(8, 9, 10, 11) → 16 × 3 = 48 marks

Chapter - 10

The uniform Plane wave and equations

Characteristics of different media

① Free space

→ Free space is characterized by $\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

② lossless or perfect dielectrics or good dielectrics

→ In this case, $\sigma \ll \omega\varepsilon$ so that $\frac{\sigma}{\omega\varepsilon} \ll 1$ therefore, this medium is characterized by $\sigma = 0$, $\varepsilon = \varepsilon_0\varepsilon_r$, $\mu = \mu_0\mu_r$.

③ lossy dielectrics (dissipative medium) or imperfect dielectrics or imperfect conductors or partially conducting medium.

→ This medium has the characteristics $\sigma \neq 0$, $\varepsilon = \varepsilon_0\varepsilon_r$, $\mu = \mu_0\mu_r$

④ Perfect or good conductors

→ For this medium, $\sigma \gg \omega\varepsilon$ so that $\frac{\sigma}{\omega\varepsilon} \gg 1$. so, this medium is characterized by $\sigma \simeq \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0\mu_r$.

Propagation constant (wave equation)

→ Propagation constant is a complex constant and is expressed as ÷ $\gamma = \alpha + j\beta$

$1\,Np = 8.686\,dB$

(i) Attenuation constant (α) (dB/m) (nepers/m) — log quantity

(ii) Phase constant (β) (rad/m)
(wave number)

The general expression for propagation constant is :

$$\gamma = \sqrt{j\omega\mu\,(\sigma + j\omega\varepsilon)}$$

The general expressions for attenuation constant and phase constant are ÷

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \quad \text{— } (*)$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \quad \text{— } (**)$$

(lossy)
→ A dissipative medium ($\sigma \neq 0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ so α and β are define by above expressions (*) and (**)

(lossless)
→ For perfect dielectric we put $\sigma = 0$ in (*) and (**)

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu\varepsilon}$$

→ For free space, $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu_0\varepsilon_0}$$

→ For perfect conductors

$\varepsilon = \varepsilon_0$, $\mu = \mu_0\mu_r$, $\sigma \gg \omega\varepsilon$, σ

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \rightarrow \frac{\sigma}{\omega\varepsilon} \gg 1$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\frac{\sigma}{\omega\varepsilon} - 1\right]} \qquad \frac{\sigma}{\omega\varepsilon} \gg 1$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2} \times \frac{\sigma}{\omega\cancel{\varepsilon}}}$$

$$= \sqrt{\frac{\mu\omega^2\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\therefore \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \#$$

Wave Impedance and Intrinsic Impedance

→ Wave Impedance (Z): The ratio of the transverse components of the electric and magnetic fields in an electromagnetive wave.

→ Intrinsic Impedance (η): Impedance of the medium that the wave propagates in. It is the characteristic of the medium.

For a medium with permittivity ε, permeability μ, and conductivity σ,

$$\text{Intrinsic impedance } (\eta) = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

* This is the general expression

* For dissipative medium

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

as $\eta = |\eta| \angle \theta$, $|\eta|$ and θ can be canclualted as

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(\frac{\sigma}{j\omega\varepsilon} + 1\right)}} = \sqrt{\frac{\mu / \varepsilon}{1 - j\frac{\sigma}{\omega\varepsilon}}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{\sqrt{(1)^2 + \left(\frac{-\sigma}{\omega\varepsilon}\right)^2}}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}$$

$$\left(\begin{array}{l} R = A + jB \\ |R| = \sqrt{A^2 + B^2} \\ \tan\theta = \frac{B}{A} \end{array} \right)$$

$$\tan 2\theta = \frac{\sigma}{\omega\varepsilon}, \quad 0^\circ < \theta < 45^\circ$$

For perfect conductors, $\sigma \gg \omega\varepsilon$ $(\omega\varepsilon \ll \sigma)$

$$\eta = \frac{\sqrt{\mu / \varepsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu / \varepsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{\sqrt{\mu / \varepsilon}}{\sqrt[4]{\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} \quad (\sigma \gg \omega\varepsilon)$$

$$= \frac{\sqrt{\mu / \varepsilon}}{\sqrt{\frac{\sigma}{\omega\varepsilon}}} = \sqrt{\frac{\mu}{\varepsilon} \times \frac{\omega\varepsilon}{\sigma}}$$

$$= \sqrt{\frac{\omega\mu}{\sigma}}$$

$$\tan 2\theta = \frac{\sigma}{\omega\varepsilon}$$

For $\omega\varepsilon \ll \sigma$, $\tan 2\theta \to \infty$ thus, $\theta \to 45^\circ$

* For perfect dielectrics, $\sigma = 0$

$$\eta = \sqrt{\mu / \varepsilon}$$

$$|\eta| = \sqrt{\mu / \varepsilon}, \quad \theta = 0^\circ$$

* For free space, $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

$$\eta = \sqrt{\mu_0 / \varepsilon_0} \qquad |\eta| = \sqrt{\mu_0 / \varepsilon_0} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}}$$

$$\approx 120\pi\,\Omega$$

Phasor

→ The phasor in the EM field is analogous to the logarithm (log) in the number field. As the log simplifies the calculation for numbers, the phasor simplifies the calculation involved in the analysis of EM waves. In general, a phasor could be a scalar or vector.

If the expression for a time-varing x-component of the electric field is :-

Imp

$$E_x = E_{x0} \cos(\omega t + \theta) \quad \text{(time-domain form)}$$

then, the phasor of E_x is written as E_{xs} and it's expression is :-

$$E_{xs} = E_{x0} e^{j\theta} \quad \text{(phasor form)}$$

For converting phasor form to time-domain form,

$$E_{xs} = E_{x0} e^{j\theta}$$

$$\begin{aligned} E_x &= \mathrm{Re}\left[(E_{xs}) e^{j\omega t}\right] \\ &= \mathrm{Re}\left[(E_{x0} e^{j\theta}) e^{j\omega t}\right] \\ &= \mathrm{Re}\left[E_{x0} e^{j(\omega t + \theta)}\right] \\ &= \mathrm{Re}\left[E_{x0}\{\cos(\omega t + \theta) + j\sin(\omega t + \theta)\}\right] \end{aligned}$$

$$E_x = E_{x0} \cos(\omega t + \theta)$$

→ The phasor replaces the cumbersome differentiation with a simple multiplication.

