AJFY 2013/14

příklady na zápočtovou písemku

1 Výchylka dráhy elektronu v příčném poli (měrný náboj)
2 Bohrův model atomu vodíku
3 Fotoefekt – náboj na kouli
4 de Broglieova vlnová délka
5 Interference – I=f(x)

6 Nekonečná jáma – E_n, Ψ_n
7 Energetický schod – R, T
8a Odhad energie zákl. stavu harmonického oscilátoru
8b Odhad energie atomu vodíku
9 Atom He – zákl. stav

10 Energie molekul
11 Energie nabité kuličky
12 Náboj=f(hmotnostní číslo)

13 Sekulární rovnováha – λ_1/λ_2
14 Statistika rozpadů – P_n <n> <stř>

VÝCHYLKA DRÁHY ELEKTRONU V PŘÍČNÉM POLI
(THOMSONŮV MĚRNÝ NÁBOJ)

a) elektrické pole

$$m\frac{d^2x}{dt^2}=0 \qquad m\frac{d^2y}{dt^2} \qquad m\frac{d^2z}{dt^2}=-eE$$

poč. podm. $x=vt;\ y=0$

$$\frac{d^2z}{dt^2}=\frac{d}{dt}\left(\frac{dz}{dt}\right)=\frac{d}{dx}\left(\frac{dz}{dx}\cdot\frac{dx}{dt}\right)\frac{dx}{dt}=\frac{d^2z}{dx^2}\cdot v^2$$

$$\frac{d^2z}{dx^2}=-\frac{e}{mv^2}E$$

$$\Delta z=-\frac{e}{mv^2}\int_0^l\left(\underbrace{\int_0^x E\,dx}\right)dx$$

per partes: $f=\int_0^x E\,dx;\ g=x$

$$\Delta z=-\frac{e}{mv^2}\left[l\int_0^l E\,dx-\int_0^l xE\,dx\right]=-\frac{e}{mv^2}\int_0^l(l-x)E\,dx$$

$$\int_0^l(l-x)E\,dx=a\left(l-\frac{a}{2}\right)E$$

$$\Delta z_E=-\frac{e}{mv^2}a\left(l-\frac{a}{2}\right)E$$

b) mag. pole

$$\frac{d^2z}{dt^2}=-\frac{e}{m}v_xB \Rightarrow \frac{d^2z}{dx^2}=-\frac{e}{mv_x^2}v_xB=-\frac{e}{mv_x}B=-\frac{e}{mv}B$$

$$\Delta z=-\frac{e}{mv}\int_0^l(l-x)B\,dx$$

$$\int_0^l(l-x)B\,dx=\frac{l^2}{2}B$$

$$\Delta z_B=-\frac{e}{mv}\cdot\frac{l^2}{2}B$$

$$-\Delta z=\frac{\Delta z_E}{\Delta z_B^2}$$

$$-\frac{e}{m}=\Delta z\cdot\frac{4a}{d\cdot l^4}\left(l-\frac{a}{2}\right)\cdot\frac{U}{B^2}$$

BOHRŮV MODEL ATOMU VODÍKU

$$\frac{m\cdot v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0\cdot r^2}$$

$$W = \frac{1}{2}m\cdot v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$r\cdot m\cdot v = n\cdot\hbar$$

$$v_n = \frac{1}{n}\cdot\frac{Ze^2}{4\pi\varepsilon_0\hbar} = \frac{1}{n}Z\alpha c$$

$$r_n = n^2\,\frac{1}{Z\alpha}\cdot\frac{\hbar}{mc} = n^2\,\frac{1}{Z\alpha}\lambda_c$$

$$W_n = -\frac{1}{n^2}\cdot\frac{Z^2}{2}\alpha^2\cdot\left(m\cdot c^2\right)$$

$$\omega_n = \frac{v_n}{r_n} = \frac{1}{n^3}Z^2\alpha^2\cdot\frac{mc^2}{\hbar}$$

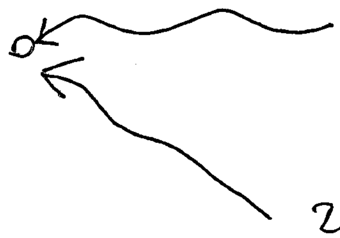
$$E_n = \frac{Ze}{4\pi\varepsilon_0 r_n^2} = \frac{1}{n^4}Z^3\alpha^3\frac{m^2c^3}{e\hbar}$$

$$B_n = \frac{1}{2}\mu_0\frac{I_n}{r_n} = \frac{1}{2\varepsilon_0 c^2}\cdot\frac{ev_n}{2\pi r_n}\cdot\frac{1}{r_n} = \frac{1}{n^5}Z^3\alpha^4\cdot\frac{m^2c^2}{e\hbar}$$

