Informed Search

Introduction to Artificial Intelligence

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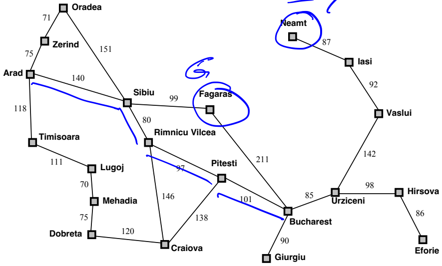
Best-First Search

Search methods differ in their strategies which node to expand next

Uninformed search:	fixed strategies without information about the cost from a given node to a goal.
Informed search:	uses information about the cost from a given node to a goal in the form of an evaluation function f, assigning each node a real number.
Best-First Search:	always expand the node with the “best” f-value.
Greedy Search:	$h(n)$ = estimated cost from state at node n to a goal state. Expand node n where $h(n)$ is minimal. Use $f = h$.

Greedy Search Example

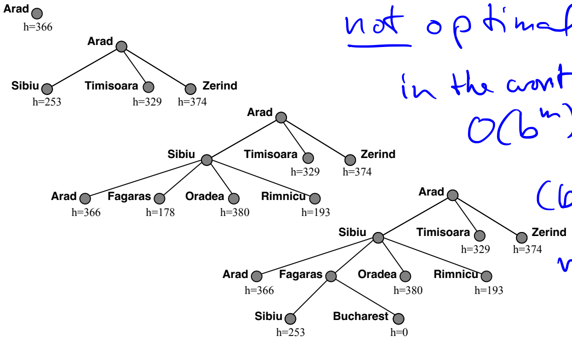


oscillates between Neamt + Iasi

Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

(not complete)



not optimal

in the worst case $O(b^m)$ time + space

(b = branching factor

m = max. depth)

A* Hart, Nilsson & Raphael (1968)

combines uniform cost search with greedy search.

$g(n)$ = actual cost from the initial state to n.

$h(n)$ = estimated cost from n to the nearest goal.

$f(n) := g(n) + h(n)$.

$f(n)$ = is the estimated cost of the cheapest path which passes through n.

Let $h^*(n)$ be the actual cost of the optimal path from n to the nearest goal.

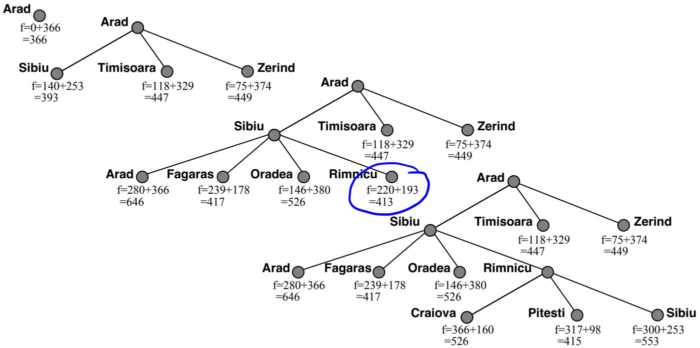
Admissible Heuristic

h is called admissible if we have for all n:

$$h(n) \leq h^*(n) \; .$$

We require for A* that h is admissible. e.g. straight line distance between cities.

Example A*



Note: in the example f is monotone nondecreasing.
The following can always be guaranteed:

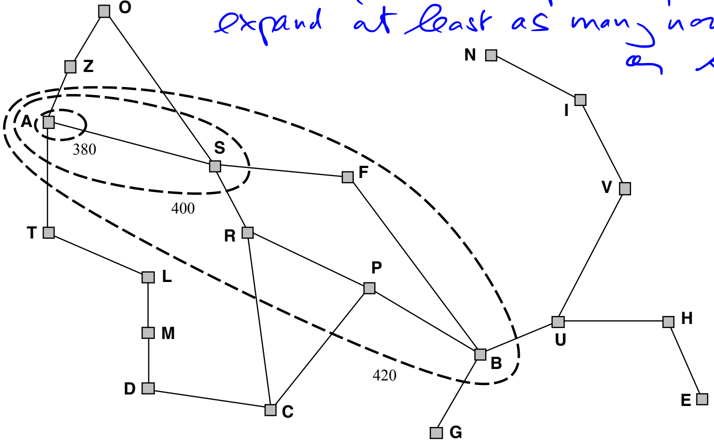
Path-Max Equation

Let n, n' be nodes, where is n parent of n'. Then let

$$f(n') = max(f(n), g(n') + h(n')).$$

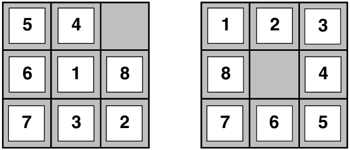
Contour Lines in A*

A* is optimally efficient, that is, any other search method, which is optimal, will expand at least as many nodes as A*.



