AJFY 2013/14
příklady na zkoušku

1 Tři odhady rozměru atomu
2 Bloudění
3 Porovnání sil
4 Nestabilita atomu
5 Stav 2s vodíku
6 Slaterova pravidla
7 Slupkový model
8 Zákony zachování
9 Kvarkové složení
10 Energie ze slunečních článků
11 Energie ze štěpení
12 Podmínka štěpení
13 Model moderování
14 Kinetika štěpné reakce
15 Energie z deuteria

ODHADY ROZMĚRU ATOMU

a) 1 kapka 0,5 % ⊙

$\varnothing$ 32 cm

$V_{kap.} = 0{,}02\ \mathrm{cm}^3$

$V_{olej} = 5\cdot 10^{-3}\cdot 0{,}02\ \mathrm{cm}^3 = 10^{-4}\ \mathrm{cm}^3 = Sd = \frac{\pi D^2}{4}\cdot d$

$d = \frac{4\cdot V_{olej}}{\pi D^2} = \frac{4\cdot 10^{-4}}{3{,}14\cdot 32^2} = 1{,}2\cdot 10^{-7}\ \mathrm{cm} = 1{,}2\cdot 10^{-9}\ \mathrm{m}$

b) výparné teplo 2,1 MJ/kg

$\sigma = 72\ \mathrm{mJ/m^2}$

$1\ \mathrm{m}^3 \ldots Q V \varrho$

$6a^2 \frac{1}{a^3} = \frac{6}{d}$

$d = \frac{6\sigma}{Q\varrho} = \frac{6\cdot 72\cdot 10^{-3}}{2\cdot 1\cdot 10^{9}} = 2\cdot 10^{-10}\ \mathrm{m}$

c) 0,1 mg $^{210}\mathrm{Po}$

aktivita $1{,}67\cdot 10^{10}$ Bq

str. doba $1{,}7\cdot 10^{7}$ s

$9400\ \mathrm{kg/m^3}$

$A = \left|\frac{dn}{dt}\right| = +\frac{n}{\tau}$

$n = A\tau$

$V_1 \approx d^3 = \frac{V}{n} = \frac{m}{\varrho n}$

$d = \sqrt[3]{\frac{m}{\varrho n}} = \sqrt[3]{\frac{10^{-7}}{10^4\cdot 1{,}67\cdot 10^{10}\cdot 1{,}7\cdot 10^{7}}} = 3{,}3\cdot 10^{-10}\ \mathrm{m}$

BLOUDĚNÍ

$$\langle \vec{R}_n^{\,2} \rangle = \langle (\sum_1^n \vec{r_i})^2 \rangle = \langle \sum r_i^{\,2} \rangle + \langle \sum r_i \cdot r_j \rangle = n \cdot l^2$$

$$\sqrt{\langle R_n^{\,2} \rangle} = \sqrt{n} \cdot l$$

$$n = \frac{t}{\tau} = \frac{\langle v \rangle \cdot t}{l}$$

$$\text{vzdál.} = \sqrt{\langle v \rangle} \cdot \sqrt{l} \cdot \sqrt{t}$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}}$$

$$V = \pi d^2 \cdot \langle v_{rel} \rangle$$

$$\tau = \frac{1}{\nu \pi d^2 \langle v_{rel} \rangle}$$

$$l = \langle v \rangle \cdot \tau = \frac{\langle v \rangle}{\langle v_{rel} \rangle \cdot \nu \pi d^2}$$

$$l = \frac{1}{\sqrt{2}\, \nu \pi d^2}$$

$$\text{vzd} = \sqrt[4]{\frac{8 k_B T}{\pi \cdot m}} \cdot \frac{1}{\sqrt[4]{2} \cdot \sqrt{n \pi d^2}} \cdot \sqrt{t}$$

POROVNÁNÍ SIL

$$F_{gr} = \varkappa \cdot \frac{mM}{r^2} \quad , \quad F_{el} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

$$F_{mag} = e\left|\vec{v} \times \vec{B}\right| = \frac{\mu_0}{4\pi} \cdot \frac{e^2}{r^2} \cdot \left|\vec{v} \times \left(\vec{V} \times \vec{r}_0\right)\right|$$

$$\frac{F_{mag}}{F_{el}} = v \cdot V \cdot \varepsilon_0 \cdot \mu_0 = \frac{v \cdot V}{c^2}$$

$$mv = MV$$

$$\frac{F_{mag}}{F_{el}} = \frac{m \cdot v^2}{M \cdot c^2}$$

$$\frac{F_{gr}}{F_{el}} = \frac{\varkappa Mm}{e^2} \cdot 4\pi\varepsilon_0$$

Gravitační síla je o 40 řádů slabší, než elektrická síla.

