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ECE350 - HW #1

1. Minterms are a canonical sum of products–it entails writing each combination of inputs which yield f=1 as a product (i.e. we would write x1x2 if f(x1,x2)=1), and then summing all these products of combinations together. It is considered “canonical” because all input combinations that yield f=1 appear only once. On the other hand, maxterms are a canonical product of sums—it entails writing each combination of inputs which yield f=0 as a sum (i.e. we would write x1+x2 if f(x1,x2) =0), and then multiplying all these sums of combinations together. Because of duality — the notion that you can substitute 0 for 1 and • for + in any statement — minterms and maxterms are complementary expressions.
2.
    a.

x1	**x2**	**x3**	**f(x1,x2,x3)**
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

    b.

x1	**x2**	**x3**	**x4**	**f(x1,x2,x3,x4)** **y1**	**y2**	**y3**	**y4**
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	0	1

0	1	1	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	0	0	1	0
1	0	0	1	0	0	1	0
1	0	1	0	0	0	1	0
1	0	1	1	0	0	1	0
1	1	0	0	0	0	1	1
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	1
1	1	1	1	0	0	1	1

c.

x1	**x2**	**x3**	**x4**	**x5**	**f(x1,x2,x3,x4,x5)**				
					y1	**y2**	**y3**	**y4**	**y5**
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1
0	0	0	1	0	0	0	0	0	1
0	0	0	1	1	0	0	0	0	1
0	0	1	0	0	0	0	0	1	0
0	0	1	0	1	0	0	0	1	0
0	0	1	1	0	0	0	0	1	0
0	0	1	1	1	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0
0	1	0	0	1	0	0	0	1	0
0	1	0	1	0	0	0	0	1	0
0	1	0	1	1	0	0	0	1	0
0	1	1	0	0	0	0	0	1	0

0	1	1	0	1	0	0	0	1	0
0	1	1	1	0	0	0	0	1	0
0	1	1	1	1	0	0	0	1	0
1	0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	0	0	1	1
1	0	0	1	0	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	0	1	0	0	0	0	0	1	1
1	0	1	0	1	0	0	0	1	1
1	0	1	1	0	0	0	0	1	1
1	0	1	1	1	0	0	0	1	1
1	1	0	0	0	0	0	0	1	1
1	1	0	0	1	0	0	0	1	1
1	1	0	1	0	0	0	0	1	1
1	1	0	1	1	0	0	0	1	1
1	1	1	0	0	0	0	0	1	1
1	1	1	0	1	0	0	0	1	1
1	1	1	1	0	0	0	0	1	1
1	1	1	1	1	0	0	0	1	1

3.
   a. x1x2x3 + x1’x2x3 + x1x2’x3 + x1x2x3’
      f(x1, x2, x3) = x2x3 + x1x3 + x1x2
   b. y1 = 0 (always); y2 = 0 (always); y3 = x1; y4 = x2
   c. y1 = 0 (always) y2 = 0 (always); y3 = 0 (always); y4 = x1 + x2 + x3; y5 = x1
4. 5 binary inputs → 32 different input combos
   To create every single output (0 or 1) of 32 inputs, we can do 2^{32}-1, which yields
   4294967295 unique functions
5.
   a. f(x1, x2, x3) = x1’x2’x3 + x1x2’x3’ + x1x2’x3 + x1x2x3’

distributive : $x2'x3(x1' + x1) + x1x3'(x2 + x2')$

$x' + x = 1$ (cancel out parentheses) : $\mathbf{f(x1,x2,x3) = x2'x3 + x1x3'}$

b. $f(x1, x2, x3) = (x1 + x2 + x3)(x1 + x2' + x3)(x1' + x2' + x3)(x1 + x2 + x3')$

Distribute parentheses $\rightarrow$ $(x1x1 + x1x2' + x1x3 + x2x1 + x2x2' + x2x3 + x3x1 + x3x2' + x3x3)$

$\rightarrow (x1'x1 + x1'x2 + x1'x3' + x2'x1 + x2'x2 + x2'x3' + x3x1 + x3x2 + x3x3')$

Apply $xx = 1$ and $x+x = 1$ $xx' = 0$ $\rightarrow (x1 + x1x2' + x1x3 + x2x1 + x2x3 + x3x2' + x3)$

$\rightarrow (x1'x2 + x1'x3' + x2'x1 + x2'x3' + x3x1 + x3x2)$

Combining $\rightarrow (x1 + x1x3 + x3)(x1'x2 + x1'x3' + x2'x1 + x2'x3' + x3x1 + x3x2)$

Absorption $\rightarrow (x1 + x3)(x1'x2 + x1'x3' + x2'x1 + x2'x3' + x3x1 + x3x2)$

Consensus $\rightarrow (x1 + x3)(x1x3 + x1'x2 + x2'x1 + x2'x3')$

Distributive $\rightarrow (x1x2x3 + x1'x1x2 + x1x2x2' + x1x2'x3' + x1x3x3 + x1'x2x3 + x2'x1x3 + x2x3'x3)$

Apply $xx = 1$ and $x+x = 1$ $xx' = 0$ $\rightarrow (x1x2x3 + x1x2'x3' + x1x3 + x1'x2x3 + x1x2'x3)$

