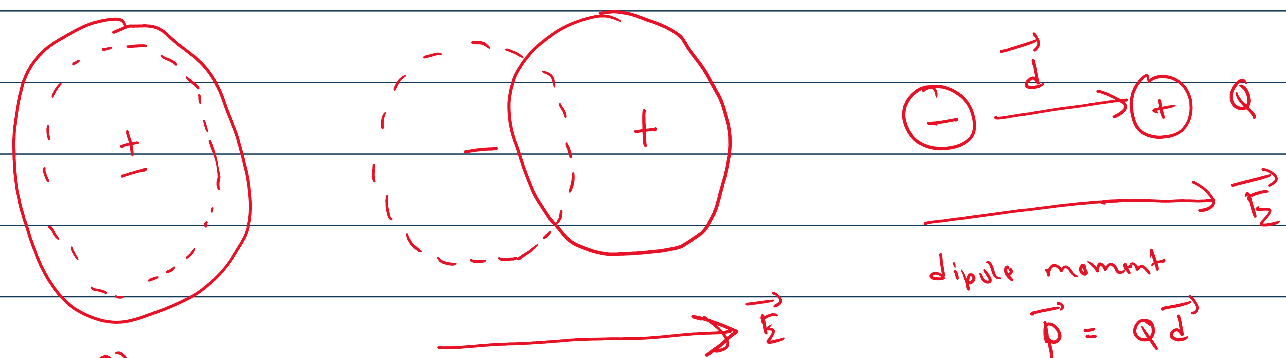
Dielectrics and Capacitance

Dielectrics

→ A dielectric is an insulating material that does not conduct electricity but can support electrostatic fields. Unlike conductors, where free electrons move easily, dielectrics have bound electrons that cannot move freely. However when placed in external electric field, the charges within the dielectric rearrange slightly leading to polarization.

Polarization

→ polarization refers to the alignment of electric dipoles within a dielectric material in response to an external electric field. It occurs because the positive and negative charges shift slightly in opposite directions, creating dipoles within the material.



(a) Unpolarized atom

~~pla~~ polarized atom (b)

(c)

the positive charge is displaced in direction of $\vec{E}$

Upon the application of electric field $\vec{E}$ by the force $\vec{F}_+ = Q\vec{E}$

Upon the application of electric field $\vec{E}$ ^ by the force $\vec{F}_+ = Q\vec{E}$ while the negative charge is displaced in the opposite direction by the force $\vec{F}_- = Q\vec{E}$. A dipole results from the displacement of charges, and the atom is said to be "polarized". This phenomenon is polarization. The direction of polarization is from '−' to '+' charges i.e in the direction of $\vec{E}$.

$\vec{p}$ (dipole moment) $= Q\vec{d}$ (C-m)

where, Q = positive one of the two bound charges composing the dipole

$\vec{d}$ = vector from −ve to +ve charge

let n be the number of dipoles per unit volume, then there are $n\Delta V$ dipoles in a volume ΔV. The total dipole moment is :-

$$\boxed{\vec{p}_{total} = \sum_{i=1}^{n\Delta V} \vec{p}_i}$$

The polarization $\vec{P}$ is defined as the dipole moment per unit volume

$$\boxed{\vec{P} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_{i=1}^{n\Delta V} \vec{p}_i} \quad (C/m^2)$$

Relationship between $\vec{D}$, $\vec{E}$ and $\vec{P}$

For isotropic dielectric materials, the linear relationship between P and E

$$\boxed{\vec{P} = \chi_e \varepsilon_0 \vec{E} \quad \text{— (i)}}$$

where χ_e (chi) is a dimensionless quantity called electric susceptibility of the material

Now,

D is now defined in more general terms, as we are taking polarization in accont,

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}} \quad \text{— (ii)}$$

Using (i) and (ii)

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = (\chi_e + 1)\varepsilon_0 \vec{E}}$$

which is in the form of $\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}$

$\therefore$ $\boxed{\varepsilon_r = \chi_e + 1}$

ε_r is relative permittivity or dielectric constant of the materials.

Dielectrics

- Isotropic (direction of $\vec{E}$ = direction of $\vec{P}$)
  - linear
    - $\vec{E}$ and $\vec{P}$ linearly related
  - non-linear
- Non isotropic (direction of $\vec{E} \neq$ direction of $\vec{P}$)
  - linear
  - non-linear

