Numericals 4

A 9.375 GHz uniform plane wave is propagating in polyethylene ($\varepsilon_r = 2.26$, $\mu_r = 1$). If the amplitude of the electric field intensity is 500 V/m and the material is assumed to be lossless, find: (a) the phase constant (b) the wavelength in the polyethylene (c) the velocity of propagation (d) the intrinsic impedance (e) the amplitude of the magnetic field intensity.

Soln:-

$1\,GHz = 10^9\,Hz$

(a) Phase constant $(\beta) = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

$= 2\pi \times 9.375 \times 10^9 \sqrt{4\pi\times10^{-7} \times 1 \times 8.854\times10^{-12} \times 2.26}$

$= 295.312\ rad/m$

(b) $\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{295.312} = 0.02127\,m = 2.127\,cm$

(c) $v = \lambda f = 0.02127 \times 9.375\times10^9 = 1.994\times10^8\ m/s$

(d) $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{0 + j\omega\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$

$= \sqrt{\frac{4\pi\times10^{-7}\times1}{8.854\times10^{-12}\times2.26}} = 250.656\ \Omega$

(e) $\eta = \frac{E_0}{H_0}$, $H_0 = \frac{E_0}{\eta} = \frac{500}{250.656} = 1.994\ A/m$

A uniform plane wave in free space is given by $E_s = (200\angle 30^\circ)\, e^{-j250z}\, \hat{a}_x$ V/m.
Find: (a) β (b) ω (c) f (d) λ (e) η (f) $\vec{H}_s$ (g) $|\vec{E}|$ at z = 8 mm, t = 6 ps.

$\vec{P} = \vec{E} \times \vec{H}$

e^{-j250z} direction of propagation

$e^{+j\beta z}$ $e^{-j\beta z}$

Soln ÷

(a) $E_s = (200\angle 30^\circ)\, e^{-j250z}$, $\beta = 250$ rad/m

(b) $\beta = \omega\sqrt{\mu_0 \varepsilon_0}$ (free space)

$$\omega = \frac{\beta}{\sqrt{\mu_0\varepsilon_0}} = 74.94 \times 10^{9} \text{ rad/s} = 74.94 \text{ G rad/s}$$

(c) $\omega = 2\pi f$, $f = \dfrac{\omega}{2\pi} = 11.9275$ GHz

(d) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{250} = 0.025$ m

(e) $$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7343\ \Omega$$

($\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$)
free space)

(f) Since -z is involved in the equation $\vec{E}_s$ the direction of $\vec{P}$ is $\hat{a}_z$

if e^{-j250z} $(+\hat{a}_z)$ else e^{j250z} $(-\hat{a}_z)$

→ (remember)

$\vec{P} = \vec{E} \times \vec{H}$

$\hat{a}_z = \hat{a}_x \times ?$

$\hat{a}_x \times \hat{a}_y = \hat{a}_z$

so direction of $\vec{H}$ is $\hat{a}_y$

(note $\vec{E_s} = E_0 e^{\gamma x} \hat{a_y}$) direction of propagation $= -\hat{a_x}$

$-\hat{a_x} = \hat{a_y} \times (-\hat{a_z})$ — $H_y = -\hat{a_z}$)

$\eta = \frac{E_o}{H_o}$ or $H_o = \frac{E_o}{\eta} = \frac{200}{376.73} = 0.531$ A/m

$\vec{H_s} = 0.531 \angle 30^\circ e^{-250z} \hat{a_y}$ A/m

(g) $\vec{E_s} = 200 \angle 30^\circ e^{-j250z} \hat{a_x}$

$= 200 e^{j30^\circ} e^{-j250z} \hat{a_x}$

$= 200 e^{j(30^\circ - 250z)} \hat{a_x}$

$\vec{E_s} = 200 e^{j(30^\circ - 250z)} \hat{a_x}$

In time domain

$\vec{E} = Re[(\vec{E_s}) e^{j\omega t}]$

$= Re[200 e^{j(30^\circ - 250z)} \cdot e^{j\omega t}]$

$= Re[200 e^{j(\omega t - 250z - 30^\circ)}]$

$= 200 Re[\cos(\omega t - 250z - 30^\circ) + j \sin(\omega t - 250z - 30^\circ)]$

$= 200 \cos(\omega t - 250z + 30^\circ) \hat{a_x}$ A/m

At $t = 6ps = 6\times10^{-12}s$, $z = 8mm = 8\times10^{-3}m$

$\vec{E} = 200 \cos(74.94 \times 10^9 \times 6\times10^{-12} - 250 \times 8\times10^{-3} + 30^\circ] \hat{a_x}$

$= 200 \cos(-1.550 + 30^\circ) \hat{a_x}$

rad

$= 200 \cos(-88.80 + 30^\circ) \hat{a_x}$

$= 200 \cos(-58.826^\circ) \hat{a_x}$

$= 103.52 \hat{a_x}$ V/m

$|\vec{E}| = 103.52 \text{ V/m}$ //

Let $\vec{E}_s$ = (1000 $\hat{a}_x$ + 400 $\hat{a}_z$) e^{-j10y} V/m for a 250-MHz uniform plane wave propagating in a perfect dielectric. If the maximum amplitude of $\vec{H}$ is 3 A/m, find β, η, λ, v, ε_r, μ_r, and $\vec{E}$ (x, y, z, t).

perfect dielectric

$\sigma = 0$

$\alpha = 0 \rightarrow E_{x0} e^{-j\beta z}$

Soln:

β = coefficient of y in equation

= 10 rad/m

$$\eta = \frac{|\vec{E}|_{max}}{|\vec{H}|_{max}}$$

$$|\vec{E}|_{max} = \sqrt{(1000)^2 + (400)^2} = 1077.03 \text{ V/m}$$

$$|\vec{H}|_{max} = 3 \text{ A/m}$$

$$\eta = \frac{1077.03}{3} = 359.011 \, \Omega$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{10} = 0.6283 \text{ m}$$

$\left(v = \frac{\omega}{\beta}\right)$

$$v = \lambda f = 0.6283 \times 250 \times 10^6$$
$$= 1.57 \times 10^8 \text{ m/s}$$

to find ε_r, μ_r we calculate first

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

or $359.011 = \sqrt{\dfrac{4\pi \times 10^{-7}\, \mu r}{8.854 \times 10^{-12}\, \varepsilon r}}$

$\left(c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} \right.$ free space $\left. \right)$

or $\dfrac{\mu r}{\varepsilon r} = 0.9081$ — (i)

perfect dielectric

Also,

$$V = \frac{1}{\sqrt{\mu r \varepsilon r \mu_0 \varepsilon_0}}$$

$v = \dfrac{1}{\sqrt{\mu r \mu_0 \varepsilon r \varepsilon_0}}$

or $1.571 \times 10^{8} = \dfrac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times \mu r \varepsilon r}}$

$\therefore \mu r \varepsilon r = 3.64165$ — (ii)

Solving (i) and (ii)

$\mu r = 1.8185$, $\varepsilon r = 2.0025$

$\vec{E}(x, y, z, t)$ requires the given expression of electric field (which is in phasor form) to be converted into time domain

Given, $\vec{E_s} = (1000\hat{a}_x + 400\,\hat{a}_z)\, e^{-j10y}$ v/m

$\vec{E}(x, y, z, t) = R_e \left[\vec{E_s} . e^{j\omega t} \right]$

$$= Re\left[(1000\,\hat{a}_x + 400\,\hat{a}_z)\, e^{-j10y}\, e^{j\omega t}\right]$$

$$= (1000\,\hat{a}_x + 400\,\hat{a}_z)\; Re\left[e^{j(\omega t - 10y)}\right]$$

$$\therefore\ \vec{E}(x,y,z,t) = (1000\,\hat{a}_x + 400\,\hat{a}_z)\cos(\omega t - 10y)\ V/m$$

Numerical example-2

Q. A uniform plane wave propagating in a medium has $\vec{E} = 2e^{-\alpha z}\sin(10^8 t - \beta z)\,\hat{a}_y$ V/m. If the medium is characterized by, $\epsilon_r = 1$, $\mu_r = 20$ & $\sigma = 3$ s/m, Find α & β.

