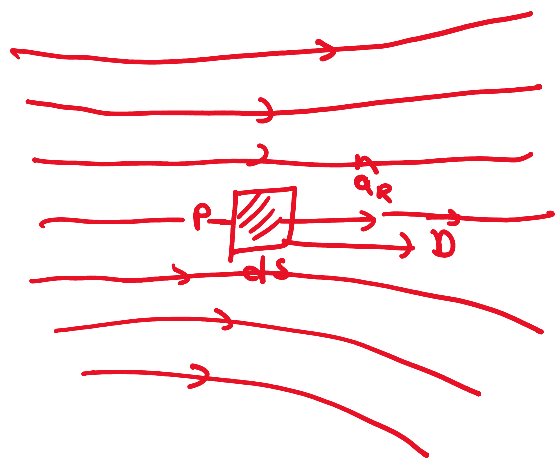
Chapter-3

Thursday, February 13, 2025 2:21 PM

Electric flux

→ Electric flux (ψ) through a surface is defined as the amount of electric field lines that pierces the surface.



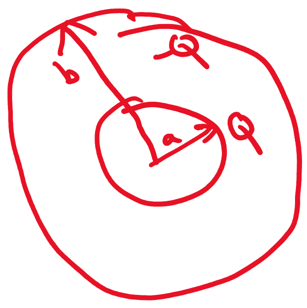
If at point P, the lines flux have direction of unit vector $\hat{a}_R$ and if an amount of flux $d\psi$ crosses the differential area ds, which is normal to $\hat{a}_R$, then electric flux density at P is :-

$$\boxed{\vec{D} = \frac{d\psi}{ds}\hat{a}_R \; (C/m^2)}$$

Thus, relation between electric flux density and electric field intensity is :-

$$\boxed{\begin{array}{l} \vec{D} = \varepsilon_0 \vec{E} \text{ (for free space)} \\ \vec{D} = \varepsilon \vec{E} \text{ (For any other medium)}; \; \varepsilon = \varepsilon_0 \varepsilon_r \end{array}}$$

$$\vec{D} = \frac{d\psi}{ds} \quad \text{or} \quad \vec{D}\cdot\vec{ds} = d\psi \quad \text{or} \quad \psi = \int_s \vec{D}\cdot\vec{ds}$$



$\psi = Q$ —— (i) (Faraday Equation)

$\vec{D} = \frac{\psi}{A} = \frac{\text{Flux}}{\text{Area}}$

For inner sp

$\vec{D}_{in} = \frac{\psi}{4\pi r^2} = \frac{\psi}{4\pi a^2}\hat{a}_r = \frac{Q}{4\pi a^2} a_r^2$

(from (i))

Electric field intensity due to point charge

$$(\vec{E}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{a}_r$$

If radius of sp is 'a'

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{a^2}\hat{a}_r$$

Now,

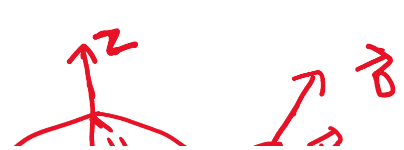
$$\frac{\vec{D}}{\vec{E}} = \frac{\frac{Q}{4\pi a^2}}{\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{a^2}} = \frac{1}{\varepsilon_0} \quad \therefore \boxed{\vec{D} = \varepsilon_0\vec{E}} \text{ proved \#}$$

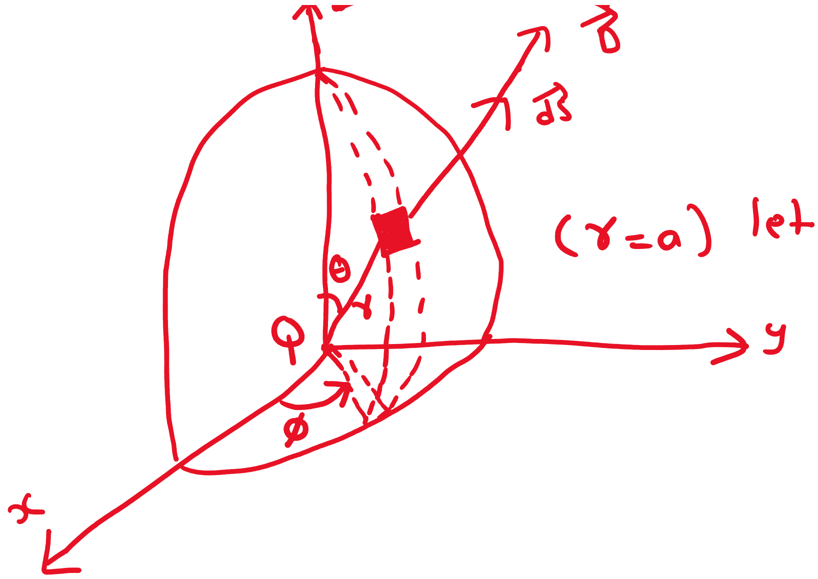
Gauss law (IOE)

