Radioactive Decay

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***Abstract*—In this experiment, we focused on studying the decay of Radon gas ^{220}Rn, which is one of the elements produced in the radioactive decay chain starting from Thorium salt ^{232}Th and ending at ^{208}Pb.**

There are multiple methods available to study radioactive decay. We used a simple yet effective method, involving measuring the ionizing capability of the charged particles produced in the decay using a Wulf's electroscope, which measures the time duration it takes to charge.

We determined the decay constant λ and the half-life $t_{1/2}$ of ^{220}Rn, and compared them to the accepted value of 55.6 seconds. The decay constant λ is a measure of the rate at which the radioactive substance decays, while the half-life $t_{1/2}$ is the time it takes for half of the initial amount of the substance to decay. By comparing the determined values to the accepted value, we can evaluate the accuracy of the experiment.

Overall, the experiment provides valuable insights into the behavior of radioactive decay and the properties of Radon gas ^{220}Rn.

I. Introduction & Theory

Radioactive decay is a natural process that occurs when the nuclei of certain isotopes become unstable, particularly those with high atomic numbers. To stabilize their nuclei, these isotopes undergo a process called radioactive decay, which involves losing energy and emitting radiation. There are three different types of radioactive decay: alpha, beta, and gamma decay.

Alpha decay occurs when an atom emits an alpha particle, which consists of two protons and two neutrons. This particle causes the atomic number to decrease by two and the atomic mass to decrease by four. In contrast, beta decay occurs when an atom emits a beta particle, which can be either an electron or a positron. This type of decay causes a change in the atomic number of the atom, while the atomic mass remains the same. Gamma decay is the third type of radioactive decay, and it occurs when an atom emits a gamma ray. Gamma rays are high-energy photons that are emitted by the nucleus of an atom. Gamma decay does not result in a change in the atomic number or the atomic mass of the atom.

Radioactivity was first discovered in 1896 by the French scientist Henri Becquerel [1]. While working with phosphorescent materials, he observed that uranium salts emitted rays that could penetrate through opaque materials. Later, Rutherford discovered that all radioactive decay processes follow the same mathematical formula, which is approximately exponential [2]. Other notable scientists who made significant contributions to the study of radioactivity are Marie Curie and Pierre Curie. In 1898, they discovered two new radioactive elements, polonium and radium [3].

All unstable isotopes follow the same decay law, expressed by the equation, regardless of the specific decay process[4]:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

In Equation 1, we see the number of unstable isotope nuclei that remain over time, represented by N, where λ is the decay constant and N_0 is the initial number of nuclei. Taking the derivative of Equation 1 with respect to time gives us:

$$\frac{dN(t)}{dt} = -N(t)\lambda \tag{2}$$

Based on the results:

$$\frac{dN(t)}{dt} \propto N(t) \tag{3}$$

In this experiment, by applying a high voltage to the circuit, a current is generated due to ionized particles in the air. But instead of measuring this current directly, we will use a simpler method.

This method involves using Wulf's electroscope. The electroscope automatically discharges after reaching a certain threshold charge and we measure the time between successive discharges. Using this method, we can avoid the difficulties associated with direct measurement of current.

It is important to note that the threshold charge required for the electroscope to discharge is constant. Consequently, the current is proportional to the inverse of the time it takes to accumulate this amount of charge:

$$I = \frac{Q}{s} \Rightarrow I \propto \frac{1}{s} \tag{4}$$

By measuring the current through the elapsed time between successive discharges, we obtain a quantity that is proportional to the amount of decay:

$$\frac{dN(t)}{dt} \propto N(t) \propto I \propto \frac{1}{s} \tag{5}$$

We can also calculate the half-life of ^{220}Rn by using Equation 1:

$$t_{1/2} = \frac{\ln(2)}{\lambda} \tag{6}$$

II. The Experimental Setup & Method

A. Aparatus

- High-voltage Power Supply $(0 - 5kV)$
- Wulf's Electroscope
- Stopwatch
- Ionisation Chamber
- Thorium Salt

B. Setup

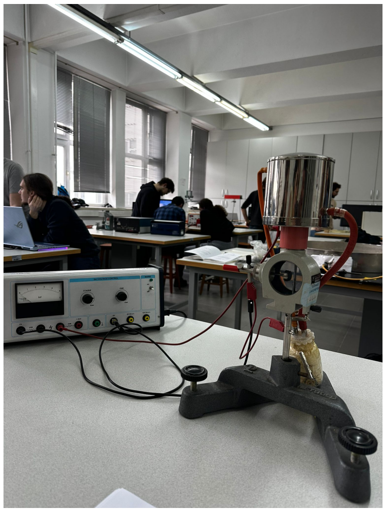

Fig. 1. The Experiment Setup with Wulf's Electroscope and High-voltage Power Supply

We are using a device called Wulf's electroscope to measure the current in the circuit during the experiment. The setup is shown in Figure 1 and Figure 2.

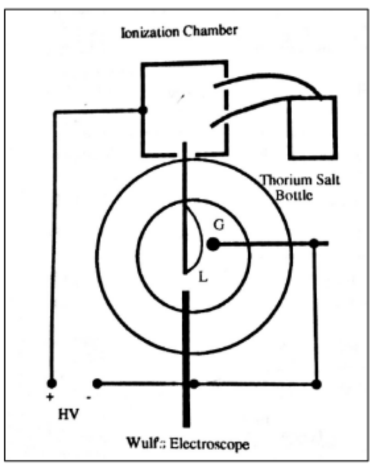

Fig. 2. Schematic Diagram of the Experiment Setup

C. Method

- To start, we apply the highest voltage to the circuit to remove any remaining charge from the electroscope. Next, we turn off the high voltage and open the ionization chamber to thoroughly clean it of any debris. When opening the ionization chamber, it's important to be careful and avoid inhaling the surrounding air, as it may contain radioactive Radon gas.
- Then we need to set the high voltage power supply to 2.5kV. Then, we should open the clamp between Thorium and the ionization chamber and send Radon gas to the chamber by squeezing the Thorium salt bottle 2 times. Once the clamps are closed, we can start the timer and record the instants when the electroscope discharges. We should note that the electroscope will discharge several times before the intervals between discharges begin to exceed 50 -120 seconds.
- To repeat this experiment, use the same voltage and vary the number of squeezes to 3, 4, and 5. It is important to discharge the electroscope and clean ionization chamber after each iteration.
- We repeated the process using 2, 3, 4, and 5 squeezes for each voltage of 3.0kV, 3.5kV, and 4kV.

III. The Data

Data table is included in the appendix.

After taking the data, we should plot $\frac{1}{s_i}$ vs T_i to find the line that best fits this data.

s_i is the time difference between two discharges:

$$s_i = t_{i+1} - t_i \tag{7}$$

and T_i is the average time:

$$T_i = \frac{t_{i+1} + t_i}{2} \tag{8}$$

where t_i symbolizes the time that we observe a discharge.

To analyze this data, we require T_i and $\frac{1}{s_i}$ values, as well as determining its uncertainty. As time is being measured, the uncertainty is comprised of two types of errors. The first stems from the margin of error of our measuring instrument, which is 0.001 seconds. The second is the systematic error, which we acknowledge as 0.30 seconds due to the human margin of error. When we consider these two types of uncertainty:

$$\sigma_{ti} = \sqrt{(0.001s)^2 + (0.30s)^2} \simeq 0.30s \tag{9}$$

and uncertainty of T_i is:

$$T_i = \frac{(t_{i+1} \pm 0.30s) + (t_i \pm 0.30s)}{2} = T_i \pm 0.30s \tag{10}$$

so σ_T is equal to:

$$\sigma_{Ti} = \pm 0.30s \tag{11}$$

According to equation 9, σ_{s_i} value is:

$$\sigma_{s_i} = \pm 0.60s \tag{12}$$

Calculating the uncertainty of $\frac{1}{s_i}$ is a bit more complicated. Using the equation below:

$$f(x, y) \longrightarrow \sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial f}{\partial y}\sigma_y\right)^2 + ...} \tag{13}$$

We find:

$$\sigma_I = \sigma_{\frac{1}{s_i}} = \sqrt{\left(\frac{\partial\left(\frac{1}{s}\right)}{\partial s}\sigma_s\right)^2} = \frac{\sigma_{s_i}}{s_i^2} = \frac{0.60s}{s_i^2} \tag{14}$$

IV. The Analysis & Result

A. Analysis

We utilized ROOT for data analysis and function fitting. In our analysis results, we included all the data, but upon removing long T times from the dataset to achieve a lower chi-square test, we observed an improved function fit and a half-life value closer to the actual value. Additionally, we obtained a lower sigma-away value, which will be further discussed in the conclusion section.