Soln:

$$\vec{E} = 2e^{-\alpha z}\sin(\underbrace{10^8}_{\omega} t - \beta z)\,\hat{a}_y\ V/m$$

$$\epsilon_r = 1,\quad \mu_r = 20,\quad \sigma = 3\ s/m$$

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{10^8 \times 8.85\times10^{-12} \times 1} = 3393 >> 1$$

medium is good conductor

$$\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \sqrt{\frac{\mu_0\mu_r\omega\sigma}{2}}$$

$$= \left(\frac{4\pi\times10^{-7} \times 20 \times 10^8 \times 3}{2}\right)^{1/2} = 61.39\ Np/m$$

$$\beta = 61.4\ rad/m$$

Q. An EM wave travels in free space with the electric field component $\vec{E} = (10\hat{a}_y + 5\hat{a}_z)\cos(\omega t + 2y - 4z)$ V/m. Find (a) ω and λ (b) Magnetic field component

soln ÷

$$\vec{E} = (10\hat{a}_y + 5\hat{a}_z)\cos(\omega t + 2y - 4z) \text{ V/m} \quad \text{— (i)}$$

$$\vec{E} = E_0 \cos(\omega t - \beta_x x - \beta_y y - \beta_z z + \phi)$$

$$\phi = 0, \quad \beta_x = 0, \quad \beta_y = -2, \quad \beta_z = 4$$

$$\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2}$$

$$= \sqrt{0 + (-2)^2 + 4^2} = 4.5 \text{ rad/m}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$= \omega\sqrt{\mu_0\varepsilon_0} \quad \text{(free space)}$$

$$\omega = \frac{\beta}{\sqrt{\mu_0\varepsilon_0}} = \frac{4.5}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}} = 1.34 \text{ G rad/s}$$

$$\beta = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{4.5} = 1.4 \text{ m}$$

$$\vec{H}_0 = \frac{1}{\omega\mu}\vec{\beta}\times\vec{E}_0$$

$10\,a_y + 5a_z$

$$\vec{\beta} \times \vec{E_0}' = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ 0 & -2 & 4 \\ 0 & 10 & 5 \end{vmatrix} =$$

$$= \hat{a_x} \begin{vmatrix} -2 & 4 \\ 10 & 5 \end{vmatrix} - \vec{a_y} \times 0 + \hat{a_z} \times 0$$

$$= -50\,\hat{a_x}$$

$$\vec{H_0} = \frac{1}{1.34\times10^{9} \times 4\pi\times10^{-7}}\,(-50\,\hat{a_x})$$

$$= -0.029\,\hat{a_x}\ A/m$$

$$\vec{H} = \vec{H_0}\cos(\omega t + 2y - 4z)$$

$$= -0.029\cos(\omega t + 2y - 4z)\,\hat{a_x}\ A/m$$

Q. In an lossless dielectric for which $\eta = 60\pi$, $\mu_r = 1$ & $\vec{H} = -0.1\cos(\omega t - z)\,\hat{a}_x + 0.5\sin(\omega t - z)\,\hat{a}_y$ A/m, calculate ε_r, ω and $\vec{E}$.

Soln ÷

$\eta = 60\pi$, $\mu_r = 1$, $\vec{H} = 0.1\cos(\omega t - z)\,\hat{a_x} + 0.5\sin(\omega t - z)\,\hat{a_y}$ A/m

$\varepsilon_r = ?$ $\omega = ?$ $E = ?$

$\therefore \quad \beta = 1 \text{ rad/m} \qquad (\beta\text{-coefficient})$

For lossless dielectric

$\sigma = 0$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

or $\quad 60\pi = \sqrt{\dfrac{4\pi \times 10^{-7} \times 1}{8.854 \times 10^{-12} \times \varepsilon_r}}$

$\therefore \quad \varepsilon_r = 4$ //

$$\beta = \omega \sqrt{\mu \varepsilon}$$

$$\omega = \frac{\beta}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1 \times 8.85 \times 10^{-12} \times 4}}$$

$= \quad 1.5 \times 10^{8} \text{ rad/s}$

$$\nabla \times \vec{H} = \vec{J_c} + J_d = \cancel{J_c}^{\,0} + \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad \left(\begin{matrix} \sigma = 0 \\ J_c = 0 \end{matrix} \right)$$

$$\Rightarrow \quad \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon} \nabla \times \vec{H}$$

$$\Rightarrow \quad \vec{E} = \frac{1}{\varepsilon} \int (\nabla \times \vec{H}) \, dt$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -0.1 \cos(\omega t - 2) & 0.5 \sin(\omega t - 2) & 0 \end{vmatrix}$$

$$= 0.5 \cos(\omega t - 2)\, \hat{a}_x - 0.1 \sin(\omega t - 2)\, \hat{a}_y$$

$$\vec{E} = \frac{1}{\varepsilon} \int \left(0.5 \cos(\omega t - 2)\, \hat{a}_x - 0.1 \sin(\omega t - 2)\, \hat{a}_y \right) dt$$

$$= \frac{1}{\varepsilon_0 \varepsilon_r} \left[0.5 \sin(\omega t - 2)\, \hat{a}_x + 0.1 \cos(\omega t - 2)\, \hat{a}_y \right]$$

$\varepsilon_0 = 8.85 \times 10^{-12}$, $\varepsilon_r = 4$, $\omega = 1.5 \times 10^{8}$ rad/s

$$\vec{E} = 94.11 \sin(\omega t - 2)\, \hat{a}_x + 18.823 \cos(\omega t - 2)\, \hat{a}_y \text{ V/m}$$

Q. An Electric field in free space is given as $\vec{E} = 800 \cos(10^8 t - \beta y)\, \hat{a}_z$ V/m. Find (a) propagation constant (β), (b) wave-length (λ) (c) Magnetic field Intensity $\vec{H}$ at $P(0.1, 1.5, 0.4)$ at $t = 8$ ns.

