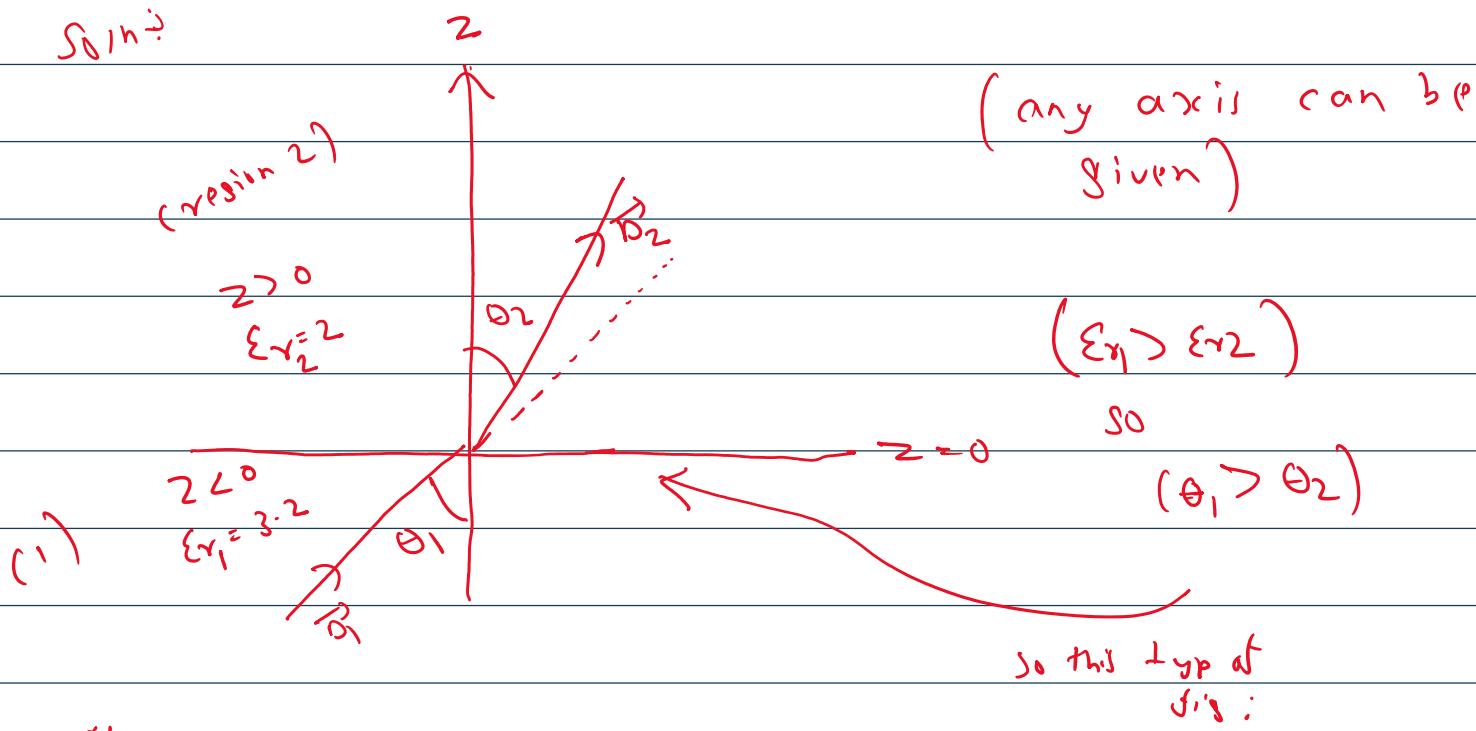


- ① Consider the region $z < 0$ be composed of a uniform dielectric material for which $\epsilon_r = 3.2$ while the region $z > 0$ is characterized by $\epsilon_r = 2$. Let $\vec{D}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ nC/m}^2$. Find ④ \vec{D}_2 , ⑤ Polarization \vec{P}_1 , ⑥ E_{n1} , ⑦ E_{t2} , ⑧ \vec{D}_2 , ⑨ θ_1 , ⑩ θ_2



Now,

$$\vec{D}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ nC/m}^2$$

$$\textcircled{4} \quad D_{n1} = \vec{D}_1 \cdot \hat{a}_z \quad (\text{question mark z-axis reference})$$

$$\text{Ans} = \left[(-30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z) \times 10^{-9} \right] \cdot \hat{a}_z$$

$$= 70 \times 10^{-9} \text{ C/m}^2$$

$$\text{Vector } \vec{D}_{N_1} = 70 \hat{a}_2 \text{ nC/m}^2$$

~~also~~ we know

$$\vec{D}_1 = \vec{D}_{N_1} + \vec{P}_1$$

$$\begin{aligned}\vec{D}_1 &= \vec{D}_1 - \vec{D}_M \\ &= (-30\hat{a}_x - 30\hat{a}_2 + 50\hat{a}_y + 70\hat{a}_2) - (70\hat{a}_2) \\ &= -30\hat{a}_x + 50\hat{a}_y \text{ nC/m}^2\end{aligned}$$

③ Polarization (\vec{P}_1) = ?

