1. Find the amplitude of the displacement current density:

a. Next to your radio, in air, where local FM station provides a carrier having $\vec{H} = 0.4\cos[2.10(3\times10^8 t - x)]\,\hat{a}_z$ A/m.

b. In air-space within a large power transformer where $\vec{B} = 1.1\cos[1.257\times10^{-6}(3\times10^8 t - y)]\,\hat{a}_x$ T

c. Inside a large oil filled power capacitor where $\vec{E} = 80\cos[6.277\times10^8 t - 2.092y]\,\hat{a}_z$ V/m, $\varepsilon_r = 6$.

d. Inside a typical metallic conductor where f = 1 KHz, $\sigma = 5\times10^7$ ℧/m, $\varepsilon_r = 1$, and $\vec{J} = 10^7 \sin(6283t - 444z)\,\hat{a}_x$ A/m²

e. Inside a capacitor where $\varepsilon_r = 600$ and $\vec{D} = 3\times10^{-6}\sin(6\times10^6 t - 0.3464x)\,\hat{a}_z$ C/m²

Soln :-

a $\nabla\times\vec{H} = \vec{J} + \vec{J}_d$ → displacement current density

For air, $\vec{J} = 0$

$$\vec{J}_d = \nabla\times\vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y$$

$$+ \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$$

$$\vec{H} = 0.4\cos[2.10(3\times10^8 t - x)]\hat{a}_z \text{ A/m}$$
$$= H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$$

$H_x = 0$, $H_y = 0$, $H_z = 0.4\cos[2.10(3\times10^8 t - x)]$

$$\vec{J}_d = 0 + 0.4\sin[2.10(3\times10^8 t - x)](0 - 2.10)$$
$$= -0.84\sin[2.10(3\times10^8 t - x)]$$

Amplitude, $J_{do} = 0.84\ A/m^2$

(b) $\nabla \times \vec{H} = \vec{J} + \vec{J}_d$

$(\vec{B} = \mu \vec{H})$

For, air, $\vec{J} = 0$

air

$$\vec{J}_d = \nabla \times \vec{H} = \nabla \times \frac{\vec{B}}{\mu} = \nabla \times \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0}(\nabla \times \vec{B})$$

$$\vec{B} = 1.1 \cos\left[1.257 \times 10^{-6} (3 \times 10^8 t - y)\right] \hat{a}_x$$
$$= B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$B_x = 1.1 \cos\left[1.257 \times 10^{-6} (3 \times 10^8 t - y)\right]$, $B_y = 0$, $B_z = 0$

$$\nabla \times \vec{B} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{a}_z$$

$$= \left(0 - \left(-1.1 \sin\left[1.257 \times 10^{-6} (3 \times 10^8 t - y)\right] \times -1.257 \times 10^{-6}\right)\right)\hat{a}_z$$

$$= -1.3827 \times 10^{-6} \sin\left[1.257 \times 10^{-6} (3 \times 10^8 t - y)\right] \hat{a}_z$$

$$\vec{J}_d = \frac{1}{\mu_0}(\nabla \times \vec{B})$$

$$= -1.10031 \sin\left[1.257 \times 10^{-6} (3 \times 10^8 t - y)\right] \hat{a}_z \ A/m^2$$

$\therefore$ Amplitude, $J_{do} = 1.10031\ A/m^2$

(c) $\vec{J_d} = \dfrac{\partial \vec{D}}{\partial t} = \dfrac{\varepsilon \, \partial \vec{E}}{\partial t} = \varepsilon_0 \varepsilon_r \dfrac{\partial \vec{E}}{\partial t}$

$(\vec{D} = \varepsilon \vec{E})$

$= 8.85 \times 10^{-12} \times 6 \times \dfrac{\partial}{\partial t}\left(80 \cos[6.277 \times 10^8 t - 2.092y] \hat{a}_z\right)$

$= 8.85 \times 10^{-12} \times 6 \times 80 \times -\sin[6.277 \times 10^8 t - 2.092y] \cdot$
$6.277 \times 10^8 \, \hat{a}_z$

$= -2.6676 \sin[6.277 \times 10^8 t - 2.092y] \, \hat{a}_z \;\; A/m^2$

Amplitude, $J_{do} = 2.6676 \; A/m^2$

(d) $\vec{J_d} = \varepsilon_0 \varepsilon_r \dfrac{\partial \vec{E}}{\partial t}$ —

remember

$\vec{J} = \sigma \vec{E} \Rightarrow \vec{E} = \dfrac{\vec{J}}{\sigma} = 0.2 \sin(6283t - 444z) \, \hat{a}_x \; V/m$

$\vec{J_d} = \varepsilon_0 \times 1 \times \dfrac{\partial}{\partial t}\left(0.2 \sin(6283t - 444z) \hat{a}_x\right)$