→ It states that the electric flux through any closed surface is equal to the total charge enclosed by the surface.

i.e $\boxed{\psi = Q}$

$\psi = Q$ —— (i) $\oint_s \vec{D}\cdot\vec{dS} = Q$ —— (ii)





Consider a point charge Q at the origin of spherical co-ordinate system, radius of sphere being a. The electrical flux density $\vec{D}$ is everywhere normal to the spherical surface and has a constant magnitude at every point on it.

The electric field intensity of the point charge is :-

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \quad \text{(spherical system)}$$

$$\therefore \vec{D} = \varepsilon_0 \vec{E} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

At surface of the sphere, $r = a$

$$\text{so, } \vec{D} = \frac{Q}{4\pi a^2}\,\hat{a}_r$$

In spherical system $\vec{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$
$= a^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

The total electric flux passing through the closed surface can be obtained as :-

$$\therefore \oint \vec{D}\cdot\vec{ds} = \oint \left(\frac{Q}{\ldots}\,\hat{a}_r\right)\left(a^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r\right)$$

can be obtained as :-

$$\psi = \oint_s \vec{D}\cdot\vec{ds} = \oint_s \left(\frac{Q}{4\pi a^2}\hat{a}_r\right)\left(a^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r\right)$$

$$(\hat{a}_r \cdot \hat{a}_r) = 1$$

$$= \oint_s \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi = \frac{Q}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

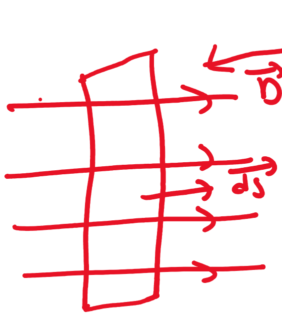
($\theta \to 0$ to π and $\phi \to 0$ to 2π in spherical)

$$= \frac{Q}{4\pi}\int_{\phi=0}^{2\pi} d\phi \times 2 = \frac{Q}{\cancel{2\pi}} \times \cancel{2\pi} = Q$$

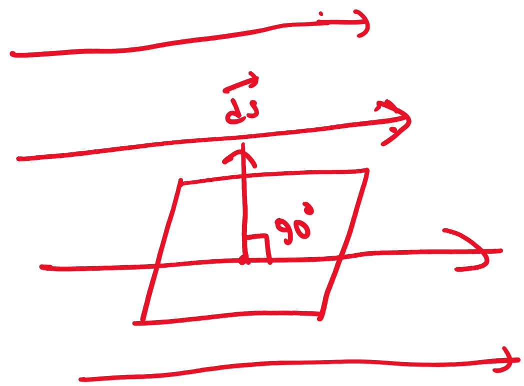
$$\therefore \boxed{\psi = Q}$$ proved

Criteria of Gauss law

① $\vec{D}$ $\vec{ds}$ either normal or tangential



$\vec{D}\cdot\vec{ds} = D\,ds\cos 0^\circ$
$= D\,ds$



in case of tangential
$\vec{D}\cdot\vec{ds} = D\,ds\cos 90^\circ = 0$

② $\vec{D}$ value is constant or zero
(throughout the electric field must be constant)