Combining $\rightarrow (x2x3 + x1x2')$

$\mathbf{f(x1,x2,x3) = (x2x3 + x1x2')}$

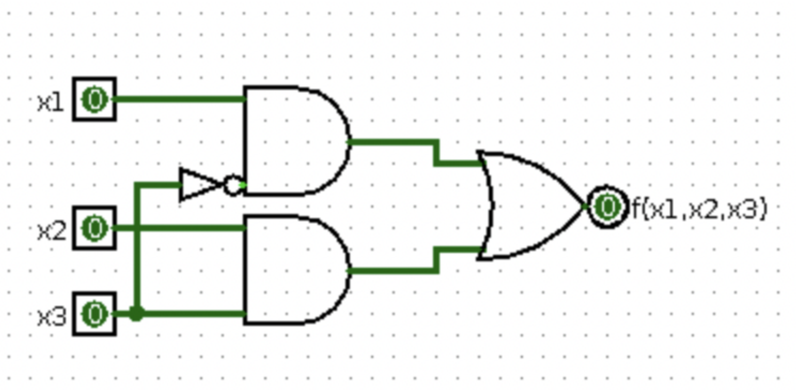
6.

a. $f(x1,x2,x3) = x1'x2x3 + x1x2'x3' + x1x2x3' + x1x2x3$

$x1x2(x3 + x3') + x1'x2x3 + x1x2'x3'$

$= x1x2 + x1'x2x3 + x1x2'x3$

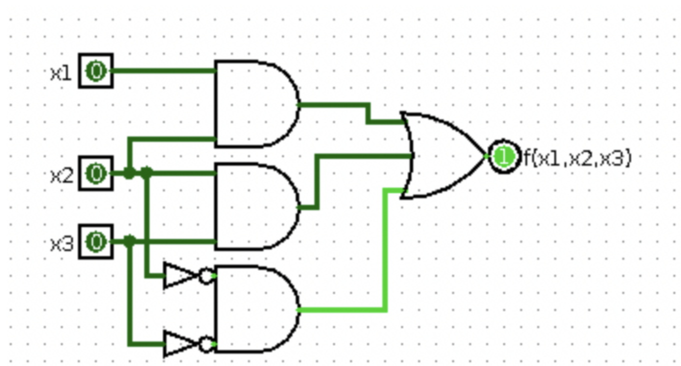
$= \mathbf{x1x3' + x2x3}$



b. $x1'x2'x3' + x1'x2x3 + x1x2'x3' + x1x2x3' + x1x2x3$

$= x2'x3'(x1' + x1) + x1x2(x3' + x3) + x2x3$

$= \mathbf{x1x2 + x2x3 + x2'x3'}$



7.

a. f(x1,x2,x3) = (x1+x2+x3)(x1+x2'+x3)(x1'+x2+x3')

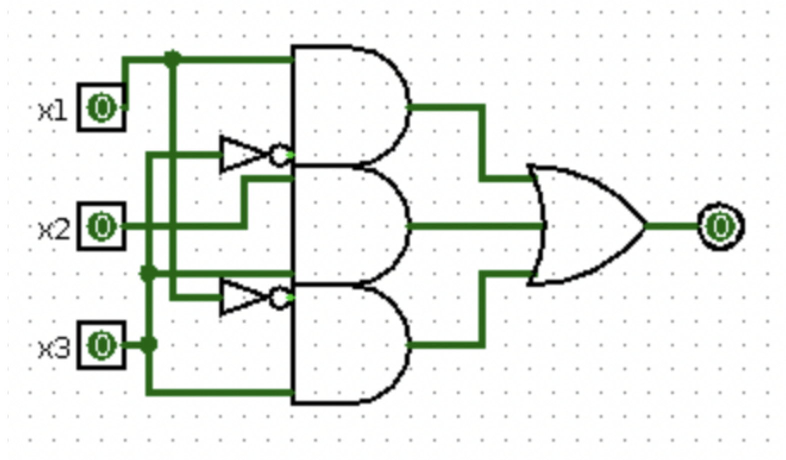
= (x1x1 + x1x2' + x1x3 + x2x1 + x2x2' + x2x3 + x3x1 + x3x2' + x3x3)(x1'+x2+x3')

= (x1 + x3)(x1'+x2+x3') = (x1x1' +x1x2 + x1x3' + x3x1' + x3x2 + x3x3')

= (x1x2 + x1x3' + x3x1' + x3x2)

= consensus → x3x2 + x3'x1 + x1x2 = x3x2 + x3'x1

f(x1,x2,x3) = x1x3' + x3x1' + x3x2



b. f(x1, x2, x3) = Π(M0, M1, M5, M7)

= (x1+x2+x3)(x1+x2+x3')(x1'+x2+x3')(x1'+x2'+x3')

= (x1x1+x1x2+x2x3' + x2x1 + x2x2 + x2x3' + x3x1 + x3x2 + x3x3')(x1'+x2+x3')(x1'+x2'+x3')

= (x1+x2)(x1'x1' + x1'x2' + x1'x3' + x2x1' + x2x2' + x2x3' +x3'x1' + x2'x3' + x3'x3')

= (x1 + x2)(x1' + x3') = x1x3' + x1'x2 + x2x3' → (consensus)

f(x1,x2,x3) = x1x3' + x1'x2