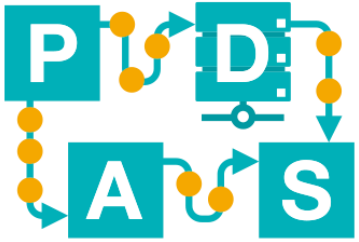
Neural Network

Lecture 7 instruction

IDS-I-L7



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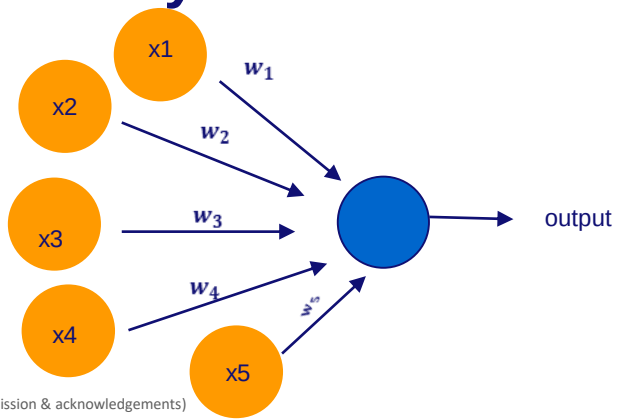
Exercise.1

- **Imagine a simple single neuron**
  - **If it has 5 input and 1 output**
  - **Each input can have value of 0 or 1**

a. Calculate the number of different input pattern ?

b. Can you formulize the pattern?



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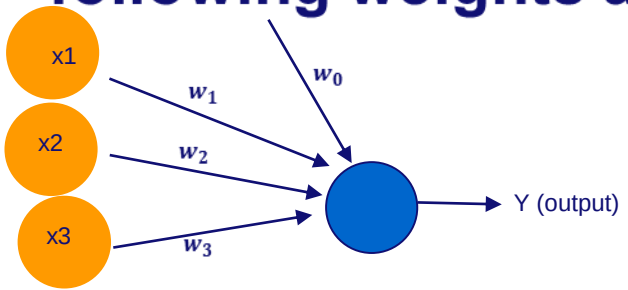
Solution.1

- **As you can see the input has binary pattern, so the pattern of input is:** 2^n *where n is the number of inputs*

	1	2	3	4	5	6	…	16
X1	0	1	0	0	0	1	…	1
X2	0	0	1	0	0	1	…	1
X3	0	0	0	1	0	0	…	1
x4	0	0	0	0	1	0	…	1

Exercise.2

- **For the previous neuron, consider three input, following weights and activation function,** for simplification ignore w_0:



$$w_1 = 2$$
$$w_2 = -4$$
$$w_3 = 1$$

$$f(x) = \begin{cases} \mathbf{0}, & x < 0 \\ \mathbf{1}, & x \geq 0 \end{cases}$$

- **Calculate the output of neurons:**

pattern	p_1	p_2	p_3	p_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

Solution.2

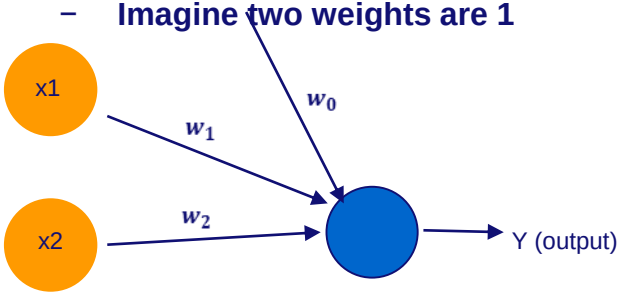
- **Use this formula for sum of the inputs:**

$$a = \sum_i w_i x_i = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3$$

- **Apply the activation function to result:**
- **Answer is:**
  - $p_1: a = 2 * 1 - 4 * 0 + 1 * 0 = 2,\ f(2) = 1$
  - $p_2: a = 2 * 0 - 4 * 1 + 1 * 1 = -3,\ f(-3) = 0$
  - $p_3: a = 2 * 1 - 4 * 0 + 1 * 1 = 3,\ f(3) = 1$
  - $p_4: a = 2 * 1 - 4 * 1 + 1 * 1 = -1,\ f(-1) = 0$