Most engineering dielectrics are linear and isotropic

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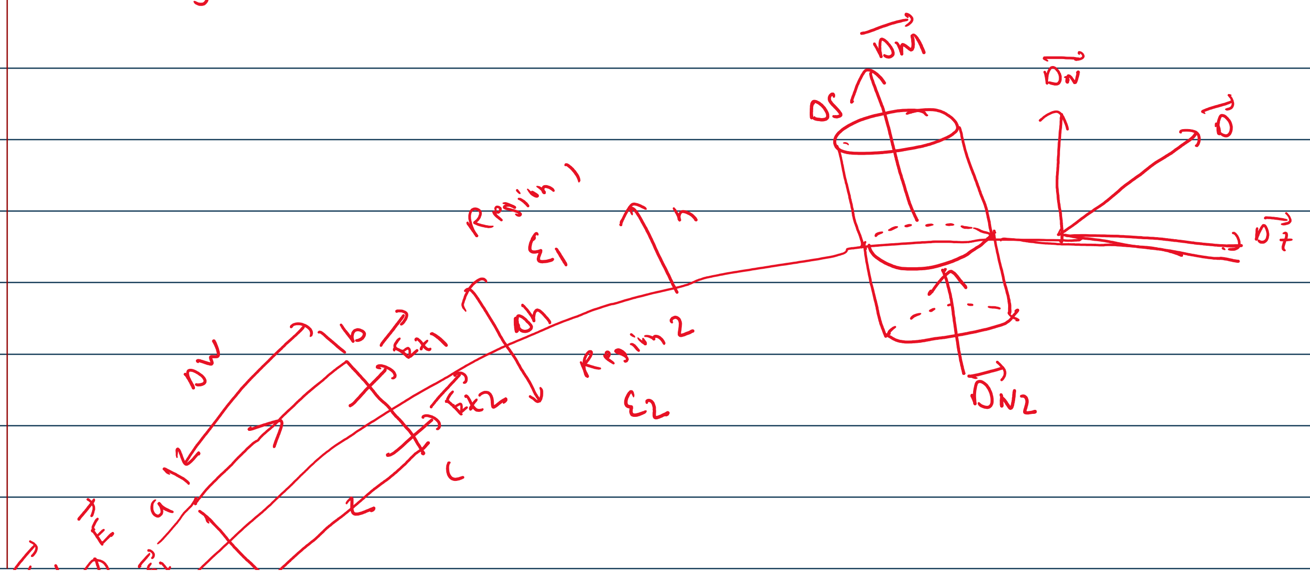
Boundary Conditions for Perfect Dielectric Materials



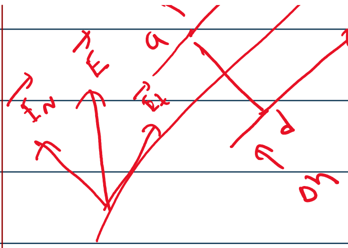


Fig : boundary condition between two perfect dielectric materials.

Consider the interface between two dielectrics having permittivities ϵ_1 and ϵ_2 and occupying the region 1 and 2 as shown in figure.

For tangential components

$$\oint_{abcda} \vec{E}\cdot\vec{dl} = 0$$

or $$\int_a^b \vec{E}\cdot\vec{dl} + \int_b^c \text{''} + \int_c^d \text{''} + \int_d^a \vec{E}\cdot\vec{dl} = 0$$

or $$E_{t1}\,\Delta w + 0 + (-E_{t2}\,\Delta w) + 0 = 0$$

$(\Delta h \to 0)$ $(\Delta h \to 0)$

$$\therefore \boxed{E_{t1} = E_{t2}} \quad \text{— (i)}$$

Which shows that the tangential component of the electric field intensity is continuous across the boundary.

Equation (i) may be written as :

$$\frac{D_{t1}}{\epsilon_1} = \frac{D_{t2}}{\epsilon_2}$$

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or, $$\boxed{D_{t1} = \frac{\epsilon_1}{\epsilon_2} D_{t2}}$$

which means that the tangential component of electric flux density is discontinuous across the boundary.

For normal component

$$\oint_S \vec{D} \cdot \vec{ds} = \Delta Q$$

or, $$\int_{top} \vec{D} \cdot \vec{ds} + \int_{bottom} \vec{D} \cdot \vec{ds} + \int_{side} \vec{D} \cdot \vec{ds} = \Delta Q$$

or, $$D_{N1} \Delta S - D_{N2} \Delta S + 0 = \rho_s \Delta S$$

$$(\Delta h \to 0)$$

or $$\boxed{D_{N1} - D_{N2} = \rho_s}$$

If no free charge exist at interface (i.e charges are not deliberately placed there), $\rho_s = 0$

$$D_{N1} - D_{N2} = 0$$

or, $$\boxed{D_{N1} = D_{N2}} \quad \text{—— (ii)}$$

This shows that the normal component of electric flux density is

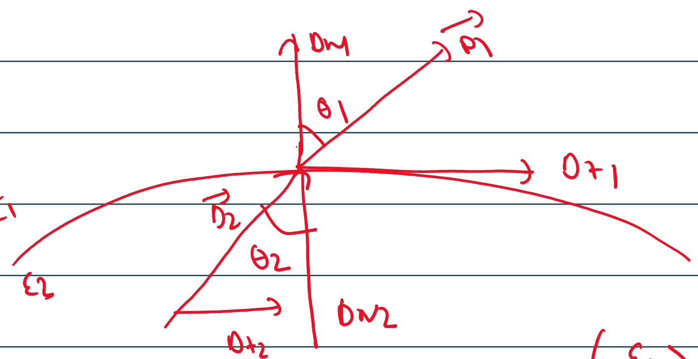
This shows that the normal component of electric flux density is continuous