After collecting the data, we plotted $\frac{1}{s_i}$ vs T_i and determined the exponential function that best fits the data using ROOT. The first parameter of the function of the best fit will provide us with the radioactive decay constant λ as indicated by Equation 1.

TABLE I
$\lambda Values$

$Dataset$	λ	σ_λ
$2.5kV, 2sq$	0.008555	0.00007732
$2.5kV, 3sq$	0.009883	0.0000662
$2.5kV, 4sq$	0.01096	0.0001083
$2.5kV, 5sq$	0.014	0.0002403
$3.0kV, 2sq$	0.01104	0.0001195
$3.0kV, 3sq$	0.009362	0.00006835
$3.0kV, 4sq$	0.008449	0.00009648
$3.0kV, 5sq$	0.009185	0.00009445
$3.5kV, 2sq$	0.008992	0.0001083
$3.5kV, 3sq$	0.01154	0.0001046
$3.5kV, 4sq$	0.008996	0.0001197
$3.5kV, 5sq$	0.01305	0.0001425
$4.0kV, 2sq$	0.009816	0.0001186
$4.0kV, 3sq$	0.01268	0.0001689
$4.0kV, 4sq$	0.009942	0.00007277
$4.0kV, 5sq$	0.01016	0.00009576

B. Result

As we acquired 16 decay constants from 16 distinct data sets, we computed the weighted average of λ values using the following formula:

$$\bar{\lambda} = \frac{\sum_{i=1}^{16} \frac{\lambda_i}{\sigma(\lambda_i)^2}}{\sum_{i=1}^{16} \frac{1}{\sigma(\lambda_i)^2}} \tag{15}$$

and we get it :

$$\bar{\lambda} = 0.00987263 \frac{1}{s} \tag{16}$$

So when we want to calculate the $\sigma_{\bar{\lambda}}$:

$${\sigma_{\bar{\lambda}}}^2 = \sum_{i=1}^{16} \frac{1}{\frac{1}{{\sigma_{\lambda_i}}^2}} \tag{17}$$

we found that:

$$\sigma_{\bar{\lambda}} = \pm 2.42438 \times 10^{-5} \frac{1}{s} \tag{18}$$

To determine the half-life of ^{220}Rn, one of the objectives of our experiment, we follow these steps:

$$t_{\frac{1}{2}} = \frac{ln2}{\bar{\lambda}} \tag{19}$$

$$t_{\frac{1}{2}} = 70.209s \tag{20}$$

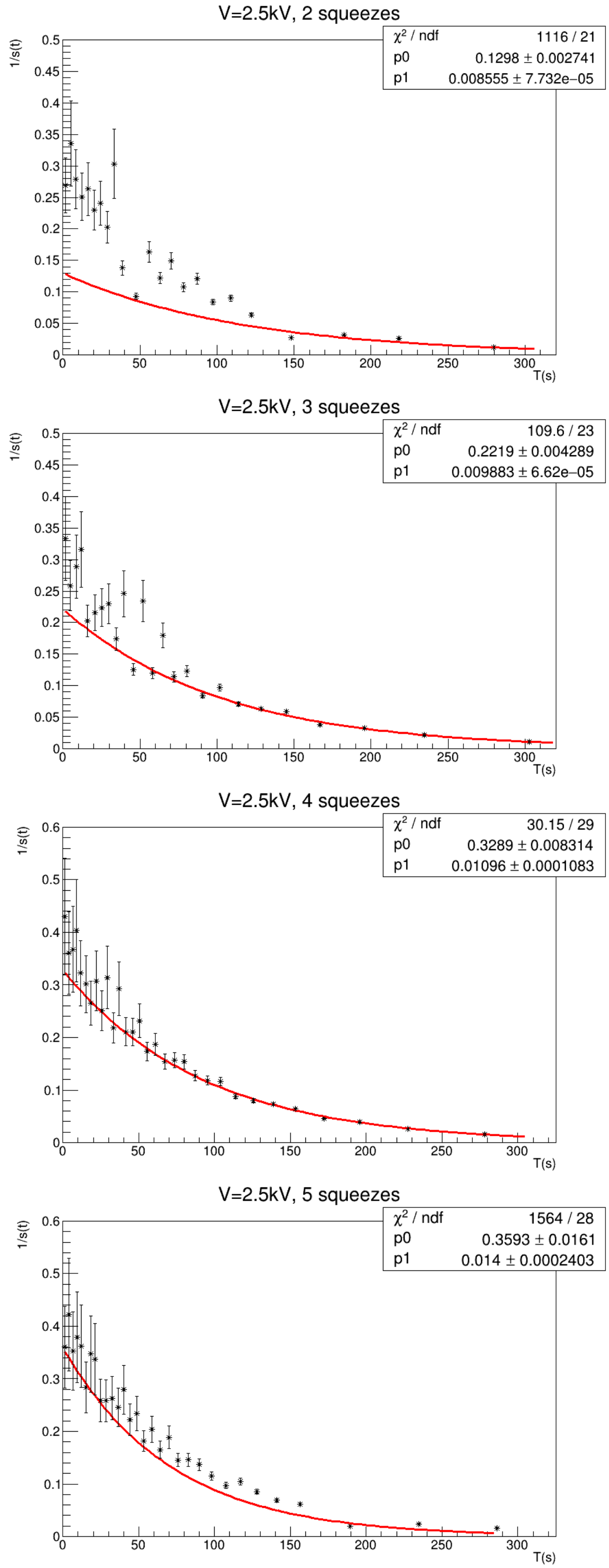

Fig. 3. V=2.5kV

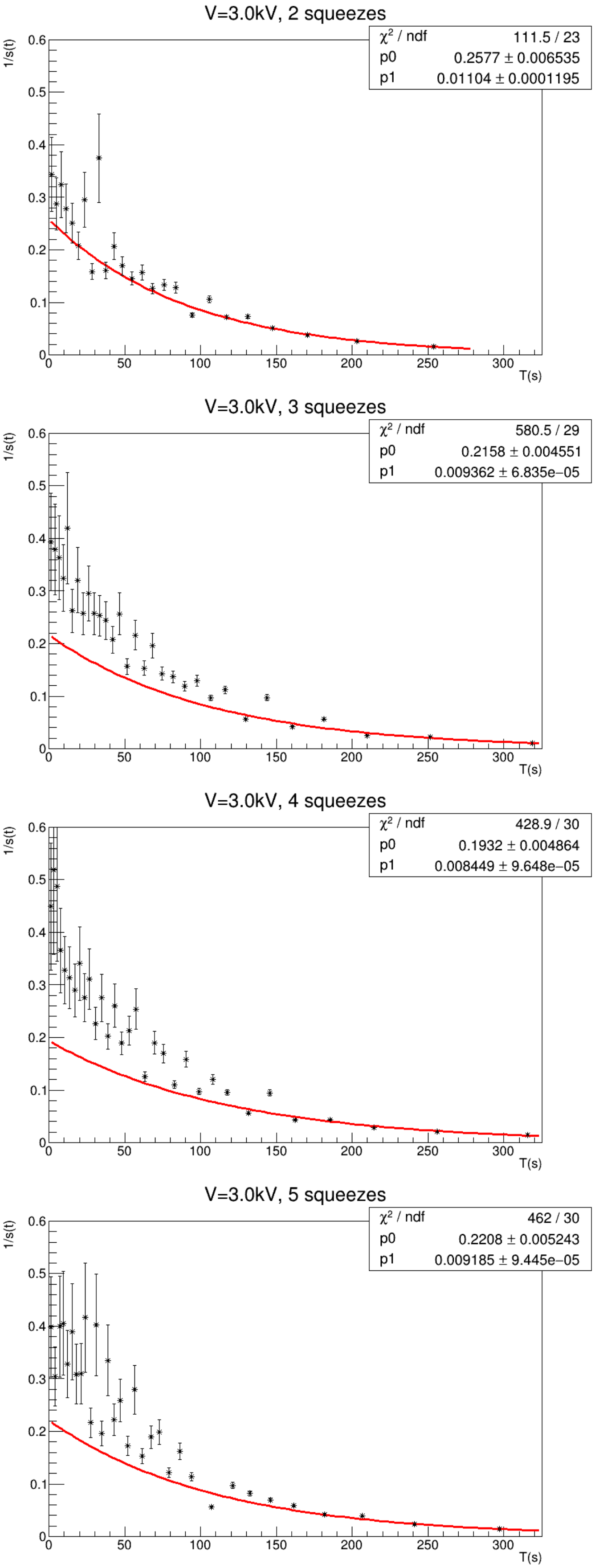

Fig. 4. V=3.0kV

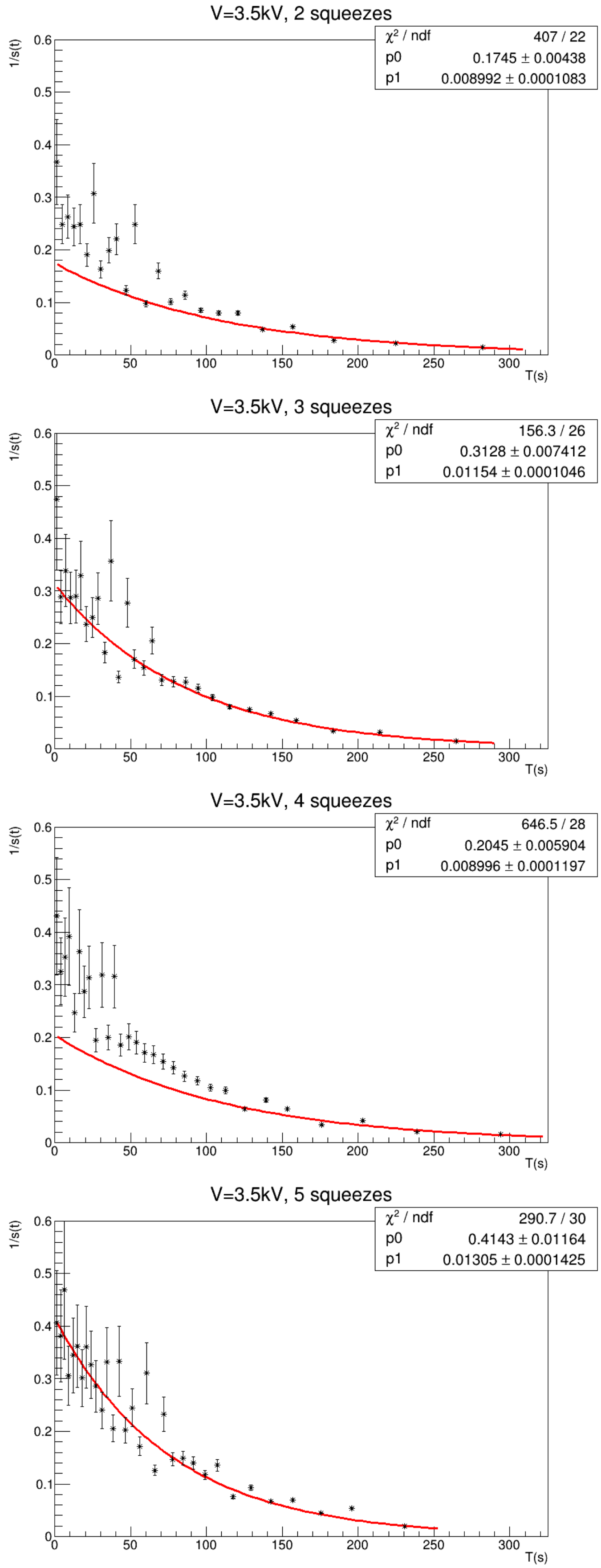

Fig. 5. V=3.5kV

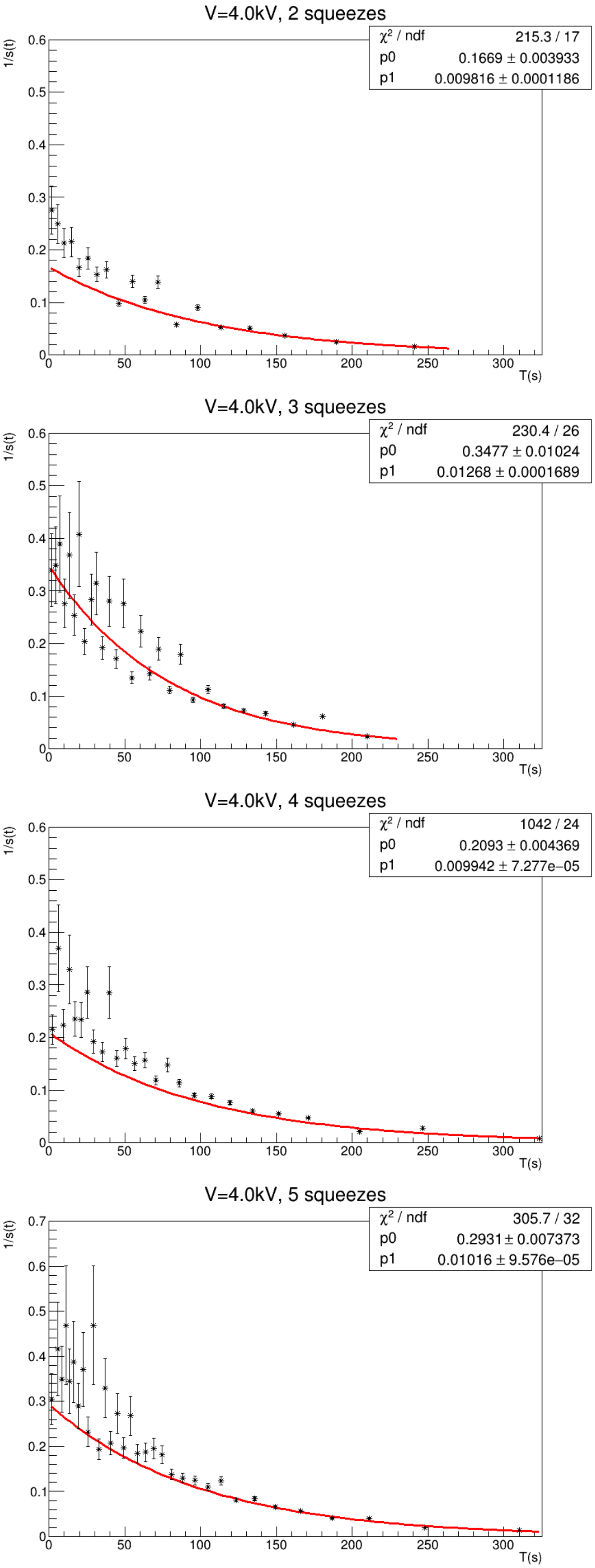

Fig. 6. V=4.0kV

Finally, $\sigma_{t_{\frac{1}{2}}}$ is equal to this:

$$\sigma t_{\frac{1}{2}} = \sqrt{\left(\frac{\partial\left(\frac{ln2}{\bar{\lambda}}\right)}{\partial\bar{\lambda}}\sigma_{\bar{\lambda}}\right)^2} = \frac{ln2}{\bar{\lambda}^2}\sigma_{\bar{\lambda}} \tag{21}$$

$$\sigma_{t_{\frac{1}{2}}} = \pm 0.1724s \tag{22}$$

And our result is this far away from the true value:

$$\frac{\left|55.6s - t_{\frac{1}{2}}\right|}{\sigma_{t_{\frac{1}{2}}}} = 84.734\sigma \tag{23}$$

This value is reasonable, but upon further analysis, we see in Table 2 that a lower χ^2 value is obtained when the long T times are removed.

