AI Übung

ÜB4

Ex 4.1)

a) $\alpha$ is valid ($\models \alpha$) iff for all interpretations $I$, $I \models \alpha$ (by definition)

iff for all int. $I$ with $I \models TRUE$, also $I \models \alpha$

(since all int. satisfy TRUE, i.e. $\{I \mid I \text{ int.}\} = \{I \mid I \text{ int.}, I \models TRUE\}$)

iff $TRUE \models \alpha$ (by def.)

b) Let $\alpha$ be an arbitrary sentence. Suppose $FALSE \not\models \alpha$.

Then there is some int. $I$ such that $\underbrace{I \models FALSE}_{\text{↯}}$, but $I \not\models \alpha$.

(since no int. satisfies FALSE i.e. $\{I \mid I \models FALSE\} = \emptyset$)

Satisfaction: $I \models \alpha$ $\alpha$ holds in $I$, $I$ is a model of $\alpha$

$I$: interpr. $\alpha$: sentence/formula

Entailment: (logical implication) $S \models \alpha$ $\iff$ for every int. $I$ s.t. $I \models S$, also $I \models \alpha$

$S$: set of formulas

(we write $\beta \models \alpha$ instead of $\{\beta\} \models \alpha$)

Validity: $\models \alpha$ $\iff$ $\{\} \models \alpha$ $\iff$ for every int. $I$, $I \models \alpha$

Ex. 4.1 c) $\alpha \supset \beta$ is valid ($\models \alpha \supset \beta$)

iff for all int. $I$, $I \models (\alpha \supset \beta)$ (definition)

iff for all int. $I$, $I \models (\neg\alpha \vee \beta)$ (def. of „$\supset$")

iff for all int. $I$, $I \models \neg\alpha$ or $I \models \beta$ (semantics of FOL)

iff —"—, $I \not\models \alpha$ or $I \models \beta$ (—"—)

iff —"—, ($I \not\models \alpha$ and $I \models \beta$)

or ($I \not\models \alpha$ and $I \not\models \beta$)

or ($I \models \alpha$ and $I \models \beta$)

iff —"—, when $I \models \alpha$ then $I \models \beta$

iff $\alpha \models \beta$

d) $\alpha \equiv \beta$ valid ($\models (\alpha \equiv \beta)$)

iff $(\alpha \supset \beta) \wedge (\beta \supset \alpha)$ valid

iff $(\neg\alpha \vee \beta) \wedge (\neg\beta \vee \alpha)$ valid

iff for all int. $I$: $I \models (\neg\alpha \vee \beta) \wedge (\neg\beta \vee \alpha)$

iff —"—: ($I \not\models \alpha$ or $I \models \beta$) and ($I \not\models \beta$ or $I \models \alpha$)

iff —"—: ($I \not\models \alpha$ and $I \not\models \beta$) or ($\cancel{I \not\models \alpha \text{ and } I \models \alpha}$)

or ($\cancel{I \models \beta \text{ and } I \not\models \beta}$) or ($I \models \beta$ and $I \models \alpha$)

iff —"—: ($I \not\models \alpha$ and $I \not\models \beta$) or ($I \models \alpha$ and $I \models \beta$)

iff —"—: $I \models \alpha$ iff $I \models \beta$

iff $\underbrace{\{I \mid I \models \alpha\}}_{Mod(\alpha)} = \underbrace{\{I \mid I \models \beta\}}_{Mod(\beta)}$

iff $\alpha$ and $\beta$ are equivalent

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Ex 4.1) e)

$\alpha \land \neg\beta$ is unsatisfiable

iff no interpr. $I$ exists s.t. $I \models \alpha \land \neg\beta$

iff for every int. $I$ : $\underbrace{I \not\models \alpha \land \neg\beta}$

$\underbrace{I \not\models \alpha \text{ or } I \not\models \neg\beta}$

$I \models \neg\alpha \qquad I \models \beta$

$I \models (\alpha \supset \beta)$

iff $\alpha \models \beta$ (from part (c)) $\square$

Ex. 4.2) Daughter(NJ, parents(M)) (1)

$\neg$ Sister(NJ, M) (2)

$\forall x \forall y$ Daughter$(x,y) \equiv$ Female$(x) \land y =$ parents$(x) \land y \neq \cancel{x}$ (3)

$\forall x \forall y$. Sister$(x,y) \equiv$ Female$(\cancel{x}) \land$ parents$(x) =$ parents$(y) \land y \neq x$ (4)

Claim: $KB \models (NJ = M)$ $KB = \{(1), (2), (3), (4)\}$

Let $I = \langle D, \phi \rangle$ be an interpretation with $I \models KB$.