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

$$I_n = \frac{e}{T} = \frac{e\omega_n}{2\pi} = \frac{ev_n}{2\pi r_n}$$

FOTOEFEKT



300 nm

$\to \frac{2}{3} \cdot 6 \doteq 4\,eV$

200 nm

$\to 6\,eV$

Fotoelektrický jev bude viditelný pouze v této vlnové délce

energie $\dfrac{2\pi\hbar c}{\lambda}$

$A \to A + eU \qquad U \approx 1{,}5\,V$

$Q = CU = 4\pi\varepsilon_0 a = 10^{-11}\,C$

Druhý zdroj elektrony uvolnit nemůže.

$$N_f = \frac{100 \cdot Q}{e} = \frac{100 \cdot 10^{-11}}{1{,}6 \cdot 10^{-19}} \doteq 10^{10}$$

$$E_f = N_f \cdot 6 \cdot eV = 10^{10} \cdot 6 \cdot 1{,}6 \cdot 10^{-19} = 10^{-8}\,J$$

$$P = \frac{E_f}{\tau} = \frac{10^{-8}}{10^{-3}} = 10^{-5}\,W$$

de BROGLIEOVA VLNOVÁ DÉLKA

$$\lambda_{dB} = \frac{2\pi\hbar}{p}$$

$$E = \frac{1}{2}mv^2 = eU \qquad mv^2 = 2eU$$

$$p^2 = \left(\frac{E}{c}\right)^2 - m^2c^2$$

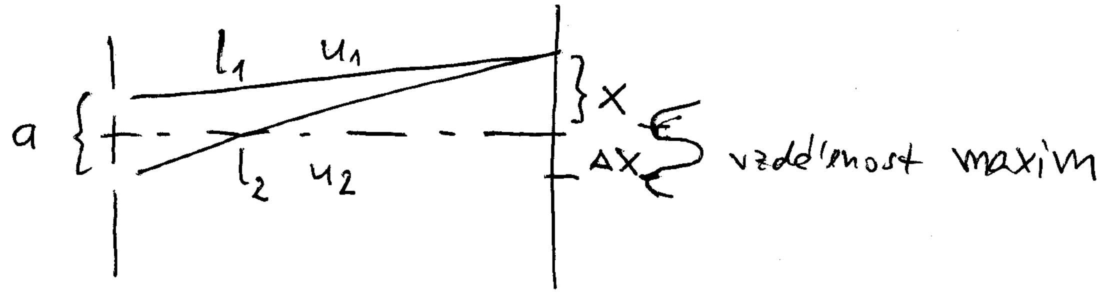
$$E = m \cdot c^2 + e \cdot U$$

$$p^2 = 2me \cdot U + \left(\frac{eU}{c}\right)^2$$

$$p = \sqrt{2meU + \frac{e^2U^2}{c^2}} = \sqrt{2meU} \cdot \sqrt{1 + \frac{eU}{2mc^2}}$$

$$\lambda_{dB} = \frac{2\pi\hbar}{\sqrt{2meU + \frac{e^2U^2}{c^2}}} = \frac{2\pi\hbar}{\sqrt{2meU}} \cdot \frac{1}{\sqrt{1 + \frac{eU}{2mc^2}}}$$

