Student Information

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Answer 1

a)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Therefore, $(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ is a contradiction.

b)

$p \rightarrow ((q \vee \neg q) \rightarrow (p \wedge q))$	**given**
$p \rightarrow (T \rightarrow (p \wedge q)$	**Table 6, Negation Laws**
$p \rightarrow (F \vee (p \wedge q))$	**Table 7, Line 1**
$p \rightarrow ((p \wedge q) \vee F)$	**Table 6, Commutative Laws**
$p \rightarrow (p \wedge q)$	**Table 6, Identity Laws**
$\neg p \vee (p \wedge q)$	**Table 7, Line 1**
$(\neg p \vee p) \wedge (\neg p \vee q)$	**Table 6, Distributive Laws**
$(p \vee \neg p) \wedge (\neg p \vee q)$	**Table 6, Commutative Laws**
$T \wedge (\neg p \vee q)$	**Table 6, Negation Laws**
$(\neg p \vee q) \wedge T$	**Table 6, Commutative Laws**
$(\neg p \vee q)$	**Table 6, Identity Laws**

Therefore, $p \rightarrow ((q \vee \neg q) \rightarrow (p \wedge q))$ and $(\neg p \vee q)$ are logically equivalent.

Answer 2

a) $\forall x \, \exists y \, W(x, y)$

b) $\neg \, (\forall y \, \exists x \, F(x, y))$

c) $\forall x \, (W(x, P) \rightarrow A(Ali, x))$

d) $\exists x \, (W(Büşra, x) \wedge F(\text{TUBITAK}, x))$

e) $\exists x\, \exists y\, \exists z\, (S(x,y) \wedge S(x,z) \wedge (y \neq z))$

f) $\forall x\, \forall y\, \forall z\, (W(x,z) \wedge W(y,z) \wedge (x = y))$

g) $\forall x\, \forall y\, \forall z\, (W(x,z) \wedge W(y,z) \wedge (x \neq y) \wedge (\neg\, (\forall t\, (W(t,z) \wedge (t \neq x) \wedge (t \neq y)))))$

Answer 3

1	$p \to q$	
2	$(q \wedge \neg r) \to s$	
3	$\neg s$	
4	$\quad p$	
5	$\quad q$	$\Rightarrow$E, 1, 4
6	$\quad\quad \neg r$	
7	$\quad\quad q \wedge \neg r$	$\wedge$I, 5, 6
8	$\quad\quad s$	$\Rightarrow$E, 2, 7
9	$\quad\quad \bot$	$\neg$E, 3, 8
10	$\quad \neg\neg r$	$\neg$I, 6–9
11	$\quad r$	$\neg\neg$E, 10
12	$p \to r$	$\Rightarrow$I, 4–11

Answer 4

1	p	
2	$p \to (q \wedge r)$	
3	$r \to s$	
4	$q \wedge r$	$\Rightarrow$E, 1, 2
5	q	$\wedge$E, 4
6	r	$\wedge$E, 4
7	s	$\Rightarrow$E, 3, 6
8	$\quad s \to \neg q$	
9	$\quad \neg q$	$\Rightarrow$E, 7, 8
10	$\quad \bot$	$\neg$E, 5, 9
11	$\neg(s \to \neg q)$	$\neg$I, 8–10

Therefore, Barış is lying.

Answer 5

1	$\forall x\,(P(x) \to (Q(x) \to R(x)))$		
2	$\exists x\,(P(x))$		
3	$\forall x\,(\neg\, R(x))$		
4	c | $P(c)$		
5		$P(c) \to (Q(c) \to R(c))$	$\forall$E, 1
6		$Q(c) \to R(c)$	$\Rightarrow$E, 4, 5
7		$\neg\, R(c)$	$\forall$E, 3
8		| $Q(c)$	
9		| $R(c)$	$\Rightarrow$E, 6, 8
10		| $\bot$	$\neg$E, 7, 9
11		$\neg\, Q(c)$	$\neg$I, 8–10
12		$\exists x\,(\neg Q(x))$	$\exists$I, 11
13	$\exists x\,(\neg Q(x))$		$\exists$E, 2, 4–12