Problem Solving Using Search

Introduction to Artificial Intelligence

G. Lakemeyer

Winter Term 2018/19

Problem Solving Using Search

Here we consider goal-oriented agents.

Given an initial (world) state, the agent wants to reach a certain goal using appropriate actions, where actions transform one state into another.

A goal is a set of world states, which the agent finds desirable (wants to reach one of them).

Example: Driving from City A to City B.

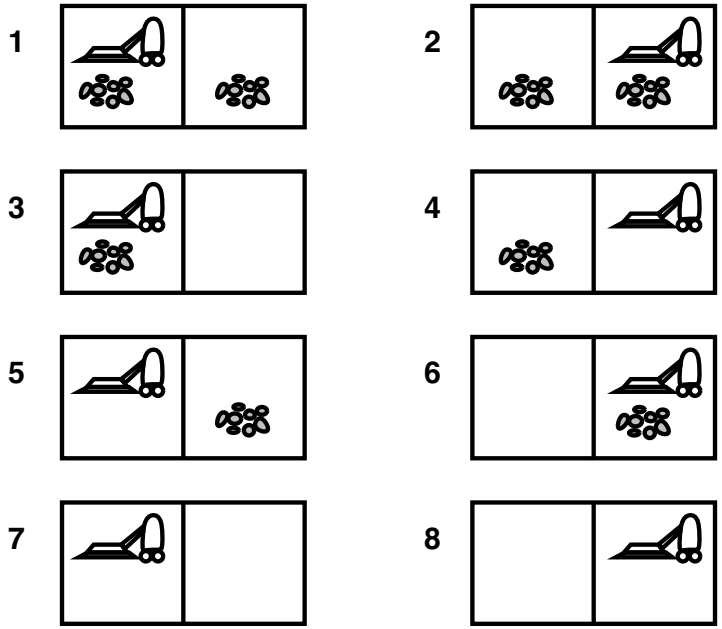
Problem formulation

- How to represent world states?
- Which actions should be considered?

Search: Finding an action sequence which transforms an initial state into a goal state.

Problem Classification

The vaccuum-world states:



- Single-state problem
  complete world knowledge,
  complete knowledge about the actions
- Multiple-state problem
  incomplete world knowledge,
  complete knowledge about the actions
- Contingency problem
  incomplete knowledge about actions,
  needs to gather information at run-time
- Exploration problem
  World states and effect of actions are both unkown. Very difficult!

↪ e.g. Mars Rover

Some Terminology (1)

Initial state:	World state which the agent believes to be in initially.
State space:	Set of all possible states.
Operator:	Description of which state is reached by an action from a given state.
Successor function S:	$S(x)$ returns the set of states reachable by any action from state x.
Goal test:	Tests whether a state is a goal state.

Some Terminology (2)

Path:	Sequence of actions.
Path cost:	Cost function over paths, usually the sum of the costs of the actions of the path, often denoted as g.
Solution:	Path from the initial state to a state that satisfies the goal test.
Search cost:	Time and memory needed to find a solution.
Total cost:	Sum of the path cost (of the solution) and search cost.

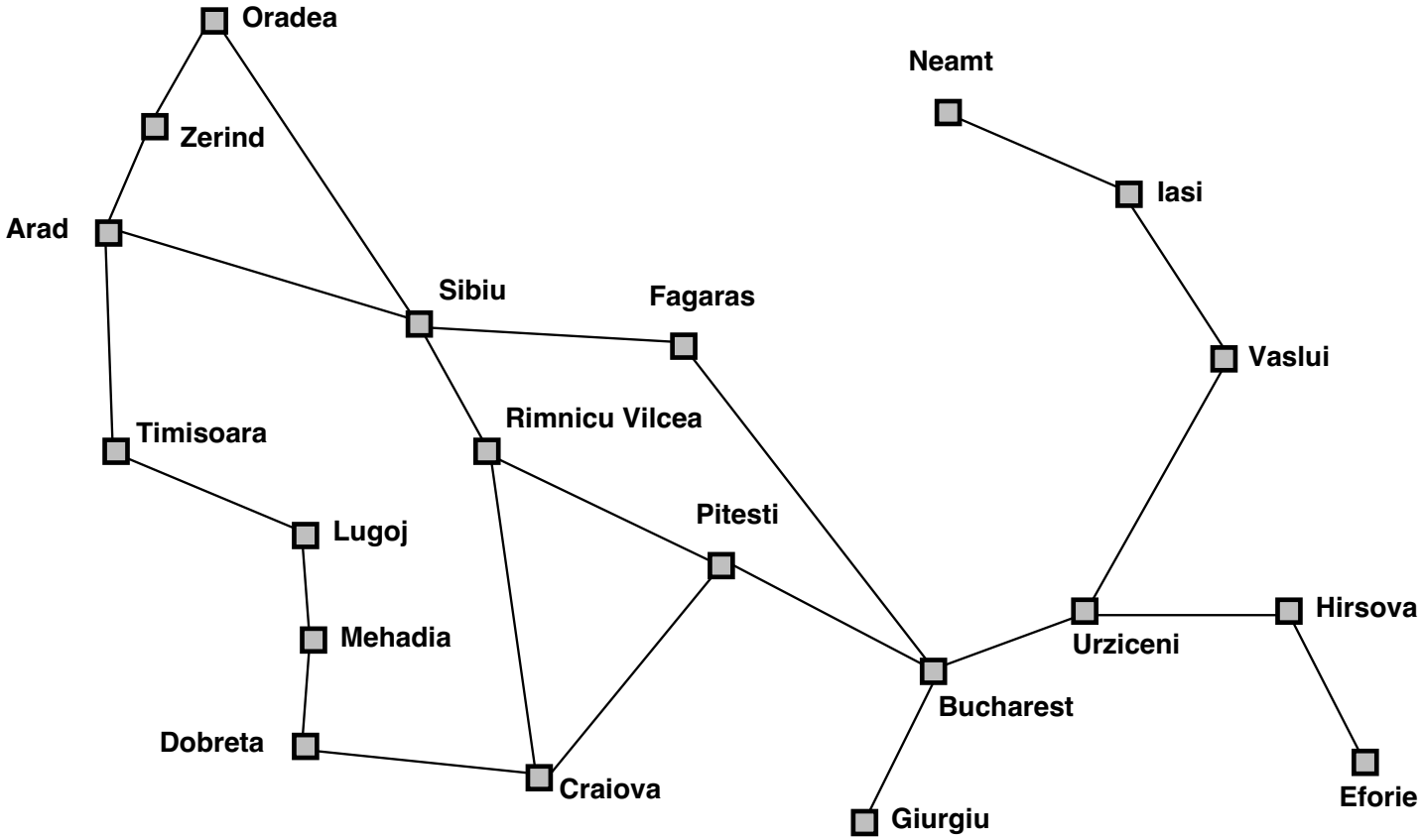
Note: For multiple-state problems, replace state by state set, operators apply to all elements of the set, and paths connect sets of states.

