1. Find $\vec{D}$ in cartesian co-ordinate system at $P(6, 8, -10)$ caused by
   (a) point charge at 30 mC at the origin.
   (b) a uniform line charge $\rho_L = 40\,\mu C/m$ on z-axis
   (c) a uniform surface charge of $\rho_s = 57.2\,\mu C/m^2$ on plane $x = 9$.

Soln:

(a) 30 mC at $(0,0,0)$ F.P → $(6, 8, -10)$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{a}_R$$

$$\vec{R} = 6\hat{a}_x + 8\hat{a}_y - 10\hat{a}_z$$

$$|\vec{R}| = 10\sqrt{2}$$

$$\vec{D} = 8.85\times10^{-12} \times 9\times10^{9} \times \frac{30\times10^{-3}}{(10\sqrt{2})^3} (6\hat{a}_x + 8\hat{a}_y - 10\hat{a}_z)$$

$$= (5.06\,\hat{a}_x + 6.75\,\hat{a}_y - 8.44\,\hat{a}_z)\,\mu C/m^2$$

(b) $\rho_L = 40\,\mu C/m$ on z-axis

$(0, 0, z)$ — F.P $(6, 8, -10)$

L

F.P

$(0, 0, -10)$

IP

$$\vec{R} = 6\hat{a}_x + 8\hat{a}_y$$

$$|\vec{R}| = 10$$

$$\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \frac{\rho_L}{2\pi\varepsilon_0 R} \hat{a}_R$$

$(6\hat{a}_x + 8\hat{a}_y)$

$$= \frac{40\times10^{-6}}{2\times3.14\times10} \times \frac{(6\hat{a}_x + 8\hat{a}_y)}{10}$$

$$= (3.82\,\hat{a}_x + 5.09\,\hat{a}_y)\times10^{-7}\ C/m^2$$

(c) $\rho_s = 57.2\ \mu C/m^2$ at $x = 9$ F.P $x = 6$

I.P

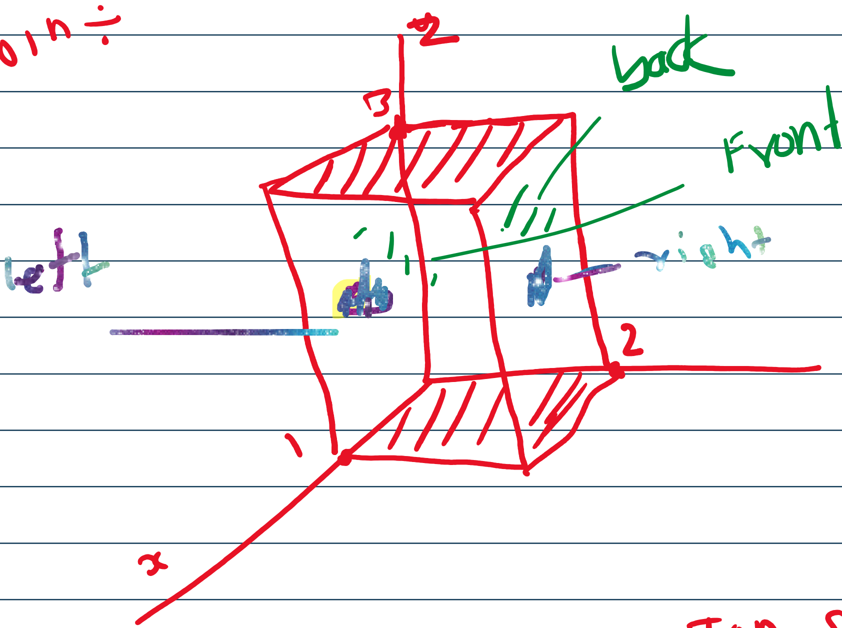
So, $\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_x)$

$$= -28.6\times10^{-6}\,\hat{a}_x\ C/m^2$$

---

(2) $\vec{D} = 4xy\,\hat{a}_x + 2x^2\,\hat{a}_y\ C/m^2$. Evaluate both sides of divergence theorem for region $x = 0$ and 1, $y = 0$ and 2 and $z = 0$ to 3. (understanding ko lagi use simple process muni haru 4 'jasto exam ma)

Soln:



Top surface

$z = 3$

$x = 0,\ x = 1$

Button surface ($z = 0$,

$x = 0,\ x = 1$

Button surface ($z = 0$) $z = 3$

$x = 0 , x = 1$ $x = 0, x = 1$

$y = 0 , y = 2$ $y = 0, y = 2$

For top and bottom z is contanst value

For left

$x = 0 , x = 1$

$z = 0 , z = 3$

$y = 0$

For right

$y = 2$

$x = 0 , x = 1$

$z = 0 , z = 3$

(y value is constant)

Front

$x = 1$

$y = 0, 2$

$z = 0, 3$

($x \to$ constant)

