Gravitational waves are ripples in the space-time fabric. The main source of these ripples is accelerated masses, and the primary origin of these masses is binary systems. Therefore, if we observe a binary system, we should expect to detect gravitational waves from it as well. By doing some calculation we can show that this ripples are actually behave as waves.

The main equation used here is the Einstein field equations (EFEs), where G is the Einstein tensor (a combination of the Ricci tensor, Ricci scalar, and the metric itself), T is the energy-momentum tensor, G is the gravitational constant, and c is the speed of light. When we are far from the source, the metric can be approximated as a background Minkowski metric plus a small perturbation caused by gravitational waves.

From this point forward, we just need to deal with math and tensor calculations. Since we are far from the source, the energy-momentum tensor on the right-hand side (RHS) becomes zero, and the left-hand side (LHS) of the equation can be purely calculated by the metric. By performing all the tensor calculations, defining a new variable as h-bar, and transitioning to a coordinate system where the Lorenz gauge is valid, we can show that h-bar obeys the wave equation. The most general solution for this equation is this, here A is the amplitude, and K is the wave 4-vector.

At the first glance, it seems that h-bar has 16 independent components, but that is not the case here. We can go the a very specific coordinate system which not only Lorentz gauge but also new conditions which is known transverse traceless gauge is valid. By the transverse gauge, we mean that the 4-velocity of the observer is orthogonal to the propagation of gravitational waves.

Due to the symmetry of the metric, we can reduce 6 degrees of freedom (d.o.f) in our metric. The Lorenz gauge provides 4 meaningful independent equations, which, when used, help reduce an additional 4 extra d.o.f. Finally, the transverse traceless gauge gives us another 4 meaningful independent equations, resulting in only 2 d.o.f in our metric.

Here, h plus and h cross are known as the so-called plus and cross polarizations of gravitational waves, respectively. So, when discussing the observation of gravitational waves, we are essentially referring to the measurement of this h.

In order to observe gravitational waves on the ground we need to have a very accurate observatories which are sensitive to change of the distance because when gravitational waves (GWs) reach us, they cause minuscule stretching and squeezing of the Earth. One of the best method for doing that is interferometry. So Gws observatories are basically Giant mikelson like interferometer. Here on the left hand side you can see one of these observatories in Hanford us. On the right hand side the scheme of this observatory is shown. The more observatories we have on the ground the more precise we can talk about the direction of Gravitational wave. At the end the out put of these observatories are a time series contained of a Gws signal and noise.

Here the PSD of the noise budget of Hanford detector is shown. The dashed black line here is the design sensitivity of it. However current sensitivity during the 3rd observation plan is shown in this pink line as you can see here the most sensitive frequency of this detector is at around 500 Hz and the most dominant noises are quantum shot noise of the laser in high frequency and seismic noise in low frequency. It basically means that in the presence of seismic noise we cannot go to lower frequency.

Lets take a look at the first detection of Gws this signal came from binary black hole merger with masses of around 35 and 30 solar masses and the luminosity distance of 440 Mpc corresponding to redshift 0.1. on the upper plots you can see the signal measured by LIGO and in the middle one the simulation of the signal is shown in the final plots amplitude and frequency of the merger is displayed. But the interesting thing here is the observation time, it is something around half a second and it is reasonable, because we are observing in such a high frequency. So we actually observe last fraction of the merger's lifetime.

In addition to ground base observatories, LISA is going to be the first space based gravitational wave observatory. It is planned to lunch at late 2030s. On the left hand side the orbit of LISA is shown. It's going to orbit around the sun with semi-majour axis of 1 AU and arms of almost 2.5 milion km the entire arrangement is 10 times bigger than the orbit of moon. LISA trails the Earth by around 20 degrees, and the plane of the triangular spacecraft formation is tilted approximately 60 degrees from the plane of the ecliptic. On the right hand side you can see the scheme of LISA. LISA is also based on interferometery but it uses time delay interferometry to reduce laser frequency noise. Similar to LIGO, the out put of LISA is also a time series contained probable signal and noise.

We talked about noise and it's worthwhile to take a look at sensitivity curve for LISA. The green line here is the instrumental noise level. Since we do not have any seismic noise in space, the most sensitive frequency of LISA is in the milihertz band. The other line on the plot show the Gws signal that can be heard by LISA. In this work we are going to talk about this little bump over here.

Gravitational waves sources in LISA band are. Firs SMBHBs which an be heard in low frequency tail of LISA. SBBHs which in contrast to LIGO, LISA is sensitive to early stage of those merger,
CGBs they are mostly white doawrf binaries and their population is expected to be around 60 million,
Verification Galactic binaries, they are CGBs that are already observed electromagnetically so they can be used as a verification for LISA performance.
Extreme mass ratio inspiral and stocastic gravitational waves background are also another probable sources of Gws in LISA Band.