Derivation

~~we know~~

$$\boxed{\vec{D}_1 = \epsilon_0 \vec{E}_1 + \vec{P}_1}$$

~~Also~~

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E} \quad (*)$$

In region -1

$$\vec{D}_1 = \epsilon_1 \vec{E}_1 = \epsilon_r \epsilon_0 \vec{E}_1 = 3.2 \epsilon_0 \vec{E}_1$$

$$\text{so } \vec{E}_1 = \frac{\vec{D}_1}{3.2 \epsilon_0} = \frac{\vec{D}_1}{\epsilon_0 \epsilon_r}$$

$$\vec{E}_1 = \frac{(-30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_2)}{3.2 \times \epsilon_0} \text{ nC/m}^2$$

$$= \frac{1}{\epsilon_0} (-9.375 \hat{a}_x + 15 \cdot 625 \hat{a}_y + 21.875) \text{ nC/m}^2$$

Now

$$\begin{aligned} (-30 \hat{a}_x + 50 \hat{a}_y + 70 \hat{a}_z) &= \epsilon_0 \times \frac{1}{\epsilon_0} (-9.375 \hat{a}_x + 15 \cdot 625 \hat{a}_y \\ &\quad + 21.875) + \vec{P}_1 \\ &\quad \checkmark \text{ nC/m}^2 \\ \vec{P}_1 &= -30 \hat{a}_x + 50 \hat{a}_y + 70 \hat{a}_z - (-9.375 \hat{a}_x + 15 \cdot 625 \hat{a}_y + 21.875 \hat{a}_z) \\ &= (-20 \cdot 625 \hat{a}_x + 34 \cdot 375 \hat{a}_y + 48 \cdot 125 \hat{a}_z) \text{ nC/m}^2 \end{aligned}$$

① Using boundary condition

$$\vec{P}_{n1} = \vec{D}_{n2}$$

$$\text{on } 70 \hat{a}_z \text{ nC/m}^2 = \epsilon_2 \vec{E}_{n2}$$

$$\text{on } 70 \hat{a}_z = \epsilon_0 \epsilon_2 \vec{E}_{n2}$$

$$\text{or } \frac{70}{\epsilon_0 \times 2} \hat{a}_z = \vec{E}_{n2}$$

$$\therefore \vec{E}_{n2} = 3.95 \times 10^3 \hat{a}_z \text{ V/m}$$

$$E_{n2} = 3.95 \times 10^3 \text{ V/m}$$

② Using boundary condition

$$\vec{r} - \vec{r}'$$

Using boundary condition

$$\vec{E}_{t_2} = \vec{E}_{t_1}$$

$$\vec{D}_{t_1} = \epsilon_1 \vec{E}_{t_1}$$

$$\vec{E}_{t_1} = \frac{\vec{D}_{t_1}}{\epsilon_0 \epsilon_r} = \frac{(-30\hat{a}_x + 50\hat{a}_y) \text{ nC/m}^2}{\epsilon_0 \times 3.2}$$

$$= -1.05 \times 10^3 \hat{a}_x + 1.76 \times 10^3 \hat{a}_y \text{ V/m}$$

$$\vec{E}_{t_1} = \vec{E}_{t_2}$$

$$\therefore E_{t_2} = \sqrt{(-1.05 \times 10^3)^2 + (1.76 \times 10^3)^2} \\ = 2.056 \times 10^3 \text{ V/m}$$

$$\textcircled{e} \quad \vec{D}_{t_2} = \epsilon_0 \epsilon_r \vec{E}_{t_2} \\ = \epsilon_0 \times 2 \times (-1.058 \times 10^3 \hat{a}_x + 1.76 \times 10^3 \hat{a}_y) \\ = (-18.726 \hat{a}_x + 31.223 \hat{a}_y) \text{ nC/m}^2$$

$$\textcircled{f} \quad \vec{D}_2 = \vec{D}_{n_2} + \vec{D}_{t_2} = (-18.726 \hat{a}_x + 31.223 \hat{a}_y + 70 \hat{a}_z) \text{ nC/m}^2$$