Equation (ii) may be written as

$$\varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}$$

or, $$\boxed{E_{N1} = \frac{\varepsilon_2}{\varepsilon_1} E_{N2}}$$

Which means that the normal comp of electric field intensity is discontinuous.



$(\varepsilon_1 > \varepsilon_2)$

refraction of D at a dielectric interface

We know

$$D_{N1} = D_{N2}$$

or $$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

or $$\frac{D_1}{D_2} = \frac{\cos\theta_2}{\cos\theta_1} \quad \text{—— (i)}$$

tangential comp discontinuous

$$D_{t1} = \frac{\varepsilon_1}{\varepsilon_2} D_{t2}$$

or $$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

or $$\frac{D_1 \sin\theta_1}{D_2 \sin\theta_2} = \frac{\epsilon_1}{\varepsilon_2} \quad \text{— (ii)}$$

From (ii) and (i) (put D_1 and D_2 from (i))

$$\frac{\cos\theta_2 \sin\theta_1}{\cos\theta_1 \sin\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

or $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

$\varepsilon_1 > \varepsilon_2$ $\tan\theta_1 > \tan\theta_2$, $\theta_1 > \theta_2$

mag of $\vec{D}$ in region 2 is :

$$D_2 = D_1 \sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1}$$

$$E_2 = E_1 \sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \cos^2\theta_1}$$

Capacitance

→ A capacitor is a device that stores electric potential energy

→ A capacitor is a device that stores electric potential energy and electric charge.

Fig: Two oppositively charge conductors M_1 and M_2 surrounded by a uniform dielectric

→ Consider two conductors M_1 and M_2 placed inside a homogeneous dielectric (an insulating material)

→ M_2 carries a total positive charge (Q) while M_1 carries an equal negative charge $(-Q)$

→ Since no other charges exist in the system, the total charge of the system remains zero.

The capacitance of this two-conductor system is :-

$$C = \frac{Q}{V_0}$$

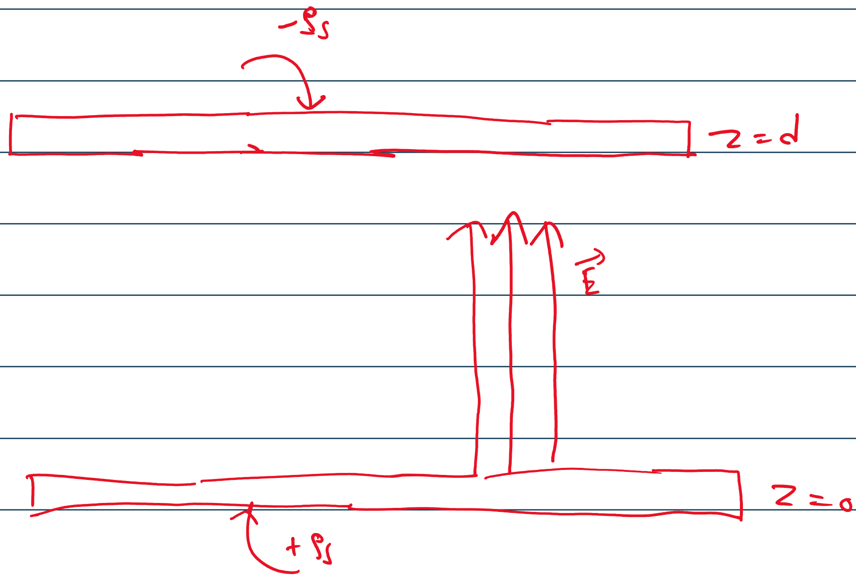
where,

Q = charge on either conductor

V_0 = Potential difference between M_1 and M_2

$$C = \frac{\oint_S \varepsilon \vec{E} \cdot d\vec{s}}{-\int \vec{E} \cdot d\vec{\ell}}$$

parallel plate capacitor



$$\boxed{C = \frac{\varepsilon S}{d} = \frac{Q}{V_0}}$$

$S \rightarrow$ surface area of each conductor

total energy stored in capacitor is :-

$$\boxed{W_E = \frac{1}{2} \int_{vol} \varepsilon E^2 \, dv = \frac{1}{2} \frac{\rho_s^2}{\varepsilon} S d}$$