(1)

(1) Planes $x = 2$ and $y = -3$ respectively carry charges $10\ nC/m^2$ and $15\ nC/m^2$. If the line $x = 0$ and $z = 2$ carries charge of $10\pi\ nC/m$. Calculate $\vec{E}$ at $(1, 1, -1)$ due to these charge distributions.

soln:

$x = 2$, $\rho_{s1} = 10\ nC/m^2$
$y = -3$, $\rho_{s2} = 15\ nC/m^2$] surface charge density

line charge

$\rho_L = 10\pi\ nC/m$ $x = 0$ and $z = 2$

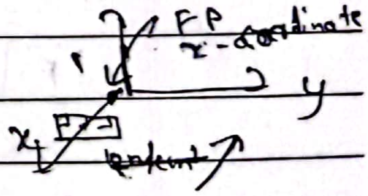
$\vec{E} = ?$ at $(1, 1, -1)$

final point (FP) $= (1, 1, -1)$

$$\vec{E}_T = \vec{E}_{s_1} + \vec{E}_{s_2} + \vec{E}_L$$

$x = 2 \rightarrow 10\ nC/m^2$

FP : $(1, 1, -1)$



Electric field due to surface charge

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\hat{a}_N$$

$$\vec{E}_{s_1} = \frac{\rho_{s_1}}{2\varepsilon_0}\hat{a}_N = \frac{10 \times 10^{-9} \cdot (-\hat{a}_x)}{2 \times 8.85 \times 10^{-12}}$$

if inward $\hat{a}_N = -\hat{a}_x$

if outward $\hat{a}_N = \hat{a}_x$

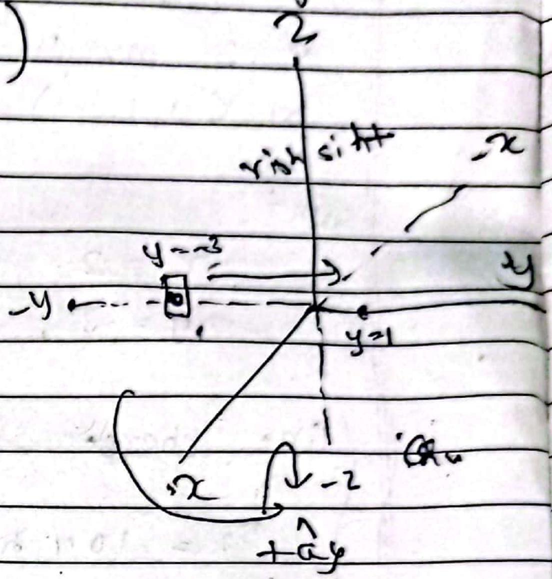
$$= -564.971\ \hat{a}_x$$

(2)

$$\vec{E}_{S_2} = \frac{\rho_{S2}}{2\varepsilon_0}\hat{a}_N$$

$$= \frac{15\times10^{-9}}{2\times 8.85\times10^{-12}} \times (\hat{a}_y)$$

$$= 847.4576\,\hat{a}_y \text{ v/m}$$



$$\vec{E}_2 = \frac{\rho_2}{2\pi\varepsilon_0 R}\hat{a}_R$$

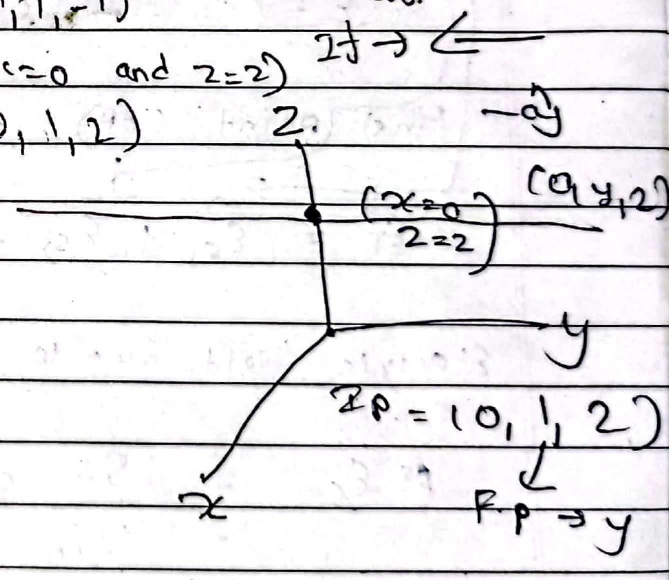
$\vec{R} = ?$ $\hat{a}_R$

FP (1, 1, -1)