$|D_{n_2}| = (P_1) \cos \theta_1$

$$\textcircled{g} \quad \theta_1 = \cos^{-1} \frac{|D_{n_1}|}{|D_1|} = \cos^{-1} \frac{70}{91.90} = 39.79^\circ$$

$$\textcircled{h} \quad \theta_2 = \cos^{-1} \frac{|D_{n_2}|}{|D_2|} = \cos^{-1} \frac{70}{91.90} = 27.479^\circ$$

$$\textcircled{y} \quad \theta_2 = \cos^{-1} \frac{|\vec{D}_{n2}|}{|\vec{D}_2|} = \cos^{-1} \frac{70}{78.902} = 27.47^\circ$$

#

1. The electric field intensity in polystyrene ($\epsilon_r = 2.55$) filling the space between the plates of a parallel-plate capacitor is 10 kV/m. The distance between the plates is 1.5 mm. Calculate:

- a. D
- b. P
- c. The surface charge density of free charge on the plates
- d. The surface density of polarization charge
- e. The potential difference between the plates.

$SQ/h \div$

$$\textcircled{a} \quad D = \epsilon_0 \epsilon_r E = \epsilon_0 \times 2.55 \times 10 \times 10^3 = 225.4 \text{ nC/m}^2$$

$$\textcircled{b} \quad P = \chi_r \epsilon_0 E = (\epsilon_r - 1) \epsilon_0 E = (2.55 - 1) \epsilon_0 \times 10 \times 10^3 \\ = 137.23 \text{ nC/m}^2$$

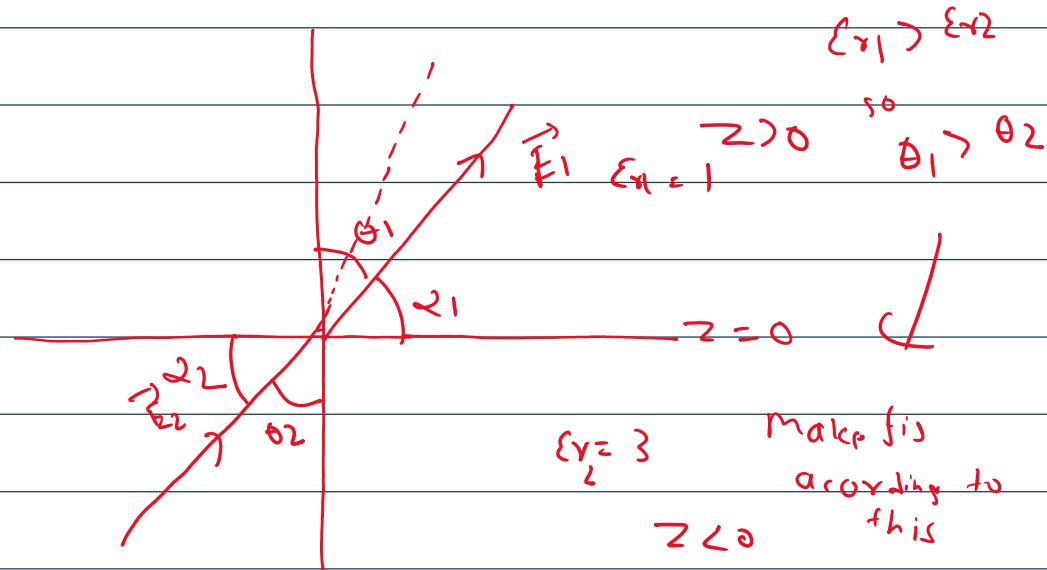
$$\textcircled{c} \quad S_s = \vec{D} \cdot \hat{a}_N = D_N = 225.4 \text{ nC/m}^2$$

$$\textcircled{d} \quad S_{sp} = \vec{P} \cdot \hat{a}_N = P_N = 137.23 \text{ nC/m}^2$$

$$\textcircled{e} \quad V = Ed = 10^4 \times (1.5 \times 10^{-3}) = 15 \text{ V}$$

- H**
2. Two extensive homogeneous isotropic dielectrics meet on plane $z=0$. For $z>0$, $\epsilon_{r1} = 4$ and for $z<0$, $\epsilon_{r2} = 3$. A uniform electric field $\vec{E}_1 = 5 \hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exists for $z \geq 0$. Find:
- \vec{E}_2 for $z \leq 0$.
 - The angles E_1 and E_2 make with the interface.
 - The energy densities (J/m^3) in both dielectrics.
 - The energy within a cube of side 2m centered at $(3, 4, -5)$.

