Stackelberg Mean-Payoff Games With One Epsilon-Optimal Adversarial Follower

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June 3rd 2020

Stackelberg Games

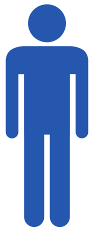
Stackelberg Games

Two (types of) Players:

Stackelberg Games

Leader

Follower



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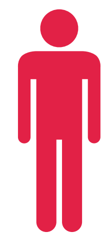
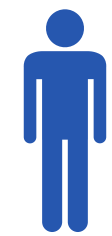
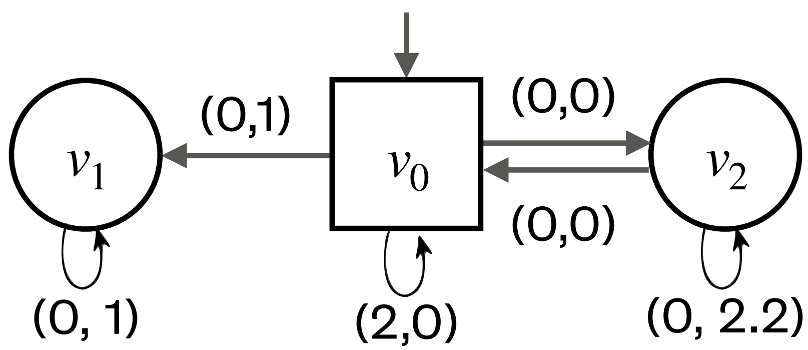
	

Two (types of) Players:

Game:

Mean-Payoff Games

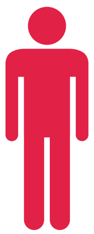


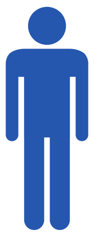
Bi-Matrix Games

	I	II
I	(1,4)	(4,2)
II	(1,3)	(3,5)

Stackelberg Games

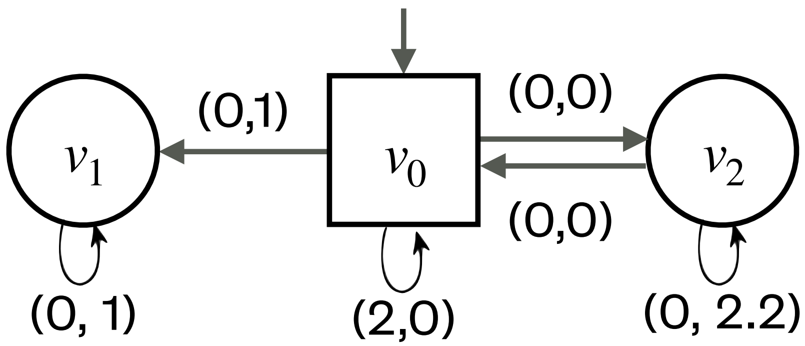
Two (types of) Players:

Leader 

Follower 

Game:

Mean-Payoff Games



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Sequential Move:

1. Leader announces her strategy

2. Follower announces his response to leader’s strategy

Stackelberg Games

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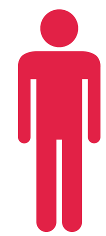
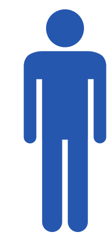


Follower



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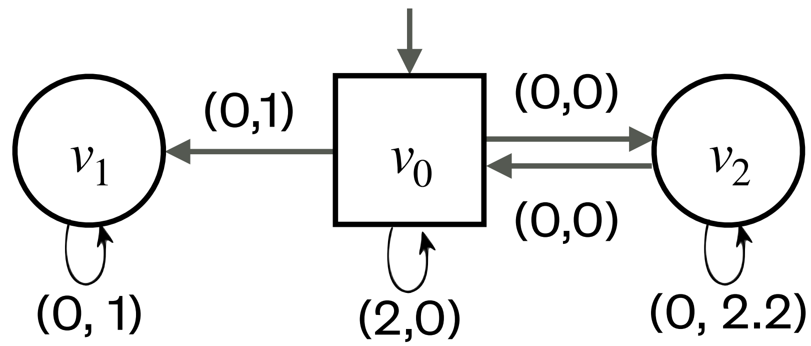
Mean-Payoff Games



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Stackelberg Mean Payoff Games

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Follower

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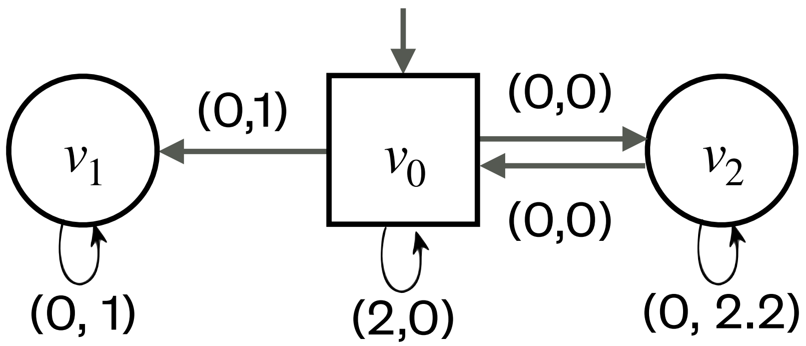

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Stackelberg Mean Payoff Games

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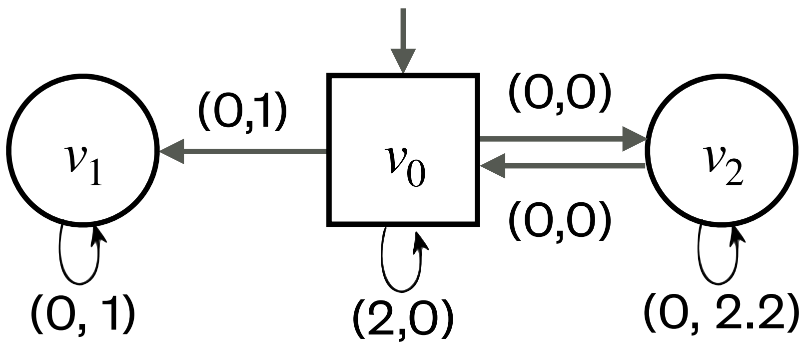

Sequential Move:

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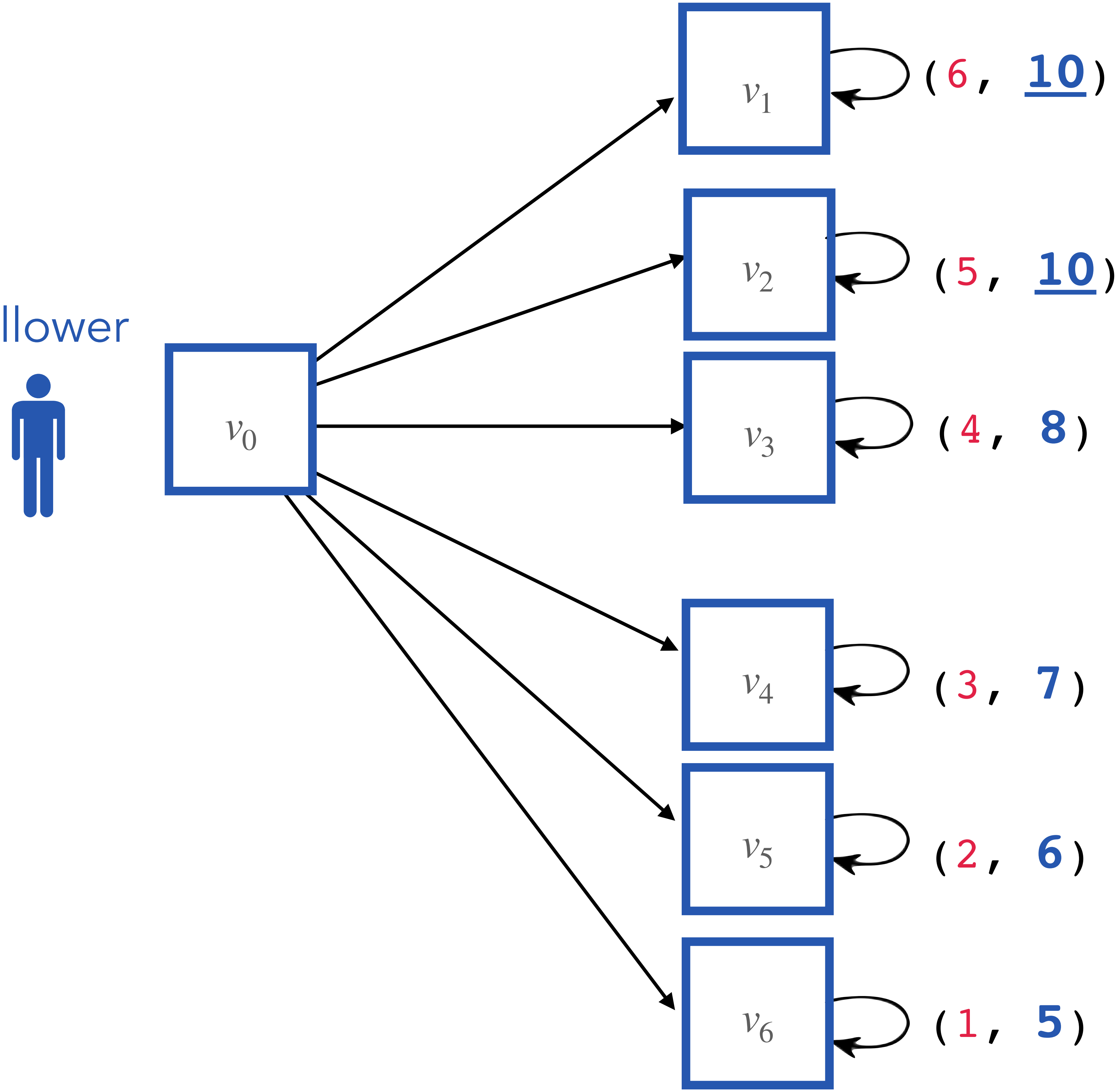
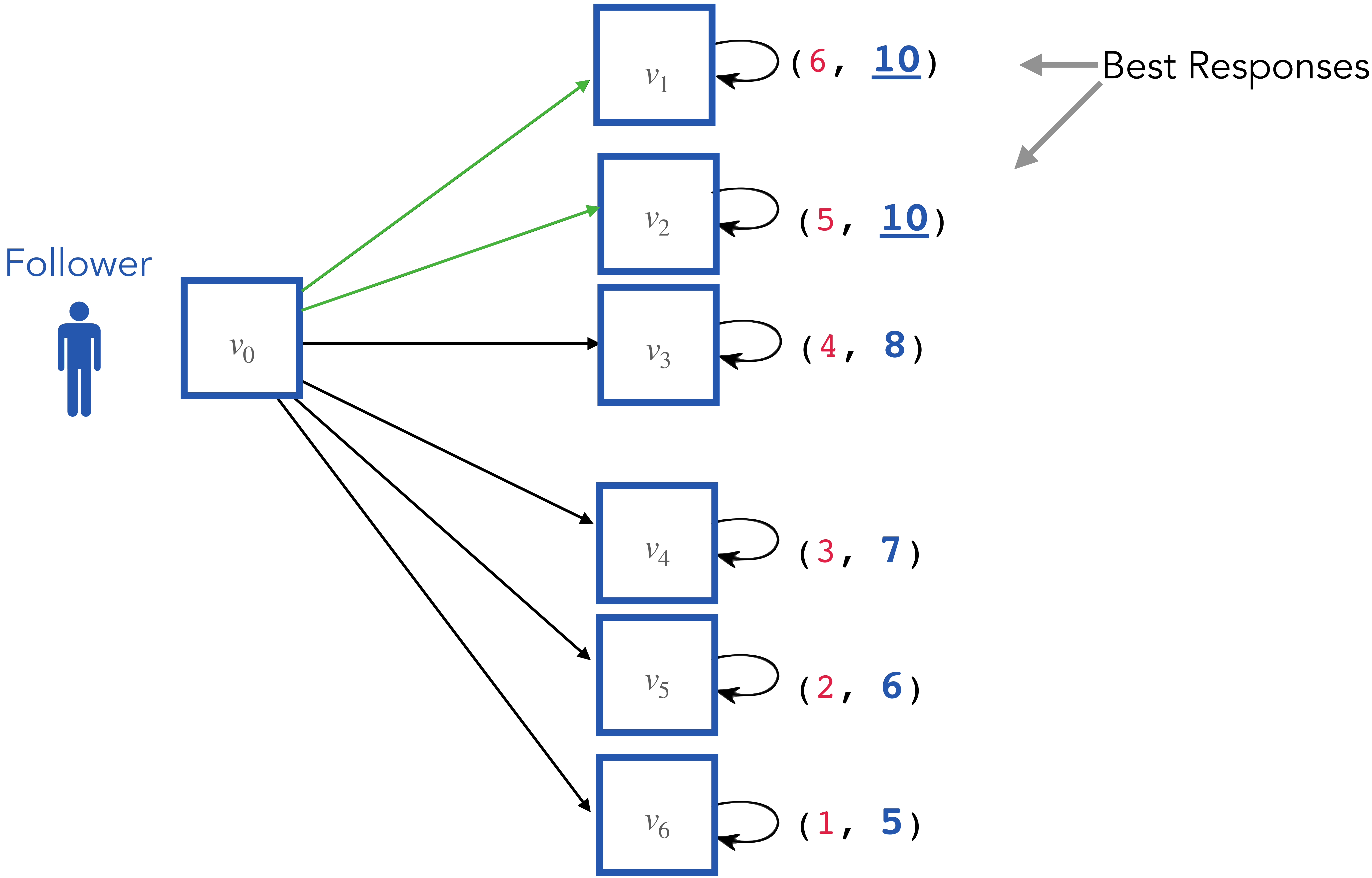
Players are rational and choose the best possible strategy

Best Response

Follower



Best Response



Follower can be

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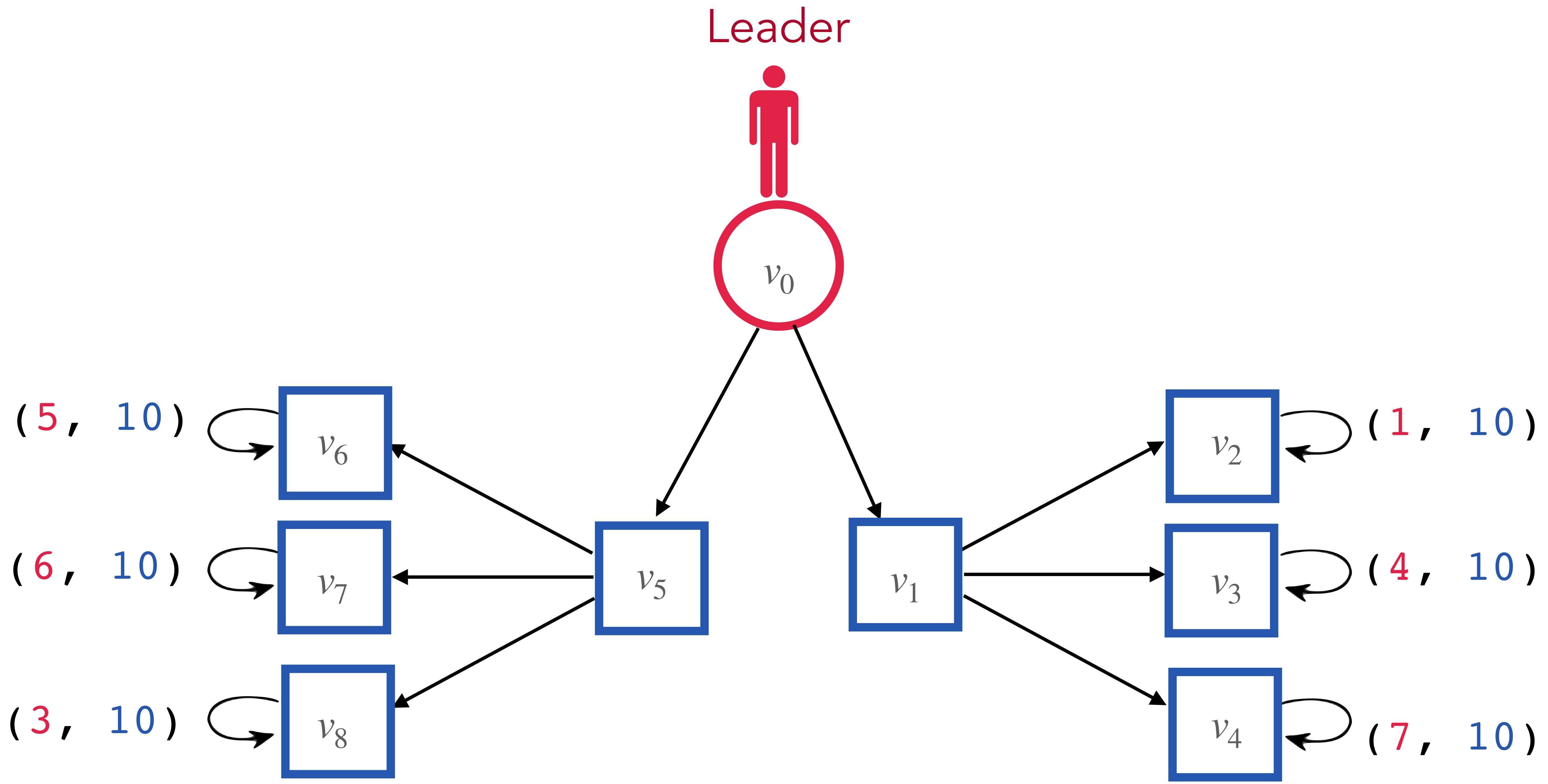
- Cooperative or

