Chapter 5 (8 Marks)

b

7. Construct the Bezier curver of order 3 and with polygon vertices A (1, 1), B(2, 3), C(4, 3) and D(6, 4) at u =0, 0.25, 0.5, 0.75, 1. Derive the blending function for parametric cubic curve. [4+6]

7. List the properties of the B-spline curve. Derive Hermite matrix in Hermite Cubic Spline Curve. Find the coordinates at U = 0.3 with respect to the control points (10,12), (15,28),(22,35) and (28,9) using Bezier function. [2+8]

b

What is Bezier Curve? Find the coordinates of Bezier curve at u = 0.25, 0.5 and 0.75 with respect to the control points (10,15), (15,25), (20,35), (25,15) using Bezier Function. [2+6]

Write the properties of Bezier curve. A cubic Bezier curve is described by the four control points. (0, 0), (3, 1), (5, 2) and (8, 1). Find the Bezier polynomial and the coordinate at f = 0.25, 0.5, 0.75. [3+8]

b

It is necessary to construct curves using parameteric equations? Justify. List down the steps for modeling curves using splines. [4+4] b

5. Mention two important properties of Bezier Curve and find the Bezier Curve which passes through (0,0,0) and (-2,1,1) and is controlled by (7,5,2) and (2,0,1). [2+6]

Find the coordinates at U=0.25, 0.5, and 0.75 with respect to the control points (10,10), (15,25), (20,30), and (25,5) using Bezier function. Draw your curve with given control points. [8]

4. Find the coordinate at U = 0.25, U = 0.5, and U = 0.75 with respect to the control points (2, 10), (6, 20), (12, 5) and (16, 15) using Bezier function. And plot Bezier curve with your calculated coordinates. [6+2]

b

Explain properties if Bezier curve. Find the coordinate at u = 0.2 with respect to the control points (1,1), (4,6) (8,-3) and (12,2) using Bezier function. [8]

b

6. Explain about parametric cubic curve? What is a Bezier Curve? Explain its properties with examples. [2+6]

5. Write down properties of Bezier curve. Find equation of Bezier curve whose control points are P0(2,6), P1(6,8) and P2(9,12). Also find co-ordinate of point at u = 0.8. [10]

b

5. Derive the equation for cubic Bezier curve. Also write down its properties. [8]

Write down properties of Bezier curve. Find equation of Bezier curve whose control points are P0(2,6), P1(6,8) and P2(9,12). Also find co-ordinate of point at u = 0.8. [10]

5. Define Hermite Interpolation in defining a curve. Use it to find the blending function of a parametric cubic curve in 2D graphics. [2+6]

pipeline. [8]

6. Explain about parametric cubic curve? What is a Bezier Curve? Explain its properties. [3+3+2]

Spline Representations

Spline Curve

A **spline curve** is a smooth curve defined by a set of control points. It is typically used to create complex shapes in a more manageable way. The term "spline" comes from a flexible strip used in shipbuilding to draw smooth curves.

Spline Surface

A **spline surface** is a smooth, flexible surface defined by a grid of control points. It can be thought of as an extension of spline curves into two dimensions. Just like spline curves, spline surfaces are used to create complex, smooth surfaces in 3D modeling and animation.

Applications in Computer Graphics

- **Modeling**: Spline curves and surfaces are extensively used in 3D modeling software to create smooth and organic shapes, such as car bodies, character models, and terrain surfaces.
- **Animation**: Splines help in defining motion paths for animations, ensuring smooth transitions and movements.
- **Rendering**: In rendering, splines can be used to represent the outlines of objects, which are then filled in to create the final image.

Spline curves and surfaces are essential tools in computer graphics, enabling the creation of detailed and smooth models and animations.

The different types of spline curve are:

1. Piecewise cubic spline
   i. Hermite spline
   ii. Cardinal spline
   iii. Kochanek-Bartels spline
2. Bezier spline
3. B-spline
4. Beta spline

Interpolation splines and **approximation splines** are two different approaches to creating curves that pass through or near a set of control points in computer graphics and numerical analysis. These methods are used to generate smooth curves, but they differ in how they relate to the given data points.

Interpolation Splines

Interpolation splines are curves that pass exactly through a given set of control points. In other words, the curve is constructed in such a way that it intersects each control point precisely.

Approximation Splines

Approximation splines are curves that do not necessarily pass through the given control points but instead aim to approximate the shape defined by those points. The goal is to create a smooth curve that captures the general trend or shape of the control points.