Heuristic Function 1

$S =$



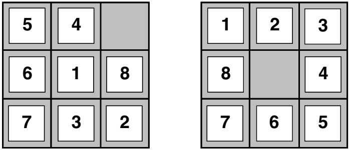
Start State **Goal State**

h_1 = number of tiles in the wrong position. $h_1(S) = 7$

h_2 = sum of the distances to the goal location for all tiles (Manhattan Distance)

$h_2(S) = 2+3+3+2+4+2+0+2 = 18$

Heuristic Function 1



Start State Goal State

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Effective branching factor b^*: If A^* generates N nodes with solution depth d, then b^* is the branching factor of a uniform tree of depth d with $N+1$ nodes, i.e.

$$N+1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$$

b^* is a measure for the goodness of h: the closer b^* is to 1 the better.

Heuristic Function 2

iterative deepening search

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

How to Find a Heuristic

General Strategy:

- Simplify the problem
- Compute the exact solution for the simplified problem
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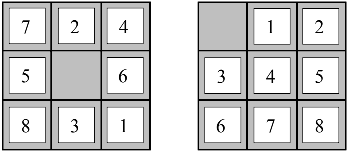
For example:

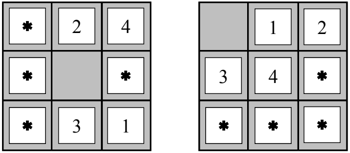
- h_1 is the solution cost for the simplified 8-puzzle where tiles can be placed at an arbitrary position with a single action.
- h_2 corresponds to the exact solution, if tiles can be moved to an arbitrary position but actions are restricted to moving a tile to a neighboring position.

Pattern Databases

if pattern $p_1, \ldots, p_k$ match current state S, then let $h(S) = \max\{h^*(p_i)\}$ when $h^*(p_i)$ is the cost of exact solution for p_i



Start State Goal State



Start State Goal State

Idea: Compute the exact solution for each *pattern* with four numbers and use that value as heuristic. When more than one pattern applies, use the maximum value.

Better than Manhattan!

Why not $h(S) = \sum h^*(p_i)$?

→ not admissible because of shared moves.

IDA* iterative deepening A* linear space

function IDA*(*problem*) **returns** a solution sequence
  inputs: *problem*, a problem
  static: *f-limit*, the current *f*- COST limit
      root, a node

  root ← MAKE-NODE(INITIAL-STATE[*problem*])
  f-limit ← *f*- COST(*root*)
  loop do
    solution, *f-limit* ← DFS-CONTOUR(*root*, *f-limit*)
    if *solution* is non-null **then return** *solution*
    if *f-limit* = ∞ **then return** failure; **end**

and good for problems with few different f-costs (e.g. 8-puzzle)

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function DFS-CONTOUR(*node*, *f-limit*) **returns** a solution sequence and a new *f*- COST limit
  inputs: *node*, a node
      f-limit, the current *f*- COST limit
  static: *next-f*, the *f*- COST limit for the next contour, initially ∞

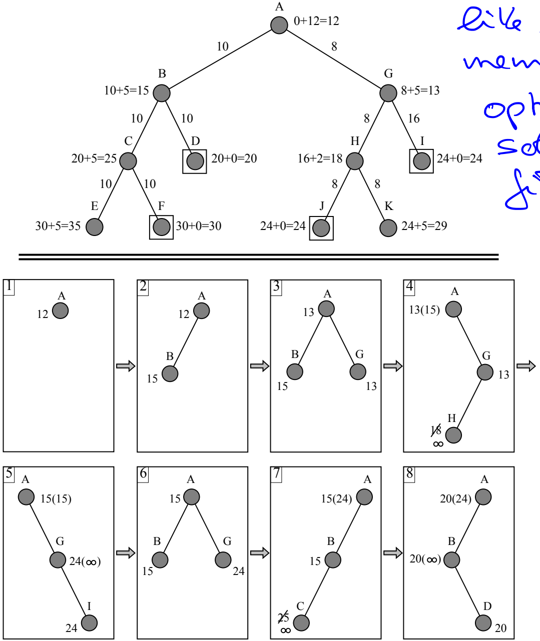
  if *f*- COST[*node*] > *f-limit* **then return** null, *f*- COST[*node*]
  if GOAL-TEST[*problem*](STATE[node]) **then return** *node*, *f-limit*
  for each node *s* **in** SUCCESSORS(*node*) **do**
    solution, *new-f* ← DFS-CONTOUR(*s*, *f-limit*)
    if *solution* is non-null **then return** *solution*, *f-limit*
    next-f ← MIN(*next-f*, *new-f*); **end**
  return null, *next-f*

