Ходжаев
Мухаммадрагаб 203. 16 - Вариант

1. $z = \sqrt{36 - 9x^2 + 4y^2} + \sqrt{y^2 - 1}$

1) $36 - 9x^2 + 4y^2 \geq 0$

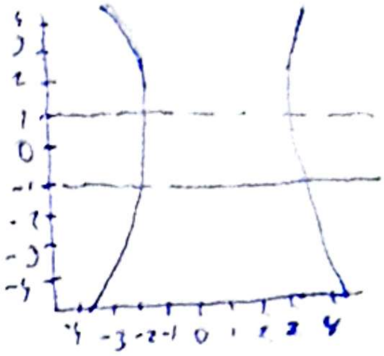
$9x^2 - 4y^2 \leq 36$

$\frac{x^2}{4} - \frac{y^2}{9} \leq 1$

2) $y^2 - 1 \geq 0$

$y^2 \geq 1$

$y \leq -1$ / $y \geq 1$



2. $z = xe^{-\sqrt{x^2+y^2}}$

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x)\, e^{-\sqrt{x^2+y^2}} + x\, \frac{\partial}{\partial x}(e^{-\sqrt{x^2+y^2}}$

$\frac{\partial}{\partial x}(x) = 1$

$\frac{\partial}{\partial x}(e^{-\sqrt{x^2+y^2}}) = e^{\sqrt{x^2+y^2}} \cdot \left(-\frac{x}{\sqrt{x^2+y^2}}\right)$

$\frac{\partial z}{\partial x} = e^{-\sqrt{x^2+y^2}} + x \cdot e^{-\sqrt{x^2+y^2}} + \left(-\frac{x}{\sqrt{x^2+y^2}}\right.$

$\frac{\partial z}{\partial x} = e^{-\sqrt{x^2+y^2}}\left(1 - \frac{x^2}{\sqrt{x^2+y^2}}\right)$

$\frac{\partial z}{\partial y} = x \cdot \frac{\partial}{\partial y}(e^{-\sqrt{x^2+y^2}})$

$\frac{\partial z}{\partial y} = x \cdot e^{-\sqrt{x^2+y^2}}\left(-\frac{y}{\sqrt{x^2+y^2}}\right)$

$\frac{\partial z}{\partial y} = -\frac{xy}{\sqrt{x^2+y^2}} \cdot e^{-\sqrt{x^2+y^2}}$

$\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$dz = e^{-\sqrt{x^2+y^2}}\left(1 - \frac{x^2}{\sqrt{x^2+y^2}}\right)dx - \frac{xy}{\sqrt{x^2+y^2}}e^{-\sqrt{x^2+y^2}}dy$

3. $f(x, y) = \sqrt{x^2 + y^3}$

$x = 1.02$ $\quad y = 1.97$

$f(x,y) \approx f(x_0, y_0) + f'_x(x_0, y_0)\Delta x + f'_y(x_0, y_0)\Delta y$

$f'_x = \frac{\partial f}{\partial x}$

$f'_y = \frac{\partial f}{\partial y}$

$f(1.02, 1.97) \approx 3 \cdot \frac{1}{2} \cdot 0.02 + 2 \cdot (-0.03)$

$f(1.02, 1.97) \approx \underline{2.95}$

$x_0 = 1$, $y_0 = 2$

$\Delta x = 1.02 - 1 = 0.02$

$\Delta y = 1.97 - 2 = -0.03$

$f(1,2) = \sqrt{1^3 + 2^3} = 3$

$\frac{\partial f}{\partial x}(1,2) = \frac{1}{2}$

$\frac{\partial f}{\partial y}(1,2) = 2.$

$f(x,y) = (x^3 + y^3)^{1/2}$

$\frac{\partial f}{\partial x} = \frac{1}{2}(x^3 + y^3)^{-1/2} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3+y^3}}$

$\frac{\partial f}{\partial y} = \frac{1}{2}(x^3 + y^3)^{-1/2} \cdot 3y^2 = \frac{3y^2}{2\sqrt{x^3+y^3}}$