soln ÷

$\vec{E} = 800\cos(10^8 t - \beta y)\,\hat{a}_z$ V/m

$\omega = 10^8 = 2\pi f$

$V = \lambda f$ for free space $c = \lambda f$

$v = \dfrac{1}{\sqrt{\mu\varepsilon}}$ $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$10^8 = 2\pi \dfrac{c}{\lambda}$

or $\lambda = \dfrac{2\pi \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}}{10^8} = \dfrac{2\pi\sqrt{\dfrac{1}{4\pi\times10^{-7}\times 8.85\times10^{-12}}}}{10^8} = 6\pi$ m

$\beta = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{6\pi} = \dfrac{1}{3}$ rad/m

$\vec{H}$ at $P(0.1, 1.5, 0.4)$ $t = 8$ ns

$\vec{E} = 800\cos(10^8 t - \beta y)\,\hat{a}_z$ V/m

$E_0 = 800$ V/m

free space

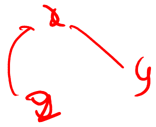
$\eta = \dfrac{E_0}{H_0}$ | $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 377$

$H_0 = \dfrac{800}{377} = 2.122$ A/m

β ⊥ to direction

$\vec{P} = \vec{E} \times \vec{H}$

$(\hat{a}_y = \hat{a}_z \times \hat{a}_x$



$\vec{H} = 2.122 \cos(10^8 t - 0.33y)\, \hat{a}_x$ A/m #

Numerical example -7 :-

Q. For a non-magnetic material having $\varepsilon_r = 2.25$ & $\sigma = 10^{-4}$ mho /m. Find the numeric value at 5 MHz for

(a) loss tangent

(b) Attenuation Constant

(c) phase Constant

(d) Intrinsic Impedence

soln ÷

(a) loss tangent $(\tan\theta) = \dfrac{\sigma}{\varepsilon\omega}$

$= \dfrac{10^{-4}}{2.25 \times \varepsilon_0 \times 2\pi \times 5\times10^6}$

$= 0.1597$

(b) $\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$= \left(j \times 2\pi f \times \mu_0 \mu_r\,(\sigma + j \times 2\pi \times f \times \varepsilon_0 \times \varepsilon_r)\right)^{1/2}$

$f = 10^6$ Hz, $\mu_r = 1$

(for non magnetic)

$= \left(j \times 2\pi \times 10^6 \times \mu_0\,(10^{-4} + j \times 2\pi \times 10^6 \times \varepsilon_0 \times 2.25)\right)^{1/2}$

4

(Store this in A)

in calc $\sqrt{|A|} \angle \left(\frac{\arg(A)}{2}\right)$

$= 0.038 \angle 70.68^\circ$

$= 0.0117 + j\,0.03$

$\gamma = \alpha + j\beta$

$\alpha = 0.0117$ Np/m $\quad \beta = 0.03$ rad/m

non magnetic

\# $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\dfrac{j \times 2\pi \times 5\times10^6 \times 1 \times 4\pi\times10^{-7}}{10^{-4} + j \times 2\pi \times 5\times10^6 \times 2.25 \times 8.85\times10^{-12}}}$

$= 249.63 \angle 4.53^\circ$ \#

Q. The electric field amplitude of a uniform plane wave propagating in the free space $\hat{a}_z$ direction is 250 V/m. If $\vec{E} = E_x \hat{a}_x$ and $\omega = 1.00$ M rad/s, find (a) the frequency (b) the wavelength (c) period (d) The amplitude of H.

Soln ÷

free space

$E_0 = 250$ V/m

$\vec{E} = E_x \hat{a}_x \qquad \omega = 1\times10^6$ rad/s

$$2\pi f = 10^6$$

(a) $$f = \frac{10^6}{2\pi} = 159.15 \times 10^3 \text{ Hz}$$

(b) $$\lambda = \frac{2\pi}{\beta} \quad ; \quad \beta = \omega\sqrt{\mu\varepsilon}$$

rad/m

for free space

$$= \omega\sqrt{\mu_0 \varepsilon_0}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{2\pi}{10^6\sqrt{\mu_0\varepsilon_0}} = 1883.65 \text{ m}$$

(c) $$T = \frac{1}{f} = \frac{1}{159.15 \times 10^3} = 6.28 \times 10^{-6} \text{ s}$$

(d) $H_0 = ?$

$$\eta = \frac{E_0}{H_0} \quad \Bigg| \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$\sigma = 0$ free space

$\mu = \mu_0 \quad \varepsilon = \varepsilon_0$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= 376.73$$

$$H_0 = \frac{E_0}{H_0} = \frac{250}{376.73} = 0.66 \text{ A/m}$$

Numerical example-9 :-

28. Numerical Example-9.mp4

Q. A plane wave propagating through a media with $\varepsilon_r = 8$, $\mu_r = 2$ has $\vec{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \hat{a}_x$ V/m. Determine

a. β

b. loss tangent

c. Intrinsic Impedence

d. $\vec{H}$ field

soln :-

$$\varepsilon_r = 8, \quad \mu_r = 2 \qquad \vec{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \hat{a}_x \text{ V/m}$$

&

$$\vec{E} = E_0\, e^{-\alpha z} \sin(\omega t - \beta z)\, \hat{a}_x$$

$$\alpha = \frac{1}{3} \qquad \omega = 10^8, \quad E_0 = 0.5$$

$$\mu_r = 2, \quad \varepsilon_r = 8$$

(a)
$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}$$
$$= 10^8 \sqrt{2 \times 8 \times \mu_0 \varepsilon_0}$$
$$= 1.334 \text{ rad/m}$$

(b)
$$\text{loss tangent } (\tan\theta) = \frac{\sigma}{\omega\varepsilon}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)}$$

$$\sigma = ?$$

$$\frac{1}{3} = 10^8 \sqrt{\frac{4\pi\times10^{-7} \times 2 \times 8 \times 8.85\times10^{-12}}{2}\left(\sqrt{1 + \left(\frac{\sigma}{10^8 \times 8 \times 8.85\times10^{-12}}\right)^2} - 1\right)}$$

$\sigma = 0.00364$

$\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{0.00364}{10^8 \times 8.85\times10^{-12}\times 8}$

$= 0.515$

(c) $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$= \sqrt{\dfrac{j\times10^8\times 2\times\mu_0}{0.00364 + j\times10^8\times 8\times\varepsilon_0}}$

$= 177.64 \angle 13.6^\circ \ \Omega$

(d) $\vec{H} = ?$

$\eta = \dfrac{E_0}{H_0}$, $H_0 = \dfrac{E_0}{\eta} = \dfrac{0.5}{177.64\angle 13.6^\circ} =$

$= 2.81\times10^{-3} \angle -13.6^\circ$

$\xrightarrow{P \text{ with } z}$ $\vec{P} = \vec{E}\times\vec{H}$

$\hat{a}_z \quad \hat{a}_x \times \hat{a}_y$

same as electric #

$\vec{H} = 2.81\times10^{-3}\angle -13.6^\circ\ e^{-2\,13}\ \sin(10^8 t - \beta z)\ \hat{a}_y$

$= 2.81\times10^{-3}\ e^{-2\,13}\ \sin(10^8 t - \beta z - 13.6)\ \hat{a}_y$ A/m #

Numerical example-10 :

A plane wave in a non-magnetic medium has $\vec{E} = 50\sin(10^8 t + 2z)\hat{a}_y$ V/m. Find:

(a) The direction of wave propagation

(b) λ, f and ε_r

(c) $\vec{H}$

Soln:

$$\vec{E} = 50\sin(10^8 t + 2z)\,\hat{a}_y \text{ V/m. Find:}$$

(a) $\vec{P} = \vec{E} \times \vec{H}$

$(-\hat{a}_z) = \hat{a}_y \times \hat{a}_x$

due to $(+2z)$

Direction of the propagation of wave is negative z-axis $(-\hat{a}_z)$

(b) $\vec{E} = 50\sin(10^8 t + 2z)\,\hat{a}_y$ V/m

$E_o = 50$, $\omega = 10^8$ $\beta = 2$ (not taking -ve)

$\omega = 2\pi f$

$10^8 = 2\pi f$

$$\therefore f = \frac{10^8}{2\pi} = 15915494.31 \text{ Hz}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2} = \pi = 3.14 \text{ m}$$

For nonmagnetic medium $(\mu_r = 1)$

$$\beta = \omega\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}$$

$$2 = 10^8 \sqrt{\mu_0 \times 1 \times \varepsilon_0 \times \varepsilon_r}$$

$$\therefore \quad \varepsilon_r = 35.95$$

now

$$\vec{H} = ?$$

$$H_0 \quad \eta = \frac{E_0}{H_0} \qquad H_0 = \frac{E_0}{\eta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \qquad \left(\begin{array}{c} \sigma \text{ not given} \\ \text{so } \sigma = 0 \end{array} \right)$$

non magnetic

$$= \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0 \times 1}{\varepsilon_0 \times 35.95}}$$

$$= 62.83$$

now

$$H_0 = \frac{50}{62.83} = 0.795$$

$$\vec{H} = 0.795 \sin(10^8 t + 2z)\, \hat{a}_x \quad \text{A/m}$$

PROBLEMS SOLVED

1. Find the skin depth δ at a frequency of 1.6 MHz in aluminum, where σ = 38.2 MS/m and μ_r = 1. Also find γ and the wave velocity.

soln:

Aliminium → good or perfect conductor

$$\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \sqrt{\pi f \mu \sigma} = \frac{1}{\delta}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 1.6\times10^{6} \times 1 \times \mu_0 \times 38.2\times10^{6}}}$$

$$= 6.4376 \times 10^{-5}\ m$$

$$\alpha = \beta = \frac{1}{\delta} = 1.553\times10^{4}$$

$$\gamma = \alpha + j\beta = 1.553\times10^{4} + j\,1.553\times10^{4}\ m^{-1}$$

wave velocity $(v) = \lambda f = \frac{2\pi f}{\beta} = \frac{2\pi \times (1.6\times10^{6})}{1.553\times10^{4}} = 647.334\ m/s$

2. Determine the propagation constant γ for a material having $\mu_r = 1$, $\varepsilon_r = 8$, and σ = 0.25 pS/m, if the wave frequency is 1.6 MHz.

soln ÷

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.25\times10^{-12}}{2\pi\times(1.6\times10^{6})\times8\times(8.854\times10^{-12})} = 10^{-9} \approx 0$$

so that $\alpha = 0$

$$\beta = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_0\mu_r\varepsilon_r\varepsilon_0}$$

$$= 9.48\times10^{-2}\ \text{rad/m}$$

Now

$$\gamma = \alpha + j\beta \simeq j\,9.84\times10^{-2}\ m^{-1}$$

→ behaves like a perfect dielectric at the given frequency.

4. **Assume that dry soil has conductivity equal to 10^{-4} S/m, $\varepsilon = 3\varepsilon_0$ and $\mu = \mu_0$. Determine the frequency at which the ratio of the magnitudes of the conduction current density and displacement current density is unity.** *[2069 Chaitra]*

Solution:

soln ÷

$$\frac{\vec{J_s}}{\vec{J_{ds}}} = \frac{\sigma\vec{E_s}}{j\omega\varepsilon\vec{E_s}} = \frac{\sigma}{j\omega\varepsilon} = \frac{\sigma+j0}{0+j\omega\varepsilon}$$

$$\frac{|\vec{J_s}|}{|\vec{J_{ds}}|} = \frac{\sqrt{\sigma^2+0^2}}{\sqrt{0^2+(\omega\varepsilon)^2}} = \frac{\sigma}{\omega\varepsilon}$$

Given,

$$\frac{|\vec{J_s}|}{|\vec{J_{ds}}|} = 1$$

or $1 = \frac{\sigma}{\omega \varepsilon}$

or $1 = \frac{\sigma}{2\pi f(3\varepsilon_0)}$ → given

$\therefore\ f = \frac{\sigma}{2\pi \times 3\varepsilon_0} = \frac{10^{-4}}{6\pi \times \varepsilon_0} = 0.599\ MHz$

7. A uniform plane wave in free space is propagating in the $-\hat{a}_y$ direction at a frequency of 10 MHz. If $\vec{E} = 400 \cos \omega t\ \hat{a}_z$ V/m at y = 0, write expressions for: (a) $\vec{E}$ (x, y, z, t) (b) $\vec{E}_s$ (x, y, z) (c) $\vec{H}_s$ (x, y, z) (d) $\vec{H}$ (x, y, z, t).

soln:-

(a) $\vec{P} = \vec{E} \times \vec{H}$

x, y, z

$-\hat{a}_y = \hat{a}_z \times (-\hat{a}_x)$

at y=0

$\vec{E} = 400 \cos \omega t\ \hat{a}_z$ $\qquad f = 10 MHz$

$E_0 = 400$ V/m $\quad \omega = 2\pi f = 2\pi \times 10 \times 10^6$

$= 2\pi \times 10^7$ rad/s

$\beta = \omega\sqrt{\mu_0 \varepsilon_0}$ (free space)

$= 2\pi \times 10^7 \sqrt{\mu_0 \varepsilon_0}$

$= 0.2093$ rad/m

at $y \neq 0$

$\vec{E} = 400 \cos(2\pi \times 10^7 t + 0.21y)\ \hat{a}_z$ V/m

$\omega t \quad + \quad \theta$

$\vec{E}(x,y,z,t) = 400\cos(2\pi\times10^{7}t + 0.21y)\,\hat{a}_z$ V/m

(b) $\vec{E}_s(x,y,z) = E_{x0}\,e^{j\theta}$

$= 400\,e^{j0.21y}\,\hat{a}_z$ V/m

(c) $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ (free space)

$= 376.7343$

$\eta = \dfrac{E_0}{H_0}$, $H_0 = \dfrac{E_0}{\eta} = \dfrac{400}{376.7343} = 1.06176$ A/m

Direction of $\vec{H}$ is $-\hat{a}_x$ from above

(d) $\vec{H}_s = -1.06176\,e^{j0.21y}\,\hat{a}_x$ A/m

In time domain

$\vec{H} = Re[\vec{H}_s \cdot e^{j\omega t}]$

$= -1.06176\,Re[e^{j(\omega t + 0.21y)}]$

$= -1.06176\cos(\omega t + 0.21y)\,\hat{a}_x$ A/m #

$\therefore H = -1.06176 \cos(\omega t + 0.21y)\, \hat{a}_x$ A/m

8. **The electric field intensity of a 300-MHz uniform plane wave in free space is given as $\vec{E}_s = (20+j50)(\hat{a}_x + 2\hat{a}_y)\, e^{-j\beta z}$ V/m. (a) Find $\omega, \lambda, v, \beta$ (b) Find $\vec{E}$ at $t = 1$ ns, $z = 10$ cm (c) What is $|\vec{H}|_{max}$?**