Follower can be

- Cooperative or
- Adversarial

Cooperative vs Adversarial



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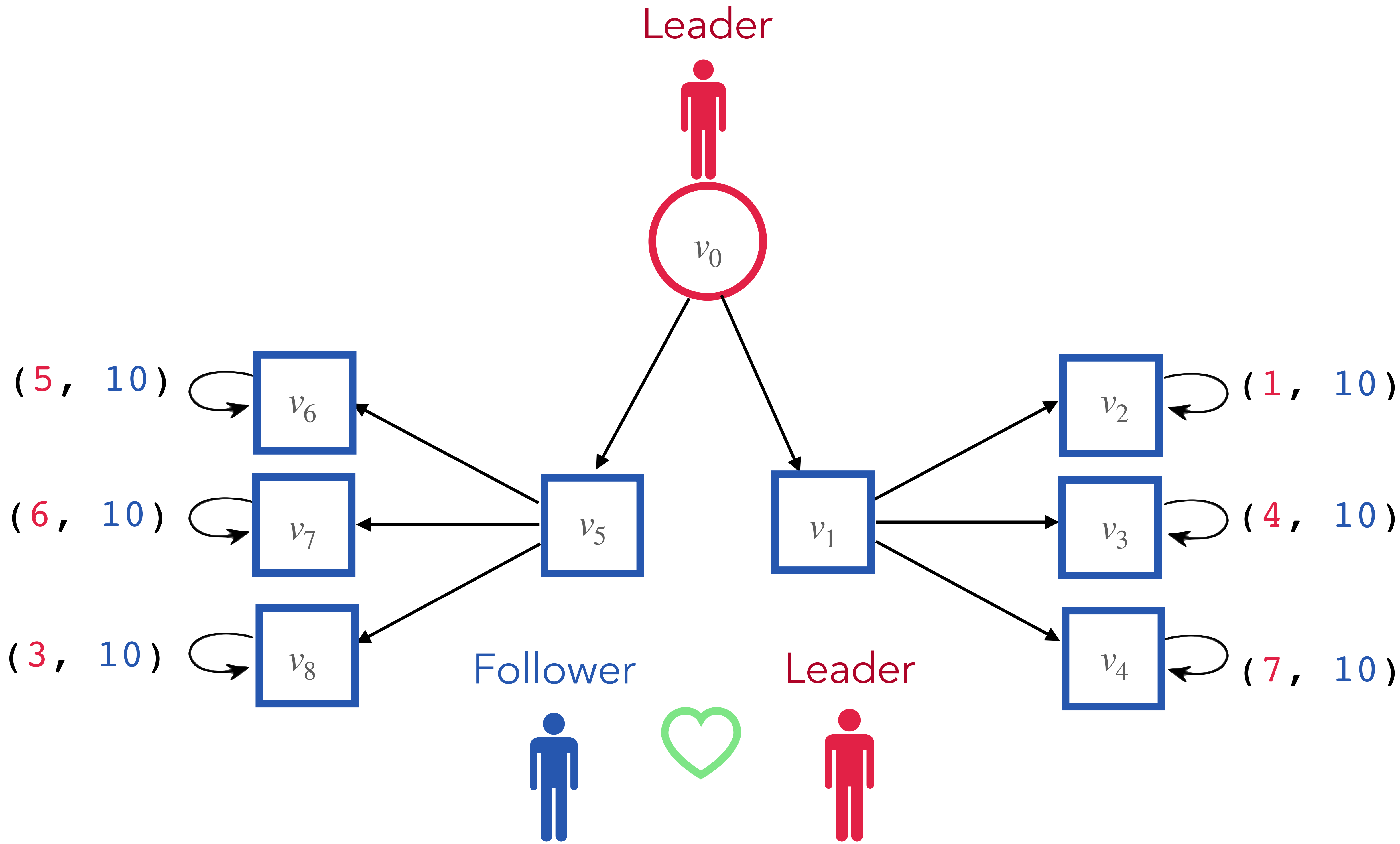

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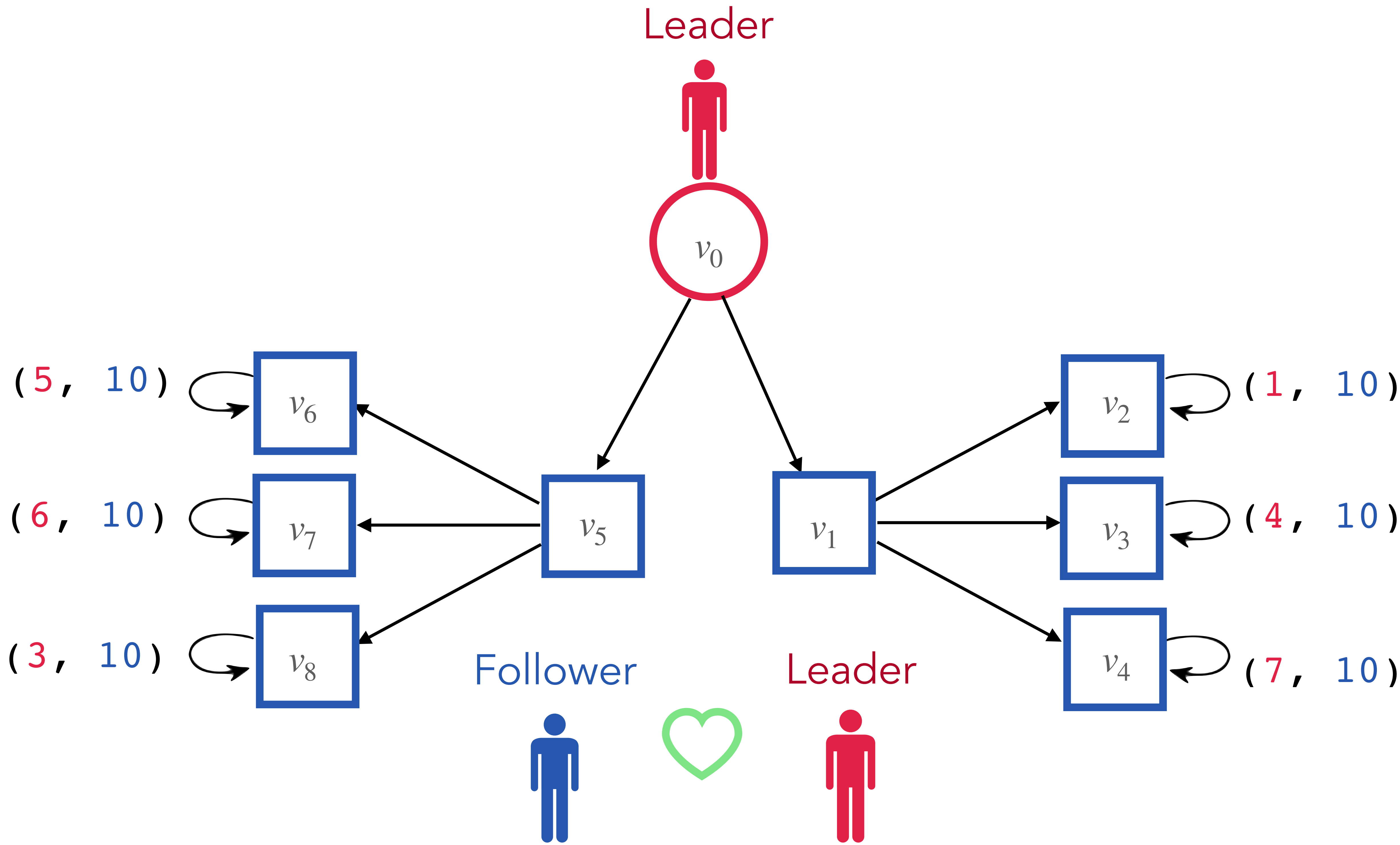
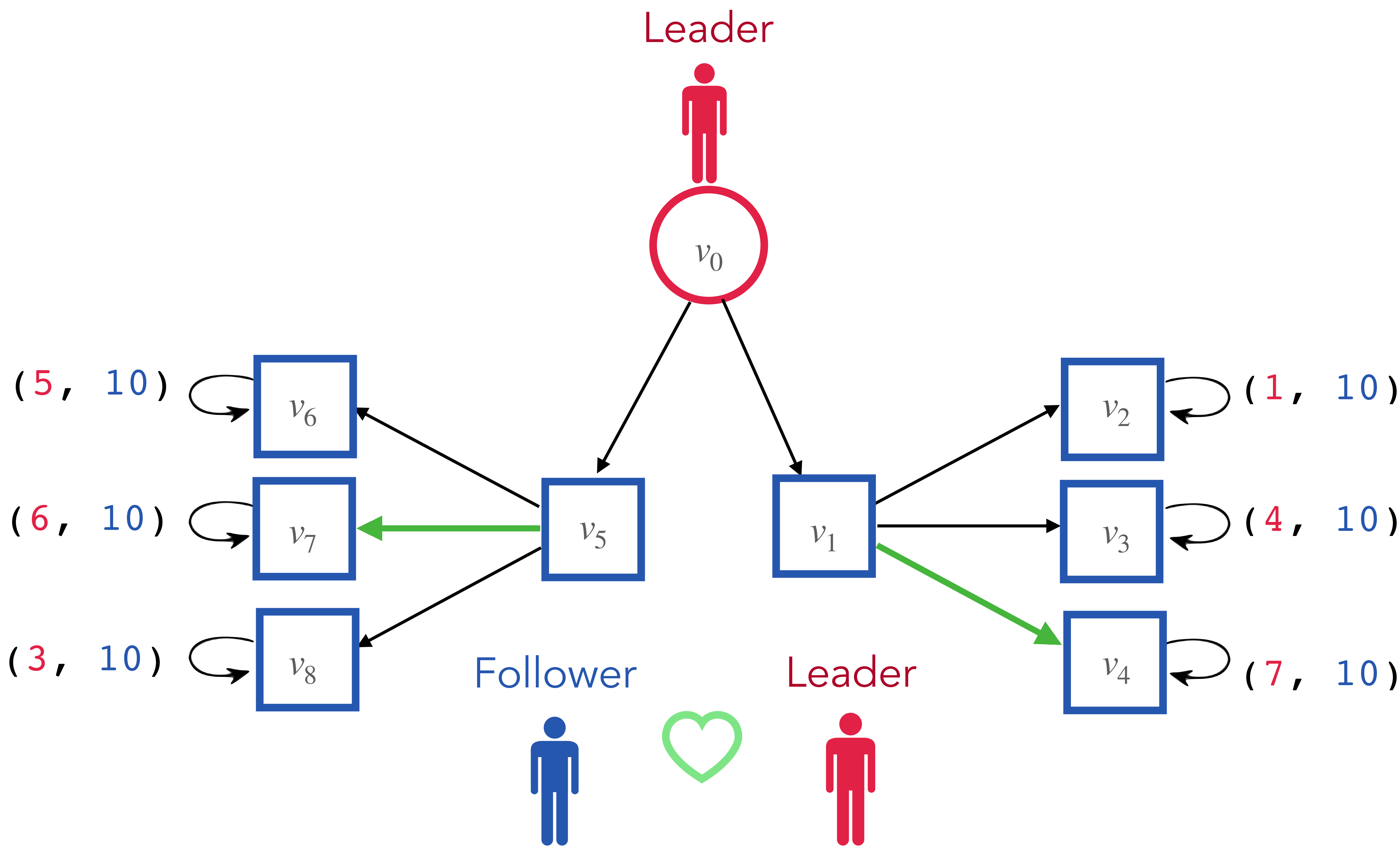


In the cooperative setting, Follower chooses Best-Response which `maximises` payoff of Leader

Cooperative vs Adversarial



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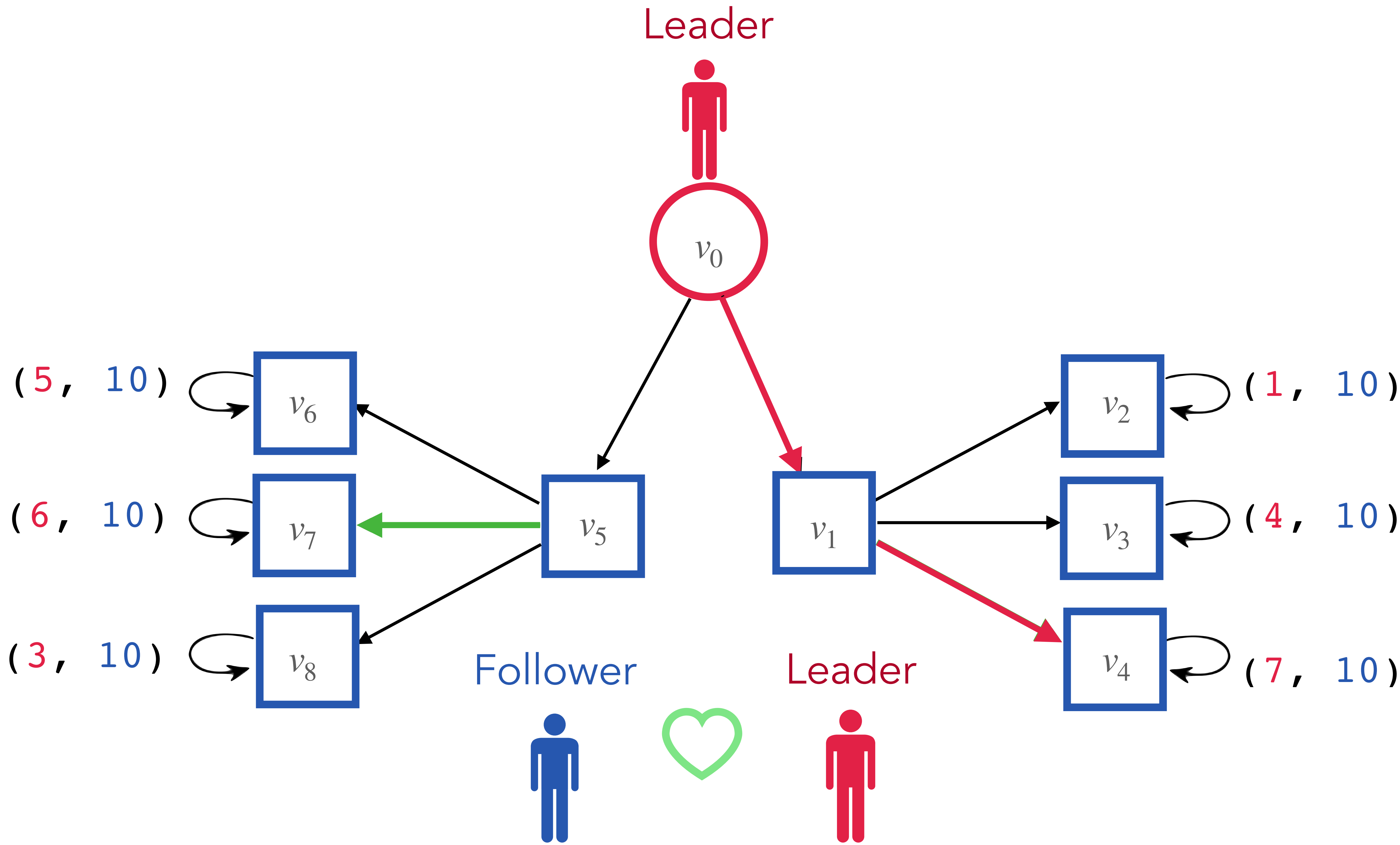