$$E_x = E_{xo} \cos(\omega t + \theta)$$

$$\frac{\partial E_x}{\partial t} = -\omega E_{xo} \sin(\omega t + \theta)$$

$$\boxed{\begin{aligned} &\therefore \frac{\partial \vec{E_x}}{\partial t} \to j\omega \vec{E_{xs}} \\ &\text{Similarly, } \int \vec{A}\, \partial t \to \frac{\vec{A_s}}{j\omega} \end{aligned}}$$

\# If $\vec{E} = 200 \sin 10^9 t \sin 20z\, \hat{a_x}$ V/m, find $\vec{E_s}$

Soln :-

$$\vec{E} = 200 \sin 10^9 t \sin 20z\, \hat{a_x}$$

$$= 200 \cos(90^\circ - 10^9 t) \sin 20z\, \hat{a_x}$$

$$= 200 \cos(\underbrace{-(90^\circ - 10^9 t)}_{\text{even function}}) \sin 20z\, \hat{a_x}$$

$$= 200 \cos(10^9 t - 90^\circ) \sin 20z\, \hat{a_x}$$

$$\downarrow$$

$\cos(\omega t + \theta)$

$\cos$ form

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

In phasor

$$\vec{E_s} = 200\, e^{-j90^\circ} \sin 20z\, \hat{a_x}$$

$$= 200 (\cos 90^\circ - j \sin 90^\circ) \sin 20z\, \hat{a_x}$$

$$= -200j \sin 20z\, \hat{a_x} \text{ V/m}$$

wave Equations

1. In perfect dielectric or lossless dielectric
   $(\sigma = 0, \mu = \mu_0 \mu_r, \varepsilon = \varepsilon_0 \varepsilon_r)$

Expression for electric field

$E_x = E_{x0} \cos(\omega t - \beta z)$ $\qquad \vec{E_x} = E_{x0} \cos(\omega t - \beta z)\, \hat{a_x}$

$E_{xs} = E_{x0}\, e^{-j\beta z}$ $\qquad \vec{E_{xs}} = E_{x0}\, e^{-j\beta z}\, \hat{a_x}$

for magnetic field

$H_y = \sqrt{\frac{\varepsilon}{\mu}}\, E_{x0} \cos(\omega t - \beta z)$ $\qquad \vec{H_y} = \sqrt{\frac{\varepsilon}{\mu}}\, E_{x0} \cos(\omega t - \beta z)\, \hat{a_y}$

$H_{ys} = \sqrt{\frac{\varepsilon}{\mu}}\, E_{x0}\, e^{-j\beta z}$ $\qquad \vec{H_{ys}} = \sqrt{\frac{\varepsilon}{\mu}}\, E_{x0}\, e^{-j\beta z}\, \hat{a_y}$

Note :

perfect dielectric : $\vec{J_s} = 0$, $\vec{J_{ds}} \neq 0$

perfect conductor : $\vec{J_s} \rightarrow \infty$, $\vec{J_{ds}} = 0$

lossy medium : $\vec{J_s} \neq 0$, $\vec{J_{ds}} \neq 0$

wave eq^n in free space

$(\sigma = 0, \mu = \mu_0, \varepsilon = \varepsilon_0)$

Same above replace $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

Wave eqn in dissipative medium (lossy dielectric)

$(\sigma \neq 0 \;,\; \varepsilon = \varepsilon_r \varepsilon_0 \;,\; \mu = \mu_r \mu_0)$

electric field

$$\vec{E}_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$$

$$\vec{E}_{xs} = E_{x0} e^{-\alpha z} e^{-j\beta z}\, \hat{a}_x$$

magnetic field

$$\vec{H}_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)\, \hat{a}_y$$

$$\vec{H}_{ys} = \frac{E_{x0}}{|\eta|} e^{-\alpha z} e^{-j(\theta + \beta z)}\, \hat{a}_y$$

$$|\eta| = \frac{\sqrt{\mu / \varepsilon}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}}$$

$$\tan 2\theta = \frac{\sigma}{\omega \varepsilon}$$

Wave equation for perfect conductors (Good conductors)

$(\sigma \simeq \infty \;,\; \varepsilon = \varepsilon_0 \;,\; \mu = \mu_r \mu_0)$

$$\vec{E}_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$$

$$\hat{H}_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)\, \hat{a}_y$$

$$|\eta| = \sqrt{\frac{\omega \mu}{\sigma}} \;,\; \theta = 45^\circ$$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\vec{E}_{xs} = E_{x0} e^{-\alpha z} e^{-j(\beta z + \theta)}\, \hat{a}_x$$

$$\vec{H}_{ys} = \frac{E_{x0}}{|\eta|} e^{-\alpha z} e^{-j(\beta z + \theta)}\, \hat{a}_y$$

Poynting vector

→ The cross product of electric field intensity ($\vec{E}$) and magnetic field intensity ($\vec{H}$) is called Poynting vector and is denoted by $\vec{S}$ or $\vec{P}$. It represents the instantaneous power density, measured in watts per square meter (W/m²)

The direction of the vector $\vec{S}$ indicates the direction of instantaneous power at a point.

$$\vec{S} = \vec{E} \times \vec{H} \quad W/m^2$$

Poynting's Theorem

→ From maxwell's equation for a conductive medium

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking scalar product with $\vec{E}$ on both side

$$\vec{E} \cdot \nabla \times \vec{H} = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \quad \text{--- (i)}$$

we know,

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\vec{E} \cdot \nabla \times \vec{H} + \vec{H} \cdot \nabla \times \vec{E}$$

$$\vec{E} \cdot \nabla \times \vec{H} = \vec{H} \cdot \nabla \times \vec{E} - \nabla (\vec{E} \times \vec{H})$$

So eqⁿ (i) will be

$$\vec{H} \cdot \nabla \times \vec{E} - \nabla (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \vec{E} \frac{\partial \vec{D}}{\partial t}$$

as, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ we have,

$$-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \nabla(\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \vec{E} \frac{\partial \vec{D}}{\partial t}$$

or $$-\nabla(\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \vec{E} \frac{\partial \vec{D}}{\partial t} + \vec{H} \frac{\partial \vec{B}}{\partial t}$$

or $$-\nabla(\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \mu \vec{H} \frac{\partial \vec{H}}{\partial t} \quad \text{— (ii)}$$

On rearranging

$$\varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} \vec{D} \cdot \vec{E}\right)$$

$$\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} \vec{B} \cdot \vec{H}\right)$$

Substituting these values in (ii)

$$-\nabla(\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \frac{\partial}{\partial t}\left(\frac{1}{2} \vec{D} \cdot \vec{E}\right) + \frac{\partial}{\partial t}\left(\frac{1}{2} \vec{B} \cdot \vec{H}\right)$$

