Unit-7
Reasoning

Introduction

- Reasoning is the act of deriving a conclusion from certain premises using a given methodology.
- Reasoning is a process of thinking, reasoning is logically arguing, reasoning is drawing inference.
- Any system must reason, if it is required to do something which has not been told explicitly.
- For reasoning, the system must find out what it needs to know from what it already knows.

Example:

Fact-1: Robins are birds.
Fact-2: All birds have wings.

Then we can ask: *Do Robins have wings?*

Hence to answer this question – some reasoning must go.

Types of Reasoning

Monotonic Reasoning

In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base. In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.

To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.

Example:

If we know that *"All birds have feathers"* and that *"Tweety is a bird,"* then we can conclude that *"Tweety has feathers."* Even if we add the premise that *"Tweety is a penguin,"* the conclusion that *"Tweety has feathers"* remains valid.

Advantages of Monotonic Reasoning

- In monotonic reasoning, each old proof will always remain valid.
- If we deduce some facts from available facts, then it will remain valid for always.

Disadvantages of Monotonic Reasoning

- We cannot represent the real world scenarios using Monotonic reasoning.
- Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.
- Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

Non-monotonic Reasoning

In non-monotonic reasoning, adding new information can invalidate previously drawn conclusions.

Logic will be said as non-monotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.

Non-monotonic reasoning deals with incomplete and uncertain models.

Example:
Let suppose the knowledge base contains the following knowledge:

- Birds can fly.
- Penguins cannot fly.
- Pitty is a bird.

So from the above sentences, we can conclude that *Pitty can fly*.

However, if we add one another sentence into knowledge base *"Pitty is a penguin"*, which concludes *"Pitty cannot fly"*, so it invalidates the above conclusion.

Advantages of Non-monotonic reasoning

- For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
- In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.

Disadvantages of Non-monotonic Reasoning

- In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
- It cannot be used for theorem proving.

Statistical Reasoning

- Statistical methods provide a method for representing beliefs that are uncertain but for which there may be some supporting (or contradictory) evidence.
- This is useful for dealing with problems where there is *randomness* and *unpredictability* (such as in games of chance).
- To do all this in a principled way requires techniques for ***probabilistic reasoning***.
- ***Probabilistic reasoning*** is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge.

Bayes' Theorem

- ***Bayes' theorem*** is also known as *Bayes' rule*, *Bayes' law*, or *Bayesian reasoning*, which determines the probability of an event with uncertain knowledge.
- Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

  $$P(A|B) = P(A \wedge B)/P(B)$$
  $$P(A \wedge B) = P(A|B)\,P(B) \ldots \ldots \ldots \text{(i)}$$

  Similarly, the probability of event B with known event A:

  $$P(B|A) = P(A \wedge B)/P(A)$$
  $$P(A \wedge B) = P(B|A)\,P(A) \ldots \ldots \ldots \text{(ii)}$$

Equating right hand side of both the equations, we will get:

$P(A|B)P(B) = P(B|A)\,P(A)$ i.e.

$$\boxed{\boldsymbol{P(A|B) = \frac{P(B|A) * P(A)}{P(B)}}}$$

Where,

$$P(B) = P(A) * P(B|A) + P(\bar{A}) * P(B|\bar{A})$$

This equation is called as ***Bayes' rule*** or ***Bayes' theorem***.

Examples:

Q. A doctor knows that the disease meningitis causes the patient to have a stiff neck 50% of the time. The doctor also knows that the probability that a patient has meningitis is 1/50,000 and the probability that any patient has a stiff neck is 1/20. Now find the probability that a patient with stiff neck has meningitis.

Solution:

Let s be the proposition that the patient has a stiff neck and m be the proposition that the patient has meningitis.

Here, we are given

$P(s|m) = 0.5$
$P(m) = 1/50{,}000$
$P(s) = 1/20$
$P(m/s) = ?$

Now using Bayes rule,

$$P(m|s) = \frac{P(s|m) * P(m)}{P(s)} = \frac{0.5 * 1/50{,}000}{1/20} = 0.0002$$

Hence, the probability that a patient with a stiff neck has meningitis is 0.0002.