TABLE II
$\lambda Values-$

$Dataset$	λ	σ_λ	$\frac{\chi^2}{ndf}$
$2.5kV, 2sq$	0.01281	0.0002798	708.6/19
$2.5kV, 3sq$	0.0107	0.0002164	92.38/21
$2.5kV, 4sq$	0.01144	0.0002431	21.73/27
$2.5kV, 5sq$	0.014	0.0002403	1564/28
$3.0kV, 2sq$	0.01175	0.0001832	87.53/22
$3.0kV, 3sq$	0.01247	0.0003267	263.8/26
$3.0kV, 4sq$	0.01169	0.0003037	209.4/27
$3.0kV, 5sq$	0.009912	0.0006025	407.5/26
$3.5kV, 2sq$	0.0119	0.0002077	166/20
$3.5kV, 3sq$	0.01188	0.0002297	81.64/24
$3.5kV, 4sq$	0.01211	0.0002407	324.7/26
$3.5kV, 5sq$	0.01305	0.0001425	290.7/30
$4.0kV, 2sq$	0.01046	0.0001953	199.1/16
$4.0kV, 3sq$	0.01268	0.0001689	230.4/26
$4.0kV, 4sq$	0.0131	0.000156	181.2/22
$4.0kV, 5sq$	0.01164	0.000134	119.2/31

When we analyze data in this manner, the resulting values are as follows:

TABLE III
$\lambda Values-$

$\bar{\lambda}$	$\sigma_{\bar{\lambda}}$	$t_{\frac{1}{2}}$	$\sigma_{t_{\frac{1}{2}}}$	$\sigma Away$
0.012137214	5.07789×10^{-5}	57.10925	0.238929999	6.316704

We are now able to plot the decay constant λ as a function of the number of squeezes for a selected voltage.

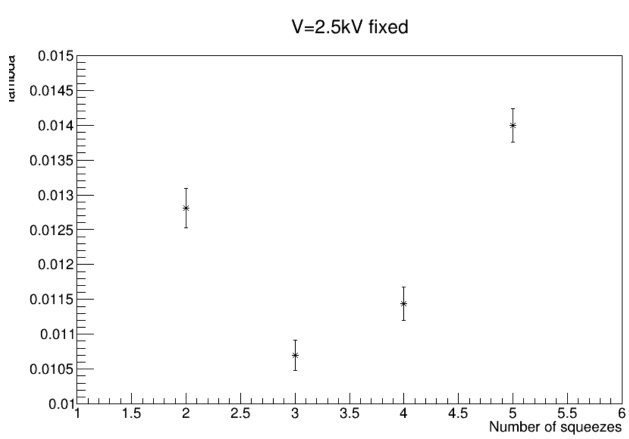

Fig. 7. λ as a function of number of squeezes

We can perform the same analysis to determine the impact of voltage.

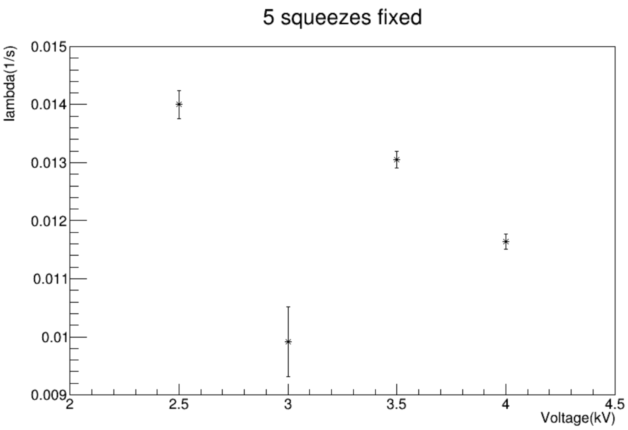

Fig. 8. λ as a function of voltage

The findings indicate that the decay rate is not impacted by either Voltage or the number of squeezes.

V. The Conclusion

During the experiment, we studied the decay of Radon gas using a Wulf's electroscope, and measured the time intervals between two emissions.

Based on our initial analysis, we have observed that $t_{1/2}$ deviates from the accepted value by 84.734 sigma, which is within acceptable limits. However, the χ^2 test results indicate a relatively poor fit. To address this, we could exclude values where T exceeds 250 seconds. By doing so, our function fits the data better and the χ^2 test results become more acceptable. Computed using this approach, $t_{1/2}$ deviates by only 6.31 sigma, which is a much better result. There are several reasons why excluding these T values leads to a better outcome. For instance, Radon gas undergoes half-life approximately 5 times over when the time exceeds 250 seconds. This implies that approximately $1/32$ of the gas has not decayed, and the data could be affected by lower density of ionized gas. Additionally, we assume a linear decrease for $1/s$ in each interval by defining T values as the midpoints of every interval. This assumption might have an impact on the accuracy of our calculations.

When ^{232}Th decays, it produces a number of other daughter nuclei, including several isotopes of radium. These daughter nuclei themselves undergo further radioactive decay, ultimately leading to the production of radon gas. The daughter nuclei produced in the decay chain of ^{232}Th can affect the ionization of radon gas because they themselves can emit alpha, beta, or gamma radiation. This extra ionization can affect the behavior of radon gas. Therefore, the presence of daughter nuclei in the decay chain of ^{232}Th can have an impact on the ionization of radon gas and the experiment.

References

[1] Henri Becquerel. "Sur les radiations émises par phosphorescence". In: *Comptes Rendus de l'Académie des Sciences* 122 (1896), pp. 420–421.

[2] Ernest Rutherford. "The laws of radioactive change". In: *Philosophical Magazine* 21 (1911), pp. 576–582.

[3] Marie Curie and Pierre Curie. "Sur une nouvelle substance fortement radioactive, contenue dans la pechblende". In: *Comptes Rendus de l'Académie des Sciences* 127 (1898), pp. 1215–1217.

[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

VI. Appendix

A. Tables

TABLE IV
2.5V, 2 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	t	s
1.855	0.269541779	0.3	0.043591662	3.71	3.71
5.202	0.335120643	0.3	0.067383507	6.694	2.984
8.486	0.279017857	0.3	0.046710579	10.278	3.584
12.27	0.251004016	0.3	0.03780181	14.262	3.984
16.162	0.263157895	0.3	0.041551247	18.062	3.8
20.2375	0.229832222	0.3	0.03169371	22.413	4.351
24.4885	0.240905806	0.3	0.034821364	26.564	4.151
29.0305	0.2027164	0.3	0.024656363	31.497	4.933
33.147	0.303030303	0.3	0.055096419	34.797	3.3
38.4225	0.137912012	0.3	0.011411834	42.048	7.251
47.4155	0.093153237	0.3	0.005206515	52.783	10.735
55.842	0.163452109	0.3	0.016029955	58.901	6.118
62.992	0.122219506	0.3	0.008962565	67.083	8.182
70.4345	0.149186931	0.3	0.013354044	73.786	6.703
78.428	0.107712193	0.3	0.00696115	83.07	9.284
87.2045	0.120933607	0.3	0.008774962	91.339	8.269
97.297	0.083920779	0.3	0.004225618	103.255	11.916
108.807	0.090057637	0.3	0.004866227	114.359	11.104
122.218	0.063621326	0.3	0.002428604	130.077	15.718
148.4025	0.027284385	0.3	0.000446663	166.728	36.651
182.697	0.031310664	0.3	0.000588215	198.666	31.938
218.1095	0.025715535	0.3	0.000396773	237.553	38.887
279.773	0.011842729	0.3	0	321.993	84.44

