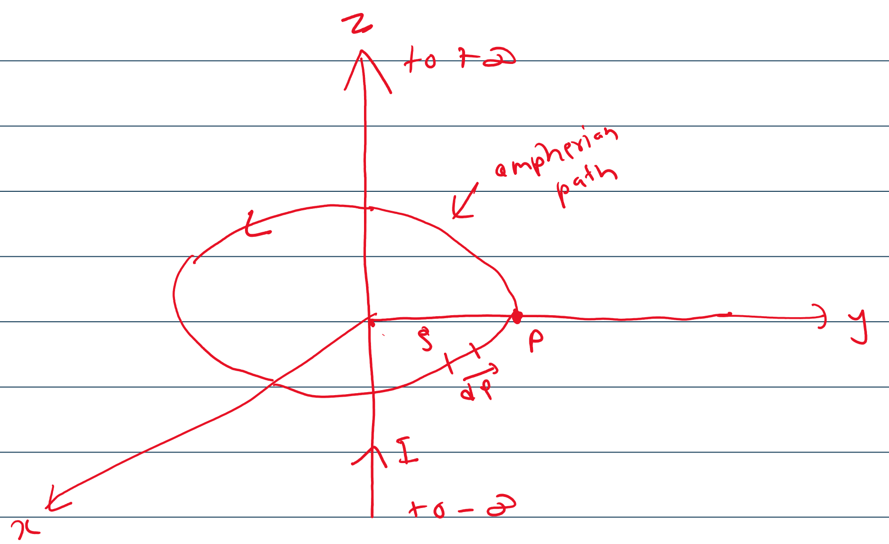
$$\oint_a \vec{H} \cdot \vec{dl} = I \qquad \oint_b \vec{H} \cdot \vec{dl} = I \qquad \oint_c \vec{H} \cdot \vec{dl} < I$$

Application of Ampere circuital law (IMP)

(i) Magnetic field Intensity due to an infinitely long Filament

→ Consider an infinitely long filament carrying a dc current I is placed on the z-axis in free space in cylindrical coordinate system as shown in the

→ Consider an infinitely long filament carrying a dc current I is placed on the z-axis in free space in cylindrical coordinate system as shown in the figure. Let P be the point where value of magnetic field intensity is to be determined.

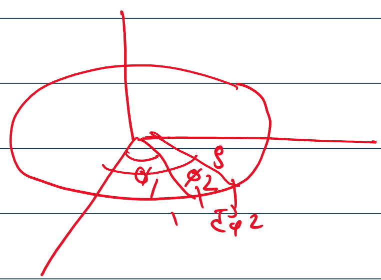


Now, consider a concentric circle as the amperian path in such a way that it passes through point P. Note that only H_ϕ component of $\vec{H}$ exists.

From Ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

or, $$\int_0^{2\pi} H_\phi \hat{a}_\phi \, \rho \, d\phi \, \hat{a}_\phi = I$$



$$dl = \rho(\phi_2 - \phi_1)$$
$$= \rho \, d\phi$$

or, $$\int_0^{2\pi} H_\phi \rho \, d\phi = I$$

or, $$H_\phi \rho \int_0^{2\pi} d\phi = I$$

or, $H_\phi \rho 2\pi = I$

$\therefore \quad H_\phi = \dfrac{I}{2\pi\rho}$

In vector form, $\boxed{\vec{H} = H_\phi \hat{a}_\phi = \dfrac{I}{2\pi\rho} \hat{a}_\phi}$

(V. Imp)

(ii) Magnetic field Intensity due to Infinitely long coaxial transmission line

→ Consider an infinitely long transmission line carrying a uniformly distributed total current I in the centre conductor and −I in the outer conductor as shown in the figure.

