Neural Networks

Introduction to Artificial Intelligence

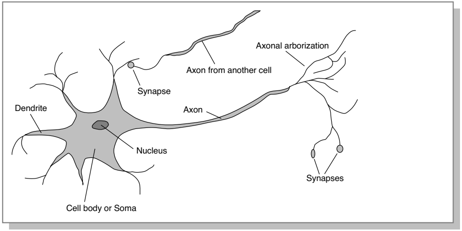
G. Lakemeyer

Winter Term 2018/19

Brain vs. Computer

Analogy to the human brain:

many processors (neurons) and connections (synapses), which process information locally and in parallel.



von Neumann computer vs. the brain:

	Computer (1995)	Human Brain
Computational units	1 CPU, $10^{\cancel{5}\,8}$ gates 13	10^{11} neurons
Storage units	$10^{\cancel{9}\,10}$ bits RAM, $10^{\cancel{10}}$ bits disk	10^{11} neurons, 10^{14} synapses
Cycle time	$10^{-\cancel{8}\,9}$ sec	10^{-3} sec
Bandwidth	$10^{\cancel{9}\,10}$ bits/sec	10^{14} bits/sec
Neuron updates/sec	$10^{\cancel{5}\,6}$	10^{14}

assumption 10^3 cycles / update

Note: The brain cycle is slow ($10^{-3}s$), but updates work in parallel. A serial simulation on a computer needs several hundred cycles.

Advantages of Neural Networks

- High processing speed through massive parallelism;
- still works if parts of the network are damaged (robustness);
- graceful degradation;
- designed for inductive learning.

It seems reasonable (obvious?) to try and recreate these advantages by designing artificial neural networks.

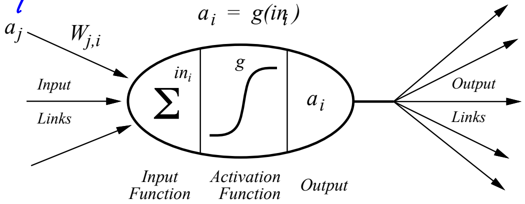
Research in this area started already in the 40-ies.
(McCulloch und Pitts 1943)

Some Terminology of Neural Networks

Units:	represent the nodes (neurons) in the network.
Input/output units:	Special nodes connected to the external environment.
Links:	Edges between nodes in the network. Each node has input and output edges.
Weights:	Each edge has a weight, usually a real number.
Activation level:	The value computed by a unit given its weighted inputs. It is passed to the neighbor nodes via the output edges.

How a Unit Works

output of unit j



$$a_i := g(in_i) = g\left(\sum_j W_{j,i} \times a_j\right)$$

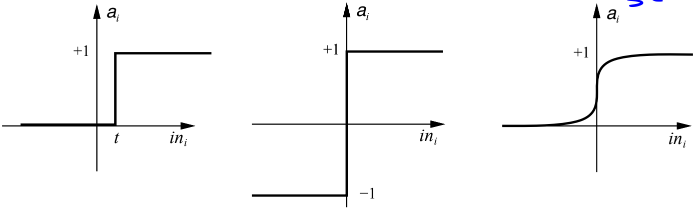
g is usually a nonlinear function.

Activation Functions

$s(x) = \frac{1}{1+e^{-x}}$

$s'(x) = s(x) * (1 - s(x))$

Some important activation functions are:



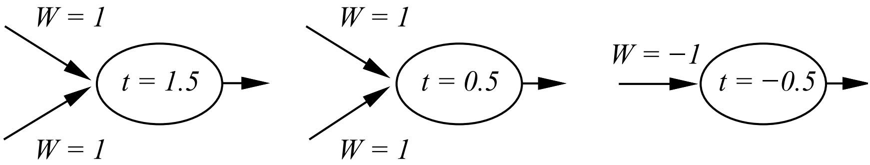
(a) Step function **(b) Sign function** **(c) Sigmoid function**

Note: t in *step* represents a threshold, that is, the minimum total weighted input necessary for the neuron to fire (similar to actual neurons in the brain).