INTERFERENCE



$$u_1+u_2=\frac{A\cdot e^{ikl_1}}{l_1}+\frac{A\cdot e^{ikl_2}}{l_2}=A\cdot\left(\frac{e^{ikl_1}}{l_1}+\frac{e^{ikl_2}}{l_2}\right)$$

$$I=|u|^2=\frac{A^2}{l^2}\cdot\left|e^{ikl_1}+e^{ikl_2}\right|^2$$

$$l_2-l_1=\sqrt{l^2+\left(x+\frac{a}{2}\right)^2}-\sqrt{l^2+\left(x-\frac{a}{2}\right)^2}=a\cdot\frac{x}{l}$$

$$I=\frac{2A^2}{l^2}\cdot\left(1+\cos\left(ak\frac{x}{l}\right)\right)$$

$$\frac{ak\,\Delta x}{l}=2\pi \qquad \Delta x=\frac{2\pi l}{ak}$$

$$E=\frac{p^2}{2m}=\frac{\hbar^2k^2}{2m} \qquad k=\frac{2\pi l}{a\Delta x}$$

$$E=\frac{\hbar^2}{2m}\cdot\left(\frac{2\pi l}{a\Delta x}\right)^2$$

NEKONEČNÁ JÁMA

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}(x) \qquad V(x) = 0 \quad x \in \langle 0, L \rangle$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad V(x) = \infty \quad x \notin \langle 0, L \rangle$$

$$\hat{H} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} + V(x)$$

$$\hat{H}\psi = E\psi$$

$$-\frac{\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + V(x) \cdot \psi = E\psi$$

$$-\frac{\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + \infty \cdot \psi = E\psi \qquad \text{vně: } \psi = 0$$

uvnitř: $-\frac{\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + 0 \cdot \psi = E\psi$

$$\psi'' + k^2\psi = 0 \qquad -\frac{2m}{\hbar^2} = -k^2$$

$$\psi = A \cdot \sin kx + B \cdot \cos kx$$

$$\psi_n = A \cdot \sin \frac{n\pi x}{l} \qquad k = \frac{2\pi}{\lambda}$$

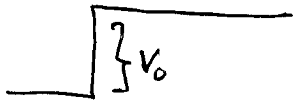
$$\frac{2\pi}{\lambda} \cdot L = n\pi \qquad L = n\frac{\lambda}{2}$$

$$\frac{2mE}{\hbar^2} = k^2 \qquad E = \frac{\hbar^2 k^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

POTENCIÁLOVÝ (ENERGETICKÝ) SCHOD

SCHRÖDINGEROVA ROVNICE

$$-\frac{\hbar^2}{2m}\cdot\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi$$



$$V(x)=\begin{cases}0 & x<0\\ V_0 & x>0\end{cases}$$

$E>V_0$: vlevo $\psi''+\frac{2mE}{\hbar^2}\psi=0$

$$\psi=A\cdot e^{ikx}+B\cdot e^{-ikx}$$

vpravo $\psi''+\frac{2m(E-V_0)}{\hbar^2}\psi=0$

$$\psi=C\cdot e^{ik'x}+D\cdot e^{-ik'x}$$

$$\hat{P}_x e^{ikx}=i\hbar\frac{\partial}{\partial x}e^{ikx}=\hbar k e^{ikx} \longrightarrow$$

$$\hat{P}_x e^{-ikx}=-i\hbar\frac{\partial}{\partial x}e^{-ikx}=\hbar k e^{-ikx} \longleftarrow$$

$$\psi_L(0)=\psi_P(0)$$

$1+r=t$ podmínka na spojitost

$$\psi_L'(0)=\psi_P'(0)$$

$$ik(1-r)=ik't=k'(1+r)$$

$$k-k'=r(k+k') \qquad r=\frac{k-k'}{k+k'} \qquad t=\frac{2k}{k+k'}$$

$$R=|r|^2=\frac{(k-k')^2}{(k+k')^2} \qquad T=\frac{k'}{k}|t|^2=\frac{k'}{k}\cdot\frac{4k^2}{(k+k')^2}$$

$$R+T=\frac{(k-k')^2+4k\cdot k'}{(k+k')^2}=1$$

$E<V_0$ vlevo stejně

$$\psi''+\frac{2m(E-V_0)}{\hbar^2}\psi=0$$

$$(ik'')^2=-k''^2$$

$$\psi_{vpravo}=t\cdot e^{ik'x}=\frac{2k}{k+ik''}\cdot e^{-k''x}$$

ODHAD ENERGIE ZÁKL. STAVU HARM. OSCILÁTORU

$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\langle x \rangle = 0$$
$$\langle p \rangle = 0$$