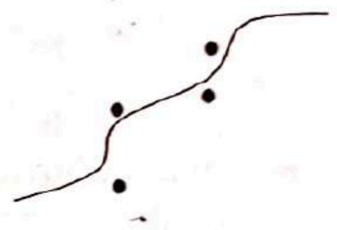


Fig 5.1(a): A set of control points interpolated with piecewise continuous polynomial sections

Fig 5.1(b): A set of six points control points approximated with piecewise continuous polynomial sections

---

Convex Hull

The **convex hull** of a set of points is the smallest convex shape (usually a polygon in 2D or a polyhedron in 3D) that completely encloses all the points. Imagine stretching a rubber band around a set of nails hammered into a board; the shape the rubber band takes is the convex hull.

Properties:

- **Convex**: The shape is convex, meaning that for any two points inside the hull, the line segment connecting them is also entirely inside the hull.
- **Smallest Enclosing Shape**: The convex hull is the smallest shape that can enclose all the points.
- **Applications**: Convex hulls are used in computer graphics for collision detection, shape analysis, mesh generation, and pattern recognition.

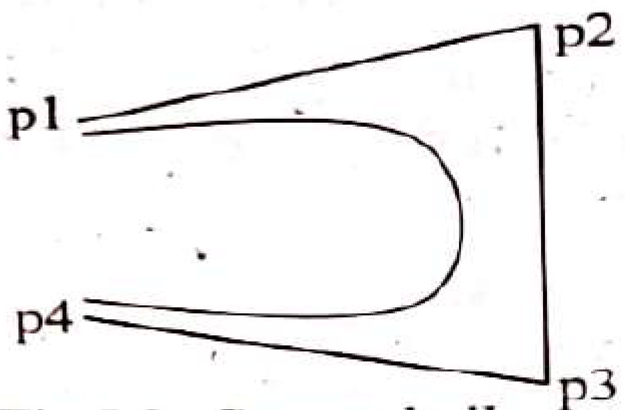


Fig 5.2: Convex hull

1. **Parametric Continuity (C Continuity)**

Parametric continuity refers to the smoothness of a curve at the junction between two curve segments, based on the mathematical properties of the curve's parameterization (usually denoted as t). The levels of parametric continuity are:

- C^0 **Continuity (Positional Continuity):**
  - The curves share the same endpoint.
  - The simplest form of continuity, ensuring that the curves meet at a common point.
  - Mathematically: $\mathbf{P}_1(t_1) = \mathbf{P}_2(t_2)$.
- C^1 **Continuity (Tangential Continuity):**
  - The curves share the same endpoint and have the same tangent direction at that point.
  - This ensures that the curve not only meets at the endpoint but also flows smoothly without any sharp corners.
  - Mathematically: $\left.\frac{d\mathbf{P}_1(t)}{dt}\right|_{t_1} = \left.\frac{d\mathbf{P}_2(t)}{dt}\right|_{t_2}$.

- C^2 **Continuity (Curvature Continuity):**
  - The curves share the same endpoint, tangent direction, and curvature at the junction.
  - This creates a very smooth transition, as the rate of change of the tangent direction is also continuous.
  - Mathematically: $\left.\frac{d^2\mathbf{P}_1(t)}{dt^2}\right|_{t_1} = \left.\frac{d^2\mathbf{P}_2(t)}{dt^2}\right|_{t_2}$.

2. **Geometric Continuity (G Continuity)**

Geometric continuity focuses on the visual smoothness of the curve rather than the strict mathematical parameterization. This form of continuity is concerned with the shape of the curve at the junction, independent of the specific parameterization.

- G^0 **Continuity:**
  - Similar to C^0 continuity, the curves simply meet at a common point.
  - Ensures positional continuity.

- G^1 **Continuity:**
  - The curves meet at the same point, and their tangents are aligned, ensuring that the curve appears smooth at the joint.
  - Unlike C^1, G^1 does not require the parametric derivative to be equal, only that the direction of the tangents is the same.
  - Mathematically: The unit tangent vectors at the junction are the same, even if their magnitudes differ.
- G^2 **Continuity:**
  - The curves have the same point, aligned tangents, and the curvature (the rate of change of the tangent) is also continuous at the junction.
  - This is similar to C^2 continuity but focuses more on the visual smoothness rather than strict parametric derivatives.