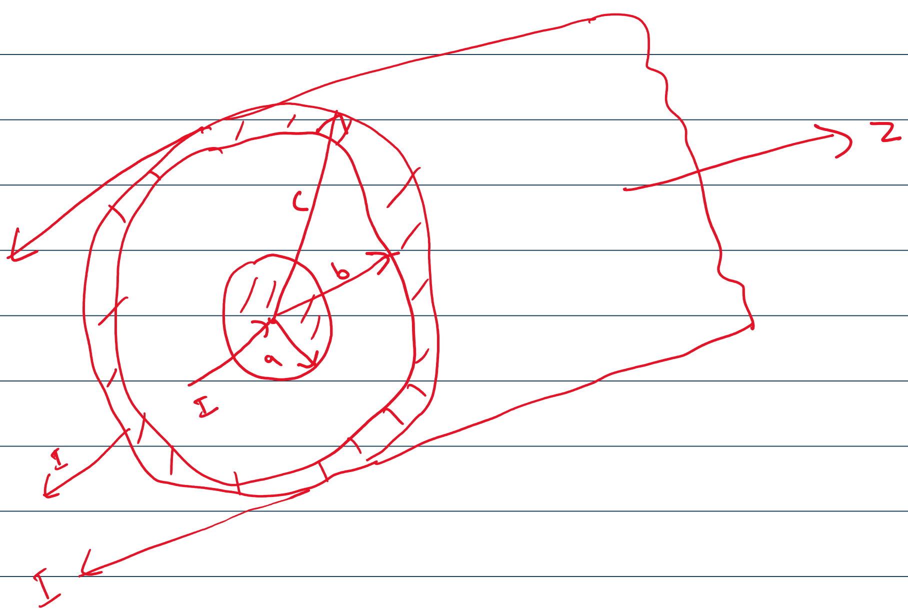
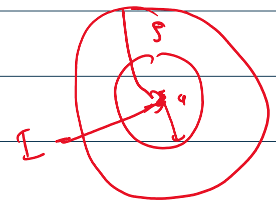


Fig: cross section of co-axial cable

As usual H is not a function of ϕ or z only H_ϕ component exists

Consider an amperian path at radius ρ.

Consider an amperion path at radius ρ.



Case I: $a < \rho < b$

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

or $$\int_0^{2\pi} H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = I$$

or $$\int_0^{2\pi} H_\phi \rho \, d\phi = I$$

or $$H_\phi \rho \int_0^{2\pi} d\phi = I$$

or $$H_\phi \rho \, 2\pi = I \quad , \quad H_\phi = \frac{I}{2\pi\rho}$$

$$\therefore \boxed{\vec{H} = H_\phi \hat{a}_\phi = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

Case - II:

$\rho < a$

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

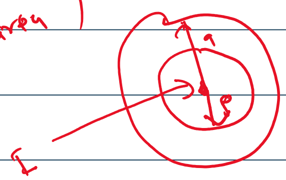
or $$H_\phi \cdot 2\pi\rho = I \frac{\rho^2}{a^2}$$

or $$H_\phi = \frac{I\rho}{2\pi a^2}$$

$$I_{enclosed} = \left(\frac{\text{Desired area}}{\text{Total area}}\right) I$$

$$= \left(\frac{\pi \rho^2}{\pi a^2}\right) I$$

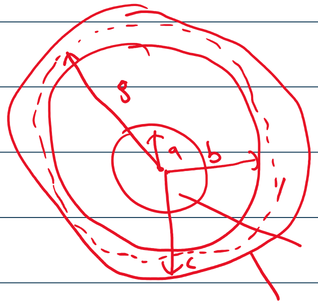
$$= \frac{\rho^2}{a^2} I$$



(amperian surface does not enclose all I)

$$\therefore \boxed{\vec{H} = H_\phi \hat{a_\phi} = \frac{I\rho}{2\pi a^2}\hat{a_\phi}}$$

case - III : $b < \rho < c$



$$\oint \vec{H} \cdot \vec{dl} = I_{\text{enclosed}}$$

$$H_\phi \cdot 2\pi\rho = I + \text{portion of } (-I)$$

$$\left(\text{portion of } -I = \left(\frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}\right) \times -I \right)$$

$$\text{or, } H_\phi \cdot 2\pi\rho = I - I\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

