Offline #3

Topic: Integration

Full Marks: 20

In a methanol-based fuel cell, DC electricity is generated from the combustion of methanol and oxygen. In such a fuel cell, the consumption of oxygen over time can be expressed using the following formula:

$$T = -\int_{x_1}^{x_2} \left(\frac{6.73x + 6.725 \times 10^{-8} + 7.26 \times 10^{-4} C_{me}}{3.62 \times 10^{-12} x + 3.908 \times 10^{-8} x C_{me}} \right) dx$$

Where

T=time in seconds

x=concentration of oxygen in $moles/cm^3$

C_{me}=concentration of methanol in $moles/cm^3$=$5 \times 10^{-4}\ moles/cm^3$

The initial concentration of oxygen is: $x(t = 0) = 1.22 \times 10^{-4}\ moles/cm^3$

Programming tasks:

1. Write a python program to evaluate the time required for the oxygen concentration in the fuel cell to drop from 75% of the initial level to 25%. Your program should accept the number of segments n as a parameter from the user and use the multiple-application trapezoid rule by partitioning the given interval into n equally spaced segments. Print the integral values and approximate relative errors for no. of segments = 1, 2,…,n. (7 Marks)

2. Solve the same problem given in (1) using Simpsons' 1/3 rule. For this case, partition the given interval into $2n$ number of equally spaced segments and use n applications of Simpson's 1/3 rule (apply single Simpson's 1/3 rule in a pair of segments). Print the integral values and approximate relative errors for no. of segments = 2, 4, …,$2n$. (7 Marks)

3. Plot time vs. oxygen concentration for the following concentrations of oxygen $x = [1.22, 1.20, 1.0, 0.8, 0.6. 0.4, 0.2] \times 10^{-4}\ moles/cm^3$. Use multiple-application Simpsons' 1/3 rule with 10 segments. (6 Marks)