NESTABILITA ATOMU

$$ma = m \cdot \frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

$$E = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{2} \cdot \frac{e^2}{4\pi\varepsilon_0 r}$$

Lamor $\quad \dfrac{dE}{dt} = -\dfrac{2}{3} \cdot \dfrac{e^2 \cdot a^2}{4\pi\varepsilon_0 c^3}$

$$\frac{dE}{dt} = \frac{1}{2} \cdot \frac{e^2}{4\pi\varepsilon_0 r^2} \frac{dr}{dt} = -\frac{2}{3} \cdot \frac{e^2}{4\pi\varepsilon_0 c^3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0 m r^2} \right)^2$$

$$r^2 \frac{dr}{dt} = -\frac{4}{3} \cdot \frac{1}{c^3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0 m} \right)^2$$

$$\int r^2 dr = -\frac{4}{3c^3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0 m} \right)^2 \int dt$$

$$\left[\frac{r^3}{\cancel{3}} \right]_{r_{at}}^{r_j} = -\frac{4}{\cancel{3} m^2 c^3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \cdot \underbrace{(t - t_0)}_{\Delta t}$$

$$r_j^3 = r_{at}^3 - \frac{4}{m^2 c^3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \cdot \Delta t$$

zanedb.

$$\Delta t = \frac{m^2 c^3}{4} \cdot \left(\frac{4\pi\varepsilon_0}{e^2} \right)^2 \cdot r_{at}^3$$

STAV 2s VODÍKU

$$\Phi_{2s} = \frac{1}{\sqrt{32\pi a_B^3}} \cdot \left(2 - \frac{r}{a_B}\right) \cdot e^{-\frac{r}{2a_B}}$$

$$p \sim |\Phi|^2 \cdot 4\pi r^2 \, dr$$

$$p \sim r^2 \left(2 - \frac{r}{a_B}\right)^2 \cdot e^{-\frac{r}{a_B}}$$

$$\frac{dp}{dr} = 2r\left(2 - \frac{r}{a_B}\right)^2 \cdot e^{-\frac{r}{a_B}}$$

$$- \frac{2r^2}{a_B} \cdot \left(2 - \frac{r}{a_B}\right) \cdot e^{-\frac{r}{a_B}}$$

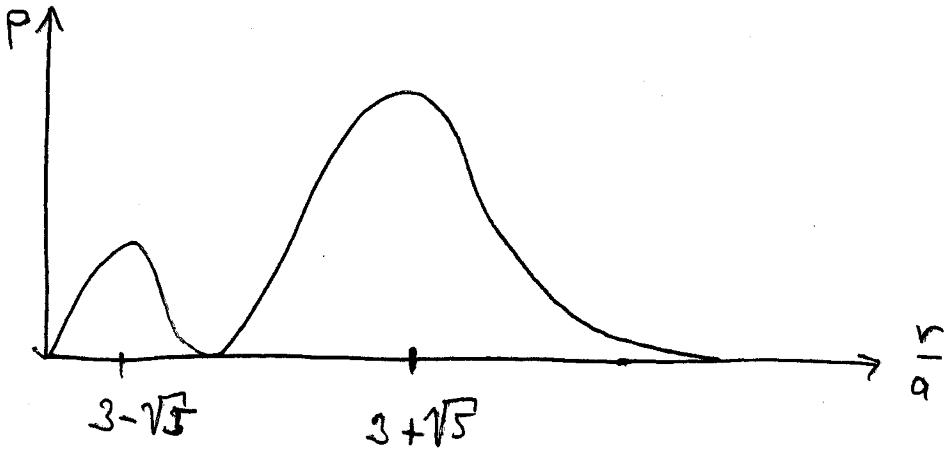
$$- \frac{r^2}{a_B} \cdot \left(2 - \frac{r}{a_B}\right)^2 \cdot e^{-\frac{r}{a_B}}$$

$$= r \cdot \left(2 - \frac{r}{a_B}\right) \cdot \left(4 - \frac{6r}{a_B} + \frac{r^2}{a_B^2}\right) \cdot e^{-\frac{r}{a_B}}$$