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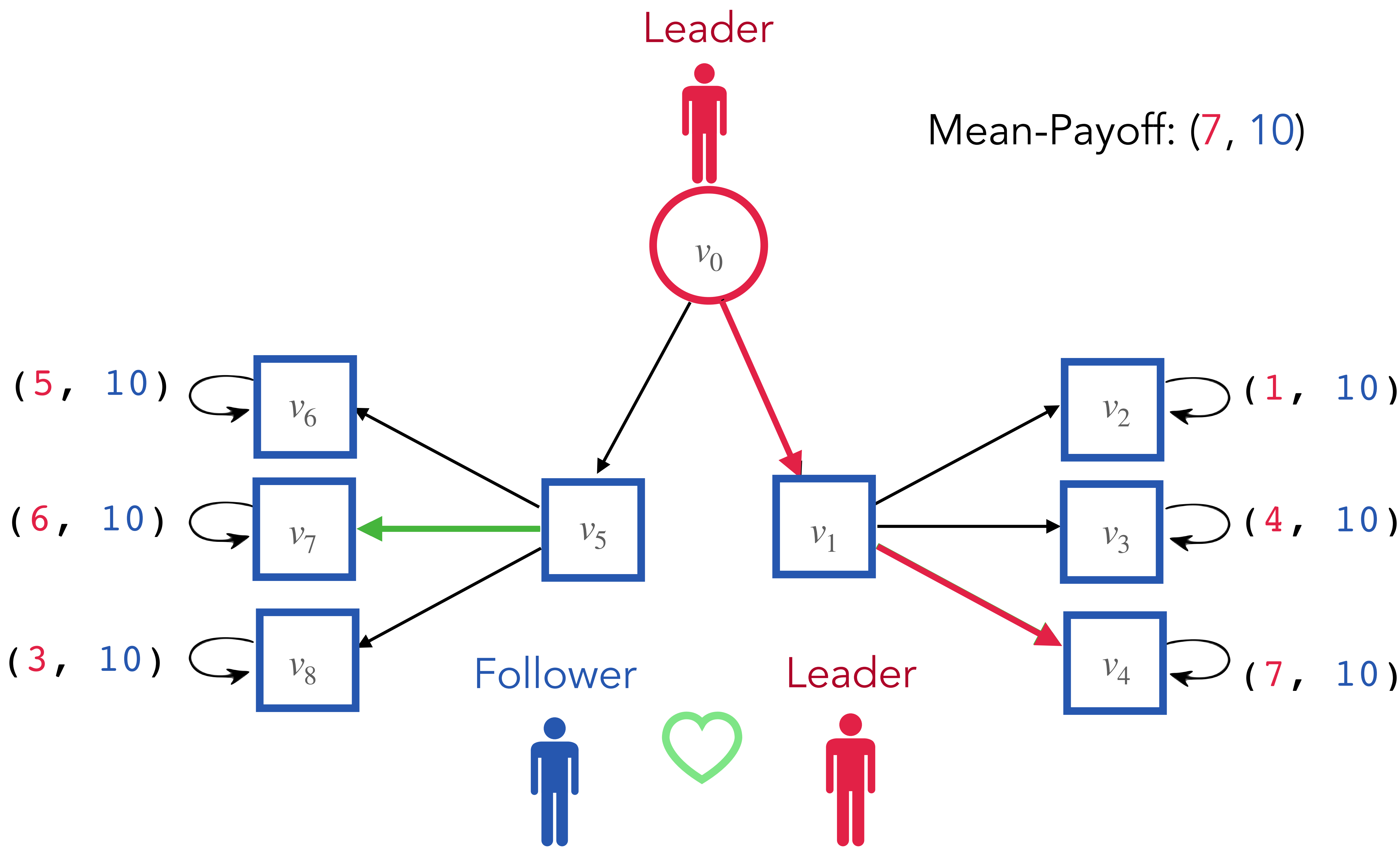

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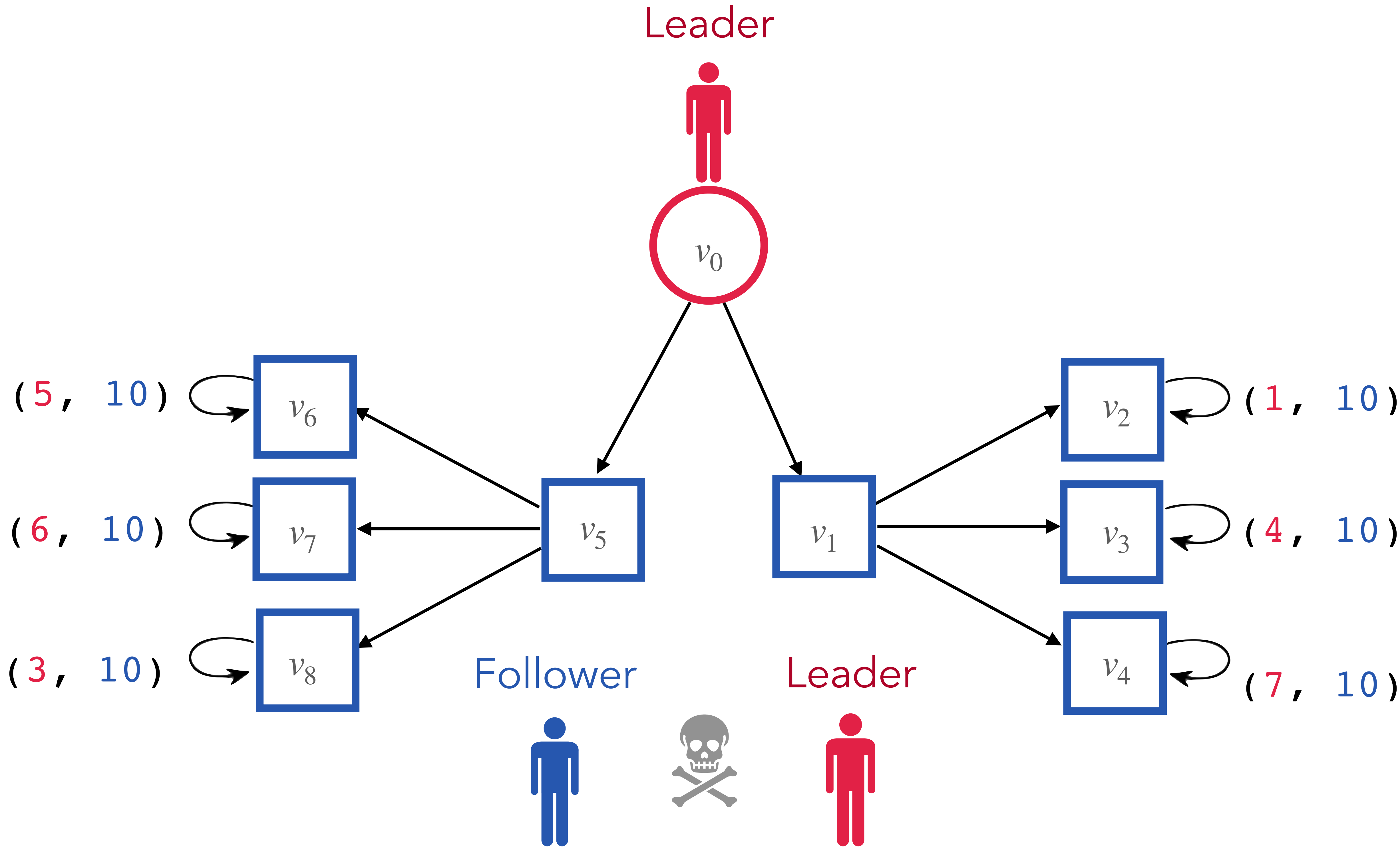
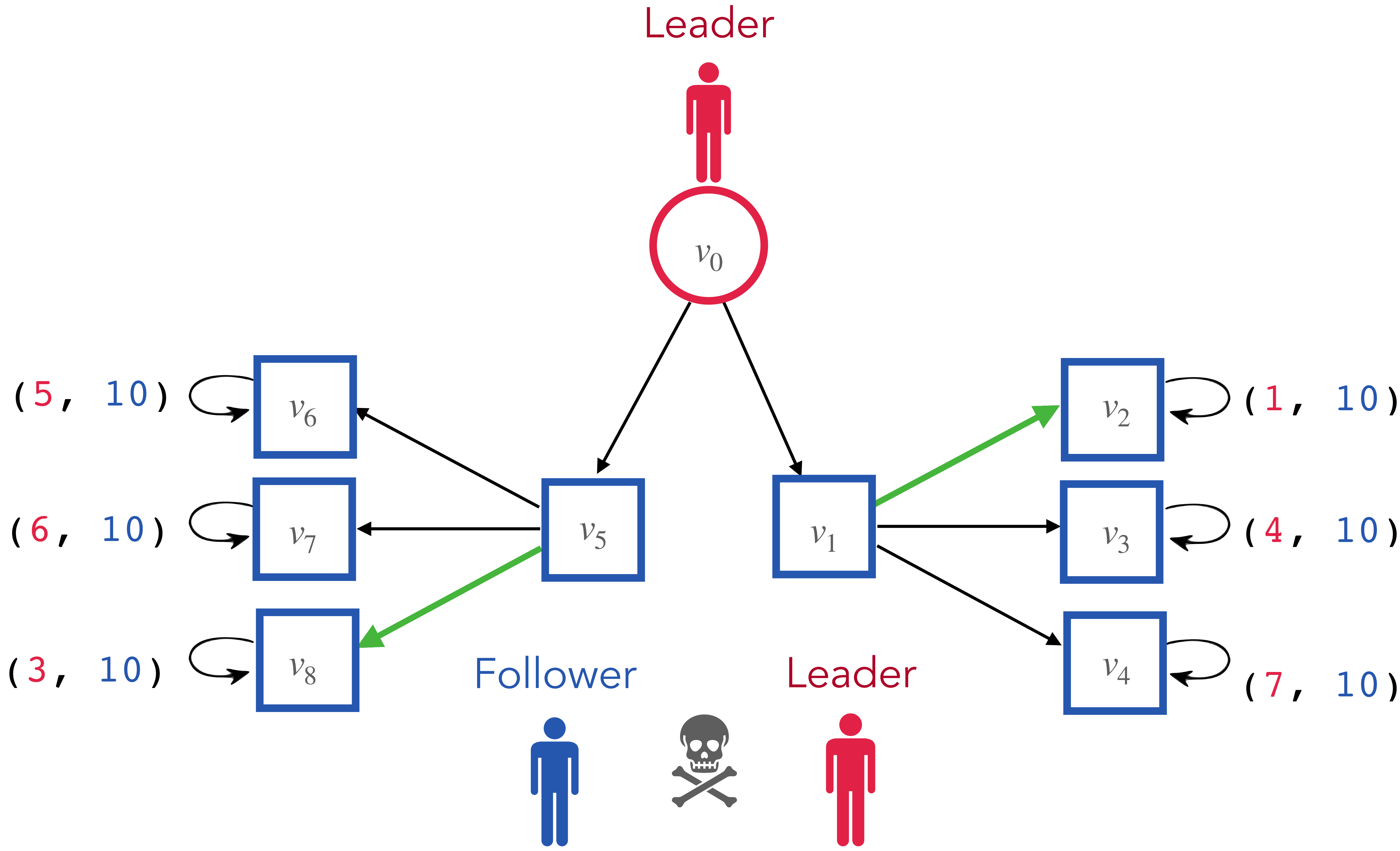


In the adversarial setting, Follower chooses Best-Response which `minimises` payoff of Leader

Cooperative vs Adversarial



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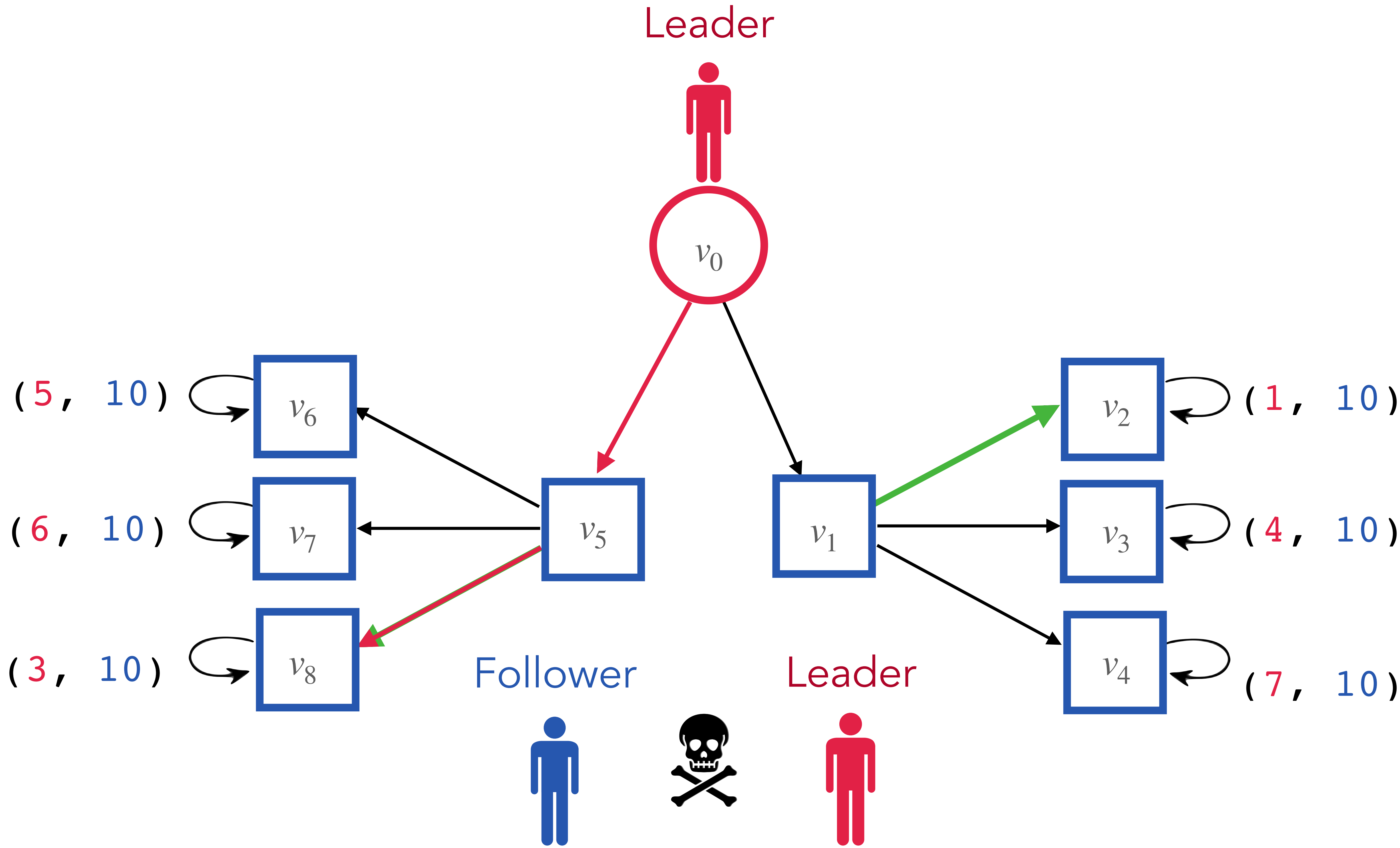

