HW 3

(1) a) The Fermi energy of a metal is 5 eV. Find the Fermi temperature T_F; the Fermi momentum p_F; the Fermi velocity for non-relativistic electrons in terms of SI units.

b) Calculate the number density of electrons $\frac{N}{V}$ for this metal.

(2) The electrons of a He atom are at the 1s orbital (the ground state of the He atom.) Use the form of the one-electron wavefunction $f(r_i) = \sqrt{\frac{\alpha^3}{\pi}} e^{-\alpha r_i}$ ($i = 1, 2$) and write the spatial part of many-body (2-e⁻'s) wavefunction for the electrons of the He atom if the spin wavefunction is an anti-symmetric wavefunction.

(3) [Blackbody Radiation] The total energy of a photon gas is given by

$$E_{Total} = \int_0^\infty \mathcal{E} \cdot \frac{1}{e^{\beta\mathcal{E}} - 1} \cdot \frac{8\pi V \cdot \mathcal{E}^2}{(hc)^3} \, d\mathcal{E}$$

Define a new variable $x = \beta\mathcal{E}$ and show that $E_{Total} \propto V T^4$ i.e.

$$E_{Total} = \tilde{C} V T^4$$

(4) a) The Boltzmann distribution

$$n = e^{\beta\mu} e^{-\beta E}$$

can be used when

$\Delta E \ll k_B T$ where

$\Delta E \equiv$ the energy difference between the nearest energy levels. Consider a 1-D Quantum Harmonic oscillator in thermal equilibrium at temperature T. Take $T = 10000\,K$. Find the frequency f of this oscillator if

$$\Delta E = 0.01 \cdot k_B T \ll k_B T$$

for $T = 10000\,K$

b) This oscillator represents vibrations of a diatomic molecule. Find the ratio of average number of molecules at the first excited vibrational state and the ground state of the vibrational state.