Solution:



④ $\vec{E}_2 = \vec{E}_{t2} + \vec{E}_{n2} \quad \text{--- (i)}$

$$\vec{E}_1 = (5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z) \times 10^3 \text{ V/m}$$

$$\begin{aligned} \vec{E}_M &= \vec{E}_1 \cdot \hat{a}_2 &= (5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z) \times 10^3 \cdot \hat{a}_2 \\ (\perp) &= 3 \times 10^3 \text{ V/m} \\ \text{z-axis} \end{aligned}$$

$$\vec{E}_M = 3 \times 10^3 \hat{a}_2 \text{ V/m} = E_M \hat{a}_2$$

Now,

$$\vec{E}_{+1} = \vec{E}_1 - \vec{E}_{N1} = (5\hat{a}_x - 2\hat{a}_y) \times 10^3 \text{ V/m}$$

using boundary condition

$$\vec{D}_M = \vec{D}_{N2}$$

$$\text{or, } \epsilon_0 \epsilon_{r1} \vec{E}_{N1} = \epsilon_0 \epsilon_{r2} \vec{E}_{N2}$$

$$\text{or, } \vec{E}_{N2} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \vec{E}_M = \frac{4}{3} \times 3 \times 10^3 \hat{a}_2$$

$$= 4 \times 10^3 \hat{a}_2 \text{ V/m}$$

Again boundary condition

$$\vec{E}_{+1} = \vec{E}_{+2}$$

$$\text{or, } (5\hat{a}_x - 2\hat{a}_y) \times 10^3 = \vec{E}_{+2}$$

$$\therefore \vec{E}_{+2} = (5\hat{a}_x - 2\hat{a}_y) \times 10^3 \text{ V/m}$$

from c.i)

$$\boxed{\vec{E}_2 = \vec{E}_{+2} + \vec{E}_{N2} = (5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_2) \times 10^3 \text{ V/m}}$$

(b) $\theta_1 = \cos^{-1} \frac{|\vec{E}_{N1}|}{|\vec{E}_1|} = \cos^{-1} \frac{3 \times 10^3}{\sqrt{38} \times 10^3} = 60.87^\circ$

$$\boxed{\gamma_1 = 90 - 60.87 = 29.121^\circ}$$

$$\theta_2 = \cos^{-1} \frac{\vec{E}_{N2}}{|\vec{E}_2|} = \cos^{-1} \frac{4 \times 10^3}{3\sqrt{5} \times 10^3} = 53.59^\circ$$

$$\angle_2 = 90^\circ - 53.59^\circ = 36.40^\circ$$

① Energy densities (w_{E1}) = $\frac{1}{2} \epsilon_1 E_1^2$

$$= \frac{1}{2} \epsilon_0 \epsilon_{r1} E_1^2$$

$$= 672 \text{ uT/m}^3$$

$$w_{E2} = \frac{1}{2} \epsilon_0 \epsilon_{r2} E_2^2 = 577 \text{ uT/m}^3$$

② At the centre $(3, 4, -5)$ at the cube of side $2m$,

$$V = 2 \times 2 \times 2 = 8 \text{ m}^3$$

($-z = -5 < 0$, implies the cube is in region 2)