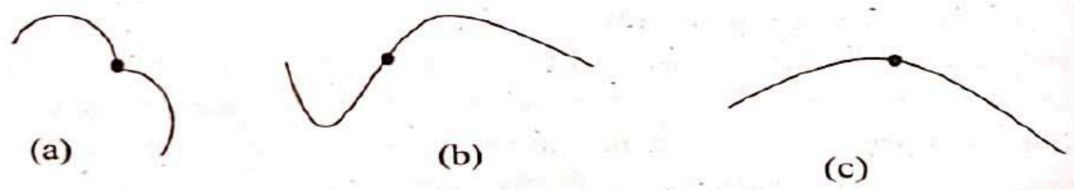


Fig 5.3: Piecewise construction of a curve by joining two curve segments using different order of continuity (a) zero-order continuity only (b) first-order continuity (c) second-order continuity.

Key Differences Between Parametric and Geometric Continuity:

- **Parametric Continuity** is stricter and based on the mathematical derivatives of the curve's parameterization. It ensures both the geometric and parametric properties are continuous across the curve.
- **Geometric Continuity** allows more flexibility by focusing on the shape of the curve rather than the exact mathematical derivatives. It is more concerned with the visual smoothness rather than the precise mathematical definition of smoothness.

Hermite Cubic Spline (Parametric Cubic Curve / Parametric Cubic Spline)

A

Hermite cubic spline is a type of parametric cubic curve that is used to smoothly interpolate between given data points. This curve is defined by not only the positions of the data points (end points) but also by the tangents (slopes) at those points. Because of this, the Hermite cubic spline is particularly useful in applications where controlling the slope at the data points is important.

Derivation of the Hermite Cubic Spline

1. General Form of a Parametric Cubic Curve

The general form of a cubic curve can be expressed as a polynomial of the parameter t (where $0 \leq t \leq 1$):

$$P(t) = \sum_{i=0}^{3} a_i t^i$$

This can be expanded as:

$$P(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad \text{(Equation ii)}$$

Here, $P(t)$ is the point on the curve at a particular value of t.

2. Curve in 3D Space

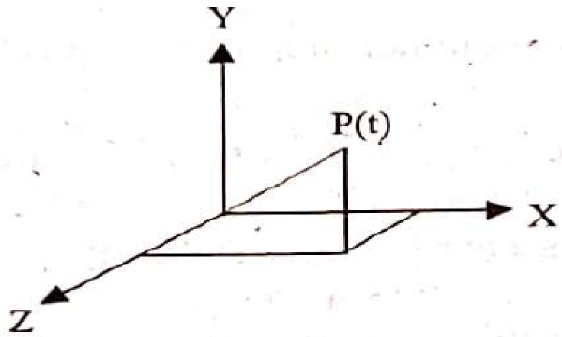
If the curve is in three-dimensional space, $P(t)$ can be separated into its x, y, and z components:

$$x(t) = a_{3x} t^3 + a_{2x} t^2 + a_{1x} t + a_{0x}$$

$$y(t) = a_{3y} t^3 + a_{2y} t^2 + a_{1y} t + a_{0y}$$

$$z(t) = a_{3z} t^3 + a_{2z} t^2 + a_{1z} t + a_{0z} \quad \text{(Equation iii)}$$

Here, $a_{3x}, a_{2x}, a_{1x}, a_{0x}$ (and similarly for y and z) are the coefficients that need to be determined.



3. Determining the Coefficients

To solve for the twelve unknown coefficients (a_{0x}, a_{1x}, etc.), we need both the positions of the end points and the tangents (slopes) at those points.

- **Positions of End Points:** The coordinates of the curve at $t = 0$ and $t = 1$ provide six equations.
- **Tangents at End Points:** The slopes (tangents) at $t = 0$ and $t = 1$ provide the remaining six equations.

4. Substituting Known Values

Substitute $t = 0$ and $t = 1$ into the expanded curve equation:

$$P(0) = a_0 \quad \text{(Equation iv)}$$

$$P(1) = a_3 + a_2 + a_1 + a_0 \quad \text{(Equation v)}$$

These equations relate the curve's starting and ending positions to the coefficients.

5. Differentiation to Find Tangents

Differentiate the curve equation with respect to t to find the tangents:

$$P'(t) = 3a_3t^2 + 2a_2t + a_1$$

Evaluate this at $t = 0$ and $t = 1$ to get:

$$P'(0) = a_1 \quad \text{(Equation vi)}$$

$$P'(1) = 3a_3 + 2a_2 + a_1 \quad \text{(Equation vii)}$$

These equations provide information about the slopes at the endpoints.