↓ Gauss law priority

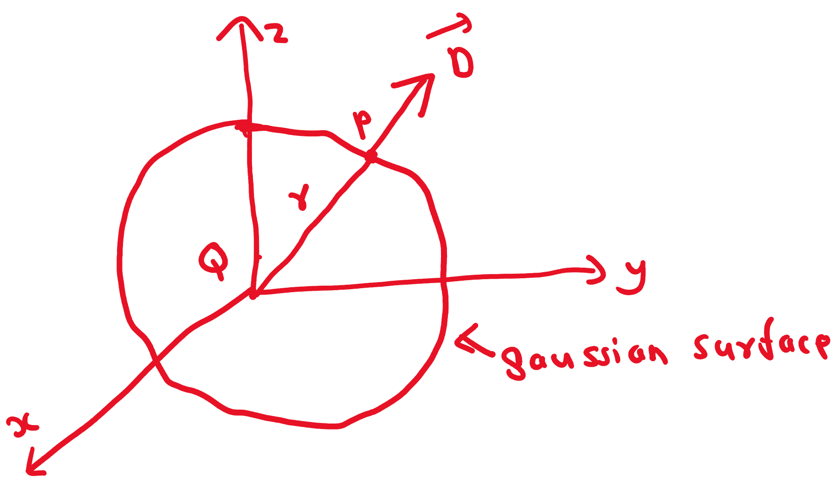
Applications of Gauss's law (chap-2 ma shodeko lamo garna bhonda yo wala para le

Applications of Gauss's law (chap-2 ma shodeko lamo garna bhanda yo wala para le derive handine exam ma)

① Field due to a point charge

→ consider point charge Q at origin of spherical system. To determine $\vec{D}$ at a point P, we choose a gaussian surface to be a spherical surface containing that point. $\vec{D}$ is normal everywhere to the surface and has same value at all points on the surface.

(so $\vec{D}$ is used in Gauss law because $\vec{E}$ doesn't satisfy criteria)



From Gauss' law

$$\psi = \oint_s \vec{D} \cdot \vec{ds} = Q \quad \text{—(i)}$$

along r direction

(surface ⊥ r which is $\hat{a}_r$ too)

we have,

$$\vec{D} = D_r \hat{a}_r \,, \quad \vec{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r \quad \text{(spherical)}$$

$$\therefore \oint_s \vec{D} \cdot \vec{ds} = \oint_s (D_r \hat{a}_r)(r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r)$$

$$= D_r r^2 \oint_s \sin\theta \, d\theta \, d\phi$$

$$= D_r r^2 \int^{2\pi} d\phi \int^{\pi} \sin\theta \, d\theta$$

$$= D_r r^2 \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta \, d\theta$$

$$= D_r \times r^2 \times 2\pi \times 2$$

$$= D_r \, 4\pi r^2$$

From (i)

$\oint \vec{D} \cdot \vec{dS} = Q$ or, $D_r 4\pi r^2 = Q$ $\therefore D_r = \dfrac{Q}{4\pi r^2}$

Thus $\vec{D} = \dfrac{Q}{4\pi r^2} \hat{a}_r$ $\quad (\vec{D} = D_r \hat{a}_r)$

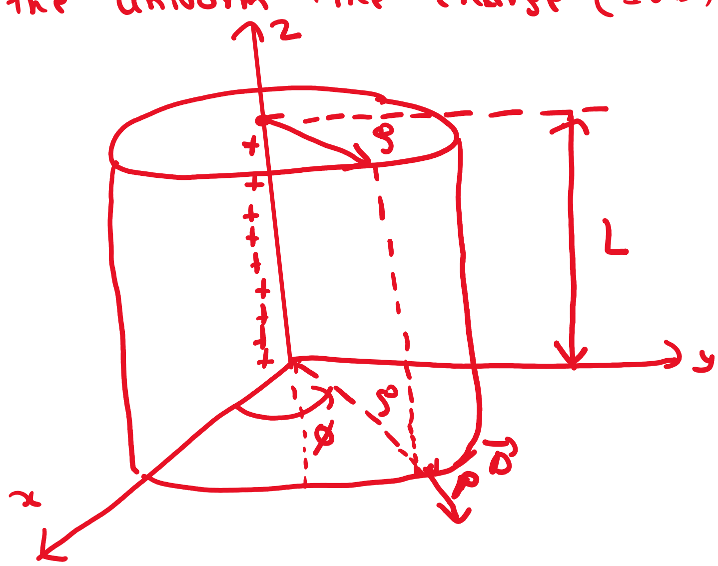
Now,

$\vec{D} = \varepsilon_0 \vec{E}$

$$\therefore \boxed{\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{a}_r}$$

Field due to the uniform line charge (IOE)

→



Consider the uniform line charge distribution having line charge ρ_L lying along the z-axis extending from $-\infty$ to $+\infty$.