Integrating above eqn throughout a volume

$$-\int_{vol} \nabla(\vec{E} \times \vec{H})\, dv$$

$$= \int_{vol} \vec{J}\cdot\vec{E}\,dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv$$

Applying divergence theorem which converts the volume integral into integral over the surface that encloses the volume, the 2.1.1.5 is written as :-

$$-\int_{vol} \nabla(\vec{E}\times\vec{H})\,dv - \oint_{area} (\vec{E}\times\vec{H})\,\vec{ds}$$

Therefore,

$$-\oint_{area} (\vec{E}\times\vec{H})\,\vec{ds}$$

$$= \int_{vol} \vec{J}\cdot\vec{E}\,dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv$$

Finally,

$$\boxed{-\oint_{area} (\vec{E}\times\vec{H})\,\vec{ds} = \int_{vol} \vec{J}\cdot\vec{E}\,dv + \frac{\partial}{\partial t}\int_{vol} \frac{1}{2}\vec{D}\cdot\vec{E}\,dv + \frac{\partial}{\partial t}\int_{vol} \frac{1}{2}\vec{B}\cdot\vec{H}\,dv}$$

which is the expression for poynting's theorem derivation

Calculation of Time Average power density (v.imp)

→ In a perfect dielectric, the $\vec{E}$ and $\vec{H}$ field amplitudes given by

$$E_x = E_{x0}\cos(\omega t - \beta z), \quad H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)$$

The power density amplitude is expressed as ÷

$$S_z = E_x H_y = \frac{E_{x0}^2 \cos^2(\omega t - \beta z)}{\eta}$$

The time average power density is denoted as $\langle S_z \rangle$

$$\langle S_z \rangle = \frac{1}{T}\int_0^T S_z \, dt$$

$$= \frac{1}{T}\int_0^T \frac{E_{x0}^2 \cos^2(\omega t - \beta z)}{\eta} \, dt \qquad \left(2\cos^2\theta = 1 + \cos 2\theta \right)$$

$$= \frac{E_{x0}^2}{T\eta}\int_0^T \frac{1 + \cos 2(\omega t - \beta z)}{2} \, dt$$

$$= \frac{E_{x0}^2}{2T\eta}\left[\int_0^T 1 \, dt + \int_0^T \cos 2(\omega t - \beta z) \, dt \right]$$

$$\boxed{\langle S_z \rangle = \frac{E_{x0}^2}{2\eta}}$$

In lossy dielectric, we have

$$E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z), \quad H_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)$$

$\therefore$ $$S_z = E_x H_y$$ $(2\cos A \cos B = \cos(A+B) + \cos(A-B))$

$$= \frac{1}{|\eta|} E_{x0}^2 e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta)$$

$$= \frac{1}{2|\eta|} E_{x0}^2 e^{-2\alpha z} [\cos(2\omega t - 2\beta z - \theta) + \cos\theta]$$

The time-avg power density $\langle S_z \rangle$ is calculated as :-

$$\langle S_z \rangle = \frac{1}{T} \int_0^T S_z \, dt$$

$$= \frac{1}{T} \left[\int_0^T \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} [\cos(2\omega t - 2\beta z - \theta) + \cos\theta] \, dt \right]$$

$$= \frac{1}{T} \cdot \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \left[\frac{1}{2\omega} \sin(2\omega t - 2\beta z - \theta) + t\cos\theta \right]_0^T$$

$$= \frac{1}{T} \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \left[T\cos\theta + \frac{1}{2\omega}(2\omega T - 2\beta z - \theta) - 0 - \frac{1}{2\omega} \sin(-2\beta z - \theta) \right]$$

Substituting $\omega = \frac{2\pi}{T}$ and simplifying, we will obtain

$$\boxed{\langle S_z \rangle = \frac{1}{2} \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z} \cos\theta}$$

Alternatively, we can obtain the above expression using

$$\boxed{\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}[\vec{E}_s \times \vec{H}_s^{*}] \quad \mathrm{W/m^2}}$$

which is general formula for computing the average power density in a propagating wave.

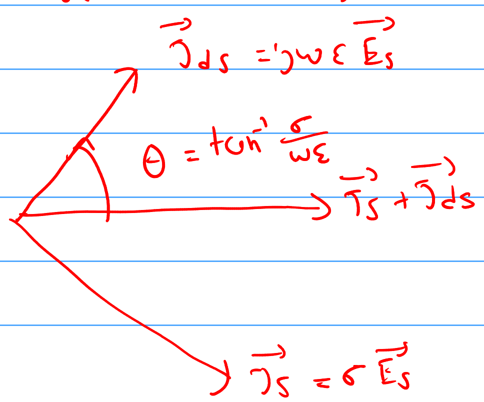
loss tangent (short-notes) (3 marks)

→ The ratio of the magnitude of the conduction current density $\vec{J}$ to that of the displacement current density $\vec{J}_d$ in a lossy medium is :-

$$\frac{|\vec{J}_s|}{|\vec{J}_{ds}|} = \frac{|\sigma \vec{E}_s|}{|j\omega\varepsilon \vec{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

where $\tan\theta$ is known as loss tangent and θ is the loss angle of the medium as illustrated in the figure.

→ $\tan\theta$ or θ may be used to determine how lossy a medium is. A medium is said to be good (lossless or perfect) dielectric if $\tan\theta$ is very small ($\sigma << \omega\varepsilon$) or a good conductor if $\tan\theta$ is very large ($\sigma >> \omega\varepsilon$)



Skin Depth

→ The electric field intensity and magnetic field intensity in a good conductor are expressed as :-

$$\vec{E}_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$$

$$\hat{H}_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z} \cos\left(\omega t - \beta z - \frac{\pi}{4}\right) \hat{a}_y$$

Both of these equations reveal that when $\vec{E}$ or $\vec{H}$ wave travels in a conducting medium, its amplitude is attenuated by factor $e^{-\alpha z}$.

For $z = 0$, $e^{-\alpha z} = e^{0} = 1$

For $z = \frac{1}{\alpha} = e^{-1} = 0.368$

This means that when wave propagates distance $z = \frac{1}{\alpha}$, the amplitude of wave decreases to 0.368 of the initial value. This distance is denoted by δ and is called "depth of penetration" or the "skin depth".

