1 Kernel as a dot product in feature space

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\begin{aligned}
X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} Y &= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\
\langle X, Y \rangle &= x_1 y_1 + x_2 y_2 \\
K(X, Y) &= (\langle X, Y \rangle + 1)^2 \\
&= 1 + 2x_1 y_1 + 2x_2 y_2 + (x_1 y_1)^2 + (x_2 y_2)^2 + 2x_1 y_1 x_2 y_2 \\
\phi(X) &= \begin{bmatrix} 1 & \sqrt{2}x_1 & \sqrt{2}x_2 & (x_1)^2 & (x_2)^2 & \sqrt{2}x_1 x_2 \end{bmatrix}^T \\
K(X, Y) &= (\langle X, Y \rangle + 1)^2 = \langle \phi(X), \phi(Y) \rangle
\end{aligned}
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