IP (x=0 and z=2)

IP (0, 1, 2)

case



$$\vec{R} = \vec{FP} - \vec{IP}$$

$$\Rightarrow (1, 1, -1) - (0, 1, 2)$$

$$= \hat{a}_x - 3\hat{a}_z$$

$$|\vec{R}| = \sqrt{(1)^2 + (-3)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\hat{a}_x - 3\hat{a}_z}{\sqrt{10}}$$

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$$\vec{E_L} = \frac{10\pi \times 10^{-9}}{2\pi \times 8.85 \times 10^{-12} \times \sqrt{10}} \quad \frac{1\hat{a_x} - 3\hat{a_z}}{\sqrt{10}}$$

$$= 56.4971\,\hat{a_x} \quad -169.4915\,\hat{a_z} \quad V/m$$

$$\vec{E_T} = \vec{E_{S_1}} + \vec{E_{S_2}} + \vec{E_L}$$

$$= -508.4746\,\hat{a_x} + 847.4576\,\hat{a_y} - 169.4915\,\hat{a_z}$$

4 Find $\vec{E}$ at origin if the point charge of 12nc is at P (2,0,6) uniform line charge at 3nc/m is at $x=2$ and $y=3$ and uniform surface charge of 0.2 nc/m² is at $x=2$.

Soln:-

F.P → (0,0,0)

2.P at pt charge (2,0,6)

$Q = 12nc$ at J

$\rho_L = 3nc/m$ at $x = 2$ and $y = 3$

$\rho_s = 0.2\,nc/m^2$ at $x = 2$

$$\vec{E_T} = \vec{E_P} + \vec{E_L} + \vec{E_S}$$

$$\vec{E_P} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{a_R}$$

$\vec{R} = (0,0,0) - (2,0,6)$
$= (-2,0,-6)$
$= -2\hat{a_x} - 6\hat{a_z}$.

$$= 9\times10^{9} \times \frac{12\times10^{-9}}{(2\sqrt{10})^2} \times \frac{(-2\hat{a_x} - 6\hat{a_z})}{2\sqrt{10}}$$

$|\vec{R}| = 2\sqrt{10}$

$$= 0.4264\,(-2\hat{a_x} - 6\hat{a_z}) = -0.8531\,\hat{a_x} - 2.5614\,\hat{a_z}\ V/m$$

(4) 7

(Surface ko direction easy method
if +1 to +3 the +ve
if +3 to +2 -ve
check ka bata kana value 3a cha

$$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 R}\hat{a}_R$$

$$= 2 \times \frac{1}{4\pi\varepsilon_0} \times 3\times10^{-9} \times \frac{\vec{R}}{R^2}$$

FP ko 2

$\vec{R}$ = FP = (0,0,0) 2P = (2,3,0)

$$\vec{R} = -2\hat{a_x} - 3\hat{a_y}$$
$$|\vec{R}| = \sqrt{13}$$

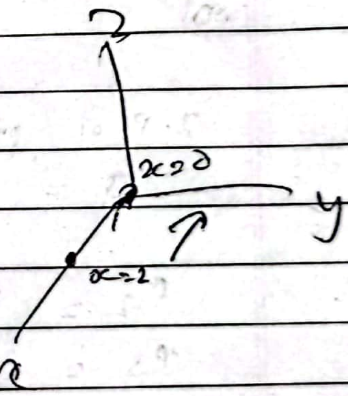
$$\vec{E}_L = \frac{54}{13}\left(-2\hat{a_x} - 3\hat{a_y}\right)$$

$$= -8.30\,\hat{a_x} - 12.46\,\hat{a_y} \text{ V/m}$$

$$\vec{E}_S = \frac{\rho_s}{2\varepsilon_0}\hat{a_N}$$

$$= \frac{0.2\times10^{-9}}{2\times8.85\times10^{-12}} \times (-\hat{a_x})$$

$$= -11.29\,\hat{a_x} \text{ V/m}$$



$$\boxed{\vec{E}_T = -\underset{20.45}{\cancel{\ \ }}\,\hat{a_x} - 12.46\,\hat{a_y} - 2.564\,\hat{a_z} \text{ V/m}}$$

surface ko z ko lagi direction ↑ +ve ↓ -ve

(8)