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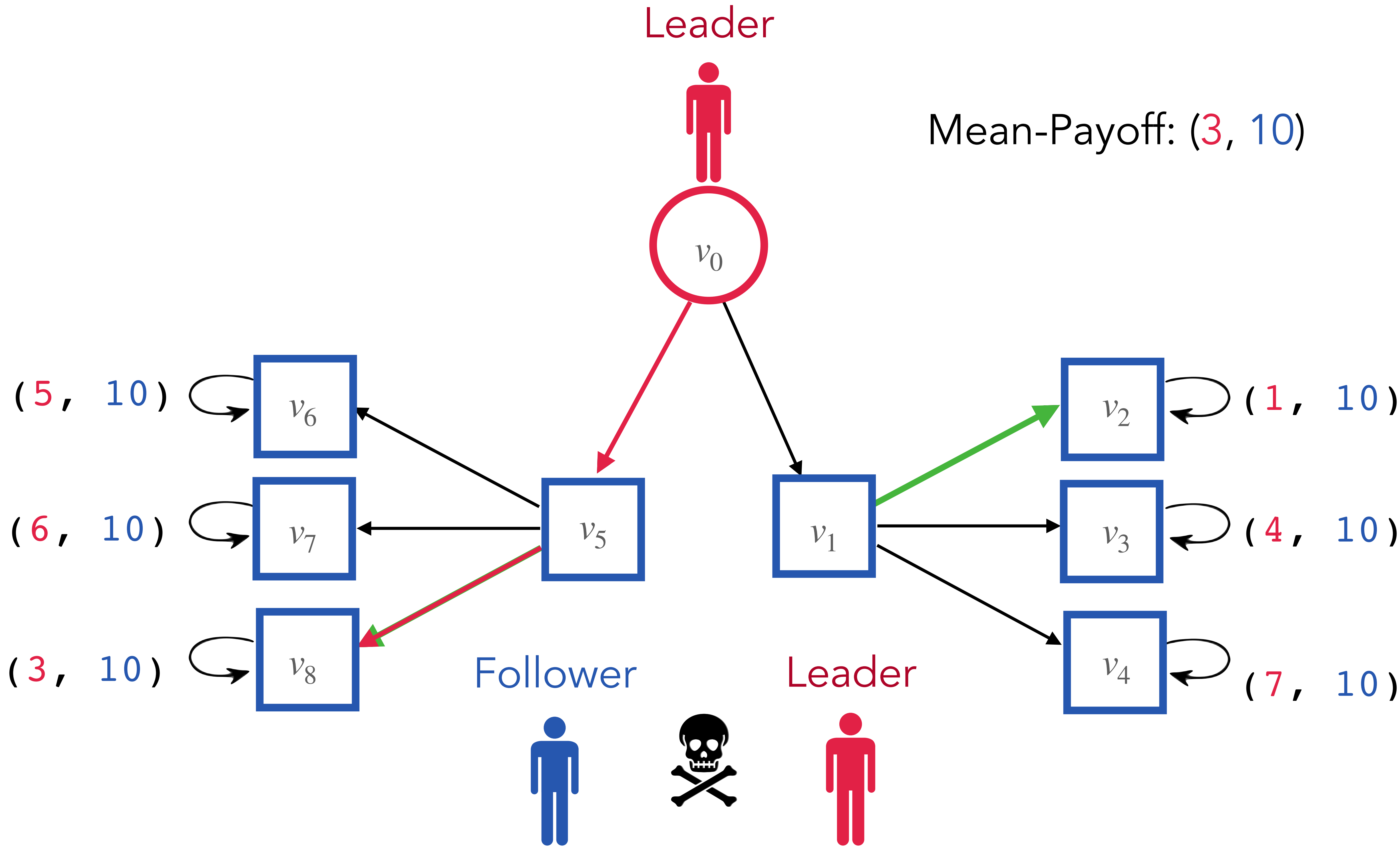

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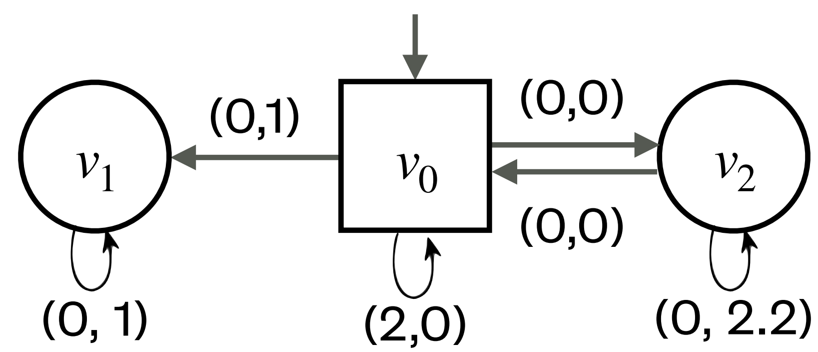
Stackelberg Mean Payoff Games with One Adversarial Follower

Two Players: Leader Follower

Mean-Payoff Game:



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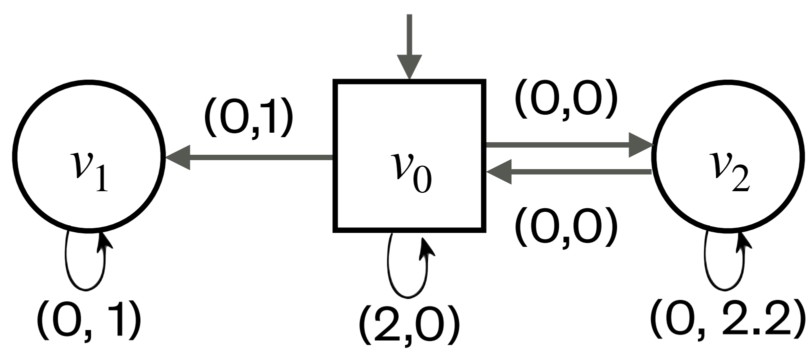
Stackelberg Mean Payoff Games with One Adversarial Follower

Leader

Adversarial Follower

Two Players:

Mean-Payoff Game:



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Best Responses May Not Exist

(Filiot, Gentilini and Raskin - ICALP 2020)

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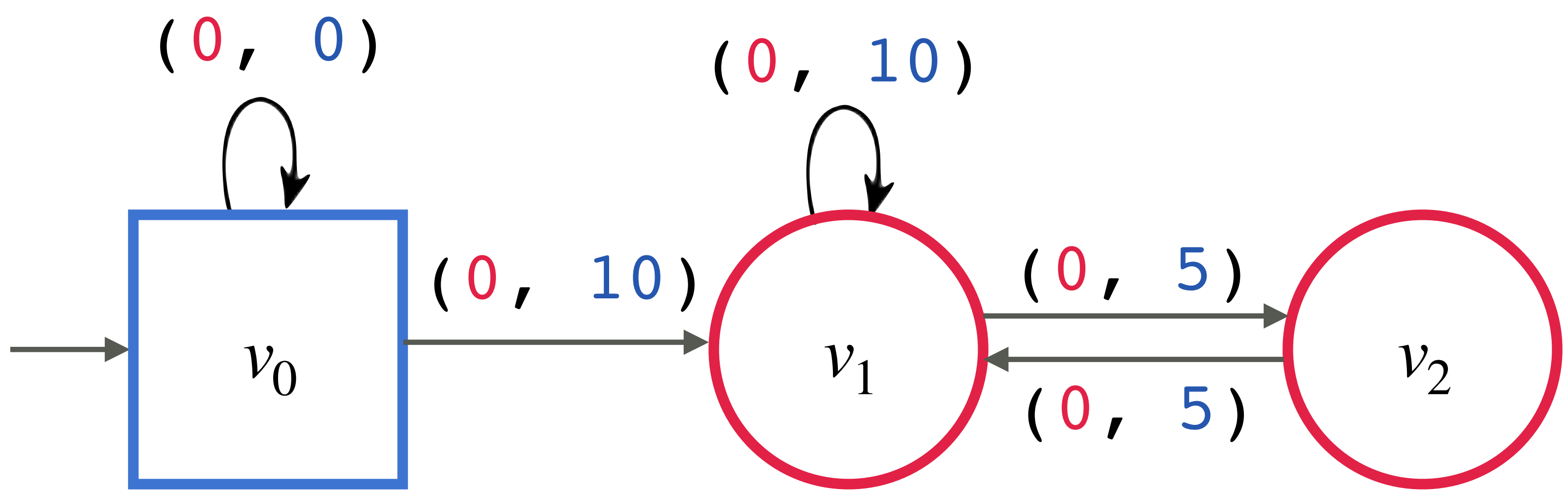

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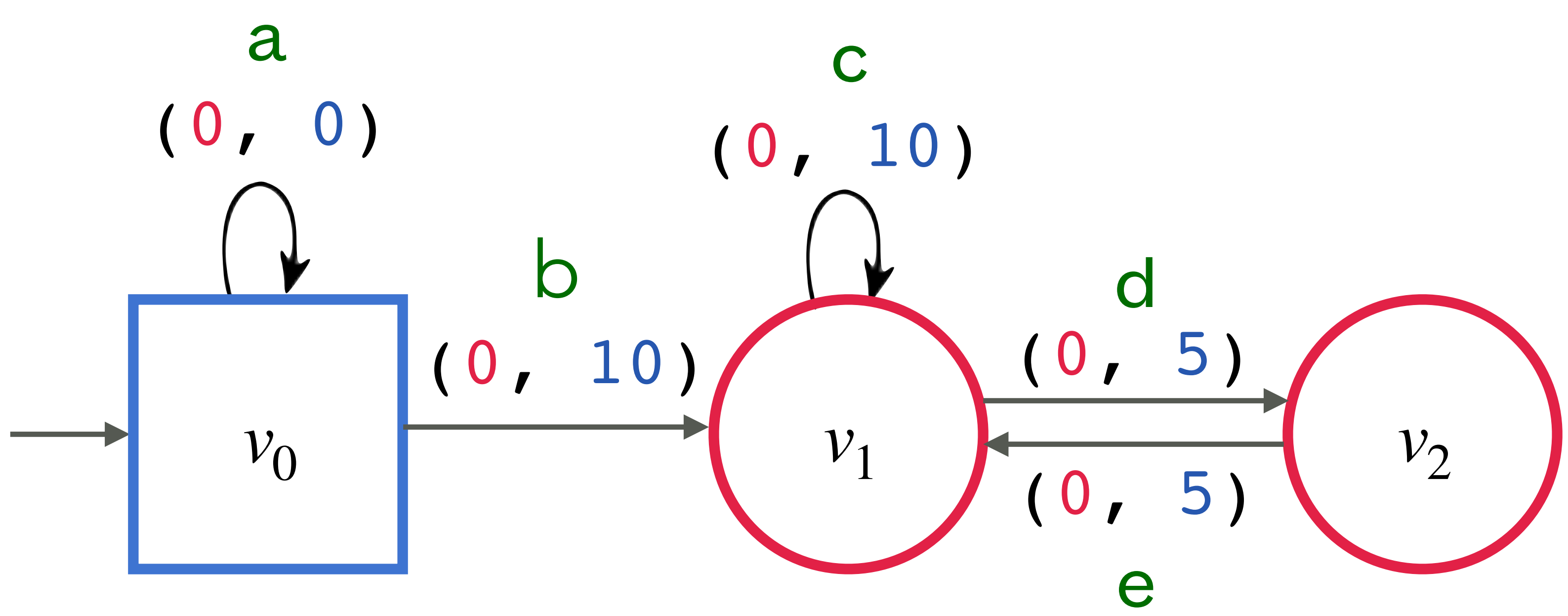

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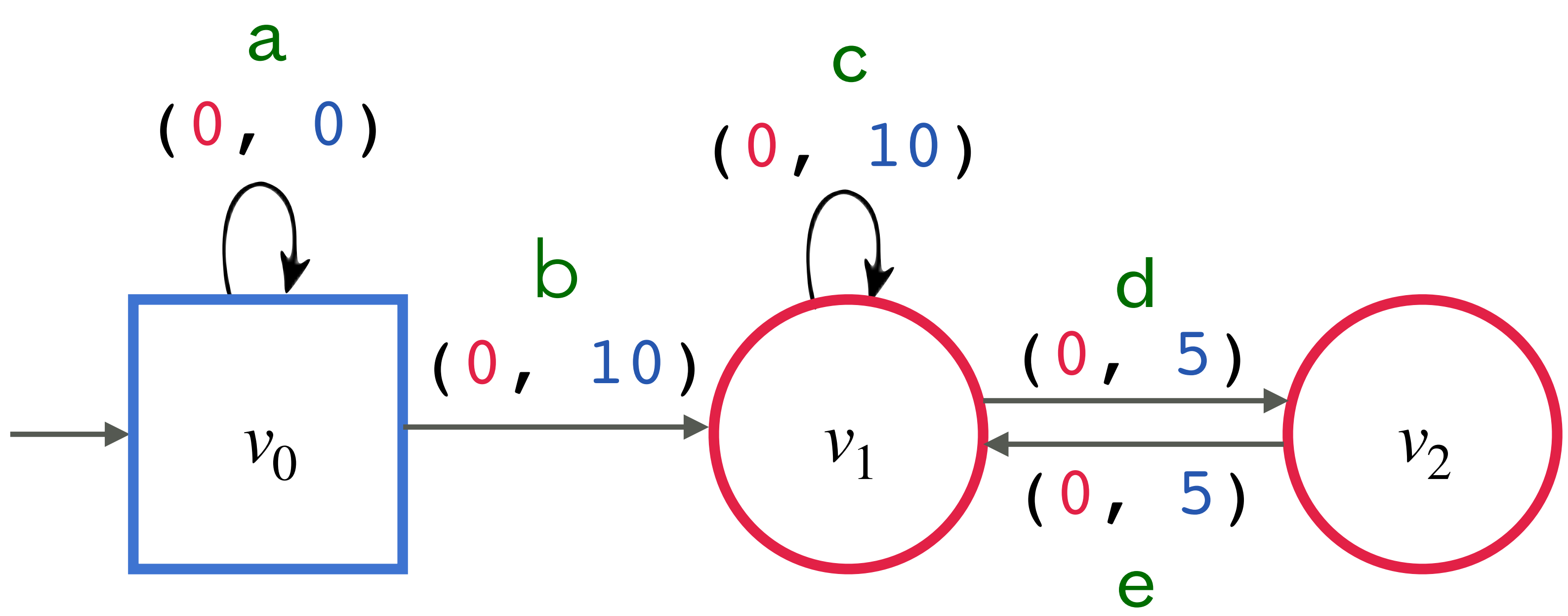

Leader strategy:

Best Responses May Not Exist

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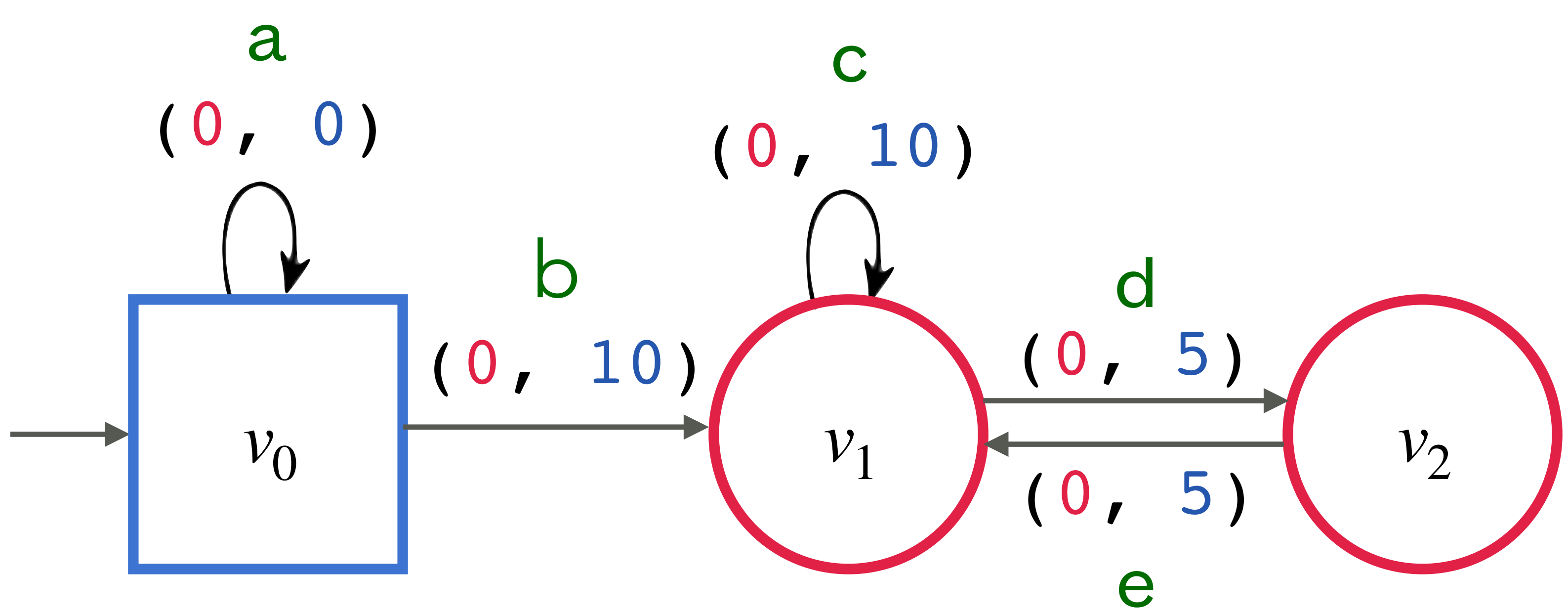