$= \varepsilon_0 \times 1 \times 0.2 \cos(6283t - 444z) \cdot 6283 \, \hat{a}_x$
$= 1.1106 \times 10^{-8} \cos(6283t - 444z) \, \hat{a}_x \;\; A/m^2$

Amplitude, $J_{do} = 1.1106 \times 10^{-8} \; A/m^2$

(e) $\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}\left(3\times10^{-6} \sin(6\times10^{6}t - 0.3464x)\,\hat{a}_z\right)$

$= 3\times10^{-6} \cos(6\times10^{6}t - 0.3464x) \times 6\times10^{6}\,\hat{a}_z$

$= 18 \cos(6\times10^{6}t - 0.3464x)\,\hat{a}_z \ \ A/m^2$

Amplitude, $J_{do} = 18\ A/m^2$

\#

2. Select the value of k so that each of the following pairs of fields satisfy Maxwell's equations in the region where $\sigma = 0$ and $\rho_v = 0$.

a. $\vec{E} = (kx - 175t)\,\hat{a}_y$ V/m, $\vec{H} = (x+35t)\,\hat{a}_z$ A/m, $\mu = 0.35$ H/m, $\varepsilon = 0.01$ F/m

b. $\vec{D} = 6x\,\hat{a}_x - 4y\,\hat{a}_y + kz\,\hat{a}_z\ \mu C/m^2$, $\vec{B} = 2\,\hat{a}_y$ mT, $\mu = \mu_o$, $\varepsilon = \varepsilon_o$

c. $\vec{E} = 60 \sin 10^6 t \sin 0.01z\,\hat{a}_x$ V/m

$\vec{H} = 0.6 \cos 10^6 t \cos 0.01z\,\hat{a}_y$ A/m, $\mu = k$, $\varepsilon = c_1$

Soln:

(a) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} = -0.35 \frac{\partial}{\partial t}\left((x+35t)\,\hat{a}_z\right)$

$= -12.25\,\hat{a}_z$ — (i)

$\nabla \times \vec{E} = k\,\hat{a}_z$ — (ii)

From (i) and (ii)

$$-12.25\,\hat{a}_z = k\,\hat{a}_z$$

$\therefore\ k = -12.25\ V/m^2$

(b) $\nabla \cdot \vec{D} = \rho_v = 0 \quad -(i)$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$$

or $$\frac{\partial (6x)}{\partial x} + \frac{\partial (-4y)}{\partial y} + \frac{\partial (kz)}{\partial z} = 0$$

or $6 - 4 + k = 0$

$\therefore\ k = -2\ C/m^3$

(c) $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{d\vec{H}}{dt}$$

$$\nabla \times \vec{E} = 0.6 \sin 10^5 t \cos 0.01z\ \hat{a}_y$$

or $$-\mu \frac{d\vec{H}}{dt} = -k \frac{\partial}{\partial t}\left(0.6 \cos 10^5 t \cos 0.01z\ \hat{a}_y\right)$$

$$= 0.6 \times 10^5 k \sin 10^5 t \cos 0.01z\ \hat{a}_y$$

From (i)

$$0.6 \sin 10^5 t \cos 0.01z\ \hat{a}_y = 0.6 \times 10^5 \times k + \sin 10^5 t \cos 0.01z\ \hat{a}_y$$

$\Rightarrow\ k = 10^{-5}\ H/m$

③

3. A straight conductor of 0.2m lies along x-axis with one end at the origin. If this conductor is subjected to the magnetic flux density $\vec{B} = 0.08\hat{a}_y$ T and velocity $\vec{v} = 2.5 \sin 10^3 t\hat{a}_z$ m/s, calculate the emf induced in the conductor.

[2071 Chaitra]

Soln :-

$$\text{Motional emf} = \oint_s (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

$$\vec{v} \times \vec{B} = 2.5 \sin 10^3 t\, \hat{a}_z \times 0.08\, \hat{a}_y$$

$$= -0.2 \sin 10^3 t\, \hat{a}_x$$

$$\vec{dl} = dx\, \hat{a}_x$$

$$\therefore \text{motional emf} = \int_{x=0}^{0.2} (-0.2 \sin 10^3 t\, \hat{a}_x)(dx\, \hat{a}_x)$$