As stated earlier, for a good conductor

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

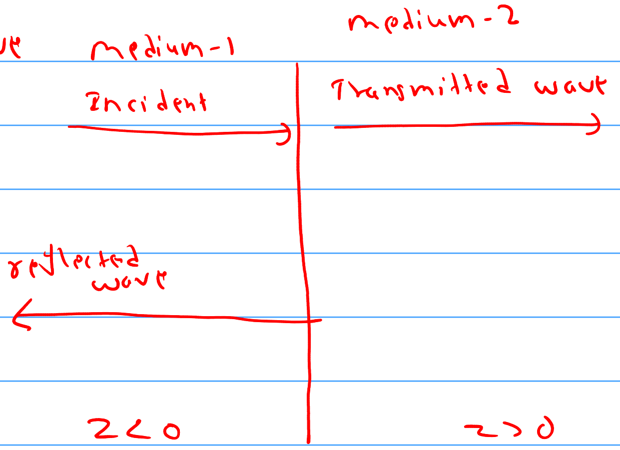
$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

→ The skin depth is a measure of depth to which an EM wave can penetrate the medium.

Skin effect

→ The phenomenon by which field intensity in a conductor rapidly decreases is known as skin effect.

Reflection of Uniform Plane Wave



Assumtions

1. Boundary, $z=0$

   $z<0$, $z>0$

2. Normal $\vec{E}$,

$$\vec{E}_N = \cancel{E_z \hat{a}_z} \to 0$$

$$\vec{E}_t = E_x \hat{a}_x + \cancel{E_y \hat{a}_y} \to 0$$

3. $H_t = H_y \hat{a}_y$

Dissipative medium

$$\vec{E}_x = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$$

$$E_{xs} = E_{x0}\, e^{-\gamma z}$$

Incident wave:

(+) forward moving

1 → medium 1

$$\overset{+}{E}_{xs_1} = \overset{+}{E}_{xo_1} e^{-\gamma_1 z}$$

$$\overset{+}{H}_{ys_1} = \frac{\overset{+}{E}_{xo_1}}{\eta_1} e^{-\gamma_1 z}$$

$$\vec{E}_{xs_1} = E_{xo_1} e^{+\gamma_1 z} \cdot \hat{a}_x$$

$$\vec{H}_{ys_1} = \frac{E_{xo_1}}{\eta_1} e^{+\gamma_1 z} (-\hat{a}_y)$$

Transmitted wave

$$\overset{+}{E}_{xs_2} = \overset{+}{E}_{xo_2} e^{-\gamma_2 z}$$

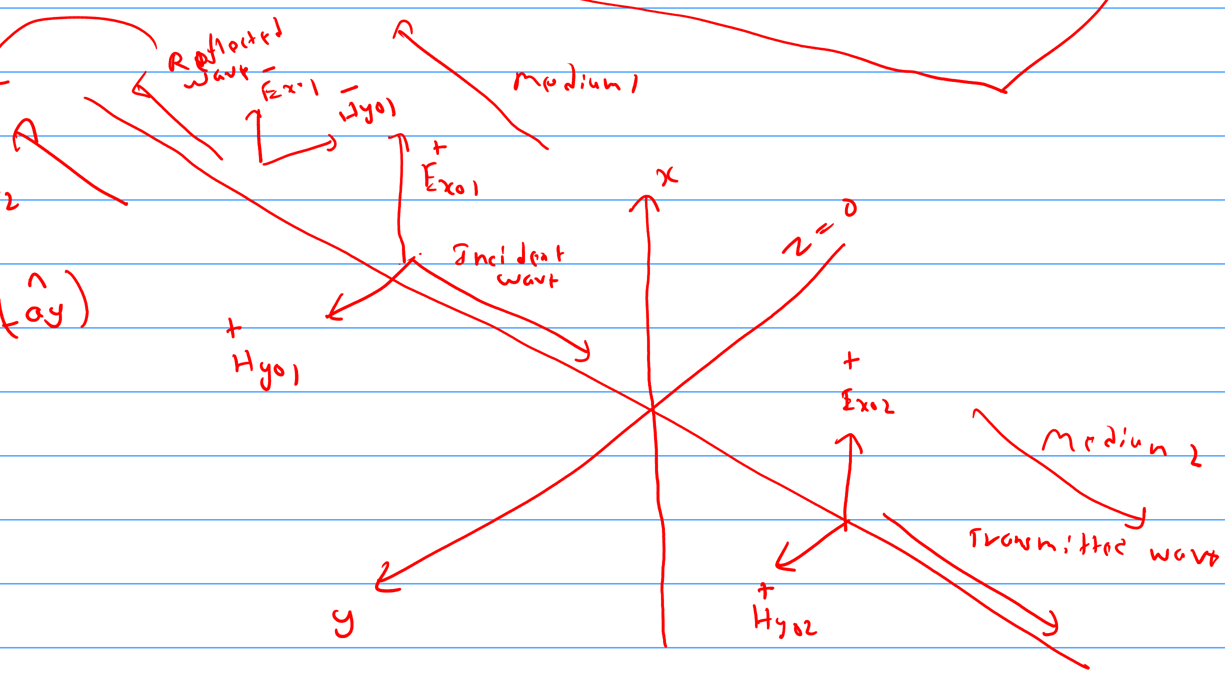
$$\overset{+}{H}_{ys_2} = \frac{\overset{+}{E}_{xo_2}}{\eta_2} e^{-\gamma_2 z}$$

reflected wave

$$\overset{-}{E}_{xs_1} = \overset{-}{E}_{xo_1} e^{+\gamma_1 z}$$

$$\overset{-}{H}_{ys_1} = \frac{\overset{-}{E}_{xo_1}}{\eta_1} e^{+\gamma_2 z}$$

Reflection of Uniform Plane Wave



$$-\hat{a}_z$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$-\hat{a}_z = \hat{a}_x \times (\hat{a}_y)$$

(Imp)

$E_{t1} = E_{t2}$

$H_{t1} = H_{t2}$

→ boundary condition previous chapter

→ Reflection coefficient (Γ)

The wave eq^n in dissipative medium is given by :-

$$E_x = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z)$$

In phasor

$$E_{xs} = E_{x0}\, e^{-\gamma z} \quad (\gamma = \text{propagation constant})$$

The incident wave in medium 1 thus may be written as :-

$$E^{+}_{xs1} = E^{+}_{x01}\, e^{-\gamma_1 z}$$

The corresponding wave eq^n for magnetic field is :-

$$H^{+}_{ys1} = \frac{E^{+}_{x01}}{\eta_1}\, e^{-\gamma_1 z}$$

In a similar way, the wave equation for reflected wave in medium 1 may be written as :-

$$E^{-}_{xs1} = E^{-}_{x01}\, e^{\gamma_1 z} \qquad H^{-}_{ys1} = -\frac{E^{-}_{x01}}{\eta_1}\, e^{\gamma_1 z}$$