TABLE V
2.5V, 3 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.502	0.332889481	0.3	0.066489244	3.004	3.004
4.938	0.258531541	0.3	0.040103135	3.868	6.872
8.6045	0.288600289	0.3	0.049974076	3.465	10.337
11.9205	0.315756236	0.3	0.0598212	3.167	13.504
15.9705	0.2027164	0.3	0.024656363	4.933	18.437
20.754	0.215796288	0.3	0.027940823	4.634	23.071
25.306	0.223713647	0.3	0.030028677	4.47	27.541
29.715	0.2299908	0.3	0.031737461	4.348	31.889
34.7645	0.173882803	0.3	0.018141138	5.751	37.64
39.6735	0.245881485	0.3	0.036274623	4.067	41.707
45.683	0.125754527	0.3	0.009488521	7.952	49.659
51.793	0.234301781	0.3	0.032938395	4.268	53.927
58.0845	0.120264582	0.3	0.008678142	8.315	62.242
65.0175	0.180147721	0.3	0.019471921	5.551	67.793
72.177	0.114051095	0.3	0.007804591	8.768	76.561
80.619	0.123213406	0.3	0.009108926	8.116	84.677
90.595	0.084488003	0.3	0.004282934	11.836	96.513
101.68	0.09676795	0.3	0.005618422	10.334	106.847
113.882	0.071073205	0.3	0.00303084	14.07	120.917
128.816	0.063299152	0.3	0.00240407	15.798	136.715
145.234	0.058692335	0.3	0.002066874	17.038	153.753
166.9705	0.037828636	0.3	0.000858603	26.435	180.188
195.655	0.03232689	0.3	0.000627017	30.934	211.122
234.534	0.021356569	0.3	0.000273662	46.824	257.946
302.825	0.011141068	0.3	0	89.758	347.704

TABLE VI
2.5V, 4 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.1635	0.42973786	0.3	0.110804777	2.327	2.327
3.711	0.361271676	0.3	0.078310334	2.768	5.095
6.454	0.367917586	0.3	0.08121801	2.718	7.813
9.053	0.403225806	0.3	0.097554631	2.48	10.293
11.8435	0.32247662	0.3	0.062394702	3.101	13.394
15.052	0.301568154	0.3	0.054566011	3.316	16.71
18.5935	0.265463233	0.3	0.042282437	3.767	20.477
22.102	0.307692308	0.3	0.056804734	3.25	23.727
25.719	0.251004016	0.3	0.03780181	3.984	27.711
29.303	0.314070352	0.3	0.059184112	3.184	30.895
33.1865	0.218197687	0.3	0.028566138	4.583	35.478
37.186	0.292740047	0.3	0.051418041	3.416	38.894
41.2625	0.211104074	0.3	0.026738958	4.737	43.631
46.006	0.210526316	0.3	0.026592798	4.75	48.381
50.539	0.231696015	0.3	0.032209826	4.316	52.697
55.581	0.173370319	0.3	0.018034361	5.768	58.465
61.1405	0.186880957	0.3	0.020954695	5.351	63.816
67.0585	0.154202005	0.3	0.014266955	6.485	70.301
73.4925	0.156666144	0.3	0.014726568	6.383	76.684
79.935	0.153798831	0.3	0.014192448	6.502	83.186
87.118	0.12716175	0.3	0.009702066	7.864	91.05
95.2765	0.118301195	0.3	0.008397104	8.453	99.503
103.7865	0.116726976	0.3	0.008175112	8.567	108.07
113.746	0.088090204	0.3	0.00465593	11.352	119.422
125.664	0.080102531	0.3	0.003849849	12.484	131.906
138.731	0.073260073	0.3	0.003220223	13.65	145.556
153.4165	0.063609185	0.3	0.002427677	15.721	161.277
172.0515	0.046405866	0.3	0.001292103	21.549	182.826
195.7355	0.038731167	0.3	0.000900062	25.819	208.645
227.564	0.026428458	0.3	0.000419078	37.838	246.483
278.1195	0.01580453	0.3	0.00014987	63.273	309.756

TABLE VII
2.5V, 5 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.3875	0.36036	0.3	0.077916	2.775	2.775
3.9585	0.422476	0.3	0.107091	2.367	5.142
6.559	0.352858	0.3	0.074705	2.834	7.976
9.2945	0.379219	0.3	0.086284	2.637	10.613
11.996	0.361533	0.3	0.078424	2.766	13.379
15.1385	0.284172	0.3	0.048452	3.519	16.898
18.3385	0.347102	0.3	0.072288	2.881	19.779
21.262	0.337154	0.3	0.068204	2.966	22.745
24.6785	0.258598	0.3	0.040124	3.867	26.612
28.5465	0.258465	0.3	0.040082	3.869	30.481
32.3805	0.263227	0.3	0.041573	3.799	34.28
36.3145	0.245761	0.3	0.036239	4.069	38.349
40.1405	0.279096	0.3	0.046737	3.583	41.932
44.182	0.222222	0.3	0.02963	4.5	46.432
48.573	0.233536	0.3	0.032723	4.282	50.714
53.4665	0.181653	0.3	0.019799	5.505	56.219
58.6765	0.203459	0.3	0.024837	4.915	61.134
64.16	0.165235	0.3	0.016381	6.052	67.186
69.8345	0.188786	0.3	0.021384	5.297	72.483
75.9175	0.145582	0.3	0.012716	6.869	79.352
82.7775	0.145964	0.3	0.012783	6.851	86.203
89.861	0.136687	0.3	0.01121	7.316	93.519
97.8455	0.115567	0.3	0.008013	8.653	102.172
107.313	0.097257	0.3	0.005675	10.282	112.454
117.2465	0.10433	0.3	0.006531	9.585	122.039
127.8725	0.085712	0.3	0.004408	11.667	133.706
140.976	0.068776	0.3	0.002838	14.54	148.246
156.3375	0.061793	0.3	0.002291	16.183	164.429
189.2715	0.020127	0.3	0.000243	49.685	214.114
234.8675	0.024092	0.3	0.000348	41.507	255.621
286.093	0.016409	0.3	0.000162	60.944	316.565

TABLE VIII
3.0V, 2 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.456	0.343407	0.3	0.070757	2.912	2.912
4.652	0.287356	0.3	0.049544	3.48	6.392
7.9335	0.324359	0.3	0.063125	3.083	9.475
11.2675	0.27894	0.3	0.046685	3.585	13.06
15.052	0.251004	0.3	0.037802	3.984	17.044
19.444	0.208333	0.3	0.026042	4.8	21.844
23.5365	0.295421	0.3	0.052364	3.385	25.229
28.3875	0.158303	0.3	0.015036	6.317	31.546
32.88	0.374813	0.3	0.084291	2.668	34.214
37.33	0.160462	0.3	0.015449	6.232	40.446
42.864	0.206782	0.3	0.025655	4.836	45.282
48.231	0.169549	0.3	0.017248	5.898	51.18
54.615	0.14556	0.3	0.012713	6.87	58.05
61.234	0.157035	0.3	0.014796	6.368	64.418
68.376	0.126326	0.3	0.009575	7.916	72.334
76.077	0.133583	0.3	0.010707	7.486	79.82
83.721	0.128172	0.3	0.009857	7.802	87.622
94.2125	0.075867	0.3	0.003453	13.181	100.803
105.528	0.10582	0.3	0.006719	9.45	110.253
117.187	0.072108	0.3	0.00312	13.868	124.121
130.956	0.073153	0.3	0.003211	13.67	137.791
147.5495	0.051237	0.3	0.001575	19.517	157.308
170.476	0.037971	0.3	0.000865	26.336	183.644
203.0805	0.025725	0.3	0.000397	38.873	222.517
253.694	0.016037	0.3	0.000154	62.354	284.871