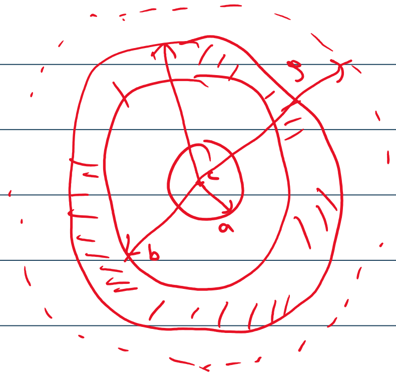
$$\text{or, } H_\phi \cdot 2\pi\rho = I\left(1 - \frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

$$\text{or, } H_\phi \cdot 2\pi\rho = I\left(\frac{c^2 - \cancel{b^2} - \rho^2 + \cancel{b^2}}{c^2 - b^2}\right)$$

$$\text{or, } H_\phi \cdot 2\pi\rho = I\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)$$

$$\text{or, } H_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)$$

$$\therefore \boxed{\vec{H} = H_\phi \hat{a}_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi}$$



Case - IV : $\rho > c$

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

$$H_\phi \cdot 2\pi\rho = I - I$$

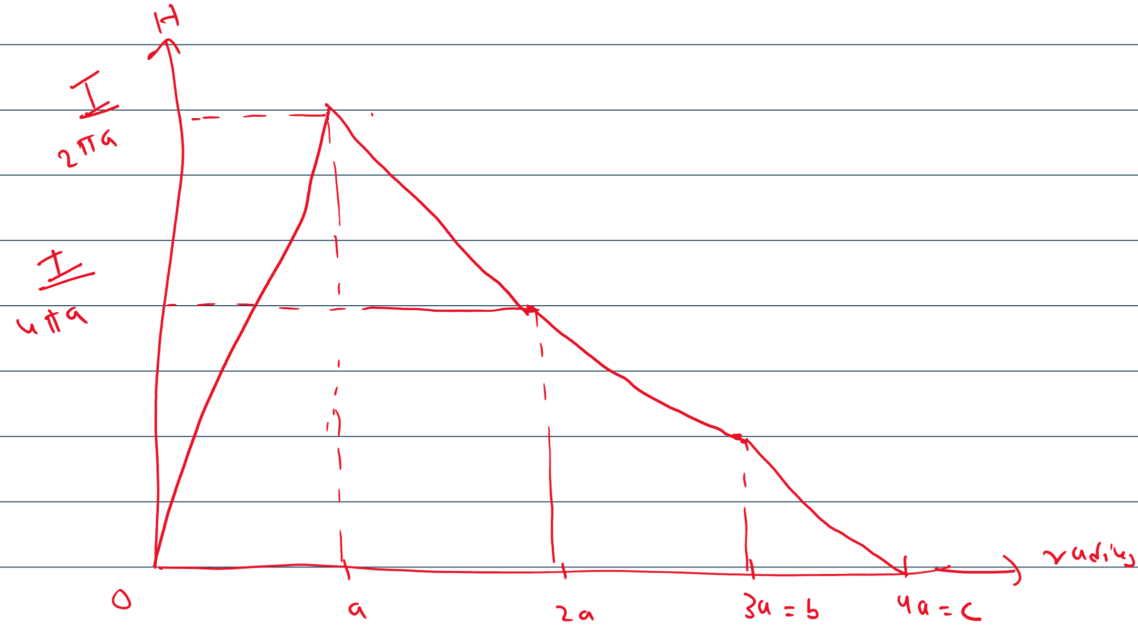
$$H_\phi \cdot 2\pi\rho = 0$$

$$H_\phi = 0$$

both current enclosed

$$\therefore \boxed{\vec{H} = 0}$$

For a co-axial cable with $b = 3a$, $c = 4a$ the variation of magnetic field strength with radius is :-



Curl and Stokes Theorem # Imp 3-4 marks (numerical more Imp)

Curl of a vector field $\vec{H}$ is a vector quantity that describes the rotation or circulating behavior of the field at a point. Mathematically it is defined using the limit of the closed line integral per unit area as the area shrinks to zero.

$$\boxed{(\text{curl } \vec{H})_N = \lim_{\Delta S_N \to 0} \frac{\oint \vec{H} \cdot d\vec{\varphi}}{\Delta S_N}}$$

where ΔS_N is the area enclosed by the path, the subscript N denotes the component normal to the surface. In vector calculus, curl is represented by the operator $\nabla \times \vec{H}$.