$$\text{Total energy (W)} = w_{E2} \times V$$

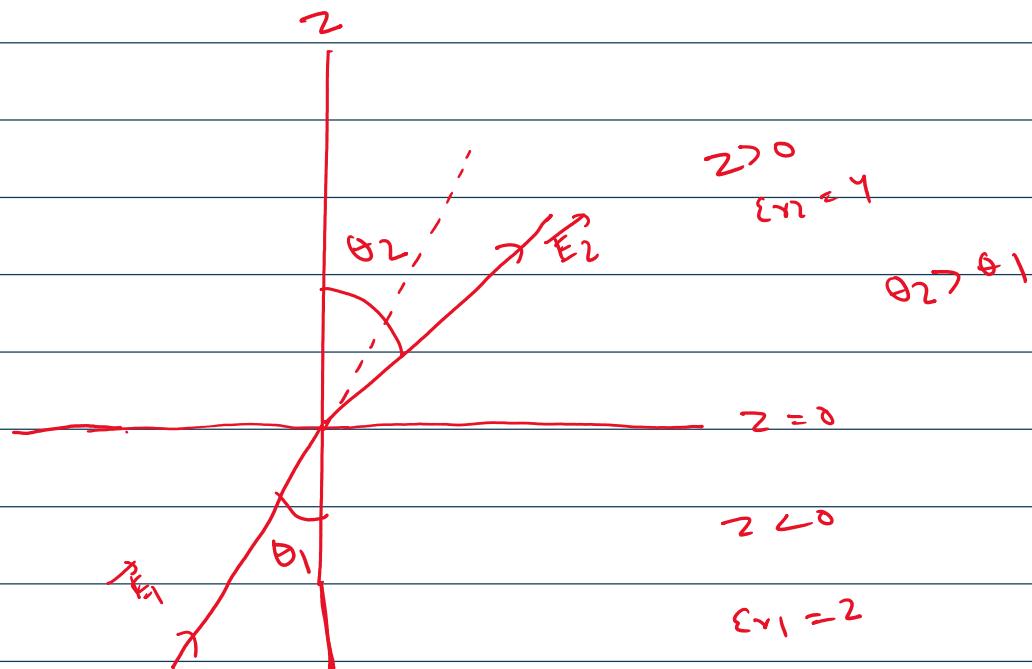
$$= 577 \times 10^{-6} \times 8$$

$$= 4.616 \times 10^{-3} \text{ J}$$

#

4. The region $z < 0$ contains a dielectric material for which $\epsilon_{r1} = 2$, while the region $z > 0$ is characterized by $\epsilon_{r2} = 4$. Let $\vec{E}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ V/m}$. Find: (a) E_{n1} (b) E_{t1} (c) θ_1 (d) D_{n2} (e) D_{t2} (f) \vec{D}_2 (g) \vec{P}_1 (h) θ_2 [2064 Shrawan]

SOLN :-



NOW,

(a) E_{n1} is component of \vec{E}_1 along z -axis

$$E_{n1} = \vec{E}_1 \cdot \hat{a}_z = 70 \text{ V/m} \quad | \quad \vec{E}_{n1} = 70 \hat{a}_z \text{ V/m}$$

(b) $\vec{E}_1 = :$

$$\vec{E}_1 = \vec{E}_{n1} + \vec{E}_{t1}$$

$$\begin{aligned} \vec{E}_{t1} &= -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z - 70\hat{a}_z \\ &= -30\hat{a}_x + 50\hat{a}_y \end{aligned}$$

$$\vec{E}_1 = -\omega_0 \epsilon_0 \vec{x} + \omega_0 \epsilon_0 \vec{y}$$

$$|E_1| = \sqrt{(-30)^2 + (50)^2} = 50\sqrt{30} \text{ V/m}$$

(c) $\theta_1 = \cos^{-1} \frac{|\vec{E}_{n1}|}{|\vec{E}_1|}$

$$|\vec{E}_{n1}| = 70 \text{ V/m} \quad |\vec{E}_1| = \sqrt{(-30)^2 + 50^2 + 70^2} \\ = 91.104 \text{ V/m}$$

$$\theta_1 = \cos^{-1} \frac{70}{91.104} = 39.79^\circ$$

(d) From boundary condition

$$\vec{D}_{n2} = \vec{D}_{n1} \\ = \epsilon_0 \epsilon_r \vec{E}_{n1} \\ = \epsilon_0 \times 2 \times 70 \hat{a}_2 \\ = 1.2395 \hat{a}_2 \text{ nC/m}^2$$

$$D_{n2} = 1.2395 \text{ nC/m}^2$$

(e) From boundary condition

⑥ From boundary condition

$$\vec{E}_{t2} = \vec{E}_{t1}$$
$$= -30\hat{a}_x + 50\hat{a}_y \text{ V/m}$$

$$\vec{D}_{t2} = \epsilon_0 \epsilon_r \vec{E}_{t2}$$
$$= \epsilon_0 \times \epsilon_r (-30\hat{a}_x + 50\hat{a}_y)$$
$$= -1.0625\hat{a}_x + 1.77\hat{a}_y \text{ nC/m}^2$$

$$|\vec{D}_{t2}| = \sqrt{(-1.0625 \times 10^{-9})^2 + (1.77 \times 10^{-9})^2}$$
$$= 2.06 \text{ nC/m}^2$$

⑦

$$\vec{D}_2 = \vec{D}_{n2} + \vec{D}_{t2} = 1.2395\hat{a}_2 + -1.0625\hat{a}_x + 1.77\hat{a}_y \text{ nC/m}^2$$
$$= -1.0625\hat{a}_x + 1.77\hat{a}_y + 1.2395\hat{a}_2 \text{ nC/m}^2$$