Leader strategy: If $a^k b$, then $(c^k de)^\omega$

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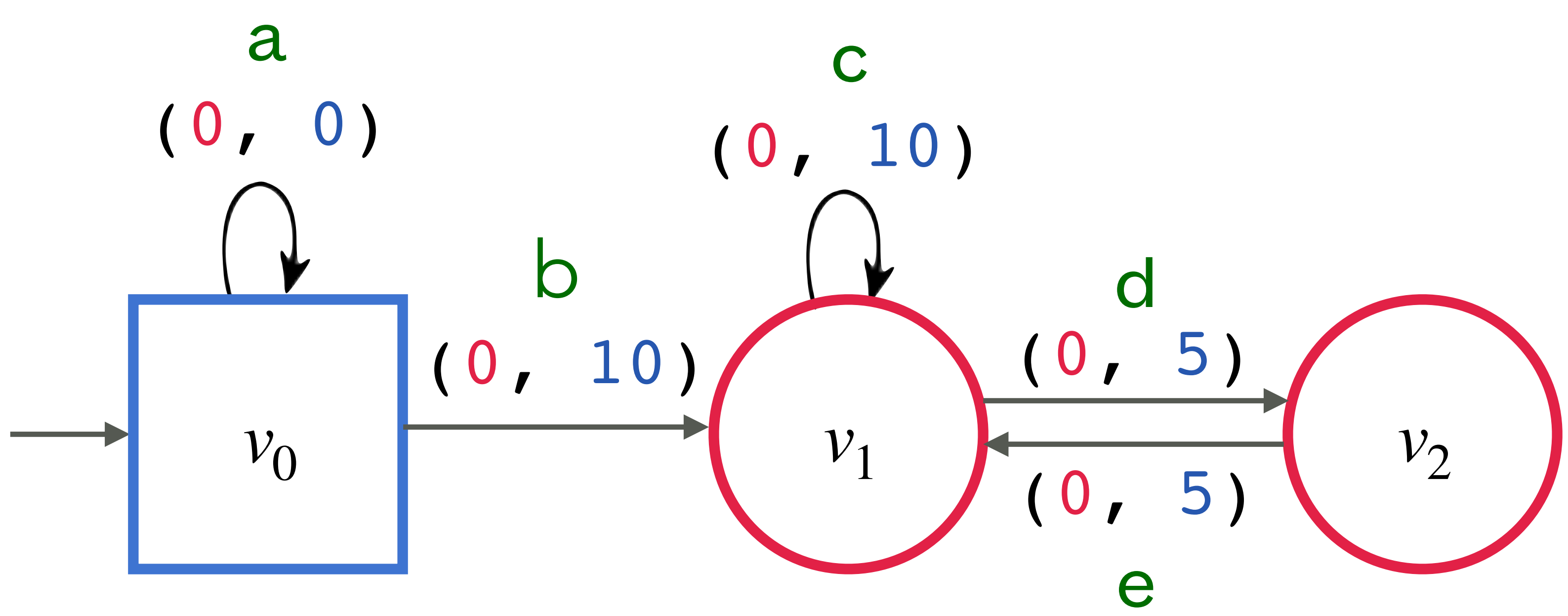

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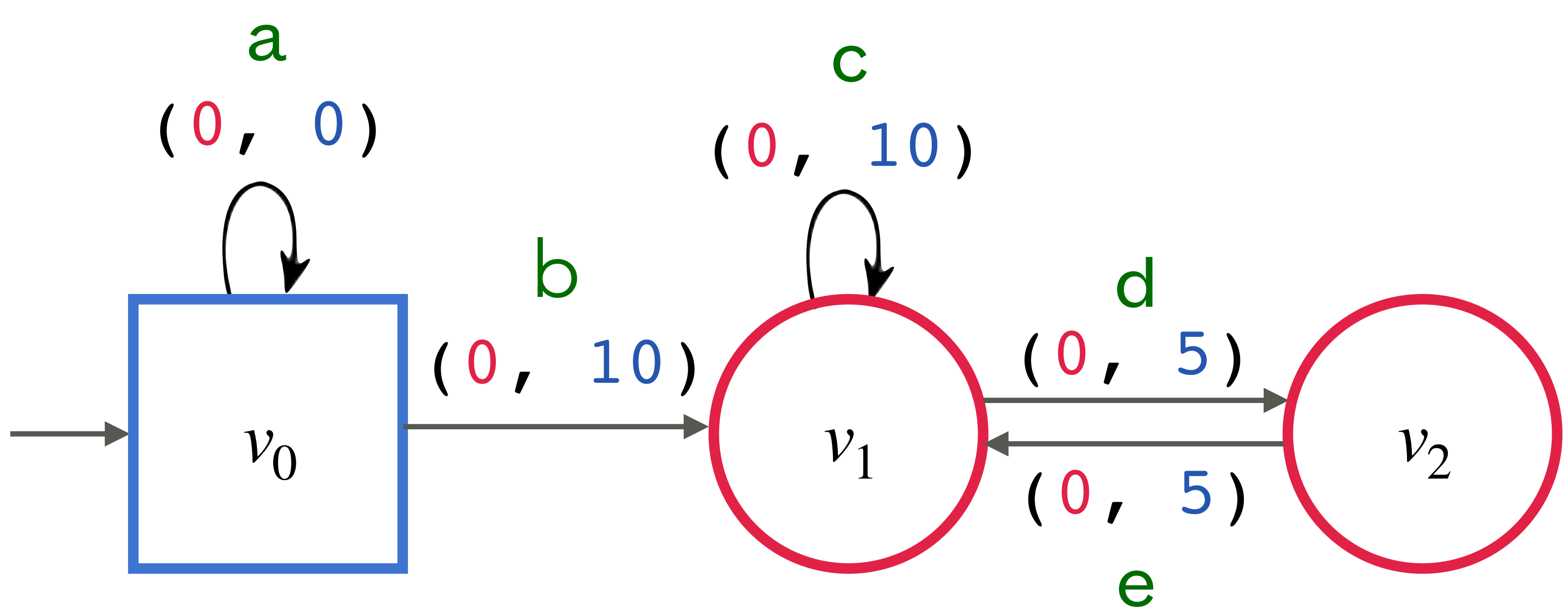

Leader strategy: If $a^k b$, then $(c^k de)^\omega$

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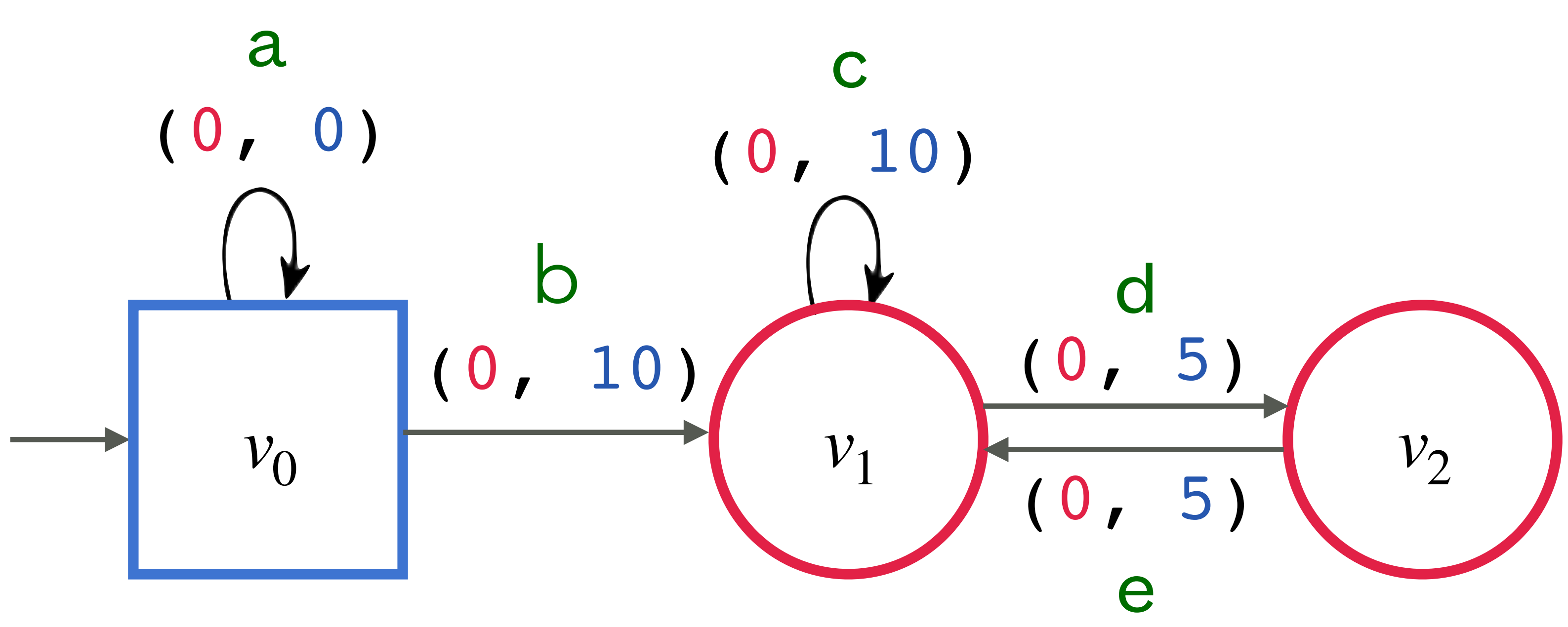

Leader strategy: If $a^k b$, then $(c^k de)^\omega$

Follower strategy: If $a^{1000} b$, then $(c^{1000} de)^\omega$

If $a^{100000} b$, then $(c^{100000} de)^\omega$

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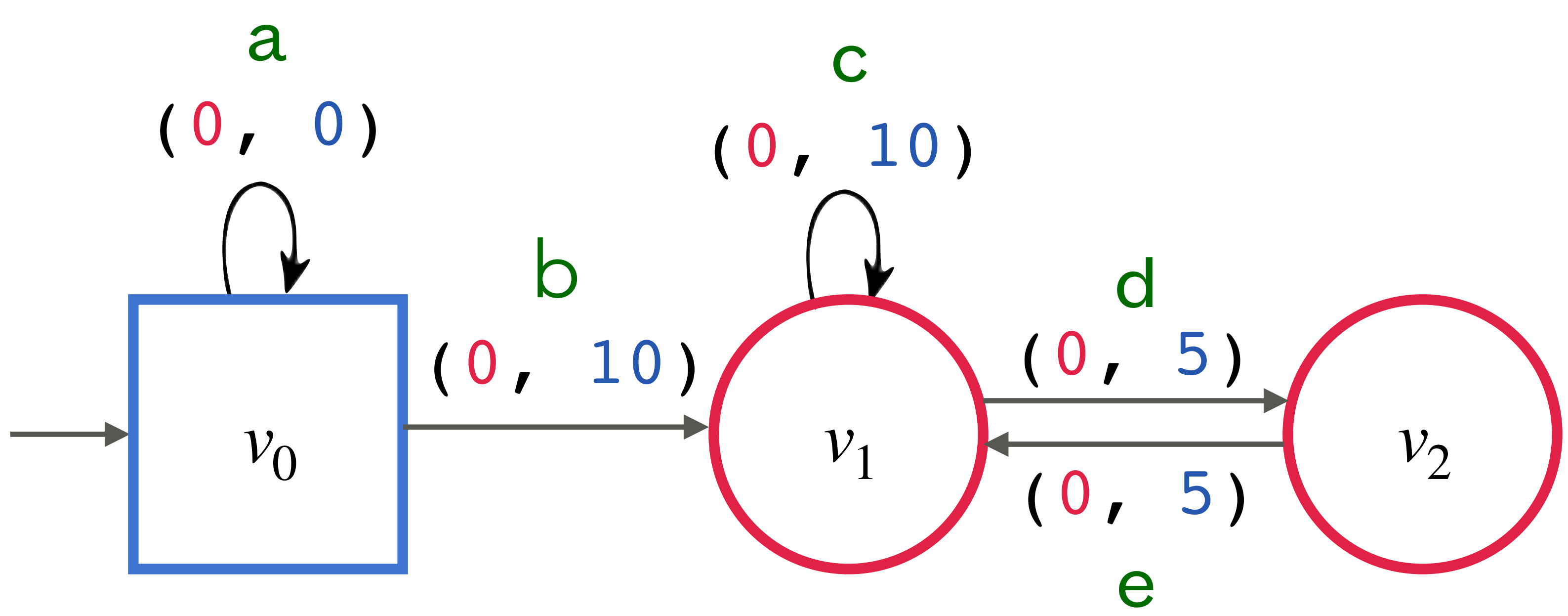

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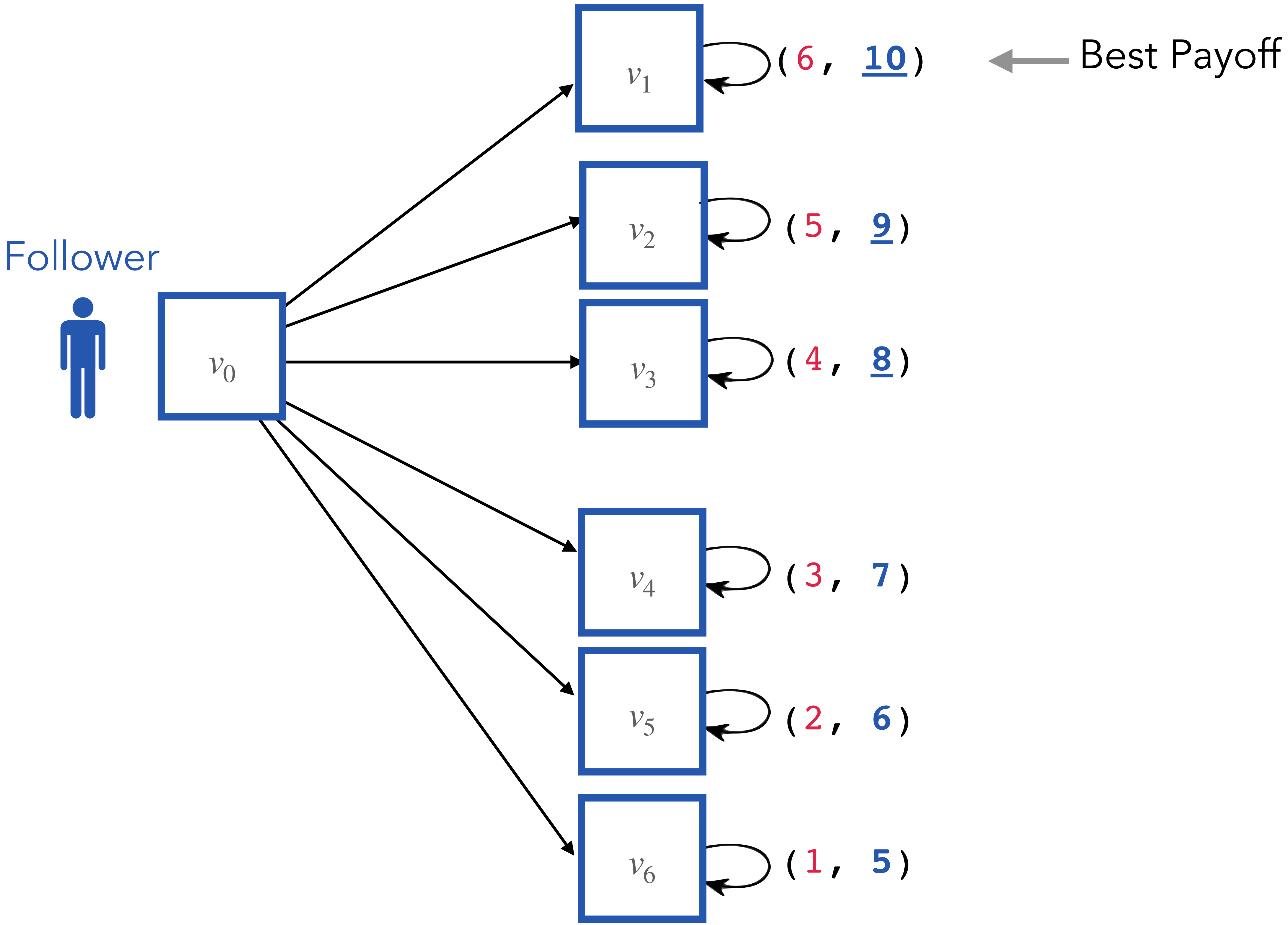
Follower strategy: If $a^{1000} b$, then $(c^{1000} de)^\omega$