And,

$$\vec{J} = \text{curl } \vec{H}$$

Since,

$$\text{curl } \vec{H} = \nabla \times \vec{H}$$

we have,

$$\vec{J} = \nabla \times \vec{H}$$

This expression is known as point form of Ampere's circuital law (applicable to only time invariant conditions) and is known as Maxwell's second equation.

$\nabla \times \vec{H}$ = rect
$\nabla \times \vec{H}$ = cyl | In paper given
$\nabla \times \vec{H}$ = spherical

also,

$$(\text{curl } \vec{E})_N = \lim_{\Delta S_N \to 0} \frac{\oint \vec{E} \cdot \vec{dl}}{\Delta S_N}$$

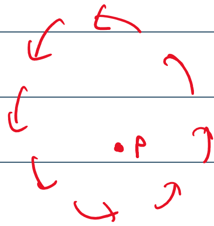
Physical Significance of curl (3 marks)

→ Curl provides the maximum value of the circulation of the field per unit area (or circulation density) and indicates the direction along which this maximum value occurs.

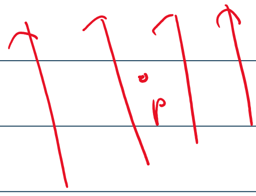
→ Imagine placing a tiny paddle wheel in the field; the curl determines how fast the wheel spins and the axis (direction) of rotation.

→ The curl vector's direction follows the right-hand rule: If the fingers curl in the direction of the field's rotation, the thumb points in the curl's direction.

→ fig (a) shows that curl of a vector field around P is directed out of the page. (b) shows a vector field with zero curl



(a)



(b)

(a)

Key Applications:

Ampère's Circuital Law (Maxwell's Second Equation): In magnetostatics, J⃗=∇×H⃗J=∇×H, where J⃗ is the current density. This shows that currents are sources of curling magnetic fields.
Zero Curl vs. Non-Zero Curl: A field with zero curl is irrotational (e.g., electrostatic fields), while non-zero curl implies rotational behavior (e.g., magnetic fields around a current-carrying wire).
Magnetism: A steady current creates a magnetic field with non-zero curl, while static charges produce electric fields with zero curl.

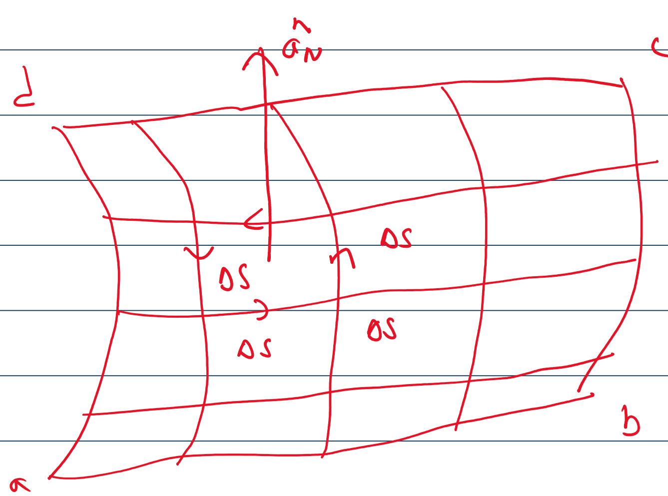
Stokes Theorem (Imp)

→ It states that the line integral of $\vec{H}$ about a closed path equals the surface integral of curl of $\vec{H}$ over the entire surface enclosed by the closed path.

Mathematically,

$$\oint \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{dS}$$

Proof:



a

Fig: For proving Stoke's theorem

Consider the surface S that comprises number of incremental surfaces each having area ΔS. Applying the definition of curl to one of these incremental surfaces,

$$\left(\text{curl } \vec{H}\right)_N = \left(\nabla \times \vec{H}\right)_N = \frac{\oint \vec{H}\cdot \vec{dl}_{\Delta S}}{\Delta S_N} \quad \text{---(i)}$$

right hand normal to surface

$\vec{dl}_{\Delta S}$ = indicates that the closed path is the perimeter of an incremental area ΔS

Eqn (i) becomes

$$\frac{\oint \vec{H}\cdot \vec{dl}_{\Delta S}}{\Delta S} = \left(\nabla \times \vec{H}\right)\cdot \hat{a}_N$$

