release date: 13.06.2024

“Ramp Up Course Mathematics — Numerical Analysis Part”
1. Exercise Assignment

Task 1 [Cancellation]
Let $|x| \ll 1$. Replace the term

$$\frac{0.5}{1+2x} - \frac{1-x}{2+2x}$$

by an equivalent term such that cancellation is avoided in finite precision arithmetics.

weight 1

Task 2 [Condition]
Compute the condition w.r.t. the $\|\bullet\|_\infty$-norm using the upper bound from the lecture for the following two problems. To do so, first define the function F which models the problem.

1. Given the radius r of a circle, compute its area.
2. Given the lengths $x, y > 0$ of a rectangle, compute its area.

weight 2

Task 3
Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 6 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & -12 & 5 \\ -1 & 3 & -1 \\ 0 & 2 & -1 \end{pmatrix}.$$

1. Compute the 1-norm and the ∞-norm of A, B and AB.
2. Compute the condition numbers $\kappa_1(A)$, $\kappa_1(B)$, $\kappa_\infty(A)$, $\kappa_\infty(B)$.
3. Let $A \in \mathbb{R}^{m\times n}$ and $x \in \mathbb{R}^n$. Let $\|\cdot\|_p$ be a vector norm which induces the matrix norm

   $$\|A\|_p := \max_{\|x\|_p=1} \|Ax\|_p$$

   Show that

   $$\|Ax\|_p \leqslant \|A\|_p \|x\|_p$$

   follows.

weight 5

Task 4 [Cholesky Decomposition]
Consider the SPD matrix

$$A = \begin{bmatrix} 16 & -8 & 12 \\ -8 & 8 & 4 \\ 12 & 4 & 35 \end{bmatrix}.$$

1. Compute the LU decomposition (without pivoting) of A.
2. Use the LU decomposition from (a) to set up an LDL^T decomposition, where D is a diagonal matrix. From this construct the Cholesky decomposition $A = GG^T$, where $G = LD^{\frac{1}{2}}$.

weight 6

To be discussed on Thursday, 20.06.2024, 11:30 am.