4. $z = \arcsin\left(\frac{u^2}{v}\right)$, $u = \sin x$ $v = \cos x$ $x = \pi$

$z = \arcsin\left(\frac{\sin^2 x}{\cos x}\right)$

$$\frac{dz}{dx} = \frac{d}{du}\left(\arcsin\left(\frac{u^2}{v}\right)\right)\cdot\frac{du}{dx} + \frac{d}{dv}\left(\arcsin\left(\frac{u^2}{v}\right)\right)\cdot\frac{dv}{dx}$$

$$\frac{d}{dw}\arcsin(w) = \frac{1}{\sqrt{1-w^2}}$$

$w = \frac{u^2}{v}$

$$\frac{d}{du}\arcsin\left(\frac{u^2}{v}\right) = \frac{1}{\sqrt{1-\frac{u^4}{v^2}}}\cdot\frac{2u}{v} = \frac{2u}{v\sqrt{1-\frac{u^4}{v^2}}}$$

$$\frac{d}{dv}\arcsin\left(\frac{u^2}{v}\right) = \frac{1}{\sqrt{1-\left(\frac{u^2}{v}\right)^2}}\cdot\left(-\frac{u^2}{v^2}\right) = -\frac{u^2}{v^2\sqrt{1-\frac{u^4}{v^2}}}$$

$u = \sin x \Rightarrow \frac{du}{dx} = \cos x$

$v = \cos x \Rightarrow \frac{dv}{dx} = -\sin x$

$$\frac{dz}{dx} = \frac{2u}{v\sqrt{1-\frac{u^4}{v^2}}}\cos x + \left(-\frac{u^2}{v^2\sqrt{1-\frac{u^4}{v^2}}}\right)\cdot(-\sin x)$$

$$\frac{dz}{dx} = \frac{2u\cos x}{v\sqrt{1-\frac{u^4}{v^2}}} + \frac{u^2\sin x}{v^2\sqrt{1-\frac{u^4}{v^2}}}$$

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-\frac{u^4}{v^2}}}\left(\frac{2u\cos x}{v} + \frac{u^2\sin x}{v^2}\right)$$

$u(\pi) = \sin\pi = 0$ · $v(\pi) = \cos\pi = -1$

$$\left.\frac{dz}{dx}\right|_{x=\pi} = \frac{1}{\sqrt{1-\frac{0^4}{(-1)^2}}}\left(\frac{2\cdot 0\cdot(-1)}{-1} + \frac{0^2\cdot 0}{(-1)^2}\right) = \frac{1}{1}\cdot(0+0) = 0$$

5. $xyz = z^2 - 2$.

$$\frac{\partial}{\partial x}(xyz) = \frac{\partial}{\partial x}(z^2-2)$$

$$y\cdot\frac{\partial}{\partial x}(xz) = 2z\frac{\partial z}{\partial x}$$

$$yz + yx\frac{\partial z}{\partial x} = 2z\frac{\partial z}{\partial x}$$

$$yx\frac{\partial z}{\partial x} - 2z\frac{\partial z}{\partial x} = -yz$$

$$\frac{\partial z}{\partial x}(yx - 2z) = -yz$$

$$\frac{\partial z}{\partial x} = \frac{-yz}{yx-2z}$$

$$\frac{\partial z}{\partial x} = \frac{yz}{2z-yx}$$

$$\frac{\partial}{\partial y}(xyz) = \frac{\partial}{\partial y}(z^2-2)$$

$$x\cdot\frac{\partial}{\partial y}(yz) = 2z\frac{\partial z}{\partial y} \; (*)$$

$$(*)\; x\left(z + y\frac{\partial z}{\partial y}\right) = 2z\frac{\partial z}{\partial y}$$

$$xz + xy\frac{\partial z}{\partial y} = 2z\frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y}(xy-2z) = -xz$$

$$\frac{\partial z}{\partial y} = \frac{-xz}{xy-2z}$$

$$\frac{\partial z}{\partial x} = \frac{yz}{2z-yx} \qquad \frac{\partial z}{\partial y} = \frac{xz}{2z-xy}.$$

$z = x\sqrt{y} - x^2 - y + 6x + 3$

$$\frac{\partial z}{\partial x} = \sqrt{y} - 2x + 6$$

$$\frac{\partial z}{\partial y} = \frac{x}{2\sqrt{y}} - 1$$

$$\begin{cases} \sqrt{y} - 2x + 6 = 0 \\ \frac{x}{2\sqrt{y}} - 1 = 0 \end{cases} \qquad \frac{x}{2\sqrt{y}} = 1 \Rightarrow \quad x = 2\sqrt{y} \qquad \sqrt{y} - 2(2\sqrt{y}) + 6 = 0 \Rightarrow \sqrt{y} = 2 \Rightarrow \underline{y = 4}$$

$$\underline{x = 2\sqrt{4} = 4.}$$

$$\frac{\partial^2 z}{\partial x^2} = -2 \Rightarrow \underline{A = -2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{2\sqrt{y}} \Rightarrow \underline{B = \frac{1}{4}} \qquad D = AC - B^2 = (-2)\cdot\left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right)^2 = \frac{3}{16} > 0$$

$$\frac{\partial^2 z}{\partial y^2} = -\frac{x}{4y^{3/2}} \Rightarrow \underline{C = -\frac{1}{8}}$$