(means N is replaced by vector)

Where $\hat{a}_N$ is a unit vector in the direction of the right hand normal to ΔS

or, $\oint \vec{H}\cdot \vec{dl}_{\Delta S} = \left(\nabla \times \vec{H}\right)\cdot \underbrace{\hat{a}_N \, \Delta S}$

or, $\oint \vec{H}\cdot \vec{dl}_{\Delta S} = \left(\nabla \times \vec{H}\right)\cdot \vec{\Delta S}$

We then calculate $\oint \vec{H}\cdot\vec{dl}$ for every ΔS of the surface S and sum the result. Some cancellation occurs because every interior wall is

We then calculate $\oint \vec{H} \cdot d\vec{l}$ for every ΔS of the surface S and sum the results. Some cancellation occurs because every interior wall is covered once in each direction. The only boundaries on which cancellation cannot occur is the outside boundary, the path enclosing S.

So,

$$\boxed{\oint \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \, d\vec{S}} \quad \text{--- (ii)}$$

$d\vec{l}$ is evaluated on the perimeter of S i.e a-b-c-d-a

eqn (ii) is Stokes theorem :

Magnetic flux and magnetic flux density

→ In free space, the magnetic flux density $\vec{B}$ is :-

$$\vec{B} = \mu_0 \vec{H}$$

$\vec{H}$ = magnetic field intensity, μ_0 = permeability of free space
$= 4\pi \times 10^{-7}$ H/m

For any other medium, $\vec{B} = \mu \vec{H}$

μ = permeability of the medium $= \mu_0 \mu_r$

Unit of $\vec{B}$ = T or Wb/m^2

Unit of $\vec{B}$ = T or Wb/m^2

magnetic flux (ϕ) is defined as the amount of magnetic field lines passing through any designated area.

It is given as $\phi = \int_S \vec{B} \cdot \vec{ds}$ Wb

The Scalar and Vector Magnetic Potentials

→ Scalar magnetic potential denoted by V_m is a Scalar quantity whose negative gradient gives the magnetic field intensity.

$$\vec{H} = -\nabla V_m \quad (\text{for } \vec{J} = 0)$$

Along a specified path, $V_{m,ab} = \int_b^a \vec{H} \cdot \vec{dl}$

→ Vector magnetic potential $\vec{A}$, defined such that $\vec{B} = \nabla \times \vec{A}$ serves as an intermediate quantity from which $\vec{B}$ and hence $\vec{H}$ can be calculated.

Force in a moving charge

electric force on stational or moving electric charge Q,

$\vec{F_e} = Q\vec{E}$ ($\vec{F_e}$, $\vec{E}$ have same direction if Q is +ve)

magnetic force experienced by a charge Q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is :-

$$\vec{F_m} = Q\vec{v} \times \vec{B}$$ ($\vec{F_m}$ is $\perp$r to both $\vec{v}$ and $\vec{B}$)

For moving charge in presence of both fields

$$\boxed{\vec{F} = \vec{F_e} + \vec{F_m} = Q(\vec{E} + \vec{v} \times \vec{B})}$$

↘ lorentz force equation

Relationship between $\vec{B}$, $\vec{H}$ and $\vec{M}$

From Ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I_T$$

or $$\oint \frac{\vec{B}}{\mu_0} \vec{dl} = I_T$$ ($\vec{B} = \mu_0 \vec{H}$)

Where, $I_T = I_b + I$

I_b → bound current

I → total free current

Now

$$I_b = \oint \vec{m} \cdot \vec{dy}$$

$$I = I_T - I_b$$

$$= \oint \frac{\vec{B}}{\mu_0} \cdot \vec{dy} - \oint \vec{m} \cdot \vec{dy}$$

$$= \oint \left(\frac{\vec{B}}{\mu_0} - \vec{m} \right) \vec{dy}$$

we can write

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{m}$$

or, $\boxed{\vec{B} = \mu_0 (\vec{H} + \vec{m})}$

For a linear isotropic media where magnetic susceptibility χ_m can be defined as:-

