Technische Universität Ilmenau
Institut für Mathematik
Fachgebiet Kombinatorik/Graphentheorie

Abschlussarbeit

zur Erlangung des akademischen Grades B. Sc.

A Mathematical Analysis of the Kickelhahnturm

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vorgelegt dem Institut für Mathematik
der Technischen Universität Ilmenau von

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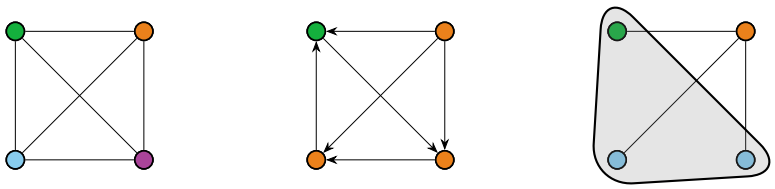
Abstract

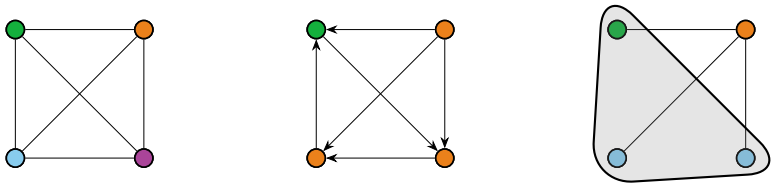
An important part of a successful thesis is always the abstract that should be provided in both german and english. Also, everybody likes colorful pictures.



Colorings of a graph, a digraph, and a hypergraph.

Zusammenfassung

Und jetzt dasselbe nochmal auf deutsch.



Färbungen eines Graphen, gerichteten Graphen, und eines Hypergraphen.

Declaration of Authorship

I hereby declare that the thesis submitted is my own unaided work. All direct or indirect sources used are acknowledged as references. This paper was not previously presented to another examination board and has not been published.

Ilmenau, 22. April 2021

Contents

Introduction

What is this about and why should I care?

Welcome to my awesome thesis. Please feel free to skip this part.

Look at the very nice Kopfzeile!

Part I

My Thesis is so substantial that it even has two parts!

Chapter 1

Preliminaries: Graphs

1.1. Basic Terminology

As usual, $\mathbb{N}$ denotes the set of positive integers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ is the set of non-negative integers. For $k, \ell \in \mathbb{N}_0$ let $[k, \ell] = \{h \in \mathbb{N}_0 \mid k \leq h \leq \ell\}$. Given a set V, we denote the **cardinality** of V by $|V|$ and the **power set** of V by 2^V. The empty set is denoted by $\varnothing$.

1.2. Structure of the Following Chapters

In Chapter blabla we will do blablubb. Then, ...

Part II

As promised: another very fabulous part!

Chapter 2

Preliminaries: Digraphs

2.1. Basic Digraph Terminology

The digraph terminology used in this thesis is mostly based on the book of Bang-Jensen and Gutin [1]. A digraph $D = (V(D), A(D))$ consists of a finite set $V(D)$ of so called **vertices** and a finite set $A(D)$ of ordered pairs of distinct vertices of D, so called **arcs** of the digraph D.

At this point, we would like to cite the following material, which—with certainty—is very enjoyable to read: [2, 3, 3, 4, 6, 5, 7]

References

[1] Bang-Jensen, J. and Gutin, G. Z.: Digraphs: Theory, Algorithms and Applications (Second Edition). Springer, 2009 (cit. on p. 6).

[2] Bang-Jensen, J. et al.: Hajós and Ore constructions for digraphs. *Electron. J. Combin.* **27** (2020), #P1.63 (cit. on p. 6).

[3] Borodin, O. V.: Criterion of chromaticity of a degree prescription (in Russian). In: Abstracts of IV All-Union Conf. on Theoretical Cybernetics (Novosibirsk). 1977, 127–128 (cit. on p. 6).

[4] Dirac, G. A.: On the Colouring of Graphs. PhD thesis. University of London, 1951 (cit. on p. 6).

[5] Schweser, T., Stehlík, M., and Stiebitz, M.: Critical digraphs with few arcs and few vertices. work in progress (cit. on p. 6).

[6] Schweser, T., Stiebitz, M., and Toft, B.: Coloring hypergraphs of low connectivity. 2018. arXiv: 1806.08567 (cit. on p. 6).

[7] Stiebitz, M. and Toft, B.: Brooks's Theorem. In: *Topics in Chromatic Graph Theory*. Edited by L. W. Beineke and R. J. Wilson, with Academic Consultant B. Toft. Cambridge University Press, 2015, 36–55 (cit. on p. 6).

List of Symbols

The number at the end of each line refers to the page where the term is defined. The items are sorted according to content as far as possible.

Basic Terminology

$\mathbb{N}$ set of positive integers, 4
$\mathbb{N}_0$ set of non-negative integers, 4
$[k, \ell]$ all $h \in \mathbb{N}_0$ with $k \leq h \leq \ell$, 4
$\varnothing$ empty set, 4
2^V power set of V, 4
$|V|$ cardinality of the set V, 4

More Fancy Terminology

$D, D', \tilde{D}$ digraphs, 6
$V(D)$ vertex set of a digraph D, 6
$A(D)$ arc set of a digraph D, 6