(1)

Coulomb's law

→ It states that the force between two charges in a vaccum or free space seperated by a distance is directly proportional to the product of two charge and inversely proportional to the square of distance between two charges.

$F \propto Q_1 Q_2$ — (i) $Q_1 \longleftarrow R \longrightarrow Q_2$

$F \propto \frac{1}{R^2}$ — (ii)

So,

$F \propto \frac{Q_1 Q_2}{R^2} \Rightarrow F = K \frac{Q_1 Q_2}{R^2}$

$\left(K = \frac{1}{4\pi\varepsilon_0} \right) = 9 \times 10^9$ [Free space)

ε_0 = permittivity of free space $= 8.854 \times 10^{-12}$ F/m

For any other medium

$K = \frac{1}{4\pi\varepsilon}$, ε = permittivity of the medium $= \varepsilon_0 \varepsilon_r$

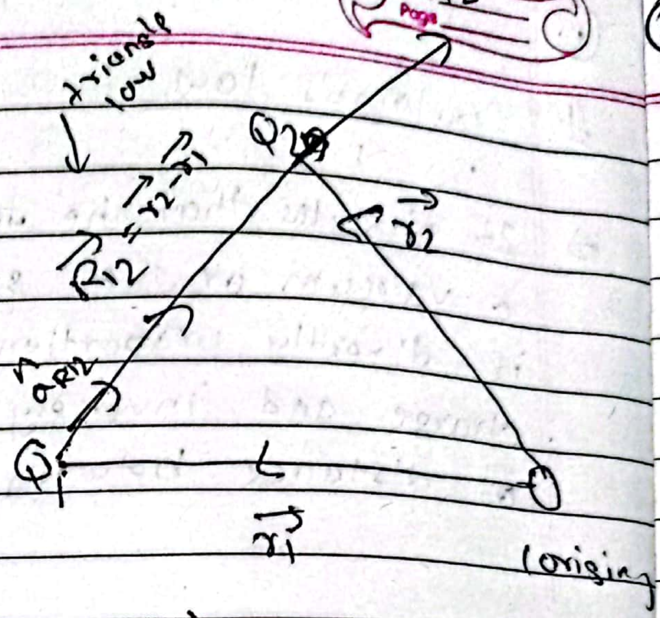
ε_0 ↓ free space; ε_r ↑ permittivity of the particular medium

$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$ (for vacuum of free space)

Vector Expression:

→ Consider as shown above, two charges Q_1 and Q_2

From columb's low,



$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 (R_{12})^2}\,\hat{a}_{R12}$$

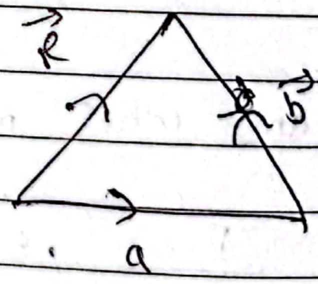
$\vec{R_{12}} = -\vec{r_1} + \vec{r_2}$

where, $\hat{a}_{R12}$ = unit vector that gives direction of $\vec{F_{12}}$

$$= \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{R_{12}}}{R_{12}}$$

or $$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\frac{\vec{R_{12}}}{R_{12}} = \frac{Q_1 Q_2 \vec{R_{12}}}{4\pi\varepsilon_0 R_{12}^3} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\frac{\vec{R_{12}}}{|\vec{R_{12}}|^3}$$

∴ $$\vec{F_{12}} = \frac{Q_1 Q_2 (\vec{r_2} - \vec{r_1})}{4\pi\varepsilon_0 |\vec{r_2} - \vec{r_1}|^3}$$



$\vec{R} = \vec{a} + \vec{b}$

(Triangle law)

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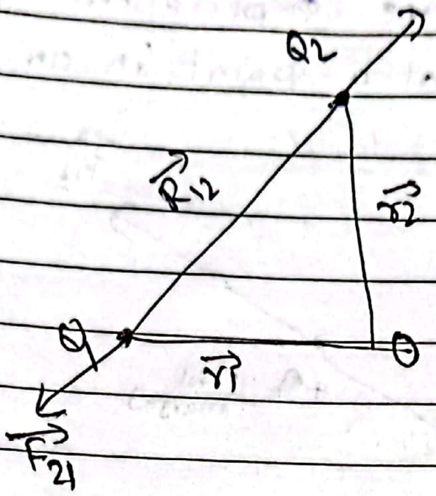
$\vec{F_{12}}$ → force exerted by 1 on 2



→ force exerted by charge Q_2 on Q_1

$$\vec{F_{12}} = -\vec{F_{21}}$$

Same sign

If Q_1 and Q_2 → both are positive.

$Q_1 \leftarrow \quad Q_2 \rightarrow$ repel

Q_1 and Q_2 → both negative

$\underset{q_1}{\leftarrow} \quad \underset{q_2}{\rightarrow}$ repel

If q_1 (+ve) q_2 (−ve) $\overset{(+)}{Q_1} \rightarrow \leftarrow \overset{(-)}{Q_2}$

(attractive)

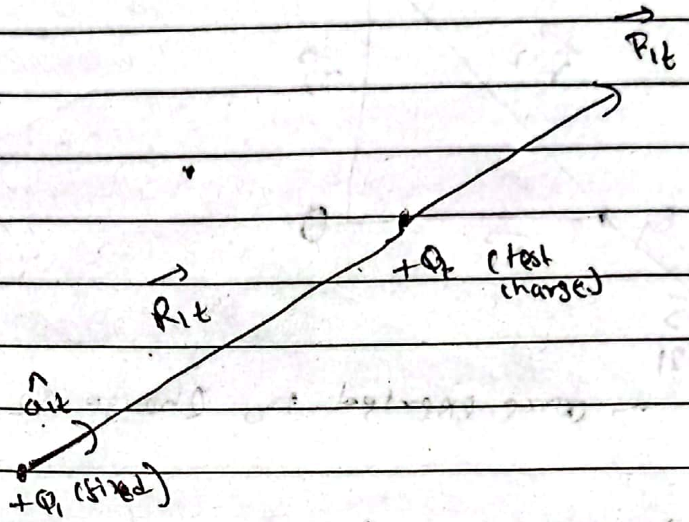
If Q_1 is multiplied by n then,

$$\vec{F_{12}} = n \cdot \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \frac{\vec{R}}{|R|}$$

(superposition and linearity)

Electrical Field Intensity.