$r_1 = 0$, $r_2 = 3 - \sqrt{5}$, $r_3 = 2$, $r_4 = 3 + \sqrt{5}$, $r_5 = \infty$

$$\left(\frac{r}{a_B}\right)^2 - 6\left(\frac{r}{a_B}\right) + 4 = 0$$

$$\frac{r}{a_B} = 3 \pm \sqrt{9 - 4}$$



SLATEROVA PRAVIDLA

$$E = \frac{(Z-\sigma)^2}{n^{*2}} \cdot R_y$$

He I

$Z = 2$ - $1s^2$

$\sigma = 0{,}3$ $\quad Z - \sigma = 1{,}7$

$n^* = 1$

$$E_1 = \frac{1{,}7^2}{1^2} \cdot R_y = 2{,}89\,R_y$$

$$E(\text{He I}) = 2E_1 = 5{,}78\,R_y$$

He II

1 el $\quad \sigma = 0$

$$E(\text{He II}) = \frac{2^2}{1^2} R_y = 4\,R_y$$

$$E_i = E(\text{He I}) - E(\text{He II}) = 1{,}78\,R_y = 24{,}20\text{ eV}$$

C I

$Z = 6 \quad 1s^2 (2s, 2p)^4$

$$2 \cdot \frac{(6-0{,}3)^2}{1^2} R_y = 2 \cdot 32{,}49\,R_y$$

$$4 \cdot \frac{(6 - 3 \cdot 0{,}35 - 2 \cdot 0{,}85)^2}{2^2} R_y = 4 \cdot 2{,}64\,R_y$$

$$E(\text{C I}) = 75{,}54\,R_y$$

C II

$Z = 6 \quad 1s^2, (2s, 2p)^3$

$$2 \cdot \frac{(6-0{,}3)^2}{1^2} R_y = 2 \cdot 32{,}49\,R_y$$

$$3 \cdot \frac{(6 - 2 \cdot 0{,}35 - 2 \cdot 0{,}85)^2}{2^2} = 3 \cdot 3{,}24\,R_y$$

$$E(\text{C II}) = 74{,}7\,R_y$$

$$E_i = E(\text{C I}) - E(\text{C II}) = 0{,}84\,R_y = 11{,}42\text{ eV}$$

SLUPKOVÝ MODEL

$$V = \frac{1}{2} k r^2 \qquad r^2 \searrow x^2 + y^2 + z^2$$

1D oscilátor $\quad E_n = \hbar\omega\left(n + \frac{1}{2}\right)$

3D oscilátor $\quad E(n_x, n_y, n_z) = \hbar\omega\left(n_x + n_y + n_z + \frac{3}{2}\right) = \hbar\omega\left(2n_r + l + \frac{3}{2}\right)$

$$V_{so} = -\frac{a}{\hbar^2}\hat{\vec{s}}\cdot\hat{\vec{l}}$$

$$\hat{\vec{j}} = \hat{\vec{l}} + \hat{\vec{s}}$$

$$\hat{\vec{j}}^2 = \hat{\vec{l}}^2 + \hat{\vec{s}}^2 + 2\hat{\vec{s}}\cdot\hat{\vec{l}}$$

