...VED AND SCRAMBLED

1. A transmission line operating at $\omega = 10^8$ rad/s has these parameter values: $R = 0.1\ \Omega/m$, $L = 0.2\ \mu H/m$, $G = 10\ \mu mho/m$, $C = 100\ pF/m$. Find:

(i) α (ii) β (iii) γ (iv) v (v) Z_o

Solution:

Soln:

$R = 0.1\,\Omega/m$, $L = 0.2\,\mu H/m$, $G = 10\,\mu\, mho/m$

$C = 100\,pF/m$, $\omega = 10^8$ rad/s

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(0.1 + j\times 10^8 \times 0.2\times 10^{-6})(10\times 10^{-6} + j\times 10^8 \times 100\times 10^{-12})}$$

$$\left((20 \angle 89.71)(0.01 \angle 89.94^\circ)\right)^{1/2}$$

$$= 0.447 \angle 89.825^\circ$$

$$\gamma = 1.365\times 10^{-3} + j\,0.4469\ m^{-1}$$

$$\gamma = \alpha + j\beta$$

$\alpha = 1.356\times 10^{-3}$ Np/m, $\beta = 0.4469$ rad/m

$$v = \lambda f = \frac{2\pi}{\beta}\times\frac{\omega}{2\pi} = \frac{\omega}{\beta} = \frac{10^8}{0.4469} = \frac{10^8}{0.4469}$$

$$= 2.22\times 10^8\ m/s$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} =$$

$$= \left(\frac{20 \angle 89.71^\circ}{0.01 \angle 89.94^\circ} \right)^{1/2} = 44.72 \angle -0.12^\circ$$

$$= 44.71 - j \times 0.093 \ \Omega$$

A certain transmission line has a characteristic impedance of 75 + j0.01 Ω and is terminated in a load impedance of 70 + j50 Ω. Compute: (a) The reflection coefficient (b) The transmission coefficient.

Soln ÷

$$Z_0 = 75 + j\,0.01 \ \Omega$$

$$Z_L = 70 + j\,50 \ \Omega$$

(a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{70 + j\,50 - 75 - j\,0.01 \ \Omega}{70 + j\,50 + 75 + j\,0.01 \ \Omega}$$

$$= 0.33 \angle 76.7^\circ$$

$$= 0.08 + j\,0.32$$

(b) $$\tau = \frac{2 \times Z_L}{Z_L + Z_0} = \frac{2 \times (70 + j\,50)}{70 + j\,50 + 75 + j\,0.01}$$

$$= 1.12 \angle 16.5^\circ = 1.08 + j\,0.32$$

A distortionless line has $Z_o = 60\ \Omega$, $\alpha = 20$ mNp/m, $v = 0.6c$, where c is the speed of light in a vacuum. Find R, L, G, C, and λ at 100 MHz.

tion:

soln ÷

distortionless line

$Z_0 = 60\Omega$, $\alpha = 20$ mNp/m, $V = 0.6C$, $f = 100$ Hz

$$\frac{R}{L} = \frac{G}{C} \quad \text{— (i)}$$

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} + j0$$

$$R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

$$V = \lambda \times f = \frac{2\pi}{\beta} \times \frac{\omega}{2\pi} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \quad \text{— (a)}$$

$$\alpha = \sqrt{RG} = \left(R \times \frac{RC}{L}\right)^{1/2} \quad \text{from (i)}$$

$$= R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

* $R = \alpha Z_0 = 20\times10^{-3} \times 60$

$= 1.2\ \Omega/m$

$\alpha = \sqrt{RG}$

* $G = \dfrac{\alpha^2}{R} = \dfrac{(20\times10^{-3})^2}{1.2} = 333\times10^{-6}\ S/m$

$Z_0 = \sqrt{\dfrac{L}{C}}\ ,\ v = \dfrac{1}{\sqrt{LC}}$

or $\dfrac{Z_0}{v} = \sqrt{\dfrac{L^2C}{C}}$

or $\dfrac{Z_0}{v} = L$

or $\dfrac{60}{0.6c} = L$

* $\therefore\ L = \dfrac{60}{0.6\times3\times10^{8}} = 333\ nH/m$

* $\dfrac{R}{L} = \dfrac{G}{C}$

or $C = \dfrac{G\times L}{R} = \dfrac{333\times10^{-6}\times 333\times10^{-9}}{1.2}$