From boundary conditions, we know that the tangential components of the electric field at the boundary $z = 0$ are equal.

$$E_{xs1} = E_{xs2}$$

(incident + reflected = Transmitted)

or $E^{+}_{xs_1} + E^{-}_{xs_1} = E^{+}_{xs_2}$

considering the maximum amplitude only.

$$E^{+}_{xo_1} + E^{-}_{xo_1} = E^{+}_{xo_2} \quad \text{—— (i)}$$

similarly for magnetic field component

$$H_{ys_1} = H_{ys_2}$$

or $H^{+}_{ys_1} + H^{-}_{ys_1} = H^{+}_{ys_2}$

-ve ma (- aaucha)

or $H^{+}_{yo_1} + H^{-}_{yo_1} = H^{+}_{yo_2}$

$$\left(\eta = \frac{\vec{E}}{\vec{H}} \qquad \vec{H} = \frac{\vec{E}}{\eta} \right)$$

or $\dfrac{E^{+}_{xo_1}}{\eta_1} - \dfrac{E^{-}_{xo_1}}{\eta_1} = \dfrac{E^{+}_{xo_2}}{\eta_2}$

or $E^{+}_{xo_2} = \dfrac{\eta_2}{\eta_1} E^{+}_{xo_1} - \dfrac{\eta_2}{\eta_1} E^{-}_{xo_1} \quad \ldots\ldots (ii)$

Now

or $E^{+}_{xo_1} + E^{-}_{xo_1} = \dfrac{\eta_2}{\eta_1} E^{+}_{xo_1} - \dfrac{\eta_2}{\eta_1} E^{-}_{xo_1}$

or $E^{-}_{xo_1} + \dfrac{\eta_2}{\eta_1} E^{-}_{xo_1} = \dfrac{\eta_2}{\eta_1} E^{+}_{xo_1} - E^{+}_{xo_1}$

or $E^{-}_{xo_1} \left(1 + \dfrac{\eta_2}{\eta_1} \right) = E^{+}_{xo_1} \left(\dfrac{\eta_2}{\eta_1} - 1 \right)$

or $$\boxed{\frac{E_{xo_1}^-}{E_{xo_1}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \quad \text{—— (iii)}$$

This ratio of the amplitude of the reflected wave to that of the incident wave is known as the reflection coefficient, Γ

$\therefore \Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\quad \Gamma = |\Gamma| e^{j\phi}$ and ϕ is relative phase shift

Transmission coefficient (τ) (Imp)

we have,

$$E_{xo_1}^+ + E_{xo_1}^- = E_{xo_2}^+$$

or $$E_{xo_1}^- = E_{xo_2}^+ - E_{xo_1}^+$$

substituting it in eqn (iii)

$$\frac{E_{xo_2}^+ - E_{xo_1}^+}{E_{o_1}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

or, $$E_{xo_2}^+ (\eta_2 + \eta_1) - E_{xo_1}^+ (\eta_2 + \eta_1) = E_{xo_1}^+ (\eta_2 - \eta_1)$$

or $$E_{xo_2}^+ (\eta_2 + \eta_1) = E_{xo_1}^+ (\eta_2 + \eta_1) + E_{xo_1}^+ (\eta_2 - \eta_1)$$

or, $$E_{xo_2}^+ (\eta_2 + \eta_1) = E_{xo_1}^+ (\eta_2 + \eta_1 + \eta_2 - \eta_1)$$

or $$\frac{E_{xo_2}^{+}}{E_{xo_1}^{+}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

This ratio of the amplitude of the transmitted wave to that of the incident wave is referred to as the transmission coefficient, τ

$$\therefore \boxed{\tau = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

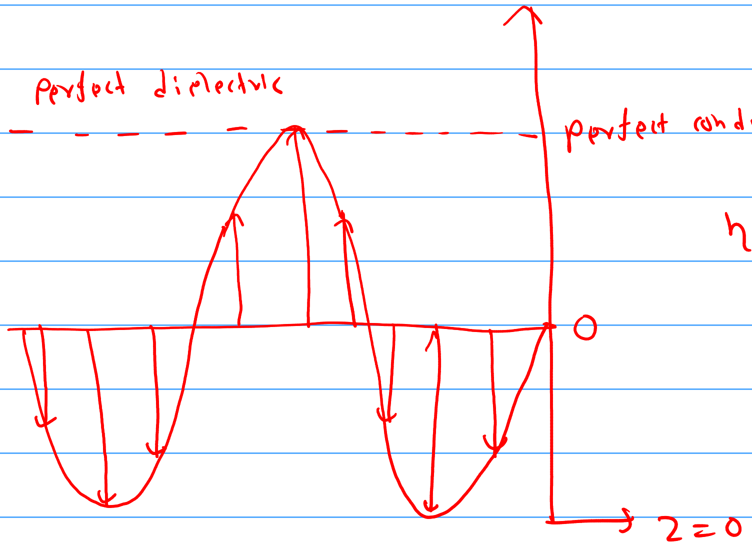
priority
1. wave eqn transmission coefficient
2. Ref and
3. standing wave

Standing Wave (Imp)

→ consider the case where the wave propagating in a perfect dielectric (medium 1) meets the surface of a perfect conductor (medium 2)

$\sigma = 0$

$\alpha = 0$



$\sigma \approx \infty$

$$\eta_2 = \left[\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}\right]^{1/2}$$

$$\eta_2 = 0$$

we have,

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}}$$

For perfect conductor, $\sigma_2 \to \infty$

$\therefore \eta_2 = 0$

as, $\frac{E^+_{x02}}{E^+_{x01}} = \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$, we can write $E^+_{x02} = 0$

This concludes no time varying field exists in the perfect conductor.