Back

$x = 0$

$y = 0, 2$

$z = 0, 3$

From eq^n

$$\oint_S \vec{D} \cdot \vec{ds} = \int_v (\nabla \cdot \vec{D}) \, dv$$

$$R.H.S = \int_{vol} (\nabla \cdot \vec{D}) \, dv$$

$$\nabla \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Given

$$\vec{D} = 4xy \, \hat{ax} + 2x^2 \, \hat{ay} \quad C/m^2$$

$$\nabla \vec{D} = \frac{\partial (4xy)}{\partial x} + \frac{\partial (2x^2)}{\partial y} + 0$$

$$\partial x \qquad \partial y$$

$$= 4y + 0$$

$$= 4y$$

$$\int_{vol} (\nabla \cdot D) dv = \int_{vol} 4y \, dv \qquad (dv = dx\,dy\,dz)$$

$$= \int_{x=0}^{1} \int_{y=0}^{2} \int_{z=0}^{3} 4y \, dx\,dy\,dz$$

$$= 4 \left[\frac{y^2}{2} \right]_0^2 \qquad = 24C$$

$$L.H.S = \oint_S \vec{D} \cdot \vec{ds} \quad (\text{sepperate into 6 surface})$$

$$= \int_{Top} \vec{D} \cdot \vec{ds} + \int_{Bottom} \vec{D} \cdot \vec{ds} + \int_{right} '' + \int_{left} '' + \int_{front} '' + \int_{back} \vec{D} \cdot \vec{ds}$$

(surface integral le yauta surface integrate garne bhayera)

$$\int_{Top} \vec{D} \cdot \vec{ds}$$

($\vec{ds}$ ko direction ↑ z axis ly top dekhi)
z constant so z=3 so mathi

(Top ma z=3 upper limit cha so direct $+\hat{a}_z$
bottom ma z = 0 lower limit so direction $-\hat{a}_z$)

along z

$$(dx\,dy)\,\hat{a}_z$$

Bottom .

$$\int_{Top} \vec{D}\cdot\vec{ds} = \int_{Top} (4xy\,\hat{a}_x + 2x^2\,\hat{a}_y)\,(dx\,dy)\,\hat{a}_z$$

$$\left(\begin{array}{l} \hat{a}_x \cdot \hat{a}_z = 0 \\ \hat{a}_y \cdot \hat{a}_z = 0 \end{array} \right)$$

so $\int_{Top} \vec{D}\cdot\vec{ds} = 0$

$$\int_{Bottom} \vec{D}\cdot\vec{ds} = \int_{Bottom} (4xy\,\hat{a}_x + 2x^2\,\hat{a}_y)\,(dx\,dy)\,-\hat{a}_z$$

$$= 0$$

y const ⊥ y-axis

$$\int_{left} \vec{D}\cdot\vec{ds} = \int_{left} (4xy\,\hat{a}_x + 2x^2\,\hat{a}_y)\,(dx\,dz)\,-\hat{a}_y$$

$$\oint_{left} \vec{D}\cdot\vec{ds} = \oint_{left} -2x^2\,dx\,dz$$

$$= -2\int_{x=0}^{x=1} x^2\,dx \int_{z=0}^{3} dz$$

$$= -2$$

$$\int_{right} \vec{D}\cdot\vec{ds} = \int_{right} (4xy\,\hat{a}_x + 2x^2\,\hat{a}_y)\,dx\,dz\,(\hat{a}_y)$$

$$= 2$$

$$\int_{front} \vec{D}\cdot\vec{ds} = \int_{front} (4xy\,\hat{a}_x + 2x^2\,\hat{a}_y)\,(dy\,dz)\,\hat{a}_x$$

$$= \int_{front} 4xy\;dy\,dz \qquad \boxed{(x = 1 \;\; \text{const})}$$

IMP!!

$$= \int \ldots\; dy\,dz$$

$= \int 4y \, dy \, dz$

$= 4 \int_{y=0}^{2} y \, dy \int_{z=0}^{3} dz$

$= 12 \times 2 = 24 \, C$

$\int_{back} = \int (4xy \hat{a}_x + 2x^2 \hat{a}_y) \, dy \, dz \, (-\hat{a}_x)$

$= \int -4xy \, dy \, dz$ $\quad (x=0)$ const

$= - 8$

$= 0$

$L.H.S = 0 + 0 + 2 + -2 + 24 + 0$

$= 24 \, C$

$\therefore L.H.S = R.H.S$ proved #

#. Given $\vec{D} = \frac{2\cos\theta}{r^3} \hat{a}_r + \frac{\sin\theta}{r^3} \hat{a}_\theta$ C/m^2. Evaluate both Sides of the divergence theorem for the region defined by $1 < r < 2$, $0 < \theta < \frac{\pi}{2}$, $0 < \phi < \frac{\pi}{2}$

Soln:-

Soln.