Exercise.3a

- **The main building blocks of any computational device are logical operators.**
  - **You have already seen representation of operator AND and OR in the lecture by a single neuron:**
    - **With the new activation function how AND works?**
    - **Imagine two weights are 1**



$$f(x) = \begin{cases} 0, & x < 2 \\ 1, & x \geq 2 \end{cases}$$

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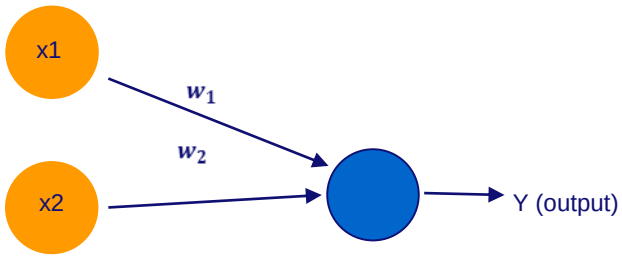
Solution.3a

- $p_1: a = 1 * 0 + 1 * 0 = 0,\ f(0) = 0$
- $p_2: a = 1 * 1 + 1 * 0 = 1,\ f(1) = 0$
- $p_3: a = 1 * 0 + 1 * 1 = 1,\ f(1) = 0$
- $p_4: a = 1 * 1 + 1 * 1 = 2,\ f(2) = 1$

$$f(x) = \begin{cases} 0, & x < 2 \\ 1, & x \geq 2 \end{cases}$$



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Exercise.3b

- How to change the neuron parameter to have logical OR function?
  - Which parameters can be changed?
  - How to change them?
  - What are the possible solutions?

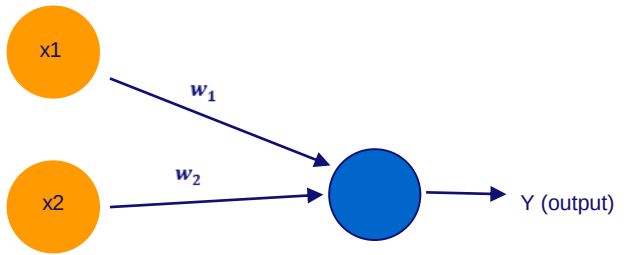
Solution.3b

- **Activation function and weights can be changed**
- **First solution:**
  - $p_1: a = 2 * 0 + 2 * 0 = 0,\ f(0) = 0$
  - $p_2: a = 2 * 1 + 2 * 0 = 2,\ f(1) = 1$
  - $p_3: a = 2 * 0 + 2 * 1 = 2,\ f(1) = 1$
  - $p_4: a = 2 * 1 + 2 * 1 = 4,\ f(2) = 1$
- **Second solution:**
  - **Changing activation function:**

$$f(x) = \begin{cases} 0, & x < 1 \\ 1, & x \geq 1 \end{cases}$$

  - **Check the functionality!**

$$f(x) = \begin{cases} 0, & x < 2 \\ 1, & x \geq 2 \end{cases}$$



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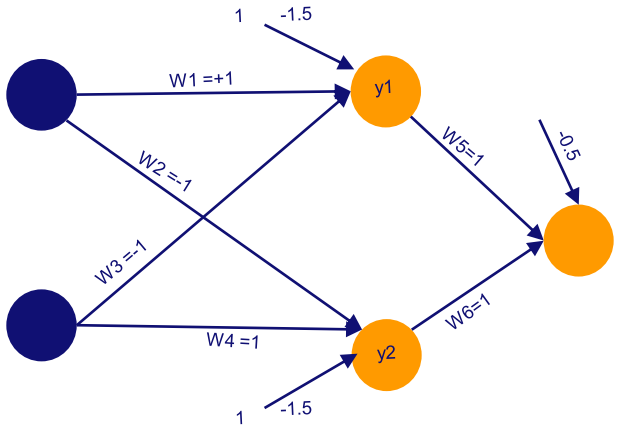
Exercise.4