platí: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

$$\Delta p = \frac{\hbar}{2L}$$

$$E = \frac{\hbar^2}{8mL^2} + \frac{1}{2} m\omega^2 L^2$$

min. energie

$$\frac{\hbar^2}{8m}\left(\frac{1}{L^2} + \frac{4m^2\omega^2}{\hbar^2} L^2\right) = \frac{\hbar^2}{8m} \cdot \left[\left(\frac{1}{L} - \frac{2m\omega}{\hbar} \cdot L\right)^2 + \frac{4m\omega}{\hbar}\right] =$$

$$= \frac{\hbar^2}{8m}\left(\frac{1}{L} - \frac{2m\omega}{\hbar} L\right)^2 + \frac{\hbar\omega}{2}$$

$$E_{min} = \frac{1}{2}\hbar\omega$$

při $\frac{1}{L} = \frac{2m\omega}{\hbar} L$

$$\frac{\hbar}{2m\omega} = L^2 \quad \Rightarrow \quad L = \sqrt{\frac{\hbar}{2m\omega}}$$

ODHAD ENERGIE ATOMU VODÍKU

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$r \approx \Delta r = a$$

$$p \approx \Delta p = \frac{\hbar}{\Delta r} = \frac{\hbar}{a}$$

$$E(a) = \frac{\hbar^2}{2ma^2} - \frac{e^2}{4\pi\varepsilon_0 a}$$

$$\min E'(a) = 0 = -\frac{\hbar^2}{ma^3} + \frac{e^2}{4\pi\varepsilon_0 a^2}$$

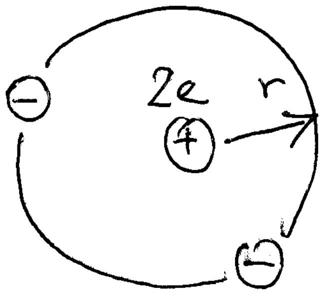
$$a = \frac{4\pi\varepsilon_0}{e^2} \cdot \frac{\hbar^2}{m} = \frac{4\pi\varepsilon_0 \hbar c}{e^2} \cdot \frac{\hbar}{mc} = a_B$$

$$E(a) = \frac{\hbar^2}{2m}\left(\frac{m}{\hbar^2} \cdot \frac{e^2}{4\pi\varepsilon_0}\right)^2 - \frac{e^2}{4\pi\varepsilon_0}\left(\frac{m}{\hbar^2} \cdot \frac{e^2}{4\pi\varepsilon_0}\right)$$

$$= \frac{1}{2} m \left(\frac{e^2}{4\pi\varepsilon_0 \hbar}\right)^2 - m\left(\frac{e^2}{\hbar\, 4\pi\varepsilon_0}\right)^2$$

$$= -\frac{1}{2}\underbrace{\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2}_{\alpha^2} \cdot mc^2 = -R_y$$

ATOM He – ZÁKL. STAV



$r \approx R$ $\qquad$ R_{12} střední vzdál. e

$$E = 2\cdot\frac{\hbar^2}{2mR^2} - 2\cdot\frac{2e^2}{4\pi\varepsilon_0 R} + \frac{e^2}{4\pi\varepsilon_0 R_{12}}$$

$$R_{12} = 2R$$

$$0 = \frac{dE}{dR} = -2\frac{\hbar^2}{mR^3} = \frac{\hbar^2}{mc}\cdot\frac{4\pi\varepsilon_0\hbar c}{e^2}\cdot\frac{4}{7}$$

$$\cancel{E = \frac{\hbar^2}{m}}$$

$$R = \frac{2\hbar^2}{m}\cdot\frac{2}{7}\cdot\frac{4\pi\varepsilon_0}{e^2} = \frac{\hbar^2}{mc}\cdot\underbrace{\frac{4\pi\varepsilon_0\hbar c}{e^2}}_{\frac{1}{\alpha}}\cdot\frac{4}{7} = \frac{4}{7}a_B$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

$$E = \frac{\hbar^2}{m}\cdot\frac{49}{16}\cdot\alpha^2\cdot\frac{m^2c^2}{\hbar^2} = \frac{7}{2}\cdot\frac{e^2}{4\pi\varepsilon_0 c}\cdot\frac{7}{4}\cdot\alpha\cdot\frac{mc^2}{\hbar} =$$

$$= \frac{49}{16}\alpha^2 mc^2 - \frac{49}{8}\alpha^2 mc^2 = -\frac{49}{16}\alpha^2\cdot mc^2 = -\frac{49}{8}\cdot R_y = -83\,\mathrm{eV}$$