Again in contrast to LIGO, LISA data are expected to be signal dominated and also expected to be long lived so lots of Gws signal are going to overlap with each other in time or corresponding frequency some of these sources have a large signal to noise ration and can be completely characterize however most of these sources do not have a large SNR so they behave like a kind of noise which is known as confusion noise or foreground noise. This paper is a simulational work for obtaingin this confusion noise due to the presence of compact Galactic binaries and stellar mass binary black holes.

This slide is an overview of the work is done in this paper for obtaining the confusion noise from the CGBs. First they use the simulation of 30 million CGBs from Radler LISA data challenge data set they also use simulation for obtaining instrumental noise using LISA code simulator. Then for each observation time they generate the idealized data set in the frequency domain. After that they apply their method which we are going to talk about it in later to find the final sensitivity and finally they use the data to present a theoretical model for confusion noise.

The method based on rather strong assumption, Bright sources with a SNR larger than a given threshold are detected and characterized without systematic bias or source confusion. So basically we need to know the SNR for each binary. SNR is the rho tot which is a sumation of hk|hk over noise orthogonal TDI variables and hk|hk is the integration of the magnitude of h over total signal. And the total signal as I mentied earlier is a combination of GW signal and noise.

Lets talk about method, First of all they simulated 30 million CGBs ad calculate the PSD of them they also did the same thing for instrumental noise in step 3 SNR of each source is calculated with respect to the instrumental noise this step is not necessary but it is going to decrease the number of calculation at the end. Next step is similar to the previous one. But instead of calculating the SNR with respect to instrumental noise, they calculate it with respect of either a running mean or median on the PSD of the data. In the next step, if the SNR of the source is being lager than a certain threshold, then that specific surce will be subtracted from the data and at the end steps 4 and 5 are repeted until no sources exceed the threshold.

Here in gray line you can see the PSD of the simulation of 30 million CGBs or actually starting data then after each iteration, some sources eliminate from the data therefor total PSD decreases.
As you can see here the confusion noise is well above the instrumental noise, so we need a model to describe it and use that model in the future works.

The empirical analitical model they used is this. Here A is amplitude which strongly depends on the population of CGBs. Alpha is an smoothness parameter f1, f2 and f_knee are scaling frequency they also find out that that f1 an f knee are strongly depend on observation time and they can be well approximated by these formulas. Here a1 b1 ak and bk are amplitude calibration parameters. The fitting parameters here are A, f2 , alpha a1 b1 ak and bk.

Here on the left hand side you can see the PSD of the residual data in gray while the colored dashed line represent the model prediction for the given observation time. On the first table here fitting parameters of the model are shown and now we have an estimation of the level of confusion noise due to the presence of CGBs (and it can be used in further studies). Next table which in my opinion is the most interesting table of this parer shows the total number of recovered sources or subtracted sources in each observation time. The more the observation time is the more sources can be recovered. Even with 6 years of observation near to only 50000 sources are recovered from the 30 million sources.

In the next 3 slides I’m going to talk about stellar mass binary black holes. As I mentioned earlier LISA is sensitive to the early stage of orbit evolution of these binaries. They appear in the high frequency tail of LISA band. An interesting fact about them is that they can be observed by both LISA and LIGO but the chance of transition from LISA to LIGO band for one binary is very low in the LISA observation time. The population of SBBHs observed by LISA depends on merger rate of these binaries and astonishingly this merger rate can be estimated by using the data of ground base observatories.

Here you can see the result after doing the process. On the right hand side the number of subtracted sources with respect to the chosen threshold is shown, however, there is not that much difference between rho0 of 5 and 15 and actually they are both well below the instrumental noise.

So far we have talked about two type of sources and calculate the power spectrum density of the noise level for each of them individually, here we are going to talk about combining these two population. Again method is the same but instead of iterating the process over one type we do it over all types of sources and compare SNR of each type with itself at the end the final result would be the super position of all the foreground signal plus absolutely instrumental noise.

This is the final result of combining the population. The blue line is the total confusion noise and the gray line is the final sensitivity in the presence of CGBs and SBBHs. On the right hand side the number of subtracted sources are shown. Here for CGBs they calculate the foreground noise with SNR of 7 but for SBBHs rho0 is written on the x axis. As you can see the contribution of SBBHs to the total sensitivity is negligeble however due to the the presence of CGBs the most sensitive frequency of LISA is moved to the right.

As conclusion, this paper used the simulation of CGBs and SBBHs population and applying an iterative process to estimate foreground noise caused by them. They also managed to fit a theoretical model to the confusion noise data of CGBs. They found that Gws signal of SBBHs is not strong enough to affect the final result however because of the presence of compact galactic binaries the most sensitive frequency of LISA is increased.