SMA* 1

simplified memory-bounded A*

like A* until memory exhausted

optimal if solution path fits into memory.



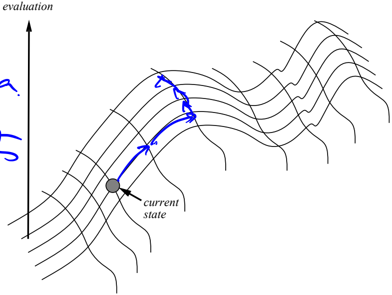
SMA* 2

```
function SMA*(problem) returns a solution sequence
  inputs: problem, a problem
  static: Queue, a queue of nodes ordered by f-cost

  Queue ← MAKE-QUEUE({MAKE-NODE(INITIAL-STATE[problem])})
  loop do
     if Queue is empty then return failure
     n ← deepest least-f-cost node in Queue
     if GOAL-TEST(n) then return success
     s ← NEXT-SUCCESSOR(n)
     if s is not a goal and is at maximum depth then
        f(s) ← ∞
     else
        f(s) ← MAX(f(n), g(s)+h(s))
     if all of n's successors have been generated then
        update n's f-cost and those of its ancestors if necessary
     if SUCCESSORS(n) all in memory then remove n from Queue
     if memory is full then
        delete shallowest, highest-f-cost node in Queue
        remove it from its parent's successor list
        insert its parent on Queue if necessary
     insert s on Queue
  end
```

Hill Climbing (dual in "gradient descent")

+ cheap method
- local maxima
↳ random restarts



beam search: start with K random nodes generate all successors and continue with K best of those.

```
function HILL-CLIMBING(problem) returns a solution state
  inputs: problem, a problem
  static: current, a node
          next, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
     next ← a highest-valued successor of current
     if VALUE[next] < VALUE[current] then return current
     current ← next
  end
```

Simulated Annealing

function SIMULATED-ANNEALING(*problem, schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  static: *current*, a node
    next, a node
    T, a "temperature" controlling the probability of downward steps

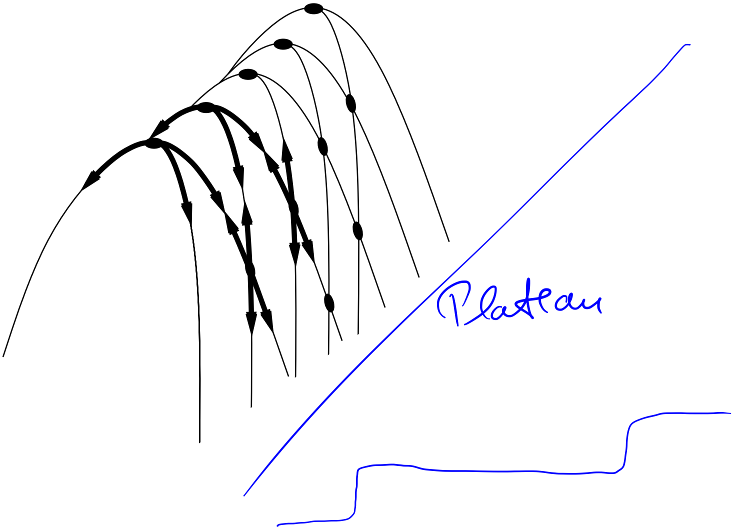
  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow$ *schedule*[t]
    if T=0 **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

if $\Delta E < 0$

$$e^{\Delta E/T} = \frac{1}{e^{|\Delta E|/T}}$$

Ridges

More problems with hill climbing.



Ridges