$$\oint \vec{D}\cdot\vec{ds} = \int_{vol} (\nabla\cdot\vec{D})\, dv$$ evaluate 1st

$$R.H.S = \int_{vol} (\nabla\cdot\vec{D})\, dv$$

$$= \nabla\cdot\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + 0$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{2\cos\theta}{r^3}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r^3}\right)$$

$$= \frac{2\times 1}{r^2}\times\cos\theta \times -r^{-2} + \frac{1}{r^4\sin\theta}\, 2\sin\theta\cos\theta$$

$$= -\frac{2\cos\theta}{r^4} + \frac{2\cos\theta}{r^4}$$

$= 0$ (spherical ko A_R hain A_r ho θn bank ko pachidi ko small ho capital naina)

$$R.H.S = \int_{vol} \nabla\vec{D}\, dV = 0$$

$$L.H.S = \oint \vec{D}\cdot\vec{ds}$$

Now,

$$ds = r^2\sin\theta\, d\theta\, d\phi\, (-\hat{a}_r)$$

$1 < r < 2$

$0 < \theta < \pi/2$

$0 < \phi < \pi/2$

For $r = 1$ small

& const

$+ \int \dots + \int \dots$

$\not\leftarrow$ small

$\underline{\underline{\text{const}}}$

$$\oint_s \vec{D}\cdot\vec{ds} = \int_{(r=1)} \vec{D}\cdot\vec{ds} + \int_{(r=2)} \ldots + \int_{\theta=0} \ldots + \int_{\theta=\pi/2} \ldots$$

const

$$+ \int_{\phi=0} \ldots + \int_{\phi=\pi/2} \vec{D}\cdot\vec{ds}$$

Now,

$$\int_{(r=1)} \vec{D}\cdot\vec{ds} = \int_s \left\{ \left(\frac{2\cos\theta}{r^3}\right)\hat{a}_r + \left(\frac{\sin\theta}{r^3}\right)\hat{a}_\theta \right\} \cdot r^2 \sin\theta \, d\theta \, d\phi \, (-\hat{a}_r)$$

$\boxed{(\text{put } r=1)}$ IMP

$$= \int_s -2\cos\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$= -\int_{\theta=0}^{\pi/2} \sin 2\theta \, d\theta \int_{\phi=0}^{\pi/2} d\phi$$

$$= -\frac{\pi}{2} \times 1 = -\frac{\pi}{2}$$

$$\int_s (\ldots\ldots\ldots\ldots) \, (+\hat{a}_r)$$

$(r=2)$

$$= \int_s \frac{2\cos\theta}{r^3} \times r^2 \sin\theta \, d\theta \, d\phi$$

$(r=2)$

$$= \frac{1}{2} \int_{\theta=0}^{\pi/2} \sin 2\theta \, d\theta \int_0^{\pi/2} d\phi$$

$$\frac{\ }{2}\int_{\theta=0}$$

$$(-\hat{a}_\theta) \quad = \frac{\pi}{4}$$

$$\int_s \vec{D}\cdot\vec{ds} = 0$$
$\theta=0$

$$\int_s \vec{D}\cdot\vec{ds} = \int_s \left(\left(\frac{2\cos\theta}{r^3}\right)\hat{a}_r + \left(\frac{\sin\theta}{r^3}\right)a_\theta\right)\cdot r\,dr\,\sin\theta\,d\phi\,\hat{a}_\theta$$
$(\theta=\frac{\pi}{2})$

$$= \int_s \frac{\sin\theta}{r^3}\, r\,dr\,\sin\theta\,d\phi$$

$$= \int_s \frac{\sin^2\theta}{r^2}\,dr\,d\phi \quad (\theta=\pi/2)$$

$$= \int_s \frac{1}{r^2}\,dr\,d\phi$$

$$= \int_1^2 \frac{1}{r^2}\,dr \int_{\phi=0}^{\pi/2} d\phi$$

$$= \frac{\pi}{4}$$

$$\int_{(\phi=0)} \vec{D}\cdot\vec{ds} = 0 \qquad \int_{(\phi=\pi/2)} \vec{D}\cdot\vec{ds} = 0 \quad \left(\begin{array}{c}\hat{a}_r\cdot\hat{a}_\phi = 0\\ \hat{a}_\theta\cdot\hat{a}_\phi = 0\end{array}\right)$$

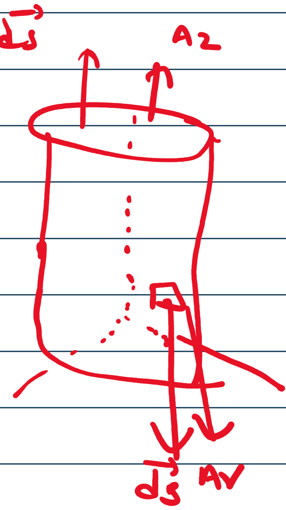
$$\cdot\ )\cdot\ \text{h}\cdot\text{s} = \int \vec{D}\cdot\vec{ds} = \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{2} = 0$$

$\therefore L.H.S = \int_S \vec{D} \cdot \vec{ds} = \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{2} = 0$

$$\boxed{\therefore L.H.S = R.H.S \quad \text{proved}}$$

\# $\vec{A} = 30 e^{-\rho} \hat{a}_\rho - 2z \hat{a}_z$ in cylindrical co-ordinates

evaluate both side of divergence theorem for the volume enclosed by $r=2$, $z=0$ and $z=5$.