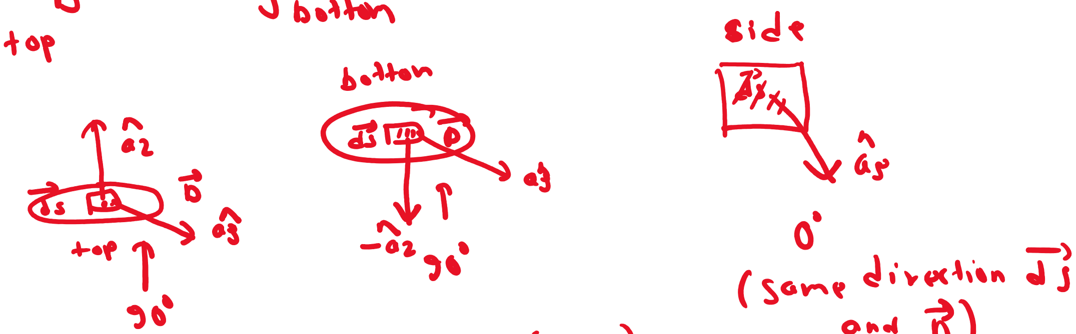
charge ρ_L lying along the z-axis extending from $-\infty$ to $+\infty$. To determine $\vec{D}$ at a point, let's take the Gaussian surface as a right circular cylinder of length L and radius ρ containing that point.

The electric flux density $\vec{D}$ is constant and perpendicular everywhere to the cylindrical surface. It is noticeable that only the radial component of $\vec{D}$ exist i.e $\vec{D} = D_\rho \hat{a}_\rho$

From Gauss law,

$$\psi = \oint_s \vec{D} \cdot \vec{ds} = Q \quad \text{—(i)}$$

or, $$\int_{top} \vec{D} \cdot \vec{ds} + \int_{bottom} \vec{D} \cdot \vec{ds} + \int_{side} \vec{D} \cdot \vec{ds} = Q$$



(same direction $\vec{ds}$ and $\vec{D}$)

(ds direction $\perp$ to surface)

we know

$$\vec{D} \cdot \vec{ds} = D\, ds \cos\theta \quad \left(\begin{matrix} \text{bottom} \\ \text{top} \end{matrix} \rightarrow 90° \rightarrow 0 \right)$$

side $\cos 0 \rightarrow 1$

so flux is only due to the side surface

along ρ direction

so, $0 + 0 + \int_{side} \vec{D} \cdot \vec{ds} = Q$, $\vec{D} = D_\rho \hat{a}_\rho$, $\vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$ (cylindrical)

or, $$\int_{side} (D_\rho \hat{a}_\rho) \, (\rho\, d\phi\, dz\, \hat{a}_\rho) = Q$$

$\int^{L} \int^{2\pi}$ ... (ρ is fixed radius)

$\int_{side}$

or, $D_\rho \rho \int_{z=0}^{L} \int_{\phi=0}^{2\pi} d\phi \, dz = Q$ (ρ is fixed radius)

or, $D_\rho \rho \int_{z=0}^{L} dz \times 2\pi = Q$

or, $2\pi D_\rho \rho \times L = Q$ or, $D_\rho = \dfrac{Q}{2\pi \rho L}$

where Q = total charge enclosed by gaussian surface $= \rho_L L$
(charge density × length)

or, $D_\rho = \dfrac{\rho_L L}{2\pi \rho L} = \dfrac{\rho_L}{2\pi \rho}$, $\vec{D} = D_\rho \hat{a}_\rho = \dfrac{\rho_L}{2\pi\rho} \hat{a}_\rho$