6. Solving for Coefficients

Using the above equations, you can solve for the coefficients a_0, a_1, a_2, a_3:

$$a_0 = P(0)$$

$$a_1 = P'(0)$$

$$a_2 = -3P(0) + 3P(1) - 2P'(0) - P'(1)$$

$$a_3 = 2P(0) - 2P(1) + P'(0) + P'(1)$$

7. Final Hermite Cubic Spline Equation

Substituting the coefficients back into the general cubic equation gives the final form of the Hermite cubic spline:

$$P(t) = (2t^3 - 3t^2 + 1)P(0) + (-2t^3 + 3t^2)P(1) + (t^3 - 2t^2 + t)P'(0) + (t^3 - t^2)P'(1)$$

Here:

- $P(0)$ and $P(1)$ are the positions of the end points.
- $P'(0)$ and $P'(1)$ are the slopes (tangents) at the end points.

The functions multiplying these terms are called **blending functions**. By varying t from 0 to 1, you can trace out the entire curve between the two points, with the curve's shape influenced by the slopes at the endpoints.

💡 calculation rough

$$3P(1) = (\cancel{a_3} + a_2 + a_1 + a_0) \times 3$$
$$P'(1) = \cancel{3a_3} + 2a_2 + a_1$$

$$3P(1) - P'(1) = a_2 + 2a_1 + 3a_0$$

$$\therefore a_2 = 3P(1) - P'(1) - 2P'(0) - 3P(0)$$

$$2P(1) = (a_3 + \cancel{a_2} + a_1 + a_0) \times 2$$
$$P'(1) = 3a_3 + \cancel{2a_2} + a_1$$

$$2P(1) - P'(1) = -a_3 + a_1 + 2a_0$$
$$\therefore a_3 = P'(0) + 2P(0) - 2P(1) + P'(1)$$

~~P(t) =~~

$$P(t) = (2P(0) - 2P(1) + P'(0) + P'(1))t^3$$
$$+ (-3P(0) + 3P(1) - 2P'(0) - P'(1))t^2$$
$$+ P'(0)t + P(0)$$

$$= (2t^3 - 3t^2 + 1)P(0) + (-2t^3 + 3t^2)P(1)$$
$$+ (t^3 - 2t^2 + t)P'(0) + (t^3 - t^2)(P'(1))$$

Bezier Curve

A Bézier curve is a parametric curve defined by a set of control points. For a simple quadratic Bézier curve, you need three control points: a start point, a control point, and an end point. For a cubic Bézier curve, you use four control points.

Key Characteristics

1. **Control Points**: Bezier curves are defined by control points. The number of control points determines the degree of the polynomial that defines the curve.
2. **Curve Degree**: If a Bezier curve has n+1 control points, it is an n-th degree polynomial curve.
3. **Specification**: Bezier curves can be specified through boundary conditions, characteristic matrices, or blending functions.

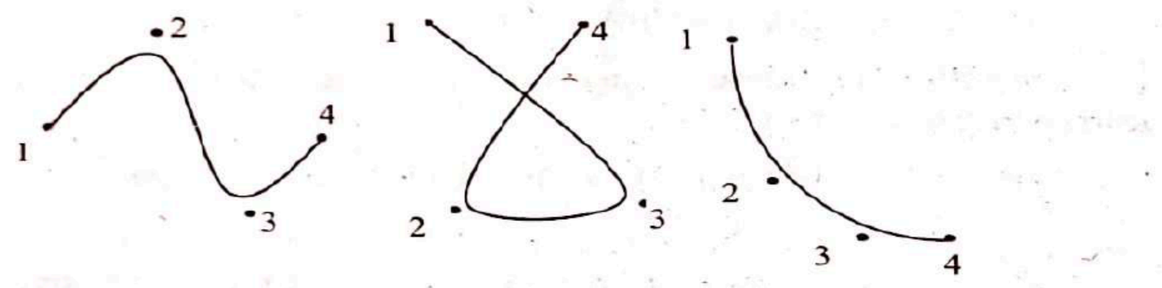


Fig 5.4: *Bezier curves generated four control points*

Properties

1. **Defined by Control Points**: A Bezier curve is defined by a set of control points. The most common Bezier curves are linear (2 control points), quadratic (3 control points), and cubic (4 control points).
2. **Smooth Transitions**: Bezier curves provide smooth transitions between points, making them ideal for modeling smooth paths and shapes.
3. **Affine Invariance**: The shape of a Bezier curve is invariant under affine transformations (translation, scaling, rotation, and shearing). This means

the curve's shape is preserved even if the coordinate system changes.

