For the point $P(3, 60^\circ, 2)$ in cylindrical co-ordinates and the potential field $V = 20(\rho+1)z^2\cos\phi$ V in free space, find at P:

(insights ko 16 number)

(i) V (ii) E (iii) D

(iv) volume charge density ρ_v

(v) Unit normal vector $\hat{a}_N$ which is in the direction of maximum rate of increase of potential.

Soln÷

(i) $V = 20(\rho+1)z^2\cos\phi$

$V(3, 60^\circ, 2) = 20(3+1)\,2^2\cos 60^\circ = 160\text{V}$

(ii) $$E = -\nabla V = -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -\left[20z^2\cos\phi\,\hat{a}_\rho + \frac{1}{\rho}20(\rho+1)z^2\cdot(-\sin\phi)\hat{a}_\phi + 20(\rho+1)\cos\phi\times 2z\,\hat{a}_z\right]$$

$$= -\left[20z^2\cos\phi\,\hat{a}_\rho - \frac{20(\rho+1)z^2\sin\phi}{\rho}\hat{a}_\phi + 40(\rho+1)z\cos\phi\,\hat{a}_z\right]$$

$$\vec{E}(3, 60^\circ, 2) = -\left[40\hat{a}_\rho - 92.37\hat{a}_\phi + 160\hat{a}_z\right]$$

$$= -40\hat{a}_\rho + 92.37\hat{a}_\phi - 160\hat{a}_z \text{ V/m}$$

$$|\vec{E}| = \sqrt{(-40)^2 + (92.37)^2 + (-160)^2} = 189.02 \text{ V/m}$$

(iii) $\vec{D} = \varepsilon_0\vec{E}$ so, $D = \varepsilon_0 E$

$D = 8.85\times10^{-2}\times 189.02$

$$D = 8.85\times10^{-2} \times 189.02$$

$$= 1.673\times10^{-9}\ C/m^2$$

(iv) $$\rho_v = \nabla\cdot\vec D = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\vec D = \epsilon_0 \vec E = -\epsilon_0\left[20z^2\cos\phi\,\hat a_\rho - \frac{20(\rho+1)z^2\sin\phi}{\rho}\hat a_\phi + 40(\rho+1)z\cos\phi\,\hat a_z\right]$$

$$\vec D = D_\rho \hat a_\rho + D_\phi \hat a_\phi + D_z\hat a_z$$

now

$$D_\rho = -20\epsilon_0 z^2\cos\phi \qquad D_\phi = \frac{20\epsilon_0(\rho+1)z^2\sin\phi}{\rho} \qquad D_z = -40\epsilon_0(\rho+1)z\cos\phi$$

now

$$\rho_v = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\cdot -20\epsilon_0 z^2\cos\phi\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\frac{20\epsilon_0(\rho+1)z^2\sin\phi}{\rho}\right)$$

$$+ \frac{\partial}{\partial z}\left(-40\epsilon_0(\rho+1)z\cos\phi\right)$$

$$= \frac{1}{\rho}\left(-20\epsilon_0 z^2\cos\phi\right) + \frac{1}{\rho^2}20\epsilon_0(\rho+1)z^2\cos\phi - 40\epsilon_0(\rho+1)\cos\phi$$

at $P(3, 60^\circ, 2)$

$$\rho_v = \epsilon_0\left(-13.33 + 17.78 - 80\right) = -6.68\times10^{-10}\ C/m^3$$

(v) $$\hat a_N = -\frac{\vec E_p}{|\vec E_p|} = -\frac{-(40\hat a_\rho - 92.376\hat a_\phi + 160\hat a_z)}{189.032}$$

remember

at point P

$$= 0.2116\hat a_\rho - 0.4886\hat a_\phi + 0.846\hat a_z$$

Find the energy stored in free space for the region $2mm < r < 3mm$, $0 < \theta < 90°$, $0 < \phi < 90°$, given the potential field (a) $V = \frac{200}{r}$ V

(b) $V = \frac{300}{r^2} \cos\theta$ V (insight 18 no)

Soln:

$$W_E = \frac{1}{2} \int_{vol} \varepsilon_0 E^2 \, dv$$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{r\theta} \hat{a}_\theta + \frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi \right]$$

gradient spherical

For (a) $V = \frac{200}{r}$ V

$$\vec{E} = -\left[\frac{\partial}{\partial r}\left(\frac{200}{r}\right) \hat{a}_r + \frac{1}{r} \frac{\partial}{r\theta}\left(\frac{200}{r}\right) \hat{a}_\theta + \frac{1}{r\sin\theta} \frac{\partial \left(200/r\right)}{r\phi} \hat{a}_\theta \right]$$

$$= -\left[200 \times (-1 r^{-2}) \hat{a}_r + 0 + 0 \right] = \frac{200}{r^2} \hat{a}_r$$

$$|\vec{E}| = E = \frac{200}{r^2}$$

$$W_E = \frac{1}{2} \int_{vol} \varepsilon_0 \left(\frac{200}{r^2}\right)^2 (r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

$$= 0.5 \times \varepsilon_0 \times (200)^2 \times \int^{3\times10^{-3}} \quad 1 \, dr \int^{\pi/2} \sin\theta \, d\theta \int^{\pi/2} d\phi$$

$$= 0.5 \times \varepsilon_0 \times (200)^2 \times \int_{r=2\times10^{-3}}^{3\times10^{-3}} \frac{1}{r^2}\, dr \int_{\theta=0}^{\pi/2} \sin\theta\, d\theta \int_{\phi=0}^{\pi/2} d\phi$$

$$= 46.36\ \mu J \quad \#$$

(b) $V = \frac{300}{r^2} \cos\theta \ V$

$$\vec{E} = -\left[\frac{\partial}{\partial r}\left(\frac{300\cos\theta}{r^2}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{300\cos\theta}{r^2}\right)\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\frac{300\cos\theta}{r^2}\right)\hat{a}_\phi \right]$$