If $a^{100000} b$, then $(c^{100000} de)^\omega$

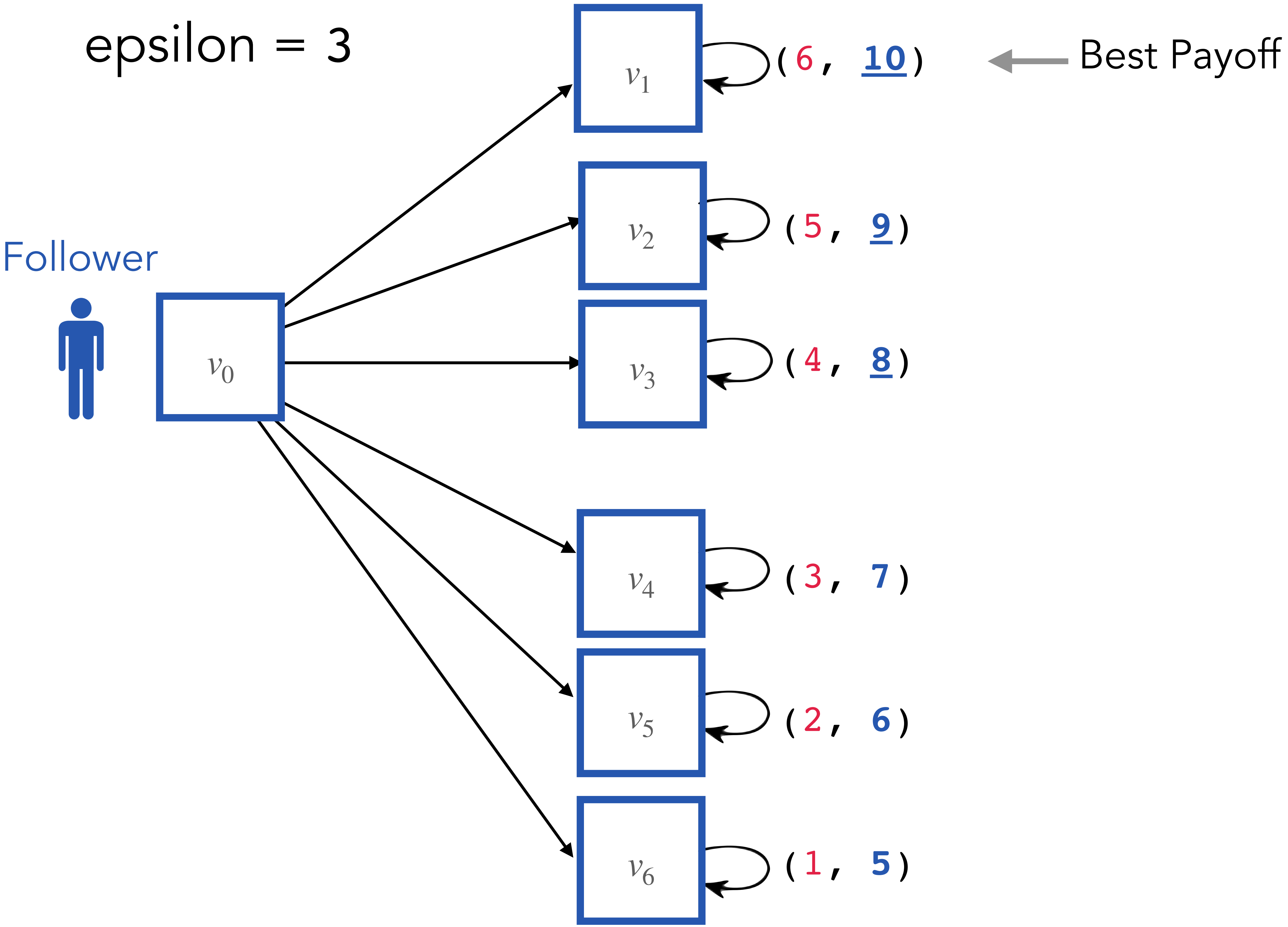
If $a^\infty b$, then the vertex v_1 is never reached.

Epsilon-Optimal Best Response



Epsilon-Optimal Best Response

epsilon = 3



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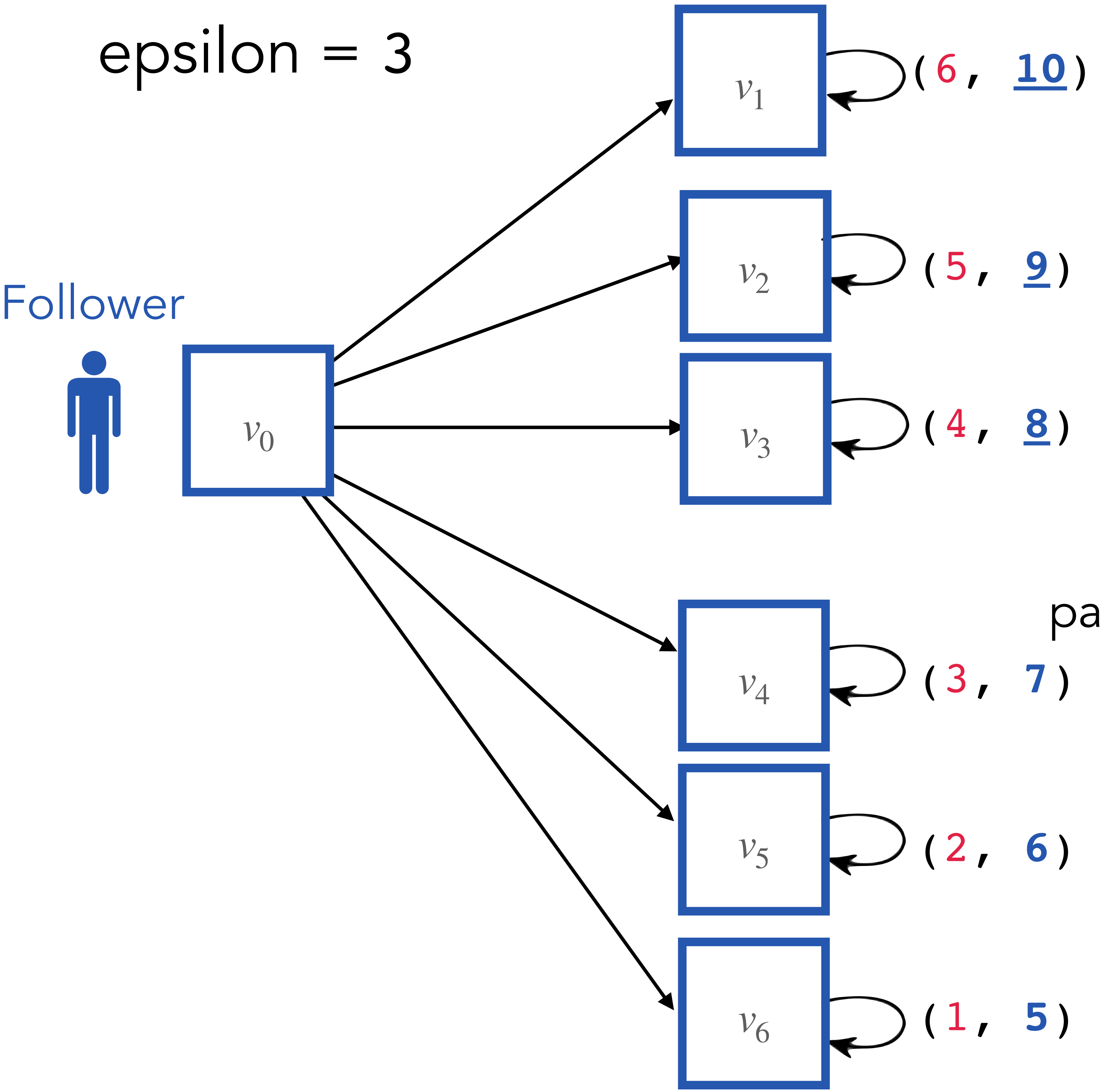

Best Payoff

Player 1 can choose any strategy that gives him a payoff > (best payoff) - epsilon

Epsilon-Optimal Best Response

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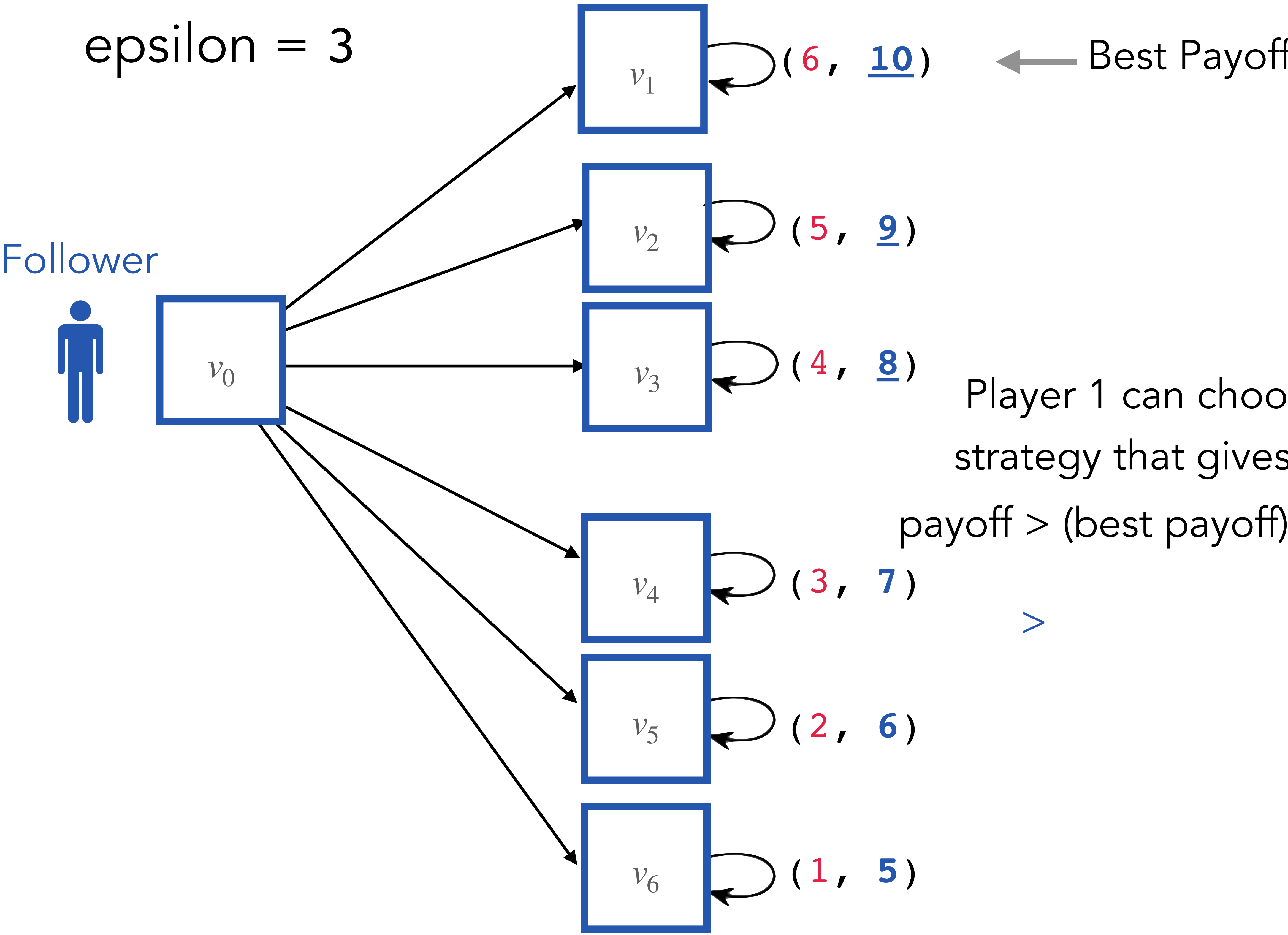

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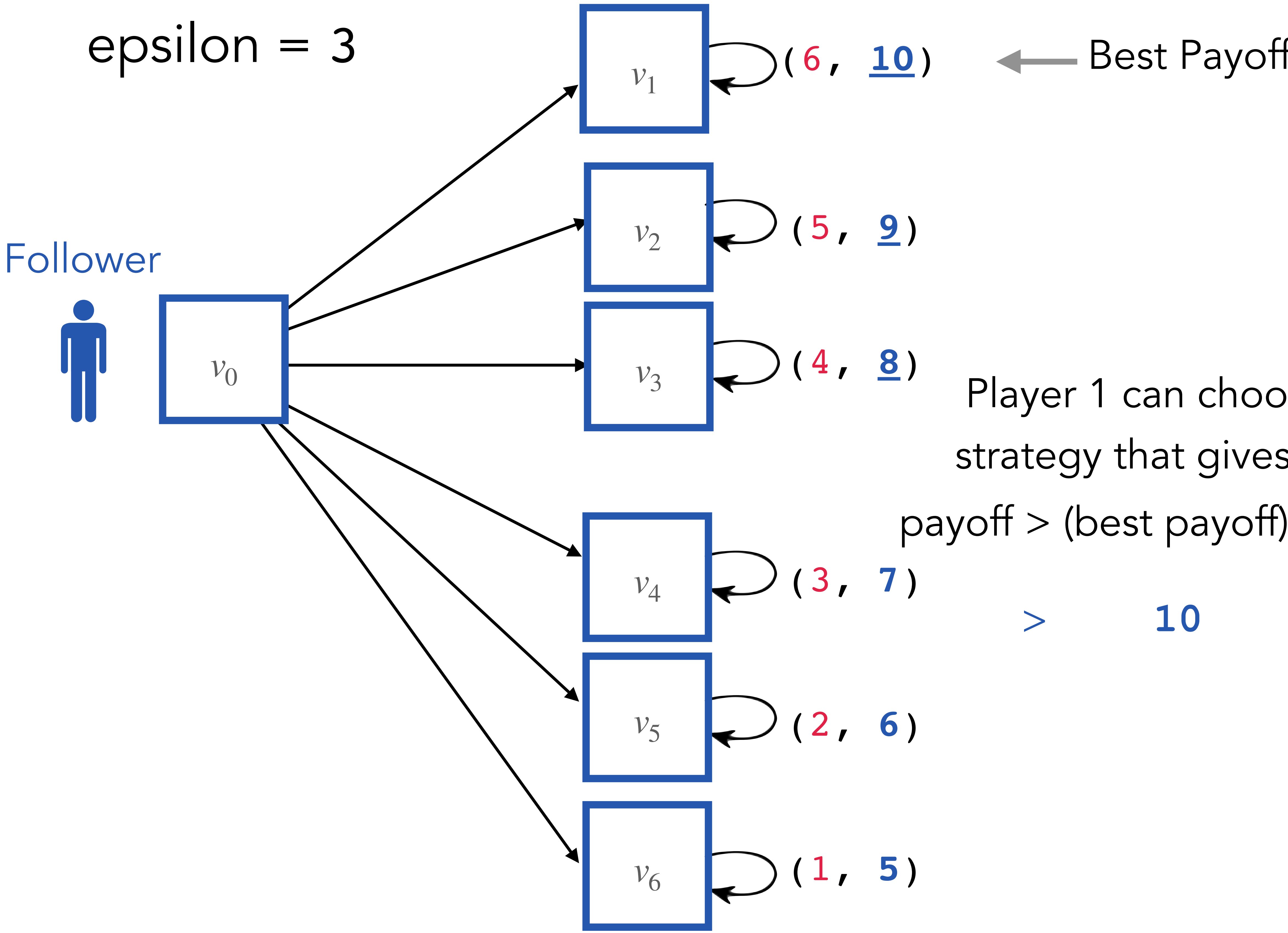

