Chapter - 9

Faraday's law of Electromagnetic Induction

→ It describes how an emf (induced potential difference) is generated in a circuit.

→ An emf appears at the two terminals of an open circuit whenever there is a change in the magnetic flux linking that circuit.

mathematically,

$$emf = -\frac{d\phi}{dt} \text{ (in volts)}$$

where,

→ ϕ represents magnetic flux through the ckt (wb)

→ $\frac{d\phi}{dt}$ rate of change of magnetic flux w.r.t. time

→ The negative sign reflects the direction of the induced emf which is governed by Lenz's law.

Conditions for Induced emf

→ Transformer Induction: time-varying magnetic flux through a stationary closed path

→ Motional (Generator) Induction: Relative motion between a steady magnetic field and a conductor.

→ net Induction → combination of the above two scenarios

Lenz's law → The induced emf opposes the change in magnetic flux that produces it. This is the statement of Lenz's law.

$$\text{so,} \quad emf = -\frac{d\phi}{dt}$$

For a coil with N turns of a filamentary conductor

$$emf = -N\frac{d\phi}{dt}$$

→ Direction of induced current is given by Flemings Right hand rule

thumb → Direction of conductor motion
Forefinger → magnetic field (B)
middle finger → induced current

The motional Induction

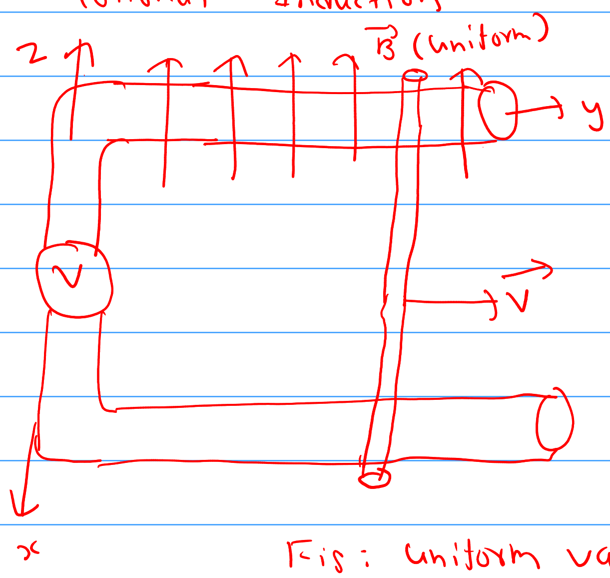

Fig: uniform value of magnetic flux density in a moving closed path

→ A specific application of Faraday's law where emf is induced by a conductor moving through a magnetic field is called motional induction.

The diagram depict a closed electric ckt with

→ two vertical parallel conductors

→ A high-resistance voltmeter

→ A horizontal sliding bar moving with velocity $\vec{v}$ to the right along the conductors

The sliding bar's motion changes the area enclosed by the circuit, altering the magnetic flux and inducing an emf which the voltmeter measures.

motional induction occurs when a conductor moves through a constant magnetic field, inducing an emf due to the changing flux. The circuit involves a time-constant $\vec{B}$ and a moving closed path (the sliding bar). A charge (q) moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences the Lorentz force.

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\frac{\vec{F}}{q} = \vec{v} \times \vec{B}$$

In the sliding bar, this force acts on both -ve and +ve charges causing charge sepration and an induced emf.

$$\boxed{\vec{E}_m = \vec{v} \times \vec{B}}$$

→ This field drives the charges, generating the emf

motional emf is calc by integrating the motional electric field around closed ckt.

$$\text{motional emf} = \oint \vec{E}_m \cdot \vec{dl}$$

$$= \oint (\vec{v} \times \vec{B}) \, \vec{dl}$$

(ii) The Transformer Induction

→ Consider a stationary closed path under the time-variant magnetic flux density $\vec{B}$ as illustrated.

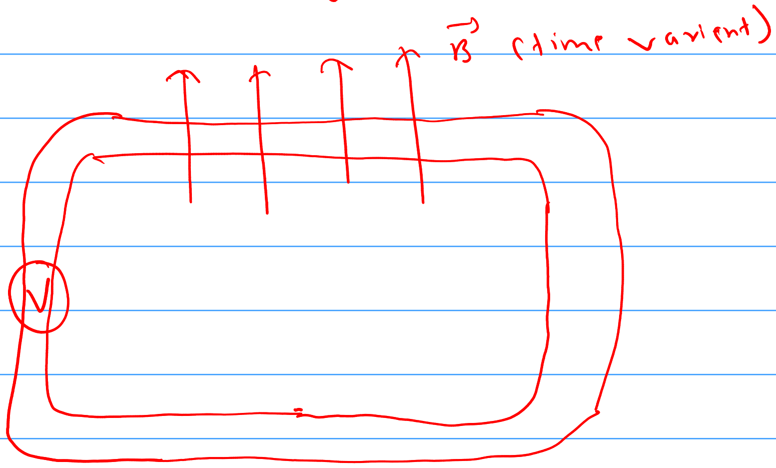

fig: Time varying magnetic flux linked with the stationary closed path

$$\text{the transformer emf} = -\frac{d\phi}{dt}$$

$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

so, transformer emf $= -\dfrac{d}{dt}\left(\int_S \vec{B} \cdot \vec{ds}\right)$

$$= -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

The net Induction

→ If both the conductor and magnetic flux density are changing with time which is the most general case, then the total emf is the sum of the emf due to the motional and the transformer induction.

