Student Information

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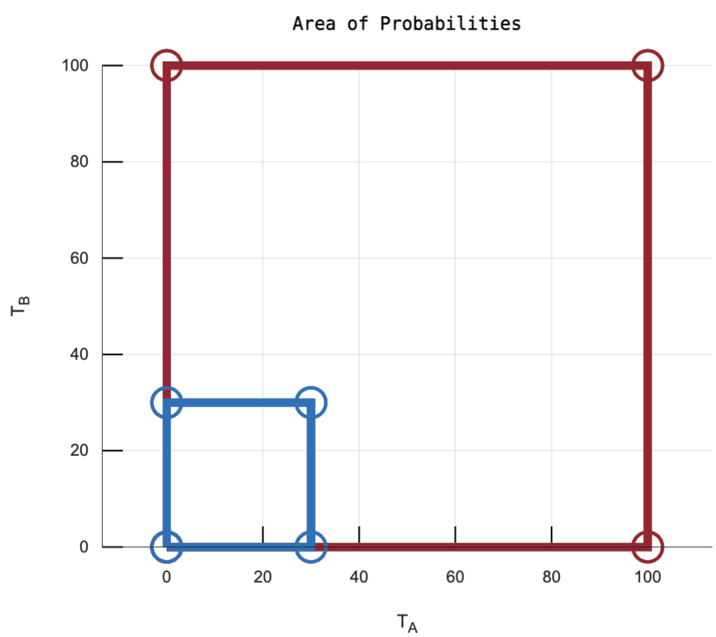
Answer 1

a)

Let's think of all the possibilities as a 100x100 square. Then, if we have a random square whose bottom left corner is in (0,0), and which aligns on the $x = x_0$ and $y = y_0$ line where $100 \geq x_0 \geq 0$ and $100 \geq y_0 \geq 0$ then we may calculate the Joint CDF F.



Joint CDF $F(x, y) = \text{P}\ \{T_A \leq x \wedge T_B \leq y\}$

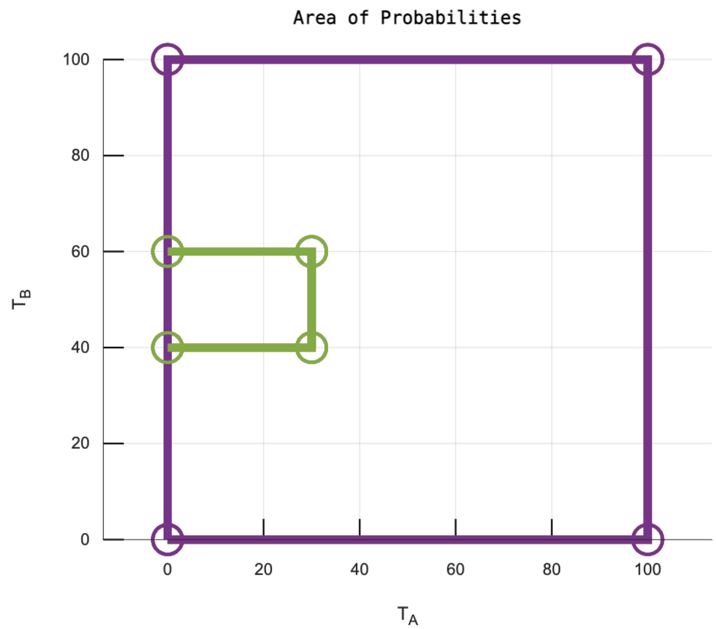
Since x and y are independent, CDF $F(x, y) = \text{P}\ \{T_A \leq x\} \cdot \text{P}\{T_B \leq y\}$

Then, CDF $F(x, y) =$ x/100 $\cdot$ y/100 = **xy/10000 = 0.0001 xy**

Finally, if we take mix partial derivatives with respect to x and y consecutively, xy/10000, we get **1/10000 = 0.0001**, which is PDF f(x,y). We would have calculated 0.0001 at first as well, according to $\frac{1}{b-a}$ rule.

b)

Let's try to understand what this question asks in a different plot.



This saying is logically equivalent to the expression P $\{(T_A \leq 30) \wedge (40 \leq T_B \leq 60)\}$.

We know that x and y are independent, then we can multiply individual probabilities to get the result again.