ENERGIE MOLEKUL

$$k = \frac{2R_y}{a_B^2} = \frac{\alpha^2 \cdot m \cdot c^2}{\left(\frac{\hbar}{\alpha \cdot m c}\right)^2} = \frac{\alpha^4 m^3 c^4}{\hbar^2}$$

$$\hbar\omega = \hbar\sqrt{\frac{k}{M}} = \alpha^2 mc^2 \sqrt{\frac{m}{M}} \approx \sqrt{\frac{m}{M}}\, R_y$$

$$E_k = \frac{1}{2} M_1 \frac{d^2 r_1}{dt^2} + \frac{1}{2} M_2 \frac{d^2 r_2}{dt^2}$$

$$E_k = \frac{1}{2} \underbrace{M_1 \frac{M_2}{M_1+M_2}}_{M} \cdot \frac{d^2 \Delta r}{dt^2}$$

$$E_{rot} = \frac{1}{2} M_1 r_1^2 \cdot \Omega^2 + \frac{1}{2} M_2 r_2^2 \cdot \Omega^2$$

$$E_{rot} = \frac{1}{2} I \Omega^2 \qquad I = M_1 \cdot r_1^2 + M_2 r_2^2 = M \cdot r^2$$

$$L = M_1 r_1^2 \cdot \Omega + M_1 r_2^2 \Omega = I\Omega$$

$$E_{rot} = \frac{L^2}{2I}$$

$$E_{rot} = \frac{\hbar^2}{2I}$$

ENERGIE NABITÉ KULIČKY

$$E\cdot 4\pi r^2=\frac{1}{\varepsilon_0}\cdot Q\cdot\frac{r^3}{R^3} \qquad \ldots r<R$$

$$E\cdot 4\pi r^2=\frac{1}{\varepsilon_0}\cdot Q \qquad \ldots r>R$$

$$E=\frac{1}{4\pi\varepsilon_0}\cdot Q\cdot\frac{r}{R_3} \qquad \ldots r<R$$

$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} \qquad \ldots r>R$$

$$W=\iiint\frac{1}{2}\varepsilon_0\cdot E^2\,dV$$

$$W=\int_0^R\frac{1}{2}\cdot\varepsilon_0\cdot\frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\frac{r^2}{R^6}\cdot 4\pi\cdot r^2\,dr+\int_R^\infty\frac{1}{2}\varepsilon_0\cdot\frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\frac{1}{r^4}\cdot 4\pi r^2\,dr=$$

$$=\frac{1}{10}\cdot\frac{Q^2}{4\pi\varepsilon_0 R}+\frac{1}{2}\cdot\frac{Q^2}{4\pi\varepsilon_0 R}=\frac{3}{5}\cdot\frac{Q^2}{4\pi\varepsilon_0\cdot R}$$

$$\frac{3}{5}\cdot\frac{e^2}{4\pi\varepsilon_0 R}=m_e\cdot c^2 \qquad \Rightarrow \qquad R=\frac{3}{5}\cdot\frac{e^2}{4\pi\varepsilon_0 m_e c^2}$$

NÁBOJ = f(HMOTNOSTNÍ ČÍSLO)

$$M(A,Z) = Z \cdot M_p + (A-Z) \cdot M_n - W(A,Z)$$

$$W(A,Z) = \alpha \cdot A - \beta A^{\frac{2}{3}} - \gamma \cdot Z^2 \cdot A^{-\frac{1}{3}} - \sigma \cdot \left(\frac{A}{2} - Z\right)^2 \cdot A^{-1}$$