now,

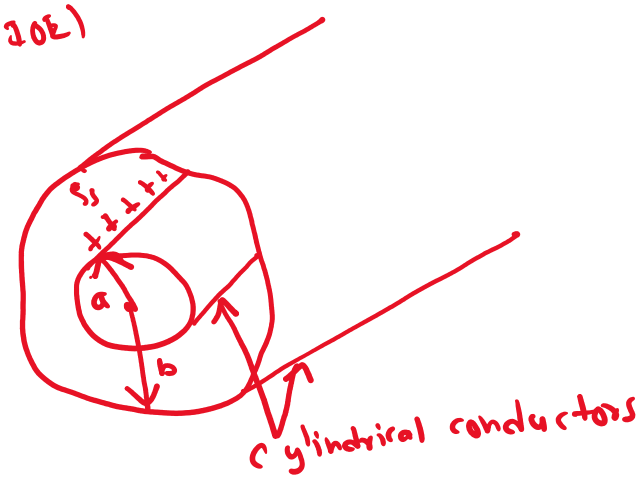
$\vec{D} = \varepsilon_0 \vec{E}$

$$\boxed{\varepsilon_0 \vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 \rho} \hat{a}_\rho}$$

General form

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 R} \hat{a}_R}$$

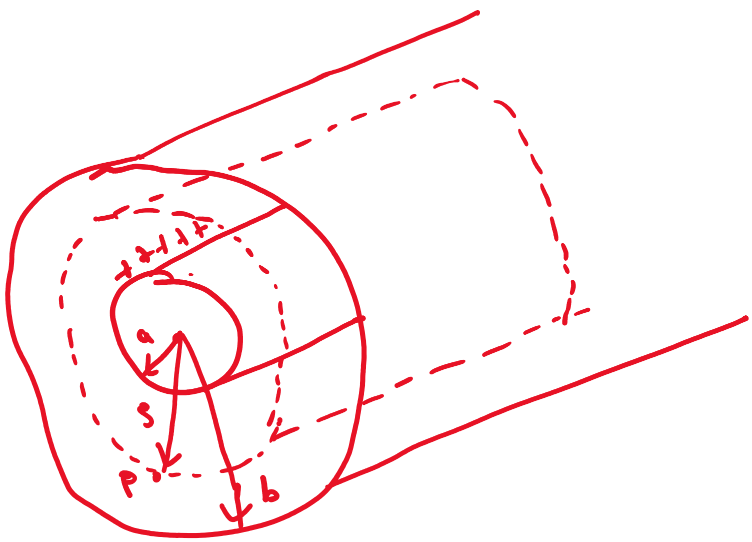
Field due to infinite co-axial cable with air as an dielectric. (IOE)



Consider a coaxial cable of an infinite length with two coaxial cylindrical conductors. The inner conductor of radius a and

Consider a coaxial cable of an infinite length with two coaxial cylindrical conductors, the inner conductor of radius a and the outer of radius b. Let ρ_s be the value of uniform surface charge density on the outer surface of the inner conductor.
Consider the gaussian surface as a cylindrical surface of radius ρ and length L. ($\vec{D} = D_\rho \hat{a}_\rho$ only radial comp)

Case I: For $a < \rho < b$



From Gauss law,

$$\psi = \oint_{cyl} \vec{D} \cdot \vec{ds} = Q \quad \text{-----(i)}$$

Here, $$\oint_{cyl} \vec{D} \cdot \vec{ds} = D_\rho 2\pi\rho L \quad \text{.....(ii)}$$

where Q = total charge enclosed by gaussian surface $= \int \rho_s ds$

or, $$Q = \rho_s \int_{\phi=0}^{2\pi} \int_{z=0}^{L} a\, d\phi\, dz$$ (inner ma charge cha)
$(\vec{ds} = \underset{=a}{\rho}\, d\phi\, dz\, \hat{a}_\rho)$

or $$Q = a \rho_s \int_{\phi=0}^{2\pi} d\phi \times L$$

Case II $\rho > b$

$$\therefore \boxed{Q = 2\pi a L \rho_s} \quad \text{—(ii)}$$

From eqⁿ (i), (ii), (iii)

$$D_\rho \, 2\cancel{\pi} \rho \cancel{L} = 2\cancel{\pi} a \cancel{L} \rho_s$$

or, $D_\rho = \dfrac{a \rho_s}{\rho}$

$$\boxed{\begin{aligned} \vec{D} &= D_\rho \hat{a}_\rho = \frac{a \rho_s}{\rho} \hat{a}_\rho \\ \vec{E} &= \frac{\vec{D}}{\varepsilon_0} = \frac{a \rho_s}{\varepsilon_0 \rho} \hat{a}_\rho \end{aligned}}$$