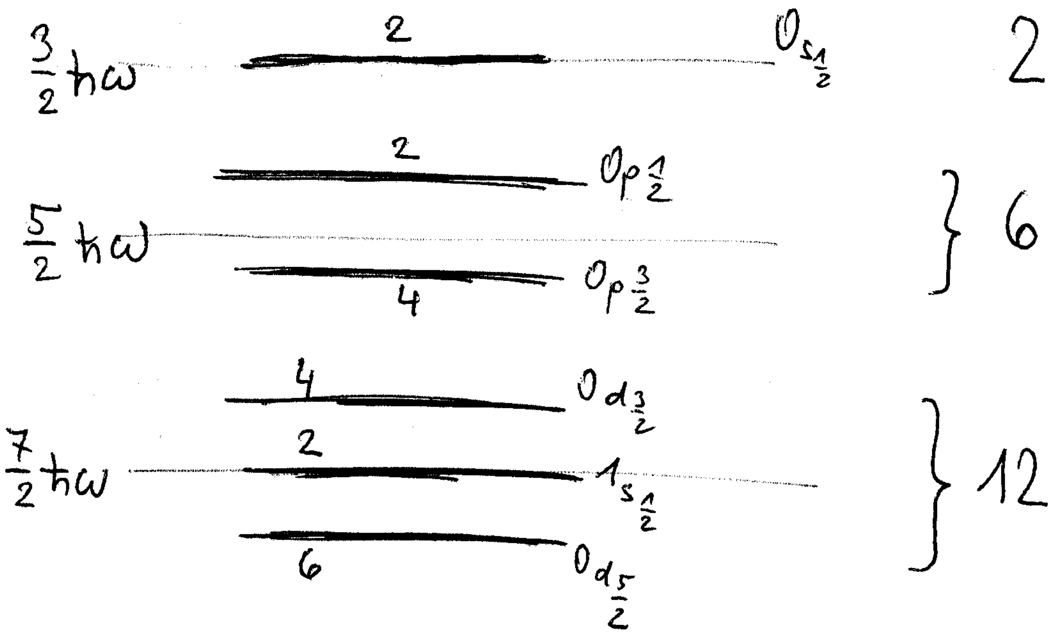
$$\frac{\hat{\vec{s}}\cdot\hat{\vec{l}}}{\hbar^2} = \frac{1}{2}\left(\vec{j}^2 - \vec{l}^2 - \vec{s}^2\right)$$

$$j(j+1) - l(l+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)$$

$$E = \hbar\omega\left(2n_r + l + \frac{3}{2}\right)$$

$$-\frac{a}{2}\left(j(j+1) - l(l+1) - \frac{3}{4}\right)$$

$a \ll \hbar\omega$



ZÁKONY ZACHOVÁNÍ

	NENÍ ZACHOVÁNO:
$p^+ \longrightarrow e^- + \tilde{\nu}_e$	náboj, baryonové č.
$n \longrightarrow p^+ + e^-$	leptonové číslo
$p^+ \longrightarrow e^+ + \pi^0$	leptonové číslo, baryonové č.
$n \longrightarrow p^+ + e^- + \nu_e$	leptonové číslo
$n \longrightarrow p^+ + e^- + \tilde{\nu}_\mu$	— leptonové
$p^+ \longrightarrow n + e^+ + \nu_e$	energie

$\Longrightarrow$ Nemůže nastat ani jeden proces

KVARKOVÉ SLOŽENÍ

BARYONY: 3 kvarky

hyperon Ω^- = (sss)

proton p = (uud)

neutron n = (udd)

MEZONY: kvark–antikvark

kaon K^+: $(u\tilde{s})$ $\cancel{(c\tilde{s})}$

$K^+ = (u\tilde{s})$

pion π^+: $(u\tilde{d})$ $\cancel{(u\tilde{s})}$ $\cancel{(c\tilde{d})}$ $\cancel{(c\tilde{s})}$

$\pi^+ = (u\tilde{d})$

mezon $\frac{J}{\psi}$: $(u\tilde{d})$ $(c\tilde{c})$ $(d\tilde{d})$ $(s\tilde{s})$ $\cancel{(u\tilde{c})}$ $\cancel{(c\tilde{u})}$ $\cancel{(d\tilde{s})}$ $\cancel{(s\tilde{d})}$

$\frac{J}{\psi} = (c\tilde{c})$

ENERGIE ZE SLUNEČNÍCH ČLÁNKŮ

Odhad

$$P_1 = \frac{1}{2}\eta E = \frac{1}{2}\cdot 0{,}20\cdot 300 = 30\ W/m^2$$

$$S = \frac{P_T}{P_1} = \frac{P_T}{\frac{1}{2}\eta E} = \frac{10^9}{30} = 33\cdot 10^6\ m^2 = \underline{\underline{33\ km^2}}$$