Soln:

$$\oint_S \vec{A} \cdot \vec{ds} = \int_{vol} (\nabla \cdot \vec{A})\, dv$$

$$L.H.S = \oint_S \vec{A} \cdot \vec{ds}$$

$$= \int_S \vec{A} \cdot \vec{ds} + \int_S \vec{A} \cdot \vec{ds} + \int_S \vec{A} \cdot \vec{ds}$$

$(\rho = 2)$ $(z=0)$ $(z=5)$

const $\hat{a}_\rho$

$$\iint (30 e^{-\rho} \hat{a}_\rho - 2z \hat{a}_z)(\rho\, d\phi\, dz)\hat{a}_\rho$$

$(\rho = 2) \rightarrow$ upper

$$= 2 \iint 30 e^{-2}\, d\phi\, dz$$

$2\pi \quad 5 \quad 254.97$

$= 2\iint 30\ldots$

$$= 2 \times 30 e^{-2} \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{5} dz = 254.97$$

$$\iint \vec{A}\cdot\vec{ds} = \iint \left(30 e^{-\rho}\hat{a}_\rho - 2z\hat{a}_z\right)(\rho\, d\rho\, d\phi)(-\hat{a}_z)$$

$(z=0)$

$$= \iint 2z\,\hat{a}_z\, \rho\, d\rho\, d\phi$$

$$\cancel{=0} = 0 \quad (z=0)$$

$(+\hat{a}_z)$

$$\iint_{(z=5)} \vec{A}\cdot\vec{ds} = \iint -2z\, \rho\, d\rho\, d\phi$$

$$= -10 \int_{\rho=0}^{2} \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi$$

$\boxed{\text{add } a_z a_i}$

$$= -10 \times 2 \times 2\pi = -125.6$$

$$\therefore \text{L.H.S} = -125.6 + 254.97$$

$$\boxed{= 129.4}$$

now,

$$\text{R.H.S} = \int_{vol} (\nabla\cdot\vec{A})\, dv$$

$$\ldots \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\, 30 e^{-\rho}\right) + 0 + \frac{\partial(-2z)}{\partial z}$$

product rule

$$= \frac{1}{\rho}\left[-\rho \cdot 30 e^{-\rho} + 30 e^{-\rho}\right] - 2$$

$$= -30 e^{-\rho} + \frac{30 e^{-\rho}}{\rho} - 2$$

$$R.H.S = \int_{vol}\left(-30 e^{-\rho} + \frac{30 e^{-\rho}}{\rho} - 2\right)(\rho\, d\rho\, d\phi\, dz)$$

$$= \int_{vol}\left(-30 e^{-\rho}\rho + 30 e^{-\rho} - 2\rho\right)(d\rho\, d\phi\, dz)$$

$$= \int_{\rho=0}^{2}\left(-30 e^{-\rho}\rho + 30 e^{-\rho} - 2\rho\right)d\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{5} dz$$

Calc

$$= 129.4$$

$\therefore \boxed{L.H.S = R.H.S}$ proved

Verify the divergence theorem for $\vec{A} = r^2 \hat{a}_r + r\sin\theta\cos\phi\, \hat{a}_\theta$ over the surface of quater of a hemisphere defined by $0 < r < 3,\ 0 < \phi < \frac{\pi}{2},\ 0 < \theta < \frac{\pi}{2}$

Soln:-

The divergence theorem is $\oint_S \vec{A}\cdot\vec{ds} = \int_{vol} (\nabla\cdot\vec{A})\, dv$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

miss

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot r^2\right) + \frac{\partial}{\partial\theta}(r\sin\theta\cos\phi\sin\theta) + 0$$

$$= \frac{1}{r^2}\times 4r^3 + \frac{1}{r\sin\theta}\cdot r\cos\phi\,\frac{\partial(\sin^2\theta)}{\partial\theta} + 0$$

$$= 4r + \frac{\cos\phi}{\sin\cancel{\theta}}\cdot 2\sin\cancel{\theta}\cdot\cos\theta$$

$$= 4r + 2\cos\theta\cos\phi$$

Now,

$$\int_{vol}(\nabla\cdot\vec{A})\,dv = \iiint (4r + 2\cos\theta\cos\phi)(r^2\sin\theta\, dr\, d\theta\, d\phi)$$

$$= \iiint (4r^3\sin\theta + 2r^2\sin\theta\cos\theta\cos\phi)\,(dr\,d\theta\,d\phi)$$

$$= \int_{r=0}^{3}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2} 4r^3\sin\theta\, dr\,d\theta\,d\phi +$$

$$\int_{r=0}^{3}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2} r^2\sin 2\theta\cos\phi\, dr\,d\theta\,d\phi$$

$$r=0 \quad \theta=0 \quad \phi=0$$

$$= 4\int_{r=0}^{3} r^3\,dr \int_{\theta=0}^{\pi/2} \sin\theta\, d\theta \int_{\phi=0}^{\pi/2} d\phi \; +$$