Choosing a State Space

It is an art to choose the right state space. Want to leave out unnecessary detail (Abstraction)!

Example: Drive from A to B.

Only represent the most important places.

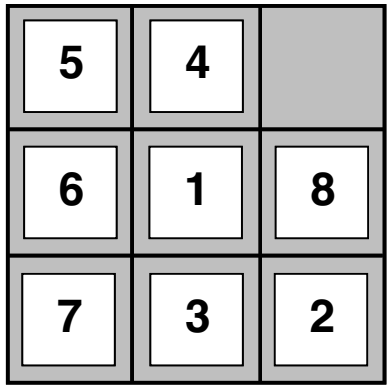


8-Puzzle

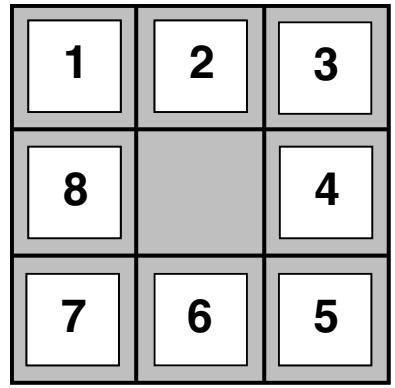
only half the states are reachable,
i.e $\frac{9!}{2}$ reach. states $= 181\,444$

24-puzzle (5 × 5)

how about 10^{25} states



Start State

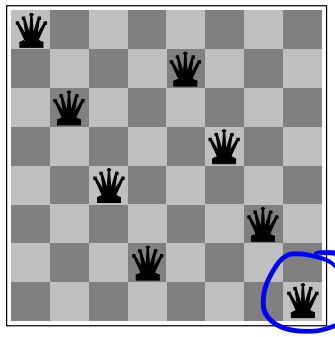


Goal State

- States: Describe the location of each tile including the blank.
- Operators: Blank moves left, right, up, or down.
- Goal test: Does the state match picture on the right?
- Path cost: Each step costs 1 unit.
- Search cost: Problem is NP-complete in the size of the puzzle.

to find the shortest solution.

8-Queens Problem



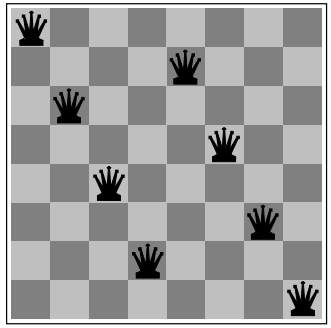
- Goal test: 8 queens on the board, all safe.
- Path cost: 0 (only the solution counts).

Representation 1

- States: any placement of 0–8 queens.
- Operators: place a queen on the board.
- Problem: Too many states (64^8)!

If we allow at most 1 queen per square the — $64 \cdot 63 \cdot \ldots \cdot 57 \approx 3 \cdot 10^{14}$

8-Queens Problem



- Goal test: 8 queens on the board, all safe.
- Path cost: 0 (only the solution counts).

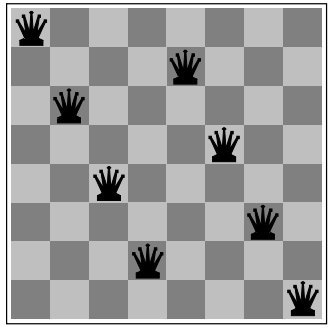
Representation 1

- States: any placement of 0–8 queens.
- Operators: place a queen on the board.
- Problem: Too many states (64^8)!

Representation 2

- States: 0–8 on board, all safe.
- Operators: place a queen starting from left.
- Problem: few state (2057), but sometimes deadend (see figure).

8-Queens Problem



- Goal test: 8 queens on the board, all safe.
- Path cost: 0 (only the solution counts).

Representation 1

- States: any placement of 0–8 queens.
- Operators: place a queen on the board.
- Problem: Too many states (64^8)!

Representation 2

- States: 0–8 on board, all safe.
- Operators: place a queen starting from left.
- Problem: few state (2057), but sometimes deadend (see figure).