4. **Polynomial Form**: Bezier curves are represented by polynomial functions. For instance, a cubic Bezier curve is a cubic polynomial.
5. **Control Points Influence**: The shape of the curve is influenced by the position of the control points. The curve starts at the first control point and ends at the last control point, with the intermediate control points determining the curvature.
6. **Curve is Contained Within the Convex Hull**: The Bezier curve is always contained within the convex hull of its control points, ensuring the curve does not extend outside the area defined by these points.
7. **Endpoints and Tangents**: The curve starts at the first control point and ends at the last. The first and last control points also define the tangent direction at the curve's start and end, respectively.
8. **Recursive Definition**: Bezier curves can be defined recursively. For example, a cubic Bezier curve can be computed using linear interpolations between points.
9. **Parameterization**: Bezier curves are parameterized by a single variable (usually denoted as (t), which ranges from 0 to 1. The curve is computed as a function of (t).
10. **Ease of Computation**: Bezier curves are relatively easy to compute and manipulate, making them suitable for real-time graphics applications.
11. The degree of polynomial defining the curve segment is one less than the number of control points.
12. If the control points are symmetric about a line then bezier curve is also symmetric about that line.
13. Bezier blending function are all positive and the sum is always 1.

Bezier curve equation

→ A bezier curve is parametically represented by :-

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t) \quad \text{—(i)}$$

Here,

t is any parameter where $0 \le t \le 1$

$P(t)$ = any point lying in the bezier curve

B_i = i^{th} control point of the bezier curve

n = degree of the curve

$J_{n,i}(t)$ = Blending function = $C(n,i)\, t^i\, (1-t)^{n-i}$

Where,

$$C(n,i) = \frac{n!}{i!\,(n-i)!}$$

Cubic Bezier Curve

→ A Cubic bezier curve is a bezier curve with degree 3

→ The total number of control points in cubic bezier curve is 4.

Substituting $n=3$ for cubic bezier curve, we get

$$P(t) = \sum_{i=0}^{3} B_i J_{3,i}(t)$$

Expanding the above equation we get

$$P(t) = B_0 J_{3,0}(t) + B_1 J_{3,1}(t) + B_2 J_{3,2}(t) + B_3 J_{3,3}(t) \quad \text{—— (ii)}$$

Now,

$$J_{3,0}(t) = \frac{3!}{0!(3-0)!} t^0 (1-t)^{3-0}$$

$$= (1-t)^3 \quad \text{—— (iii)}$$

$$J_{3,1}(t) = \frac{3!}{1!\,2!} t^1 (1-t)^{3-1}$$

$$= 3t(1-t)^2 \quad \text{—— (iv)}$$

$$J_{3,2}(t) = \frac{3!}{2!(3-2)!} t^2 (1-t)^{3-2}$$

$$= 3t^2(1-t) \quad \text{—— (v)}$$

$$J_{3,3}(t) = \frac{3!}{3!(3-3)!} t^3 (1-t)^{3-3}$$

$$= t^3 \quad \text{—— (vi)}$$

Using (iii), (iv), (v), (vi) in (ii) we get

$$\boxed{P(t) = B_0 (1-t)^3 + B_1\, 3t(1-t)^2 + B_2\, 3t^2(1-t) + B_3 t^3}$$

This is the required parametric equation for cubic bezier curve.

Given a bezier curve with 4 control points - $B_0[1,0]$, $B_1[3,3]$, $B_2[6,3]$, $B_3[8,1]$. Determine any 5 points lying on the curve. Also draw a rough sketch of the curve.

Soln:

we have,

The given curve is defined by 4 control points so the curve is a cubic bezier curve

The parametric equation for a cubic bezier curve is :-

$$P(t) = B_0(1-t)^3 + 3t(1-t^2)B_1 + B_2\,3t^2(1-t) + B_3 t^3$$

Substituting ~~value~~ B_0, B_1, B_2, B_3 we get

$$P(t) = [1,0](1-t)^3 + [3,3]\,3t(1-t^2) + [6,3]\,3t^2(1-t) + [8,1]\,t^3 \quad \text{--- (i)}$$

Now,

to get 5 points lying on curve assume any 5 values lying in the range $0 \leq t \leq 1$

lets values of t are 0, 0.2, 0.5, 0.7, 1

For $t = 0$

put $t = 0$ in (i) we got

$P(0) = [1, 0] (1-0)^3$

$P(0) = [1, 0]$

For $t = 0.2$

$P(0.2) = [1,0] (1-0.2)^3 + [3,3]\, 3 \times 0.2 (1-0.2)^2$
$+ [6,3]\, 3 \times 0.2^2 (1-0.2) + [8,1]\, 0.2^3$