$$\int_{r=0}^{3} r^2\,dr \int_{\theta=0}^{\pi/2} \sin 2\theta\, d\theta \int_{\phi=0}^{\pi/2} \cos\phi\, d\phi$$

$$= 131.17$$

Now,

$$L.H.S = \oint_S \vec{A}\cdot\vec{dS}$$

$$= \iint_{(r=0)} \vec{A}\cdot\vec{dS} + \ldots \ldots \int \vec{A}\cdot\vec{dS}$$

$$r=3 \quad \theta=0 \quad \theta=\pi/2 \quad \phi=0 \quad \phi=\pi/2$$

$$= \phi=0 \text{ and } \phi=\pi/2 = 0$$

$$= \iint_{(r=0)} \left(r^2\hat{a}_r + r\sin\theta\cos\phi\,\hat{a}_\theta\right)\left(r^2\sin\theta\, d\phi\, d\theta\right)(-\hat{a}_r)$$

$$= \iint 0$$

$$\int_{(r=3)} \vec{A}\cdot\vec{dS} = \iint r^4 \sin\theta\, d\theta\, d\phi \quad (+\hat{a}_r)$$

$$= 3^4 \int_{\theta=0}^{\pi/2} \sin\theta\, d\theta \int_{\phi=0}^{\pi/2} d\phi$$

$$= 127.17$$

$= 131.17$

Now

$\int \vec{A}\cdot\vec{ds} = \oint 0$
$(-\hat{a}_\theta)$

$(\theta = 0)$

cont $(+\hat{a}_\theta)$

$\int_{\theta=\pi/2} \vec{A}\cdot\vec{ds} = \iint (r^2\hat{a}_r + r\sin\theta\cos\phi\,\hat{a}_\theta)(r\,dr\,\sin\theta\,d\phi)\,\hat{a}_\theta$

$= \iint r^2\sin^2\theta\cos\phi\,dr\,d\phi$

$= \iint r^2 \cdot 1 \cdot \cos\phi\,dr\,d\phi$

$\theta = \frac{\pi}{2}$

$= \int_{r=0}^{3} r^2\,dr \int_{\phi=0}^{\pi/2} \cos\phi\,d\phi = 9$

$\oint_S \vec{A}\cdot\vec{ds} = 131.17$

$\therefore$ L.H.S = R.H.S proved #

1. A point charge of 20 nC is located at (4, –1, –3) & a uniform line charge ρ_L of –25 nC/m lies along the intersection of planes x = –4 and z = 6. Calculate the electric flux density $\vec{D}$ in cylindrical coordinate system at point (3, 1, 0). *[2062 Bhadra]*

Sol^n:

point) line

Soln :

point | line

$Q = 20nC$ at $(4,-1,-3)$ | $\rho_L = -25\, nC/m$

F.P = (3,1,0) | $x = -4$ and $z = 6$

F.P = (3,1,0)

Soln :

point charge :

$$\vec{E}_P = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{Q}{4\pi\varepsilon_0 R^3}\vec{R}$$

$$\vec{R} = (3,1,0) - (4,-1,-3) = -\hat{a}_x + 2\hat{a}_y + 3\hat{a}_y$$

$$|\vec{R}| = \sqrt{14}$$

$$= \frac{20\times10^{-9}\times 9\times10^{9}\times}{(\sqrt{14})^3}(-\hat{a}_x + 2\hat{a}_y + 3\hat{a}_y)$$

$$= -3.43\,\hat{a}_x + 6.87\,\hat{a}_y + 10.30\,\hat{a}_z \ V/m$$

now,

$$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 R}\hat{a}_R$$

I.P = (-4,1,6) F.P = (3,1,0)

$$\vec{R} = (7\hat{a}_x - 6\hat{a}_z)$$

$$|\vec{R}| = \sqrt{85}$$

now

$$\vec{E}_L = \frac{-25\times10^{-9}\times 2\times 9\times10^{9}\times}{85}(7a_x - 6\hat{a}_z)$$

$$= -37.05\,\hat{a}_x + 31.76\,\hat{a}_z \ V/m$$

now, $\vec{E} = \vec{E}_P + \vec{E}_L = -40.48\hat{a}_x + 6.87\hat{a}_y + 42.06 \ V/m$

10^{-12}

now

now,

$\vec{D} = \varepsilon_0 \vec{E} = -358.248\, \hat{a}_x + 60.7995\, \hat{a}_y + 372.231 \; \overset{10^{-12}}{\downarrow}\!\!\; pC/m^2$

now, convert to spherical like chap-1 #

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2. A point charge of 12 nC is located at the origin. Four uniform line charges are located in the x = 0 plane as follows: 80 nC/m at y = –1 and –5m, –50 nC/m at y = –2 and –4 m. Find the electric flux density $\vec{D}$ in spherical coordinate system at P(0, –3, 2). *[2075 Chaitra]*

Solution:

Soln÷

line charge

point charge at (0,0,0) | 1: $x=0$, $y=-1$ | 2: $x=0$, $y=-5$ | 3: $x=0$, $y=-2$ | 4: $x=0$, $y=-4$

$Q = 12\,nC$ | $\rho_L = 80\,nC/m$ | $\rho_2 = -50\,nC/m$

Now,

$\vec{E}_L = \vec{E}_{L1} + \vec{E}_{L2} + \vec{E}_{L3} + \vec{E}_{L4}$

$= \dfrac{\rho_1}{2\pi\varepsilon_0 R_1}\,\hat{a}_{R1} + \dfrac{\rho_2}{2\pi\varepsilon_0 R_2}\,\hat{a}_{R2} + \dfrac{\rho_3}{2\pi\varepsilon_0 R_3}\,\hat{a}_{R3} + \dfrac{\rho_4}{2\pi\varepsilon_0 R_4}\,\hat{a}_{R4}$

$\vec{R}_1 = (0,-3,2) - (0,-1,2) = -2\hat{a}_y \qquad R_1 = 2$

$\vec{R}_2 = (0,-3,2) - (0,-5,2) = 2\hat{a}_y \qquad R_2 = 2$

$\vec{R}_3 = (0,-3,2) - (0,-2,2) = -\hat{a}_y \qquad R_1 = 1$

$R_4 = (0,-3,2) - (0,-4,2) = \hat{a}_y \qquad R_1 = 1$

$\vec{E}_L = \dfrac{1}{2\pi\varepsilon_0}\left(\dfrac{\rho_1}{R_1^2}\vec{R}_1 + \dfrac{\rho_2}{R_2^2}\vec{R}_2 + \dfrac{\rho_3}{R_3^2}\vec{R}_3 + \dfrac{\rho_4}{R_4^2}\vec{R}_4\right)$

$= 2\times 9\times 10^9 \left(\cancel{\dfrac{80\times10^{-9}}{2^2}(-2\hat{a}_y)} + \cancel{\dfrac{80\times10^{-9}}{2^2}}(2\hat{a}_y) + \cancel{\dfrac{-50\times10^{-9}}{1}}\times(-\hat{a}_y)\right.$

$$+ \frac{-50\times10^{-9}}{1} \cancel{x}(\hat{a}_y) \Big)$$

$$= 0$$

$$\vec{E}_p = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R \qquad \vec{R} = (0,-3,2) - (0,0,0)$$

$$= -3\hat{a}_y + 2\hat{a}_z$$

$$R = \sqrt{13}$$

$$= \frac{12\times10^{-9}\times9\times10^{9}}{(\sqrt{13})^3} \times (-3\hat{a}_y + 2\hat{a}_z) = -6.91\hat{a}_y + 4.60\hat{a}_z \text{ V/m}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_p = -6.91\hat{a}_y + 4.60\hat{a}_z \text{ V/m}$$

Now,

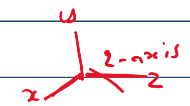
$$\vec{D} = \varepsilon_0\vec{E} = -611.535 + 407.1 \text{ pC/m}^2$$

\# A uniform line charge of 15 nC/m lies along z-axis and a uniform sheet of charge ρ_s of 4 nC/m^2 is located at $z = 1$. Find the electric flux density $\vec{D}$ in spherical co-ordinate system at point (2,0,0).

Soln ÷

line charge $\rho_l = 15$ nC/m (z-axis) | $\vec{D}$ at (2,0,0)

surface charge $\rho_s = 4$ nC/m^2 ($z = 1$)



Now,

$$\vec{E}_L = \frac{\rho_l}{2\pi\varepsilon_0 R}\hat{a}_R = \frac{\rho_l}{2\pi\varepsilon_0 R^2}\vec{R} \qquad \vec{R} = (2,0,0) - (0,0,0)$$

$$= 2\hat{a}_x$$

$$R = 2$$

$$= 2\times9\times10^{9}\times15\times10^{-9}\times(2\hat{a}_x)$$

$R = 2$

$$= \frac{2 \times 9 \times 10^{9} \times 15 \times 10^{-9}}{(2)^2} \times (2\hat{a}_x)$$

$$= 135\ \hat{a}_x$$

now,

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\hat{a}_N = \frac{4 \times 10^{-9}}{2 \times 8.85 \times 10^{-12}} \times (-\hat{a}_z) \qquad z = 1 \to 0$$

$$= -225.9\ \hat{a}_z$$

now,

$$\vec{E} = \vec{E}_s + \vec{E}_\rho = 135\ \hat{a}_x - 225.9\ \hat{a}_z$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} = 1.19\ \hat{a}_x - 2\hat{a}_z\ \ nC/m^2}$$

→ convert to sph as in chap-1

Surface charge densities of 200, -50 and ρ $\mu C/m^2$ are located at $r = 3, 5, 7$ cm respectively. Find $\vec{D}$ at (a) $r = 1$ cm (b) $r = 4.8$ cm (c) $r = 6.9$ cm. Find ρ if $\vec{D} = 0$ at $r = 9$ cm