$$\vec{m} = \chi_m \vec{H}$$

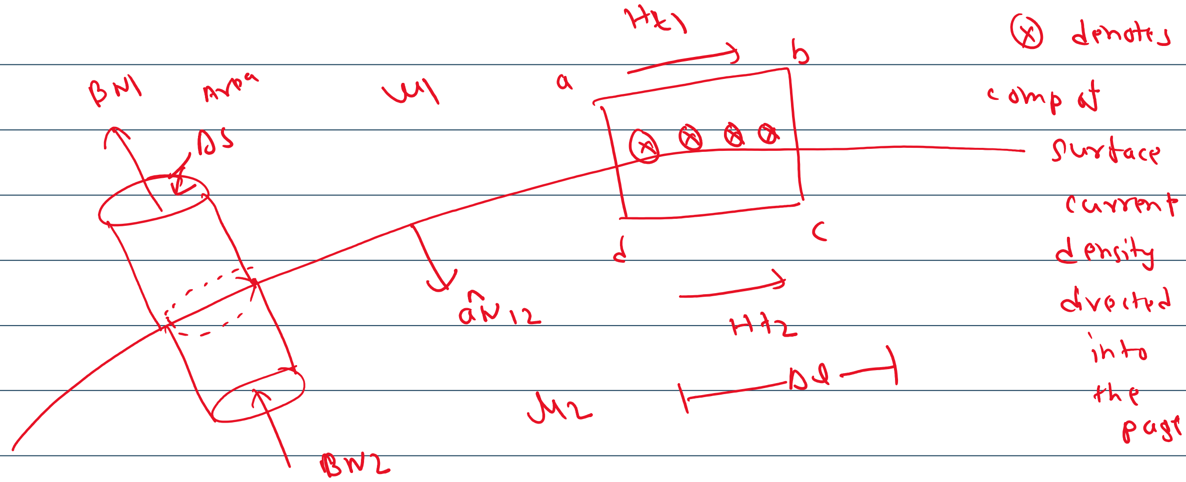
This leas to obtain

$$\boxed{\begin{aligned} \vec{B} &= \mu_0 (\vec{H} + \chi_m \vec{H}) \\ &= \mu_0 \mu_r \vec{H} \\ \text{where, } \mu_r &= 1 + \chi_m \end{aligned}}$$

Thus, $\mu = \mu_0 \mu_r$

Magnetic Boundary condition (Imp)

→ consider a boundary as shown in fig between two isotropic homogeneous linear materials with permeabilities μ_1 and μ_2. To find the boundary condition, we construct cylindrical guassian surface and rectangular closed path at boundary.



For calc magnetic boundary condition between two media

Now,

normal comp using Gauss's law

$$\oint \vec{B} \cdot \vec{dS} = 0$$

or $B_{N_1} \Delta S - B_{N_2} \Delta S = 0$

or $B_{N1}\Delta S - B_{N2}\Delta S = 0$

or: $\boxed{B_{N2} = B_{N1}}$

$B = \mu H$

So, $\mu_2 H_{N2} = \mu_1 H_{N1}$

or, $\boxed{H_{N2} = \frac{\mu_1}{\mu_2} H_{N1} = \frac{\mu_{r1}}{\mu_{r2}} H_{N1}}$

For tangential comp

$$\oint_{abcda} \vec{H} \cdot \vec{dl} = I$$

or, $$\int_a^b \vec{H} \cdot \vec{dl} + \int_b^c + \int_c^d + \int_d^a \vec{H} \cdot \vec{dl} = I$$

or, $H_{t1}\Delta l + 0 - H_{t2}\Delta l + 0 = K \Delta l$

or, $H_{t1} - H_{t2} = K$

K is component of $\vec{K}$ (surface current density) and is normal to the plane of the closed path. The value of K is zero if neither material is a conductor.

The equivalent vector form of above eqn is :-

$$\boxed{H_{t1} - H_{t2} = \hat{a}_{N12} \times \vec{K}}$$

Where $\hat{a}_{N12}$ is the unit normal at the boundary directed from region 1 to 2.

For $K = 0$,

$$\boxed{H_{t1} = H_{t2}}$$

$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$$

$$\boxed{B_{t1} = \frac{\mu_1}{\mu_2} B_{t2} = \frac{\mu_{r1}}{\mu_{r2}} B_{t2}}$$