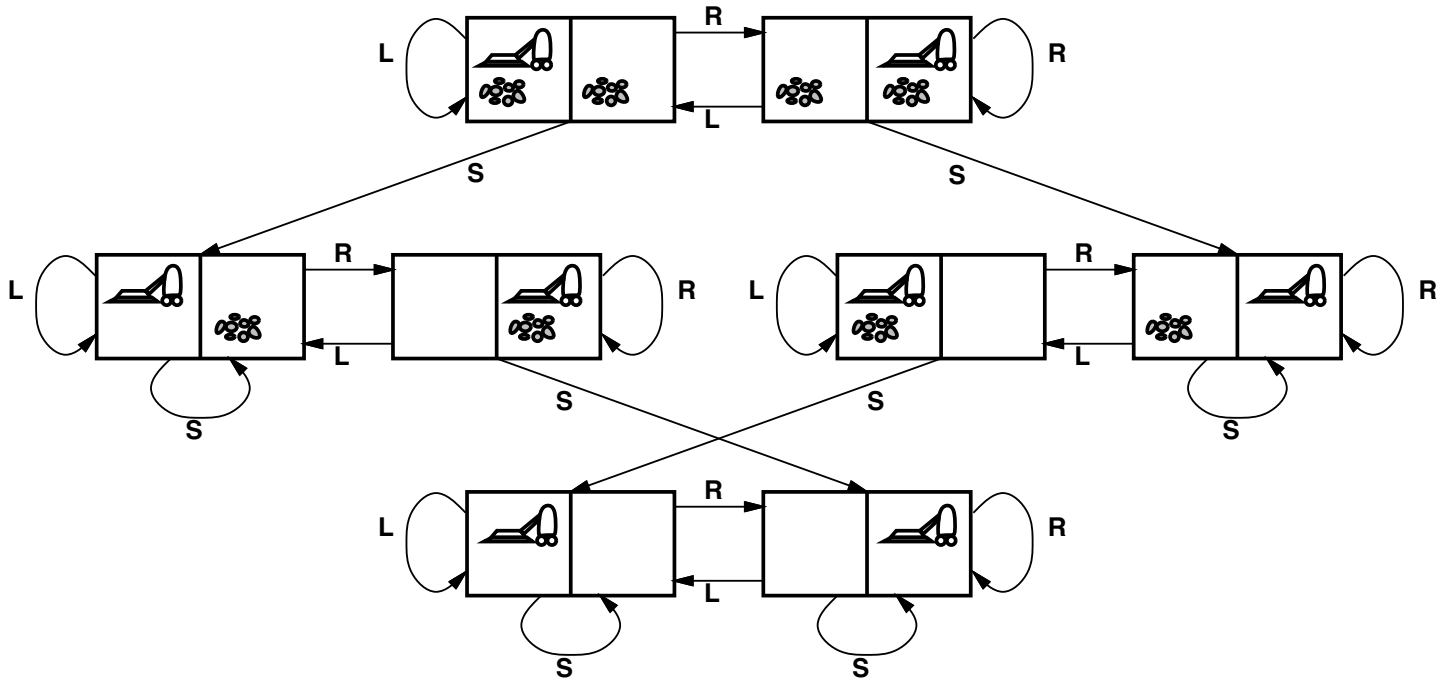
Representation 3

- States: 8 queens on board, one per column.
- Operators: move a queen under attack in the same column.

8^8 state

Vacuum World 1

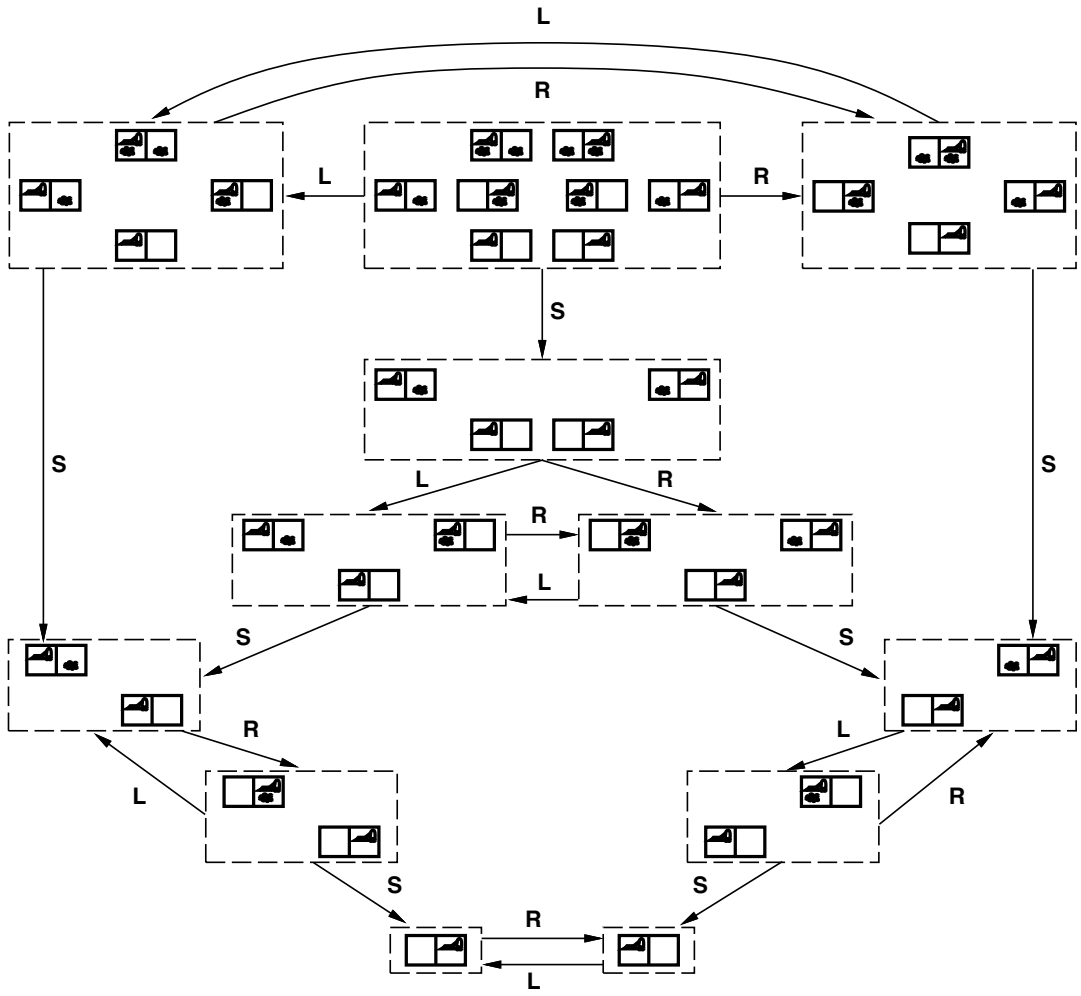
Complete World Knowledge (single-state problem):



- States: All combinations of robot location and presence of dust (8).
- Operators: left (L), right (R), suck (S).
- Goal test: no dust in any room.
- Path cost: 1 unit per action.