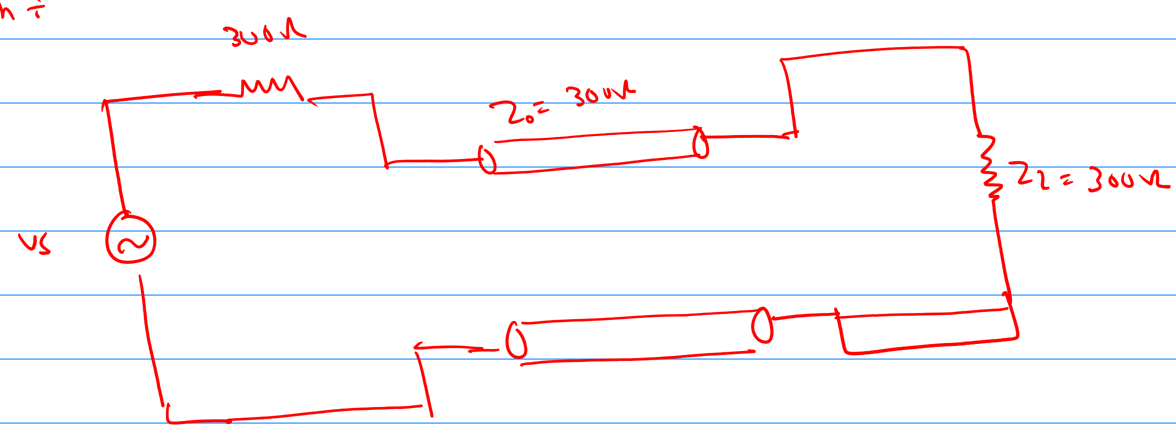
$= 92.40\ pF/m$

$$\Rightarrow \lambda = \frac{v}{f} = \frac{0.6 \times 3\times10^8}{100 \times 10^6} = 1.8\,m$$

Numerical example-4 :→

Assume a two wire 300Ω lossless line ($Z_0 = 300\Omega$) such as the lead in wire form the antenna to the TV. Suppose the length of the line is 2m. & the value of L & C is such that the velocity on the line is 2.5×10^8 m/s. The line is terminated at with a receiver having the input resistance of 300Ω. The antenna is represented by the thevenin's equivalent $Z = 300\Omega$ in series with the voltage source $|V_s| = 60V$ at 100MHz. Determine the load voltage, load current & power delivered to the load by the line.

soln :



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 300}{300 + 300} = 0$$

100 MHz

ac

$$V_s = 60 \cos(2\pi \times 10^8 t)\ V$$

$$\beta l = \frac{2\pi}{\lambda} \times 2m = \frac{4\pi f}{v} = \frac{4\pi \times 10^8}{2.5 \times 10^3}$$

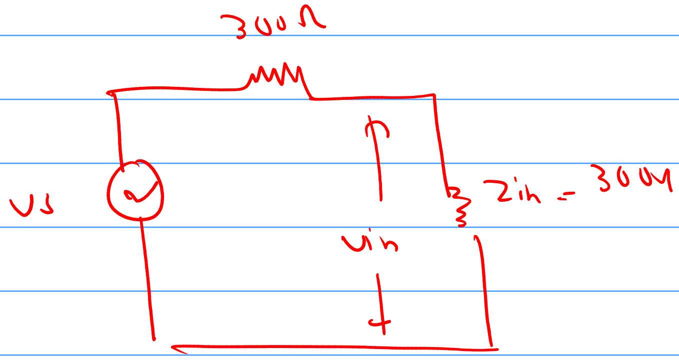
$$= 1.6\pi$$

$$Z_{in} = Z_0 \cdot \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

$$= 300 \left[\frac{300 + j \times 300 \times \tan 1.6\pi^c}{300 + j \times 300 \times \tan 1.6\pi^c} \right]$$

$$= 300\,\Omega$$

Now,



$$V_{in} = V_s \times \frac{Z_{in}}{Z_{in} + 300} = V_s \times \frac{300}{300 + 300}$$

$$V_{in} = 60 \cos (2\pi \times 10^8 t) \times \frac{1}{2}$$

$$= 30 \cos (2\pi 10^8 t)\ V$$

$\Gamma = 0$

$$V_L = 30 \cos (2\pi \times 10^8 t - \beta l)$$

$$= 30 \cos (2\pi \times 10^8 t - 1.6\pi^c)$$

Input current $(I_{in}) = \dfrac{V_{in}}{Z_{in}} = \dfrac{30 \cos (2\pi \times 10^8 t)}{300}$