→ measure of the force experienced by a unit positive charge at a point in an electric field.



let Q_1 and Q_t both positive charge, then the direction of force on Q_t due to Q_1 is given by $\hat{a}_{1t}$ (directed from Q_1 to Q_t)

From Columb's law,

$$\vec{F}_{1t} = \frac{Q_1 Q_t}{4\pi\varepsilon_0} \frac{\hat{a}_{1t}}{R_{1t}^2}$$

electric field intensity is the force per unit test charge

$$\vec{E}_{1t} = \frac{\vec{F}_{1t}}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2} \hat{a}_{1t}$$

where $\hat{a}_{1t} = \frac{\vec{R}_{1t}}{R_{1t}}$

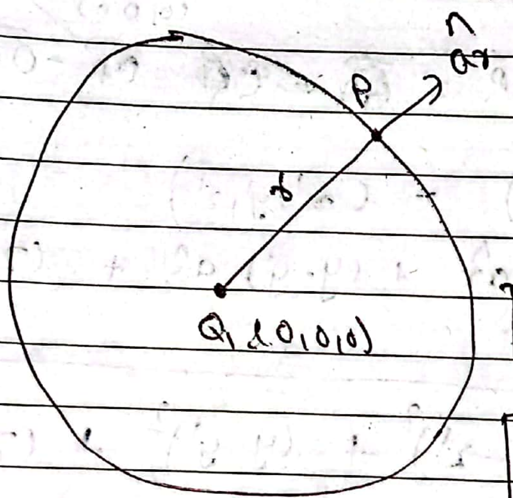
(5)

$\therefore$ $\vec{E}_{1t} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_1}{|\vec{R}_{1t}|^3} (\vec{R}_{1t})$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{|\vec{R}|^3} (\vec{R})}$$ General

Unit V/m

Qn. In spherical co-ordinate system



$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} \hat{a}_r}$$

$$\boxed{\vec{E} = E_r \hat{a}_r}$$

$$\boxed{E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2}}$$

Electric field Intensity due to a Point charge.

→ Consider a point charge Q is located at (x', y', z') and (x, y, z) be the point where electric field intensity is to be determined

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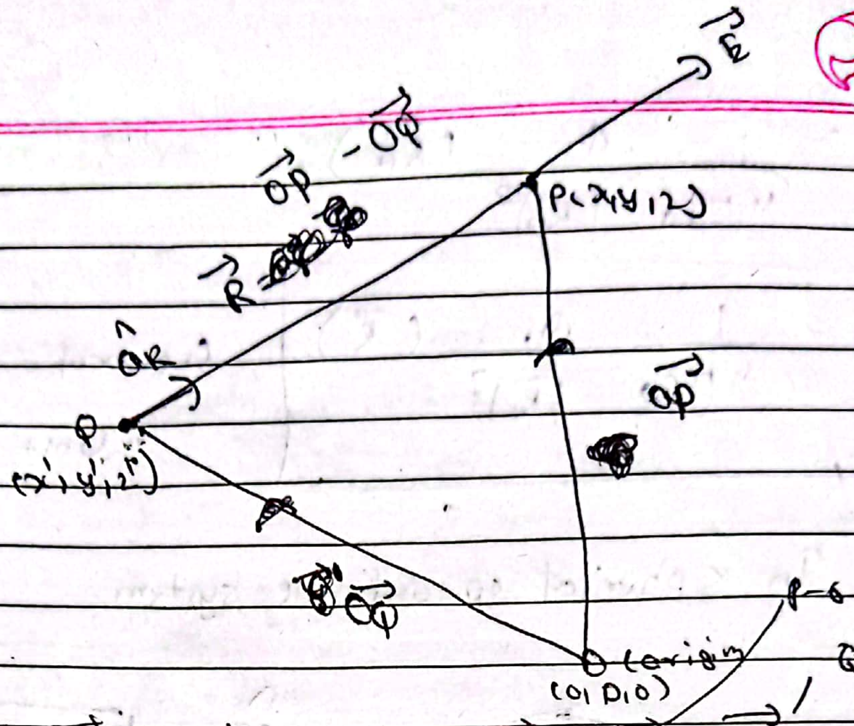


$$\vec{QP} = \vec{R} = \vec{OP} - \vec{OQ}$$

$$= (x,y,z) - (x',y',z')$$

$$= (x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z = \vec{R}$$

$$|\vec{R}| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

Now

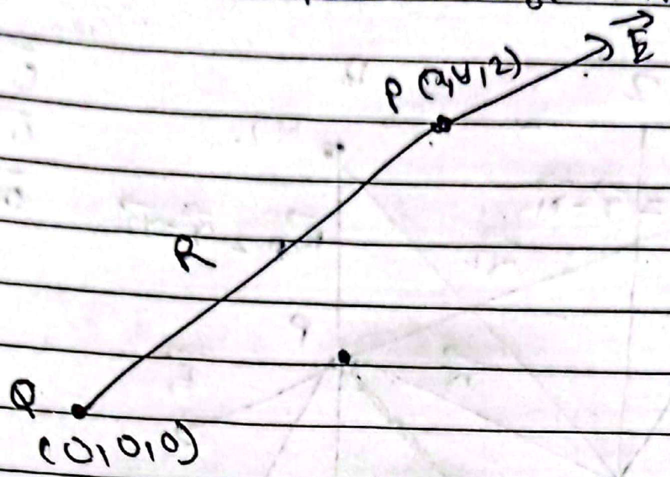
$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{a}_R}$$

$$= \frac{1}{4\pi\varepsilon_0} Q \cdot \frac{(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z}{\left(\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}\right)^3}$$