⑧

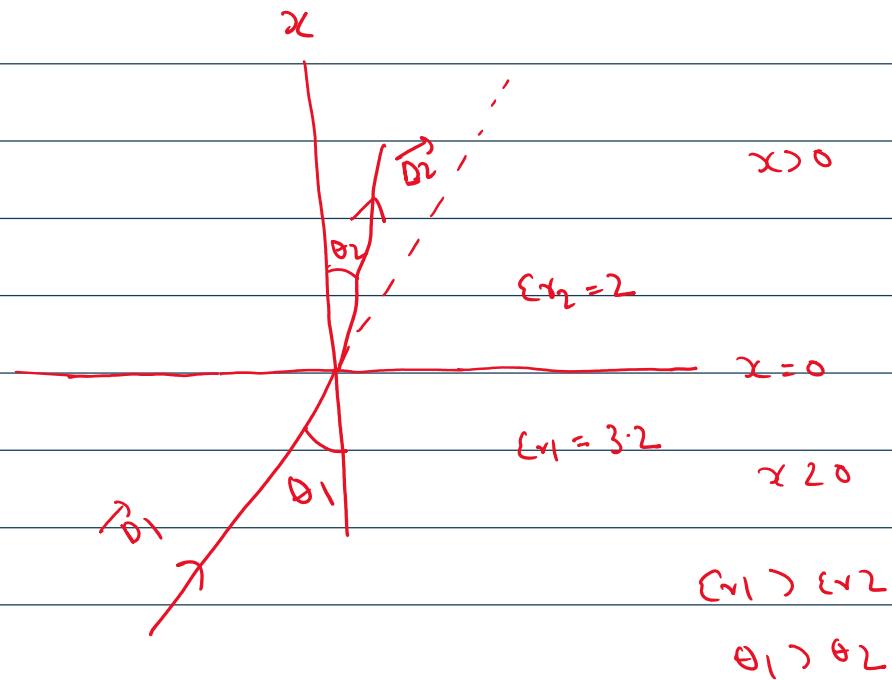
$$\vec{P}_1 = \chi_e \epsilon_0 \vec{E}_1$$
$$= (\epsilon_r - 1) \epsilon_0 \vec{E}_1$$
$$= (2-1) \times \epsilon_0 \times (-30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_2)$$
$$= -265.62\hat{a}_2 + 442.70\hat{a}_y + 619.79\hat{a}_2 \text{ pC/m}^2$$

⑨

$$\theta_2 = \cos^{-1} \frac{|\vec{D}_{n2}|}{|\vec{D}_2|} = \cos^{-1} \frac{1239}{2407.46} = 59.025^\circ$$

- H**
5. The region $x < 0$ is composed of a uniform dielectric material for which $\epsilon_{r1} = 3.2$ while region $x > 0$ is characterized by $\epsilon_{r2} = 2$. The electric flux density at region $x < 0$ is $\vec{D} = 30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ nC/m}^2$, then find polarization (\vec{P}) and electric field intensity (\vec{E}) in both regions.

[2068 Chaitra]



Now,

Since boundary is at $x = 0$,

$$\begin{aligned} D_{n1} &= \vec{D}_1 \cdot \hat{a}_x = (30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z) \times 10^{-9} \cdot \hat{a}_x \\ &= 30 \times 10^{-9} \text{ C/m}^2 \end{aligned}$$

$$\vec{D}_{n1} = D_{n1}\hat{a}_x = 30\hat{a}_x \text{ nC/m}^2$$

$$\begin{aligned} \text{Now, } \vec{D}_{t1} &= \vec{D}_1 - \vec{D}_{n1} \\ &= 30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z - 30\hat{a}_x \end{aligned}$$

$$\text{Now, } D_{+1} = D_1 - n\epsilon_1$$

$$= 30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z - 30\hat{a}_x \\ = 50\hat{a}_y + 70\hat{a}_z \text{ nC/m}^2$$

$$\vec{D}_{+1} = \epsilon_1 \vec{E}_{+1}$$

$$\text{or } \vec{E}_{+1} = \frac{\vec{D}_{+1}}{\epsilon_0 \epsilon_r} = \frac{(50\hat{a}_y + 70\hat{a}_z) \times 10^{-9}}{\epsilon_0 \times 3.2}$$

$$= 1.76\hat{a}_y + 2.47\hat{a}_z \text{ kV/m}$$

From boundary condition, $\vec{D}_{n2} = \vec{D}_{n1}$

$$\text{or } \vec{E}_{n2} = \frac{\vec{D}_{n2}}{\epsilon_0 \epsilon_r} = \frac{\vec{D}_{n1}}{\epsilon_0 \epsilon_r} = \frac{30\hat{a}_x \text{ nC/m}^2}{\epsilon_0 \times 2}$$

$$= 1.49 \text{ kV/m}$$

From boundary condition $\vec{E}_{+2} = \vec{E}_{+1}$

$$= 1.76\hat{a}_y + 2.47\hat{a}_z \text{ kV/m}$$

Now

$$\vec{E}_1 = \frac{\vec{D}_1}{\epsilon_0 \epsilon_r} = \frac{(30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z) \times 10^{-9}}{\epsilon_0 \times 3.2}$$

$$= (1059.3\hat{a}_x + 1765.5\hat{a}_y + 2471.7\hat{a}_z) \text{ V/m}$$

$$= (1059.32 \hat{a}_x + 1765.53 \hat{a}_y + 2471.75 \hat{a}_z) \text{ V/m}$$

$$\begin{aligned}\vec{E}_2 &= \vec{E}_{h2} + \vec{E}_{t2} \\ &= (1.694 \times 10^3 \hat{a}_x) + (1.76 \times 10^3 \hat{a}_y + 2.47 \times 10^3 \hat{a}_z) \\ &= 1694 \hat{a}_x + 1764 \hat{a}_y + 2470.58 \hat{a}_z \text{ V/m}\end{aligned}$$

Now!