$$= 0.1 \cos (2\pi \times 10^8 t)$$

$$I_L = \frac{V_L}{Z_L} = \frac{30\cos(2\pi\times10^8 t - 1.6\pi c)}{300}$$

$$= 0.1\cos(2\pi\times10^8 t - 1.6\pi)\ A$$

lossless

$$P_{in} = P_L = V_{in\,(rms)} \times I_{in\,(rms)}$$

$$= \frac{30}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} = 1.5\,W$$

3. A standard air filled rectangular waveguide with dimention a = 8.636, b = 4.318 cm is fed by a 4 GHz from a coaxial cable. Determine whether a TE_{10} mode wil be propagated. If so, calculate the phase velocity and the group velocity.

Solution:

soln:-

cut off frequency, $f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

TE_{10} $(m=1, n=0)$

For TE_{10}, we have $m=1$, $n=0$

since the waveguide is air filled, $v = c = 3\times10^8$ m/s

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{8.636\times10^{-2}}\right)^2 + \left(\frac{0}{4.318\times10^{-2}}\right)^2}$$

$\Rightarrow$ $f_c = 1.737 \times 10^9$ Hz $= 1.737$ GHz

As 4 GHz is greater than f_c, the TE_{10} will propagated.

Phase velocity, $V_p = \dfrac{V}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$= \dfrac{3\times10^8}{\sqrt{1-\left(\frac{1.737}{4}\right)^2}}$ → Both GHz

$= 3.33 \times 10^8$ m/s

Group velocity, $V_g = \dfrac{V^2}{V_p} = \dfrac{(3\times10^8)^2}{3.33\times10^8} = 2.7\times10^8$ m/s

4. **A 6 GHz signal is to be propagated in TE_{10} mode in rectangular waveguide. If its group velocity is 90 percent of free space velocity, what must be the breadth of the waveguide.**

Soln:

$V_g = 0.9\,c$

$V_g = \dfrac{V^2}{V_p}$

or $$0.9c = \frac{v^2}{\frac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

or $$0.9c = v\left(\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right)$$

We know that $v = c$

or $$0.9 = \sqrt{1-\frac{f_c^2}{f^2}}$$

or $$0.81 = 1 - \frac{f_c^2}{f^2}$$

$$\therefore \frac{f_c^2}{f^2} = 0.19 \quad \text{— (i)}$$

Also, $$f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE_{10}

$m = 1$, $n = 0$

or $$f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2}$$

$$\therefore f_c = \frac{c}{2a} \quad \text{— (ii)}$$

or,
$$\frac{\left(\frac{c}{2a}\right)^2}{(f)^2} = 0.19$$

or
$$\frac{\left(\frac{3\times10^8}{2}\right)^2 \times \frac{1}{a^2}}{(6\times10^9)^2} = 0.19$$

or
$$a^2 = \frac{\left(\frac{3\times10^8}{2}\right)^2}{(6\times10^9)^2 \times 0.19} \qquad \therefore a = 0.05735\,m$$

5. **Consider a rectangular waveguide with $\varepsilon_r = 2$, $\mu = \mu_o$ with dimensions a = 1.07 cm, b = 0.43 cm. Find the cutoff frequency for TM_{11} mode and the dominant mode.**

[2074 Chaitra]

Soln:

$$f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where, $v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{2\mu_o\varepsilon_o}} = 2.1213\times10^8\, m/s$

For TM_{11}, m=1, n=1

$$f_c = \frac{2.1213\times10^8}{2}\sqrt{\left(\frac{1}{1.07\times10^{-2}}\right)^2 + \left(\frac{1}{0.43\times10^{-2}}\right)^2}$$

$\therefore f_c = 26.58 \text{ GHz}$

For rectangular waveguide, the dominant mode is TE_{10}
$m=1, n=0$

$$f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{2.1213\times10^8}{2}\sqrt{\left(\frac{1}{1.07\times10^{-2}}\right)^2 + 0} = 9.9126 \text{ GHz}$$

An air-filled rectangular waveguide of inside dimensions 7×3.5 cm operates in the dominant TE_{10} mode. (a) Find the cutoff frequency (b) Determine the phase velocity of the wave in the guide at a frequency of 3.5 GHz. (c) Determine the guide wavelength at the same frequency.