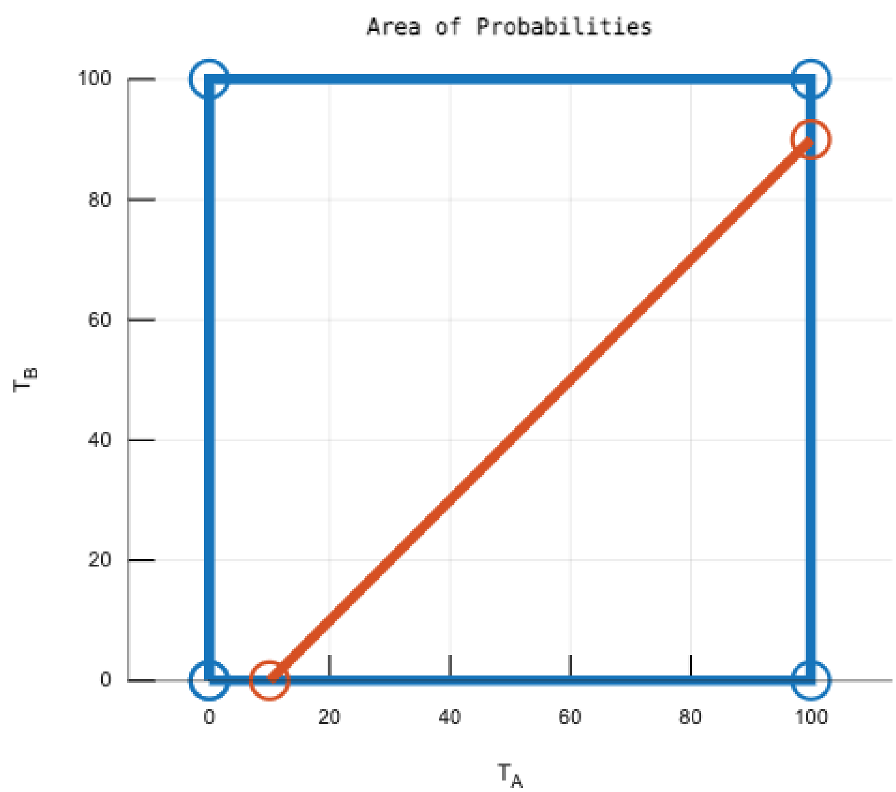
$$= \text{P}\ \{(T_A \leq 30) \wedge (40 \leq T_B \leq 60)\} = \text{P}\ \{(T_A \leq 30)\} \cdot \text{P}\{(40 \leq T_B \leq 60)\}$$
$$= 30/100 \cdot 20/100$$
$$= \mathbf{0.06}$$

c)

For this part, we should plot the line where server A processes the packet and send a response exactly later than 10ms. If we think of T_A as x and T_B as y,

$$x - y \leq 10$$
$$y \geq x - 10$$

Now, our plot is like this:



The points below the line represent the outcomes where A is later than B by more than 10 milliseconds. We are interested in the points above because we are asked to find points where A is no later than 10 milliseconds.

The probability that getting the points above is

$$1 - \frac{\frac{90 \cdot 90}{2}}{10000}$$

which makes **0.595**.

d)

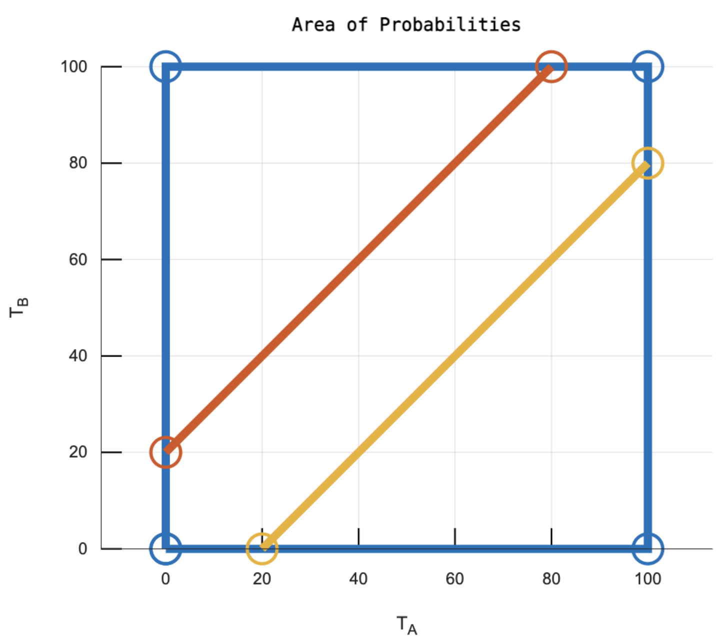
Servers are considered to have failed the task if $|T_A - T_B| \geq 20$. Then, servers pass the task when $|T_A - T_B| \leq 20$.

$$-20 \leq T_A - T_B \leq 20$$

We split this into two equations, which are $-20 \leq T_A - T_B$ and $T_A - T_B \leq 20$. If we think of T_A as x and T_B as y,

$$\begin{aligned} -20 &\leq x - y \\ y &\leq x + 20 \end{aligned} \qquad\qquad \begin{aligned} x - y &\leq 20 \\ x - 20 &\leq y \end{aligned}$$

Now, our plot is like this:



We are interested in the region closed by two lines and the square; in other words, the area between the lines.

$$1 - \frac{\frac{80 \cdot 80}{2} + \frac{80 \cdot 80}{2}}{10000}$$

which makes **0.36**.