Q. One percent of women over 50 have breast cancer. 90% percent of women who have breast cancer test positive on mammograms. 80% percent of women will have false positives, i.e., they will test positive even though they do not have breast cancer. What is the probability that a woman over 50 has cancer if she has a positive mammogram result?

Solution:

Let's denote the events as follows: $C =$ woman has cancer, $M =$ woman has positive mammogram result.

We are given that,

$P(C) = 1\% = 0.01,$ $\quad P(\bar{C}) = 99\% = 0.99$
$P(M|C) = 0.9,$ $\quad P(M|\bar{C}) = 0.08$
$P(C|M) = ?$

By using Bayes' rule,

$$P(C|M) = \frac{P(M|C) * P(C)}{P(M)}$$

$$P(M) = P(C) * P(M|C) + P(\bar{C}) * P(M|\bar{C})$$
$$= 0.01 * 0.9 + 0.99 * 0.08$$
$$= 0.0882$$

$$\therefore P(C|M) = \frac{0.9 * 0.01}{0.0882} = 0.1020$$

The probability that a woman over 50 has breast cancer, given that she tested positive on a mammogram, is approximately 10.20%.

Q. Consider in Nepal, 51% of adults are males and rest are females. Consider one adult is randomly selected for a survey of drinking alcohol. It is found that 15% of males drink alcohol where as 2% of female drink alcohol. Now find the probability that the selected adult is a male.

Solution:

Let, M be the adult male, $\bar{M}$ be the adult female (*or not male*) and A be the adult who is drinking alcohol.

Here, we are given

$P(M) = 0.51$ $\quad$ $P(\bar{M}) = 1 - 0.51 = 0.49$

$P(A|M) = 0.15$ $\quad$ $P(A|\bar{M}) = 0.02$

$P(M|A) = ?$

Now,

$$P(A) = P(M) * P(A|M) + P(\bar{M}) * P(A|\bar{M}) = 0.51 * 0.15 + 0.49 * 0.02 = 0.0863$$

By using Bayes' rule,

$$P(M|A) = \frac{P(A|M) * P(M)}{P(A)} = \frac{0.15 * 0.51}{0.0863} = 0.8864$$

Application of Bayes' theorem:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

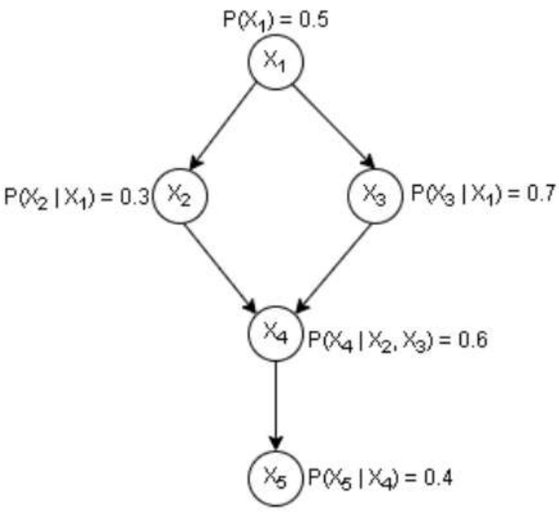
Bayesian Network

Bayesian network is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph.

- Nodes in the graph represent the random variables and the directed edges between nodes represent conditional dependencies.
- The edges exists between nodes iff there exists conditional probability i.e. a link from X to Y means Y is dependent of X.
- Each nodes are labelled with probability.

It is also called Belief network or Causal network or Probabilistic network or Bayes' network.

Example:



$P(X_1) = 0.5$

$P(X_3|X_1) = 0.7$

$P(X_2|X_1) = 0.3$

$P(X_4|X_2, X_3) = 0.6$

$P(X_5|X_4) = 0.4$

Fig: Bayesian Network

Bayesian network is based on *Joint probability distribution* and *conditional probability*. It can answer any query about the domain by using Joint distribution.

If we have variables $x_1,\ x_2,\ x_3, \ldots\ldots, x_n$, then the probabilities of a different combination of $x_1,\ x_2,\ x_3, \ldots\ldots, x_n$, are known as ***Joint probability distribution***.

$P[x_1,\ x_2,\ x_3, \ldots\ldots, x_n]$, it can be written as the following way in terms of the joint probability distribution.