$$= -\left[300\cos\theta \times (-2r^{-3})\,\hat{a}_r + \frac{300}{r^2}\cdot\frac{1}{r}(-\sin\theta)\,\hat{a}_\theta + 0 \right]$$

$$= \frac{600\cos\theta}{r^3}\hat{a}_r + \frac{300\sin\theta}{r^3}\hat{a}_\theta$$

$$E = \sqrt{\left(\frac{600\cos\theta}{r^3}\right)^2 + \left(\frac{300\sin\theta}{r^3}\right)^2}$$

$$= \sqrt{\frac{600^2\cos^2\theta}{r^6} + \frac{300^2\sin^2\theta}{r^6}} = \left(\frac{600^2\cos^2\theta + 300^2\sin^2\theta}{r^6}\right)^{1/2}$$

$$= \frac{1}{r^3}\left(600^2\cos^2\theta + 300^2\sin^2\theta\right)^{1/2}$$

$$= \frac{1}{r^3}\left(600^2\cos^2\theta + 300^2\sin^2\theta\right)^{1/2}$$

$$= \frac{1}{r^3}\left\{300^2\left(4\cos^2\theta + \sin^2\theta\right)\right\}^{1/2} = \frac{300}{r^3}\left(4\cos^2\theta + \sin^2\theta\right)^{1/2}$$

$$W_E = \frac{1}{2}\int_{vol} \varepsilon_0 \frac{(300)^2}{r^6}\left(4\cos^2\theta + \sin^2\theta\right)\left(r^2\sin\theta\, dr\, d\theta\, d\phi\right)$$

$$= \frac{1}{2}\varepsilon_0\, 300^2\left[\int_{vol}\frac{1}{r^6}4\cos^2\theta\, r^2\sin\theta\, dr\, d\theta\, d\phi + \int \frac{1}{r^6}\sin^2\theta\, r^2\sin\theta\, dr\, d\theta\, d\phi\right]$$

$$= \frac{1}{2}\varepsilon_0\, 300^2\left[\underbrace{4\int_{vol}\frac{1}{r^4}\cos^2\theta\sin\theta\, dr\, d\theta\, d\phi}_{I_1} + \underbrace{\int_{vol}\frac{1}{r^4}\sin^3\theta\, dr\, d\theta\, d\phi}_{I_2}\right]$$

$$I_1 = 4\int_{2\times10^{-3}}^{3\times10^{-3}}\frac{1}{r^4}dr\int_{\theta=0}^{\pi/2}\cos^2\theta\sin\theta\, d\theta\, , \int_{\phi=0}^{\pi/2} d\phi$$

$$= 4 \times 29320987.65 \times \frac{1}{3} \times \frac{\pi}{2}$$

$$= 61.409\ M$$

$$I_2 = \int_{2\times10^{-3}}^{3\times10^{-3}}\frac{1}{r^4}dr\int_{\theta=0}^{\pi/2}\sin^3\theta\, d\theta\int_{\phi=0}^{\pi/2} d\phi$$

$$= 29.32\,M \times 666.666\,m \times \frac{\pi}{2} = 30.70\ M$$

$$W_E = \frac{1}{2}\varepsilon_0\, 300^2\left[I_1 + I_2\right]$$

$W_E = \frac{120 \ 300}{2} \left[\quad \right]$

$= 36.699 \ J$ #

#

Calculate the potential difference V_{AB} for the line charge $\rho_L = 0.25 \times 10^{-9} \ C$ on z-axis where point A is $(2m, \frac{\pi}{2}, 0)$ and point B is $(4m, \frac{\pi}{2}, 5m)$

Soln ÷

Formula

Potential due due line charge

$$\boxed{V = \frac{-\rho_L}{2\pi\epsilon_0} \ln(r) + C}$$

Now,

Given

$\rho_L = 0.25 \times 10^{-9} \ C$ on z-axis $(0, 0, z)$ $\qquad V_{AB} = ?$

For simplicity converting A and B to rectangular co-ordinates

For $(2, \frac{\pi}{2}, 0) \leftrightarrow (\rho, \phi, z)$ $\quad x = \rho \cos\phi = 0 \quad z = z = 0 \quad (x, y, z) = (0, 2, 0)$

$y = \rho \sin\phi = 2 \qquad$ A

For $(4, \frac{\pi}{2}, 5) \rightarrow (\rho, \phi, z)$ $\quad x = \rho \cos\phi = 0 \quad$ B $(x, y, z) = (0, 4, 5)$

$y = \rho \sin\phi = 4$

$z = z = 5$

now

$$V_{AB} = V_A - V_B = \left[\frac{-\rho_L}{2\pi\epsilon_0} \ln(r_A) + C \right] - \left[\frac{-\rho_L}{2\pi\epsilon_0} \ln(r_B) + C \right]$$

$$V_{AB} = V_A - V_B = \left[-\frac{\rho_\ell}{2\pi\varepsilon_0} \ln(r_A) + C \right] - \left[-\frac{\rho_\ell}{2\pi\varepsilon_0} \ln(r_B) + C \right]$$

$$= -\frac{\rho_\ell}{2\pi\varepsilon_0} \ln(r_A) + \cancel{C} + \frac{\rho_\ell}{2\pi\varepsilon_0} \ln(r_B) - \cancel{C}$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0} \left(\ln(r_B) - \ln(r_A) \right) = \frac{\rho_\ell}{2\pi\varepsilon_0} \ln \frac{r_B}{r_A}$$

$\vec{r}_A = \vec{F.P} - I.P = (0,2,0) - (0,0,0) = 2\hat{a_y}$, $r_A = 2$

$\vec{r}_B = (0,4,5) - (0,0,5) = 4\hat{a_y}$, $r_B = 4$

$$V_{AB} = \frac{0.25 \times 10^{-9}}{2\pi\varepsilon_0} \ln\left(\frac{4}{2}\right) = 3.119\,V$$