Answer 2

a)

We use Central Limit Theorem.

$$\begin{aligned} n &= 150 \\ p &= 0.6 \\ \mu = 150 \cdot 0.6 &= 90 \\ \sigma = \sqrt{p \cdot (1-p)} = \sqrt{0.6 \cdot 0.4} &= 0.49 \end{aligned}$$

Let X be the number of frequent shoppers.

We are asked to find the probability that at least 65% of the customers in the sample are frequent shoppers. $150 \cdot 0.65 = 97.5$. In other words, we should find P{ $X \geq 97.5$ }.

$$P\{X \geq 97.5\} = 1 - P\{X < 97.5\}$$

To use Standard Normal CDF from Octave Online, we should standardize our values.

$$\frac{97.5 - \mu}{\sqrt{n} \cdot \sigma} = \frac{97.5 - 90}{\sqrt{150} \cdot 0.49} = \frac{7.5}{6} = 1.25$$

$$P\{X < 97.5\} = \Phi(1.25)$$

Finally, our result complement of this.

According to the *stdnormal_cdf*(1.25) function from Octave Online,

$$\begin{aligned} P\{X \geq 97.5\} &= 1 - \Phi(1.25) \\ 1 - 0.8944 &= \mathbf{0.1056} \end{aligned}$$

b)

We use Central Limit Theorem.

$$\begin{aligned} n &= 150 \\ p &= 0.1 \\ \mu &= 150 \cdot 0.1 = 15 \\ \sigma &= \sqrt{p \cdot (1-p)} = \sqrt{0.1 \cdot 0.9} = 0.3 \end{aligned}$$

Let X be the number of rare shoppers.

We are asked to find the probability that no more than 15% of the customers in the sample are rare shoppers. $150 \cdot 0.15 = 22.5$. In other words, we should find P{ $X \leq 22.5$ }.

To use Standard Normal CDF from Octave Online, we should standardize our values.

$$\frac{22.5-\mu}{\sqrt{n} \cdot \sigma} = \frac{22.5-15}{\sqrt{150} \cdot 0.3} = \frac{7.5}{3.67} = 2.04$$

$$P\{X \leq 22.5\} = \Phi(2.04)$$

According to the *stdnormal_cdf*(2.04) function from Octave Online,

$$P\{X \leq 22.5\} = \mathbf{0.9793}$$

Answer 3

$$\mu = 175$$
$$\sigma = 7$$

Let X be the height of a person. We are asked to find $P\{170 \leq X \leq 180\}$.

Since the population follows a normal distribution, we can use Standard Normal CDF from Octave Online, but first, we should standardize our values.

$$\frac{170-\mu}{\sigma} = \frac{170-175}{7} = \frac{-5}{7} = -0.71$$
$$\frac{180-\mu}{\sigma} = \frac{180-175}{7} = \frac{5}{7} = 0.71$$

According to the $stdnormal\_cdf(-0.71)$ and $stdnormal\_cdf(0.71)$ function from Octave Online,