⑦

Electric field due to point charge kept at origin:



from previous

$$\vec{R} = (x-x')\hat{a_x} + (y-y')\hat{a_y} + (z-z')\hat{a_z}$$

Here, Q is at origin so

$$\vec{R} = x\hat{a_x} + y\hat{a_y} + z\hat{a_z}$$

$$|\vec{R}| = \sqrt{x^2+y^2+z^2}$$

$$\hat{a_R} = \frac{x a_x + y\hat{a_y} + z\hat{a_z}}{\sqrt{x^2+y^2+z^2}}$$

$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0} Q \frac{x\hat{a_x} + y\hat{a_y} + z\hat{a_z}}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

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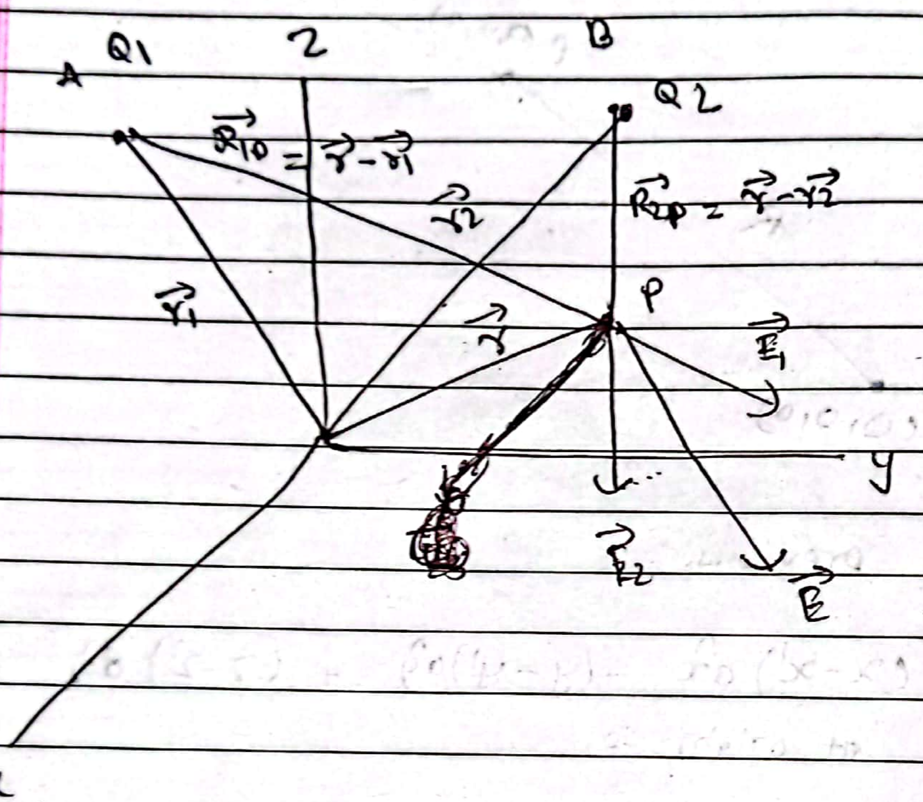
Electric Field Intensity due to n Point charges:



$\vec{OA} = \vec{r_1}$

$\vec{OB} = \vec{r_2}$

$\vec{OP} = \vec{r_3}$

$$\boxed{\vec{E_1} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{|\vec{r} - \vec{r_1}|^2} \cdot \hat{a}_{1p}}$$

$\vec{R}_{1p} = \vec{AP} = \vec{OP} - \vec{OA}$

$= \vec{r} - \vec{r_1}$

$\vec{R}_{1p} = \vec{r} - \vec{r_1}$

$$\boxed{\vec{E_2} = \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{|\vec{r} - \vec{r_2}|^2} \hat{a}_{2p}}$$

$$\boxed{\hat{a}_{1p} = \frac{\vec{r} - \vec{r_1}}{|\vec{r} - \vec{r_1}|}}$$

$\vec{E} = \vec{E_1} + \vec{E_2}$

~~a 2p~~

$$\boxed{\hat{a}_{2p} = \frac{\vec{r} - \vec{r_2}}{|\vec{r} - \vec{r_2}|}}$$

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$$= \frac{Q_1}{4\pi\varepsilon_0 |\vec{r}-\vec{r_1}|^2}\hat{a}_{1p} + \frac{Q_2}{4\pi\varepsilon_0 |\vec{r}-\vec{r_2}|^2}\hat{a}_{2p}$$

If there are n point charges then

$$\vec{E} = \vec{E_1} + \vec{E_2} + \dots + \vec{E_n}$$

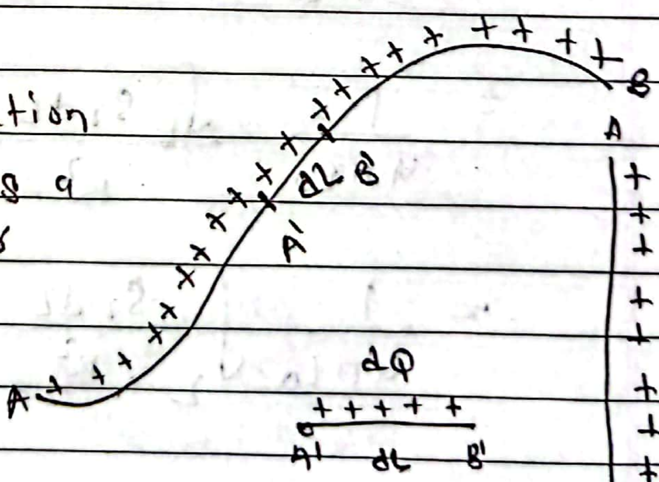
$$= \frac{Q_1}{4\pi\varepsilon_0 |\vec{r}-\vec{r_1}|^2}\hat{a}_{1p} + \frac{Q_2}{4\pi\varepsilon_0 |r-\vec{r_2}|^2}\hat{a}_{2p} + \dots + \frac{Q_n}{4\pi\varepsilon_0 |\vec{r}-\vec{r_n}|^2}\hat{a}_{np}$$

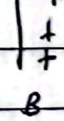
$$\therefore \boxed{\vec{E} = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0 |\vec{r}-\vec{r_m}|^2}\hat{a}_{mp}} \qquad \boxed{\hat{a}_{np} = \frac{\vec{r}-\vec{r_n}}{|\vec{r}-\vec{r_n}|}}$$

Charge Distribution

1. Line charge

→ ~~Charge charge~~ distribution of electric charge along a one-dimensional line or curve in space.