Mathematically, one can always use a threshold of 0 by having an additional input with activation level $a_0 = -1$ and $W_{0,i} = t$. Then

$$a_i = step_t\left(\sum_{j=1}^{n} W_{j,i} \times a_j\right) = step_0\left(\sum_{j=0}^{n} W_{j,i} \times a_j\right)$$

Representing Boolean Functions



AND **OR** **NOT**

Thus neural nets can at least represent any arbitrary Boolean function.

[We will see below that they can represent much more.]

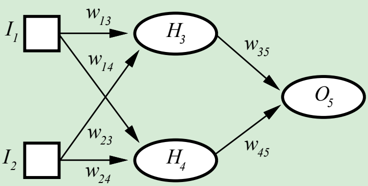
Network Topologies

Feed-forward: Units and links represent a directed acyclic graph.

Recurrent: Units and links represent arbitrary directed graphs.

Nets are often arranged in layers, that is, nodes of one layer are linked only to nodes of the next layer. There are no links between nodes of a layer. (When counting layers, the input layer is ignored.)

Example of a feed-forward 2-layer network:



Of all networks feed-forward nets are the best understood. (The output is solely a function of the inputs and the weights.) Recurrent networks are often unstable, oscillate, or have chaotic behavior.

Note: The brain is massively recurrent.

Recurrent Network Types – Hopfield Nets

- Recurrent nets with symmetric bi-directional links ($W_{i,j} = W_{j,i}$).
- All units are both input and output units.
- Nets function as associative memory.
- After training a new input is matched against the "closest" example seen during training.

unit represent pixels for image

Example:

Training with n photographs.
New input = part of a photograph used during training.
Then the network reconstructs the whole image.

Note: Each weight is a partial representation of every image!

Theorem

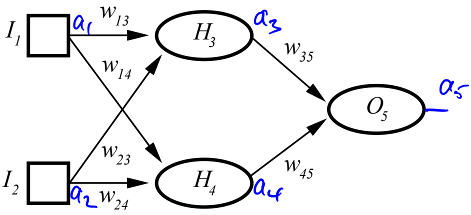
A Hopfield Net can reliably store $0,138 \times N$ examples, where N is the number of units.

Recurrent Network Types – Boltzmann Machines

G. Hinton + T. Sejnowski
1982

- Also use symmetric weights.
- There are units which are neither input nor output units.
- Stochastic Activation function.
- State transitions are like simulated annealing search for the configuration that best approximates the training set.
  (Formally identical to a certain kind of belief nets.)

Feed-Forward (FF) Nets



Hidden Units:
Units without direct connection to the external environment. Hidden units are organized in one or more hidden layers.

Perceptrons:
FF Networks without hidden units.

FF Nets represent complex nonlinear functions.

- With 1 hidden layer every continuous function is representable.
- With 2 hidden layers every function is representable.

When the topology and activation function g are fixed, the representable functions have a specific parametrized form (parameters = weights).

Example:

$a_5 = g(W_{3,5}a_3 + W_{4,5}a_4) = g(W_{3,5}g(W_{1,3}a_1 + W_{2,3}a_2) + W_{4,5}g(W_{1,4}a_1 + W_{2,4}a_2))$

$\Rightarrow$ Learning = Search for the correct parameters = nonlinear regression.

Note : g must be non-linear.

Optimal Network structure?

Choosing the right network is a difficult problem!

Also the optimal net may be exponentially large (relative to the input).

- Net is too small: the desired function is not representable.
- Net is too big: Net memorizes examples without generalization (analogous to memorizing in decision trees) Overfitting

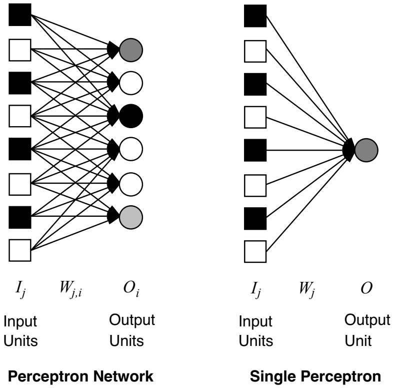
There is no good theory of how to choose the right network, only some heuristics.