Also,

$$\frac{E^-_{x01}}{E^+_{x01}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

For, $\eta_2 = 0$

$$\frac{E^-_{x01}}{E^+_{x01}} = -1$$

$$\therefore E^-_{x01} = -E^+_{x01}$$

reflected wave has same amplitude as that of incident wave but is opposite in sign (direction). Since the entire incident wave is reflected, the total field in the perfect dielectric (medium 1) is given by:-

$$\begin{aligned} E_{xs1} &= E^+_{xs1} + E^-_{xs1} \\ &= E^+_{x01} e^{-\gamma_1 z} + E^-_{x01} e^{+\gamma_1 z} \\ &= E^+_{x01} e^{-(\alpha_1 + j\beta_1)z} + E^-_{x01} e^{(\alpha_1 + j\beta_1)z} \end{aligned}$$

In the perfect dielectric, there is no ohmic losses i.e $\sigma_1 = 0$
so eq^n above becomes

$$E_{xs1} = E_{x01}^{+} e^{-j\beta_1 z} + E_{x01}^{-} e^{+j\beta_1 z}$$

Using $E_{x01}^{-} = -E_{x01}^{+}$

$$E_{xs1} = E_{x01}^{+} e^{-j\beta_1 z} - E_{x01}^{+} e^{j\beta_1 z}$$

$$= E_{x01}^{+} \left(\frac{e^{-j\beta_1 z} - e^{j\beta_1 z}}{2j} \right) \times 2j$$

$$= -E_{x01}^{+} \left(\frac{e^{j\beta_1 z} - e^{-j\beta_1 z}}{2j} \right) \times 2j \qquad \left(\sin\theta = \frac{1}{2j}\left[e^{j\theta} - e^{-j\theta} \right] \right)$$

$$E_{xs1} = -2j\, E_{x01}^{+} \sin \beta_1 z$$

In time domain

$$E_{x1} = R_e \left[E_{xs1} e^{j\omega t} \right]$$

$$= R_e \left[-2j\, E_{x01}^{+} \sin(\beta_1 z)\, e^{j\omega t} \right]$$

$$= -2 E_{x01}^{+} \sin(\beta_1 z)\, R_e \left[j\, e^{j\omega t} \right]$$

$$= -2 E_{x01}^{+} \sin(\beta_1 z)\, R_e \left[j\cos\omega t + j^2 \sin\omega t \right]$$

$$= \quad " \quad " \quad " \quad " \quad " \; R_e \left[j\cos\omega t - \sin\omega t \right]$$

$$= -2 E_{x01}^{+} \sin(\beta_1 z)\, (-\sin\omega t)$$

$$E_{x1} = 2 E_{x01}^{+} \sin(\beta_1 z) \cdot \sin(\omega t) \qquad \text{--- (iv)}$$

The wave represented by eq^n (iv) is known as standing wave. The max amplitude that the wave may attain at different z is :-

$$z = \frac{n\lambda_1}{2}$$

The expression for the magnetic field in the standing wave may be derived as :-

$$H_{ys1} = H^{+}_{ys1} + H^{-}_{ys1}$$

$$= \frac{E^{+}_{x01} e^{-\gamma_1 z}}{\eta_1} - \frac{E^{-}_{x01} e^{\gamma_1 z}}{\eta_1}$$

$$= \frac{E^{+}_{x01} e^{-(\alpha_1 + j\beta_1) z}}{\eta_1} - \frac{E^{-}_{x01} e^{(\alpha_1 + j\beta_1) z}}{\eta_1}$$

For perfect dielectric $(\alpha_1 = 0)$

$$H_{ys1} = \frac{E^{+}_{x01} e^{-j\beta_1 z}}{\eta_1} - \frac{E^{-}_{x01} e^{j\beta_1 z}}{\eta_1}$$

$$= \frac{E^{+}_{x01} e^{-j\beta_1 z}}{\eta_1} + \frac{E^{+}_{x01} e^{j\beta_1 z}}{\eta_1}$$

$$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$$

$$= \frac{E^{+}_{x01}}{\eta_1} \left(\frac{e^{-j\beta_1 z} + e^{j\beta_1 z}}{2} \right) 2$$

$$= \frac{2 E^{+}_{x01}}{\eta_1} \cos(\beta_1 z)$$

In time domain

$$H_{y1} = Re\left[H_{ys1}\, e^{j\omega t} \right]$$

$$= Re\left[\frac{2 E_{x01}^{+}}{\eta_1} \cos \beta_1 z \times e^{j\omega t} \right]$$

$$= \frac{2 E_{x01}^{+}}{\eta_1} \cos(\beta_1 z)\; Re\left[e^{j\omega t} \right]$$

$$= \text{'' '' '' ''}\; Re\left[\cos \omega t + j \sin \omega t \right]$$

$$\therefore H_{y1} = \frac{2 E_{x01}^{+}}{\eta_1} \cos(\beta_1 z) \cos \omega t \qquad \text{— (v)}$$

From eq^n (iv) and (v) it is seen that when the electric field attains the maximum amplitude the magnetic field attains the minimum (zero) and vice versa.

Standing Wave Ratio (SWR) (Imp) (eq^n derive)

$$SWR = \frac{E_{xs1,max}}{E_{xs1,min}}$$

we have,

$$E_{xs1}^{+} = E_{x01}^{+}\, e^{-j\beta_1 z}$$

$$E_{xs1}^{-} = E_{x01}^{-}\, e^{j\beta_1 z}$$

$$\bar{E}_{xs1} = \Gamma E^{+}_{x01} e^{j\beta_1 z} \qquad \left(\Gamma = \frac{\bar{E}_{x01}}{E^{+}_{x01}} \right)$$

$$= |\Gamma| e^{j\phi} E^{+}_{x01} e^{j\beta z}$$

now,

$$E_{xs1} = E^{+}_{xs1} + \bar{E}_{xs1}$$

$$= E^{+}_{x01} e^{-j\beta_1 z} + |\Gamma| e^{j\phi} E^{+}_{x01} e^{j\beta_1 z}$$

$$= E^{+}_{x01} \left(e^{-j\beta_1 z} + |\Gamma| e^{j\phi} e^{j\beta z} \right)$$

$$= E^{+}_{x01} \left(e^{-j\beta_1 z} + |\Gamma| e^{j(\beta z + \phi)} \right)$$

The E_{xs1} will be max when

$$E_{xs1,max} = (1 + |\Gamma|) E^{+}_{x01}$$

E_{xs1} will be minimun when

$$E_{xs1,min} = (1 - |\Gamma|) E^{+}_{x01}$$

The ratio of the maximum to the minimum electric field is known as standing wave ratio, SWR.

$$SWR = \frac{E_{xs1,max}}{E_{xs1,min}} = \frac{(1+|\Gamma|) E^{+}_{x01}}{(1-|\Gamma|) \bar{E}_{x01}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Since $|\Gamma| \leq 1$, we have $1 \leq SWR \leq \infty$. The higher the SWR, the greater the portions of the standing wave in the wave comprising of both the travelling and the standing waves.