FOTOVOLTAIKA: $\frac{20\,000}{30} = 666{,}7$ Kč/W

TEMELÍN: 100 Kč/W

$$\frac{666{,}7}{100} = \underline{\underline{6{,}7}} \Rightarrow$$ Investice víc než 6 Temelínů

energie z 1 jádra ^{235}U: $E = 200$ MeV

ENERGIE ZE ŠTĚPENÍ

energie z 1 kg ^{235}U:

$$E_1 = \frac{N_A}{A_U} \cdot E \cdot e = \frac{6 \cdot 10^{23}}{0{,}235} \cdot 200 \cdot 10^6 \cdot 1{,}6 \cdot 10^{-19} = 8{,}2 \cdot 10^{13}\ \mathrm{J}$$

potřebná energie 1000 MW elny:

$$E_r = 365{,}25 \cdot 86400 \cdot 1000 \cdot 10^6\ \mathrm{J} = 3{,}2 \cdot 10^{16}\ \mathrm{J}$$

spotřeba uranu:

$$m_u = \frac{E_r}{\eta \cdot E_1} = \frac{3{,}2 \cdot 10^{16}}{0{,}30 \cdot 8{,}2 \cdot 10^{13}} = 1300\ \mathrm{kg}$$

spotřeba nafty:

$$m_n = \frac{E_r}{3\eta \cdot \frac{E_1}{200\,000}} \doteq 80 \cdot 10^6\ \mathrm{kg}$$

potřebná energie jaderné pumy:

$$E' = 0{,}5 \cdot 10^6 \cdot 4 \cdot 10^9 = 2 \cdot 10^{15}\ \mathrm{J}$$

spotřeba uranu na jadernou pumu:

$$m_p = \frac{E'}{E_1} = \frac{2 \cdot 10^{15}}{8{,}2 \cdot 10^{13}} = 24{,}4\ \mathrm{kg}$$

PODMÍNKA ŠTĚPENÍ

$$W(A,Z) = \alpha A - \beta A^{\frac{2}{3}} - \gamma \frac{Z^2}{A^{\frac{1}{3}}} - \sigma \frac{\left(\frac{A}{2}-Z\right)^2}{A}$$

$$M(A,Z) > M(A_1, Z_1) + M(A_2, Z_2)$$

$$A = A_1 + A_2, \quad Z = Z_1 + Z_2$$

$$M(A,Z) = ZM_p + (A-Z)M_n - W(A,Z)$$

$$W(A,Z) < W(A_1, Z_1) + W(A_2, Z_2)$$

$$\beta A^{\frac{2}{3}} + \gamma \frac{Z^2}{A^{\frac{1}{3}}} + \sigma \frac{\left(\frac{A}{2}-Z\right)^2}{A} > \beta A_1^{\frac{2}{3}} + \gamma \frac{Z_1^2}{A_1^{\frac{1}{3}}} + \sigma \frac{\left(\frac{A_1}{2}-Z_1\right)^2}{A_1}$$

$$+ \beta (A_2^{\frac{2}{3}} + \gamma \frac{Z_2^2}{A_2^{\frac{1}{3}}} + \sigma \frac{\left(\frac{A_2}{2}-Z_2\right)^2}{A_2}$$

$$A_1 = \frac{3}{5}A; \qquad A_2 = \frac{2}{5}A$$

$$Z_1 = \frac{3}{5}Z; \qquad Z_2 = \frac{2}{5}Z$$

$$\beta A^{\frac{2}{3}}\left[1 - \left(\frac{3}{5}\right)^{\frac{2}{3}} - \left(\frac{2}{5}\right)^{\frac{2}{3}}\right] > \gamma \frac{Z^2}{A^{\frac{1}{3}}}\left[\left(\frac{3}{5}\right)^{\frac{5}{3}} + \left(\frac{2}{5}\right)^{\frac{5}{3}} - 1\right]$$

$$\frac{Z^2}{A} > \frac{\beta}{\gamma} \cdot \frac{\left[\left(\frac{3}{5}\right)^{\frac{2}{3}} + \left(\frac{2}{5}\right)^{\frac{2}{3}} - 1\right]}{\left[1 - \left(\frac{3}{5}\right)^{\frac{5}{3}} - \left(\frac{2}{5}\right)^{\frac{5}{3}}\right]}$$

