0.1 Event

$$\underline{x}_i = \begin{pmatrix} \underline{x}_0 \\ \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{pmatrix} = \begin{pmatrix} x_0 + i\,c\,t_0 \\ x_1 + i\,c\,t_1 \\ x_2 + i\,c\,t_2 \\ x_3 + i\,c\,t_3 \end{pmatrix} \tag{1}$$

If we have to give the event a name, we exchange x with the name or (if it's importaant to indicate the nature of the value) add the name in scriptstyle:

$$\underline{x_a}_i = \begin{pmatrix} \underline{x_a}_0 \\ \underline{x_a}_1 \\ \underline{x_a}_2 \\ \underline{x_a}_3 \end{pmatrix} = \begin{pmatrix} x_{a0} + i\,c\,t_{a0} \\ x_{a1} + i\,c\,t_{a1} \\ x_{a2} + i\,c\,t_{a2} \\ x_{a3} + i\,c\,t_{a3} \end{pmatrix} \tag{2}$$

0.2 Distance between Event *a* and Event *b*

$$\underline{\Delta x_{a,b}}_i = \begin{pmatrix} \underline{\Delta x_{a,b}}_0 \\ \underline{\Delta x_{a,b}}_1 \\ \underline{\Delta x_{a,b}}_2 \\ \underline{\Delta x_{a,b}}_3 \end{pmatrix} = \begin{pmatrix} \Delta x_{a,b0} + i\ c\ \Delta t_{a,b0} \\ \Delta x_{a,b1} + i\ c\ \Delta t_{a,b1} \\ \Delta x_{a,b2} + i\ c\ \Delta t_{a,b2} \\ \Delta x_{a,b3} + i\ c\ \Delta t_{a,b3} \end{pmatrix} = \begin{pmatrix} (x_{b0} - x_{a0}) + i\ c\ (t_{b0} - t_{a0}) \\ (x_{b1} - x_{a1}) + i\ c\ (t_{b1} - t_{a1}) \\ (x_{b2} - x_{a2}) + i\ c\ (t_{b2} - t_{a2}) \\ (x_{b3} - x_{a3}) + i\ c\ (t_{b3} - t_{a})_3 \end{pmatrix} \tag{3}$$

This value is oriented (has a sign ...) and dependent on the observer (coordinate system).

0.3 Squared distance between two Events

$$\begin{aligned} (\underline{\Delta x}_i)^2 = \underline{\Delta x}_i \underline{\Delta x}_i &= (\underline{\Delta x}_0)^2 + (\underline{\Delta x}_1)^2 + (\underline{\Delta x}_2)^2 + (\underline{\Delta x}_3)^2 \\ &= (\Delta x_0^2 + \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2) - c^2\,(\Delta t_0^2 + \Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2) \\ &\quad + i\,2\,c\,(\Delta x_0 \Delta t_0 + \Delta x_1 \Delta t_1 + \Delta x_2 \Delta t_2 + \Delta x_3 \Delta t_3) \end{aligned} \tag{4}$$

This value is not oriented and independent of the observer.

0.4 Velocity

The quotients of distance between two closely neighbored events are the components of the velocity tensor:

$$\underline{v}_{ij} = \frac{\underline{\Delta x}_i}{\underline{\Delta x}_j} = \frac{x_i + i\,c\,t_i}{x_j + i\,c\,t_j} = \begin{pmatrix} v_{00} & v_{01} & v_{02} & v_{03} \\ v_{10} & v_{11} & v_{12} & v_{13} \\ v_{20} & v_{21} & v_{22} & v_{23} \\ v_{30} & v_{31} & v_{32} & v_{33} \end{pmatrix} \tag{5}$$

As the distances are observer-dependent this tensor is dependent on the observer as well.