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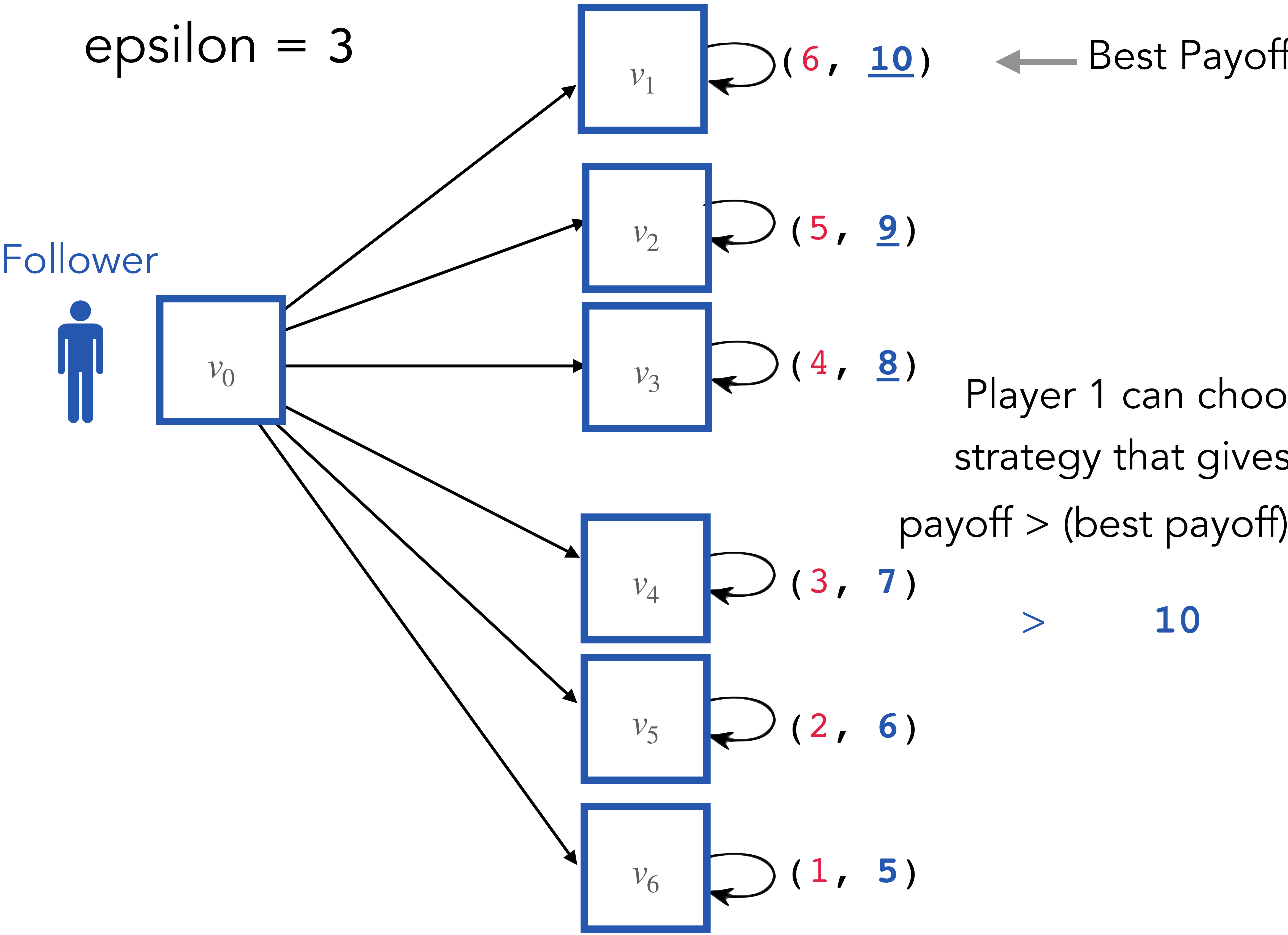

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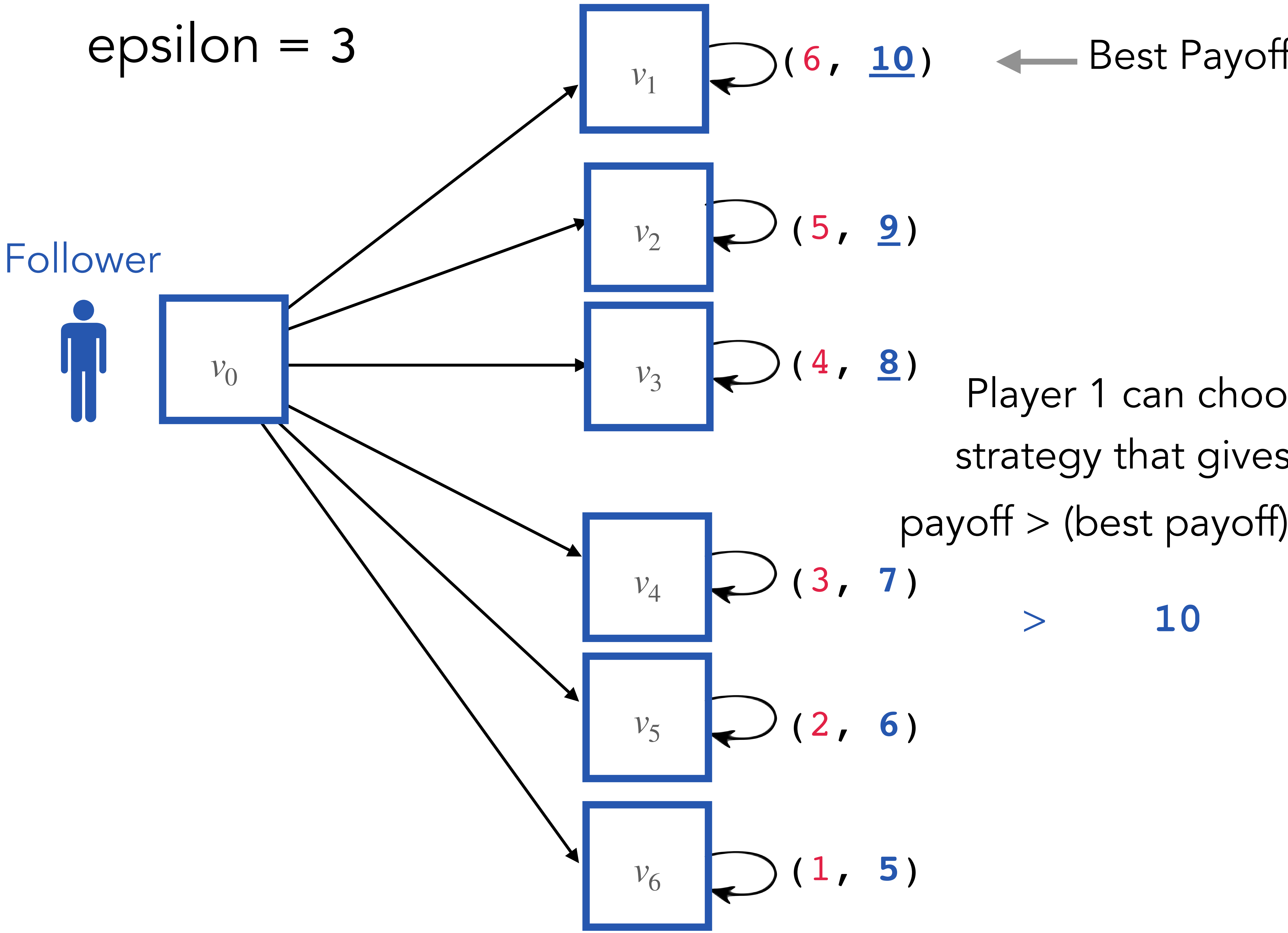

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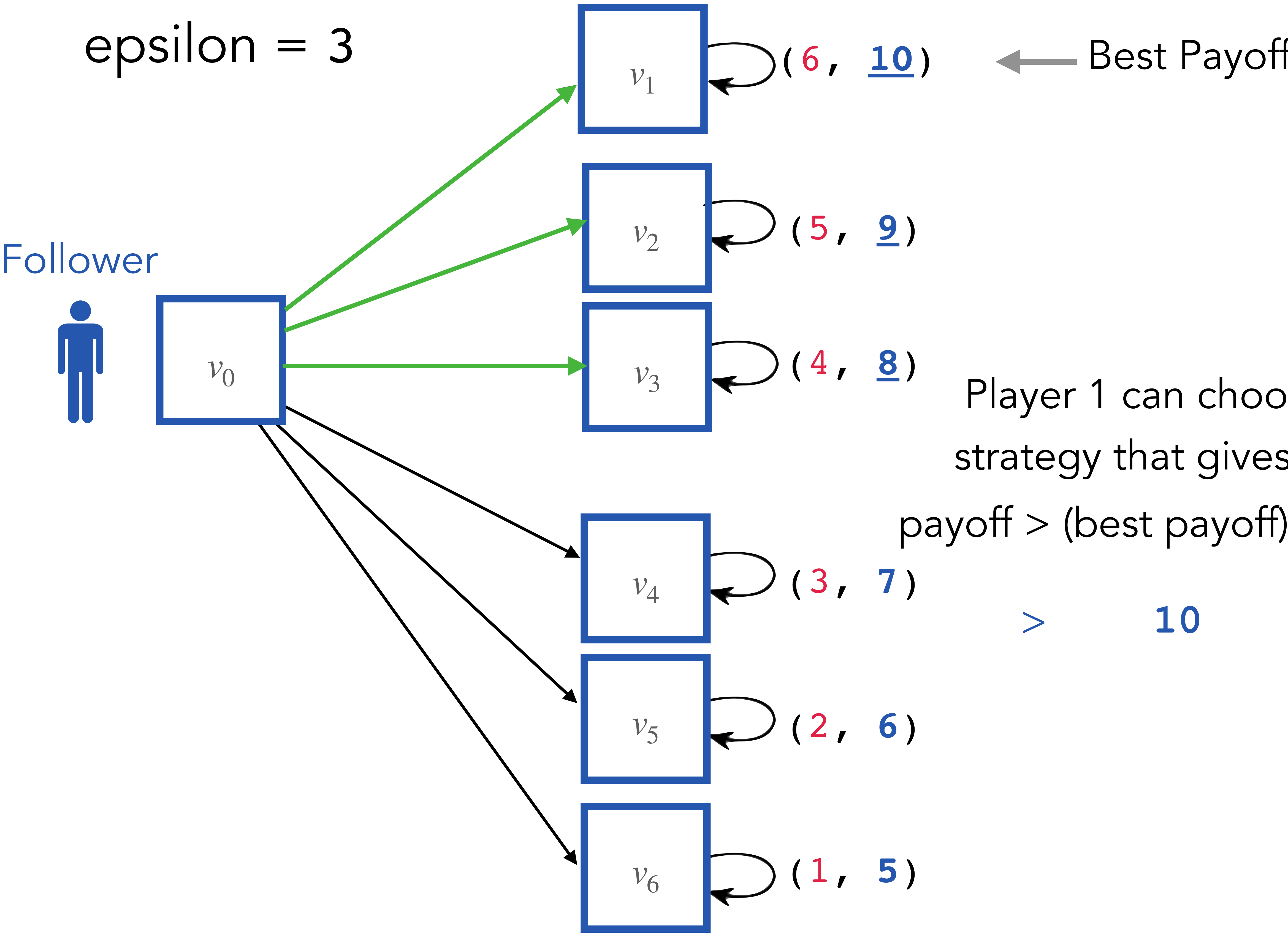

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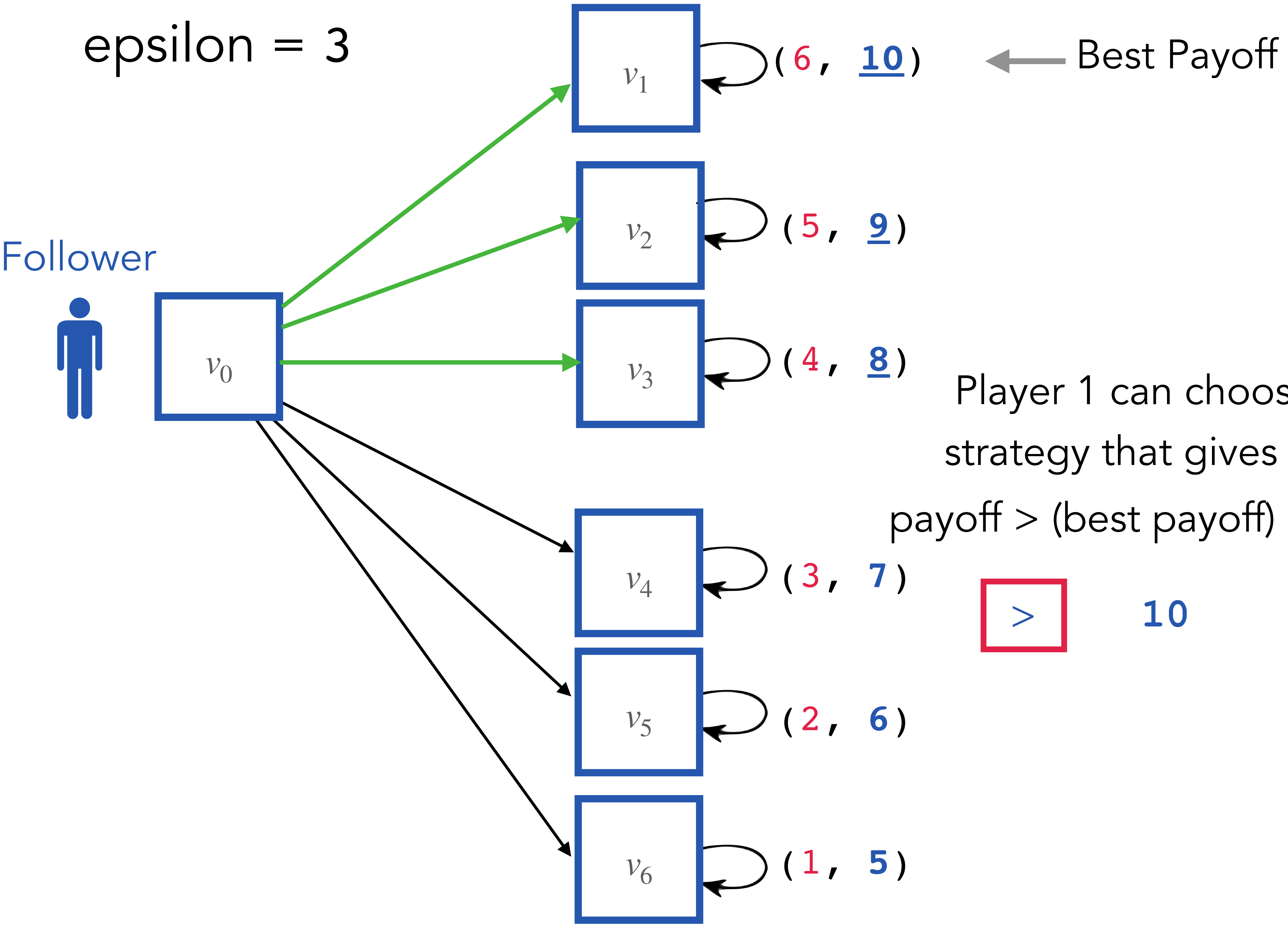