MODEROVÁNÍ

před srážkou $\xrightarrow{v_0}$ O

po srážce $\xleftarrow{v_1}$ $O \xrightarrow{V}$

$$\text{I.}\quad \frac{1}{2}v_0^2 = \frac{1}{2}v_1^2 + \frac{1}{2}AV^2$$

$$\text{II.}\quad v_0 = v_1 + AV$$

$$\text{I.}\quad v_0^2 - v_1^2 = AV^2$$

$$\text{II.}\quad v_0 - v_1 = AV$$

$$V = \frac{v_0^2 - v_1^2}{v_0 - v_1} = v_0 + v_1$$

$$v_0 - v_1 = A(v_0 + v_1)$$

$$v_1(A+1) = -v_0(-1+A)$$

$$v_1 = -v_0 \cdot \frac{A-1}{A+1}$$

$$T_1 = \left(\frac{A-1}{A+1}\right)^2 \cdot T_0$$

$$T_n = \left(\frac{A-1}{A+1}\right)^{2n} \cdot T_0$$

$$\shortparallel$$

$$T_f$$

$$\left(\frac{A-1}{A+1}\right)^{2n} = \frac{T_f}{T_0}$$

$$2n \ln \frac{A+1}{A-1} = \ln \frac{T_0}{T_f}$$

$$n = \frac{1}{2}\,\frac{\ln \frac{T_0}{T_f}}{\ln \frac{A+1}{A-1}}$$

KINETIKA ŠTĚPNÉ REAKCE

$$dn = (K-1)\,n\,\frac{dt}{t}$$

$$n = n_0 \cdot e^{(K-1)\cdot\frac{t}{\tau}}$$

$$\frac{dn}{dt} = \frac{K\cdot(1-\beta)-1}{\tau}\cdot n + \frac{n_z}{\tau_z}$$

$$\frac{dn_z}{dt} = \beta\cdot\frac{K}{\tau}\cdot n - \frac{n_z}{\tau_z}$$

$$n = A\cdot e^{\alpha t}, \quad n_z = B\cdot e^{\alpha t}$$

$$\alpha A = \frac{K\cdot(1-\beta)-1}{\tau}\cdot A + \frac{1}{\tau_z}\cdot B$$

$$\alpha B = K\cdot\frac{\beta}{\tau}\cdot A - \frac{1}{\tau_z}\cdot B$$

$$\alpha^2 - \left[\frac{K(1-\beta)-1}{\tau} + \cancel{\frac{1}{\tau_z}}\right]\cdot\alpha - \frac{1}{\tau_z}\cdot\frac{K-1}{\tau} = 0$$

$$K = 1+\kappa$$

$$\alpha_{1,2} = -\left(\frac{\beta-\kappa}{2\tau}\right) \pm \sqrt{\left(\frac{\beta-\kappa}{2\tau}\right)^2 + \frac{\kappa}{\tau\cdot\tau_z}}$$

$$\alpha_1 = \frac{\kappa}{(\beta-\kappa)\cdot\tau_z}$$

$$\alpha_2 = -\frac{\beta-\kappa}{\tau} - \frac{\kappa}{(\beta-\kappa)\cdot\tau_z}$$

ENERGIE Z DEUTERIA

Odhad:

$$E = 28{,}3 - 2\cdot 2{,}22 = 23{,}9\ \text{MeV}$$

$$1000\cdot\frac{N_A}{A_D} = 500\cdot N_A$$

$$E_1 = 500\cdot\frac{N_A}{N_D}\cdot 23{,}9\ \text{MeV} = 2{,}4\cdot 10^{14}\ \text{J}$$

$$E_{rok} = 6\cdot 10^{9}\cdot 2\cdot 10^{3}\cdot 3{,}15\cdot 10^{7} = 3{,}8\cdot 10^{20}\ \text{J}$$

$$V = 0{,}6\cdot 4\pi R^2\cdot 3\cdot 10^{3} = 10^{18}\ \text{m}^3$$

$$M_D = 1{,}5\cdot 10^{-4}\cdot\frac{1}{9}\cdot 10^{3}\cdot 10^{18} = 1{,}7\cdot 10^{6}\ \text{kg}$$

$$E_D = \eta\cdot M_D\cdot E_1 = 10^{-2}\cdot 1{,}7\cdot 10^{16}\cdot 2{,}4\cdot 10^{14} = 4{,}1\cdot 10^{28}\ \text{J}$$

$$\frac{E_D}{E_{rok}} = 10^{8}\ \text{let}$$