TABLE IX
3.0V, 3 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.2705	0.393546	0.3	0.092927	2.541	2.541
3.8595	0.379219	0.3	0.086284	2.637	5.178
6.554	0.363372	0.3	0.079224	2.752	7.93
9.471	0.324465	0.3	0.063166	3.082	11.012
12.2035	0.419639	0.3	0.105658	2.383	13.395
15.3025	0.262123	0.3	0.041225	3.815	17.21
18.7685	0.320821	0.3	0.061756	3.117	20.327
22.2705	0.257268	0.3	0.039712	3.887	24.214
25.9055	0.295596	0.3	0.052426	3.383	27.597
29.5395	0.2574	0.3	0.039753	3.885	31.482
33.4575	0.2531	0.3	0.038436	3.951	35.433
37.482	0.244021	0.3	0.035728	4.098	39.531
41.941	0.207469	0.3	0.025826	4.82	44.351
46.2985	0.256739	0.3	0.039549	3.895	48.246
51.441	0.156495	0.3	0.014694	6.39	54.636
56.952	0.215889	0.3	0.027965	4.632	59.268
62.527	0.153421	0.3	0.014123	6.518	65.786
68.3345	0.196194	0.3	0.023095	5.097	70.883
74.3815	0.142918	0.3	0.012255	6.997	77.88
81.5305	0.136968	0.3	0.011256	7.301	85.181
89.3815	0.119033	0.3	0.008501	8.401	93.582
97.449	0.129299	0.3	0.010031	7.734	101.316
106.491	0.096618	0.3	0.005601	10.35	111.666
116.1255	0.11212	0.3	0.007543	8.919	120.585
129.484	0.056186	0.3	0.001894	17.798	138.383
143.5335	0.097078	0.3	0.005654	10.301	148.684
160.453	0.042484	0.3	0.001083	23.538	172.222
181.064	0.056548	0.3	0.001919	17.684	189.906
209.691	0.025272	0.3	0.000383	39.57	229.476
251.228	0.022986	0.3	0.000317	43.504	272.98
318.684	0.01094	0.3	0	91.408	364.388

TABLE X
3.0V, 4 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.113	0.449236	0.3	0.121088	2.226	2.226
3.1885	0.519481	0.3	0.161916	1.925	4.151
5.177	0.487329	0.3	0.142494	2.052	6.203
7.57	0.365764	0.3	0.08027	2.734	8.937
10.461	0.328084	0.3	0.064583	3.048	11.985
13.578	0.313873	0.3	0.05911	3.186	15.171
16.895	0.290023	0.3	0.050468	3.448	18.619
20.087	0.340599	0.3	0.069605	2.936	21.555
23.3645	0.276319	0.3	0.045811	3.619	25.174
26.7815	0.311042	0.3	0.058048	3.215	28.389
30.5965	0.226501	0.3	0.030782	4.415	32.804
34.621	0.275179	0.3	0.045434	3.634	36.438
38.913	0.20202	0.3	0.024487	4.95	41.388
43.306	0.260688	0.3	0.040775	3.836	45.224
47.8665	0.189215	0.3	0.021481	5.285	50.509
52.8575	0.212902	0.3	0.027196	4.697	55.206
57.1735	0.25413	0.3	0.038749	3.935	59.141
63.133	0.125251	0.3	0.009413	7.984	67.125
69.758	0.189897	0.3	0.021637	5.266	72.391
75.3435	0.169348	0.3	0.017207	5.905	78.296
82.837	0.110108	0.3	0.007274	9.082	87.378
90.529	0.15868	0.3	0.015108	6.302	93.68
98.8295	0.097097	0.3	0.005657	10.299	103.979
108.1365	0.120265	0.3	0.008678	8.315	112.294
117.5115	0.095831	0.3	0.00551	10.435	122.729
131.5715	0.056545	0.3	0.001918	17.685	140.414
145.689	0.094787	0.3	0.005391	10.55	150.964
162.475	0.043437	0.3	0.001132	23.022	173.986
185.5105	0.043386	0.3	0.001129	23.049	197.035
214.321	0.028925	0.3	0.000502	34.572	231.607
256.0185	0.020482	0.3	0.000252	48.823	280.43
315.623	0.014207	0.3	0.000121	70.386	350.816

TABLE XI
3.0V, 5 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.2535	0.398883	0.3	0.095465	2.507	2.507
4.149	0.304507	0.3	0.055635	3.284	5.791
7.042	0.39968	0.3	0.095847	2.502	8.293
9.526	0.405515	0.3	0.098665	2.466	10.759
12.2835	0.327976	0.3	0.064541	3.049	13.808
15.0915	0.38956	0.3	0.091054	2.567	16.375
17.9925	0.309119	0.3	0.057333	3.235	19.61
21.226	0.309406	0.3	0.057439	3.232	22.842
24.043	0.41632	0.3	0.103993	2.402	25.244
27.552	0.216638	0.3	0.028159	4.616	29.86
31.1025	0.402414	0.3	0.097162	2.485	32.345
34.894	0.196155	0.3	0.023086	5.098	37.443
38.935	0.335121	0.3	0.067384	2.984	40.427
42.6775	0.222173	0.3	0.029616	4.501	44.928
46.8615	0.258598	0.3	0.040124	3.867	48.795
51.695	0.172414	0.3	0.017836	5.8	54.595
56.3865	0.279096	0.3	0.046737	3.583	58.178
61.4375	0.153398	0.3	0.014119	6.519	64.697
67.339	0.189251	0.3	0.021489	5.284	69.981
72.4985	0.19861	0.3	0.023667	5.035	75.016
79.1315	0.121492	0.3	0.008856	8.231	83.247
86.331	0.162127	0.3	0.015771	6.168	89.415
93.8155	0.113623	0.3	0.007746	8.801	98.216
107.175	0.05581	0.3	0.001869	17.918	116.134
121.2775	0.09721	0.3	0.00567	10.287	126.421
132.5285	0.081867	0.3	0.004021	12.215	138.636
145.778	0.070008	0.3	0.002941	14.284	152.92
161.346	0.05934	0.3	0.002113	16.852	169.772
181.6645	0.042043	0.3	0.001061	23.785	193.557
206.391	0.038959	0.3	0.000911	25.668	219.225
240.927	0.023039	0.3	0.000318	43.404	262.629
297.141	0.014488	0.3	0.000126	69.024	331.653

TABLE XII
3.5V, 2 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.3615	0.367242	0.3	0.08092	2.723	2.723
4.7315	0.248942	0.3	0.037183	4.017	6.74
8.64	0.263158	0.3	0.041551	3.8	10.54
12.59	0.243902	0.3	0.035693	4.1	14.64
16.649	0.24888	0.3	0.037165	4.018	18.658
21.2825	0.190512	0.3	0.021777	5.249	23.907
25.5325	0.307598	0.3	0.05677	3.251	27.158
30.2255	0.162999	0.3	0.015941	6.135	33.293
35.801	0.199362	0.3	0.023847	5.016	38.309
40.576	0.220556	0.3	0.029187	4.534	42.843
46.8935	0.123442	0.3	0.009143	8.101	50.944
52.9525	0.248942	0.3	0.037183	4.017	54.961
60.086	0.097561	0.3	0.005711	10.25	65.211
68.337	0.159949	0.3	0.01535	6.252	71.463
76.4135	0.101	0.3	0.006121	9.901	81.364
85.7385	0.114299	0.3	0.007839	8.749	90.113
96.005	0.084861	0.3	0.004321	11.784	101.897
108.1145	0.080418	0.3	0.00388	12.435	114.332
120.591	0.079885	0.3	0.003829	12.518	126.85
137.0425	0.049056	0.3	0.001444	20.385	147.235
156.6275	0.053234	0.3	0.0017	18.785	166.02
183.9045	0.027957	0.3	0.000469	35.769	201.789
224.825	0.021705	0.3	0.000283	46.072	247.861
281.95	0.014667	0.3	0.000129	68.178	316.039