\# If $V = \dfrac{60\sin\theta}{r^2}\,V$ in free space and point P is located at $r = 3m$, $\theta = 60°$, $\phi = 25°$ find (a) V_P (b) $\vec{E_P}$ (c) dV/dN at P (d) $\hat{a_N}$ at P (e) ρ_v at P

Soln÷

(a) $V_P = \dfrac{60\sin\theta}{r^2}\Bigg|_{(3,60°,25°)} = V_P = \dfrac{60\sin 60°}{3^2} = 5.77\,V$

spherical

(b) $$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a_\phi} \right]$$

$$= -\left[60\sin\theta\,(-2r^{-3})\,\hat{a_r} + \frac{1}{r} \times \frac{60}{r^2} \times \cos\theta\,\hat{a_\theta} + 0 \right]$$

$$= -\left[\frac{-120 \sin\theta}{r^3} \hat{a}_r + \frac{60 \cos\theta}{r^3} \hat{a}_\theta \right]$$

$$= \frac{120 \sin\theta}{r^3} \hat{a}_r - \frac{60 \cos\theta}{r^3} \hat{a}_\theta$$

$$\vec{E}_P (3, 60^\circ, 25^\circ) = \frac{120 \sin 60}{3^3} \hat{a}_r - \frac{60 \cos 60^\circ}{3^3} \hat{a}_\theta$$

$$= (3.849 \, \hat{a}_r - 1.11 \, \hat{a}_\theta) \text{ V/m}$$

① $\frac{dV}{dN}$ at P | $\boxed{\left.\frac{dV}{dN}\right|_P = |\vec{E}_P|}$ $= \sqrt{(3.84)^2 + (-1.11)^2}$

$= 4.006$ V/m

derivation remember

② $\boxed{\hat{a}_N = -\hat{a}_{E_P} = -\frac{\vec{E}_P}{|\vec{E}_P|}}$ Imp formula

$$= -\frac{(3.849 \, \hat{a}_r - 1.11 \, \hat{a}_\theta)}{4.006}$$

$$= -0.961 \, \hat{a}_r + 0.277 \, \hat{a}_\theta$$

③ ρ_v at P = ?

$$\rho_v = \nabla \vec{D}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \left(\frac{120 \sin\theta}{r^3} \hat{a}_r - \frac{60 \cos\theta}{r^3} \hat{a}_\theta \right)$$

spherical

$$\rho_v = \nabla \vec{D} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\frac{120 \, \varepsilon_0 \sin\theta \, r^2}{r^3} \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left(\frac{-60 \, \varepsilon_0 \cos\theta \sin\theta}{r^3} \right)$$

$$= \frac{1}{r^2} 120 \varepsilon_0 \sin\theta \left(-1 r^{-2} \right) + \frac{1}{2 r \sin\theta} \left(\frac{-60 \varepsilon_0}{r^3} \cdot \frac{\partial (\sin 2\theta)}{r \theta} \right)$$

$$= \frac{-120 \varepsilon_0 \sin\theta}{r^4} - \frac{30 \varepsilon_0}{r^4 \sin\theta} \cos 2\theta \times 2$$

$$= \frac{-120 \varepsilon_0 \sin\theta}{r^4} - \frac{60 \varepsilon_0 \cos 2\theta}{r^4 \sin\theta}$$

ρ_v at $P(3, 60^\circ, 25^\circ) = -7.573$ pC/m^3

\# A uniform sheet of charge $\rho_s = 40 \varepsilon_0$ C/m^2 is located in the plane $x=0$ in free space. A uniform line charge $\rho_\ell = 0.6$ nC/m lies along the line $x=9, y=4$ in free space. Find the potential at point $P(6, 8, -3)$ if $V = 10$V at $A(2, 9, 3)$.

given to find constant

Soln ÷

$V = V_L + V_S$

Potential due to surface charge:

$V = -\frac{\rho_s}{2\varepsilon_0} x + C$ → for $x=0$ plane

$-\frac{\rho_s}{2\varepsilon_0} y + C$ → for $y=0$ plane

$-\frac{\rho_s}{2\varepsilon_0} z + C$ → for $z=0$ plane

formula

now,

now,

$$V_L = \frac{-\rho_L}{2\pi\varepsilon_0} \ln(r) + C \qquad V_S = \frac{-\rho_S}{2\varepsilon_0} x + C$$

$$V = V_L + V_S = \frac{-\rho_L}{2\pi\varepsilon_0} \ln(r) - \frac{\rho_S}{2\varepsilon_0} x + C \quad \text{—— (i)}$$

To find C we know at (2, 9, 3), V = 10 V

For (2, 9, 3) FP

$$\vec{r} = F.P - I.P = (2,9,3) - (9,4,3) \qquad r = \sqrt{(-7)^2 + 5^2}$$

$$= -7\hat{a_x} + 5\hat{a_y} \qquad = \sqrt{74}$$

F.P I.P

$$x = 2 - 0 = 2$$

From (i)

$$10 = \frac{-0.6 \times 10^{-9}}{2\pi\varepsilon_0} \ln(\sqrt{74}) - \frac{40\varepsilon_0}{2\varepsilon_0}(2) + C$$

$\therefore$ $C = 73.210$

now eqn (i) is

$$V = \frac{-\rho_L}{2\pi\varepsilon_0} \ln(r) - \frac{\rho_S}{2\varepsilon_0} x + 73.210$$

At P (6, 8, -3)

then,

$$\vec{r} = (6,8,-3) - (9,4,-3) = -3\hat{a_x} + 4\hat{a_y}$$

$$r = 5$$

$$x = 6 - 0 = 6$$

$$\boxed{V = \frac{-0.6\times10^{-9}}{2\pi\varepsilon_0}\ln(5) - \frac{40\varepsilon_0}{2\varepsilon_0}(6) + 73.218 = -64.1482\,V}$$

\# Given the potential field $V = \frac{100xz}{x^2+4}$ V in free space. (a) Find $\vec{D}$ at the surface $z=0$ (b) show that the $z=0$ surface is an equipotential surface (c) Assume that the $z=0$ surface is a conductor and find the total charge on that portion of the conductor defined by $0<x<2$, $-3<y<0$.