- You have already seen the logical XOR neural network in the lecture.
  - XOR network can be created based on basic operators:
    - Such the one in the lecture
    - a) Try to create a neural network of XOR function(Not using first fact)
    - b)Take a way part! set different parameter for your network to act as XOR function

$$XOR = (x_1 \text{ OR } x_2) \text{ AND NOT}(x_1 \text{ AND } x_2)$$

Solution.4

- ***A*** *xor* ***B*** **= (*A* ∧ ¬ *B*) ∨ (¬ *A* ∧ *B*)**



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Exercise.5

- **Imagine a car renting company wants to deploy a new system for assessing worthiness of its customers.**
  - **The new system is using feed forward neural network with supervised learning algorithms.**
- **Suggest what should the company do before the system can be used?**

Solution.5

- First thing in order to have a supervised learning is some historical data about its customer and the one who already took cars in the past.
  - This data will be used as training set for neural network
  - The amount of data and the extensiveness of data is very important
  - The network is not be able to predict and works accurate for the customer that their similar situation were not in the historical data.

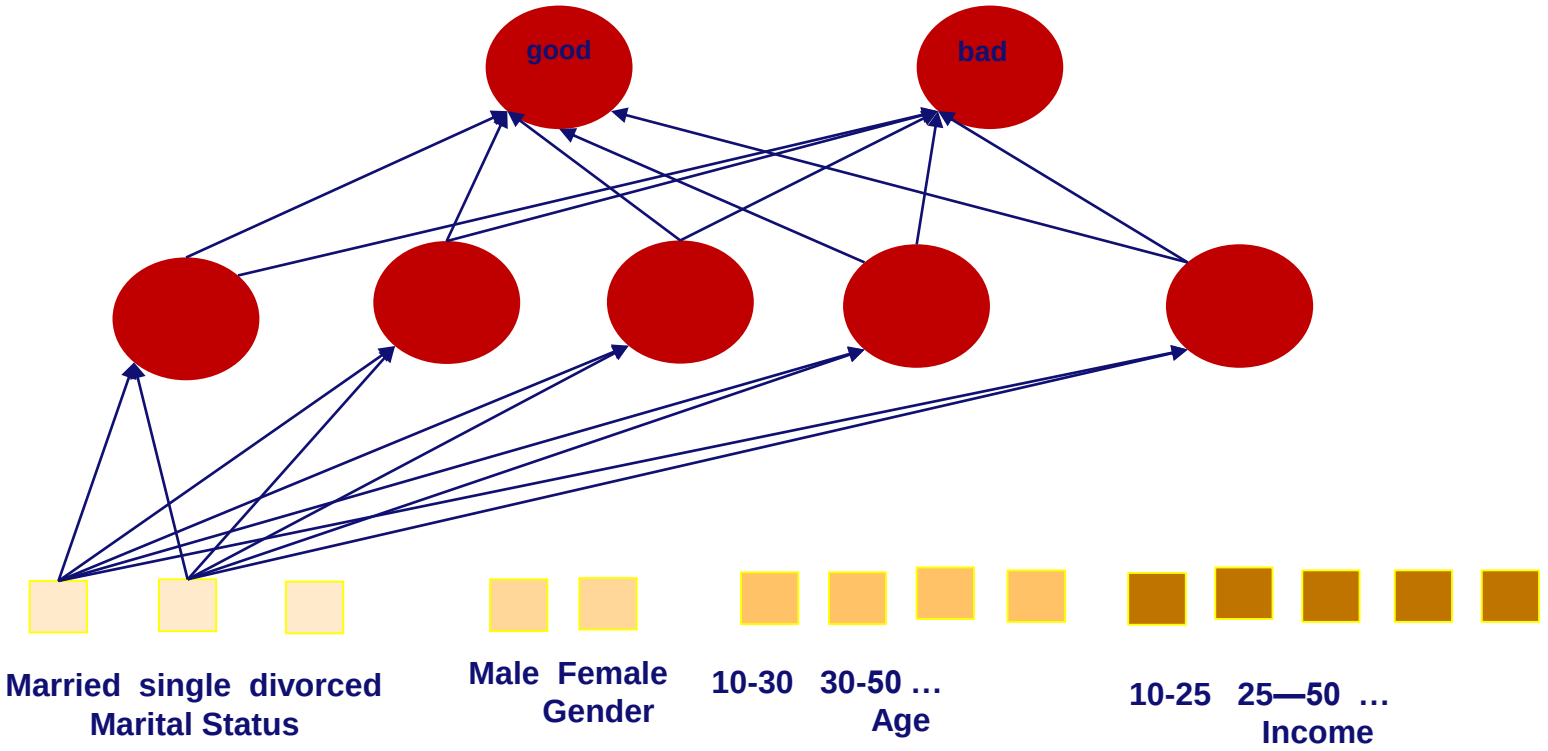
Exercise.6

- Now imagine the previous car company