Vacuum World 2

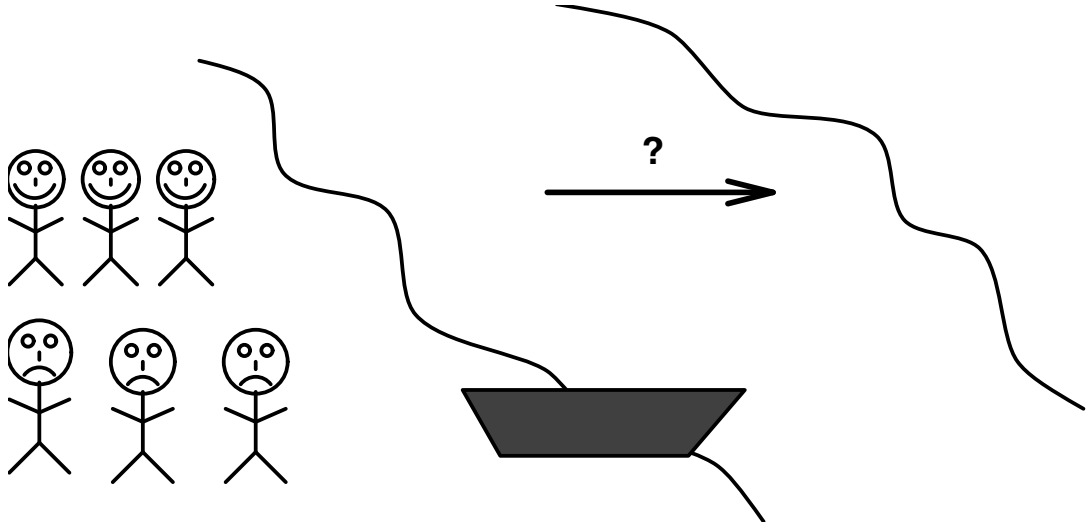
Incomplete Knowledge (multiple-state problem).



- State set: All subsets of the 8 states.
- Goal test: No dust anywhere in the elements of the state set.

Missionaries and cannibals (MaC)

John McCarthy



3 missionaries and 3 cannibals want to cross a river. There is a row boat holding at most 2 people. Should the cannibals ever outnumber the missionaries, the missionaries will be eaten. How do all six make it across safely.

Non-Intended MaC Solutions

1. All six walk across the bridge nearby.
2. They take a helicopter.
3. There are three more missionaries on the other side.
4. . . .

Contradicts the underlying assumption that the description of the puzzle contains all relevant information.

Gricean principle

Sabotaging the Correct MaC Solution

1. There is a hole in the boat.
2. The missionaries are too weak to row.
3. One missionary goes overboard and drowns.
4. . . .

Contradicts the assumptions of what happens normally.

Formalizing the MaC-Problem

- States: Triple (x,y,z) with $0 \leq x, y \leq 3$ and $0 \leq z \leq 1$, where x, y, and z denote how many missionaries, cannibals, and boats are currently on the left river bank.
- Initial state: (3,3,1) 

- Operators: Either one missionary, one cannibal, two missionaries, two cannibals, or one of each cross the river in a boat. (i.e. 5 operators)

  Note: not every state is reachable [e.g. (0,0,1)].
- Goal state: (0,0,0)
- Path cost: 1 unit per river crossing.

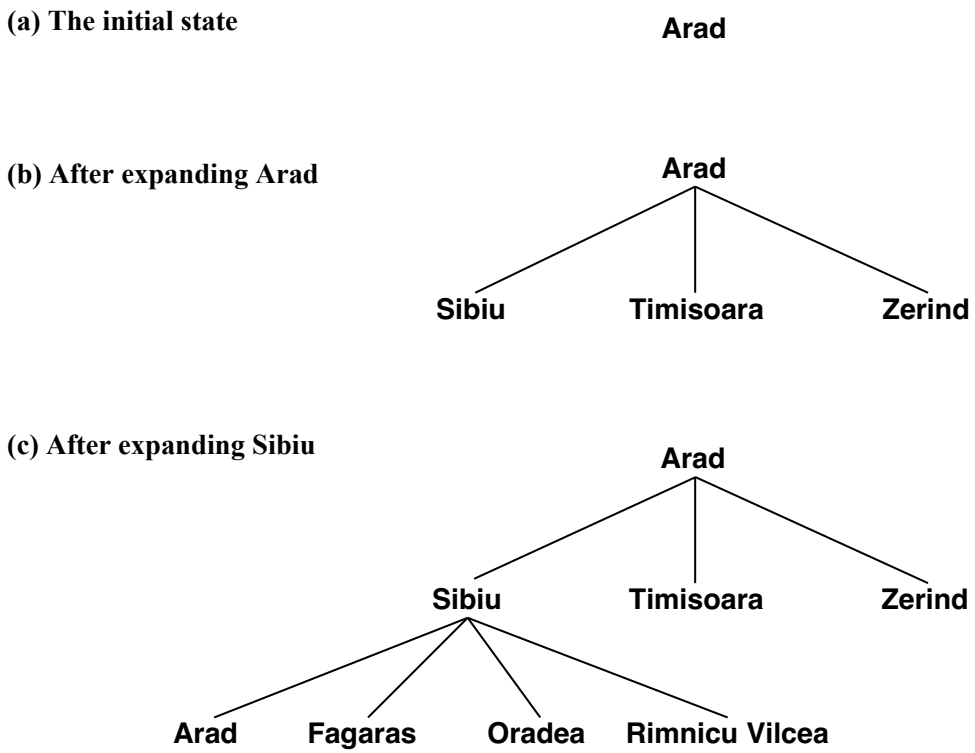
Note: Solution due to Saul Amarel (1968).

Search in General

Starting from the initial state we always expand one state, i.e. we create all successor states of that state.