Case II $\rho > b$

$$\Psi = \oint_{(s)} \vec{D} \cdot d\vec{s} = Q = 0$$

$\vec{D} = 0$, $\vec{E} = 0$

Case III, $\rho < a$, $Q = 0$

$\vec{D} = 0$, $\vec{E} = 0$

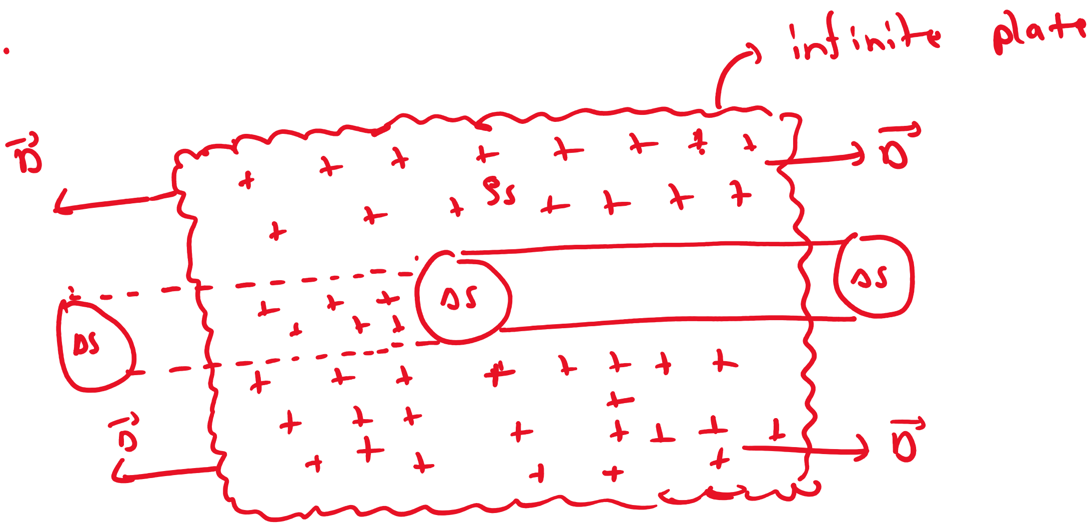
Expression of ρ_s for outer cylinder

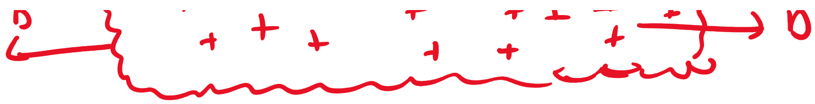
$$Q_{outer} = -Q_{inner}$$

or, $2\pi b L \rho_{s,outer} = -2\pi a L \rho_{s,inner}$

$$\therefore \boxed{\rho_{s,outer} = \frac{-a}{b} \rho_{s,inner}}$$

Field near an infinite plate with surface charge density ρ_s.





let ρ_s be the surface charge density on each side of the infinite plate. Thus fluxes are emanating from an infinite conductor plate on each side as shown. Consider the gaussian surface as a cylindrical surface enclosing a small area ΔS on the plate.

from Gauss law

$$\Psi = \oint_{cyl} \vec{D} \cdot \vec{dS} = Q \quad \text{—— (i)}$$

$$\int_{top} \vec{D} \cdot \vec{dS} + \int_{bottom} \vec{D} \cdot \vec{dS} + \int_{side} \vec{D} \cdot \vec{dS} = Q$$

For side $\vec{D}$ and $\vec{dS} = 90^\circ$ top bottom $\vec{D}$ and $\vec{dS}$ is 0°

So,

$$\int_{top} \vec{D} \cdot \vec{dS} + \int_{bottom} \vec{D} \cdot \vec{dS} = Q$$

$$\int_{top} D\, dS + \int_{bottom} D\, dS = Q$$

$$D \Delta S + D \Delta S = Q$$

or, $2 D \Delta S = Q$

But, $Q = \rho_s \Delta S + \rho_s \Delta S$ (for both side)

or, $2 D \Delta S = 2 \rho_s \Delta S$

or, $\boxed{D = \rho_s}$

In vector

$\vec{D} = \rho_s \hat{a}_N$

In vector

$$\vec{D} = \rho_s \hat{a}_N$$

$$\boxed{\therefore \vec{E} = \frac{\rho_s \hat{a}_N}{\varepsilon_0}}$$

When only one side of plate is considered then,

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_N}$$

(theory sodcha ya bata sab)

Divergence is a concept in vector calculus that helps us understand how a vector field behaves at a specific point. Imagine you have a vector field, which could represent things like fluid flow, air movement, or electromagnetic fields. The divergence of this field at a point tells us whether the point acts as a source, a sink, or neither.