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An infinite line charge $\rho_L = 2\,nC/m$ lies along X-axis in free space while point charge of $8\,nC$ each are located at $(0,0,1)$ and $(0,0,-1)$. Find $\vec{E}$ in cylindrical co-ordinate system at $(2,3,-4)$

Soln:-

Point charge

$Q_1 = 8\,nC$ at $(0,0,1)$ $\qquad Q_2 = 8\,nC$ $(0,0,-1)$

Line charge

$\rho_L = 2\,nC/m$ along X-axis

IP (Xaxis, 0, 0) no y and z

IP $(2,0,0)$ $\qquad$ FP $(2,3-4)$

$$\vec{E}_1 = \frac{\rho_L}{2\pi\varepsilon_0 R}\,\hat{a}_R$$

$$\vec{R} = 3\hat{a}_y - 4\hat{a}_z \qquad |\vec{R}| = 5$$

$$E_1 = \frac{2\times10^{-9}}{\cancel{2\pi\times8.85\times10^{-12}}} \qquad 2\times2\times10^{-9}\times9\times10^{9}\times\frac{1}{5^2}\,(\vec{R})$$

$$= 1.44\,(3\hat{a}_y - 4\hat{a}_z) = 4.32\,\hat{a}_y - 5.76\,\hat{a}_z$$

⑥

$$\vec{E}_{P_1} = \frac{1}{4\pi\varepsilon_0}\ \frac{Q}{R^3} \times \vec{R}$$

$$\vec{R} = (2,3,4) - (0,0,1)$$
$$= 2\hat{a_x} + 3\hat{a_y} + 3\hat{a_z}$$

$$|\vec{R}| = \sqrt{22}$$

$$\vec{E}_{P_1} = 0.697\ (2\hat{a_x} + 3\hat{a_y} + 3\hat{a_z})$$

$$\Rightarrow 1.39\ \hat{a_x} + 2.091\ \hat{a_y} + 2.091\ \hat{a_z}$$

$$\vec{E}_{P_2} = \rightarrow \quad \vec{R} = (2,3,4) - (0,0,-1)$$
$$= 2\hat{a_x} + 3\hat{a_y} + 5\hat{a_z}$$

$$|\vec{R}| = \sqrt{38}$$

$$= 9\times10^9 \times \frac{8\times10^{-9}}{(\sqrt{38})^3}\ (2\hat{a_x} + 3\hat{a_y} + 5\hat{a_z})$$

$$= 0.61\ \hat{a_x} + 0.922\ \hat{a_y} + 1.53\ \hat{a_z}$$

$$\vec{E}_T = 2.017\ \hat{a_x} + 7.34\ \hat{a_y} - 2.139\ \hat{a_z}$$

convert to cylindrical like in chap 1

(7)

A point charge at 30 nC is located at the origin while plane y = 3 carries charge 10 nC/m². Find $\vec{E}$ at (0, 4, 3).

soln ÷

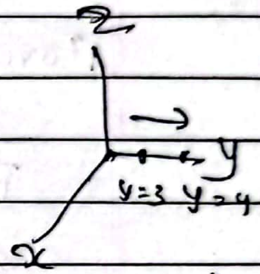
↓ point

IP → (0, 0, 0) Q = 30 nC FP (0, 4, 3)

IP → ∞ Q = 10 nC/m² FP (0, 4, 3)

↓

surface



$$\vec{E} = \vec{E}_p + \vec{E}_s \quad \text{—— (i)}$$

$$\vec{E}_p = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{Q}{4\pi\varepsilon_0 R^3}\vec{R}$$