Soln ÷

(a) $\vec{D}$ at $z=0$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -\left[100z\frac{\partial}{\partial x}\left(\frac{x}{x^2+4}\right)\hat{a}_x + 0 + \frac{100x}{x^2+4}\hat{a}_z\right]$$

$$\left(\frac{d\,u/v}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right)$$

↑ quotient rule

$$= -\left[100z\left(\frac{(x^2+4)\cdot1 - x\cdot2x}{(x^2+4)^2}\right)\hat{a}_x + \frac{100x}{x^2+4}\hat{a}_z\right]$$

$$= -\left[100z\left\{\frac{4-x^2}{(x^2+4)^2}\right\}\hat{a}_x + \left\{\frac{100x}{x^2+4}\right\}\hat{a}_z\right]\ \text{V/m}$$

$$\vec{D} = \varepsilon_0 \vec{E} = -\varepsilon_0 \left[100z \left\{ \frac{4-x^2}{(x^2+4)^2} \right\} \hat{a}_x + \left\{ \frac{100x}{x^2+4} \right\} \hat{a}_z \right] \ C/m^2$$

$$\boxed{\vec{D}\Big|_{z=0} = \frac{-100\,\varepsilon_0\, x}{x^2+4}\,\hat{a}_z \ C/m^2}$$

(b) $V = \frac{100xz}{x^2+4}$ V

At $z=0$, $V=0$ for all values of x and y. Therefore $z=0$ surface is on equipotential surface.

(c) total charge on the conductor defined by $0 < x < 2$, $-3 < y < 0$

$$\varphi = \oint_S \vec{D}\cdot\vec{ds}$$

For $z=0$ $\vec{D} = \frac{-100\,\varepsilon_0\, x}{x^2+4}\,\hat{a}_z \ C/m^2$

$$\varphi = \int_S \left(\frac{-100\varepsilon_0\, x}{x^2+4} \right) \hat{a}_z \ \left(dx\,dy\,\hat{a}_z \right)$$

$\vec{ds}$ along $\underline{z}$

$$= \int_{x=0}^{2} \int_{y=-3}^{0} \left(\frac{-100\,\varepsilon_0\, x}{x^2+4} \right) dx\,dy$$

$$= \int_{x=0}^{2} \frac{-300\,\varepsilon_0\, x}{x^2+4}\, dx = -920.58 \text{ pC}$$

\# Given the potential $V = \frac{10}{r^2} \sin\theta \cos\phi$,

r^2

(a) Find the electric flux density $\vec{D}$ at $(2, \frac{\pi}{2}, 0)$

(b) Calculate the work done in moving a $10\mu C$ from point $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$

Soln:

(a) $\vec{D} = \varepsilon_0 \vec{E}$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right)$$

$$= -\left[\frac{\partial}{\partial r}\left(\frac{10}{r^2}\sin\theta\cos\phi\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{10}{r^2}\sin\theta\cos\phi\right)\hat{a}_\theta + \right.$$

$$\left.\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\frac{10}{r^2}\sin\theta\cos\phi\right)\hat{a}_\phi\right]$$

$$= -\left[10\sin\theta\cos\phi\,(-2r^{-3})\hat{a}_r + \frac{1}{r}\cdot\frac{10}{r^2}\cos\phi\cos\theta\,\hat{a}_\theta + \frac{1}{r\cancel{\sin\theta}}\frac{10}{r^2}\cancel{\sin\theta}(-\sin\phi)\hat{a}_\phi\right]$$

$$= \left[\left(\frac{20}{r^3}\sin\theta\cos\phi\right)\hat{a}_r - \left(\frac{10}{r^3}\cos\theta\cos\phi\right)\hat{a}_\theta + \left(\frac{10}{r^3}\sin\phi\right)a_\phi\right]$$

At $(2, \frac{\pi}{2}, 0)$

$\vec{E} = 2.5\,\hat{a}_r$ V/m So, $\vec{D} = \varepsilon_0\vec{E} = 2.5\,\varepsilon_0\,\hat{a}_r$ C/m^2

(5) work done in moving a $10\mu C$ from point A (Q) to B

$$W = -Q\int_A^B \vec{E}\cdot d\vec{l} = Q\,V_{BA} = Q\,(V_B - V_A)$$ imp formula #

A to B

$$V_B = \frac{10}{4^2}\sin 90^\circ \cos 60^\circ \qquad V_A = \frac{10}{1^2}\sin 30^\circ \cos 120^\circ$$

(B (4, 90°, 60°) A (1, 30°, 120°)

now,

$$W = Q\,(V_B - V_A) = 10\mu\left(\frac{10}{4^2}\sin 90\cos 60^\circ - \frac{10}{1^2}\sin 30^\circ\cos 120^\circ\right)$$

$$= 28.125\,\mu J \quad \#$$

Two point charges $-4\mu C$ and $5\mu C$ are located at (2, −1, 3) and (0, 4, −2) respectively. Find the potential at (1, 0, 1) assuming zero potential at infinity.

soln:-

$$V = V_{p1} + V_{p2}$$

$$V_{p1} = \frac{Q_1}{4\pi\varepsilon_0 R_1} + C \qquad V_{p2} = \frac{Q_2}{4\pi\varepsilon_0 R_2} + C$$

$$V = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} + C \quad \text{— (i)}$$

