Discrete Structures: 1.4-1.7 Small Review

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Section 1.4 Quick Review

In the statement "x is greater than 3," x is called a subject and "greater than 3" is called the predicate.
$\forall x$ means "for all x."
$\exists x$ means "there exists an x."
$P(x)$ is a propositional function that has an input x. Given this input, it will have a value based on if x satisfies the given predicate.

Example Problems for Section 1.4

Problem 1

Let $P(x)$ denote the statement "$x \leq 4$." What are these truth values?
a) $P(0)$
b) $P(4)$
c) $P(6)$

Solution

a) **T**
In this case, x would be 0, which is less than or equal to 4.
Therefore, the answer to this would be **T**.
b) **T**
c) **F**

Problem 2

Suppose that the domain of the propositional function $P(x)$ consists of the integers -2, -1, 0, 1, and 2. Write out the proposition using disjunctions, conjunctions, and negations.
$\exists x \neg P(x)$

Solution

$\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$
Here, $\exists x \neg P(x)$ means that "there exists an x such that $\neg P(x)$ is true." Therefore, it follows that at least one element in the domain satisfies $\neg P(x)$, which leads to the solution (where we use the "or" operator because we are looking for existence, which is true if one of the terms in the statement is true).

Problem 3

Translate the statement into logical expressions using predicates, quantifiers, and logical connectives.
One of your tools is not in the correct place, but it is in excellent condition.

Solution

$\exists x(\neg P(x) \wedge C(x))$ where x is a tool in the toolbox, $P(x)$ means that tool x is in the correct position, and $C(x)$ means that the tool is in excellent condition.
The key to this problem is utilizing the existential operator, as the problem is stating that "One of your

Problem 3 continued on next page. . .

tools" satisfies the conditions, implying the use of the existential operator. The rest then is similar to Section 1.1 where we translated propositional logic.

Problem 4

Express the statement using quantifiers. Then form the negation of the statement. Next, express the negation in simple English.

There is someone in this class who does not have a good attitude.

Solution

Given statement:
$\exists x(\neg A(x))$ where $A(x)$ means that x has a good attitude.
Negation:
$\neg\exists x(\neg A(x)) \equiv \forall x(A(x))$
(Note the use of De Morgan's Law for Quantifiers that flips the "there exists" into a "for all" and negates the inside)
In simple English:
Everyone in the class has a good attitude.

Section 1.5 Quick Review

Pretty much the same as 1.4 except with multiple quantifiers. Examples: $\forall x \forall y$ means "for all x and y."
$\forall x \exists y$ means "for all x there exists y."
$\exists x \forall y$ means "there exists an x such that for all y."

Problem 1

Translate the statement into English, where the domain for each variable consists of all real numbers. $\forall x \forall y(((x \geq 0) \wedge (y \geq 0)) \rightarrow (xy \geq 0))$

Solution

For all real numbers x and y, if x is greater than equal to 0 and y is greater than equal to 0, then x times y is greater than or equal to 0.

Problem 2

Let $I(x)$ be the statement "x has an internet connection." Write the below statement using quantifiers. Everyone except one student in your class has an internet connection.

Solution

$\exists x(\neg I(x) \wedge \forall y(x \neq y \rightarrow I(y))$
This can be thought of as "there exists a student x such that x does not have an internet connection and every other student except them has one."

Problem 3

Rewrite the statement.
$\neg\exists y(\exists x R(x,y) \vee \forall x S(x,y)$

Solution

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\begin{aligned}
\neg\exists y(\exists x R(x,y) \vee \forall x S(x,y)) &\equiv \forall y \neg(\exists x R(x,y) \vee \forall x S(x,y) && \text{(De Morgan's Law of Quantifiers)} \\
&\equiv \forall y(\neg\exists x R(x,y) \wedge \neg\forall x S(x,y) && \text{(De Morgan's Law)} \\
&\equiv \forall y(\forall x \neg R(x,y) \wedge \exists x \neg S(x,y)) && \text{(De Morgan's Law of Quantifiers)}
\end{aligned}
$$