Wave eq^n derivation

v. Imp

(exam
free space
dissapitative
perfect dielectric)

① In perfect dielectric or lossless dielectric

$(\sigma = 0, \mu = \mu_0 \mu_r, \varepsilon = \varepsilon_0 \varepsilon_r)$

For electric field

Consider,

$$\nabla \times (\nabla \times \vec{E_s}) = \nabla(\nabla \cdot \vec{E_s}) - \nabla^2 \vec{E_s} \quad \text{—— (i)}$$

For source-free region $(\rho_v = 0)$, $\nabla \cdot \vec{E_s} = 0$

$$\nabla \times (\nabla \times \vec{E_s}) = -\nabla^2 \vec{E_s} \quad \text{—— (ii)}$$

Maxwell's eq^n in phasor is :-

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

Substituting this value in eqn (ii) we get

$$\nabla \times (-j\omega\mu \vec{H_s}) = -\nabla^2 \vec{E_s}$$

or, $-j\omega\mu (\nabla \times \vec{H_s}) = -\nabla^2 \vec{E_s}$ — (iii)

maxwell eqn in phasor

$$\nabla \times \vec{H_s} = \vec{J_s} + \vec{J_{ds}}$$

where

J_s = conduction current density , $\vec{J_{ds}}$ = displacement current density

For perfect dielectric medium , $\vec{J_s} = 0$

$$\nabla \times \vec{H_s} = 0 + j\omega\varepsilon \vec{E_s} = j\omega\varepsilon \vec{E_s}$$

Now eqn (iii) becomes

$$-j\omega\mu (j\omega\varepsilon \vec{E_s}) = -\nabla^2 \vec{E_s}$$

or, $\omega^2\mu\varepsilon \vec{E_s} = -\nabla^2 \vec{E_s}$

or, $\nabla^2 E_s = -\omega^2\mu\varepsilon \vec{E_s}$

which is known as vector Helmholtz equation

considering x-comp only

$$\nabla^2 E_{xs} = -\omega^2\mu\varepsilon \vec{E_{xs}}$$

or, $\nabla^2 \vec{E}_{xs} = -\omega^2 \mu \varepsilon \vec{E}_{xs}$

or, $\nabla^2 E_{xs} = -\omega^2 \mu \varepsilon E_{xs}$

or, $\dfrac{\partial^2 E_{xs}}{\partial x^2} + \dfrac{\partial^2 E_{xs}}{\partial y^2} + \dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu \varepsilon E_{xs}$

Let E_{xs} varies along z-direction only

$0 + 0 + \dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu \varepsilon E_{xs}$

or, $\dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu \varepsilon E_{xs}$

or, $\dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu \varepsilon E_{xs}$

The soln of the above eqn is :-

$E_{xs} = E_{xo} e^{-j\beta z}$, $\beta = \omega\sqrt{\mu\varepsilon}$ for perfect dielectric

In time domain,

$E_x = Re\left[E_{xs} e^{j\omega t}\right]$

$= Re\left[E_{xo} e^{-j\beta z} \cdot e^{j\omega t}\right]$

$= E_{xo} Re\left[e^{j(\omega t - \beta z)}\right]$

$= E_{xo} Re\left[\cos(\omega t - \beta z) + j\sin(\omega t - \beta z)\right]$

$= E_{xo} \cdot \cos(\omega t - \beta z)$

In vector form $\vec{E_x} = E_{xo} \cos(\omega t - \beta z)\, \hat{a}_x$

For magnetic field

Maxwell eqn ÷

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

Taking y-comp of mag and x-comp of electric.

$$\nabla \times \vec{E_{xs}} = -j\omega\mu \vec{H_{ys}}$$

or $$\left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\vec{E_{xs}} = -j\omega\mu \vec{H_{ys}}$$

1 (a) $\vec{E_{xs}}$ varies along z-direction only

or $$\frac{\partial \vec{E_{xs}}}{\partial z} = -j\omega\mu \vec{H_{ys}}$$

or $$\frac{\partial E_{xs}}{\partial z} = -j\omega\mu H_{ys}$$

or $$\frac{\partial\left(E_{x0} e^{-j\beta z}\right)}{\partial z} = -j\omega\mu H_{ys}$$

or $$-j\beta E_{x0} e^{-j\beta z} = -j\omega\mu H_{ys}$$

or $$H_{ys} = \frac{\beta}{\omega\mu} E_{x0} e^{-j\beta z}$$

or $$H_{ys} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu} E_{x0} e^{-j\beta z}$$

$$\therefore \quad H_{ys} = \sqrt{\frac{\varepsilon}{\mu}} E_{x0} e^{-j\beta z}$$

In time domain

$$H_y = Re\, I\, (H_{ys})\, e^{jwt}]$$

$$= Re\left[\sqrt{\frac{\varepsilon}{\mu}}\; E_{xo}\, e^{-j\beta z} \cdot e^{jwt} \right]$$

$$= \sqrt{\frac{\varepsilon}{\mu}}\; E_{xo}\; Re\left[\cos(\omega t - \beta z) + j \sin(\omega t - \beta z) \right]$$

$$= \sqrt{\frac{\varepsilon}{\mu}}\; E_{xo} \cos(\omega t - \beta z)$$

In vector

$$\vec{H_y} = \sqrt{\frac{\varepsilon}{\mu}}\; E_{xo} \cos(\omega t - \beta z)\; \hat{a_y} \;//$$

\# Free space ($\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$) → $\vec{E_x}$ → same

$$\vec{H_y} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\; E_{xo} \cos(\omega t - \beta z)\; \hat{a_y}$$

→ Same process just replace $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

\# Wave p^n in Dissipative medium (lossy dielectric)

($\sigma \neq 0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$)

p. 10

For electric field

let

$$\nabla \times (\nabla \times \vec{E_s}) = \nabla(\nabla \cdot \vec{E_s}) - \nabla^2 \vec{E_s} \quad \text{--- (i)}$$

For source free region $(\rho_v = 0)$, $\nabla \cdot \vec{E_s} = 0$

$$\nabla \times (\nabla \times \vec{E_s}) = -\nabla^2 E_s \quad \text{--- (ii)}$$

maxwell's eqn in phasor is

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

substituting this value in (ii)

$$\nabla \times (-j\omega\mu \vec{H_s}) = -\nabla^2 \vec{E_s}$$

or $$-j\omega\mu (\nabla \times \vec{H_s}) = -\nabla^2 \vec{E_s} \quad \text{--- (iii)}$$

