AI Übung

2. ÜB

Ex 2.1)

Greedy search: always expand a node n with the least value for $h(n)$ (heuristic function = estimate for the cost to reach the closest goal from n)

a) $h(n) = g(n) \Longrightarrow$ uniform cost search
b) $h(n) = d(n) \Rightarrow$ breadth-first search

$g(n) = d(n)$ (all actions have cost 1)

c) $h(n) = \frac{1}{1+d(n)} \Rightarrow$ depth-first search

Ex 2.3)

$h_1 \hat{=}$ misplaced-tiles heuristic
$h_2 \hat{=}$ Manhattan-distance heuristic
$h_3 \hat{=}$ Gaschnig's heuristic

relaxations of the following rule:

"A tile can move to square X if X is adjacent and X is empty/blank"
↳ $\hat{=}$ original rule for 8-puzzle

"A tile can move to square X if X is adjacent" $\hat{=}$ relaxation corresponding to h_2

Example:

"A tile can move to a square X" $\hat{=}$ relaxation corresponding to h_1
"A tile can move to a square if X is blank" $\hat{=}$ —"— h_3

Initial state

6	2	8
	3	5
4	1	7

→

6	2	3
8		5
4	1	7

→

1	2	3
8		5
4	6	7

→

1	2	3
8		4
7	6	5

← goal state

$h_1 = 7$
↳ (# of misplaced tiles)
$h_2 = 3+0+2+3+1+3+2+3 = 17$
$h_3 = 9$
$h^* = 19$

$h_1 = 5$
$h_2 = 3+0+0+3+1+3+2+0 = 12$
$h_3 = 7$
$h^* = \infty$

$h_1 = 3$
$h_2 = 0+0+4+0+3+1+0+2+0 = 6$
$h_3 = 4$
$h^* = 12$

$h_1 = 0$
$h_2 = 0$
$h_3 = 0$
$h^* = 0$

Gaschnig heuristic
⤳ cyclic decompos.
blank $\hat{=}$ 0 (*)

$(0)(1)(2)\ldots(8)$
$0+0+0+\ldots+0 = 0$

(*)
$(083)(16)(2)(475)$
$2+3+0+4 = 9 = h_3$

$(0)(16)(2)(3)(475)(8)$
$0+3+0+0+4+0 = 7$

$(0)(1)(2)(3)(475)(6)(8)$
$0+0+0+0+4+0+0 = 4$

Idea: Regard states as permutations of tiles (denoting "blank" by 0), identify cyclic decomposition

⇓

Known: $h_1 \leq h_2$

Also: $h_1 \leq h_3$ (mathematical proof later)

because: Since one "Gaschnig move" brings at most one tile to its correct place, at least one Gaschnig move is needed for each misplaced tile, i.e.

$$\underbrace{\text{number of Gaschnig moves}}_{= h_3} \geq \underbrace{\text{number of misplaced tiles}}_{= h_1}$$

But: $h_2 \not\leq h_3$ and $h_3 \not\leq h_2$ (not always)

counterexamples above — counter examples

e.g. by interchanging two or four pairs of adjacent tiles and maybe moving the blank.

swap

			h_1	h_2	h_3	h^*
2	1	3				
8		4	4	4	6	16
6	7	5				

swap

			h_1	h_2	h_3	h^*
2	1	4	8	8	12	24
7		3				
8	5	6				

move 5,6

			h_1	h_2	h_3	h^*
2	1	4	7	8	10	26
7	5	3				
8	6					

$\Pi(n) \hat{=}$ permutation representing (state at) node n, blank denoted by 0.

$\pi =_{\ldots} \gamma_1 \ldots \gamma_l \hat{=}$ cyclic decomposition of permutation Π

$|\gamma| \hat{=}$ length of cycle γ

$\#_3(\gamma) \hat{=}$ number of Gaschnig's moves needed for cycle γ

$h_3(n) \equiv$ ——— " ——— for node n

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@ Ex 2.3)

2. ÜB

$$h_3(n) = \#_3(\gamma_1) + \dots + \#_3(\gamma_\ell) \text{ where } \Pi(n) = \gamma_1 \cdot \dots \cdot \gamma_\ell$$

$$\#_3(\gamma) = \begin{cases} 0, & \text{if } |\gamma| = 1 \\ |\gamma| - 1, & \text{if } |\gamma| > 1 \text{ and } 0 \in \gamma \\ |\gamma| + 1, & \text{if } |\gamma| > 1 \text{ and } 0 \notin \gamma \end{cases}$$

Note: $\#_1(\gamma) \hat{=}$ number of misplaced tiles in cycle γ

$h_1(n) \hat{=}$ ——— " ——— in node n

$$h_1(n) = \#_1(\gamma_1) + \dots + \#_1(\gamma_\ell) \text{ where } \Pi(n) = \gamma_1 \dots \gamma_\ell$$

$$\#_1(\gamma) = \begin{cases} 0, & \text{if } |\gamma| = 1 \\ |\gamma| - 1, & \text{if } |\gamma| > 1 \text{ and } 0 \in \gamma \\ |\gamma|, & \text{if } |\gamma| > 1 \text{ and } 0 \notin \gamma \end{cases}$$

$\Rightarrow \#_1(\gamma) \leq \#_3(\gamma)$

$\Rightarrow h_1(n) \leq h_3(n)$

$$h_3(n) = h_1(n) + \left| \{ \gamma_i \mid |\gamma_i| > 1 \text{ and } 0 \notin \gamma_i \} \right|$$

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 2 & 8 & 5 & 7 & 1 & 4 & 0 \end{pmatrix}$$

Note: Computing the cyclic decomposition is cheap (linear time)

$\Rightarrow h_3(n)$ can be computed efficiently.

Note: The $n \times n$ - puzzle is NP-complete.