$$\rho_L = \frac{Q}{L}$$

↳ line charge density

C/m

$$\rho_L = \frac{dQ}{dL}$$

$$\therefore \rho_L \, dL = dQ$$

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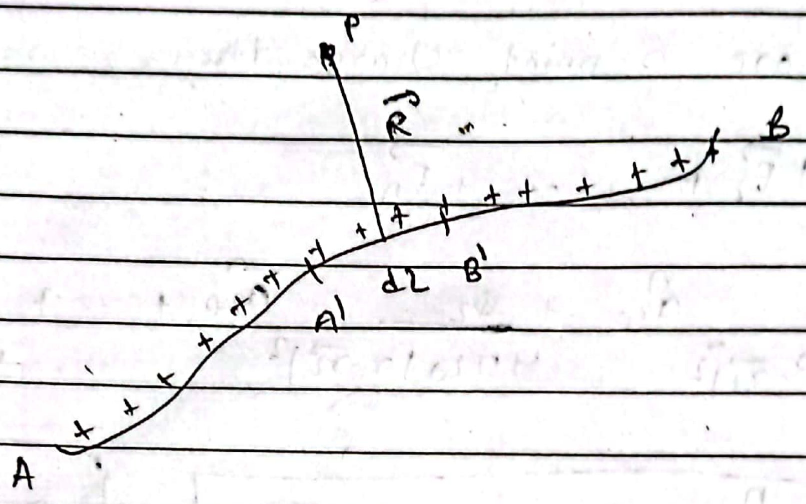
$$\boxed{\phi = \int_{AB\,||\,L} \rho_L \, dL} \quad \boxed{\phi = \int_L \rho_L \, dL}$$



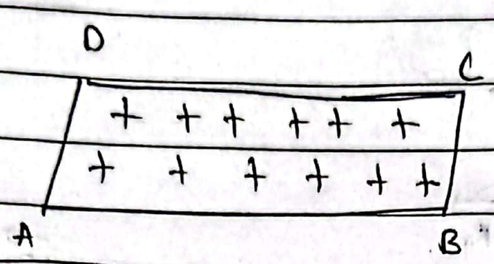
Electric ~~fiend~~ field intensity at P due to line charge

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{a}_R$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\int_L \rho_L dL}{R^2} \hat{a}_R$$

$$= \frac{1}{4\pi\varepsilon_0} \int_L \frac{\rho_L dL}{R^2} \hat{a}_R$$

2. Surface charge Density (ρ_s)



$$\boxed{\rho_s = \frac{Q}{S}} \quad C/m^2 \qquad \rho_s = \frac{dQ}{dS}$$

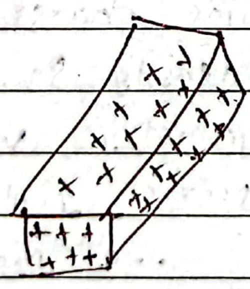
$$dQ = \rho_s \, dS$$

$$\boxed{Q = \int_S \rho_s \, dS \quad (ABCD)}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{a}_R$$

$$= \int \frac{1}{4\pi\varepsilon_0} \int_S \frac{\rho_s \, dS}{R^2} \hat{a}_R$$

3. Volume charge Density (ρ_v)



$$\rho_v = \frac{Q}{V} \quad (C/m^3)$$

$$\rho_v = \frac{dQ}{dV}$$

$$dQ = \rho_v \, dV$$

$$Q = \int_{vol} \rho_v \, dV$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{vol} \frac{\rho_v \, dv}{R^2} \hat{a}_R$$

(Imp)

Electric field intensity due to infinitely long line charge

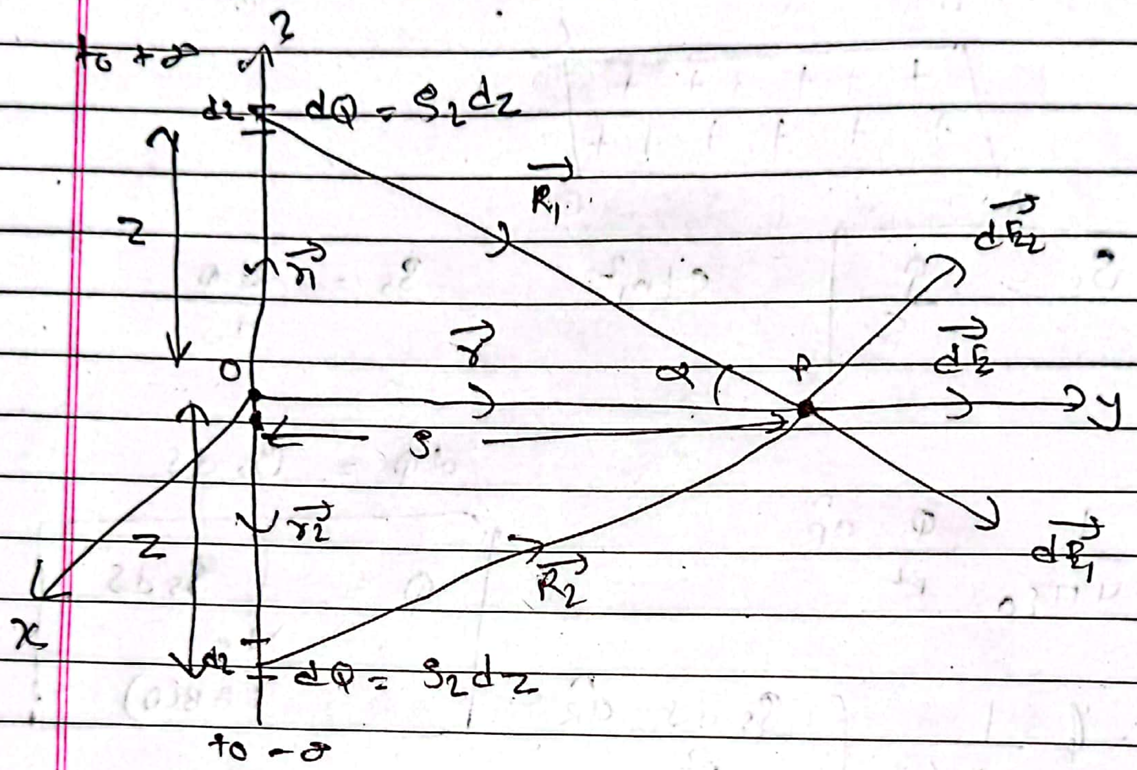


Fig: A uniform infinite line charge extending along the entire z-axis

Consider a uniform infinite line charge is placed in z-axis having line charge density ρ_L. Since the problem has cylindrical symmetry we work with cylindrical co-ordinates. Electric field only vary if we vary ρ and mantain ρ and ϕ constant rest no vary.