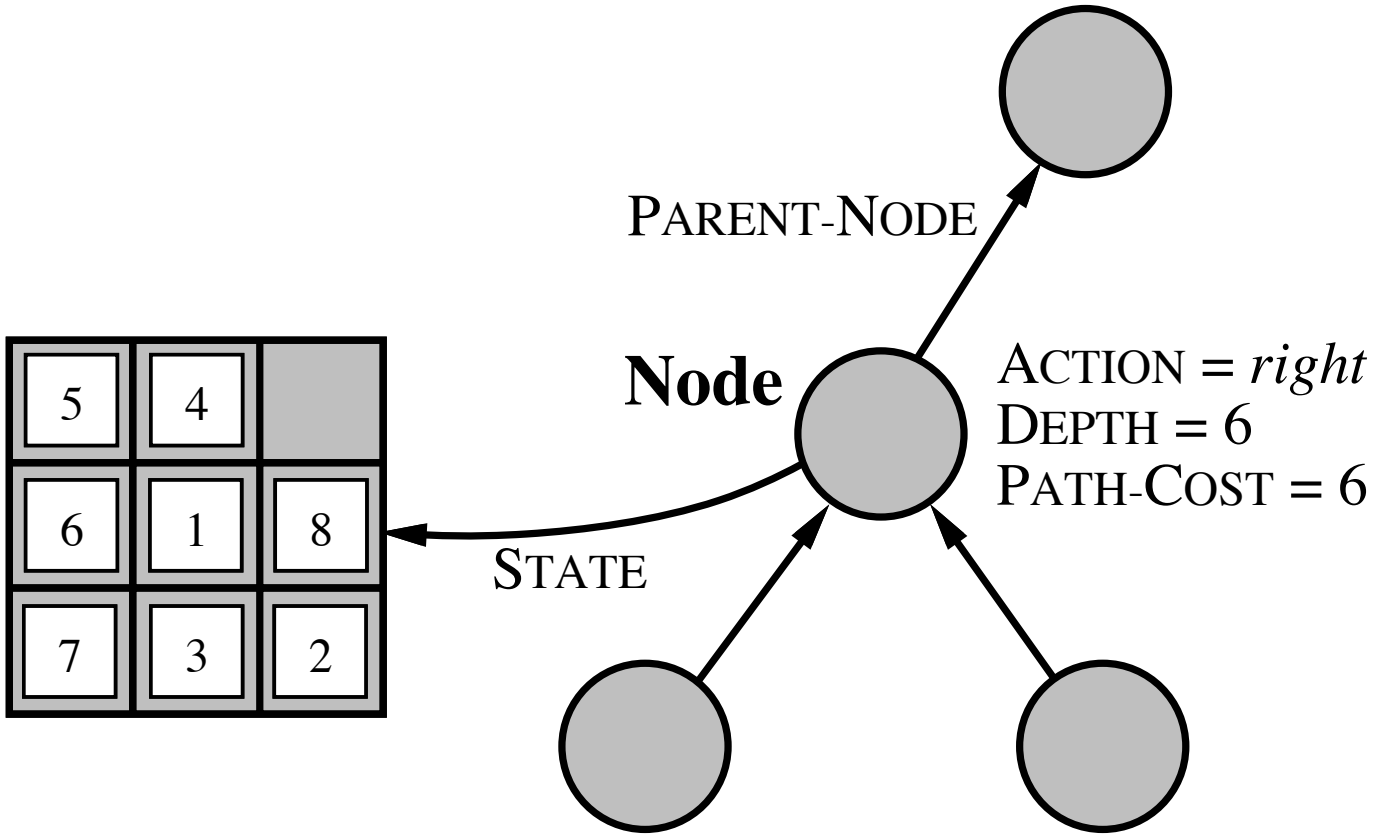
Search tree = the tree induced by expansion.

Nodes contain world states; edges = actions.



function GENERAL-SEARCH(*problem, strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end

Search Nodes



Evaluating Search Strategies

- Completeness: Does it always find a solution if one exists?
- Time Complexity: How long does it take to find a solution in the worst case?
- Space Complexity: How much memory is used in the worst case?
- Optimality: Does it always find the best solution?

Uninformed vs. Informed Search

Uninformed (blind) search:

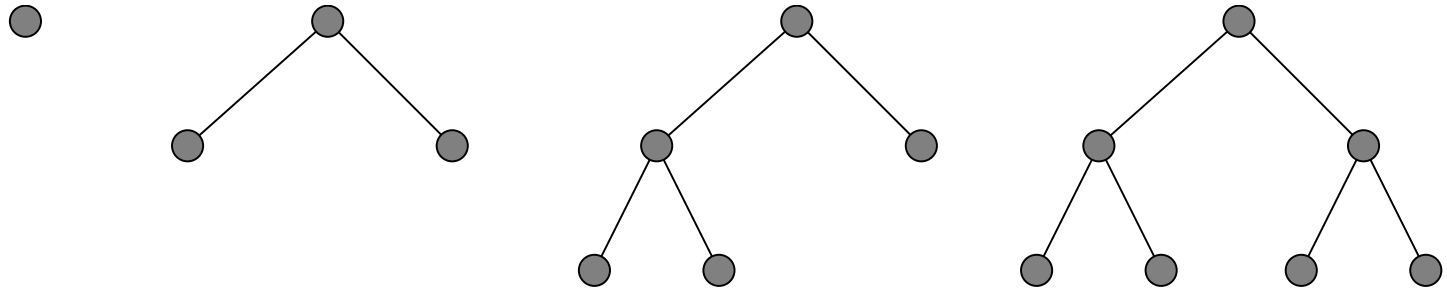
- No information available about the length or the cost of a solution.
- The main methods are:
  breadth-first search, uniform cost search, depth-first search,
  depth-limited search, iterative deepening, bidirectional search.

Informed (heuristic) search:

- Here the user has information about the length or the cost of a solution,
  which often helps in speeding up the search.
- We will look at:
  Greedy search, A* and some variants, hill-climbing, and
  simulated annealing.

Breadth-First Search

Expand the nodes in the same order in which they are generated.



- Always finds the shallowest solution (completeness).
- Solutions are optimal, provided all actions have identical non-negative cost.

Cost of Breadth-First Search

time = space complexity !

The Cost is very high!. Let b be the max. branching factor, d the depth of a solution. Then at most

$$b + b^2 + b^3 + \ldots + b^d + b^{d+1} - b$$

nodes need to be generated, i.e. $O(b^{d+1})$.

