Lehr- und Forschungsgebiet Informatik 5
RWTH Aachen University

Prof. G. Lakemeyer, Ph.D.
J. Claßen, M. Liebenberg
28.11.2018

Introduction to Artificial Intelligence (Winter 2018)

Assignment 4

Submit your solution electronically via the L2P until 12.12.2018.

Homework assignments are optional but strongly recommended.

Exercise 4.1 (25 points)

Prove each of the following statements.

1. α is valid if and only if $\textsc{True} \models \alpha$.
2. For any α, $\textsc{False} \models \alpha$.
3. $\alpha \models \beta$ if and only if the sentence $\alpha \supset \beta$ is valid.
4. α and β are equivalent[1] if and only if the sentence $\alpha \equiv \beta$ is valid.
5. $\alpha \models \beta$ if and only if the sentence $\alpha \wedge \neg\beta$ is unsatisfiable.

Exercise 4.2 (25 points)

Formalize "*Norma Jeane Baker is a daughter of Marilyn Monroe's parents.*" and "*Norma Jeane is not a sister of Marilyn.*" together with the needed background knowledge on relationships as first-order sentences. Provide a semantical[2] proof that "*Norma Jeane is Marilyn.*" is an entailment thereof.

Exercise 4.3 (15 points)

Use resolution to prove the following logical consequences:

1. Implication introduction: $\{\} \models (P \supset (Q \supset P))$
2. Implication distribution: $\{(P \supset (Q \supset R))\} \models ((P \supset Q) \supset (P \supset R))$
3. Contradiction realization: $\{(Q \supset P), (Q \supset \neg P)\} \models \neg Q$

[1] For any sentence ϕ, let $Mod(\phi) = \{I \mid I \text{ is an interpretation such that } I \models \phi\}$. Two sentences α and β are equivalent iff $Mod(\alpha) = Mod(\beta)$.

[2] i.e. *not* a resolution proof

Exercise 4.4 **(35 points)**

Are the following statements correct or not?

1. $\{\, \exists x P(x)\, ,\, \exists x Q(x)\, \} \quad \models \quad \exists x[P(x) \wedge Q(x)]$
2. $\{\, \forall x P(x) \vee \forall x Q(x)\, \} \quad \models \quad \forall x[P(x) \vee Q(x)]$
3. $\models \; (\forall x P(x) \wedge \forall x Q(x)\,) \equiv \forall x[P(x) \wedge Q(x)]$
4. $\models \; \neg\phi \quad$ where ϕ is $\forall x[P(x) \supset Q(x, g(x))] \wedge \exists x[P(g(x)) \wedge \neg Q(g(x), g(g(x)))]$
5. $$\begin{aligned} \{\, \forall x \exists y[\quad & (\, P(y) \supset P(f(x))\,) \\ & \wedge (\, P(f(x)) \supset Q(x, f(y))\,) \\ & \wedge (\, Q(x, f(x)) \vee P(y)\,) \quad]\,\} \quad \models \quad \forall x \exists y Q(x, y) \end{aligned}$$
6. $\{\, \forall x \exists y P(x, y)\, ,\, \neg\exists z P(z, a)\, \} \;\models\; \exists y \forall x P(x, y)$

Prove each of your claims either by means of resolution (when the statement is correct) or by specifying a suitable interpretation as a counterexample (in case the statement is wrong).