Let P be the point on the y-axis where electric field intensity has to be calculated.

(13)

Electric field intensity at point P due to dQ is :-

$$d\vec{E}_1 = \frac{dQ}{4\pi\epsilon_0 R_1^2}\hat{a}_{R_1} = \frac{dQ}{4\pi\epsilon_0 R_1^2}\frac{\vec{R_1}}{R_1} = \frac{dQ}{4\pi\epsilon_0 R_1^3}\vec{R_1}$$

By triangle law

$\vec{r_1} + \vec{R_1} = \vec{r}$ $\qquad \vec{R_1} = \vec{r} - \vec{r_1}$

$= \rho\hat{a}_\rho - z\hat{a}_z$ → Cylindrical system

$$R_1 = |\vec{R_1}| = \sqrt{\rho^2 + (-z)^2} = \sqrt{\rho^2 + z^2}$$

So

$$d\vec{E}_1 = \frac{dQ\,(\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\epsilon_0\,(\sqrt{\rho^2+z^2})^3} = \frac{dQ\,(\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\epsilon_0\,(\rho^2+z^2)^{3/2}}$$

$$d\vec{E}_2 = \frac{dQ}{4\pi\epsilon_0 R_2^3}\vec{R_2}$$

$\vec{r_2} + \vec{R_2} = \vec{r}$ $\qquad \therefore \vec{R_2} = \cancel{\vec{r} + \vec{r_2}}$

$= \cancel{\rho\hat{a}_\rho - z\hat{a}_z}$

$$R_2 = \vec{r} - \vec{r_2}$$

$$= \rho\hat{a}_\rho - [z(-\hat{a}_z)] = \rho\hat{a}_\rho + z\hat{a}_z$$

$$R_2 = |\vec{R_2}| = \sqrt{\rho^2 + z^2}$$

$$\therefore d\vec{E}_2 = \frac{dQ\,(\rho\hat{a}_\rho + z\hat{a}_z)}{4\pi\epsilon_0\,(\rho^2+z^2)^{3/2}}$$

$$d\vec{E} = d\vec{E_1} + d\vec{E_2}$$

$$= \frac{dQ}{4\pi\varepsilon_0} \frac{(\rho\hat{a}_\rho - z\hat{a}_z)}{(\rho^2+z^2)^{3/2}} + \frac{dQ}{4\pi\varepsilon_0} \frac{(\rho\hat{a}_\rho + z\hat{a}_z)}{(\rho^2+z^2)^{3/2}}$$

$$= \frac{dQ}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}} [\rho\hat{a}_\rho - \cancel{z\hat{a}_z} + \rho\hat{a}_\rho + \cancel{z\hat{a}_z}]$$

$$= \frac{2\rho\, dQ}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}} \hat{a}_\rho$$

Finally Electric field intensity at point P due to infinetely long line charge can be obtained by integrating $d\vec{E}$ from 0 to ∞ (since we have calulate $d\vec{E}$ as a result at two dQ)

$$\vec{E} = \int_0^\infty d\vec{E} \qquad (dQ = \rho_L dz)$$

$$= \int_0^\infty \frac{2\rho\, dQ}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}} \hat{a}_\rho = \frac{2\rho\rho_L \hat{a}_\rho}{4\pi\varepsilon_0} \int_0^\infty \frac{dz}{(\rho^2+z^2)^{3/2}} \quad \text{---(1)}$$

let $I = \int_0^\infty \frac{dz}{(\rho^2+z^2)^{3/2}}$ convert z to α form

put $z = \rho\tan\alpha$ $\quad$ $\left(\because \tan\alpha = \frac{z}{\rho}\right)$

$dz = \rho\sec^2\alpha\, d\alpha$

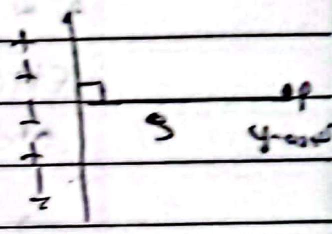
$z = 0 \Rightarrow \alpha = 0$ and $z = \infty \Rightarrow \alpha = \frac{\pi}{2}$

$$\sec^2 - \tan^2 = 1$$

$$I = \int_0^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{\cancel{\rho^3 \sec^3\alpha} \; (\rho^2 + \rho^2 \tan^2\alpha)^{3/2}}$$

$$= \int_0^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{(\rho^2 \sec^2\alpha)^{3/2}} = \int_0^{\pi/2} \frac{\rho \sec^2\alpha \, d\alpha}{\rho^3 \sec^3\alpha}$$

$$= \frac{1}{\rho^2} \int_0^{\pi/2} \cos\alpha \, d\alpha = \frac{1}{\rho^2} [\sin\alpha]_0^{\pi/2}$$