Polarization vector in region 1

$$\begin{aligned}(\vec{P}_1) &= \chi_e \epsilon_0 \vec{E}_1 \\ &= (\epsilon_r - 1) \cdot \frac{\vec{D}_1}{\epsilon_0 \epsilon_r} \\ &= \frac{\epsilon_r - 1}{\epsilon_r} \vec{D}_1\end{aligned}$$

$$= \frac{3.2 - 1}{3.2} (30 \hat{a}_x + 50 \hat{a}_y + 70 \hat{a}_z) \times 10^9$$

$$= 20 \cdot 625 \hat{a}_x + 34 \cdot 375 \hat{a}_y + 44.8 \cdot 125 \hat{a}_z \text{ nC/m}^2$$

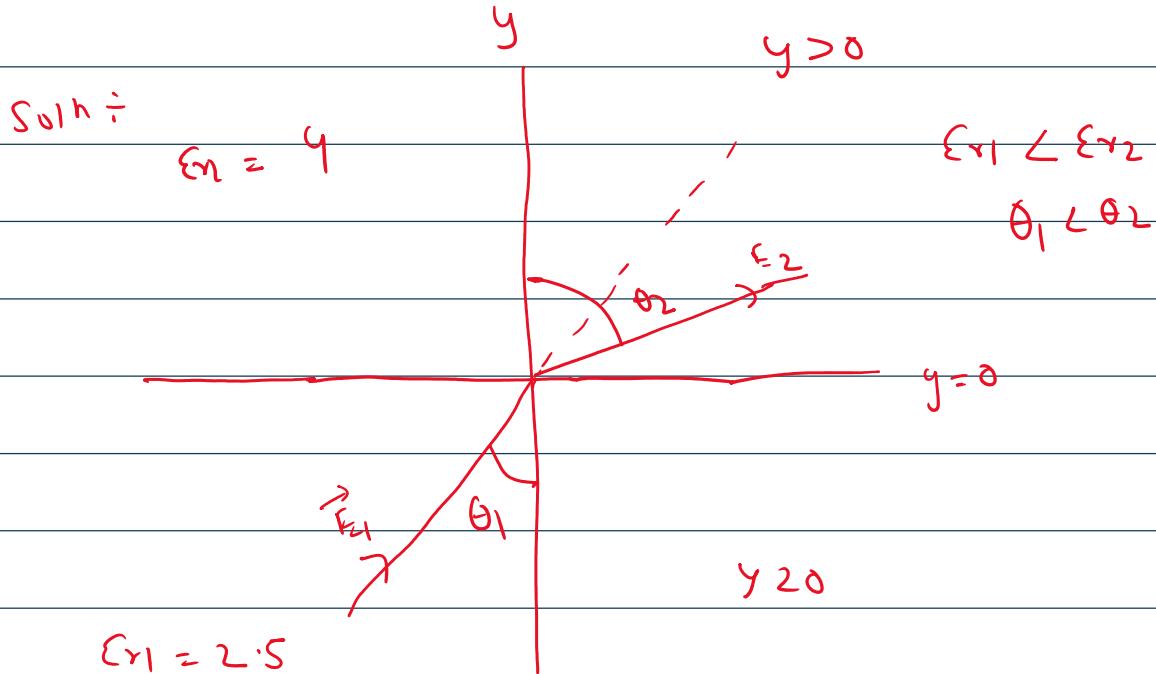
$$\begin{aligned}\vec{P}_2 &= \chi_e \epsilon_0 \vec{E}_2 \\ &= (\epsilon_r - 1) \epsilon_0 \vec{E}_2 \\ &= (2 - 1) \times \epsilon_0 \times (1964 \hat{a}_x + 1764.7 \hat{a}_y + 2470.58 \hat{a}_z)\end{aligned}$$

$$= 17.38 \hat{a}_x + 15.624 \hat{a}_y + 21.87 \hat{a}_z \text{ nC/m}^2$$

$$= 17.38 \hat{a}_x + 15.624 \hat{a}_y + 21.87 \hat{a}_z \text{ nC/m}^2$$

The region $y > 0$ contains a dielectric material for which $\epsilon_{r1} = 2.5$ while the region $y > 0$ is characterized by $\epsilon_{r2} = 4$. Let, $\vec{\epsilon}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ V/m}$ and find ④ $\vec{\epsilon}_m$ ⑤ $\vec{\epsilon}_t$ ⑥ $|E_t|$ ⑦ $|E_i|$ ⑧ θ_1