Solution:

soln ÷

$$f = 300 \times 10^6 \text{ Hz}$$

$$\vec{E}_s = (20 + j50)(\hat{a}_x + 2\hat{a}_y)\, e^{-j\beta z} \text{ V/m}$$

$$\omega = 2\pi f = 2\pi \times 300 \times 10^6 = 1.885 \times 10^9 \text{ rad/s}$$

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0} = 6.287468 \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 0.999318 \text{ m}$$

For free space, $v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

(b) $\vec{E}_s = (20 + j50)(\hat{a}_x + 2\hat{a}_y)\, e^{-j\beta z}$

$\vec{E}$ at $t = 1$ns, $z = 10$ cm

$$\vec{E}_s = 53.85 \angle 68.2^\circ (\hat{a}_x + 2\hat{a}_y)\, e^{-j\beta z}$$

$$= 53.85 \cdot e^{j68.2^\circ} (\hat{a}_x + 2\hat{a}_y)\, e^{-j\beta z}$$

$$= 53.85 (\hat{a}_x + 2\hat{a}_y)\, e^{j(68.2^\circ - \beta z)} \text{ V/m}$$

time domain

$$\vec{E} = Re\left[(\vec{E_s})\, e^{j\omega t} \right]$$

$$= Re\left[\left\{ 53.852\,(\hat{a}_x + 2\hat{a}_y)\, e^{j(68.2^\circ - \beta z)} \right\} e^{j\omega t} \right]$$

$$= 53.582\,(\hat{a}_x + 2\hat{a}_y)\, Re\left[e^{j(\omega t - \beta z + 68.2^\circ)} \right]$$

$$= 53.582\,(\hat{a}_x + 2\hat{a}_y)\cos(\omega t - \beta z + 68.2^\circ)$$

At $t = 1\times10^{-9}\,s$, $z = 0.1\,m$

$$\vec{E} = 53.582\,(\hat{a}_x + 2\hat{a}_y)\cos(1.885\times10^{9} \times 10^{-9} - 6.287\times0.1 + 68.2^\circ)$$

radian

$$\vec{E} = 53.582\,(\hat{a}_x + 2\hat{a}_y)\cos(1.23623 + 68.2^\circ)$$

$$\vec{E} = \quad " \quad " \quad " \quad \cos(71.978^\circ + 68.2^\circ)$$

degree

$$\therefore \quad \vec{E} = -41.15\,(\hat{a}_x + 2\hat{a}_y)\ V/m$$

(c) $|\vec{H}|_{max}$

$$\eta = \frac{|\vec{E}|_{max}}{|\vec{H}|_{max}}$$

or

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{|\vec{E}|_{max}}{|\vec{H}|_{max}}$$

$\vec{E} = 53.852\,(\hat{a}_x + 2\hat{a}_y)\cos(\omega t - \beta z + 61.2°)$

or $|\vec{E}|_{max} = 53.852\sqrt{1^2+2^2} = 120.4167$ V/m

Now,

$376.730 = \dfrac{120.4167}{|\vec{H}|_{max}}$

$\therefore$ $|\vec{H}|_{max} = 0.3196$ A/m

9. A wave propagating in a lossless dielectric has the components, $\vec{E} = 500\cos(10^7 t - \beta z)\,\hat{a}_x$ V/m and $\vec{H} = 1.1\cos(10^7 t - \beta z)\,\hat{a}_y$ A/m. If the wave is travelling at v = 0.5c, find: (a) μ_r (b) ε_r (c) β (d) λ (e) η

Soln:

$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

$\lambda = \dfrac{2\pi}{\beta}$ $\qquad \eta = \dfrac{E_0}{H_0} = \dfrac{500}{1.1} = 454.545\ \Omega$

Now,

$\eta = 454.545$

or $\sqrt{\dfrac{\mu}{\varepsilon}} = 454.545$

or $\sqrt{\dfrac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = 454.545$

$\therefore$ $\dfrac{\mu_r}{\varepsilon_r} = 1.455$ ---(i) now,

$$V = 0.5C \quad \swarrow \text{speed of light in vaccum}$$

$$= 0.5 \times \underbrace{\left(\frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)}_{3\times10^8} = 1.5 \times 10^8$$

Now

$$\cancel{1.5\times10^8 =} \quad V = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

$$1.5\times10^8 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r}}$$

$$1.5 \times \cancel{10^8} = 3\times\cancel{10^8} \times \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$0.5 = \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$0.25 = \frac{1}{\mu_r \varepsilon_r} \quad \text{— (ii)}$$

from (i) and (ii)

$$0.25 = \frac{1}{1.455\, \varepsilon_r . \varepsilon_r}$$

$$\varepsilon_r = 1.65 \quad , \mu_r = 2.413$$

$$\beta = 10^7 \sqrt{\mu_0 \varepsilon_0 \times 1.65 \times 2.413} = 0.0667 \text{ rad/m}$$

$\lambda = \frac{2\pi}{\beta} = 94.201 \, m$

11. Uniform plane wave, $\vec{E}_s = (5\hat{a}_x + j10\,\hat{a}_y)\, e^{-j2z}$ V/m, has a frequency of 50 MHz in a lossless dielectric material for which $\mu_r = 1$. Find (a) $\beta, \omega, v, \lambda, \varepsilon_r, \eta$ (b) $\vec{E}$ at the origin for $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π.

(similar)

$\vec{E}_s = (5\hat{a}_x + j\,10\,\hat{a}_y)\, e^{-j2z}$

$= 5 \cdot e^{-j2z}\,\hat{a}_x + j \cdot 10\, e^{-j2z}\,\hat{a}_y$

$= 5\, e^{-j2z}\,\hat{a}_x + 10\angle 90^\circ\, e^{-j2z}\,\hat{a}_y$

$= 5\, e^{-j2z}\,\hat{a}_x + 10\, e^{j(90^\circ - 2z)}\,\hat{a}_y$

$\vec{E} = 5\cos(\omega t - 2z)\,\hat{a}_x + 10\cos(\omega t - 2z + 90^\circ)\,\hat{a}_y$

at origin

$\vec{E}(x,y,z,t) = \vec{E}(0,0,0,t)$

$= 5\cos(\omega t)\,\hat{a}_x + 10\cos(\omega t + 90^\circ)\,\hat{a}_y$

$= 5\cos(\omega t)\,\hat{a}_x - 10\sin(\omega t)\,\hat{a}_y$

not putting diff ωt in this eq^n

$\vec{E} = \text{Re}\left[(\vec{E}_s)\, e^{j\omega t}\right]$

$= \text{Re}\left[(5e^{-j2z}\,\hat{a}_x)\, e^{j\omega t}\right] + \text{Re}\left[\{10e^{j(90^\circ - 2z)}\,\hat{a}_y\} e^{j\omega t}\right]$