$$= \frac{1}{\rho^2} [1 - 0] = \frac{1}{\rho^2}$$

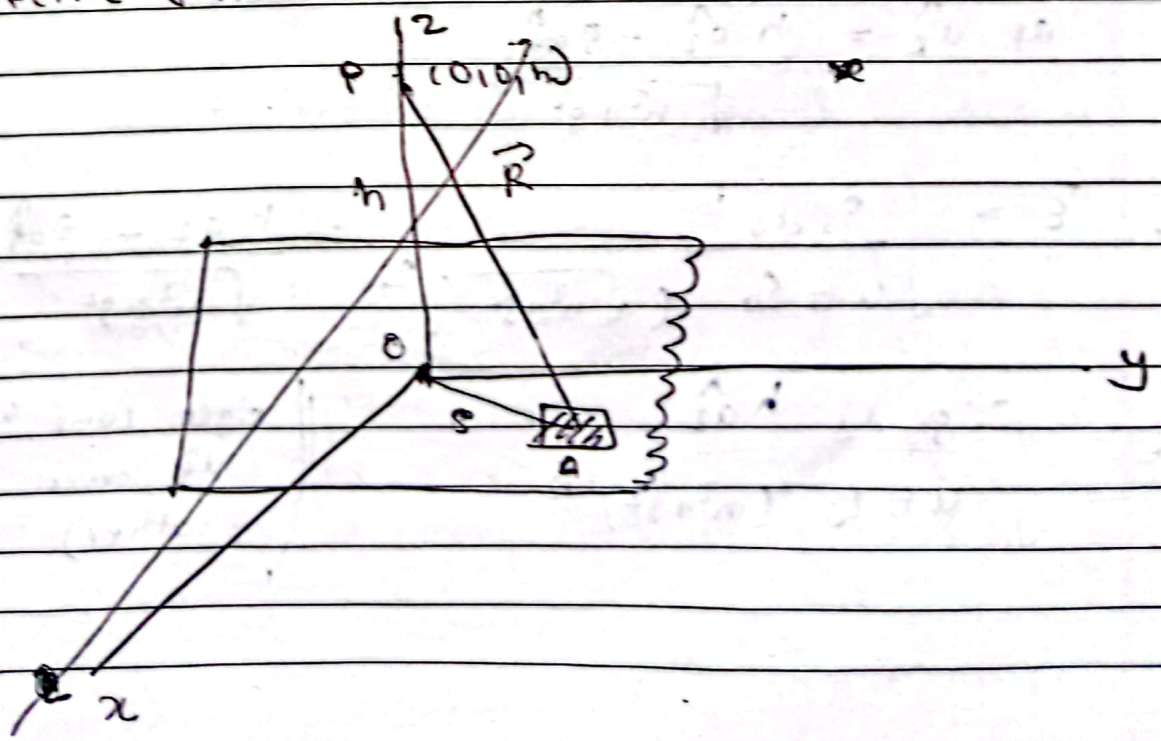
$$\therefore \boxed{\vec{E} = \frac{2\rho\, \rho_L}{4\pi\varepsilon_0} \hat{a}_\rho \cdot \frac{1}{\rho^2} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \hat{a}_\rho}$$

final

In general $\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 R} \hat{a}_R}$

Imp

Electric field due to Infinite sheet of charge:



$$\cancel{\vec{E} = \int_{s} \frac{\rho_s\, ds\, h\, \hat{a}_z}{4\pi\epsilon_0 (h^2+\rho^2)^{3/2}}}$$

$\cancel{ds =}$

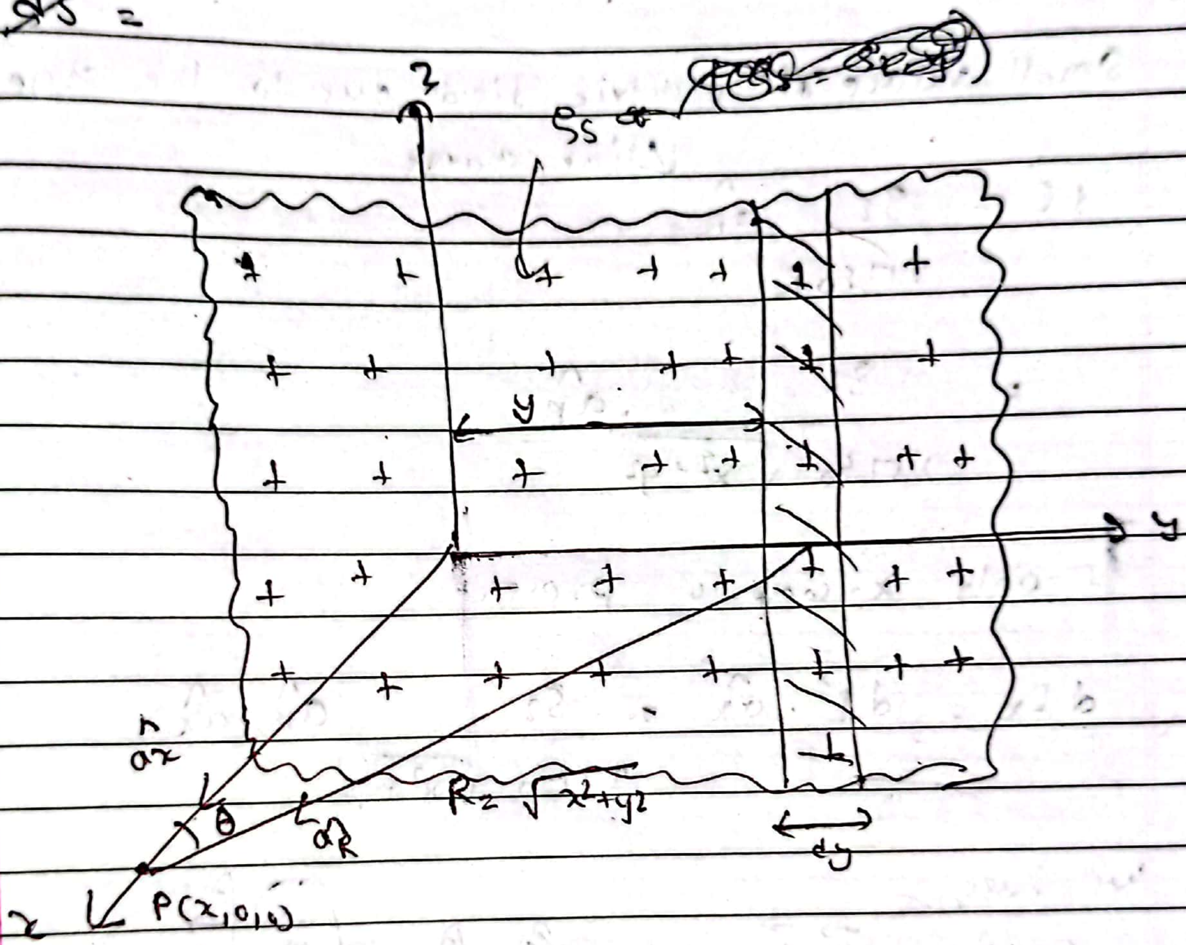


Fig: An infinite sheet of charge in the yz plane

Consider an infinite sheet of charge having a uniform surface charge density ρ_s (C/m²) placed in yz plane i.e. $x=0$ plane. Due to symmetry of the sheet of charge, we can be sure that field cannot vary with y or with z, but only along x-axis.

let P $(x, 0, 0)$ be the point on x-axis where the value of electric field intensity is to be determined.