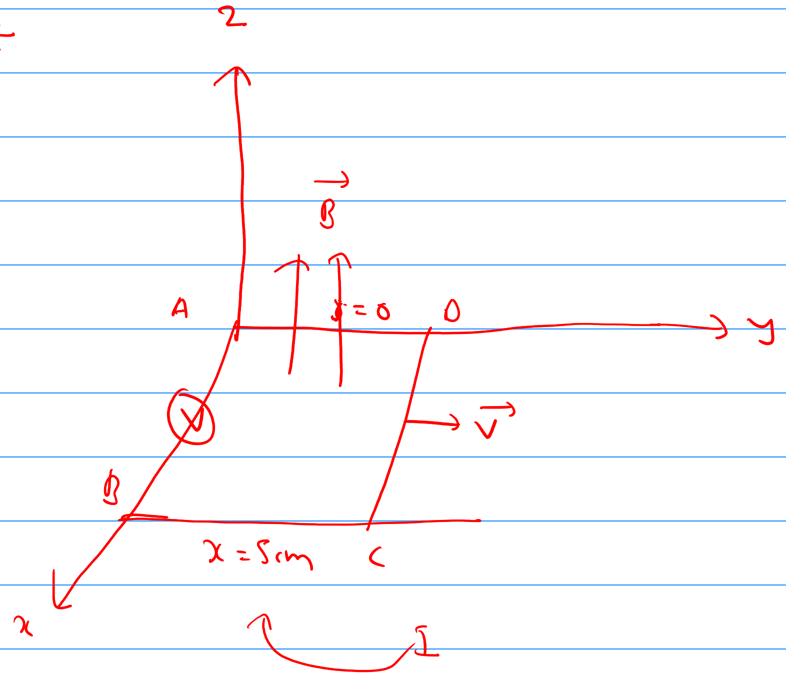
$$= -0.2 \sin 10^3 t\, [x]_0^{0.2}$$

$$= 0.04 \sin 10^3 t \text{ V}$$

4. Consider two parallel conductors placed at x=0 and x=5 cm in a magnetic field $\vec{B} = 6\hat{a}_z \frac{\text{mWb}}{\text{m}^2}$. A high resistance voltmeter is connected at one end and a conducting bar is sliding at other end with velocity $\vec{v} = 18\hat{a}_y$ m/s. Calculate the induced voltage and show the polarity of induced voltage across the voltmeter.

[2073 Chaitra]

Soln ÷



The direction of $\vec{v}$ and $\vec{B}$ suggests that current flows in the direction B - A - D - C - B

$$\text{induced emf} = \oint (\vec{v} \times \vec{B}) \, d\vec{l}$$

$$\vec{v} \times \vec{B} = 18\,\hat{a}_y \times 6\,\hat{a}_z \, \frac{mWb}{m^2} = 18 \times 6 \times 10^{-2}\,\hat{a}_x \ wb/m^2$$

$$= 1.08\,\hat{a}_x \ wb/m^2$$

$$d\vec{l} = dx\,\hat{a}_x$$

$$\text{induced emf} = \int_{x=0.05}^{0} (1.08\,\hat{a}_x)(dx\,\hat{a}_x)$$

$$= 1.08 \int_{x=0.05}^{0} dx = -0.054V$$

$$= -0.054V$$

Find the amplitude of displacement current density in an air space with a large power transformer where $\vec{H} = 10^6 \cos(377t + 1.2566\times10^{-6}z)\hat{a}_y$ A/m. *[2068 Magh]*

Soln ÷

$\nabla \times \vec{H} = \vec{J} + \vec{J_d}$

For air, $\vec{J} = 0$

$\vec{J_d} = \nabla \times \vec{H} = 1.2566 \sin(377t + 1.2566\times10^{-6}z)\hat{a}_x$ A/m^2

∴ Amplitude of displacement current density $(J_{do}) = 1.2566$ A/m^2

6. **Given the magnetic flux density $\vec{B} = 2\times10^{-4}\cos(10^6t)\sin(0.01x)\hat{a}_z$ T, find: (a) the magnetic flux passing through the surface z = 0, 0 < x < 20m, 0 < y<3m at t = 1 μs. (b) $\oint \vec{E}.\vec{dl}$ around the perimeter of the surface specified above at t = 1 μs.**

Soln ÷

(a) $\phi = \int_s \vec{B}.\vec{ds}$

$\vec{B} = 2\times10^{-4}\cos(10^6t)\sin(0.01x)\hat{a}_z$

$\vec{ds} = dxdy\,\hat{a}_z$; $t = 1\times10^{-6}$ s

Sol

$$\phi = \int_s 2\times10^{-4}\cos(10^6\times1\times10^{-6})\sin(0.01x)\,\hat{a}_z \cdot (dx\,dy\,\hat{a}_z)$$

$$= 2\times10^{-4}\cos(1)\int_{y=0}^{3}\int_{x=0}^{20}\sin(0.01x)\,dx\,dy$$

$$= 2.09\times10^{-5}\ \text{wb}$$

(b) $$\oint \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\frac{\partial \vec{B}}{\partial t} = \frac{\partial\left[2\times10^{-4}\cos(10^6 t)\sin(0.01x)\hat{a}_z\right]}{\partial t}$$