From maxwell eqn, we have

$$\nabla \times \vec{H_s} = \vec{J_s} + \vec{J_{ds}}$$

$$\nabla \times \vec{H_s} = \sigma \vec{E_s} + j\omega\varepsilon \vec{E_s} = (\sigma + j\omega\varepsilon) \vec{E_s}$$

now (iii) becomes

$$-j\omega\mu [(\sigma + j\omega\varepsilon) \vec{E_s}] = -\nabla^2 \vec{E_s}$$

or $$\nabla^2 \vec{E_s} = j\omega\mu (\sigma + j\omega\varepsilon) \vec{E_s}$$

or $$\nabla^2 \vec{E_s} = j\omega\mu (\sigma + j\omega\varepsilon) \vec{E_s}$$

or $\nabla^2 \vec{E_s} - \gamma^2 \vec{E_s} = 0$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$ is propagation constant of the medium

we only take x-component of the electric field intensity which will result

$\nabla^2 \vec{E_{xs}} - \gamma^2 \vec{E_{xs}} = 0$, or $\nabla^2 E_{xs} - \gamma^2 E_{xs} = 0$

or $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z^2}\right) E_{xs} - \gamma^2 E_{xs} = 0$

let's assume that E_{xs} varies along the z direction only

$0 + 0 + \frac{\partial^2 E_{xs}}{\partial z^2} - \gamma^2 E_{xs} = 0$

or $\frac{\partial^2 E_{xs}}{\partial z^2} - \gamma^2 E_{xs} = 0$

The solution of above differential equation is ÷

$E_{xs} = E_{x0}\, e^{-\gamma z}$

or $E_{xs} = E_{x0}\, e^{-(\alpha + j\beta) z}$

or $E_{xs} = E_{x0}\, e^{-\alpha z} . e^{-j\beta z}$ (This is phasor notation)

In time domain, we have

$E_x = Re[E_{xs}\, e^{j\omega t}]$

$$= Re\left[\left(E_{x0}\, e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right]$$

$$= E_{x0}\, e^{-\alpha z}\, Re\left[e^{j(\omega t-\beta z)}\right]$$

$$= E_{x0}\, e^{-\alpha z}\left[\cos(\omega t-\beta z) + j\sin(\omega t-\beta z)\right]$$

$$= E_{x0}\, e^{-\alpha z}\cos(\omega t-\beta z) \quad \#$$

In vector form,

$$\vec{E_x} = E_{x0}\, e^{-\alpha z}\cos(\omega t-\beta z)\,\hat{a}_x$$

Expression for magnetic field

Consider maxwell's equation in phasor

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

Taking only the y-component of the magnetic field, and x-component of the electric field, we have

$$\nabla \times \vec{E_{xs}} = -j\omega\mu \vec{H_{ys}}$$

or, $$\left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\vec{E_{xs}} = -j\omega\mu \vec{H_{ys}}$$

let's assume that $\vec{E_{xs}}$ varies along the z-direction only

or, $$\frac{\partial}{\partial z}\hat{a}_z \times \vec{E_{xs}} = -j\omega\mu \vec{H_{ys}}$$

or, $$\frac{\partial}{\partial z}\hat{a}_z \times E_{xs}\,\hat{a}_x = -j\omega\mu H_{ys}\,\hat{a}_y$$

or, $\dfrac{\partial E_{xs}}{\partial z} = -j\omega\mu H_{ys}$

Here $E_{xs} = E_{xo}e^{-\gamma z}$

$$\dfrac{\partial (E_{xo}e^{-\gamma z})}{\partial z} = -j\omega\mu H_{ys}$$

or, $-\gamma E_{xo}e^{-\gamma z} = -j\omega\mu H_{ys}$

or,

or, $\gamma E_{xo}e^{-\gamma z} = j\omega\mu H_{ys}$

or, $\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}\, E_{xo}e^{-\gamma z} = j\omega\mu H_{ys}$

or, $\sqrt{\dfrac{\sigma + j\omega\varepsilon}{j\omega\mu}}\, E_{xo}e^{-\gamma z} = H_{ys}$

or, $H_{ys} = \dfrac{E_{xo}}{\eta}e^{-\gamma z}$

where $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$, $|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}}$, $\tan 2\theta = \dfrac{\sigma}{\omega\varepsilon}$

Hence,

$$H_{ys} = \dfrac{E_{xo}}{|\eta|e^{j\theta}}e^{-\gamma z} = \dfrac{E_{xo}}{|\eta|}e^{-j\theta}e^{-(\alpha + j\beta)z}$$

$$H_{ys} = \dfrac{E_{xo}}{|\eta|}e^{-\alpha z}e^{-j(\theta + \beta z)}$$

$\therefore\ H_{ys} = \dfrac{E_{xo}}{|\eta|}e^{-\alpha z}e^{-j(\theta + \beta z)}$

In time domain

$$H_y = Re[H_{ys} e^{j\omega t}]$$

$$= Re\left[\left\{\frac{E_{xo}}{|\eta|} e^{-\alpha z} e^{-j(\theta+\beta z)}\right\} e^{j\omega t}\right]$$

$$= \frac{E_{xo}}{|\eta|} e^{-\alpha z} Re[e^{-j\theta - j\beta z + j\omega t}]$$

$$= \text{" " "} \quad Re[e^{j(\omega t - \beta z - \theta)}]$$

$$= \text{" " "} \quad Re[\cos(\omega t - \beta z - \theta) + j\sin(\omega t - \beta z - \theta)]$$

$$= \frac{E_{xo}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)$$

In vector form

$$\vec{H}_y = \frac{E_{xo}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)\, \hat{a}_y$$

4. Wave Equation for Perfect Conductors (Good Conductors)

($\sigma \simeq \infty$, $\varepsilon = \varepsilon_o$, $\mu = \mu_r\mu_o$)

In order to determine wave equation for perfect conductions, we first write the wave equation for a dissipative medium, and then substitute θ, α, β which defines perfect conductors.

For a **dissipative medium,**

$$\vec{E}_x = E_{xo}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x, \quad \vec{H}_y = \frac{E_{xo}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta)\, \hat{a}_y$$

A perfect or good conductor is a special case of dissipative medium for which $|\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$, $\theta = 45^o \left(\frac{\pi^c}{4}\right)$, and $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

Thus, for perfect conductors

$$\boxed{\begin{array}{l} \vec{E}_x = E_{xo}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x \\ \vec{H}_y = \frac{E_{xo}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \frac{\pi^c}{4})\, \hat{a}_y;\ |\eta| = \sqrt{\frac{\omega\mu}{\sigma}},\ \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \end{array}}$$