Small value of electric field due to the strip is :-

line charge

$$d\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 R}\,\hat{a}_R$$

$$= \frac{\rho_L}{2\pi\varepsilon_0\sqrt{x^2+y^2}}\,\hat{a}_R$$

As only x-comp is present

$$dE_x = d\vec{E}\cdot\hat{a}_x = \frac{\rho_L}{2\pi\varepsilon_0\sqrt{x^2+y^2}}\,\hat{a}_R\cdot\hat{a}_x$$

we have,

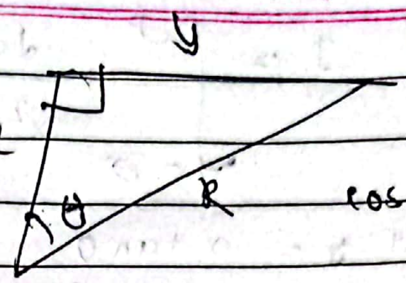
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\vec{R}}{|\vec{R}|}\cdot\frac{\vec{x}}{|\vec{x}|}$$

$$= \hat{a}_R\cdot\hat{a}_x$$

→ $\frac{\text{vector}}{\text{mag}}$

$$dE_x = \frac{\rho_L}{2\pi\varepsilon_0\sqrt{x^2+y^2}}\cos\theta$$

From fig



$$\cos\theta = \frac{x}{R}$$

$$dE_x = \frac{\rho_L}{2\pi\varepsilon_0 \sqrt{x^2+y^2}} \cdot \frac{x}{\sqrt{x^2+y^2}}$$

= ~~$\rho_L \cos$~~

= ~~from fig~~ ~~Sheet~~

We know,

$$\rho_L = \rho_S \cdot dy$$

$\downarrow$ C/m $\quad$ $\downarrow$ C/m² $\quad$ $\downarrow$ m

$$dE_x = \frac{\rho_S\, dy \cdot x}{2\pi\varepsilon_0 (x^2+y^2)} = \frac{x\,\rho_S\, dy}{2\pi\varepsilon_0 (x^2+y^2)}$$

Now,

Total electric field due to entire infinite sheet

$$E_x = \int_{-\infty}^{\infty} dE_x = \int_{y=-\infty}^{\infty} \frac{x\,\rho_S\, dy}{2\pi\varepsilon_0 (x^2+y^2)}$$

$$= \frac{\rho_S x}{2\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{dy}{(x^2+y^2)} \quad \text{——— (i).}$$

(20)

let $I = \int_{-\infty}^{\infty} \frac{dy}{x^2+y^2}$

put $y = x\tan\theta$ $\longrightarrow$ w.r.t θ $\boxed{\tan\theta = \frac{y}{x}}$

$\therefore dy = x\sec^2\theta \, d\theta$

$y = -\infty \Rightarrow \theta = -\frac{\pi}{2}$ and $y = \infty \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_{-\pi/2}^{\pi/2} \frac{x\sec^2\theta \, d\theta}{x^2 + x^2\tan^2\theta}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{x\cancel{\sec^2\theta}\, d\theta}{x^2\cancel{\sec^2\theta}} = \int_{-\pi/2}^{\pi/2} \frac{1}{x} d\theta$$

$$= \frac{1}{x}[\theta]_{-\pi/2}^{\pi/2} = \frac{\pi}{x}$$

Now, from (i)

$$\boxed{E_x = \frac{\rho_s \cancel{x}}{2\cancel{\pi}\varepsilon_0} \frac{\cancel{\pi}}{\cancel{x}} = \frac{\rho_s}{2\varepsilon_0}}$$

