How to prove a statement by the Resolution method:

① convert into CNF (i.e. clausal form)
  ↳ propositional logic (Resolution 5)
  ↳ first order logic (Resolution 14)

formulas ≙ sets of clauses
literals ≙ clause
"
pos./neg. atom

② Resolution: CNF(S) → [ ] iff S unsat.

set of formulas, formula

$S \models \alpha$

$\Leftrightarrow$ for every interpr. I, if $I \models S$ then $I \models \alpha$

$\Leftrightarrow$ $S \cup \{\neg\alpha\}$ is unsat.

Ex. 4.3) b) $\{P \supset (Q \supset R)\} \models ((P \supset Q) \supset (P \supset R))$

iff $\underbrace{\{(P \supset (Q \supset R))\} \cup \{\neg((P \supset Q) \supset (P \supset R))\}}$ unsat.

$\hat{=}\ (P \supset (Q \supset R)) \land \neg((P \supset Q) \supset (P \supset R))$

$\overset{1.}{\equiv}\ (\neg P \lor (\neg Q \lor R)) \land \neg(\neg(\neg P \lor Q) \lor (\neg P \lor R))$

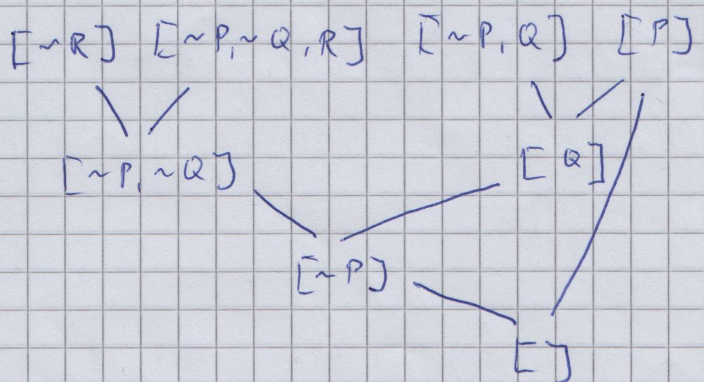
(cf. "Transformation to CNF"-steps)

$\overset{2.}{\equiv}\ (\neg P \lor (\neg Q \lor R)) \land (\neg\neg\neg P \lor \neg\neg Q) \land (\neg\neg P \land \neg R)$

$\overset{3.}{\equiv}\ (\neg P \lor \neg Q \lor R) \land (\neg\neg\neg P \lor \neg\neg Q) \land (\neg\neg P) \land (\neg R)$

$\overset{4.}{\equiv}\ (\neg P \lor \neg Q \lor R) \land (\neg P \lor Q) \land P \land \neg R$

$\hat{=}\ \{[\sim P, \sim Q, R], [\sim P, Q], [P], [\sim R]\}$



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Ex 4.3) c)

$$\{(Q \supset P), (Q \supset \neg P)\} \models \neg Q$$

iff $\underbrace{\{(Q \supset P), (Q \supset \neg P)\} \cup \{\neg\neg Q\}}$ unsat.

$$(Q \supset P) \wedge (Q \supset \neg P) \wedge \neg\neg Q$$

$$\overset{1.}{\equiv} (\neg Q \vee P) \wedge (\neg Q \vee \neg P) \wedge \neg\neg Q$$

$$\overset{2.}{\equiv} \text{—— " ——} \overset{3.}{\equiv} \text{— " —}$$

$$\overset{4.}{\equiv} (\neg Q \vee P) \wedge (\neg Q \vee \neg P) \wedge Q$$

$$\hat{=} \{[\sim Q, P], [\sim Q, \sim P], [Q]\}$$

[~Q, P]   [~Q, ~P]   [Q]

[~P]

[P]

[ ]

~~[illegible]~~ Counterexample:

To show that $S \not\models \alpha$, provide some (one) interpretation I

s.t. $\underbrace{I \models S, \text{ but } I \not\models \alpha}$

$\Leftrightarrow S \cup \{\neg\alpha\}$ is satisfiable, i.e. not unsatisfiable

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Ex. 4.4) a) $I = \langle D, \phi \rangle \quad D = \{1, 2\}$

False

$\phi(P) = \{1\}, \phi(Q) = \{2\}$

$\Rightarrow I \models \exists x\, P(x)$, $\Rightarrow \exists x\, Q(x)$

But: $I \not\models \exists x\, P(x) \wedge Q(x)$

$(\phi(P) \cap \phi(Q) = \emptyset)$

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b)

Skolemization: Introduce new function symbols

for $\exists$-quantifier variables

~~variable~~ $\exists x\, P(x) \rightsquigarrow P(f)$ (f constant)

$\forall x\, \exists y\, Q(y) \rightsquigarrow \forall x\, Q(f'(x))$ (f' unary)

Note: The resulting formula is not equivalent, but sat-equiv.

~~but [illegible]~~

$\models \alpha \equiv \beta$: every I: $I \models \alpha$ iff $I \models \beta$

ex. I: $I \models \alpha$ iff

ex. I': $I' \models \beta$

~~[illegible]~~ alternative solution for Ex. 4.4 a) with resolution:

cf. L2P for conversion into CNF:

$[P(f)] \quad [Q(f')] \quad [\sim P(z), \sim Q(z)]$

z/f'

$[\sim Q(f)] \quad [\sim P(f')]$

no more resolution steps possible

$\Rightarrow$ Statement is wrong

e.g. $[P(a)], [P(f(a))], [P(f(f(a)))], \ldots$

BUT: does not work in general, because we may generate new clauses indefinitely

$\Rightarrow$ instead, provide a counterexample

Reason: FOL is semi-decidable

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Ex 4.4/b) cf. L2P for conversion into CNF

Correct

[P(x), Q(y)] [~P(f)] [~Q(f)]

x/f

[Q(y)]

y/f

[ ]

c) "⊃": [P(x)] [Q(y)] [~P(f), ~Q(f)]

y/f

[~P(f)]

x/f

[ ]

Correct

"⊂": [P(x)] [Q(x)] [~P(f), ~Q(f')]

x/f'

[~P(f)]

x/f

[ ]

Ex 4.4) d) Correct

[~P(x), Q(x, g(x))]   [P(g(f))]   [~Q(g(f), g(g(f)))]

~~x/g(f)~~

[Q(g(f), g(g(f)))]

[ ]

x/g(f)

[~P(g(f))]

[ ]

e) Correct

[~P(f'(x)), P(f(x))]   [~P(f(x)), Q(x, f(f'(x)))]   [Q(x, f(x)), P(f'(x))]   [~Q(f'', z)]

[~P(f'(x)), Q(x, f(f'(x)))]

[Q(x, f(f'(x))), Q(x, f(x))]

x/f''
z/f(f'')

[Q(f'', f(f'(f'')))]

z/f(f'(f''))

[ ]

f) Wrong $I = \langle D, \phi \rangle$

$D = \{1, 2, 3\}$ $\phi(a) = 1$

$\phi(P) = \{\langle 1,2 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle\}$

$I \models \forall x \, \exists \, P(x, y)$ ✓

$I \models \neg \exists z \, P(z, a)$ ✓

$I \not\models \exists y \, \forall x \, P(x, y)$ ✓