Soln ÷

(a) at $r = 1$ cm (no charge is enclosed so flux = 0)

$$\vec{D} = \frac{\text{flux}}{\text{Area}}\hat{a}_r = \frac{0}{4\pi(0.01)^2}\hat{a}_r = 0$$

(b) at $r = 4.8$ cm → It encloses the sphere at $r = 3$ cm → charge enclosed

so,

$$\phi = \psi = \rho_s \times \text{Area} \qquad (\rho_s \cdot C \cdot D \times \text{Area} = \text{charge})$$

(at 3) → cm

$$= 200 \times 10^{-6} \times 4\pi(0.03)^2 = 2.26\mu C$$

now

$$\vec{D}\ (\text{at } r = 4.8\text{cm}) = \frac{\text{flux}}{}\hat{a}_r = \frac{\psi}{}\hat{a}_r = \frac{2.26 \times 10^{-6}}{}\hat{a}_r$$

now

$\vec{D}$ (at $r = 4.8cm$) $= \frac{\text{flux}}{\text{area}} \hat{a}_r = \frac{\Psi}{A} \hat{a}_r = \frac{2.26 \times 10^{-6}}{4\pi (0.048)^2} \hat{a}_r$

$= 78.05 \hat{a}_r \ \mu C/m^2$

(c) at $r = 6.9cm$ (it enclose 5 sphere with $r=3$ and $r=5$)

charge enclosed

$\Psi = Q =$ charge density $\times$ Area

$= 200\mu \times 4\pi \times (0.03)^2 + (-50)\mu \times 4\pi \times (0.05)^2$

$= 691.15 \ nC$

$\vec{D}$ ($r = 6.9cm$) $= \frac{\Psi}{A} \hat{a}_r = \frac{691.15 nC}{4\pi (0.069)^2} \hat{a}_r = 11.55 \hat{a}_r \ \mu C/m^2$

(c) At $r = 9$, $\vec{D} = \frac{\Psi}{A} \hat{a}_r = 0$ (given)

so,

$\Psi = 0 = Q$

(charge enclosed by $r = 9cm$ sphere is ÷

$Q =$ by 3cm + by 5cm + by 7cm

$(\rho_s \times A)$ (SA) $(\rho_s \times A)$

$0 = 4\pi (200\mu \times 0.03^2 - 50\mu \times 0.05^2 + S\mu \times 0.07^2)$

$\therefore \ S = -1\cancel{7}1.22 \ \mu C/m^2$ #

\# let $\vec{D} = \frac{r}{3} \hat{a}_r$ nc/m² in free space.

(a) Find $\vec{E}$ at $r = 0.2m$

(b) Find the total charge within sphere $r = 0.2$ m

(c) Find the total electric flux leaving the sphere at $r = 0.3m$

soln ÷

(a) $\vec{D} = \varepsilon_0 \vec{E} \rightarrow \vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{r}{3} \hat{a}_r$ nc/m² $= 37.64 \, r \, \hat{a}_r$ c/m²

(a) $\vec{D} = \varepsilon_0 \vec{E} \rightarrow \vec{E} = \dfrac{\vec{D}}{\varepsilon_0} = \dfrac{\frac{r}{3} \hat{a}_r \ nC/m^2}{\varepsilon_0} = 37.64\, r\, \hat{a}_r \ C/m^2$

$\therefore \vec{E}\big|_{r=0.2m} = 7.529\, \hat{a}_r \ V/m$

(b) Total charge within the sphere

$\varphi = \int_{vol} \rho_v \, dv = \int_{vol} (\nabla \cdot \vec{D}) \, dv$

ρ_v: volume charge density

$(\nabla \cdot \vec{D})$: convert to spherical (r cha tesiale)

alternate

Total flux = ψ = total charge enclosed = Q (Gauss law)

total: within sphere

$\psi = \oint_S \vec{D} \cdot \vec{ds} \rightarrow$ simpler integration

Now,

$\psi = \iint \left(\frac{r}{3} \times 10^{-9} \, \hat{a}_r \right) \left(r^2 \sin\theta \, d\theta \, d\phi \ \hat{a}_r \right)$

along r
wala

$= \iint \dfrac{r^3}{3} \times 10^{-9} \sin\theta \, d\theta \, d\phi$

$= \dfrac{r^3}{3} \times 10^{-9} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$

$= \dfrac{4\pi r^3}{3} \times 10^{-9}$

at $r = 0.2$

$\psi = 33.51\, p \ Wb = \varphi$

$\therefore Q = 33.51\, pC$

$\psi = 33.51\,p\ wb = Q$

$\therefore Q = 33.51\,pC$

(c) Total flux leaving sphere at $r = 0.3$

$$\psi_{r=0.3} = \frac{4\pi r^3}{3} \times 10^{-9} = \frac{4\pi \times 0.3^2}{3} \times 10^{-9} = 113.08\,p\ wb = Q$$