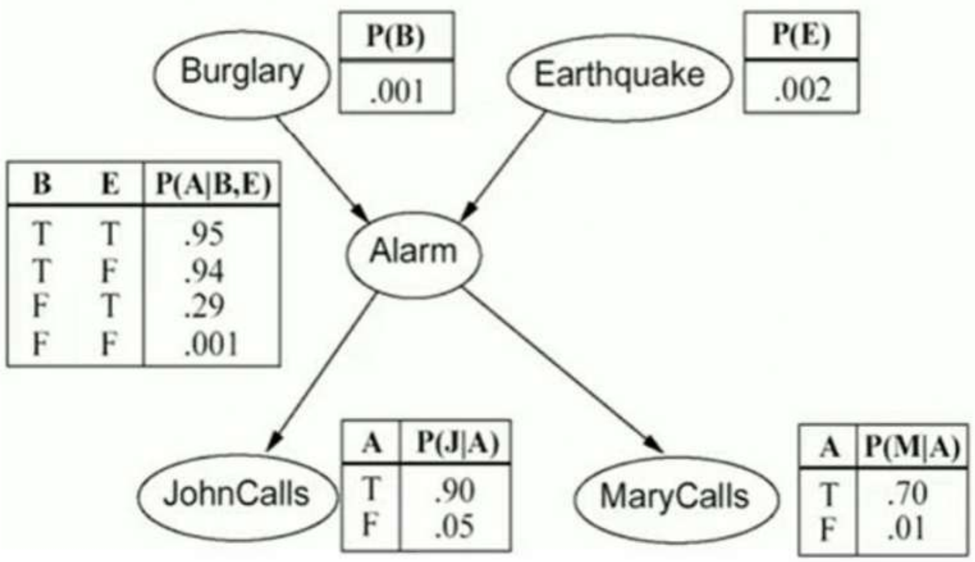
$$= P[\, x_1 \mid x_2,\ x_3, \ldots\ldots, x_n] P[x_2,\ x_3, \ldots\ldots, x_n]$$
$$= P[\, x_1 \mid x_2,\ x_3, \ldots\ldots, x_n] P[\, x_2 \mid x_3, \ldots\ldots, x_n] \ldots\ldots P[\, x_{n-1} \mid x_n] P[\, x_n]$$

In general for each variable X_i, we can write the equation as:

$$\boldsymbol{P[\, X_i \mid X_{i-1}, \ldots\ldots, x_1] = P[\, X_i \mid Parents(X_i)]}$$

Example: Burglar Alarm System

You have a new burglar alarm installed at home. The alarm is not only capable of detecting a burglary, but it can also detect mild earthquakes. You have two next-door neighbors, John and Mary, who promised to call you at work when they hear the alarm. John always calls when he hears the alarm, but sometimes confuses telephone ringing with the alarm and calls too. Mary, on the other hand, enjoys listening to loud music and occasionally misses the alarm.



What is the probability that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both John and Merry call?

$$P(j \wedge m \wedge a \wedge \neg b \wedge \neg e) = P(j \mid a)\, P(m \mid a)\, P(a \mid \neg b, \neg e)\, P(\neg b)\, P(\neg e)$$
$$= 0.90 \times 0.70 \times 0.001 \times 0.999 \times 0.998$$
$$= 0.00062$$

Uncertainty in Reasoning

The world is an uncertain place; often the Knowledge is imperfect (Incomplete, Inconsistent, Changing) which causes uncertainty Therefore reasoning must be able to operate under uncertainty.

AI system must have ability to reason under conditions of uncertainty.

Uncertainties	***Desired action***
Incompleteness Knowledge	Compensate for lack of knowledge
Inconsistencies Knowledge	Resolve ambiguities and contradictions.
Changing Knowledge	Update the knowledge base over time.