$$= -2\times10^{-4}\sin(0.01x)\cdot\sin(10^6 t)\cdot10^6\,\hat{a}_z$$

$$= -2\times10^{2}\sin(10^6 t)\sin(0.01x)\,\hat{a}_z$$

Sol $(d\vec{l} = dx\,dy\,\hat{a}_z)$

$$\oint \vec{E}\cdot d\vec{l} = -\int_s \left(-2\times10^{2}\sin(10^6 t)\sin(0.01x)\hat{a}_z\right)(dx\,dy\,\hat{a}_z)$$

$$= 2\times10^{2}\sin(10^6\times10^{-6})\int_{x=0}^{20}\sin(0.01x)\,dx\int_{y=0}^{3}dy$$

$$= 0.036\ \text{V}\ \#$$

The displacement current density is $5\cos(2\times10^8 t - kz)\hat{a}_z \ \mu A/m^2$ in a material for which $\sigma = 0$, $\varepsilon = 5\varepsilon_0$ and $\mu = 4\mu_0$ (a) use the displacement current density to find $\vec{D}$ and $\vec{E}$ (b) Find $\vec{B}$ and $\vec{H}$ (c) what must be the value of k.

soln:

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \int \vec{J}_d \cdot dt$$

$$= \int 5\cos(2\times10^8 t - kz)\hat{a}_x \times 10^{-6}\, dt$$

let $u = 2\times10^8 t - kz$

$$\frac{du}{dt} = 2\times10^8$$

$$= \int 5\cos(u)\times10^{-6} \times \frac{du}{2\times10^8}\ \hat{a}_x$$

$$= \frac{5\times10^{-6}}{2\times10^8}\sin(u) = 2.5\times10^{-14}\sin(2\times10^8 t - kz)\hat{a}_x$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{2.5\times10^{-14}\sin(2\times10^8 t - kz)}{5\varepsilon_0}\ \hat{a}_x$$

$\vec{E} = 5.647 \times 10^{-4} \sin(2\times10^8 t - kz)\, \hat{a}_x$

(b) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$$\nabla \times \vec{E} = \left(\frac{\partial H_x}{\partial z}\right)\hat{a}_y + \left(-\cancel{\frac{\partial H_x}{\partial y}}^{\,0}\right)\hat{a}_z$$

$$= \frac{\partial}{\partial z}\left(5.647\times10^{-4}\sin(2\times10^8 t - kz)\,\hat{a}_y\right)$$

$$= 5.647\times10^{-4}\cos(2\times10^8 t - kz)\cdot(-k)\,\hat{a}_y$$

$$\frac{\partial \vec{B}}{\partial t} = 5.647\times10^{-4}\, k\cos(2\times10^8 t - kz)\,\hat{a}_y$$

$$\vec{B} = 5.647\times10^{-4} k \int \cos(2\times10^8 t - kz)\, dt\,\hat{a}_y$$

$$\vec{B} = 5.647\times10^{-4}\, k \times \frac{\sin(2\times10^8 t - kz)}{2\times10^8}\,\hat{a}_y$$

$$= 2.8235\times10^{-12}\, k \sin(2\times10^8 t - kz)\,\hat{a}_y$$

Now,

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{\vec{B}}{4\mu_0} = 5.617\times10^{-7}\, k\sin(2\times10^8 t - kz)\,\hat{a}_y$$

Now,

$$\nabla \times \vec{H} = \vec{J} + \vec{J_d} = \sigma \vec{E} + \vec{J_d} \qquad (\sigma = 0)$$

So,

$$\nabla \times \vec{H} = \vec{J_d}$$

$$\nabla \times \vec{H} = \left(0 - \frac{\partial H_y}{\partial z} \right) \hat{a_x}$$

$$= - \frac{\partial}{\partial z} \left(5.617 \times 10^{-7} k \sin (2 \times 10^8 t - kz) \right) \hat{a_x}$$

$$= -5.617 \times 10^{-7} k \times \cos (2 \times 10^8 t - kz) \times (-k) \hat{a_x}$$

$$= 5.617 \times 10^{-7} k^2 \cos (2 \times 10^8 t - kz) \hat{a_x}$$

Now

$$\nabla \times \vec{H} = \vec{J_d}$$

or $$5.617 \times 10^{-7} k^2 \cos (2 \times 10^8 t - kz) \hat{a_x} = 5 \cos (2 \times 10^8 t - kz) \times 10^{-6} \hat{a_x}$$

or $$5.617 \times 10^{-7} \times k^2 = 5 \times 10^{-6}$$

$\therefore$ $k = 2.983$