Now, To determine the value of C_1 we have $V = 0$ at (∞, ∞, ∞)

(1)

Qn ma given

For (∞, ∞, ∞)

$$\vec{R}_1 = (\infty, \infty, \infty) - (2,-1,3) \quad ; \; R_1 = \infty$$
$$\vec{R}_2 = (\infty, \infty, \infty) - (0,4,-2) \quad ; \; R_2 = \infty$$

$$0 = \frac{-4\times10^{-6}\times9\times10^{9}}{\infty} + \frac{5\times10^{-6}\times9\times10^{9}}{\infty} + C$$

$\therefore \; C = 0$

Eqn (i) is :-

$$V = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

For $(1,0,1)$ as field point

$$\vec{R}_1 = (1,0,1) - (2,-1,3) = -\hat{a}_x + \hat{a}_y - 2\hat{a}_z$$
$$R_1 = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$
$$\vec{R}_2 = (1,0,1) - (0,4,-2) = \hat{a}_x - 4\hat{a}_y + 3\hat{a}_z$$
$$R_2 = \sqrt{(1)^2 + (-4)^2 + 3^2} = \sqrt{26}$$

Now,

$$V = \frac{-4\times10^{-6}\times9\times10^{9}}{\sqrt{6}} + \frac{5\times10^{-6}\times9\times10^{9}}{\sqrt{26}} = -5871.72\,V$$

\# let a uniform surface charge density of $5\,nC/m^2$ be present at $z=0$ plane, a uniform line charge density of $8\,nC/m$ be located at $x=0$, $z=4$ and a point charge of $2\,\mu C$ be present at $P(2,0,0)$. If $V=0$ at $M(0,0,5)$, find the potential at N at $x=0$, $z=4$ and a point charge at $2\mu C$ be present at $P(2,0,0)$. If $V=0$ at $M(0,0,5)$, find the potential at $N(1,2,3)$.

to find C

Soln÷

Given now,

$$V = V_s + V_\varphi + V_p$$

$$= -\frac{\rho_s}{2\varepsilon_0} z - \frac{\rho_\varphi}{2\pi\varepsilon_0} \ln(R_\varphi) + \frac{Q}{4\pi\varepsilon_0 R_p} + C \quad \text{— (i)}$$

We know, $V=0$ at $M(0,0,5)$

So,

$z = F.P - I.P = 5 - 0 = 5$

$R_\varphi = (0,0,5) - (0,0,4) = \hat{a}_z$, $R_\varphi = 1$

$R_p = (0,0,5) - (2,0,0) = -2\hat{a}_x + 5\hat{a}_z$, $R_p = \sqrt{29}$

now,

$$0 = -\frac{5\times10^{-9}}{2\varepsilon_0}\times 5 - \frac{8\times10^{-9}}{2\pi\varepsilon_0}\ln(1) + \frac{2\times10^{-6}}{4\pi\varepsilon_0\sqrt{29}} + C$$

$\therefore C = -1927.82$

So, eqn (i) becomes

$$V = \frac{Q}{4\pi\varepsilon_0 R_p} - \frac{\rho_\varphi}{2\pi\varepsilon_0}\ln(R_\varphi) - \frac{\rho_s z}{2\varepsilon_0} - 1927.82$$

At $N(1,2,3)$

$\vec{R_p} = (1,2,3) - (2,0,0) = -\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$; $R_p = \sqrt{14}$

$R_\varphi = (1,2,3) - (0,2,4) = \hat{a}_x - \hat{a}_z$; $R_\varphi = \sqrt{2}$

$z = 3 - 0 = 3$

$R_4 = (1,2,3) - (0,2,4) = \hat{a_x} - \hat{a_z}$; $R_4 = \sqrt{2}$

$z = 3 - 0 = 3$

$$V = \frac{2\times10^{-6}\times9\times10^{9}}{\sqrt{14}} - 2\times8\times10^{-9}\times9\times10^{9}\ln(\sqrt{2}) - \frac{5\times10^{-9}\times3}{2\times8.85\times10^{-12}}$$

$$- 1.928\times10^{-3}$$

$$= 1.985\ kV$$

Two uniform charges 8nC/m are located at $x=1$, $z=2$ at $x=-1$, $y=2$ in free space. If the potential at origin is 100V, find V at (4,1,3).

Soln:

$$V = V_{L1} + V_{L2} = -\frac{\rho_{L1}}{2\pi\varepsilon_0}\ln(R_1) - \frac{\rho_{L2}}{2\pi\varepsilon_0}\ln(R_2) + C \quad \text{---(i)}$$

At (0,0,0), V = 100 so,

$\vec{R_1} = (0,0,0) - (1,0,2) = -\hat{a_x} - 2\hat{a_z}$; $R_1 = \sqrt{5}$

$\vec{R_2} = (0,0,0) - (-1,2,0) = \hat{a_x} - 2\hat{a_y}$; $R_2 = \sqrt{5}$

Now,

$$100 = -\frac{8n}{2\pi\varepsilon_0}\ln(\sqrt{5}) - \frac{8n}{2\pi\varepsilon_0}\ln(\sqrt{5}) + C$$

$\therefore C = 331.555$

Now (i)

$$V = -\frac{\rho_{L1}}{2\pi\varepsilon_0}\ln(R_1) - \frac{\rho_{L2}}{2\pi\varepsilon_0}\ln(R_2) + 331.555$$

For $(4,1,3)$ as field point

$$\vec{R}_1 = (4,1,3) - (1,1,2) = 3\hat{a_x} + \hat{a_z} \; ; \; R_1 = \sqrt{10}$$

$$\vec{R}_2 = (4,1,3) - (-1,2,3) = 5\hat{a_x} - \hat{a_y} \; ; \; R_2 = \sqrt{26}$$

Now,

$$V = \frac{-8n}{2\pi\varepsilon_0}\left(\ln(\sqrt{10}) + \ln(\sqrt{26})\right) + 331.555$$

$$= -68.46\,V \quad \#$$

In free space, a line charge $\rho_l = 80\,nC/m$ lie along entire z-axis, while point charge of $100\,nC$ is located at $(0,1,0)$. Find the potential difference V_{PQ} given that $P(2,1,0)$ and $Q(3,2,5)$.