$D > 0$ и $A < 0$, $(4,4)$ локальный максимум.

$$z(4,4) = 8 - 16 - 4 + 24 + 3 = \underline{\underline{15}}.$$

$$\boxed{z_{max} = 15 \quad (4,4)}$$

7. $z = 3x^3 + 3y^2 - 2x - 2y + 1$

$D: x \geq 0 \qquad 0 \leq y \leq 1 - x$

$$\frac{\partial z}{\partial x} = 6x - 2 = 0 \Rightarrow x = \frac{1}{3} \qquad x = \frac{1}{3} \geq 0 \qquad y = \frac{1}{3} \leq 1 - \frac{1}{3} = \frac{2}{3} \text{ (верно).}$$

$$\frac{\partial z}{\partial y} = 6y - 2 = 0 \Rightarrow y = \frac{1}{3} \qquad z\left(\frac{1}{3}, \frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2 - 2\cdot\frac{1}{3} - 2\cdot\frac{1}{3} + 1 = \frac{1}{3}$$

Наибольшее значение $\boxed{z}$:

(1,0) (правый угол)

(0,1) (верхний угол)

Наименьшего значения $\boxed{\frac{1}{3}}$:

$\left(\frac{1}{3}, \frac{1}{3}\right)$ (внутр.-я крит.-я точка).

8. $\int_0^4 dy \int_{y/2}^{6-y} f(x,y)\,dx \qquad 0 \le y \le 4 \qquad \frac{y}{2} \le x \le 6-y$

$$\int_0^2 dy \int_0^{2x} f(x,y)\,dy + \int_2^6 dx \int_0^{6-x} f(x,y)\,dy.$$

9. $\iint_D \left(\frac{4}{5}xy + 9x^2y^2\right) dx\,dy \qquad x=1 \qquad y=\sqrt{x} \qquad y=-x^3$

$$\int_0^1 dx \int_{-x^3}^{\sqrt{x}} \left(\frac{4}{5}xy + 9x^2y^2\right) dy$$

$$\int_0^1 \left(\frac{2}{5}x^2 + 3x^{7/2} - \frac{2}{5}x^7 + 3x^{11}\right) dx$$

$$\left(\frac{2}{15}(1)^3 + \frac{2}{3}(1)^{9/2} - \frac{1}{20}(1)^8 + \frac{1}{4}(1)^{12}\right) - 0 = \frac{2}{15} + \frac{2}{3} - \frac{1}{20} + \frac{1}{4} = \underline{\underline{1}}$$

10.

$x^2 - 2x + y^2 = 0$

$x^2 - 4x + y^2 = 0$

$y = 0$

$y = \frac{x}{\sqrt{3}}$

$x = r\cos\theta, \; y = r\cos\theta$

$x^2 - 2x + y^2 = 0 \Rightarrow r^2 - 2r\cos\theta = 0 \Rightarrow r = 2\cos\theta$

$x^2 - 4x + y^2 = 0 \Rightarrow r^2 - 4r\cos\theta = 0 \Rightarrow r = 4\cos\theta$

$y = 0 \qquad \theta = 0$

$y = \frac{x}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta\ \frac{\pi}{6}$

$r = 2\cos\theta$

$r = 4\cos\theta$

$\theta = 0 \;\; и \;\; \theta = \frac{\pi}{6}$

$$S = \iint_D r\,dr\,d\theta$$

$0 \le \theta \le \frac{\pi}{6} \qquad 2\cos\theta \le r \le 4\cos\theta$

$$S = \int_0^{\pi/6} \left(\int_{2\cos\theta}^{4\cos\theta} r\,dr\right) d\theta \;(*)$$

$$\int_{2\cos\theta}^{4\cos\theta} r\,dr = 6\cos\theta$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$(*)\; S = \int_0^{\pi/6} 6\cos\theta\,d\theta$

$$S = 6\int_0^{\pi/6} \frac{1+\cos 2\theta}{2}\,d\theta = 3\left(\frac{\pi}{6} + \frac{\sin(\pi/3)}{2}\right)$$

$$\boxed{\frac{\pi}{2} + \frac{3\sqrt{3}}{4}}$$