soln ÷

(a) $f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ For TE_{10} $m=1, n=0$

air filled

$$f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + 0} = \frac{c}{2a} = \frac{3\times10^8}{2\times7\times3.5\times10^{-2}} = 2.1428 \text{ GHz}$$

(b) $V_p = \dfrac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$= \dfrac{3\times10^8}{\sqrt{1-\left(\frac{2.1428\times10^9}{3.5\times10^9}\right)^2}} = 3.794\times10^8\ \text{m/s}$

(c) $\lambda_{guided} = \dfrac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{c/f}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$= \dfrac{(3\times10^8)\,(3.5\times10^9)}{\left(1-\left(\frac{2.1428\times10^9}{3.5\times10^9}\right)^2\right)^{1/2}} = 10.84\ \text{cm}$

7. Standard air-filled waveguides have been designed for the radar bands. One type, designated WG-16, is suitable for X-band applications. Its dimensions are: a = 2.29 cm and b = 1.02 cm. If it is desired that a WG-16 waveguide operate only in the dominant TE_{10} mode and that the operating frequency be at least 25% above the cutoff frequency of the TE_{10} mode but no higher than 95% of the next higher cutoff frequency, what is the allowable operating-frequency range?

Soln ÷

For $a = 2.29 \times 10^{-2}$ m, $b = 1.02 \times 10^{-2}$ m, the two modes having the lowest cut off frequencies are TE_{10} and TE_{20}.

$$f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For air filled $v = c$, corresponding to TE_{10} and TE_{20} modes, we calculate

$$f_{c10} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 2.29 \times 10^{-2}} = 6.55 \times 10^9 \text{ Hz}$$

$$f_{c20} = \frac{c}{a} = \frac{3 \times 10^8}{2.29 \times 10^{-2}} = 13.10 \times 10^9 \text{ Hz}$$

Thus, the allowable operating-frequency range under the specified conditions is ÷

$$1.25 f_{c10} \le f \le 0.95 f_{c20}$$

or, $$8.19 \times 10^9 \text{ Hz} \le f \le 12.45 \times 10^9 \text{ Hz}$$

An air line characteristic impedance of 70 Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

ution:

soln ÷

air line can be regarded as lossless line

$R = 0 = G$

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

For $R = G = 0$

$\gamma = j\omega\sqrt{LC}$

now

$\alpha + j\beta = j\omega\sqrt{LC}$

$\alpha = 0, \quad \beta = \omega\sqrt{LC}$ — (i)

Z_0 (characteristic impedance) $= \sqrt{\dfrac{R + j\omega L}{G + j\omega L}}$

$Z_0 = \sqrt{\dfrac{L}{C}}$

or $R_0 + jX_0 = \sqrt{\dfrac{L}{C}} + j0$

$\therefore Z_0 = R_0 = \sqrt{\dfrac{L}{C}}, \quad X_0 = 0$ — (ii)

Dividing (i) by (ii)

$$\frac{\beta}{R_0} = \frac{\omega\sqrt{LC}}{\sqrt{\frac{L}{C}}} = \omega C$$

or $$C = \frac{\beta}{\omega R_0} = \frac{\beta}{\omega Z_0} = \frac{3}{2\pi \times 100 \times 10^6 \times 70} = 68.2\ pF/m$$

From eqn (i)

$$R_0 = \sqrt{\frac{L}{C}}$$

or $$L = R_0^2 C = Z_0^2 C = (70)^2 \times (68.2 \times 10^{-12})$$
$$= 334.4\ nH/m$$

$\therefore$ $C = 68.2\ pF/m$, $L = 334.4\ nH/m$

11. A telephone line has R = 30 Ω/km, L = 100 mH/km, G = 0, and C = 20 μF/km. At f = 1 kHz, obtain: (a) the characteristic impedance of the line (b) the propagation constant (c) the phase velocity.