There are three different approaches to reasoning under uncertainties:

- Symbolic Reasoning
- Statistical reasoning
- Fuzzy logic reasoning

1. Symbolic Reasoning

- *Symbolic Reasoning* refers to the approach where knowledge is represented using symbols and manipulated through formal logic rules to derive conclusions.
- The basis for intelligent mathematical software is the integration of the "power of symbolic mathematical tools" with the suitable "proof technology".
- Mathematical reasoning enjoys a property called *monotonic*, that says "If a conclusion follows from given premises A, B, C, … then it also follows from any larger set of premises, as long as the original premises are included."
- Human reasoning is *not monotonic*. People arrive to conclusions only tentatively, based on partial or incomplete information, reserve the right to retract those conclusions while they learn new facts. Such reasoning is *non-monotonic*, precisely because the set of accepted conclusions have become smaller when the set of premises is expanded.

Types of Non-monotonic Reasoning:

a. ***Default Reasoning:*** This type of reasoning uses default rules or assumptions that are assumed to be true unless there is evidence to the contrary.

b. ***Circumscription:*** Circumscription is a non-monotonic logic to formalize the common sense assumption. Circumscription is a formalized rule of conjecture (guess) that can be used along with the rules of inference of first order logic.

c. ***Truth Maintenance System (TMS):*** It is assumption-based reasoning. The TMS maintains the consistency of a knowledge base as soon as new knowledge is added. It considers only one state at a time so it is not possible to manipulate environment.

2. Statistical Reasoning

- In the logic based approaches described, we have assumed that everything is either believed false or believed true.
- However, it is often useful to represent the fact that we believe such that something is probably true, or true with probability (say) 0.65.
- This is useful for dealing with problems where there is randomness and unpredictability (such as in games of chance) and also for dealing

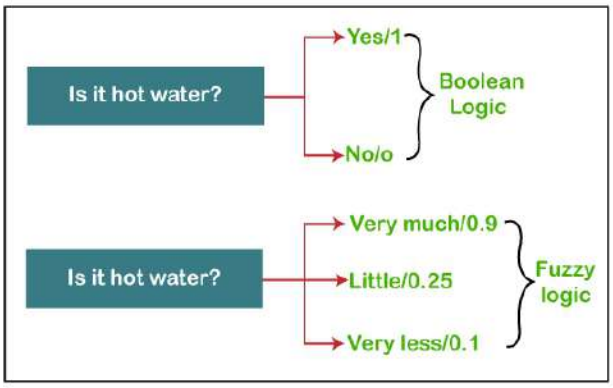
with problems where we could, if we had sufficient information, work out exactly what is true.

- To do all this in a principled way requires techniques for ***probabilistic reasoning***.

3. Fuzzy Logic

The word fuzzy means things that are not very clear or vague. In real life, everyone comes across a situation where they can't decide if a statement is true or false. Whenever such a scenario arrives, fuzzy logic provides valuable flexibility for reasoning by considering the uncertainties of the situation.

Fuzzy logic is a form of many-valued logic; it deals with reasoning that is approximate rather than fixed and exact. In contrast with traditional logic theory, where binary sets have two-valued logic: true or false, fuzzy logic variables have a truth value that ranges in degree between 0 and 1.



Architecture of Fuzzy Logic:

The Fuzzy Rule-Based System has four major components, which are explained with the help of the architecture diagram below:

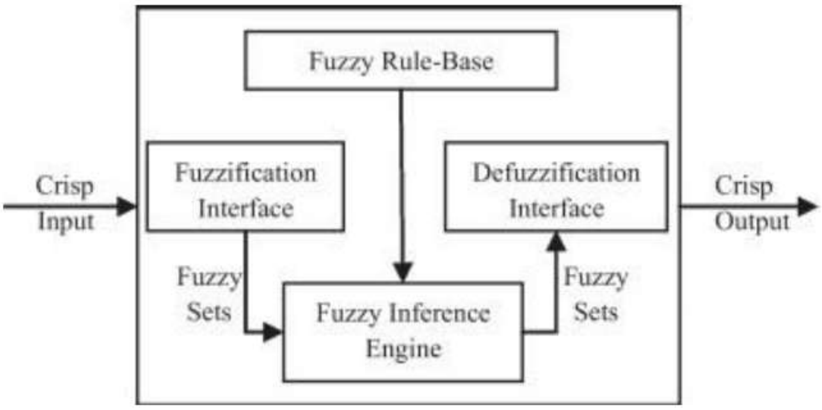


Fig: Fuzzy Logic Architecture

- ***Rule Base:*** Rule Base consists of a large set of rules programmed and fed by experts that govern the Fuzzy System's decision-making. The rules are sets of "If-Then" statements that decide the event occurrence based on condition.
- ***Fuzzification:*** It is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.