Heuristics of optimal brain damage:
Start with a network with a maximal number of connection. After the 1st training reduce the number of connections using information theory. Iterate.

Example:

Network to recognize zip codes. 3/4 of the initial connections could be removed. (There are also methods to move from a network with few nodes to a network with more nodes.)

Perceptrons



$$O = Step_0\left(\sum_j W_j I_j\right) = Step_0(W \cdot I)$$

How Do Perceptrons Learn?

```
function NEURAL-NETWORK-LEARNING(examples) returns network
  network ← a network with randomly assigned weights
  repeat
    for each e in examples do
      O ← NEURAL-NETWORK-OUTPUT(network, e)
      T ← the observed output values from e
      update the weights in network based on e, O, and T
    end
  until all examples correctly predicted or stopping criterion is reached
  return network
```

1 iteration = "epoch"

Error = $T - O$ with

O = predicted (actual) output

T = correct output

learning rate $(0 < \alpha \leq 1)$

value of input unit j

Update-Rule: $W_j := W_j + \alpha \times I_j \times \text{Error}$

Theorem: [Rosenblatt 1960]

Learning using the update rule always converges to the correct output for representable functions.

What can Perceptrons represent?

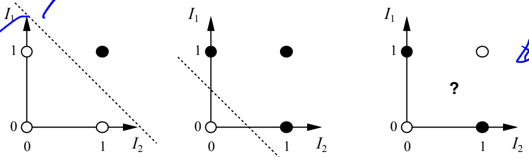
Answer: Not much!

Perceptrons can represent exactly the class of linearly separable functions.

separate 0's from 1's

Examples:

not linearly separable

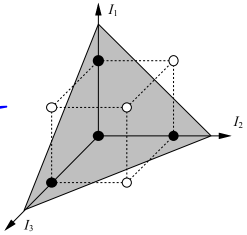


(a) I_1 **and** I_2 **(b)** I_1 **or** I_2 **(c)** I_1 **xor** I_2

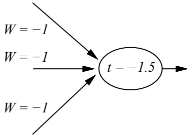
$I_1 + I_2 = 1.5$

I_1 I_2 $t = 1.5$

AND

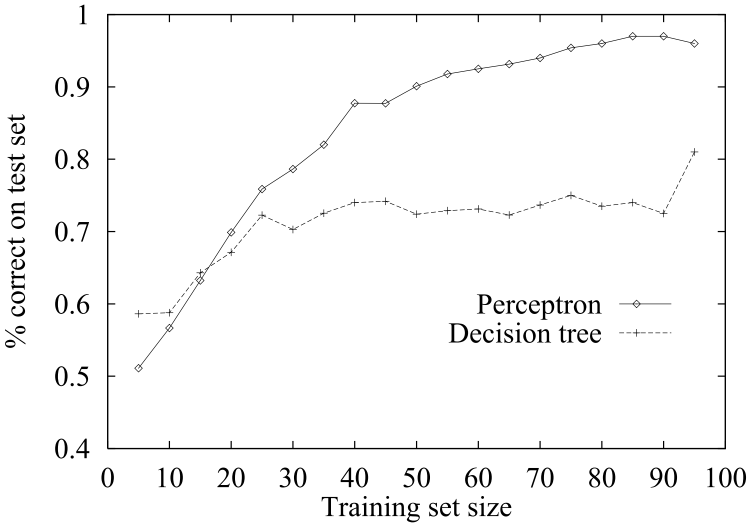


(a) Separating plane

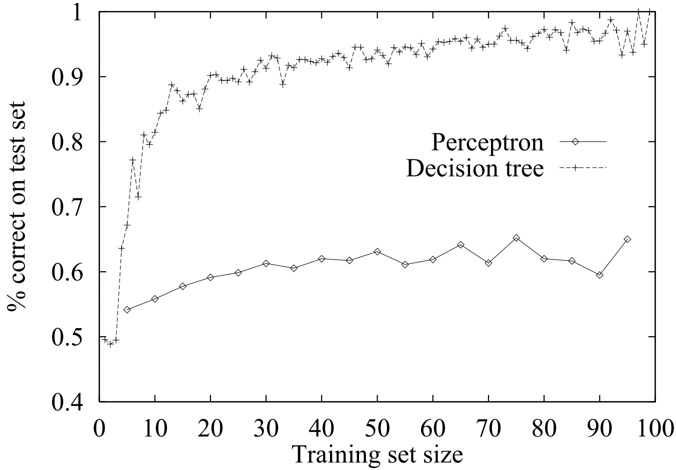


(b) Weights and threshold

Learning Behavior I: Majority function

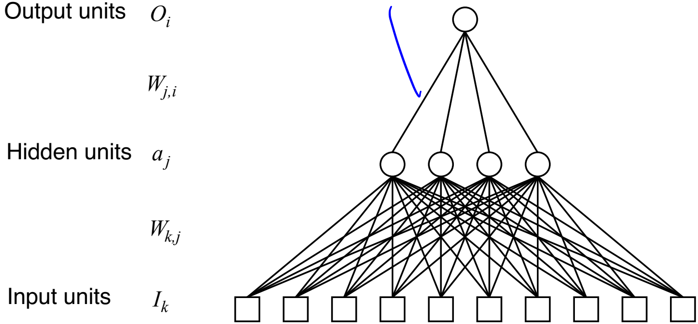


Learning Behavior II: Restaurant Example



Multi-layer Feed-Forward Networks

$W_{j,i}$



local encoding of attributes

(This topology is suitable for the restaurant example.)

Back-Propagation

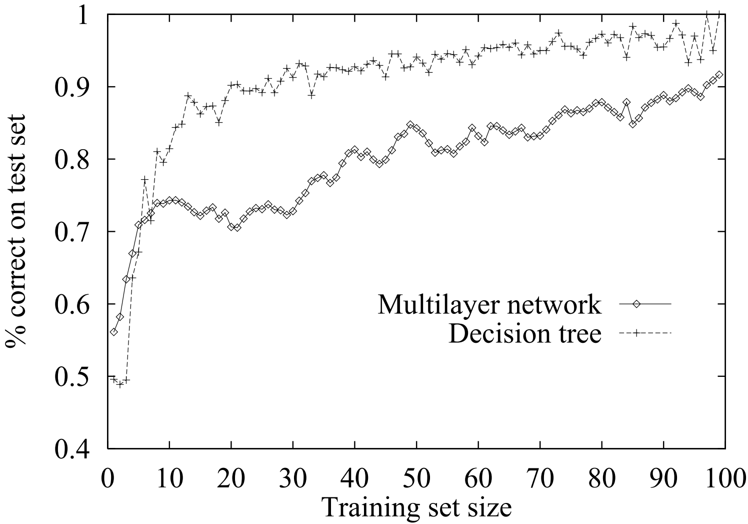
```
function BACK-PROP-UPDATE(network, examples, α) returns a network with modified weights
   inputs: network, a multilayer network
           examples, a set of input/output pairs
           α, the learning rate

   repeat
      for each e in examples do
         /* Compute the output for this example */
            O ← RUN-NETWORK(network, I^e)
         /* Compute the error and Δ for units in the output layer */
            Err^e ← T^e − O
         /* Update the weights leading to the output layer */
            W_{j,i} ← W_{j,i} + α × a_j × Err_i^e × g′(in_i)
         for each subsequent layer in network do
            /* Compute the error at each node */
               Δ_j ← g′(in_j) Σ_i W_{j,i} Δ_i
            /* Update the weights leading into the layer */
               W_{k,j} ← W_{k,j} + α × I_k × Δ_j
         end
      end
   until network has converged
   return network
```

Err_i^e = Error $T_i^e - O_i^e$ of the i-th output unit. Err^e = error vector.