Solution:

Soln:

$R = 30\Omega/km = 30 \times 10^{-3}\ \Omega/m$

$L = 100\ mH/km = 100 \times 10^{-6}\ H/m$

$G = 0$, $C = 20 \times 10^{-6} \times 10^{-3}\ F/m = 20 \times 10^{-9}\ F/m$

$f = 1\ kHz$

$$\omega = 2\pi f = 2\pi \times 10^3 = 6283.1853\ rad/s$$

(a) $Z_0 = \sqrt{\dfrac{R + jwL}{G + jwC}}$

$$= \left(\frac{30\times10^{-3} + j\,6283.1853 \times 100\times10^{-6}}{0 + j\,6283.1853 \times 20\times10^{-9}}\right)^{1/2}$$

$$= \left(\frac{0.629 \angle 87.266^\circ}{1.2566\times10^{-4}\angle 90^\circ}\right)^{1/2}$$

$$= \sqrt{5005.5705 \angle -2.734^\circ}$$

$$= \sqrt{5005.5705}\ \angle \frac{-2.734}{2}$$

$$= 70.75 \angle -1.367^\circ\ \Omega$$

(b) $\gamma = \sqrt{(R + jwL)(G + jwC)}$

$$= \sqrt{(0.629 \angle 87.266^\circ)(1.2566\times10^{-4}\angle 90^\circ)}$$

$$= \sqrt{7.9040\times10^{-5} \angle 177.266^\circ}$$

$$= \sqrt{7.9040\times10^{-5}}\ \angle \frac{177.266^\circ}{2}$$

$$= 8.890\times10^{-3} \angle 88.633^\circ$$

(c) $v = \dfrac{w}{\beta}$

$\omega = 6283.1853$ rad/s

$\gamma = (2.1208 \times 10^{-4} + j\, 8.8874 \times 10^{-3})\ m^{-1}$

$\beta = 8.8874 \times 10^{-3}$ rad/m

$$V = \frac{6283.1853}{8.8874 \times 10^{-3}} = 7.069 \times 10^{5}\ m/s$$

12. **A transmission line operating at 500 MHz has $Z_0 = 80\ \Omega$, $\alpha = 0.04$ Np/m, $\beta = 1.5$ rad/m. Find the line parameters R, L, G, and C.**

soln:

$Z_0 = 80\Omega$, $\alpha = 0.04$ Np/m, $\beta = 1.5$ rad/m

$f = 500$ MHz

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

or, $$\gamma^2 = (R + j\omega L)(G + j\omega C) \quad \text{—— (i)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega L}}$$

or, $$Z_0^2 = \frac{R + j\omega L}{G + j\omega L} \quad \text{—— (ii)}$$

From (i) and (ii)

$$\gamma^2 = Z_0^2 (R + jwL)(G + jwC)$$

or $$\frac{\gamma^2}{Z_0^2} = (G + jwC)^2$$

or, $$\frac{1}{Z_0^2}\left[(\alpha + j\beta)^2\right] = (G + jwC)^2$$

or, $$\frac{1}{Z_0^2}\left[(\alpha^2 - \beta^2) + j2\alpha\beta\right] = \left[(G^2 - \omega^2 C^2) + j(2G\omega C)\right]$$

Evaluating real and imaginary parts

$$\frac{1}{Z_0^2}(\alpha^2 - \beta^2) = G^2 - \omega^2 C^2$$

or $$\frac{1}{(80)^2}(0.04^2 - 1.5^2) = G^2 - (2\pi \times 500 \times 10^6)^2 C^2$$

or $$G^2 - 9.8695 \times 10^{18} C^2 = -3.5131 \times 10^{-4} \quad \text{— (iii)}$$

$$\frac{1}{Z_0^2}(2\alpha\beta) = 2G\omega C$$

or $$GC = 2.9840 \times 10^{-15} \quad \text{— (iv)}$$

From eqn (iii), (iv)

$$\left(\frac{2.9840 \times 10^{-15}}{C}\right)^2 - 9.8695 \times 10^{18} C^2 = -3.5131 \times 10^{-4}$$

or, $8.9042 \times 10^{-30} - 9.8695 \times 10^{18} C^4 + 3.5131 \times 10^{-4} C^2 = 0$

or, $9.8695 \times 10^{18} C^4 - 3.5131 \times 10^{-4} C^2 - 8.9042 \times 10^{-30} = 0$

which is in form of $ax^2 + bx + c = 0$

$C^2 = 3.5620 \times 10^{-23} \Rightarrow C = 5.9682 \times 10^{-12}$ F/m

$= 5.9682$ pF/m

From (iv)

$$G = \frac{2.9840 \times 10^{-15}}{C} = \frac{2.9840 \times 10^{-15}}{5.9682 \times 10^{-12}} = 5 \times 10^{-4} \text{ S/m}$$

using eqn (i)

$$Z_0^2 (G + j\omega C) = (R + j\omega L)$$

or, $R = Z_0^2 G = (80)^2 \times (5 \times 10^{-4}) = 3.2\ \Omega/m$

$L = Z_0^2 C = 80^2 \times (5.9682 \times 10^{-12})$

$= 3.8196 \times 10^{-8}$ H/m

$= 38.196$ nH/m

Hence, $R = 3.2\ \Omega/m$, $L = 38.196$ nH/m,
$G = 5 \times 10^{-4}$ S/m, $C = 5.9682$ pF/m