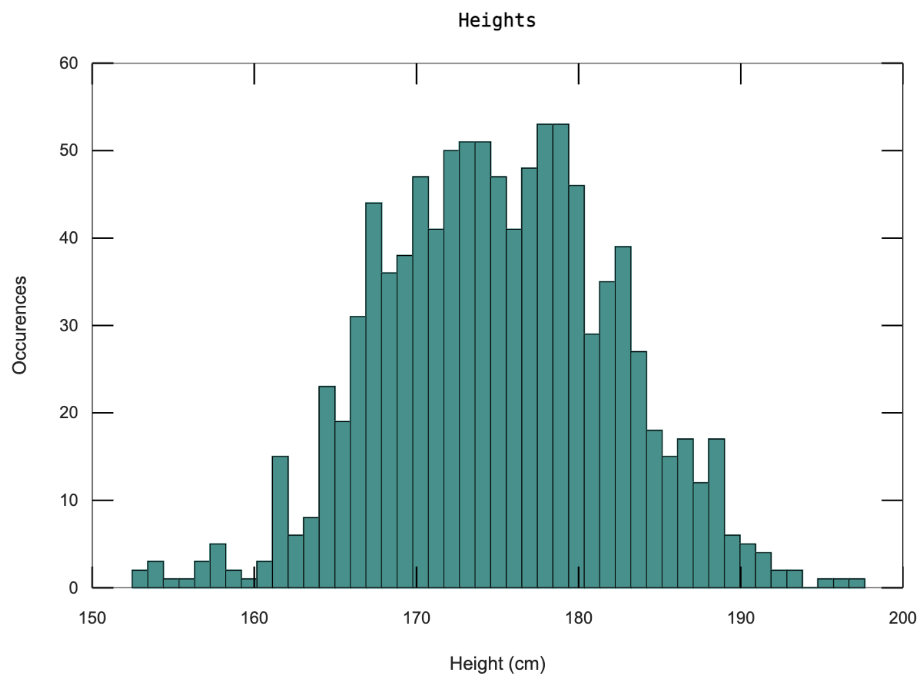
$$\Phi(0.71) - \Phi(-0.71)$$

$$0.7611 - 0.2389 = \mathbf{0.5223}$$

Answer 4

a)

Although this one is not the best distribution, we can see that there is an accumulation near 175s, which is μ. If we increase N, then we could have gotten a better result.



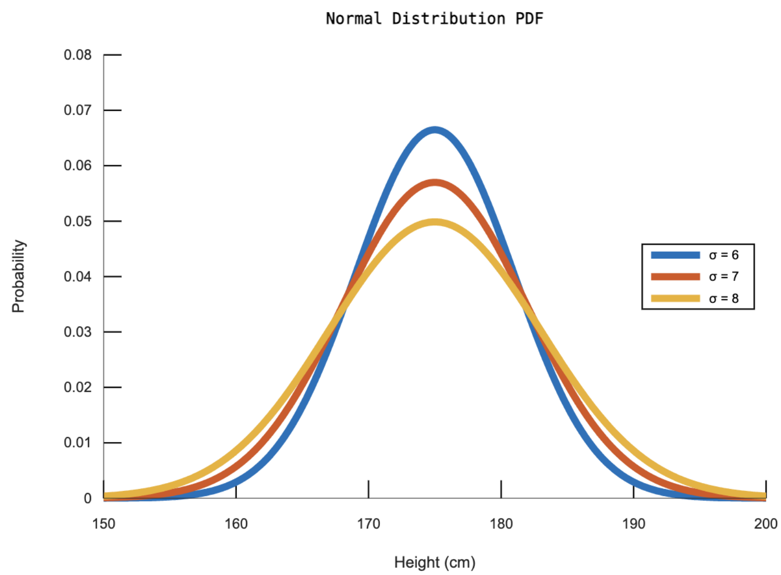

```
mu = 175;
sigma = 7;
N = 1000;

heights = normrnd(mu, sigma, N, 1);

hist(heights,47)
xlabel('Height (cm)')
ylabel('Occurences')
title('Heights')
```

b)

As we increase σ, the deviation increases and the plot has a more pressed looking.




```
mu = 175;

sigma_values = [6 7 8];

x = linspace(150,200,1000);

figure
hold on
    for i = 1:length(sigma_values)
        sigma = sigma_values(i);
        y = normpdf(x, mu, sigma);
        plot(
            x, y,
            'LineWidth', 5,
            'DisplayName', ['\sigma = ' num2str(sigma)]
        )
end
hold off

xlabel('Height (cm)')
ylabel('Probability')
title('Normal Distribution PDF')
legend('Location', 'east')
```

c)

There is not much to say about this question, it is self-explanatory.

```
octave:1> source("Q4c.m")
At least 45% of adults have heights between 170 cm and 180 cm
with the probability 0.9740
At least 50% of adults have heights between 170 cm and 180 cm
with the probability 0.7440
At least 55% of adults have heights between 170 cm and 180 cm
with the probability 0.2740
```

```
mu = 175;
sigma = 7;
N = 1000;

p45 = zeros(1, N);
p50 = zeros(1, N);
p55 = zeros(1, N);

for i = 1:N
    heights = normrnd(mu, sigma, 1, 150);

    heightsBetween = sum(heights >= 170 & heights <= 180);

    percentBetween = heightsBetween / 150;

    p45(i) = percentBetween >= 0.45;
    p50(i) = percentBetween >= 0.50;
    p55(i) = percentBetween >= 0.55;
end

p45Result = sum(p45) / N;
p50Result = sum(p50) / N;
p55Result = sum(p55) / N;

fprintf('At least 45%% of adults have heights between 170 cm and 180 cm with the probability %.4f\n', p45Result);
fprintf('At least 50%% of adults have heights between 170 cm and 180 cm with the probability %.4f\n', p50Result);
fprintf('At least 55%% of adults have heights between 170 cm and 180 cm with the probability %.4f\n', p55Result);
```