$(+\hat{a}_y)$

$$\vec{R} = (0,4,3) - (0,0,0) = 4\hat{a}_y + 3\hat{a}_z$$

$$R = \sqrt{(4)^2+(3)^2} = 5$$

$$\vec{E}_p = \frac{30\times10^{-9}}{4\times\pi\times 8.85\times10^{-12}\times 5^3}(4\hat{a}_y + 3\hat{a}_z)$$

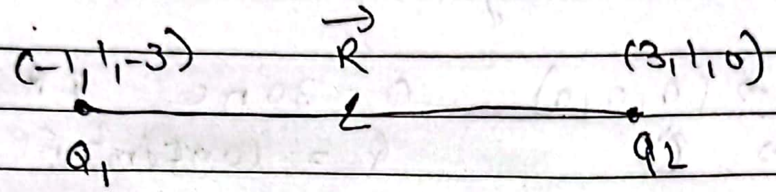
$$= 8.6365\,\hat{a}_y + 6.477\,\hat{a}_z \quad V/m$$

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\hat{a}_N = \frac{10\times10^{-9}}{2\times8.85\times10^{-12}}(\hat{a}_y) = 564.971\,\hat{a}_y \; V/m$$

$$\boxed{\vec{E} = 573.611\,\hat{a}_y + 6.48\,\hat{a}_z \quad V/m}$$

Two point charges $Q_1 = 50\ \mu C$ and $Q_2 = 10\ \mu C$ are located at $(-1, 1, -3)$ m and $(3, 1, 0)$ m respectively. Find the force on Q_1.

soln :-



Force on Q_1 due to Q_2 is :-

$$\vec{F_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a_R} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3}\vec{R}$$

$$\vec{R} = F.P - I.P$$
$$= (-1, 1, -3) - (3, 1, 0)$$
$$= -4\hat{a_x} - 3\hat{a_z}$$

$$|R| = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\vec{F_{21}} = \frac{50\times10^{-6} \times 10\times10^{-6}}{4\pi \times 8.85\times10^{-12} \times 5^3} \times (-4\hat{a_x} - 3\hat{a_z})$$

$$= 0.0359\,(-4\hat{a_x} - 3\hat{a_z})\ N \ \#$$

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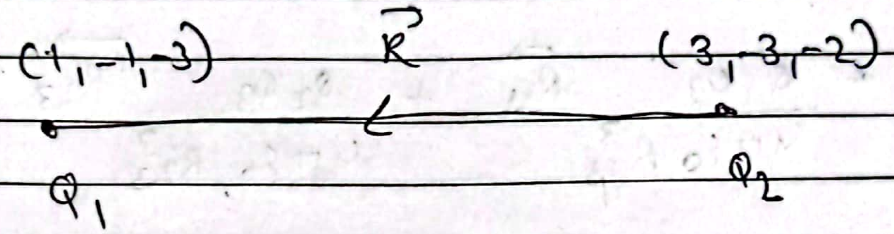
Point charge $Q_1 = 300\,\mu C$ located at $(1,-1,-3)\,m$ experiences a force $\vec{F_1} = 8\hat{a_x} - 8\hat{a_y} + 4\hat{a_z}$ N due to point charge Q_2 at $(3,-3,-2)$ m. Determine Q_2.

Soln:



$$\vec{F_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3}\vec{R}$$

$$\vec{R} = (1,-1,-3) - (3,-3,-2)$$
$$= -2\hat{a_x} + 2\hat{a_y} - \hat{a_z}$$

$$|R| = \sqrt{(-2)^2 + (2)^2 + (-1)^2}$$
$$= 3$$

So,

$$8\hat{a_x} - 8\hat{a_y} + 4\hat{a_z} = \frac{300\times10^{-6}\times Q_2}{4\pi\times8.85\times10^{-12}\times(3)^3}(-2\hat{a_x} + 2\hat{a_y} - \hat{a_z})$$

$\therefore$ $Q_2 = -40\times10^{-6} = -40\,\mu C$

A point charge $Q_1 = 10\,\mu C$ is located at $(1,2,3)$ while $Q_2 = -5\,\mu C$ is located at $(1,2,10)$. Find the co-ordinates at which a point charge Q_3 experiences no force.