To show: $I \models (NJ = M)$.

$I = \langle D, \phi \rangle$ an interpretation

- D, the "domain" is a non-empty set (of domain elements)
- $\phi$, the interpretation function:
  - -> constants: $\phi(NJ) \in D$, $\phi(M) \in D$
  - -> functions: $\phi(\text{parents}) : D \to D$
  - -> predicates: $\phi(\text{Female}) \subseteq D$, $\phi(\text{Daughter}) \subseteq D \times D$, $\phi(\text{Sister}) \subseteq D \times D$

Let $a = \phi(NJ)$, $b = \phi(M)$, $c = I\|\text{parents}(M)\| = \phi(\text{parents})(\phi(M))$

$= \phi(\text{parents})(b)$ (5)

$\downarrow$

With $I \models (1)$ have $\langle a, c \rangle \in \phi(\text{Daughter})$ (6)

$I \models (2)$ have $\langle a, b \rangle \notin \phi(\text{Sister})$ (7)

$I \models (3)$ : $\langle d, d' \rangle \in \phi(\text{Daughter})$ iff

$$d \in \phi(\text{Female}),\ d' = \phi(\text{parents})(d),\ d \neq d'$$

$$\overset{(6)}{\Longrightarrow} a \in \phi(\text{Female}),\ c = \phi(\text{parents})(a),\ a \neq c \qquad (8)$$

$I \models (4)$ : $\langle d, d' \rangle \in \phi(\text{Sister})$ iff

$$d \in \phi(\text{Female}),\ \phi(\text{parents})(d) = \phi(\text{parents})(d'),\ d \neq d'$$

$$\overset{(7)}{\Longrightarrow} \underbrace{a \notin \phi(\text{Female})}_{\text{impossible because of (8)}} \ \underline{\text{or}}\ \underbrace{\phi(\text{parents})(a) \neq \phi(\text{parents})(b)}_{\text{impossible because of } \phi(\text{parents})(a) \overset{(8)}{=} c \overset{(5)}{=} \phi(\text{parents})(b)} \ \underline{\text{or}}\ a = b$$

$$\Longrightarrow a = b \quad \Longrightarrow \phi(NJ) = \phi(M) \Longrightarrow I \models (NJ = M)$$

$\square$

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## ÜB4

$KB = \{\alpha_1, \dots, \alpha_k\}$

$KB \models \alpha$

iff $\models (\alpha_1 \wedge \dots \alpha_k \supset \alpha)$

iff $\alpha_1 \wedge \dots \alpha_k \wedge \neg\alpha$ unsatisfiable

(iff $KB \cup \{\neg\alpha\}$ is unsat.)

**Resolution:** $\underbrace{CNF(S)}_{\text{clausal form of } S} \rightarrow [\,]$ iff $S$ is unsatisfiable

Ex 4.3) a) $\{\} \models (P \supset (Q \supset P))$

(cf Folie 5/37 Transformation to CNF)

$\{\} \cup \{\neg(P \supset (Q \supset P))\}$

$\hat{=}$ $\cancel{\{\}}$ $\neg(P \supset (Q \supset P))$

$\overset{1.}{\equiv}$ $\neg(\neg P \vee (\neg Q \vee P))$

$\overset{2.}{\equiv}$ $(\neg\neg P \wedge (\neg\neg Q \wedge \neg P))$

$\overset{3}{\equiv}$ —— " ——

$\overset{4.}{\equiv}$ $P \wedge Q \wedge \neg P$

$\hat{=}$ $\{[P], [Q], [\neg P]\}$

[P]  [Q]  [~P]

[ ]