  - If we want to classify potential company customers as good customers or bad ones for renting high price car with special offers:
  - We have a training dataset describing past customers using the following attributes:
    - Age {[18..30[, [30..50[, [50..65[,[65+]},
    - Marital status {married, single, divorced},
    - Gender {male, female},
    - Income {[10K..25K[, [25K..50K[, [50K..65K[, [65K..100K[, [100K+]}.
- Design a neural network in order to predict the good customers?

Solution.6

- We have two output classes good and bad
  - Then two nodes in the output layer
  - We have 4 variables
  - And for each: {3,2,4,5} values
  - We would have 14 neurons in the input layer
  - In the hidden layer we probably can have less number than input
    - However there is no predefined answer

Solution.6



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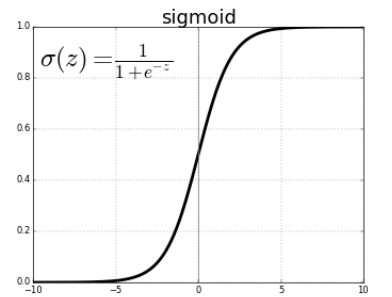
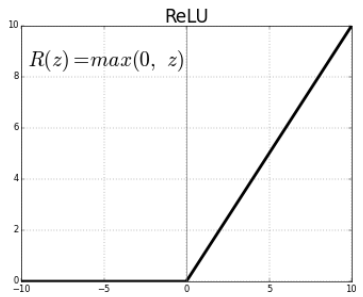
Exercise.7

- Assume that we have a 2-layer net
  - one hidden layer
  - with weights $W_0{}^1, w_1{}^1, W_0{}^2$ and $w_1{}^2$.
  - Assume that we use activation function named Relu in the hidden layer, and no activation on the output layer.
- Write down an equation for the output of the j'th node in the hidden layer.





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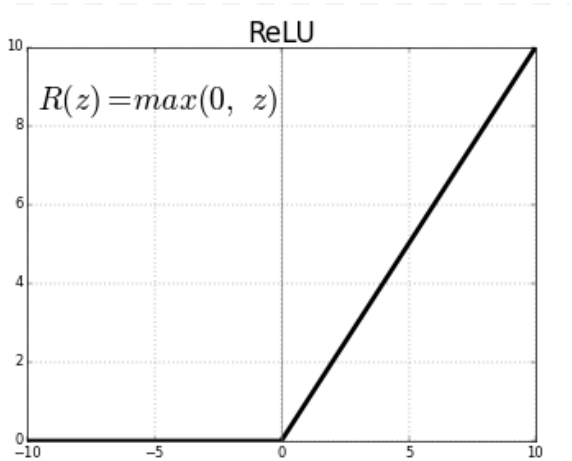
Solution.7

$$output\ of\ jth\ neuron\ in\ the\ hidden\ layer = max(W_1{}^1 * X + w_0{}^1, 0)$$