Given.

$Q_1 = 10\mu C$ at $(1,2,3)$ $Q_2 = -5\mu C$ $(1,2,10)$

Force erected on Q_3 due to Q_1 and Q_2 is ÷

$$\vec{F} = \vec{F_{13}} + \vec{F_{23}}$$

$$= \frac{Q_1 Q_3}{4\pi\varepsilon_0 R_{13}^3} \vec{R_{13}} + \frac{Q_2 Q_3}{4\pi\varepsilon_0 R_{23}^3} \vec{R_{23}} \quad \text{—— (i)}$$

let Q_3 be located at $(1,2,z)$ such tat $\vec{F} = 0$

F.P because $(1,2$ same in both points $(1,2,10)$ and $(1,2,$

$\vec{R_{13}} = F.P - I.P$

$= (1,2,z) - (1,2,3)$

$= (z-3)\hat{a_z}$

$R_{13} = z-3$

$\vec{R_{23}} = (1,2,z) - (1,2,10) = (z-10)\hat{a_z}$

$R_{23} = z-10$

$$0 = \frac{(10\times10^{-6})\times Q_3}{4\pi\varepsilon_0 (z-3)^3}(z-3)\hat{a_z} + \frac{(-5\times10^{-6})\times Q_3}{4\pi\varepsilon_0 (z-10)^3}(z-10)\hat{a_z}$$

(11)

$$\frac{Q_3 \times 5\times10^{-6}\,\hat{a}_z}{4\pi\varepsilon_0}\left[\frac{2}{(z-3)^2} - \frac{1}{(z-10)^2}\right] = 0$$

Thus,

$$\frac{2}{(z-3)^2} - \frac{1}{(z-10)^2} = 0$$

or, $$z^2 - 34z + 191 = 0$$

Solving for z, we get $z = 7.1,\ 26.899$

Required location of Q_3 are (1, 2, 26.899) and (1, 2, 7.1) #

A uniform sheet of charge with $\rho_s = \left(\frac{1}{3\pi}\right)$ nC/m^2 is located at $z = 5$m and a uniform line charge with $\rho_l = \left(\frac{-25}{9}\right)$ nC/m at $z = -3$m, $y = 3$m. Find $\vec{E}$ at $(x, -1, 0)$ m.

soln:

$$\vec{E} = \vec{E_l} + \vec{E_s} \quad \cdots\cdots (i)$$

P.T.O

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$\rho_L = \left(\frac{-25}{9}\right)$ nC/m at $z = -3$, $y = 3$

I.P $= (x, 3, -3)$ F.P $(x, -1, 0)$

$\vec{R} = (x, -1, 0) - (x, 3, -3)$
$= -4\hat{a}_y + 3\hat{a}_z$

$|\vec{R}| = \sqrt{(-4)^2 + (3)^2} = 5$

$$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 R^2}\vec{R}$$

$$= \frac{-25/9 \times 10^{-9}}{2\pi \times 8.85 \times 10^{-12} \times 5^2}(-4\hat{a}_y + 3\hat{a}_z)$$

$= 8\hat{a}_y - 6\hat{a}_z$ V/m

$$\vec{E}_S = \frac{\rho_S}{2\varepsilon_0}\hat{a}_N = \frac{\frac{1}{3\pi} \times 10^{-9}}{2 \times 8.85 \times 10^{-12}}(-\hat{a}_z)$$

$= -6\hat{a}_z$ V/m

From (i)

$\vec{E} = \vec{E}_L + \vec{E}_S = 8\hat{a}_y - 12\hat{a}_z$ V/m

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Two points charges 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate the electric force on 10nC located at (0, 3, 1) and electric field at that point. (or electric field intensity)

Soln:

$Q_1 = 1mC$ $\quad Q_2 = -2mC$

$(3, 2, -1)$ $\quad (-1, -1, 4)$

Force at Q_3 due to 1 and 2 is:

$$\vec{F} = \vec{F}_{13} + \vec{F}_{23}$$

$$\vec{R}_{13} = (0, 3, 1) - (3, 2, -1) = -3\hat{a_x} + \hat{a_y} + 2\hat{a_z}$$

$$R_{13} = \sqrt{14}$$

$$\vec{R}_{23} = (0, 3, 1) - (-1, -1, 4) = \hat{a_x} + 4\hat{a_y} - 3\hat{a_z}$$

$$R_{23} = \sqrt{26}$$

$$\vec{F} = \frac{Q_1 Q_3}{4\pi\varepsilon_0 R_{13}^3}\vec{R}_{13} + \frac{Q_2 Q_3}{4\pi\varepsilon_0 R_{23}^3}\vec{R}_{23}$$

$$= \cancel{-5.15\times10^{-9}\hat{a_x} + 1.71\times10^{-9}\hat{a_y}} + 3.43\times10^{-9} +$$

$$= -5.15\times10^{-3}\hat{a_x} + 1.71\times10^{-3}\hat{a_y} + 3.43\times10^{-3}\hat{a_z}$$

$$-1.35\times10^{-3}\hat{a_x} - 5.43\times10^{-3}\hat{a_y} + 4.073\times10^{-3}\hat{a_z}$$

$\vec{F} = -6.512 \times 10^{-3}\,\hat{a_x} - 3.712 \times 10^{-3}\,\hat{a_y} + 7.809 \times 10^{-3}\,\hat{a_z}$ N

Now,

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{\vec{F}}{10 \times 10^{-9}}$$

$$= -651.2\,\hat{a_x} - 371.28\,\hat{a_y} + 780.93\,\hat{a_z} \text{ kV/m}$$

~~Two point charges 1nC and -2nC~~

Calculate electric field intensity at a point A (1,2,3) in free space by a charge $Q_1 = 5nC$ at point P(2,3,5) and another charge $Q_2 = 4nC$ at R(3,0,3)

Soln:

$Q_1 = 5nC$ at P (2,3,5)

$Q_2 = 4nC$ at R (3,0,3)

$$\vec{E_1} = \frac{Q_1}{4\pi\varepsilon_0 R_1^3}\vec{R_1}$$

$\vec{R_1} = (1,2,3) - (2,3,5)$

$= -\hat{a_x} - \hat{a_y} - 2\hat{a_z}$

$|\vec{R_1}| = \sqrt{6}$

$= -3.061\,\hat{a_x} - 3.061\,\hat{a_y} - 6.123\,\hat{a_z}$

(15)

$$\vec{E_2} = \frac{Q_2}{4\pi\varepsilon_0 R_2^3}\vec{R_2}$$

$\vec{R_2} = (1,2,3) - (3,0,3)$ $\quad |R_2| = 2\sqrt{2}$

$= 0 - 2\hat{a_x} + 2\hat{a_y}$

$\vec{E_2} = -3.18\,\hat{a_x} + 3.18\,\hat{a_y}.$

$\vec{E} = \vec{E_1} + \vec{E_2} = -6.24\,\hat{a_x} + 0.199\,\hat{a_y} - 6.123\,\hat{a_z}$ N/C

$\rho_{s1} = 3n\ cm^2$ at $z = -4$

$\rho_{s2} = 6n\ cm^2$ at $z = 1$ $\rightarrow$ convert to (m)

$\rho_{s3} = -8n\ cm^2$ at $z = 4$

Find $\vec{E}$ at

(a)

$P_A(2, 5, -5)$

$$\vec{E_A} = \frac{\rho_{s1}}{2\varepsilon_0}(-\hat{a_z}) + \frac{\rho_{s2}}{2\varepsilon_0}(-\hat{a_z}) + \frac{\rho_{s3}}{2\varepsilon_0}(-\hat{a_z})$$

(b) $P_C(-1, -5, 2)$

$$\vec{E_C} = \frac{\rho_{s1}}{2\varepsilon_0}(\hat{a_z}) + \frac{\rho_{s2}}{2\varepsilon_0}(\hat{a_z}) + \frac{\rho_{s3}}{2\varepsilon_0}(-\hat{a_z})$$

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