Mathematically, divergence is represented as:

$$\boxed{\mathrm{div}\vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A} \cdot d\vec{S}}{\Delta v}}$$

This equation means we're looking at the net outflow of the vector field $\vec{A}$ through a small closed surface surrounding a point, as the volume of that surface shrinks to zero.

where $\vec{A}$ may represent velocity, temperature gradient, force, or any other vector field.

Note that, $\boxed{\mathrm{div}\,\vec{D} = \nabla \cdot \vec{D}}$

Physical Interpretation:

Positive Divergence: If the divergence at a point is positive, it means the point is a source. In other words, the vector field is spreading out from that point. Think of it like water flowing out of a faucet.

Negative Divergence: If the divergence is negative, the point is a sink. This means the vector field is converging or being absorbed at that point. Imagine water going down a drain.

Zero Divergence: If the divergence is zero, the point is neither a source nor a sink. The amount of the vector field entering the point is equal to the amount leaving it. This is like water flowing smoothly without any net creation or destruction.

In the image, Figure 3.7 illustrates these concepts:

Figure 3.7 (a): Shows positive divergence where the vectors are spreading out from point PP.

Figure 3.7 (b): Shows negative divergence where the vectors are converging towards point PP.

Figure 3.7 (c): Shows zero divergence where the vectors are neither spreading out nor converging at point PP.

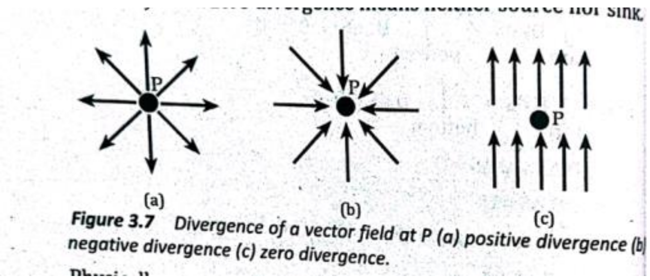


Figure 3.7 *Divergence of a vector field at P (a) positive divergence (b) negative divergence (c) zero divergence.*

Understanding divergence helps in analyzing various physical phenomena, such as fluid dynamics, electromagnetism, and heat transfer, by identifying sources and sinks within the field.

Maxwell's First Equation (Electrostatics)

From Gauss's law

$$\oint_s \vec{D} \cdot \vec{dS} = Q$$

or,
$$\frac{\oint_s \vec{D} \cdot \vec{dS}}{\Delta V} = \frac{Q}{\Delta V}$$

let volume shrink to zero

$$\text{or} \lim \frac{\oint \vec{D} \cdot \vec{dS}}{\Delta V} = \lim_{\Delta V \to 0} \frac{Q}{\Delta V}$$

or $\lim_{\Delta V \to 0} \frac{\oint \vec{D} \cdot d\vec{s}}{\Delta V} = \lim_{\Delta V \to 0} \frac{q}{\Delta V}$

or $\boxed{\text{div } \vec{D} = \rho_v}$

which is maxwell 1st equation also called the point form or differential form of Gauss law

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Divergence Theorem:

→ The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.

$$\oint_s \vec{D} \cdot \vec{ds} = \int_{vol} \nabla \cdot \vec{D} \, dv$$

Proof:

From Gauss law

$$\oint_s \vec{D} \cdot \vec{ds} = Q \quad (\text{total charge})$$

or, $$\oint_s \vec{D} \cdot \vec{ds} = \int_{vol} \rho_v \, dv$$

or, $$\oint_s \vec{D} \cdot \vec{ds} = \int_{vol} (\nabla \cdot \vec{D}) dv \quad (\because \nabla \cdot \vec{D} = \text{div } \vec{D} = \rho_v)$$

$$\therefore \boxed{\oint_s \vec{D} \cdot \vec{ds} = \int_{vol} (\nabla \cdot \vec{D}) dv}$$ proved #