(ii) \vec{D}_{m1} , $|D_m|$, \vec{D}_2 (iii) \vec{P}_2 (iv) θ_2



$$\vec{\epsilon}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z$$

plane $y=0$ separates two dielectric material to this plane.

$$\text{Sol } |\vec{\epsilon}_m| = \vec{\epsilon}_1 \cdot \hat{a}_y \\ = 50 \text{ nC/m}^2$$

$$|E_1| = \epsilon_1 \cdot a_y \\ = 50 \text{ V/m}$$

$$\vec{E}_M = \epsilon_M \cdot a_y \\ = 50 a_y \text{ V/m}$$

$$\vec{E}_1 = \vec{E}_1 + \vec{E}_M \\ \vec{E}_1 = \vec{E}_1 - \vec{E}_M$$

$$\vec{E}_1 = \vec{E}_1 - \vec{E}_M \\ = -30 a_x + 70 a_y \text{ V/m}$$

$$|E_1| = 76.158 \text{ V/m}$$

$$|\epsilon_1| = 91.104 \text{ V/m}$$

Now,

$$|\vec{E}_M| = |\vec{E}_1| \cos \theta_1$$

$$\text{Or } 50 = 91.104 \cos \theta_1$$

$$\therefore \theta_1 = 56.7^\circ$$

Boundary condition

$$\vec{D}_{n1} = \vec{D}_{n2} = \epsilon_0 \epsilon_r \vec{E}_M$$

$$= \epsilon_0 \times 2.5 \times 50 \hat{a_y}$$

$$= 1.107 \hat{a_y} \text{ nC/m}^2$$

\downarrow boundary

$$\epsilon_{t1} = \epsilon_{t2}$$

$$\vec{\epsilon}_{t2} = -30 \hat{a_x} + 70 \hat{a_2} \text{ V/m}$$

$$\begin{aligned}\vec{D}_{t2} &= \epsilon_0 \epsilon_r \vec{\epsilon}_{t2} \\ &= \epsilon_0 \times 4 \times (-30 \hat{a_x} + 70 \hat{a_2}) \text{ C/m}^2 \\ &= (-1.062 \hat{a_x} + 2.479 \hat{a_2}) \text{ nC/m}^2\end{aligned}$$

$$|D_{t2}| = 2.697 \text{ nC/m}^2$$

Now,

$$\begin{aligned}\vec{D}_2 &= \vec{D}_{t2} + \vec{D}_{N2} \\ &= (-1.062 \hat{a_x} + 2.479 \hat{a_2} + 1.107 \hat{a_y}) \text{ nC/m}^2\end{aligned}$$

$$\vec{P}_2 = \chi_r \epsilon_0 \vec{E}_2$$

$$\vec{E}_2 = \vec{E}_{t2} + \vec{E}_{N2}$$

$$\text{Now } \vec{D}_{N1} = \vec{D}_{N2}$$

$$\epsilon_0 \epsilon_{r1} \vec{E}_{N1} = \epsilon_0 \epsilon_{r2} \vec{E}_{N2}$$

$$\vec{E}_{N2} = \frac{\epsilon_{r1} \vec{E}_M}{\epsilon_{r2}}$$

$$\omega_2 = \frac{\omega_1}{\epsilon_{r2}}$$

$$= \frac{2.5}{9} \cdot 50 \hat{a}_y$$

$$= 31.25 \hat{a}_y$$

$$\vec{E}_2 = -30 \hat{a}_x + 31.25 \hat{a}_y + 70 \hat{a}_z$$

$$\begin{aligned}\vec{P}_2 &= (\epsilon_r - 1) \epsilon_0 \vec{E}_2 \\ &= (4 - 1) \epsilon_0 (-30 \hat{a}_x + 31.25 \hat{a}_y + 70 \hat{a}_z) \\ &= (-0.77 \hat{a}_x + 0.83 \hat{a}_y + 1.85 \hat{a}_z) nC/m^2\end{aligned}$$

Now,

$$\theta_2 = \cos^{-1} \frac{|\vec{E}_{n2}|}{|\vec{E}_2|} = \cos^{-1} \frac{31.25}{\sqrt{(-30)^2 + 31.25^2 + 70^2}}$$

$$= 67.69^\circ \#$$