TABLE XIII
3.5V, 3 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.053	0.474834	0.3	0.13528	2.106	2.106
3.84	0.288351	0.3	0.049888	3.468	5.574
7.049	0.338983	0.3	0.068946	2.95	8.524
10.266	0.287026	0.3	0.04943	3.484	12.008
13.7325	0.289939	0.3	0.050439	3.449	15.457
16.974	0.329598	0.3	0.065181	3.034	18.491
20.5995	0.237135	0.3	0.03374	4.217	22.708
24.709	0.249875	0.3	0.037463	4.002	26.71
28.4595	0.285796	0.3	0.049008	3.499	30.209
32.9425	0.182916	0.3	0.020075	5.467	35.676
37.076	0.357143	0.3	0.076531	2.8	38.476
42.143	0.136351	0.3	0.011155	7.334	45.81
47.61	0.277778	0.3	0.046296	3.6	49.41
52.3435	0.170445	0.3	0.017431	5.867	55.277
58.5275	0.153822	0.3	0.014197	6.501	61.778
64.2115	0.205465	0.3	0.02533	4.867	66.645
70.479	0.130412	0.3	0.010204	7.668	74.313
78.23	0.127649	0.3	0.009777	7.834	82.147
86.0975	0.126566	0.3	0.009611	7.901	90.048
94.3895	0.115168	0.3	0.007958	8.683	98.731
103.848	0.097714	0.3	0.005729	10.234	108.965
115.2575	0.07946	0.3	0.003788	12.585	121.55
128.275	0.074349	0.3	0.003317	13.45	135
142.4425	0.067182	0.3	0.002708	14.885	149.885
159.295	0.053135	0.3	0.001694	18.82	168.705
183.422	0.033974	0.3	0.000693	29.434	198.139
214.1995	0.031132	0.3	0.000582	32.121	230.26
264.6705	0.01453	0.3	0.000127	68.821	299.081

TABLE XIV
3.5V, 4 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.1605	0.430849	0.3	0.111378	2.321	2.321
3.8545	0.326052	0.3	0.063786	3.067	5.388
6.805	0.352858	0.3	0.074705	2.834	8.222
9.497	0.392157	0.3	0.092272	2.55	10.772
12.7975	0.246853	0.3	0.036562	4.051	14.823
16.198	0.363636	0.3	0.079339	2.75	17.573
19.3145	0.287109	0.3	0.049459	3.483	21.056
22.648	0.31407	0.3	0.059184	3.184	24.24
26.807	0.19478	0.3	0.022764	5.134	29.374
30.941	0.319081	0.3	0.061088	3.134	32.508
35.008	0.2	0.3	0.024	5	37.508
39.0915	0.315756	0.3	0.059821	3.167	40.675
43.3665	0.18577	0.3	0.020706	5.383	46.058
48.5415	0.201329	0.3	0.02432	4.967	51.025
53.651	0.190404	0.3	0.021752	5.252	56.277
59.2015	0.170969	0.3	0.017538	5.849	62.126
65.11	0.16756	0.3	0.016846	5.968	68.094
71.3275	0.154631	0.3	0.014346	6.467	74.561
78.0695	0.142511	0.3	0.012186	7.017	81.578
85.5365	0.12631	0.3	0.009573	7.917	89.495
93.737	0.117869	0.3	0.008336	8.484	97.979
102.763	0.104515	0.3	0.006554	9.568	107.547
112.5725	0.099493	0.3	0.005939	10.051	117.598
125.349	0.064508	0.3	0.002497	15.502	133.1
139.2585	0.081189	0.3	0.003955	12.317	145.417
153.2005	0.064238	0.3	0.002476	15.567	160.984
175.927	0.03346	0.3	0.000672	29.886	190.87
202.921	0.04149	0.3	0.001033	24.102	214.972
238.6995	0.021073	0.3	0.000266	47.455	262.427
293.7715	0.015952	0.3	0.000153	62.689	325.116

TABLE XV
3.5V, 5 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.23	0.406504	0.3	0.099147	2.46	2.46
3.7685	0.382117	0.3	0.087608	2.617	5.077
6.1435	0.468823	0.3	0.131877	2.133	7.21
8.8435	0.306091	0.3	0.056215	3.267	10.477
11.9275	0.344709	0.3	0.071294	2.901	13.378
14.761	0.361533	0.3	0.078424	2.766	16.144
17.803	0.301386	0.3	0.0545	3.318	19.462
20.8495	0.36036	0.3	0.077916	2.775	22.237
23.7665	0.326904	0.3	0.06412	3.059	25.296
27.046	0.285714	0.3	0.04898	3.5	28.796
30.8795	0.239981	0.3	0.034554	4.167	32.963
34.4715	0.331455	0.3	0.065917	3.017	35.98
38.4135	0.205465	0.3	0.02533	4.867	40.847
42.3475	0.333222	0.3	0.066622	3.001	43.848
46.323	0.20202	0.3	0.024487	4.95	48.798
50.8395	0.244918	0.3	0.035991	4.083	52.881
55.7985	0.17138	0.3	0.017623	5.835	58.716
60.324	0.310945	0.3	0.058012	3.216	61.932
65.899	0.12604	0.3	0.009532	7.934	69.866
72.0165	0.232504	0.3	0.032435	4.301	74.167
77.5675	0.147037	0.3	0.012972	6.801	80.968
84.3185	0.149231	0.3	0.013362	6.701	87.669
91.2355	0.140193	0.3	0.011793	7.133	94.802
99.061	0.117398	0.3	0.008269	8.518	103.32
107.0085	0.135556	0.3	0.011025	7.377	110.697
117.3345	0.07533	0.3	0.003405	13.275	123.972
129.3225	0.093449	0.3	0.00524	10.701	134.673
142.182	0.066587	0.3	0.00266	15.018	149.691
156.9055	0.069305	0.3	0.002882	14.429	164.12
175.203	0.045114	0.3	0.001221	22.166	186.286
195.6755	0.053251	0.3	0.001701	18.779	205.065
230.6585	0.019536	0.3	0.000229	51.187	256.252

TABLE XVI
4.0V, 2 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.815	0.275482	0.3	0.045534	3.63	3.63
5.634	0.249501	0.3	0.03735	4.008	7.638
9.985	0.213038	0.3	0.027231	4.694	12.332
14.6555	0.215193	0.3	0.027785	4.647	16.979
19.9895	0.166085	0.3	0.016551	6.021	23
25.72	0.183824	0.3	0.020275	5.44	28.44
31.696	0.153563	0.3	0.014149	6.512	34.952
38.037	0.162075	0.3	0.015761	6.17	41.122
46.192	0.098619	0.3	0.005835	10.14	51.262
54.847	0.13947	0.3	0.011671	7.17	58.432
63.2	0.104866	0.3	0.006598	9.536	67.968
71.5815	0.13837	0.3	0.011488	7.227	75.195
83.8745	0.057607	0.3	0.001991	17.359	92.554
98.105	0.090074	0.3	0.004868	11.102	103.656
113.1915	0.052436	0.3	0.00165	19.071	122.727
132.411	0.051632	0.3	0.001599	19.368	142.095
155.5955	0.037036	0.3	0.000823	27.001	169.096
189.373	0.024658	0.3	0.000365	40.554	209.65
241.0125	0.015943	0.3	0.000153	62.725	272.375