$= 5\,\text{Re}\left[e^{j(\omega t - 2z)}\right]\hat{a}_x + 10\,\text{Re}\left[e^{j(\omega t - 2z + 90^\circ)}\right]\hat{a}_y$

$\vec{E}(x, y, z, t) = 5\cos(\omega t - 2z)\,\hat{a}_x + 10\cos(\omega t - 2z + 90^\circ)\,\hat{a}_y$ V/m

11. An EM wave travels in free space with the electric field component $\vec{E} = (10\,\hat{a}_y + 5\,\hat{a}_z)\cos(\omega t + 2y - 4z)$ V/m. Find: (a) ω and λ (b) The magnetic field component (c) The time average power in the wave. *[2069 Chaitra]*

Soln÷

$$\vec{E} = (10\,\hat{a}_y + 5\,\hat{a}_z)\cos(\omega t + 2y - 4z)\ \text{V/m}$$

comparin with

$$\vec{E} = E_0 \cos(\omega t - \beta_x x - \beta_y y - \beta_z z + \phi)$$

$$\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2} = \sqrt{0^2 + (-2)^2 + (4)^2}$$

$\beta_x = -2 \qquad \beta_z = 4 \qquad\qquad = 4.472$ rad/m

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0} =$$

a, $\omega = \dfrac{\beta}{\sqrt{\mu_0 \varepsilon_0}} = \dfrac{4.472}{\sqrt{\mu_0 \varepsilon_0}} = 1.32 \times 10^9$ rad/s

$= 1.34$ Grad/s

$\beta = \dfrac{2\pi}{\lambda}$, $\lambda = \dfrac{2\pi}{\beta} = 1.405$ m

(b) $\vec{H_0} = ?$

$$\vec{H_0} = \frac{1}{\omega\mu}\,\vec{\beta} \times \vec{E_0} = \frac{1}{\omega\mu_0}\,\vec{\beta} \times \vec{E_0}$$

free space

$$\vec{\beta} \times \vec{E_0} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & -2 & 4 \\ 0 & 10 & 5 \end{vmatrix} = -50\hat{a}_z$$

$$\vec{H_0} = \frac{1}{\underset{(1.34 \times 10^9)}{\omega \times \mu_0}} \times (-50\hat{a}_z) = -0.02965\,\hat{a}_x \text{ A/m}$$

$$\vec{H} = \vec{H_0} \cos(\omega t + 2y - 4z)$$

$$= -0.02965 \cos(\omega t + 2y - 4z)\,\hat{a}_x \text{ V/m}$$

(c)

$$\langle S_{avg} \rangle = \frac{1}{2} Re[\vec{E_s} \times \vec{H_s}^*] \text{ W/m}^2$$

For free space

$$\langle S \rangle = \frac{E_0^2 \hat{a}_\beta}{2\eta}$$

$$\hat{a}_\beta = \frac{\vec{\beta}}{\beta} = \frac{-2\hat{a}_y + 4\hat{a}_z}{4.472} = -0.447\hat{a}_y + 0.894\hat{a}_z$$

$$E_0 = \sqrt{(10)^2 + (5)^2} = 11.18 \text{ V/m}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.730$$

$$\langle S \rangle = \frac{(11.18)^2 \times (-0.447\hat{a}_y + 0.894\hat{a}_z)}{2 \times 376.730}$$

$$= -0.073\hat{a}_y + 0.147\hat{a}_z \text{ W/m}^2$$

12. Consider a 30-MHz uniform plane wave propagating in free space and given by the electric field vector $\vec{E} = 5(\hat{a}_x + \sqrt{3}\,\hat{a}_y)\cos\,[6\pi\times10^7 t - 0.05\pi(3x - \sqrt{3}y + 2z)]$ V/m. Find β, direction of propagation, and $\vec{H}$.

soln ÷

$$\vec{E} = 5\,(\hat{a}_x + \sqrt{3}\,\hat{a}_y)\cos\,[6\pi\times10^7 t - 0.05\pi\,(3x - \sqrt{3}y + 2z)]$$
V/m

comparing with

$$\vec{E} = E_0 \cos\,(\omega t - \beta_x x - \beta_y y - \beta_z z + \phi)$$

$\beta_x = 0.05\pi\times 3 = 0.15\pi$, $\beta_y = -0.05\pi\times\sqrt{3}$
$= -0.08\pi$

$\beta_z = 0.05\pi\times 2 = 0.1\pi$

$$\vec{\beta} = 0.15\pi\,\hat{a}_x - 0.08\pi\,\hat{a}_y + 0.1\pi\,\hat{a}_z$$

$$\beta = \sqrt{(0.15\pi)^2 + (-0.08\pi)^2 + (0.1\pi)^2}$$

$= 0.62$ rad/m

$$\hat{a}_\beta = \frac{\vec{\beta}}{\beta} = 0.75\,\hat{a}_x - 0.4329\,\hat{a}_y + 0.5\,\hat{a}_z$$

direction of propagation is given by this vector //

$$\vec{H_0} = \frac{1}{\omega\mu} \vec{\beta} \times \vec{E_0} = \frac{1}{\omega\mu_0} \vec{\beta} \times \vec{E_0}$$

$$\vec{\beta} \times \vec{E_0} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0.15\pi & -0.08\pi & 0.1\pi \\ 5 & 5\sqrt{3} & 0 \end{vmatrix}$$

$$= -2.72\,\hat{a}_x + 1.5705\,\hat{a}_y + 5.4407\,\hat{a}_z$$

$$\vec{H_0} = \frac{1}{6\pi \times 10^7 \times \mu_0} \left(-2.72\hat{a}_x + 1.5705\,\hat{a}_y + 5.4407\,\hat{a}_z \right)$$

$$= -0.01148\,\hat{a}_x + 6.6302 \times 10^{-3}\,\hat{a}_y + 0.02296\,\hat{a}_z \quad A/m$$

The expression for magnetic field is therefore given as ÷

$$\vec{H} = \vec{H_0} \cos\left[6\pi \times 10^7 t - 0.05\pi\,(3x - \sqrt{3}y + 2z)\right]$$

$$= \left(-0.01148\,\hat{a}_x + 6.6302 \times 10^{-3}\,\hat{a}_y + 0.02296\,\hat{a}_z\right)$$

$$\cos\left[6\pi \times 10^7 t - 0.05\pi\,(3x - \sqrt{3}y + 2z)\right]$$

An EM wave travels in free space with the electric field component $\vec{E}_s = 100\, e^{j(0.866y+0.5z)}\, \hat{a}_x$ V/m. Determine: (a) ω and λ (b) The magnetic field component (c) The time average power in the wave.