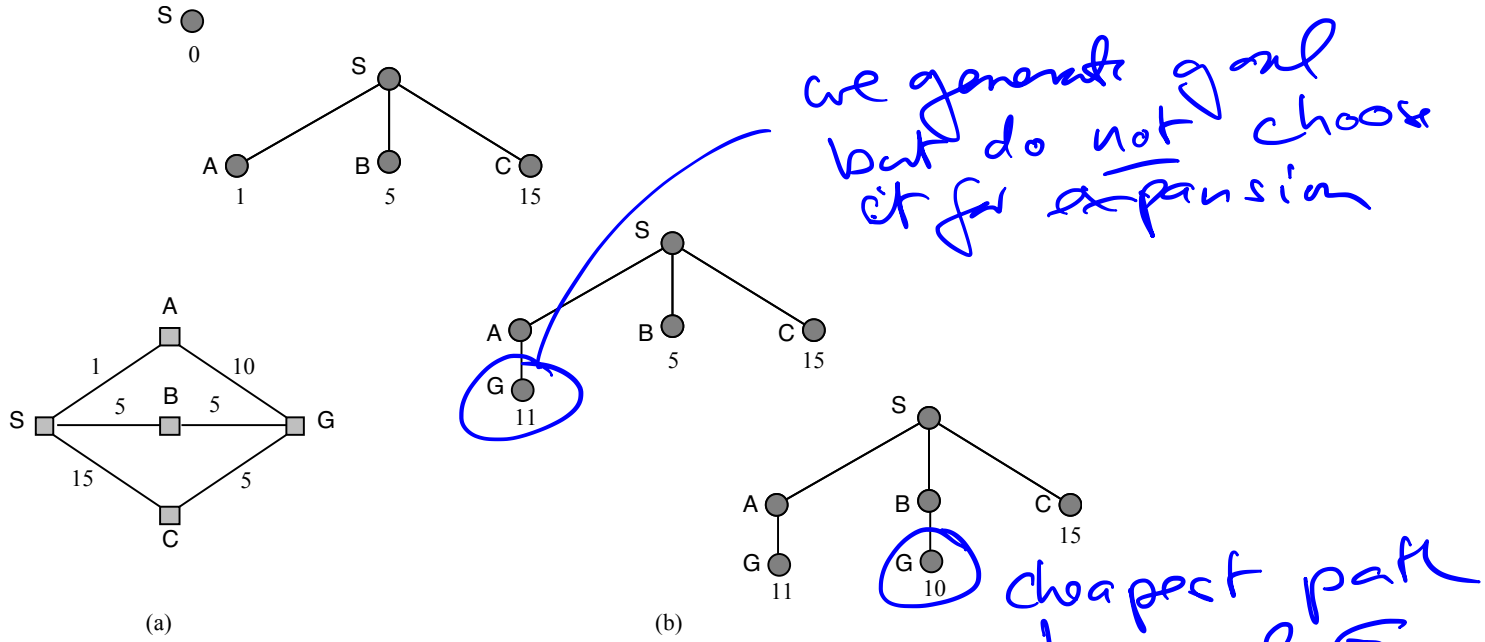
→ depends on doing goal test only when choosing a node for expansion

Example: $b = 10$, 10000 nodes/s; 1000bytes/node:

Depth	Nodes	Time	Memory
2	1100	.11 seconds	1 megabyte
4	111,100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabytes
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3,523 years	1 exabyte

Uniform cost search

A modified breadth-first search, which always expands node n with the least path cost $g(n)$.



Note: Always finds the cheapest solution, provided

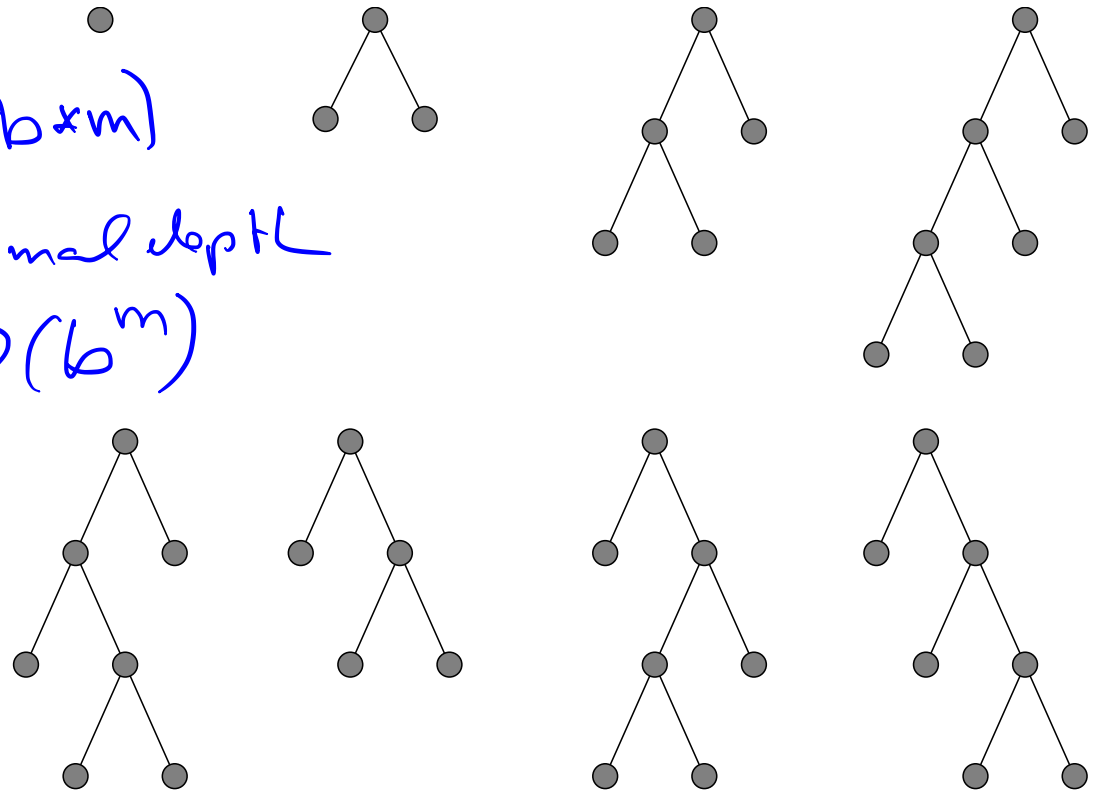
$$g(successor(n)) \geq g(n) \text{ for all } n.$$

optimality guaranteed for non-negative action costs.