16. A lossless transmission line with $Z_o = 60\ \Omega$ is 400 m long. It is terminated with load $Z_L = 40 + j80\ \Omega$ and is operated with frequency of 1 MHz. Let v=0.8c. Find:

a. The reflection coefficient (Γ)

b. The standing wave ratio (s or SWR)

c. Input Impedance (Z_{in})

[2063 Kartik]

Solution:

soln:

① $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$= \dfrac{(40 + j80) - (60 + j0)}{(40 + j80) + (60 + j0)} = 0.6439 \angle 65.4^\circ$

② $SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.6439}{1 - 0.6439} = 4.62$

③ $Z_{in} = Z_0 \left[\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$

$\beta l = \dfrac{2\pi}{\lambda} l = \dfrac{2\pi f l}{v} = \dfrac{2\pi \times 10^6 \times 400}{0.8 \times 3 \times 10^8} = \dfrac{10\pi^c}{3}$

$Z_{in} = 60 \angle 0^\circ \left[\dfrac{40 + j80 + j\, 60\angle 0^\circ \tan \frac{10\pi^c}{3}}{60\angle 0^\circ + j(40 + j80) \tan \frac{10\pi^c}{3}} \right]$

$= 108 \angle -60.9^\circ\ \Omega$

$\begin{bmatrix} 104.7876\angle 138.598° \end{bmatrix} = 108\angle -60.9°\,\Omega$

17: A certain transmission line 2 m long operating at $\omega = 10^6$ rad/s has $\alpha = 8$ dβ/m, $\beta = 1$ rad/m, and $Z_o = 60 + j40\ \Omega$. If the line is connected to a source of $10\angle 0°$ V, $Z_g = 40\ \Omega$ and terminated by a load of $20 + j50\ \Omega$, determine the input impedance.

soln ÷

(a) $\alpha = 8\ d\beta/m = \frac{8}{8.686}\ Np/m = 0.921\ Np/m$

$\beta = 1\ rad/m$, $\gamma = \alpha + j\beta$

$= 0.921 + j \times 1\ m^{-1}$

$\gamma l = 2(0.921 + j) = 1.842 + 2j$

$\tanh \gamma l = \tanh(1.842 + j2)$

$= \frac{\sinh(2 \times 1.842)}{\cosh 2 \times 1.84 + \cos 2 \times 2} + j \frac{\sinh 2 \times 2}{\cosh 2 \times 1.84 + \cos 2 \times 2}$

$\sinh 3.68 = \frac{e^{3.68} - e^{-3.68}}{2} = 19.8105$

$\cosh 3.68 = \frac{e^{3.68} + e^{-3.68}}{2} = 19.8358$

$\tanh \gamma l = \frac{19.8105}{19.8358 + \cos 4} + j \frac{\sin 4}{19.8358 + \cos 4}$

$= 1.0327 - j\,0.0394$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

$$= (60 + j40) \left[\frac{(20+50j) + (60+j40)(1.0327 - j\,0.03294)}{(60+40j) + (20+j50)(1.0327 - j\,0.03294)} \right]$$

$$= 71.7887 \angle 0.573^\circ$$

$$\therefore Z_{in} = 60.322 + j\,38.9206\ \Omega \quad \#$$

19. Calculate the cut-off frequencies of the first four propagating modes for an air-filled copper wave guide with dimension a = 2.5 cm, b = 1.2 cm. [2073 Shrawan]