$= [1,0] \times 0.512 + [3,3] \times 0.384 + [6,3] \times 0.096$
$+ [8,1] \times 8 \times 10^{-3}$

$= [0.512, 0] + [1.152, 1.152] + [0.576, 0.288] +$
$[0.064, 0.008]$

$\therefore P(0.2) = [2.304, 1.448]$

For $t = 0.5$

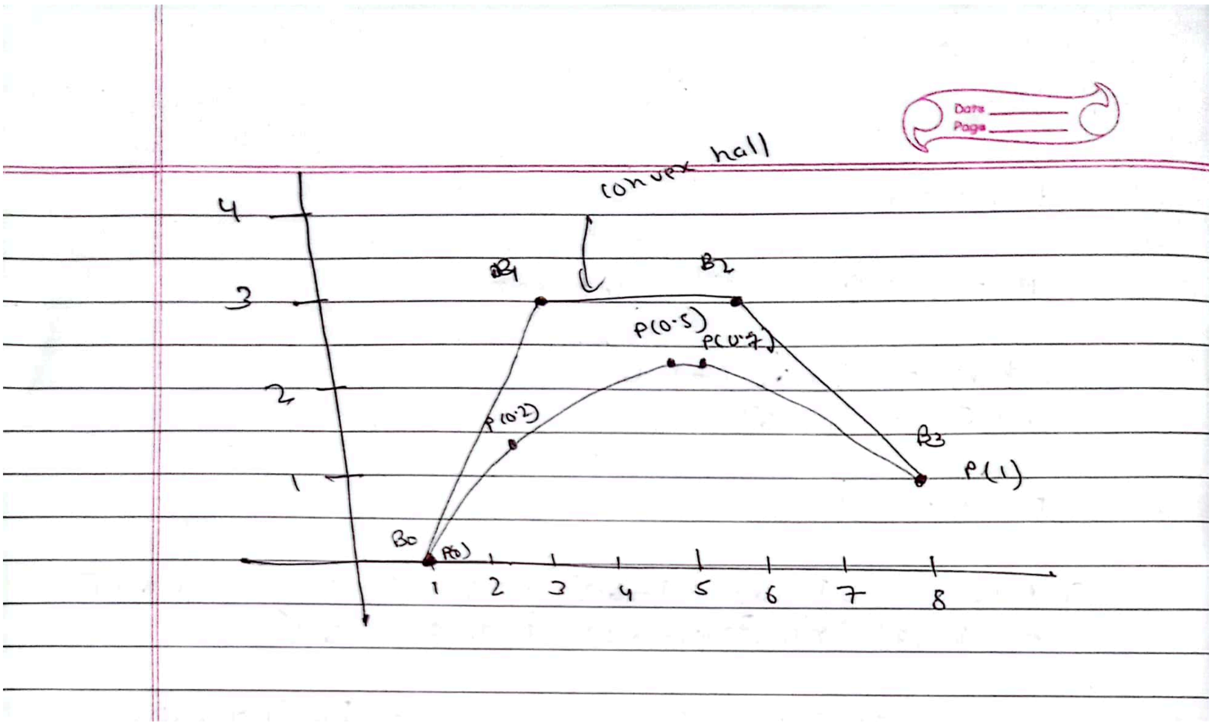
$P(0.5) = [4.5, 2.375]$

For $t = 0.7$

$P(0.7) = [5.984, 2.333]$

For $t = 1$

$P(1) = [8, 1]$



convex hall
B1
B2
P(0.5)
P(0.7)
P(0.2)
B3
P(1)
B0
P(0)
4
3
2
1
1
2
3
4
5
6
7
8

Find equation of Bezier curve with control points B_0 (2,6), B_1 (6,8), B_2 (9,12). Also find coordinates of point at $u = 0.8$