In vector form. If P is at $(-x,0,0)$

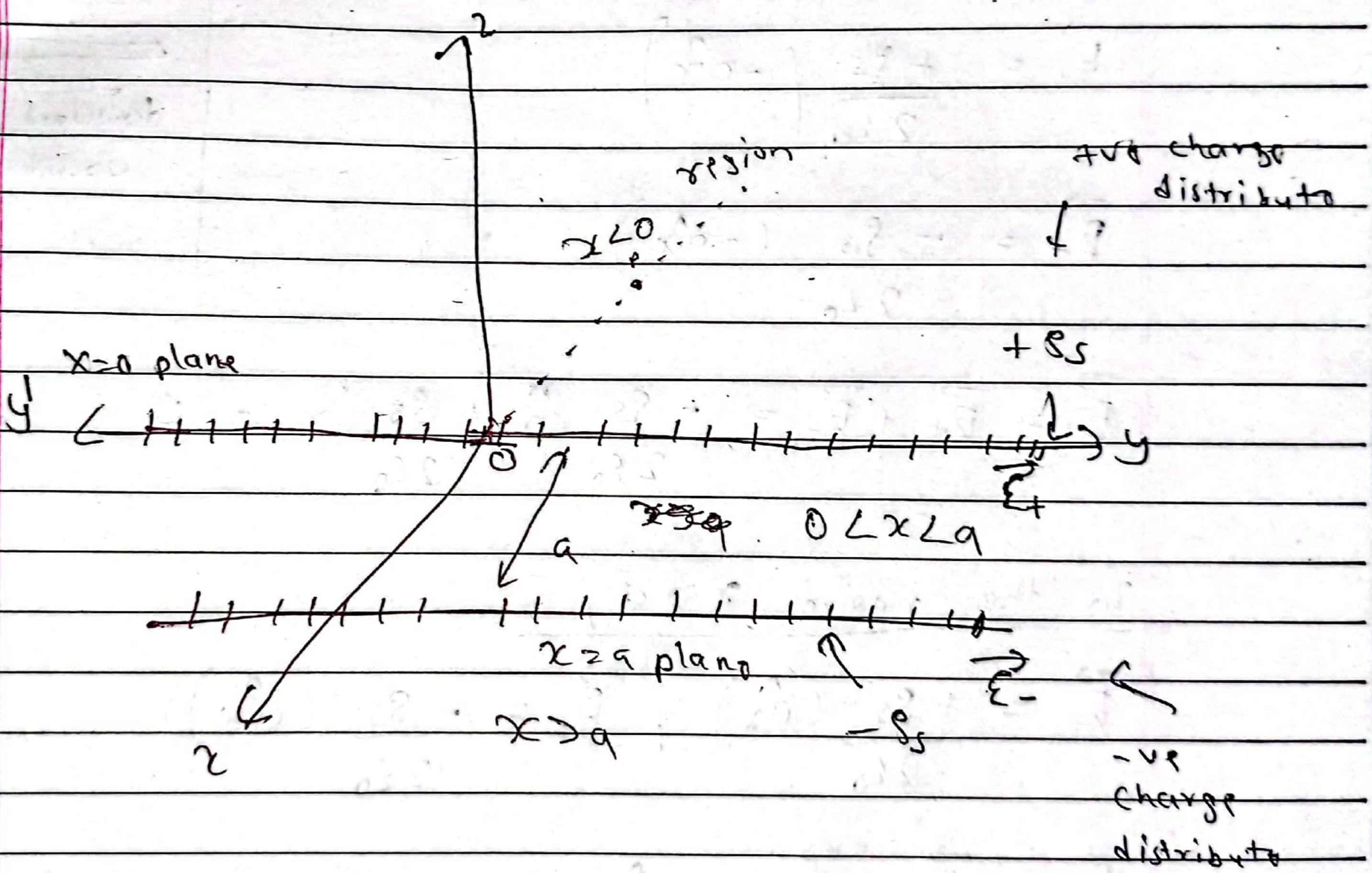
$$\boxed{\vec{E_x} = \frac{\rho_s}{2\varepsilon_0}\hat{a_x}} \qquad \boxed{\vec{E_x} = \frac{\rho_s}{2\varepsilon_0}(-\hat{a_x})}$$

In general

$$\boxed{\vec{E_s} = \frac{\rho_s}{2\varepsilon_0}\hat{a_N}}$$ where, $\hat{a_N}$ = unit vector normal to sheet and directed away from it.

Imp

Electric field due to parallel plate capacitor (3-marks)



consider two infinite sheet of charge placed on x = 0 and x = a plane. let $+\rho_s$ and $-\rho_s$ be surface charge density on these sheets. This arrangement is parallel plate capacitor with air in between. let $\vec{E_+}$ and $\vec{E_-}$ be electric field

(22)

$$\vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_N$$

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due to infinite sheets having +ve and −ve charges respectively.

pathadi −ve infinite charge sheet hannd

ko $\hat{a}_N$

agadi ko +ve

<u>In the region $x < 0$</u>

$$\cancel{\vec{E}_+ = \frac{+\rho_s}{2\epsilon_0}(-\hat{a}_x) + \frac{(-\rho_s)}{2\epsilon_0}(-\hat{a}_x)}$$

$$=$$

dos $\hat{a}_N$

$\hat{a}_N$

downward −ve

upward +ve

$$\vec{E}_+ = \frac{+\rho_s}{2\epsilon_0}(-\hat{a}_x)$$

$$\vec{E}_- = \frac{-\rho_s}{2\epsilon_0}(-\hat{a}_x)$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = -\frac{\rho_s}{2\epsilon_0}\hat{a}_x + \frac{\rho_s}{2\epsilon_0}\hat{a}_x = 0.$$

<u>In the region $x > a$,</u>

$$\vec{E}_+ = \frac{+\rho_s}{2\epsilon_0}(\hat{a}_x), \quad \vec{E}_- = \frac{-\rho_s}{2\epsilon_0}(\hat{a}_x)$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 0$$

<u>In the region $0 < x < a$</u>

$$\vec{E}_+ = +\frac{\rho_s}{2\epsilon_0}(+\hat{a}_x)$$

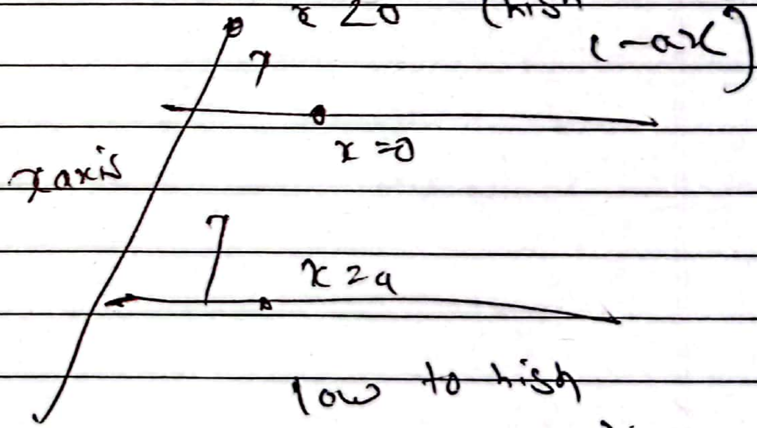
$$\vec{E}_- = \frac{-\rho_s}{2\epsilon_0}(-\hat{a}_x) = \frac{\rho_s \hat{a}_x}{2\epsilon_0}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\cancel{2}\rho_s}{\cancel{2}\epsilon_0}\hat{a}_x$$

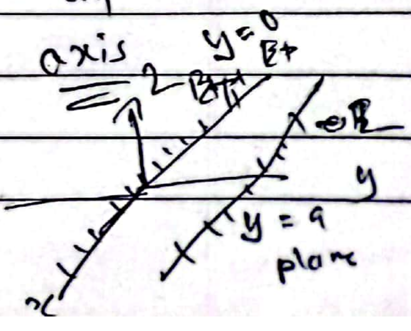
$$= \frac{\rho_s \hat{a}_x}{\epsilon_0}$$

Hence, electric field exists only between the capacitor.

(~~numerical ko logic simple~~



(Can be asked for along y = a axis also use similar method) for all axis



(~~Same result as~~ similar process as above)