Solution:

soln

$$f_{c\,min} = \frac{v}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

air filled

$$f_{c\,min} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

dominant mode of rectangular wave is TE_{10}

$$f_{c10} = \frac{c}{2a} = \frac{3\times10^8}{2\times2.5\times10^{-2}} = 6\times10^9\ Hz$$

$$= 6\ GHz$$

Now,

$$f_{c01} = \frac{c}{2}\sqrt{\left(\frac{0}{a}\right)^2 + \left(\frac{1}{1.2\times10^{-2}}\right)^2}$$

$$= 12.5 \text{ GHz} > 6 \text{ GHz}$$

TE_{01} mode will propagate

$$f_{c11} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 13.865 \text{ GHz} > 6 \text{ GHz}$$

TE_{11} will propagate

$$f_{c21} = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 17.327 \text{ GHz} > 6 \text{ GHz}$$

So, TE_{21} mode will propagate

$$f_{c20} = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = 12 \text{ GHz} > 6 \text{ GHz}$$

So, TE_{20} mode will propagate.

Hence, the cut-off frequencies of the first four propagating modes are 12 GHz, 12.5 GHz, 13.865 GHz, 17.327 GHz.

22. If a transmission line having characteristics impedance Z_o = 200 Ω is operating at frequency 15 MHz with propagation constant $\gamma = j\,0.5\ m^{-1}$, then determine

a. Velocity of propagation
b. Wavelength
c. Inductance
d. Capacitance

[2068 Shrawan]

Solution:

$$\text{Velocity } (v) = f\lambda = f \times \frac{2\pi}{\beta} = \frac{15 \times 10^6 \times 2\pi}{0.5} = 1.8849 \times 10^8 \text{ m/s}$$

We have, $\gamma = \alpha + j\beta = j0.5 \text{m}^{-1}$

$\alpha = 0$ So, it is a lossless transmission line.

$\beta = 0.5 \text{ m}^{-1}$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

As $\alpha = 0$, we have $R = 0, G = 0$.

So, $\beta = \omega\sqrt{LC}$

or, $\omega\sqrt{LC} = 0.5$

or, $\sqrt{LC} = \frac{0.5}{\omega} = \frac{0.5}{2\pi f}$

$$LC = \left(\frac{0.5}{2\pi f}\right)^2 \text{ .... (i)}$$

Characteristics impedance is given as

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0 + j\omega L}{0 + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = Z_o^2 = (200)^2$$

$$\therefore L = (200)^2 C \text{ .... (ii)}$$

From equations (i) and (ii),

$$(200)^2 C^2 = \left(\frac{0.5}{2\pi f}\right)^2$$

$$\therefore C = \frac{0.5}{2\pi f} \times \frac{1}{200}$$

$$= \frac{0.5}{2\pi \times 15 \times 10^6 \times 200} = 2.653 \times 10^{-11} = 26.533 \text{ pF}$$

From equation (ii),

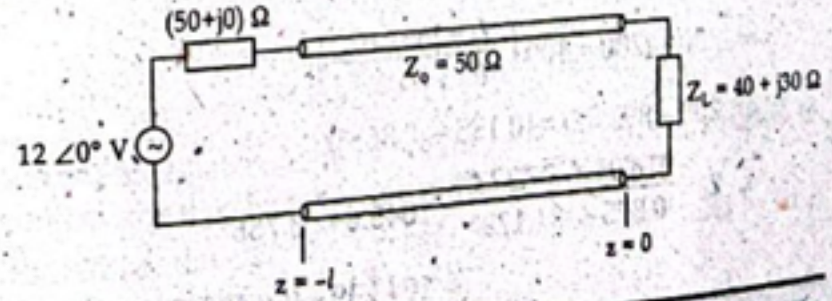
$L = (200)^2 \times (2.653 \times 10^{-11}) = 1.0612 \times 10^{-6} \text{ H} = 1.0612 \text{ mH}$

$$\text{Wavelength } (\lambda) = \frac{2\pi}{\beta} = 12.56 \text{ m}$$

22. A load of 100 ...

24. A 50 Ω lossless line has a length of 0.4λ. The operating frequency is 300 MHz. A load $Z_L = 40 + j30\ \Omega$ is connected at $z = 0$, and the Thevenin equivalent source at $z = -l$ is $12\angle 0°$ in series with $Z_{Th} = 50 + j0\ \Omega$. Find: (a) The reflection coefficient (Γ) (b) The voltage standing wave ratio (VSWR) (c) The input impedance (Z_{in}). *[2072 Kartik]*

Solution:



$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(40 + j30) - 50}{(40 + j30) + 50} = 0.333\angle 90°$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.333}{1 - 0.333} = 1.998$$