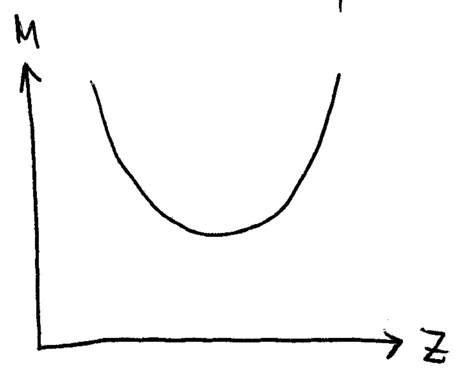
$$M(A,Z) = \min$$

$$0 = \frac{\partial M}{\partial Z} = M_p - M_n + 2\gamma \cdot Z \cdot A^{-\frac{1}{3}} - 2\sigma \cdot \left(\frac{A}{2} - Z\right) \cdot A^{-1}$$

$$Z \cdot \left(2\gamma \cdot A^{-\frac{1}{3}} + 2\sigma \cdot A^{-1}\right) = M_n - M_p + \sigma$$

$$Z = \frac{M_n - M_p + \sigma}{2\sigma \cdot A^{-1} + 2\gamma \cdot A^{-\frac{1}{3}}} = \frac{A}{\frac{2\sigma}{M_n - M_p + \sigma} + \frac{2\gamma}{M_n - M_p + \sigma} \cdot A^{\frac{2}{3}}}$$

pro liché A: 1 parabola



pro sudá A: 2 paraboly

SEKULÁRNÍ ROVNOVÁHA

$1g\ ^{226}Ra$ v sekulární rovnováze $6{,}5\ \mu g\ ^{222}Rn$

$$\frac{dn_1}{dt} = -\lambda_1 n_1$$

$$\frac{dn_2}{dt} = -\lambda_1 n_2 + \lambda_1 n_1$$

$$n_1 = n_{10} e^{-\lambda_1 t}$$

$$\underbrace{\frac{dn_2}{dt} + \lambda_2 n_2}_{=0} = \lambda_1 n_{10} e^{-\lambda_1 t}$$

$n_2 = Ce^{-\lambda_2 t}$ + part.

$$Ae^{-\lambda_1 t}$$

$$-\lambda_1 \cancel{Ae^{-\lambda_1 t}} + \lambda_2 \cancel{Ae^{-\lambda_1 t}} = \lambda_1 n_{10} e^{-\lambda_1 t}$$

$$A = \frac{\lambda_1 n_{10}}{\lambda_2 - \lambda_1}$$

$$n_2 = Ce^{-\lambda_2 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} n_{10} e^{-\lambda_1 t}$$

$$n_{20} = C + \frac{\lambda_1}{\lambda_2 - \lambda_1} \cdot n_{10} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$\frac{n_2}{n_1} = \frac{n_{20}}{n_{10}} \cdot e^{(\lambda_1 - \lambda_2)t} + \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(1 - e^{(\lambda_1 - \lambda_2)t}\right)$$

$$\lambda_2 > \lambda_1 \qquad \frac{n_2}{n_1} \to \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

STATISTIKA ROZPADŮ

Binomické rozdělení

$$P_n = \binom{N}{n} \cdot p^n \cdot (1-p)^{N-n}$$

$N \to \infty$

$$P_n \approx \frac{N^n p^n}{n!} \cdot (1-p)^{\frac{Np}{p}}$$

$N \cdot p \to \mu$ $\qquad \frac{n^n}{n!}\left[(1-p)^{\frac{1}{p}}\right]^{\mu}$

$(1-p)^{\frac{1}{p}}$ $\qquad p = \frac{1}{k}$

$$\left(1-\frac{1}{k}\right)^k \to \frac{1}{e}$$

$$\left(1+\frac{1}{k}\right)^k \to e$$

$P_n = \frac{\mu^n}{n!} \cdot e^{-\mu}$ $\qquad$ Poissonovo rozdělení

$$\langle n \rangle = \sum n P_n$$

$$\langle n \rangle = \sum_0^{\infty} n \cdot \frac{\mu^n}{n!} \cdot e^{-\mu} = \mu e^{-\mu} \sum_1^{\infty} \frac{\mu^{n-1}}{(n-1)!} = \mu$$

$$\sigma^2 = \langle n^2 \rangle - \langle n \rangle^2 - \mu^2$$

$\langle (n-\mu)^2 \rangle$ $\qquad \langle n(n-1) \rangle + \langle n \rangle$

$$\sigma^2 = \mu$$