- ***Inference Engine:*** It determines the matching degree of the current fuzzy input with respect to each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.
- ***Defuzzification:*** It is used to convert the fuzzy sets obtained by inference engine into a crisp value. There are several defuzzification methods available and the best suited one is used with a specific expert system to reduce the error.

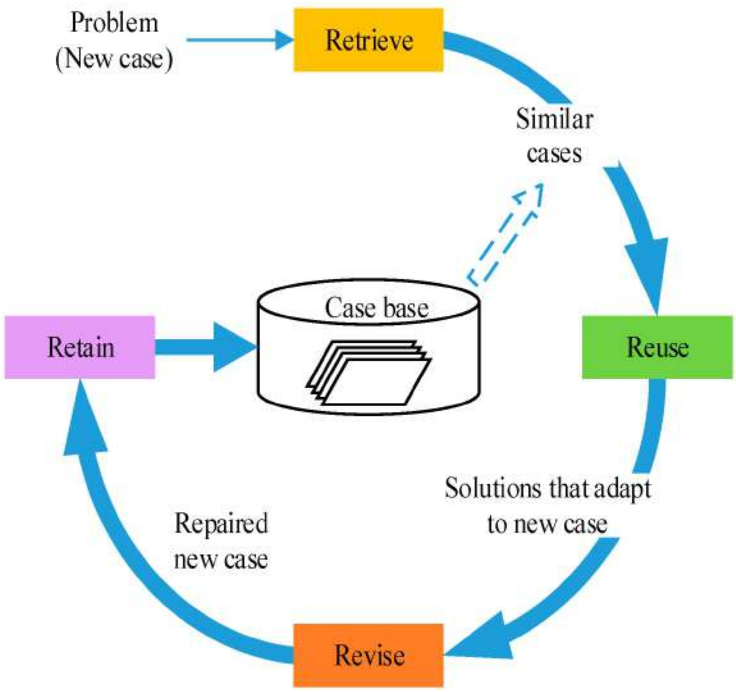
Case Based Reasoning (CBR)

Case-Based Reasoning (CBR) is a process of solving new problems based on the solutions of similar past problems. It is a method of thinking in which we employ a previous experience or case to address a new problem.

Example: An auto mechanic who fixes an engine by recalling another car that exhibited similar symptoms is using case-based reasoning.

CBR works by first storing past experiences in a case base. When a new problem arises, the system searches for similar cases in the case base and uses the knowledge from those cases to solve the new problem. The system evaluates the similarity between the new problem and past experiences to select the most appropriate case to solve the new problem.

4-step process for CBR:



1. ***Case retrieval:*** One or more source cases most similar to the new case are retrieved from the case base.
2. ***Case re-use:*** Information and knowledge from similar cases are re-used to establish solutions adapted to new case.
3. ***Case revision:*** The proposed solution is evaluated, and the solution is adjusted if it does not meet the requirements.
4. ***Case retention:*** The parts of this experience that may be useful for solving problems in the future are retained.

Advantages of CBR

- CBR systems can reuse past solutions to similar problems, which can save time and effort compared to developing a solution from scratch.
- CBR systems can adapt to changing situations or contexts by selecting and modifying relevant cases.
- CBR systems improve their problem-solving abilities over time as they learn from new cases and expand their database.
- Since solutions are based on previous cases, CBR is also able to provide transparent explanations for certain solution approaches.
- Since case-based reasoning is based on human problem-solving strategies, it is easy to understand how solutions are reached.

Disadvantages of CBR

- The efficiency of a CBR system relies heavily on the quality and completeness of the case base. If the stored cases are inaccurate or incomplete, the system may produce suboptimal solutions.
- As the size of the case base increases, it may take significantly longer to retrieve and adjust cases, reducing the efficiency of the system.
- Solutions that worked well in the past may become irrelevant over time, especially in rapidly evolving fields. In some cases, this could lead to outdated solutions being proposed.