Depth-First Search

not complete if depth is unbounded.
not optimal

Always expand the node at the deepest level.



space : $O(b*m)$

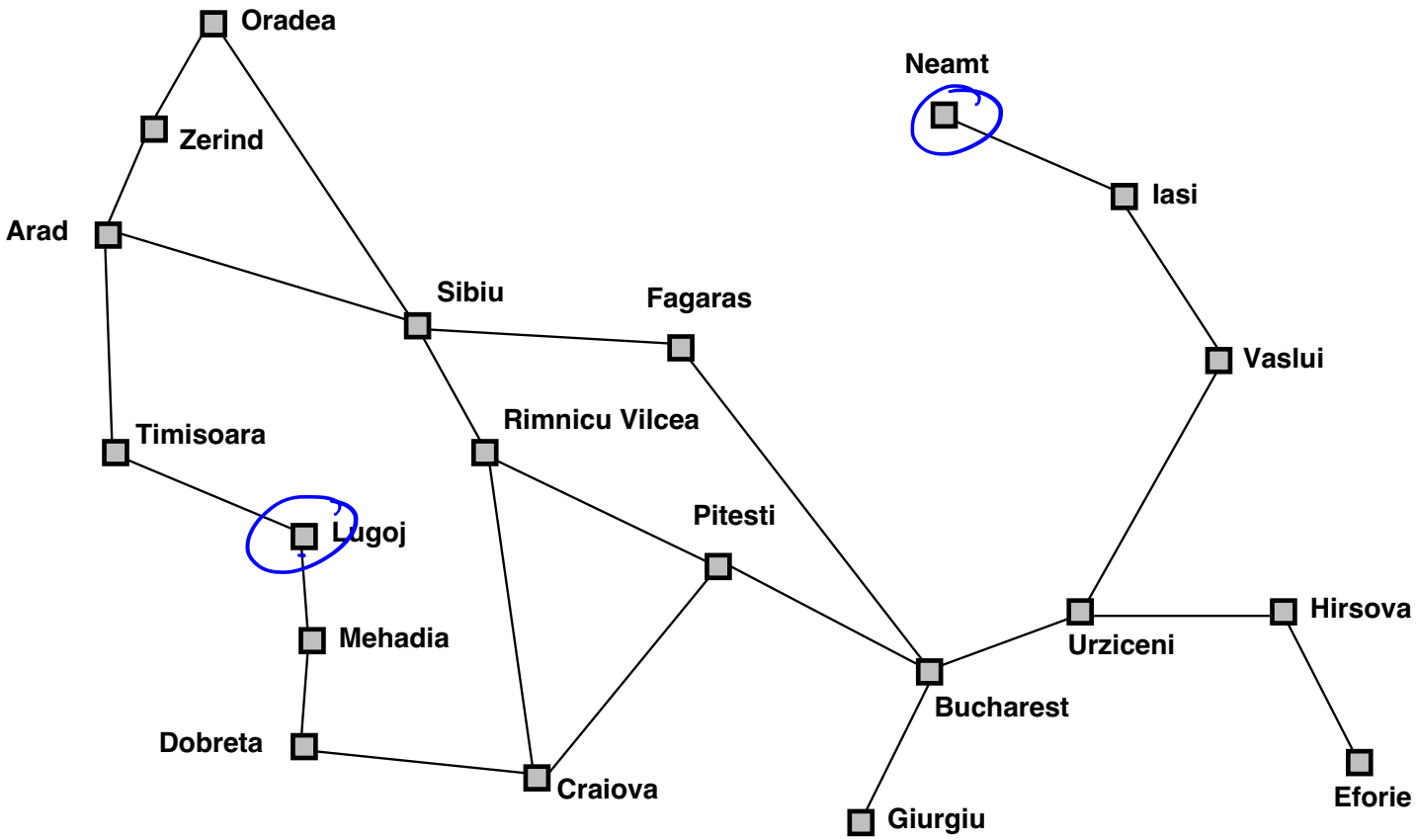
m : maximal depth

time : $O(b^m)$

Depth-limited Search

Perform depth-first search, but only up to a pre-specified depth.

Route planning: For n cities the maximal depth is $n - 1$.



Actually, a maximal depth of 9 suffices (called the diameter of the problem).

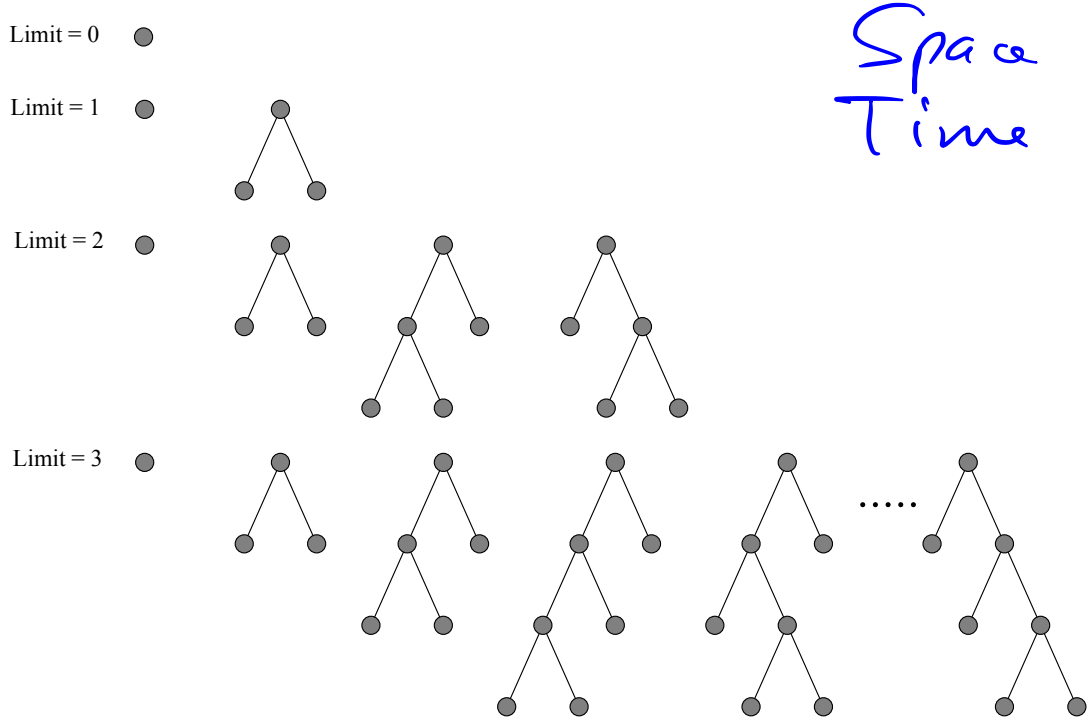
(difficult to find in general)

Iterative Deepening 1

- Combines depth- and breadth-first search.
- Optimal and complete like breadth-first search, but requires less space.