$\Delta_i = Error_i \times g'(in_i)$

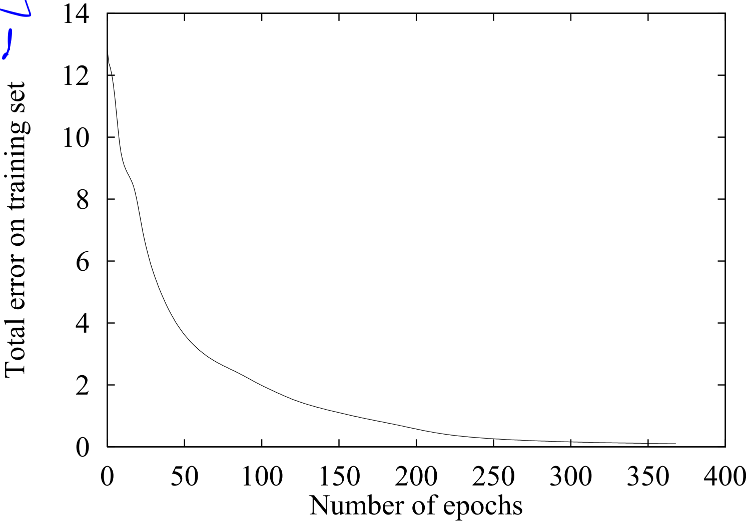
Restaurant Example



Restaurant Example

100 training examples

Sum of the squared error

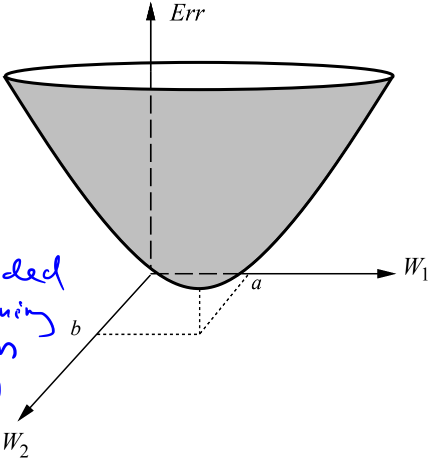


Backprop = Gradient Descent

Gradient Descent

is like Hill Climbing except that we are looking for minima instead of maxima.

may end up in local minima.
(can be avoided with pre-training of hidden layers + lots of data)



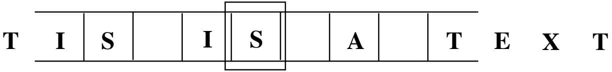
Summary

- What are they good for?
  Neural nets are useful for attribute-based representations (like decision trees), in particular also for attributes with continuous values.
- Size of a net:
  $2^n/n$ units to represent any Boolean function.
  ($\Rightarrow 2^n$ weights). In practice one often needs much less.
- Efficiency of Learning:
  no useful theoretical results.
  Also a problem with local minima. Simulated annealing sometimes helpful (see Boltzmann Maschines)
- Generalization: Good if the right network is used :-)
- Noise: no problem.
- Transparency:
  bad! Often one has no idea why a network yields a certain output.
  Decision trees are much better in this regard!
- Using additional knowledge: bad. ~~No~~ Few theoretical results.

Application: Pronunciation

NETtalk synthesizes speech from written text (in English) [Sejnowski and Rosenberg 1987]

Input: seven-character window sliding over the written text.:



uses 29 input units per character (for the 26 letters of the alphabet, blank, comma, and 1 unit for other characters)

Hidden layer: 80 units

Output: Units to encode phonemes.

Training set: text with 1024 words. After 50 epochs 95% correctness.
Problems: words with identical spelling but different pronunciation (e.g. lead).

Result on test set: 78% correctness.
(Today there are better systems not based on neural nets, but NETtalk paved the way for the commercial success of neural nets.)

Application: Recognizing Zip Codes

Neural net to recognize handwritten numbers (zip codes) [Le Cun 89]

Input: 16 $\times$ 16 pixels per digit.

3 hidden layers: 768; 192; 30 units.

Output: 10 units for digits 0–9.

first „convolutional neural network"

Limitation of connections was crucial:

- each unit of the 1st layer was connected with a 5 $\times$ 5 array of the input (25 connections).
- First layer was organized in 12 groups with 64 units each. All units of the same group use identical weights.
- The whole network used only 9760 different weights instead of 200,000.

Training set: 7300 examples.

Test set: 2000 examples; 99% correctness!
Is used by US Postal Service, realized on VLSI.