Online: Measure dipstick height

Full Marks: 10

A spherical oil tank is 8 ft in diameter. We want to design a dipstick to measure the volume of oil in the tank. Knowing the height to which the dipstick would become wet with oil, one would know the height h of the oil. The volume V of the oil left in the tank is:

$$V = \frac{\pi h^2 (3r - h)}{3}$$

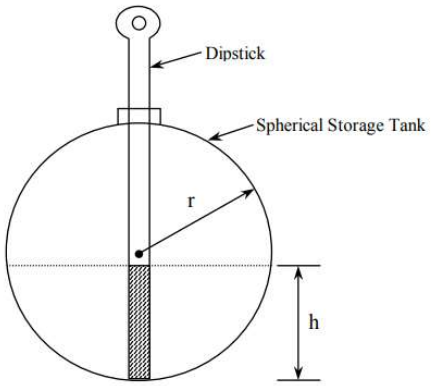

Figure 1 Oil in a spherical storage tank.

To design the given dipstick, we need to know the height h for a given volume V. Assume $V = 5\ ft^3$ and find a non-linear equation for solving h. Use newton-Raphson method to find the value of h with an error precision of at most 0.05%. Show the root estimate and absolute approximate relative error after every iteration. Show the graph of your $f(h)$ and explain how you chose the initial guess.