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution sequence
  inputs: *problem*, a problem

  for *depth* $\leftarrow$ 0 **to** ∞ **do**
    if DEPTH-LIMITED-SEARCH(*problem, depth*) succeeds **then return** its result
  end
  return failure



Space $O(b \times d)$
Time $O(b^d)$?

Iterative Deepening 2

Number of expansions:

$$(d)b + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

Compared to breadth-first search: $b + b^2 + b^3 + \ldots + b^{d-1} + b^d + b^{d+1} - b$.

Note: iterative deepening is better!

Example: $b = 10$, $d = 5$

Breadth-first search:

$$10 + 100 + 1000 + 10000 + 100000 + 999990 = 1111110$$

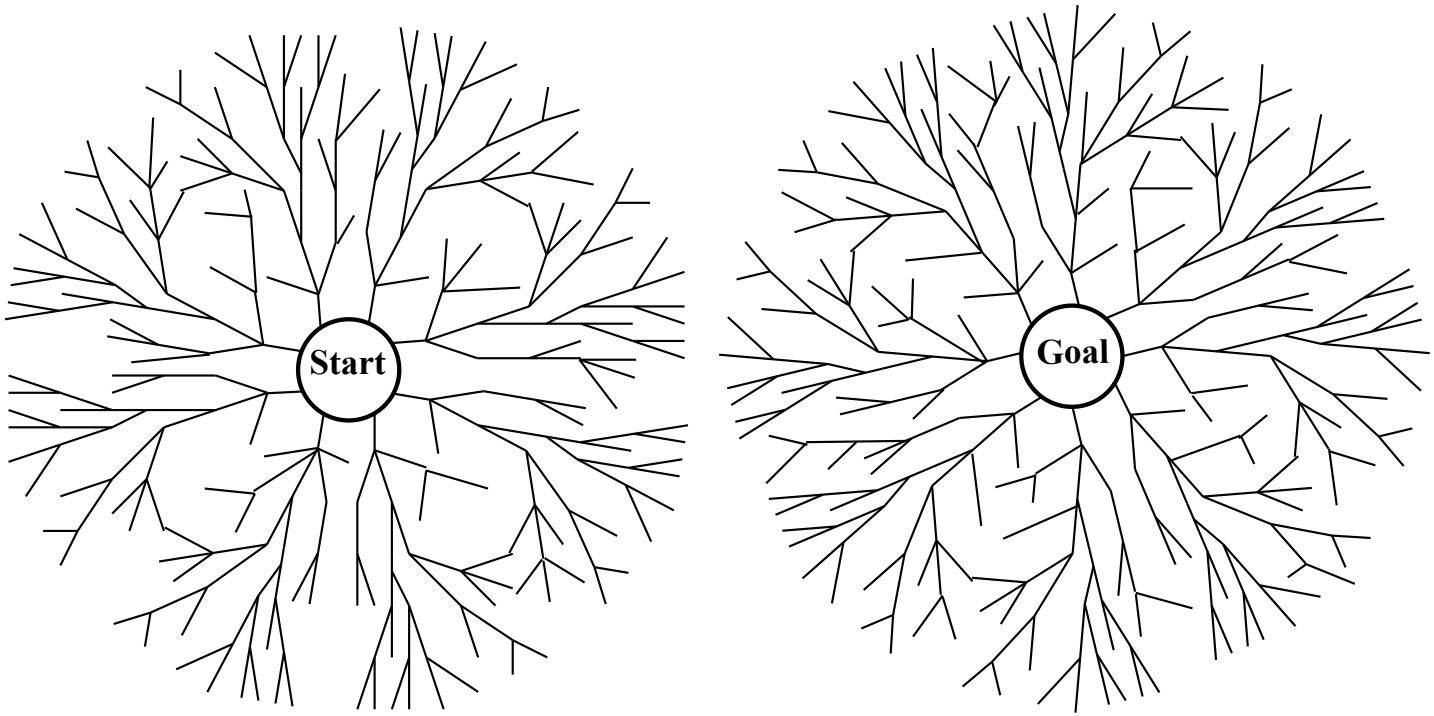
Iterative deepening:

$$50 + 400 + 3000 + 20000 + 100000 = 123450$$

Time complexity: $O(b^d)$
Space complexity: $O(b \times d)$

Note: Preferred method if search depth unknown!

Bidirectional Search



Assuming that forward and backward search are symmetric, we can ideally obtain search times of $O(2 \times b^{d/2}) = O(b^{d/2})$.

Example: For $b = 10$, $d = 6$ we have 2222 nodes instead of 1111111!

Problems with Bidirectional Search

- Operators not always reversible. (Need to compute the predecessor node.)
- Sometimes there are very many goal states, which are described only incompletely. E.g., what are the predecessors of “checkmate” in chess?
- We need efficient methods to decide when the two search methods have met.
- Which kind of search method should one use for each direction? (In the figure it is breadth-first search for both, but that is not always optimal.)

Comparison

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^{d+1})$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^{d+1})$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

assumes action cost $\geq \varepsilon > 0$

C^* = optimal cost to a goal.

b = maximal branching factor

d = depth of a solution path

m = maximal search depth

l = depth restriction

[a]: complete if b is finite; [b]: complete if step costs $\geq \epsilon > 0$;
[c]: optimal if step costs identical; [d]: if both directions use BFS.