6.

(a) The persistence length (l_p) means the flexibility of a polymer chain. For a polymer chain with large persistence length means the chain is high stiffness.

(b) We know the persistence length is ca. $l_p = 6\,nm$, so the Kuhn length of PF8 is ca. $b = 2l_p = 12\,nm$

(c) For a PF8 polymer, number of monomer is $N = 200$, and length of each monomer is ca. $l = 0.7\,nm$. So $r_{max} = Nl = 140\,nm$, and end-to-end distance

$$< r^2 >= 2l_p r_{max} - 2l_p^2 \left[1 - \exp\left(-\frac{r_{max}}{l_p}\right)\right] = 1608\,nm^2$$

(d) We know that (from lecture) the gyration radius of a Gaussian-like chain,

$$< R_g^2 >= \frac{1}{6}N'b^2,$$

and for rod-like chain,

$$< R_g^2 >= \frac{1}{12}L^2.$$

- Case $r_{max} >> l_p$:
  $l_p/r_{max} << 1$ and $(l_p/r_{max})^{n+1} << (l_p/r_{max})^n$ for $n > 0$. So

  $$\begin{aligned}\frac{< R_g^2 >}{r_{max}^2} &= \frac{1}{3}\left(\frac{l_p}{r_{max}}\right) - \left(\frac{l_p}{r_{max}}\right)^2 + 2\left(\frac{l_p}{r_{max}}\right)^3 - 2\left(\frac{l_p}{r_{max}}\right)^4 \left[1 - \exp\left(-\frac{r_{max}}{l_p}\right)\right] \\ &= \frac{1}{3}\left(\frac{l_p}{r_{max}}\right)\end{aligned}$$

  Therefore the gyration radius of the Gaussian-like chain

  $$< R_g^2 >= \frac{1}{3}(l_p r_{max}) = \frac{1}{3}N'b\left(\frac{b}{2}\right) = \frac{1}{6}N'b^2$$

- Case $l_p >> r_{max}$:
  Set $x = r_{max}/l_p << 1$, and we know $\exp(-x) \approx 1 - x + x^2/2 - x^3/6 + x^4/24 + O(x^5)$ for $x << 1$. So,

  $$\begin{aligned}\frac{< R_g^2 >}{r_{max}^2} &= \frac{1}{3}\left(\frac{l_p}{r_{max}}\right) - \left(\frac{l_p}{r_{max}}\right)^2 + 2\left(\frac{l_p}{r_{max}}\right)^3 - 2\left(\frac{l_p}{r_{max}}\right)^4 \left[1 - \exp\left(-\frac{r_{max}}{l_p}\right)\right] \\ &= \frac{1}{3x} - \frac{1}{x^2} + \frac{2}{x^3} - \frac{2}{x^4}\left(x - x^2/2 + x^3/6 - x^4/24 + O(x^5)\right) \\ &= \frac{1}{12} \qquad \text{(if } x << 1\text{)}\end{aligned}$$

  Therefore the gyration radius of a rod-like chain,

  $$< R_g^2 >= \frac{1}{12}r_{max}^2 = \frac{1}{12}(N'b)^2 = \frac{L^2}{12}$$