Soln:

due to point and line

due to point and line

$$V_{PQ} = V_P - V_Q$$

$$= \left[\frac{Q}{4\pi\varepsilon_0 R_P} - \frac{\rho_l}{2\pi\varepsilon_0}\ln(r_P) + C\right] - \left[\frac{Q}{4\pi\varepsilon_0 R_Q} - \frac{\rho_l}{2\pi\varepsilon_0}\ln(r_Q) + C\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 R_P} - \frac{Q}{4\pi\varepsilon_0 R_Q} - \frac{\rho_l}{2\pi\varepsilon_0}\ln(r_P) + \frac{\rho_l}{2\pi\varepsilon_0}\ln(r_Q)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_P} - \frac{1}{R_Q}\right] + \frac{\rho_l}{2\pi\varepsilon_0}\left[\ln\left(\frac{r_Q}{r_P}\right)\right]$$

$\vec{R_P}$ = field point − source point

$$= (2,1,0) - (0,1,0) = 2\hat{a_x} \; ; \; R_P = 2$$

$$\vec{R_Q} = (3,2,5) - (0,1,0) = 3\hat{a_x} + \hat{a_y} + 5\hat{a_z} \; ; \; R_Q = \sqrt{35}$$

$\vec{r_p} = (2,1,0) - (0,0,0) = 2\hat{a_x} + \hat{a_y}$; $r_p = \sqrt{5}$

$\vec{r_Q} = (3,2,5) - (0,0,5) = 3\hat{a_x} + 2\hat{a_y}$; $r_Q = \sqrt{13}$

now

$$V_{PQ} = 100\times10^{-9} \times 9\times10^{9}\left[\frac{1}{2} - \frac{1}{\sqrt{35}}\right] + 2\times 80\times10^{-9}\times 9\times10^{9}\, \ln\left(\frac{\sqrt{13}}{\sqrt{5}}\right)$$

$$= 985.84\,V \quad \#$$

\# For a potential field $V = r^2 z^2 \sin\phi$ at $P(1, 45°, 1)$ in cylindrical co-ordinate system, determine (a) V (b) $\vec{E}$ (c) $\vec{D}$ (d) ρ_v (e) Unit vector in direction of $\vec{E}$

cylindrical $\boxed{\rho = r}$

soln:-

(a) $V = r^2 z^2 \sin\phi \quad (1, 45°, 1)$

$= 1^2\, 1^2 \sin 45° = 0.707V$

(b) $$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{\partial V}{\partial z}\hat{a_z}\right]$$

$$= -\left[z^2 \sin\phi\, 2r\, \hat{a_r} + \frac{1}{r}\times r^2 z^2 \cos\phi\, \hat{a_\phi} + r^2 \sin\phi\, 2z\, \hat{a_z}\right]$$

$$= -\left[2rz^2 \sin\phi\, \hat{a_r} + rz^2 \cos\phi\, \hat{a_\phi} + 2zr^2 \sin\phi\, \hat{a_z}\right]$$

at $P(1, 45°, 1)$

$$\vec{E} = -(1.41\hat{a_r} + 0.707\hat{a_\phi} + 1.41\hat{a_z}) \ V/m$$

(i) $\vec{D} = \varepsilon_0 \vec{E} = -\varepsilon_0 (1.41\hat{a_r} + 0.707\hat{a_\phi} + 1.41\hat{a_z}) \quad C/m^2$

(ii) $\rho_v = \nabla \vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(r D_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

$$\vec{D} = D_r\hat{a_r} + D_\phi \hat{a_\phi} + D_z\hat{a_z}$$

$$\vec{D} = \varepsilon_0 \vec{E} = -\varepsilon_0 (2rz^2\sin\phi\,\hat{a_r} + rz^2\cos\phi\,\hat{a_\phi} + 2zr^2\sin\phi\,\hat{a_z})$$

$D_r = -2\varepsilon_0 r z^2 \sin\phi$, $D_\phi = -\varepsilon_0 r z^2 \cos\phi$, $D_z = -2\varepsilon_0 r^2 z \sin\phi$

$$\rho_v = \frac{1}{r}\frac{\partial}{\partial r}(r \cdot -2\varepsilon_0 r z^2 \sin\phi) + \frac{1}{r}\frac{\partial(-\varepsilon_0 r z^2\cos\phi)}{\partial\phi} + \frac{\partial(-2\varepsilon_0 r^2 z\sin\phi)}{\partial z}$$

$$= -\frac{1}{\cancel{r}} 2\varepsilon_0 z^2 \sin\phi \, 2\cancel{r} + \frac{1}{\cancel{r}}(-\varepsilon_0 \cancel{r} z^2)(-\sin\phi) + (-2\varepsilon_0 r^2\sin\phi)$$

$$= -4\varepsilon_0 z^2\sin\phi + z^2\varepsilon_0\sin\phi - 2\varepsilon_0 r^2\sin\phi$$

$$= -\varepsilon_0 (4z^2\sin\phi - z^2\sin\phi + 2r^2\sin\phi)$$

ρ_v at $P(1, 45°, 1)$

$$\rho_v = -31.30 \ pC/m^3$$

(iii) Unit vector in the direction of $\vec{E}$

$$|\vec{E}| = \sqrt{(-1.41)^2 + (-0.707)^2 + (-1.41)^2} = 2.12 \ V/m$$

$$\hat{a_N} = -\frac{\vec{E}}{|\vec{E}|} = \frac{1.41\hat{a_r} + 0.707\hat{a_\phi} + 1.41\hat{a_z}}{2.12}$$

$$\hat{a}_N = -\frac{\vec{E}}{|\vec{E}|} = \frac{1.41\,\hat{a}_r + 0.707\,\hat{a}_\phi + 1.41\,\hat{a}_z}{2.12}$$

$$= 0.667\,\hat{a}_r + 0.333\,\hat{a}_\phi + 0.667\,\hat{a}_z$$

\# Find the potential at point P (2,3,3) due to a 1nC charge located at Q (3,4,4), 1nC/m uniform line charge located at x=2, y=1 if potential at (3,4,5) is 0V.