Soln ÷

number of control points $(n) = 3$

degree $= n-1 = 2$

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$$

$$P(t) = \sum_{i=0}^{2} B_i J_{2,i}(t)$$

$$= B_0 J_{2,0}(t) + B_1 J_{2,1}(t) + B_2 J_{2,2}(t)$$

$$J_{n,i} = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}$$

$$J_{2,0}(t) = \frac{2!}{0!\,(2-0)!} t^0 (1-t)^{2-0} = (1-t)^2$$

$$J_{2,1}(t) = \frac{2!}{1!\,(2-1)!} t^1 (1-t)^{2-1} = 2t(1-t)$$

$$J_{2,2}(t) = \frac{2!}{2!\,0!} t^2 (1-t)^{2-2} = t^2$$

$$P(t) = B_0 (1-t)^2 + B_1 2t(1-t) + B_2 t^2 \quad \text{—— (i)}$$

$P(t) = (2,6)(1-t)^2 + (6,8)\,2t(1-t) + (9,12)t^2$ #

At $t = 0.8$ $(1-t)$

$P(0.8) = (2,6)(1-0.8)^2 + (6,8)\,2\times0.8(1-0.8)$
$+ (9,12)\times 0.8^2$

$\therefore$ $P(0.8) = (7.76, 10.48)$

Find Bezier curve which passes through $(0,0,0)$ and $(-2,1,1)$ and is controlled by $(7,5,2)$ and $(2,0,1)$

Start

end

Soln:-
Given,

$B_0 = (0,0,0)$ $B_3 = (2,0,1)$
$B_1 = (7,5,2)$ $B_4 = (-2,1,1)$

Cubic parametric equation

$P(t) = B_0(1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2(1-t)$
$+ B_3 t^3$

$= (0,0,0)(1-t)^3 + (7,5,2)\,3t(1-t)^2 + (2,0,1)$
$3t^2(1-t) + t^3(-2,1,1)$

$= (0,0,0) + (21t(1-t)^2, 15t(1-t)^2, 6t(1-t)^2)$
$+ (6t^2(1-t), 0, 3t^2(1-t)) + (-2t^3, t^3, t^3)$

$\Rightarrow$ Solve the rest #

A cubic Bezier curve is described by the four control points (0,0), (2,1), (5,2) and (6,1). Find the tangent to the curve at $t = 0.5$

Soln:

$P(t) = B_0 (1-t)^3 + B_1\, 3t(1-t)^2 + B_2\, 3t^2 (1-t) + t^3 B_3$

$P(t) = (0,0)(1-t)^3 + (2,1)\, 3t(1-t)^2 + (5,2)\, 3t^2(1-t) + t^3 (6,1)$

~~at~~

$P'(t) = -(1-t)^2 B_0 + (3 - 12t + 9t^2) B_1 + B_2 (6t - 9t^2) + 3t^2 B_3$

At $t = 0.5$

$P'(0.5) = -(1-0.5)^2 (0,0) + (2,1)(3 - 12\times0.5 + 9\times0.5^2) + (5,2)(6\times0.5 - 9\times0.5^2) + (6,1)\, 3\times0.5^2$

$= (6.75, 1.5)$

tangent to bezier curve at $t = 0.5 = \frac{\Delta y}{\Delta x} = \frac{1.5}{6.75}$

$= 0.222$

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Page |

Develop an equation of resulting Bezier curve for the coordinates of the four control points relative to current wcs by A (1,1), B (2,3), C (4,3) and D (6,4) find the points on the curve and draw the plot for u = 0, 0.2, 0.4, 0.6, 0.8 and 1.

soln:-

$$P(t) = B_0 (1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2 (1-t) + B_3 t^3$$

To find x(t) and y(t)

$$x(t) = B_{0x} (1-t)^3 + B_{1x} 3t(1-t)^2 + B_{2x} 3t^2 (1-t) + B_{3x} t^3$$

$$= 1(1-t)^3 + 6t(1-t)^2 + 12t^2(1-t) + 6t^3 \quad \text{--- (2)}$$

$$y(t) = B_{0y} (1-t)^3 + B_{1y} 3t(1-t)^2 + B_{2y} 3t^2 (1-t) + B_{3y} t^3$$

$$= 1(1-t)^3 + 9t(1-t)^2 + 9t^2(1-t) + 4t^3 \quad \text{--- (3)}$$

t	x(t)	y(t)
0	1	1
0.2	1.712	1.984
0.4	2.616	2.632
0.6	3.604	3.088
0.8	4.808	3.496
1	6	4

(plot as previous)

💡 Parametric cubic curve blending funtion is bezier ko blending function

[3+7]

. What is Parametric Cubic Curve and why do you need it? Write down the step for cubic spline interpolation. [3+5]

What is a Parametric Cubic Curve?

A **parametric cubic curve** is a type of curve in computer graphics and mathematical modeling where the position of a point on the curve is expressed as a function of a parameter, typically denoted as t. The curve is described by cubic polynomials, which means the highest degree of the polynomial is 3.