Consider a co-axial cable of length 50cm having inner radius of 1mm and an outer radius of 4mm with the space between conductors filled with air. Total charge on the inner conductor is 30nC. Find (i) charge density on the inner and outer conductor

Soln ÷

$(Q_{inner} = -Q_{outer})$

(i) $Q_{inner} = 2\pi a L \rho_{s,inner}$

$$\rho_{s,inner} = \frac{Q_{inner}}{2\pi a L} = \frac{30\,n}{2\pi \times 10^{-3} \times 50\times 10^{-2}} = 9.55\,\mu C/m^2$$

$Q_{outer} = 2\pi b L \rho_{s,outer}$

$$\rho_{s,outer} = \frac{Q_{outer}}{2\pi b L} = \frac{-Q_{inner}}{2\pi b L} = \frac{-30\,n}{2\pi \times 4\times 10^{-3} \times 0.5} = -2.3\,\mu C/m^2$$

let $\vec{D} = 5r^2\,\hat{a}_r\ mC/m^2$ for $r \le 0.08m$ and $\vec{D} = \frac{0.205}{r^2}\,\hat{a}_r\ \mu C/m^2$ for $r > 0.08m$. (a) Find ρ_v for $r = 0.06$ (b) ρ_v for $r = 0.1m$ (c) What surface charge density could be located at $r = 0.08m$ to cause $\vec{D} = 0$ for $r = 0.08m$?

Soln ÷

Now,

(a) $\rho_v = \nabla \cdot \vec{D}$ $(r = 0.06\,m)$

$\rightarrow mC/m^2$

$= \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r + 0 + 0 \right]$ ✓ C/m^2

$= \frac{1}{r^2} \cdot \frac{\partial}{\partial r} (r^2 \cdot 5r^2) = \frac{5}{r^2} \cdot 4 \cdot r^3 = \frac{20r}{10^3}$

$r = 0.06\,m$

$\rho_v = \frac{20 \times 0.6}{10^{-3}} = 1.2 \times 10^{-3}\, C/m^3$ (ρ_v)

(b) ρ_v for $r = 0.1\,m$ $\rightarrow \vec{D} = \frac{0.205}{r^2} \times 10^{-6}\, \hat{a}_r\, C/m^2$

$\rho_v = \nabla \cdot \vec{D}$

$= \left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r} (r^2 \cdot D_r) \right) = \frac{1}{r^2} \cdot \frac{\partial}{\partial r} \left(\frac{r^2 \cdot 0.2054}{r^2} \right)$

$= \frac{1}{r^2} \times 0 = 0$

$\therefore \rho_v = 0$

(c) ρ_s at $r = 0.08\,m$ to cause $\vec{D} = 0$ for $r = 0.08\,m$

$Q = \int_{vol} \rho_v\, dv$ ✓ $r = 0.06$ na 0.08 same naaha

✓ previous calc

$= \rho_v = \nabla \cdot \vec{D} = \frac{20r}{10^3}$

$$\Phi = \iiint (20r \times 10^{-3})(r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

$$= \iiint 20 r^3 \times 10^{-3} \sin\theta \, dr \, d\theta \, d\phi$$

ϕ at $r = 0.08$

$$= 20 \times 10^{-3} \int_0^{0.08} r^3 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= 2.57\mu$$

$$\rho_s = \frac{\Phi}{A} = \frac{2.57\mu}{4\pi \times (0.08)^2} = 32\mu \text{ C/m}^2$$

In order to cause $\vec{D} = 0$ charge should be placed at $r = 0.08$ m

$= -\Phi = 2.57\mu$. So, $\rho_s = -32\mu \text{ C/m}^2$

\# $\vec{D} = \dfrac{5 \sin\theta \cos\phi}{r} \hat{a}_r$ C/m², find (a) the volume charge density (b) the total charge confined in the region $r < 2$m (c) the value of $\vec{D}$ at the surface $r = 2$.

Soln÷

Now,

(a) $\rho_v = \nabla \cdot \vec{D}$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot 5 \sin\theta \cos\phi / r \right)$$

$$= \frac{1}{r^2} \times 5 \sin\theta \cos\phi \cdot 1 = \underline{5 \sin\theta \cos\phi} \text{ C/m}^2$$

$$= \frac{1}{r^2} \times 5 \sin\theta \cos\phi \cdot 1 = \frac{5 \sin\theta \cos\phi}{r^2} \; C/m^2$$

(b) $\psi = \int_{vol} \rho_v \, dv$

$$= \iiint \frac{5 \sin\theta \cos\phi}{\cancel{r^2}} \left(\cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi \right)$$

$$= 5 \int_0^2 dr \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta \int_{\phi=0}^{2\pi} \cos\phi \, d\phi$$

$$= 5 \times 2 \times 1.57 \times 0 = 0$$

(c) $\vec{D} = \frac{5 \sin\theta \cos\phi}{r} \hat{a}_r \; C/m^2$

$(r = 2)$

$$\vec{D} = \frac{5}{2} \sin\theta \cos\phi \; \hat{a}_r \; C/m^2$$