Summer term 2024

Ramp-up Mathematics — Analysis
Homework Sheet 2

Exercise 2.1
Show that the operator norm of $A \in \mathbb{R}^{m\times n}$ with respect to the ℓ^∞-norm in the domain and range is the row-sum norm from Example 3.4.2.

Exercise 2.2
Let $A \in \mathbb{R}^{n\times n}$ and $x \in \mathbb{R}^n$. Show the identities

$$x^T Ax = \operatorname{trace}(xx^T A),$$
$$\|x\|_2^2 = \operatorname{trace}(xx^T).$$

Hint: Use that the trace is cyclic, i.e. $\operatorname{trace}(ABC) = \operatorname{trace}(CAB)$ (if the dimensions fit).

Exercise 2.3
Let $f : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ be given by $f(A) = A^3 - A + A^T$. What is $Df(A)[H]$ for some $H \in \mathbb{R}^{n\times n}$?

Exercise 2.4
Let $f(A, x) = Ax$ for $A \in \mathbb{R}^{m\times n}$, $x \in \mathbb{R}^n$. What is the derivative of f with respect to x? What is the derivative with respect to A? (Let's denote the former by $D_x f(A, x)$ and the latter by $D_A f(A, x)$.)

Exercise 2.5
Let $B \in \mathbb{R}^{n\times n}$ and $f : \mathbb{R}^n \to \mathbb{R}$ be given by $f(x) = x^T Bx$. What is $Df(x)$ and what is $\nabla f(x)$?