Soln:

$$\vec{E}_s = 100\, e^{j(0.866y+0.5z)}\, \hat{a}_x \quad \text{— (i)}$$

Comparing eqn with

$$\vec{E}_s = E_0\, e^{j(\beta_x x + \beta_y y + \beta_z z)}\, \hat{a}_x$$

$E_0 = 100$, $\beta_x = 0$, $\beta_y = 0.866$, $\beta_z = 0.5$

$$\beta = \sqrt{0^2 + 0.866^2 + 0.5^2} = 1$$

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0} \qquad \omega = \frac{\beta}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8 \text{ rad/s}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1} = 6.283 \text{ m}$$

b) $$\vec{H} = \frac{1}{\omega\mu}\vec{\beta}\times\vec{E}_0 = \frac{1}{\omega\mu_0}\vec{\beta}\times\vec{E}_0$$

$$\vec{\beta}\times\vec{E}_0 = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0.866 & 0.5 \\ 100\, e^{j(0.866y+0.5z)} & 0 & 0 \end{vmatrix}$$

$$= 50\, e^{j(0.866y+0.5z)}\, \hat{a}_y - 86.6\, e^{j(0.866y+0.5z)}\, \hat{a}_z$$

$$\vec{H_s} = \frac{1}{3\times10^8 \times \mu_0}\left[e^{j(0.866y+0.5z)} \left(50\hat{a}_y - 86.6\,\hat{a}_z \right) \right]$$

$$= (0.1326\,\hat{a}_y - 0.2297\,\hat{a}_z)\, e^{j(0.866y+0.5z)} \quad A/m$$

(c) The time average power is :-

$$\langle S \rangle_{avg} = \frac{1}{2} Re\left[\vec{E_s} \times \vec{H_s}^* \right] = \frac{E_0^2\, \hat{a}_\beta}{2\eta}$$

$$E_0 = 100,\quad \eta = 377,\quad \hat{a}_\beta = \frac{\vec{\beta}}{\beta} = \frac{0.866\hat{a}_y + 0.5\hat{a}_z}{1}$$

$$= 0.866\,\hat{a}_y + 0.5\,\hat{a}_z$$

$$\langle S \rangle = \frac{100^2}{2\times377}(0.866\,\hat{a}_y + 0.5\,\hat{a}_z)$$

$$= 11.4854\,\hat{a}_y + 6.6312\,\hat{a}_z \quad W/m^2$$

14. In a lossless dielectric for which $\eta = 60\pi$, $\mu_r = 1$ and $\vec{H} = -0.1 \cos(\omega t - z)\,\hat{a}_x + 0.5 \sin(\omega t - z)\,\hat{a}_y$ A/m, calculate ε_r, ω, and $\vec{E}$. [2072 Chaitra]

Solution:

Soln :-

$$\vec{H} = -0.1\cos(\omega t - 2)\,\hat{a}_x + 0.5\sin(\omega t - 2)\,\hat{a}_y \text{ A/m}$$

$$\beta = 1 \text{ rad/m} \quad , \mu_r = 1 \quad , \eta = 60\pi$$

For lossless dielectric

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 377 \times \frac{1}{\sqrt{\varepsilon_r}}$$

$$\sqrt{\varepsilon_r} = \frac{377}{60\pi}$$

$$\varepsilon_r = \left(\frac{377}{60\pi}\right)^2 = 4$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$\omega = \frac{\beta}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0 \times 1 \times 4}} = 1.5 \times 10^8 \text{ rad/s}$$

For electric field

$$\sigma = 0 \quad , \nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} = \cancel{\sigma \vec{E}}^{\,0} + \varepsilon\frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \nabla \times \vec{H} = \varepsilon\frac{\partial \vec{E}}{dt}$$

or $$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon}(\nabla \times \vec{H})$$

$$\vec{E} = \frac{1}{\varepsilon} \int (\nabla \times \vec{H})\, dt$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -0.1 \cos(\omega t - z) & 0.5 \sin(\omega t - z) & 0 \end{vmatrix}$$

$$= .0.5 \cos(\omega t - z)\, \hat{a}_x - 0.1 \sin(\omega t - z)\, \hat{a}_y$$

$$\vec{E} = \frac{1}{\varepsilon} \int \left[0.5 \cos(\omega t - z)\, \hat{a}_x - 0.1 \sin(\omega t - z)\, \hat{a}_y \right] dt$$

$$= \frac{1}{\varepsilon_0 \varepsilon_r} \int 0.5 \cos(\omega t - z)\, dt\, \hat{a}_x - \frac{0.1}{\varepsilon_0 \varepsilon_r} \int \sin(\omega t - z)\, \hat{a}_y\, dt$$

$$= \frac{0.5}{\varepsilon_0 \varepsilon_r} \cdot \frac{1}{\omega} \sin(\omega t - z)\, \hat{a}_x + \frac{0.1}{\varepsilon_0 \varepsilon_r} \cdot \frac{1}{\omega} \cos(\omega t - z)\, \hat{a}_y$$

$$\therefore \vec{E} = 94.119 \sin(\omega t - z)\, \hat{a}_x + 18.823 \cos(\omega t - z)\, \hat{a}_y \ \text{V/m}$$

Alternative Method

Given, $\vec{H}_1 = -0.1 \cos(\omega t - z)\, \hat{a}_x$ A/m

$\vec{H}_2 = 0.5 \sin(\omega t - z)\, \hat{a}_y$ A/m

The direction of $\vec{E}_1$ is $\hat{a}_y$. For amplitude of $\vec{E}_1$, we use

$$\eta = \frac{E_{o1}}{H_{o1}}$$

or, $E_{o1} = 60\pi \times 0.1$ [$\because \eta = 60\,\pi$ given]

or, $E_{o1} = 18.84$ V/m

$\therefore \vec{E}_1 = 18.84 \cos(\omega t - z)\, \hat{a}_y$

The direction of $\vec{E}_2$ is $\hat{a}_x$. For amplitude of $\vec{E}_2$, we use

$$\eta = \frac{E_{o2}}{H_{o2}}$$

or, $E_{o2} = 60\pi \times 0.5 = 94.24$ V/m

$\therefore \vec{E}_2 = 94.24 \sin(\omega t - z)\, \hat{a}_x$

Hence, $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= 18.84 \cos(\omega t - z)\, \hat{a}_y + 94.24 \sin(\omega t - z)\, \hat{a}_x \text{ V/m}$$

19. A time harmonic uniform plane wave $\vec{E}$ (x, y, z, t) with polarization in $\hat{a}_z$ direction & frequency 150 MHz is moving in free space in negative y direction & has maximum amplitude 2 V/m. Determine:

150 mHz

a. **The angular frequency**

b. **Phase constant β**

c. **Expression for $\vec{E}$ (x, y, z, t)**

d. **Expression for $\vec{H}$ (x, y, z, t)** *[2068 Shrawan]*

soln:- $f = 150$ mHz $\quad \vec{E} = E_0 \cos(\omega t + \beta y)\, \hat{a}_z$

$E_0 = 2$ V/m

$-$ve y direction move

(a) $\omega = 2\pi f = 2\pi \times 150 \times 10^6 = 300\pi \times 10^6$ rad/s

b) $\beta = \dfrac{2\pi}{\lambda} \qquad \lambda = \dfrac{v}{f} = \dfrac{3\times 10^8 \text{ (free space)}}{150\times 10^6} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$= 2$ m