TABLE XVII
4.0V, 3 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.472	0.339674	0.3	0.069227	2.944	2.944
4.3775	0.348797	0.3	0.072995	2.867	5.811
7.0945	0.38956	0.3	0.091054	2.567	8.378
10.1865	0.276472	0.3	0.045862	3.617	11.995
13.3535	0.368053	0.3	0.081278	2.717	14.712
16.679	0.254194	0.3	0.038769	3.934	18.646
19.8715	0.407997	0.3	0.099877	2.451	21.097
23.5465	0.204123	0.3	0.025	4.899	25.996
27.757	0.28393	0.3	0.04837	3.522	29.518
31.108	0.314465	0.3	0.059333	3.18	32.698
35.307	0.191644	0.3	0.022037	5.218	37.916
39.699	0.280426	0.3	0.047183	3.566	41.482
44.4075	0.170911	0.3	0.017526	5.851	47.333
49.1415	0.276472	0.3	0.045862	3.617	50.95
54.6515	0.13508	0.3	0.010948	7.403	58.353
60.5935	0.223164	0.3	0.029881	4.481	62.834
66.3265	0.143164	0.3	0.012298	6.985	69.819
72.454	0.189753	0.3	0.021604	5.27	75.089
79.571	0.111557	0.3	0.007467	8.964	84.053
86.837	0.179598	0.3	0.019353	5.568	89.621
94.989	0.093145	0.3	0.005206	10.736	100.357
104.8	0.112537	0.3	0.007599	8.886	109.243
115.4335	0.080769	0.3	0.003914	12.381	121.624
128.5335	0.072364	0.3	0.003142	13.819	135.443
142.868	0.06734	0.3	0.002721	14.85	150.293
161.1855	0.045903	0.3	0.001264	21.785	172.078
180.1795	0.061717	0.3	0.002285	16.203	188.281
209.7675	0.02327	0.3	0.000325	42.973	231.254

TABLE XVIII
4.0V, 4 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
2.3235	0.215193	0.3	0.027785	4.647	4.647
5.997	0.37037	0.3	0.082305	2.7	7.347
9.5795	0.223964	0.3	0.030096	4.465	11.812
13.3285	0.329707	0.3	0.065224	3.033	14.845
16.9705	0.235239	0.3	0.033202	4.251	19.096
21.2385	0.233372	0.3	0.032678	4.285	23.381
25.1315	0.285633	0.3	0.048952	3.501	26.882
29.482	0.192308	0.3	0.022189	5.2	32.082
34.9815	0.172444	0.3	0.017842	5.799	37.881
39.632	0.285551	0.3	0.048924	3.502	41.383
44.5	0.160411	0.3	0.015439	6.234	47.617
50.4085	0.179115	0.3	0.019249	5.583	53.2
56.526	0.150331	0.3	0.01356	6.652	59.852
63.0445	0.156617	0.3	0.014717	6.385	66.237
70.453	0.118596	0.3	0.008439	8.432	74.669
78.0525	0.147776	0.3	0.013103	6.767	81.436
85.8545	0.113161	0.3	0.007683	8.837	90.273
95.831	0.08996	0.3	0.004856	11.116	101.389
107.0645	0.088098	0.3	0.004657	11.351	112.74
119.324	0.075942	0.3	0.00346	13.168	125.908
134.2085	0.060237	0.3	0.002177	16.601	142.509
151.519	0.055494	0.3	0.001848	18.02	160.529
171.0045	0.04773	0.3	0.001367	20.951	181.48
204.808	0.021433	0.3	0.000276	46.656	228.136
246.362	0.027433	0.3	0.000452	36.452	264.588
323.4565	0.008494	0.3	$4.33E-05$	117.737	382.325

TABLE XIX
4.0V, 5 SQUEEZES

T	$1/s$	σ_T	$\sigma_{1/s}$	s	t
1.639	0.305064	0.3	0.055838	3.278	3.278
3.978	0.714286	0.3	0.306122	1.4	4.678
5.88	0.415973	0.3	0.10382	2.404	7.082
8.5135	0.349284	0.3	0.0732	2.863	9.945
11.0125	0.468384	0.3	0.13163	2.135	12.08
13.531	0.34459	0.3	0.071245	2.902	14.982
16.2725	0.387447	0.3	0.090069	2.581	17.563
19.29	0.289519	0.3	0.050293	3.454	21.017
22.3655	0.370782	0.3	0.082488	2.697	23.714
25.8645	0.232504	0.3	0.032435	4.301	28.015
29.0815	0.468823	0.3	0.131877	2.133	30.148
32.7315	0.193536	0.3	0.022474	5.167	35.315
36.832	0.329598	0.3	0.065181	3.034	38.349
40.7585	0.207512	0.3	0.025837	4.819	43.168
45.001	0.272777	0.3	0.044644	3.666	46.834
49.376	0.196696	0.3	0.023213	5.084	51.918
53.7845	0.267881	0.3	0.043056	3.733	55.651
58.368	0.184026	0.3	0.020319	5.434	61.085
63.76	0.186916	0.3	0.020963	5.35	66.435
69.0025	0.194742	0.3	0.022755	5.135	71.57
74.3285	0.181258	0.3	0.019713	5.517	77.087
80.7215	0.137571	0.3	0.011355	7.269	84.356
88.1985	0.130124	0.3	0.010159	7.685	92.041
96.0565	0.124517	0.3	0.009303	8.031	100.072
104.632	0.109649	0.3	0.007214	9.12	109.192
113.2495	0.123229	0.3	0.009111	8.115	117.307
123.458	0.081288	0.3	0.003965	12.302	129.609
135.5345	0.084381	0.3	0.004272	11.851	141.46
149.12	0.065274	0.3	0.002556	15.32	156.78
165.6725	0.056227	0.3	0.001897	17.785	174.565
186.6745	0.04129	0.3	0.001023	24.219	198.784
211.1255	0.040514	0.3	0.000985	24.683	223.467
247.9365	0.020434	0.3	0.000251	48.939	272.406
310.0555	0.01328	0.3	0.000106	75.299	347.705

B. Codes

The code I use to fit the exponential function to the data:

```
{
  TCanvas *c1 = new TCanvas();
  c1->SetWindowSize(900, 600);

  // Defining the graph object with error
      bars. Read from a txt file.
  TGraphErrors *mygraph = new
      TGraphErrors("datafile.txt");

  // Setting the title of the graph
  mygraph->SetTitle("Title");

  // Drawing the graph
  mygraph->Draw("A*");
  gStyle->SetOptFit(1);

  // Defining and fitting an exponential
      function in a given range
  TF1* expo_fit = new TF1("expo_fit",
      "[0]*exp(-[1]*x)", 0, 325);
  expo_fit->SetParameters(0.30, 0.012);
  expo_fit->SetLineColor(kRed);
  expo_fit->SetLineWidth(3);

  // Fitting and plotting
  mygraph->Fit(expo_fit);

  // Naming the axes
  mygraph->GetXaxis()->SetTitle("T(s)");
  mygraph->GetYaxis()->SetTitle("1/s(t)");

  // Setting the limits of "x-axis" if
      required for visibility
  mygraph->GetXaxis()->SetLimits(0, 325);

  // Setting the limits of "y-axis" if
      required for visibility
  mygraph->SetMinimum(0);
  mygraph->SetMaximum(0.6);
}
```

Code for fixed voltage and fixed squeeze graph:

```
{
  TCanvas* c1 = new TCanvas();
  c1->SetWindowSize(900, 600);

  TGraphErrors* mygraph = new
      TGraphErrors("fixed.txt");

  mygraph->SetTitle("title");

  mygraph->Draw("A*");

  mygraph->GetXaxis()->SetTitle("Voltage(kV)
      or squeezes");
  mygraph->GetYaxis()->SetTitle("lambda(1/s)");

  mygraph->GetXaxis()->SetLimits(2, 4.5);

  mygraph->SetMinimum(0.009);
  mygraph->SetMaximum(0.015);
}
```