net emf = transfor emf + motional emf

$$= -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} + \oint (\vec{v} \times \vec{B}) \, \vec{dl}$$

Ampere's law Conflicting with the continuity Equation:

→ The point form of Ampere's circuital law as it applies to steady magnetic fields is ÷

$$\nabla \times \vec{H} = \vec{J} \quad \text{--- (i)}$$

Taking divergence on both sides

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

L.H.S = 0 because the divergence of the curl is zero

$$0 = \nabla \cdot \vec{J}$$

or $\nabla \cdot \vec{J} = 0 \quad \text{--- (ii)}$

But the equation of continuity says :-

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{--- (iii)}$$

Comparing (ii) and (iii) we see that Ampere's circuital law conflicts with the continuity equation. Hence, we conclude that eqn (i) which was derived for the direct current (time-invariant conditions) shows incompleteness when we use it for the time-variant conditions.

Displacement Current

→ Ampere's circuital law initially stated

$$\nabla \times \vec{H} = \vec{J}$$

(fails for time-varying fields eg: capacitors where I is discontinuous. Maxwell resolved this by introducing displacement current)

Maxwell added a term $\vec{G}$ to account for time-varying electric fields

$$\nabla \times \vec{H} = \vec{J} + \vec{G} \quad \text{— (i)}$$

Taking divergence on both sides

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}$$

or

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}$$

div of curl = 0

we know,

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \vec{G} = \frac{\partial \rho_v}{\partial t} = \frac{\partial (\nabla \cdot \vec{D})}{\partial t} \qquad \left(\rho_v = \nabla \cdot \vec{D} \right)$$

Gauss law

or,

$$\nabla \cdot \vec{G} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

So,

$$\vec{G} = \frac{\partial \vec{D}}{\partial t} \qquad \left(\text{displacement current density, } \vec{J}_d \right)$$

Final modified form

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{— (ii)}$$

Integral form

$$\int_S (\nabla \times \vec{H})\, d\vec{s} = \int_S \vec{J} \cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t}\, d\vec{s}$$

Applying the stokes theorem

$$\oint \vec{H} \cdot d\vec{l} = \int (\nabla \times \vec{H})\, d\vec{s}$$

$$\boxed{\text{or} \quad \oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t}\, d\vec{s} \qquad \therefore \; I_{total} = I_{conduction} + I_{displacement}}$$

Significance of Displacement current

- → ensures continuity in circuits with capacitors
- → enables propagation of EM waves in free space (radio, TV, wireless signals)
- → completes maxwell's equations, allowing them to predict light as EM waves

Maxwell's equations

Table 1 : Maxwell's equations for static electric and magnetic fields

	Differential (or point form)	Integral form	Remarks
1	$\nabla \cdot \vec{D} = \rho_v$	$\oint \vec{D} \cdot \vec{ds} = \int_{vol} \rho_v \, dv$	Gauss law
2	$\nabla \times \vec{H} = \vec{J}$	$\oint \vec{H} \cdot \vec{dl} = \int_s \vec{J} \cdot \vec{ds}$	Amphere's law
3	$\nabla \times \vec{E} = 0$	$\oint \vec{E} \cdot \vec{dl} = 0$	conservative nature of electrostatic field
4	$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$	non-existence of magnetic monopole. Gauss law for magnetic

Table 2 : Generalized forms of Maxwell equations (Time varying fields)

	Differential (or point form)	Integral form	Remarks
1	$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot \vec{ds} = \int_{vol} \rho_v \, dv$	Gauss law

②	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \vec{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \vec{ds}$	Ampere's law
③	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot \vec{ds}$	Faraday's law
④	$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$	non-existence of magnetic monopole

Generalized forms of maxwell's equations in phasor form

	Differential form	Integral form
①	$\nabla \cdot \vec{D_s} = \rho_{vs}$	$\oint \vec{D_s} \cdot \vec{ds} = \int_{vol} \rho_{vs} \, dv$
②	$\nabla \times \vec{H_s} = \vec{J_s} + j\omega \vec{D_s}$	$\oint \vec{H_s} \cdot \vec{dl} = \int_S (\vec{J_s} + j\omega \vec{D_s}) \vec{ds}$
③	$\nabla \times \vec{E_s} = -j\omega \vec{B_s}$	$\oint \vec{E_s} \cdot \vec{dl} = -j\omega \int_S \vec{B_s} \cdot \vec{ds}$
④	$\nabla \cdot \vec{D_s} = 0$	$\oint \vec{B_s} \cdot \vec{ds} = 0$

(time domain to phasor domain in chapter-10)