For lossless line,

$$Z_{in} = Z_o \frac{(Z_L + jZ_o \tan\beta l)}{(Z_o + jZ_L \tan\beta l)}$$

$$\beta l = \frac{2\pi}{\lambda} \times 0.4\lambda = 2.513^c$$

$$\therefore Z_{in} = 26.1428\angle 13.048°\ \Omega$$

25. A lossless line having

25. **A lossless line having an air dielectric has a characteristics impedance of 400 Ω. The line is operating at 200 MHz and $Z_{in} = 200 - j200\ \Omega$. Find: (a) SWR (b) Z_L if the line is 1 m long. (c) the distance from the load to the nearest voltage maximum.**

[2074 Ashwin]

Solution:

We have,

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right) \text{.........(i)}$$

$Z_o = 400\ \Omega$, $Z_{in} = 200 - j200\ \Omega$

Let, $v = c = 3 \times 10^8$ m/s

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi \times 200 \times 10^6}{3 \times 10^8} = \frac{4\pi}{3} \text{ rad/m}$$

$$\beta l = \frac{4\pi}{3} \times 1 = \left(\frac{4\pi}{3}\right)^c$$

So, from equation (i),

$$200 - j200 = 400\left[\frac{Z_L + j400 \tan\left(\frac{4\pi}{3}\right)^c}{400 + jZ_L \tan\left(\frac{4\pi}{3}\right)^c}\right]$$

or, $200 - j200 = 400\left(\dfrac{Z_L + j692.82}{400 + jZ_L\, 1.73}\right)$

or, $200 - 892.82j = (0.135 - j0.865)Z_L$

$$\therefore\ Z_L = \frac{914.94\angle{-77.373^\circ}}{0.875\angle{-81.129^\circ}} = 1045.64\angle 3.756^\circ$$

$$= 1043.39 + j68.497\ \Omega$$

Using, $\text{SWR} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$= \frac{1043.39 + j68.497 - 400}{1043.39 + j68.497 + 400} = \frac{647.025\angle 6.076^\circ}{1445.014\angle 2.716^\circ} = 0.447\angle 3.36^\circ$$

So, $\text{SWR} = \dfrac{1 + 0.447}{1 - 0.447} = 2.616$

For finding the distance to the nearest voltage maximum,

$$z_{max} = -\frac{1}{2\beta}(\phi + 2n\pi);\ n = 0, \pm 1, \pm 2,$$

Taking $n = 0$,

$$z_{max} = -\frac{\phi}{2\beta}$$

As $\Gamma = 0.447\angle 3.36^\circ = |\Gamma|\angle\phi$, $\phi = 3.36^\circ$

$$\text{So, } z_{max} = \frac{-3.36^\circ}{2\left(\frac{4\pi}{3}\right)^c} = \frac{-3.36^\circ}{\left(2 \times \frac{4\pi}{3} \times \frac{180}{\pi}\right)^\circ} = -7 \times 10^{-3} \text{ m}$$

26. **An air-filled rectangular waveguide has dimensions a = 2 cm, b = 1 cm. Determine the range of frequencies over which the guide will operate single mode (TE_{10}).** *[2076 Ashwin]*

Solution:

$$f_{cmn} = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{c10} = \frac{v}{2} \times \frac{1}{a}$$

As waveguide is air filled, $v = c = 3 \times 10^8$ m/s

$$\therefore \; f_{c10} = \frac{3 \times 10^8}{2} \times \frac{1}{2 \times 10^{-2}} = 7.5 \times 10^9 \text{ Hz} = 7.5 \text{ GHz}$$

The next higher mode will be either TE_{20} or TE_{01}, as

$$f_{c20} = \frac{v}{2} \times \frac{2}{a} = \frac{3 \times 10^8}{2} \times \frac{2}{2 \times 10^{-2}} = 15 \times 10^9 \text{ Hz} = 15 \text{ GHz}$$

$$f_{c01} = \frac{v}{2} \times \frac{1}{b} = \frac{3 \times 10^8}{2} \times \frac{1}{1 \times 10^{-2}} = 15 \times 10^9 \text{ Hz} = 15 \text{ GHz}$$

Note: Students are advised to find out f_c for other modes too to confirm the next higher mode will be TE_{20} or TE_{01}.

Therefore, the operating range over which the guide will be single mode is $7.5 \text{ GHz} < f < 15 \text{ GHz}$

10

01

20

remember