soln :-

$$V = V_p + V_l = \frac{Q}{4\pi\varepsilon_0 R} - \frac{\rho_l}{2\pi\varepsilon_0}\ln(r) + C \quad \text{—(i)}$$

at point R (3,4,5) ; V = 0

$\vec{R} = (3,4,5) - (3,4,4) = \hat{a}_z$; R = 1

$\vec{r} = (3,4,5) - (2,1,5) = \hat{a}_x + 3\hat{a}_y$; $r = \sqrt{10}$

$$0 = \frac{9\times10^9 \times 1\times10^{-9}}{1} - 2\times9\times10^9 \times 10^{-9}\ln(\sqrt{10}) + C$$

$\therefore$ C = 11.723

now,

$$V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{\rho_l}{2\pi\varepsilon_0}\ln(r) + 11.723$$

at point P (2,3,3)

$\vec{R} = (2,3,3) - (3,4,4) = -\hat{a}_x - \hat{a}_y - \hat{a}_z$; $R = \sqrt{3}$

$\vec{r} = (2,3,3) - (2,1,3) = 2\hat{a}_y$; r = 2

$$V = \frac{\cancel{10^{-9}} \times 9 \times \cancel{10^{9}}}{\sqrt{3}} - \cancel{10^{-9}} \times 2 \times 9 \times \cancel{10^{9}} \ln(2) + 11.723$$

$$= 4.442\ V \quad \#$$

\# Two dipoles with dipole moments $-5\hat{a}_z$ nC m and $9\hat{a}_z$ nC m are located at points $(0,0,-2)$ and $(0,0,3)$ respectively. Find the potential at the origin.

derivation #

Soln ÷

$$\boxed{V = \sum_{k=1}^{2} \frac{\vec{P}_k \cdot \hat{a}_{rk}}{4\pi\varepsilon_0 r_k^2}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{P}_1 \cdot \hat{a}_{r1}}{r_1^2} + \frac{\vec{P}_2 \cdot \hat{a}_{r2}}{r_2^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{P}_1 \cdot \vec{r}_1}{r_1^3} + \frac{\vec{P}_2 \cdot \vec{r}_2}{r_2^3}\right)$$

now,

$$\vec{P}_1 = -5\times10^{-9}\,\hat{a}_z \quad , \quad \vec{P}_2 = 9\times10^{-9}\,\hat{a}_z$$

F.P

$$\vec{r}_1 = (0,0,0) - (0,0,-2) = 2\hat{a}_z \quad ; \quad r_1 = 2$$

$$r_2 = (0,0,0) - (0,0,3) = -3\hat{a}_z \quad ; \quad r_2 = 3$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{-5\times10^{-9}\,\hat{a}_z \cdot 2\hat{a}_z}{2^3} + \frac{9\times10^{-9}\,\hat{a}_z \times (-3\hat{a}_z)}{3^3}\right)$$

$$= -20.23\ V \quad \#$$

\# A point charge $Q_A = -1\mu C$ is at A $(0,0,1)$ and $Q_B = 1\mu C$ is at B $(0,0,-1)$. Find $\vec{E}$ in spherical co-ordinate system at P $(1,2,?)$

B(0,0, -1) Find $\vec{E}$ in spherical co-ordinate system at P (1,2,3)

Soln ÷

The system of charges mentioned in the question create a dipole

Derivation

For diplole

$$\boxed{\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3} \left(2\cos\theta \, \hat{a}_r + \sin\theta \, \hat{a}_\theta\right)}$$

$Q = 1\mu C$ (considering mag of either charge)

$d = 2$ (two charge)

P (1,2,3) $r = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right) = 36.7^\circ$

$$\boxed{\therefore E(r,\theta,\phi) = 550.25\,\hat{a}_r + 205.274\,\hat{a}_\theta \text{ V/m}}$$

\# $V = \rho^2 z \sin\phi$. calc energy within the region $1 < \rho < 4$

$-2 < z < 2$

$0 < \phi < \pi/3$

Soln ÷

$W_E = \frac{1}{2}\int_{vol} \varepsilon_0 E^2 \, dV$

cylindrical

cylindrical

$$\bar{E} = -\nabla V$$

$$= -\left(\frac{\partial V}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z \right)$$

$$= -\left(z \sin\phi \, 2\rho \, \hat{a}_\rho + \frac{1}{\rho} \rho^2 z \cos\phi \, \hat{a}_\phi + \rho^2 \sin\phi \, \hat{a}_z \right)$$

$$= -\left(2\rho z \sin\phi \, \hat{a}_\rho + \rho z \cos\phi \, \hat{a}_\phi + \rho^2 \sin\phi \, \hat{a}_z \right)$$

$$E = \sqrt{4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi}$$

or $$E^2 = 4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi$$

$$dv = \rho \, d\rho \, d\phi \, dz$$

$$W_E = \frac{1}{2} \varepsilon_0 \int_{vol} \left(\underbrace{4\rho^2 z^2 \sin^2\phi}_{I_1} + \underbrace{\rho^2 z^2 \cos^2\phi}_{I_2} + \underbrace{\rho^4 \sin^2\phi}_{I_3} \right) \rho \, d\rho \, d\phi \, dz$$