$\beta = \frac{2\pi}{2} = \pi$ rad/m

polarization $\hat{a}_z$

moving in free space with $-y$ direction so $+y$ here

(c) $\vec{E}(x,y,z,t) = E_0 \cos(\omega t + \beta y)\,\hat{a}_z$

$= 2\cos(\omega t + \beta y)\,\hat{a}_z$

(d) $H(x,y,z,t) = H_0 \cos(\omega t + \beta y)$

$H_0 = \frac{E_0}{\eta} = \frac{2}{377} = 5.305 \times 10^{-3}$ A/m

$\therefore H(x,y,z,t) = 5.305\times10^{-3} \cos(\omega t + \beta y)$

$\vec{P} = \vec{E} \times \vec{H}$

$(-\hat{a}_y) = \hat{a}_z \times (-\hat{a}_x)$

x y y

$\therefore \vec{H}(x,y,z,t) = 5.305\times10^{-3} \cos(\omega t + \beta y)\,(-\hat{a}_x)$

$= -5.305\times10^{-3} \cos(\omega t + \beta y)\,\hat{a}_x$ A/m

20. For a non-magnetic material having $\varepsilon_r = 2.25$ and $\sigma = 10^{-4}$ mho/m. Find the numeric value at 5 MHz for

a. the loss tangent

b. the attenuation constant

c. phase constant

d. the intrinsic impedance

[2068 Baishakh]

Sol^n ÷

(a) loss tangent ($\tan\theta$) $= \dfrac{\sigma}{\omega\varepsilon} = \dfrac{\sigma}{\omega\varepsilon_0\varepsilon_r}$

$$\omega = 2\pi f = 2\pi \times 5\times10^{6} = 3.1415 \times 10^{7} \text{ rad/s}$$

$$\tan\theta = \frac{10^{-4}}{3.1415\times10^{-7} \times 8.854\times10^{-2} \times 2.25} = 0.1597$$

(b) For non magnetic material, $\mu_r = 1$ so, $\mu = \mu_0$

so, $$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\alpha = 2\pi \times 5\times10^{6} \left(\frac{4\pi\times10^{-7} \times 2.25 \times 8.85\times10^{-12}}{2}\left(\left(1 + (0.1597)^2\right)^{1/2} - 1\right)\right)^{1/2}$$

$$= 0.011275 \text{ Np/m}$$

(c) $$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$= 0.15755 \text{ rad/m}$$

This can be done by another way as

Propagation constant $(\gamma) = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\therefore \gamma = \sqrt{j2\pi\times5\times10^{6}\times4\pi\times10^{-7}(1\times10^{-4}+j3.1415\times10^{7}\times10^{6}\times2.25\times8.854\times10^{-12})}$

$= \sqrt{39.478\angle90^\circ \times 6.335119\times10^{-4}\angle80.91^\circ}$

$= \sqrt{0.02500978} \angle 170.9178^\circ$

$= \sqrt{0.02500978} \angle \dfrac{170.9178^\circ}{2}$

$= 0.158144\angle85.458^\circ = 0.0125 + j\,0.15755 \text{ m}^{-1}$

Hence, attenuation constant $(\alpha) = 0.0125$ Np/m

Phase constant $(\beta) = 0.15755$ rad/m

(d) $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$= \sqrt{\dfrac{j \times 2\pi \times 5\times10^{6} \times \mu_0 \times 1}{10^{-4} + j \times 2\pi \times 5\times10^{6} \times 2.25 \times \varepsilon_0}}$

$= \sqrt{62288.029 \angle 9.07^\circ}$

$= 249.57 \angle 4.535^\circ \ \Omega$

21. **At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\varepsilon_o$, $\mu = 2.1\mu_o$ and $\sigma = 0.08$ S/m. It $\vec{E}_s = 6e^{-\gamma x}\hat{a}_z$ V/m, compute: (a) propagation constant (b) wavelength (c) $\vec{H}_s$.** *[2074 Chaitra]*

Solution:

$$\gamma = \sqrt{j\omega\mu\,(\sigma + j\omega\varepsilon)}$$

$$= \sqrt{[j(\pi\times10^{8})\times(2.6389\times10^{-6})][0.08 + j(\pi\times10^{8})(3.1874\times10^{-11})]}$$

$$= 8.1744 \angle 48.5672^\circ = 5.4093 + j6.1286 \ \ /\text{m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.1286} = 1{,}0252 \text{ m}$$

The amplitude of $\vec{H}_s$ is calculated as

$$H_o = \frac{E_o}{\eta} = \frac{6}{\eta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{829.0467\angle 90^\circ}{0.0806\angle 7.1345^\circ}} = 101.4196\angle 41.43^\circ\ \Omega$$

$$\therefore H_o = \frac{6}{101.4196\angle 41.43^\circ} = 0.0591\angle -41.43^\circ \text{ A/m}$$

The direction of $\vec{H}_s$ is calculated as

$\vec{P} = \vec{E} \times \vec{H}$

$+\hat{a}_x = +\hat{a}_z \times ?$

Therefore, the direction of $\vec{H}_s$ is $-\hat{a}_y$

Now, the expression for $\vec{H}_s$ is

$$\vec{H}_s = 0.0591\angle{-41.43^\circ}\, e^{-\gamma x}(-\hat{a}_y)$$

$$= -0.0591\, e^{-j41.43^\circ}\, e^{-(5.4093 + j6.1286)x}\, \hat{a}_y$$

$$= -0.0591\, e^{-5.4093x}\, e^{-j(6.1286x + 41.43^\circ)}\, \hat{a}_y \text{ A/m}$$

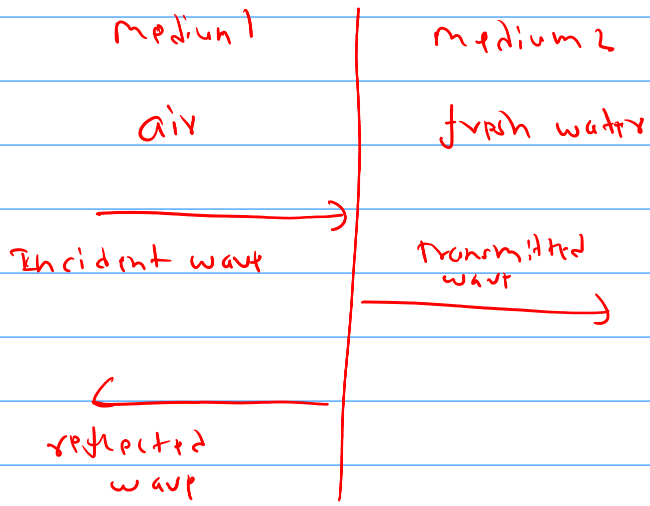
22. Find the reflection coefficient for the interface between air and fresh water ($\varepsilon = 81\varepsilon_o$, $\sigma \cong 0$), in case of normal incidence.

[2072 Chaitra]

Solution:

soln:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$



$$\eta_1 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

For air, $\sigma = 0$,

$\mu = \mu_0$, $\varepsilon = \varepsilon_0$

so $\eta_1 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 376.734\ \Omega$

For fresh water

$\sigma \approx 0$, $\varepsilon = 81\varepsilon_0$, $\mu \approx \mu_0$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu_0}{81\,\varepsilon_0}} = 41.859\,\Omega$$

$\eta_{(r)}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{41.589 - 376.734}{41.859 + 376.734} = -0.8$$