For a 2D curve, the parametric cubic equations can be written as:

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

where $x(t)$ and $y(t)$ are the coordinates of the curve, and t is the parameter, usually varying from 0 to 1.

Why Do We Need Parametric Cubic Curves?

1. **Smoothness**: Cubic curves provide smooth transitions between points, which is crucial in computer graphics for creating realistic animations, shapes, and paths.
2. **Flexibility**: They offer a good balance between complexity and control, allowing for precise adjustments to the shape of the curve.
3. **Interpolation**: In interpolation (such as cubic spline interpolation), parametric cubic curves can be used to ensure that a curve passes through a given set of points smoothly.
4. **Bezier Curves**: They are used to create Bezier curves, which are fundamental in vector graphics, font design, and many other areas of graphics and CAD.

Steps for Cubic Spline Interpolation in Computer Graphics

Cubic spline interpolation is a mathematical technique used to estimate or predict the values of a function between known data points. It's especially useful when you want a smooth curve that passes through all the data points, unlike linear interpolation, which simply connects the dots with straight lines. Here's how it works, broken down into simple steps:

1. **Understanding the Problem**

Suppose you have a set of data points (x_1, y_1), (x_2, y_2), ..., (x_n, y_n). You want to find a smooth curve that goes through all these points. This curve is made up of several cubic polynomials, one between each pair of points.

2. **Defining the Cubic Splines**

For each interval between two consecutive data points (x_i, y_i) and (x_{i+1}, y_{i+1}), we define a cubic polynomial:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

Here, $S_i(x)$ is the cubic spline function in the interval $[x_i, x_{i+1}]$, and a_i, b_i, c_i, and d_i are the coefficients that we need to determine.

3. **Ensuring Continuity**

To make sure the curve is smooth and continuous:

- The spline should pass through the given data points, meaning $S_i(x_i) = y_i$ and $S_i(x_{i+1}) = y_{i+1}$.
- The first derivative (slope) of the splines at each data point should be the same from both sides, ensuring the curve is smooth where the splines meet.
- The second derivative (curvature) should also be the same at each data point.

4. **Setting Up the Equations**

With these conditions, you get a system of equations that allows you to solve for the unknown coefficients a_i, b_i, c_i, and d_i for each interval. This system will have:

- $n - 1$ equations for the condition that the spline passes through each data point.
- $n - 2$ equations for the first derivative being equal at the interior points.
- $n - 2$ equations for the second derivative being equal at the interior points.
- Two additional boundary conditions, often set by assuming the second derivative at the endpoints is zero (this is called the "natural spline" condition).

5. Solving the Equations

You solve this system of equations, usually using matrix methods, to find the coefficients for each cubic polynomial. These coefficients then fully define your spline functions.

6. Constructing the Spline

After solving for the coefficients, you construct the cubic splines for each interval. The final result is a piecewise-defined function, where each piece is a cubic polynomial that smoothly joins with the others.

7. Using the Spline

Once you have the spline functions, you can use them to estimate the value of the function at any point between your original data points. The spline will give you a smooth curve that not only passes through all your data points but also has a continuous first and second derivative, meaning it's very smooth.

Differentiate between Interpolation and approximation. Explain the process of performing curve modeling using splines. [3+5]

Aspect	Interpolation	Approximation
Definition	Finding a function that exactly passes through a set of known data points.	Finding a function that closely fits a set of data points but does not necessarily pass through them all.
Objective	To predict values within the range of the known data points.	To create a function that best represents the overall trend of the data.
Data Points	The function must pass through all given data points.	The function is allowed to deviate from some or all data points.
Usage	Used when the data points are considered accurate and	Used when the data may contain noise, or when a simpler function

Aspect	Interpolation	Approximation
	reliable.	is desired.
Types	Polynomial interpolation, spline interpolation, etc.	Least squares approximation, Fourier approximation, etc.
Flexibility	Less flexible as it strictly passes through all data points.	More flexible as it aims to fit the general trend, ignoring small deviations.
Computational Complexity	Can be computationally expensive for large datasets or high-degree polynomials.	Generally less computationally intensive, especially with simple models like linear approximation.

2nd part - explain bezier splines ko derivation

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b

4. List out the properties of Bezier curve. What is order of continuity? Explain. [8]

the
order of continuity in curve modeling refers to how smoothly a curve or surface transitions at the junctions where different segments or patches meet. It ensures that there are no visible breaks or abrupt changes in the shape, making the transitions appear smooth and natural.

c1 c2 c3 continuity explain