$$I_1 = \int_{vol} 4\rho^3 z^2 \sin^2\phi \, d\rho \, d\phi \, dz$$

$$= 4 \int_{\rho=1}^{4} \rho^3 d\rho \int_{\phi=0}^{\pi/3} \sin^2\phi \, d\phi \int_{z=-2}^{2} z^2 \, dz$$

$$= 417.64$$

$$I_2 = \int_{vol} \rho^3 z^2 \cos^2\phi \, d\rho \, d\phi \, dz$$

$$I_2 = \int_{vol} \rho^3 z^2 \cos^2\phi \, d\rho \, d\phi \, dz$$

$$= \int_1^4 \rho^3 d\rho \int_0^{\pi/3} \cos^2\phi \, d\phi \int_{-2}^{2} z^2 dz = 251.63$$

$$I_3 = \int_{vol} \rho^5 \sin^2\phi \, d\rho \, d\phi \, dz$$

$$= \int_1^4 \rho^5 d\rho \int_{\phi=0}^{\pi/3} \sin^2\phi \, d\phi \int_{-2}^{2} dz = 838.36$$

$$W_E = \frac{1}{2}\varepsilon_0 \left[417.64 + 251.63 + 838.36 \right]$$

$$= 6.67 \times 10^{-9} \text{ J}$$

\# The point charges $-1nC$, $4nC$ and $3nC$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ respectively. Find the energy in the system.

Soln:-

derivation

$$\boxed{W = \frac{1}{2}\sum_{m=1}^{3} Q_m V_m}$$

$$= \frac{1}{2}\left(Q_1V_1 + Q_2V_2 + Q_3V_3 \right)$$

$V_1 \rightarrow$ potential ~~test~~ location at Q_1 due to Q_2 and Q_3

$V_2 \rightarrow$ " " " " " Q_2 " " Q_1 " Q_3

$V_3 \rightarrow$ " " " " " " " " " Q_3 " " " Q_1 " " Q_2

$(0,0,1) - (1,0,0)$

at Q_2 due to Q_3

$V_3 \rightarrow$ ' ' ' ' ' ' ' ' ' Q_3 ' ' ' ' Q_1 ' ' ' Q_2 at Q_2 due to Q_3

$$= \frac{Q_1}{2}\left[\frac{Q_2}{4\pi\varepsilon_0 (1)} + \frac{Q_3}{4\pi\varepsilon_0 (1)}\right] + \frac{Q_2}{2}\left[\frac{Q_1}{4\pi\varepsilon_0 (1)} + \frac{Q_3}{4\pi\varepsilon_0 \sqrt{2}}\right]$$

V at Q_1 due to Q_2 — $(0,0,0) - (0,0,1)$

V at Q due to Q_3 — $(0,0,0) - (1,0,0)$

at Q_2 due to Q_1 — $(0,0,1) - (0,0,0)$

$$+ \frac{Q_3}{2}\left[\frac{Q_1}{4\pi\varepsilon_0 (1)} + \frac{Q_2}{4\pi\varepsilon_0 \sqrt{2}}\right]$$

V at Q_3 due to Q_1 — $(1,0,0) - (0,0,0)$

V at Q_3 due to Q_2 — $(1,0,0) - (0,0,1)$

$= 13.37 \text{ nJ}$

A The conducting planes $2x+3y=12$ and $2x+3y=18$ are at potentials 100V and 0V respectively. let $\varepsilon = \varepsilon_0$ and find

(a) V at P(5,2,6) $\vec{E}$ at P (5,2,6)

Soln:-

Two plane are parallel if $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = k$

Here

$$\frac{2}{2} = \frac{3}{3} = 1 \quad \text{(so it is parallel)}$$

Two planes are parallel, so we expect variation in potential in the direction normal to them in the direction normal to them

Using two boundary conditions, our general potential function can be written as:-

$$V(x,y) = A(\cancel{2x+3y}-12) + 100 = A(\cancel{2x+3y}-18) + 0$$

or, $-12A + 18A = -100$

or, $A = \frac{-100}{6}$

$$\therefore V(x,y) = \frac{-100}{6}(2x+3y-18)$$

at A P(5,2,6) $\therefore V = 33.33V$

(b) $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$$= -\left[-\frac{200}{6}\hat{a}_x - \frac{300}{6}\hat{a}_y + 0\right]$$

$$= \frac{100}{3}\hat{a}_x + 50\hat{a}_y \text{ V/m}$$

At P(5,2,6)

$$\boxed{\vec{E} = \frac{100}{3}\hat{a}_x + 50\hat{a}_y \text{ V/m}}$$

\# The potential on the plane $x-2y+5z=2$ is 50V. Point (2,3,-7) lies in parallel conducting plane having potential -360V.

(a) Find V at (-4,4,6) (b) $\vec{E}$ at (x,y,z) lies in parallel conducting plane having potential −360V.

ⓐ Find V at $(-1, 4, 6)$ ⓑ $\vec{E}$ at (x, y, z)

Soln ÷

The equation of parallel conducting plane can be expressed as

$$V = A(x - 2y + 5z - 2) + 50$$

As $V = -360$ and $P(2, 3, -7)$ lies on it, we have

$$-360 = A(2 - 2\times3 + 5\times(-7) - 2) + 50$$

$\therefore A = 10$

so, $V = 10(x - 2y + 5z - 2) + 50$

At $A(-1, 4, 6)$ $V = 10(1 - 2\times4 + 5\times6 - 2) + 50$

$$= 240\ V$$

ⓑ $\vec{E}(x, y, z) = ?$

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= (-10\hat{a}_x + 20\hat{a}_y - 50\hat{a}_z)\ V/m$$