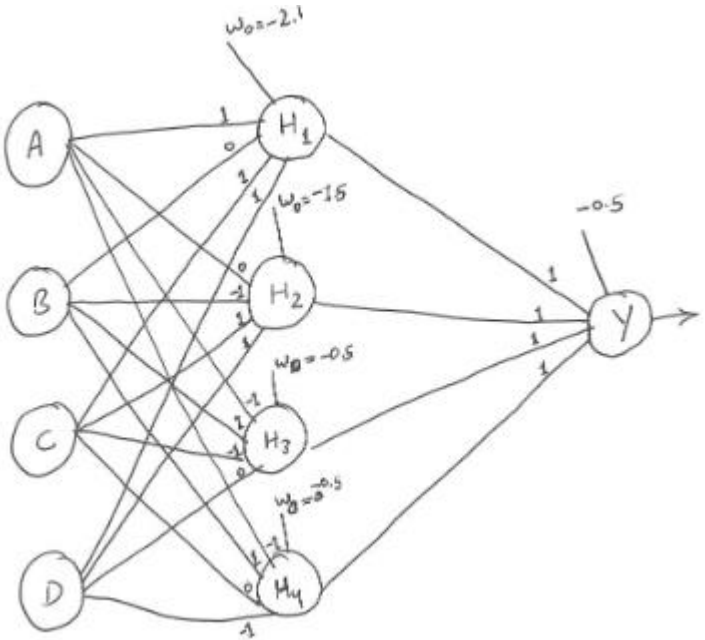
Exercise.8

- **Create a neural network with only one hidden layer (of any number of units) that implements $(A \vee \neg B) \oplus (\neg C \vee \neg D)$.**
- **Draw your network, and show all weights of each unit.**

Solution.8

- XOR can be written in terms of AND and OR:
  - p xor q = $(p \wedge \neg q) \vee (\neg p \wedge q)$
  - Given this, we can rewrite the formula as
  - $(A \wedge C \wedge D) \vee (\neg B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg D)$
- This formula can be represented by a neural network with one hidden layer and
- four nodes in the hidden layer
  - one unit for each parenthesis

Solution.8



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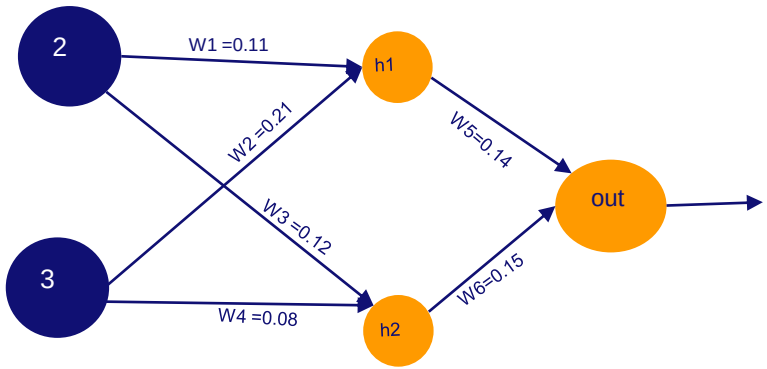
Exercise.9

- **There is a feedforward network with specified weights and inputs, calculate the value of the hidden layer neurons and the output layer neuron.**



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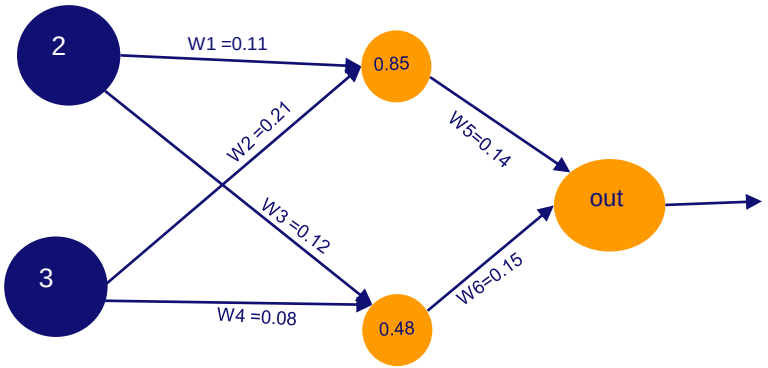
Solution.9



$$\mathbf{H1: 2 \times 0.11 + 3 \times 0.21 = 0.85}$$

$$\mathbf{Out: 0.85 \times 0.14 + 0.48 \times 0.15 = 0.191}$$

$$\boldsymbol{H2: 2 \times 0.12 + 3 \times 0.08 = 0.48}$$