"Ancient Egyptian algebra"

Deliverable: The entire worksheet will be *graded*. Submit work on a **separate sheet**. **Any work that does not strictly follow the *Laws of Horus* will be marked as zero, unless it is extremely amusing. Penmanship counts.** Take care to be neat, since the hieroglyphs are difficult to draw sometimes.

Instructional objectives: You will learn to apply some – seemingly arbitrary – computational rules on expressions. These rules will later be adapted to describe the computation of differentials.

Title source: Futurama (April 27, 1999) *A Fishful of Dollars*, https://vimeo.com/60696810

The Laws of Horus. 𓀀 and 𓁐 denote any expressions, made out of atomic variables x, y, z, constant numbers like $e, 1, 2, \pi, i, \mathtt{a}$, etc, arithmetic operations of addition, division, multiplication, exponentiation, and unary transcendental functions ($\sin, \cos, \tan, \arctan, \arcsin, \ln$) Also $\mathtt{a}$ is a constant. The Symbol of the Ibis, 𓅞, is an operator that takes expressions to expressions, and satisfies the following Laws:

Law 1. $\text{𓅞}\left(\text{𓀀}^{\text{𓁐}}\right) = \text{𓀀}^{\text{𓁐}}\left(\ln(\text{𓀀})\text{𓅞}\left(\text{𓁐}\right) + \frac{\text{𓁐}}{\text{𓀀}}\text{𓅞}\left(\text{𓀀}\right)\right)$

In the special case when $\text{𓁐} = \mathtt{a}$ is constant, $\text{𓅞}\left(\text{𓀀}^{\mathtt{a}}\right) = \mathtt{a}\text{𓀀}^{\mathtt{a}-1}\text{𓅞}\left(\text{𓀀}\right)$

Law 2. $\text{𓅞}(\mathtt{a} \cdot \text{𓀀}) = \mathtt{a} \cdot \text{𓅞}\left(\text{𓀀}\right)$

Law 3. $\text{𓅞}\left(\text{𓀀} + \text{𓁐}\right) = \text{𓅞}\left(\text{𓀀}\right) + \text{𓅞}\left(\text{𓁐}\right)$

Law 4. $\text{𓅞}(\mathtt{a}) = 0$

Law 5. $\text{𓅞}(\text{𓀀}\text{𓁐}) = \text{𓁐}\text{𓅞}\left(\text{𓀀}\right) + \text{𓀀}\text{𓅞}\left(\text{𓁐}\right)$

Law 6. $\text{𓅞}\left(\frac{\text{𓀀}}{\text{𓁐}}\right) = \frac{\text{𓁐}\text{𓅞}\left(\text{𓀀}\right) - \text{𓀀}\text{𓅞}\left(\text{𓁐}\right)}{\text{𓁐}^2}$

Law 7. $\text{𓅞}\left(\sin \text{𓀀}\right) = \left(\cos \text{𓀀}\right)\text{𓅞}\left(\text{𓀀}\right), \qquad \text{𓅞}\left(\cos \text{𓀀}\right) = -\left(\sin \text{𓀀}\right)\text{𓅞}\left(\text{𓀀}\right)$

Law 8. $\text{𓅞}\left(\ln \text{𓀀}\right) = \frac{\text{𓅞}(\text{𓀀})}{\text{𓀀}}$

Law 9. $\text{𓅞}\left(\arctan \text{𓀀}\right) = \frac{\text{𓅞}\left(\text{𓀀}\right)}{1 + \text{𓀀}^2}, \qquad \text{𓅞}\left(\arcsin \text{𓀀}\right) = \frac{\text{𓅞}\left(\text{𓀀}\right)}{\sqrt{1 - \text{𓀀}^2}}$

Once 𓅞 reaches an atomic variable, it does not simplify further: e.g., $\text{𓅞}(x) = \text{𓅞}x$. We say in that case that the bird has found the food, and we leave the bird together with its food as a single bird-with-food symbol, $\text{𓅞}x, \text{𓅞}y$, etc.

For example,

$$\begin{aligned}
\text{𓅞}(x^3+4)^{27} &= 27(x^3+4)^{26}\text{𓅞}(x^3+4) && \text{Law 1 with } \text{𓀀} = x^3+4 \text{ and } \mathtt{a} = 27\\
&= 27(x^3+4)^{26}(\text{𓅞}(x^3) + \text{𓅞}(4)) && \text{Law 3}\\
&= 27(x^3+4)^{26} \cdot 3x^2\text{𓅞}x && \text{Law 1 with } \text{𓀀} = x^3 \text{ and } \mathtt{a} = 3 \text{ (and Law 4)}
\end{aligned}$$

Use the *Laws of Horus* to simplify the following, so that the bird gets the food. Apply one law per step, and cite each law as you go.

1. $\text{𓅞}(x^3)$

2. $\text{𓅞}(ye^x)$

3. $\text{𓅞}(x^2+1)$

4. $\text{𓅞}\left(x\cos(x^2+1)\right)$

5. $\text{𓅞}\left(\frac{\cos(x^2+1)}{(x^3+4)^{27}}\right)$

6. $\text{𓅞}\left(x^2+y^2\right)$