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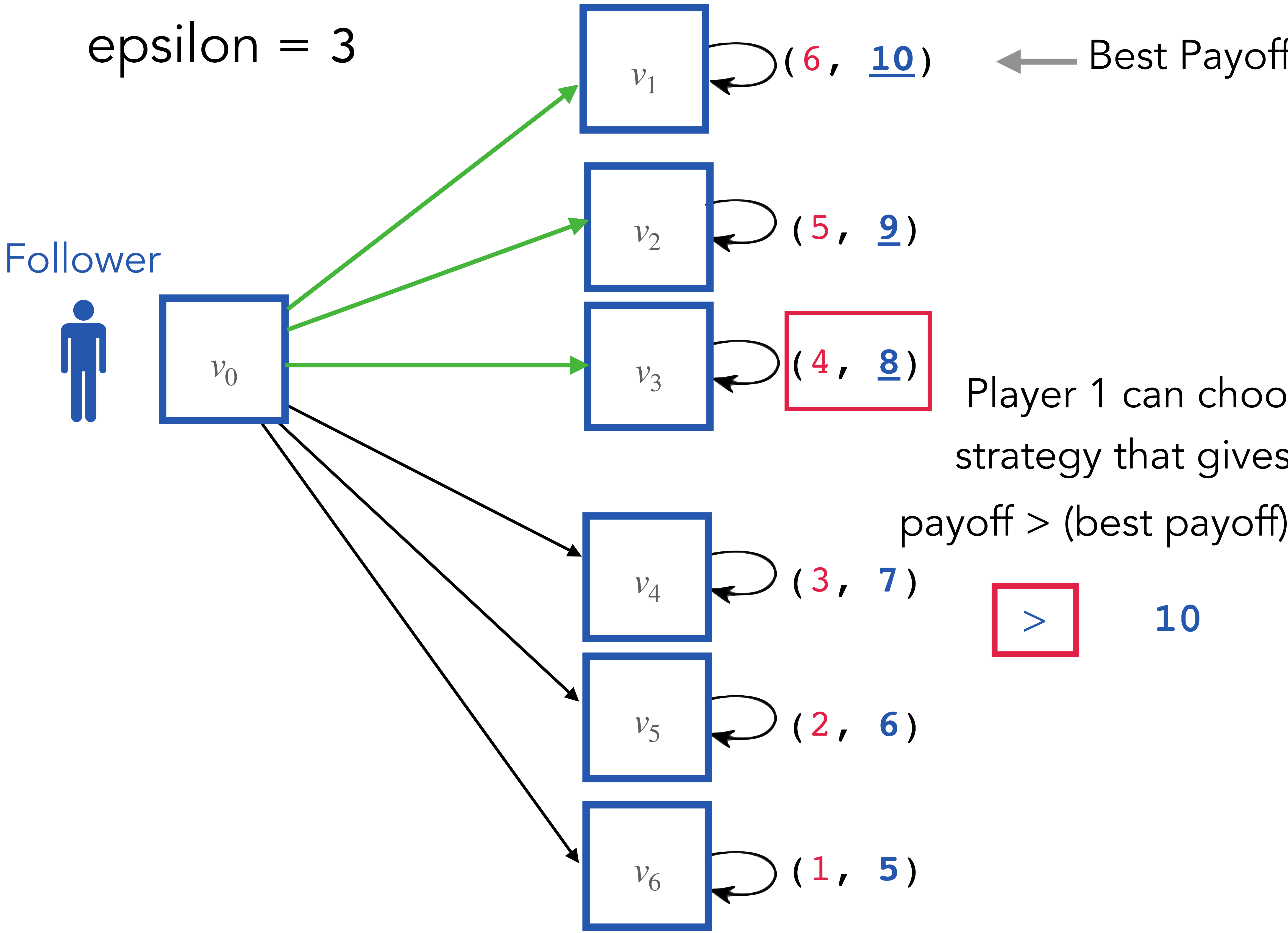

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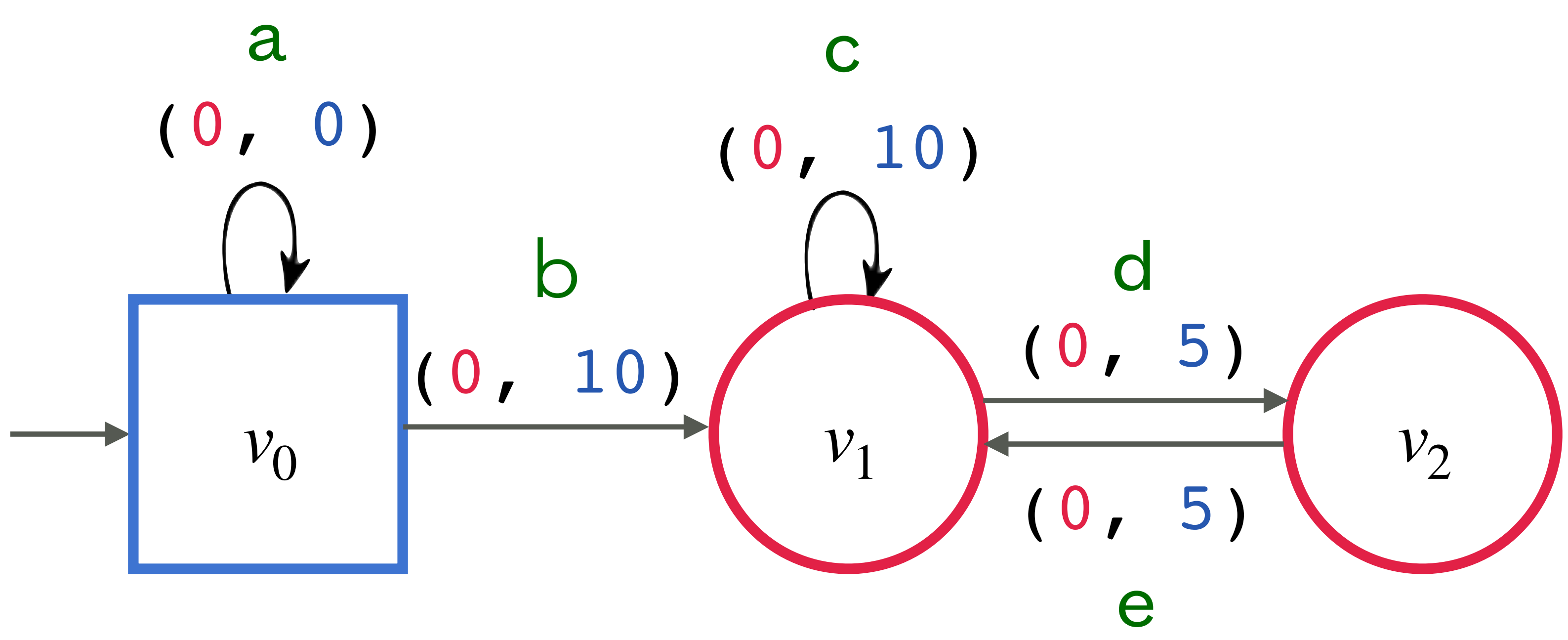
Epsilon-Optimal Best Responses Always Exist

(Filiot, Gentilini and Raskin - ICALP 2020)



Leader strategy: If $a^k b$, then $(c^k de)^\omega$

Epsilon-Optimal Best Responses Always Exist

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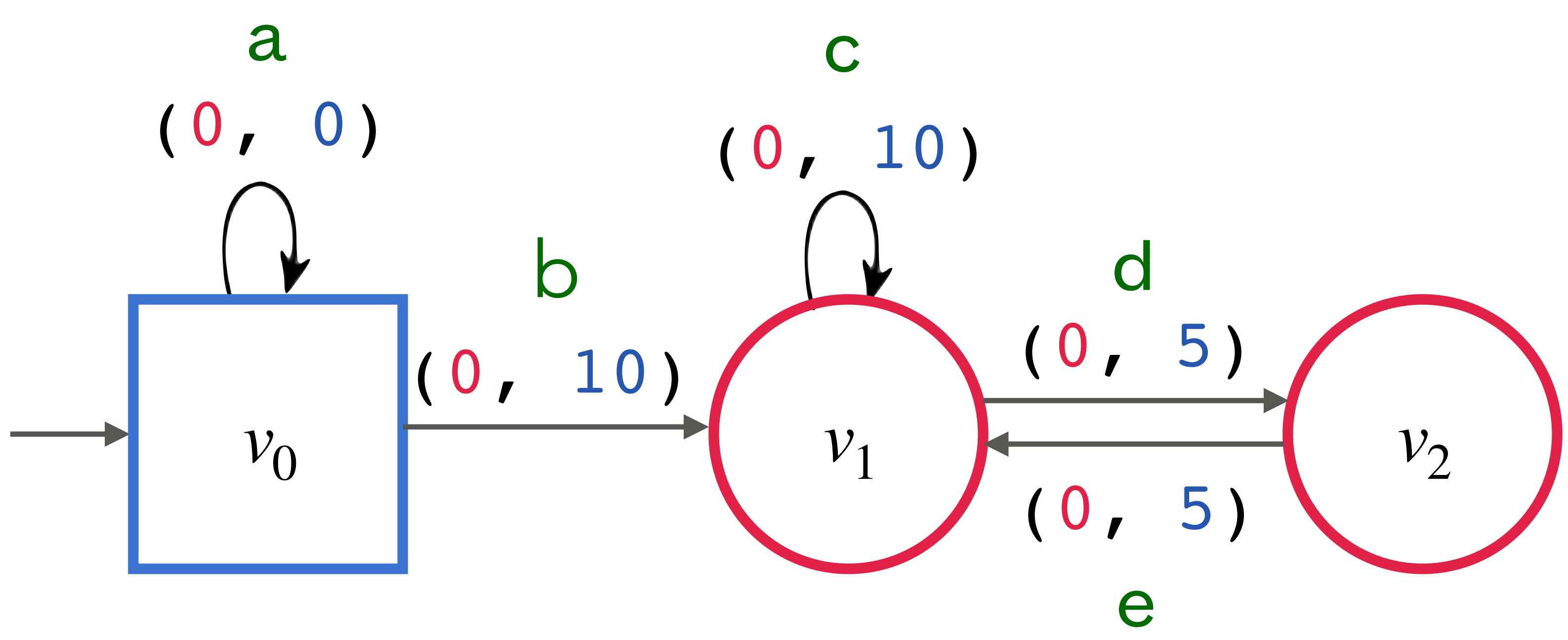

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Follower strategy:

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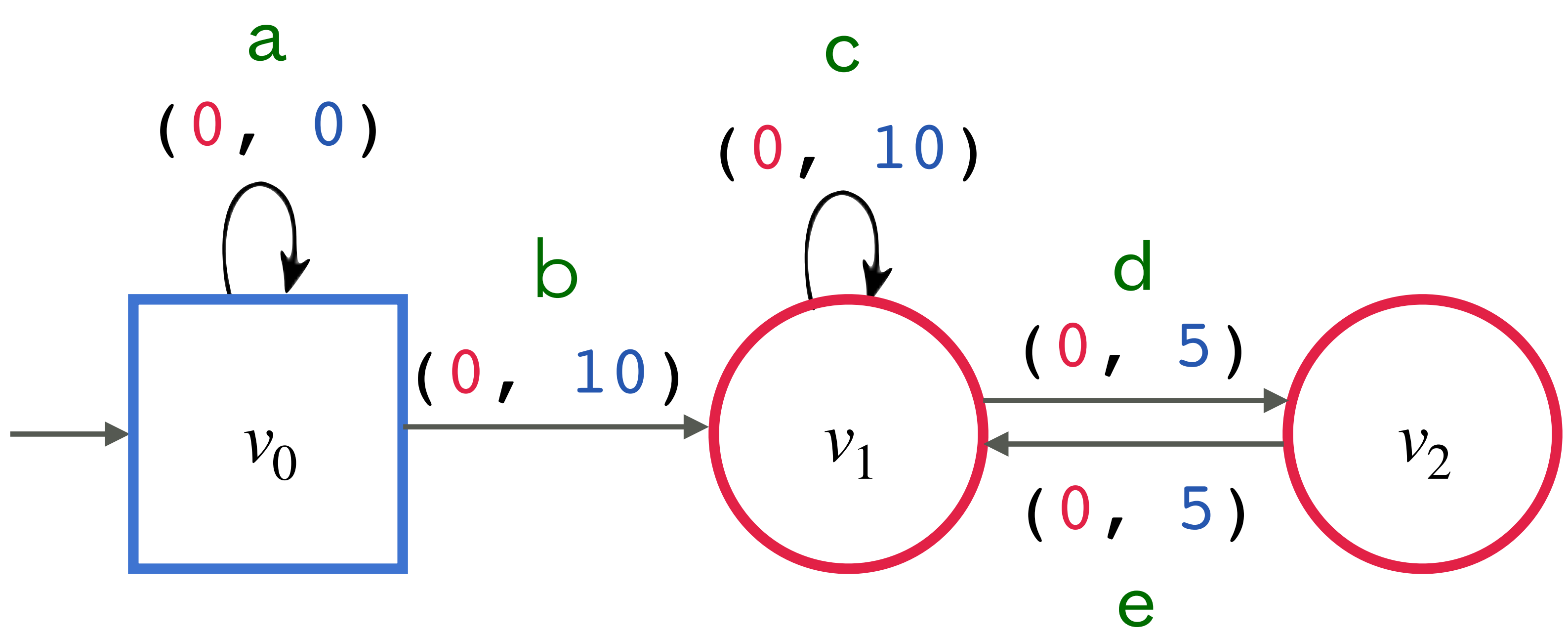

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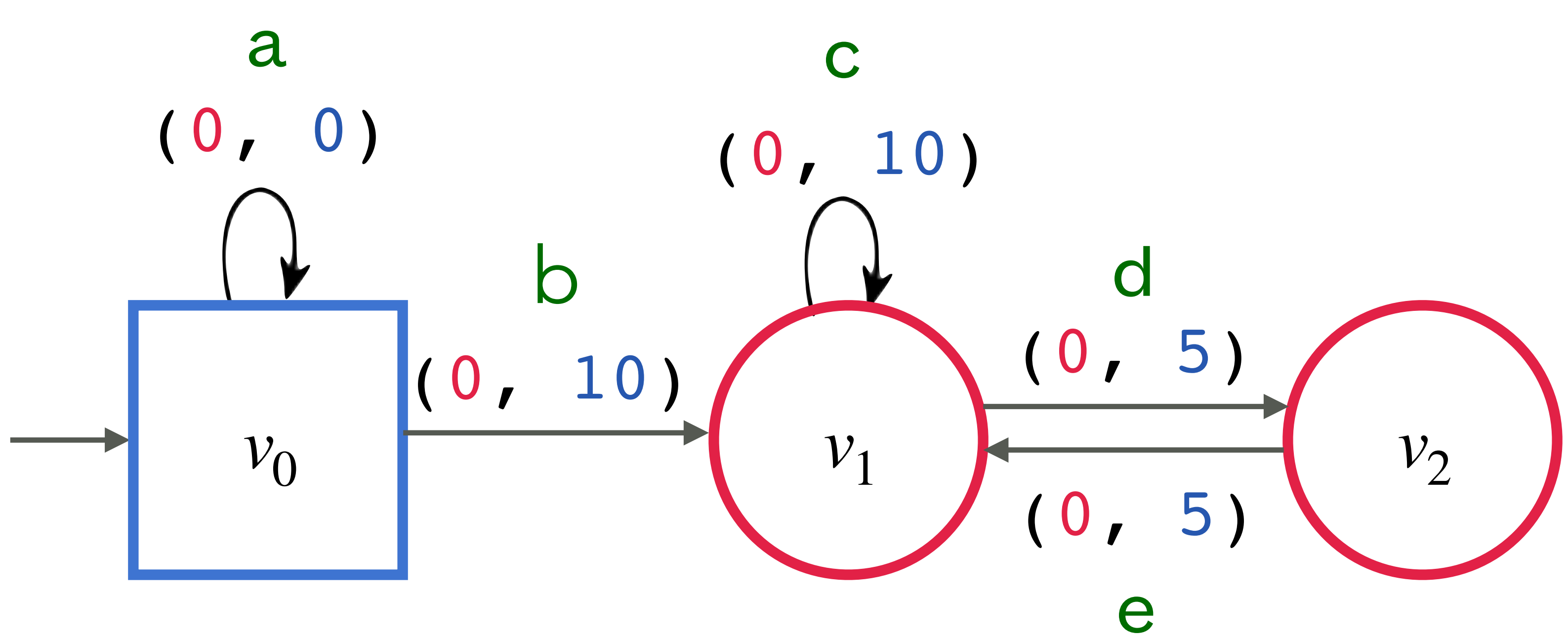

Leader strategy: If $a^k b$, then $(c^k de)^\omega$

Follower strategy: For $\epsilon = 0.1$, play $a^{1000}b$

For $\epsilon = 0.001$, play $a^{100000}b$

Adversarial Stackelberg Value ($\mathbf{ASV}$)

(Filiot, Gentilini and Raskin - ICALP 2020)

$\mathbf{ASV}$ is the largest mean-payoff value the Leader can obtain when the Follower plays an **adversarial** best response.

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$$\mathbf{ASV}(\sigma_0)(v) = \sup_{\epsilon > 0} \inf_{\sigma_1 \in \mathbf{BR}^{\epsilon}(\sigma_0)} \text{Mean-Payoff } [\text{Outcome}(\sigma_0, \sigma_1)]$$

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σ_0 : Leader Strategy

σ_1 : Follower Strategy

$\mathbf{BR}^{\epsilon}(\sigma_0)$: Epsilon-Optimal Best Response of Follower to Leader's strategy σ_0

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$$\mathbf{ASV}(v) = \sup_{\sigma_0} \mathbf{ASV}(\sigma_0)(v)$$

σ_0 : Leader Strategy

σ_1 : Follower Strategy

$\mathbf{BR}^{\epsilon}(\sigma_0)$: Epsilon-Optimal Best Response of Follower to Leader's strategy σ_0

Follower is almost rational and choose the epsilon-optimal best response

Follower is almost rational and choose the epsilon-optimal best response

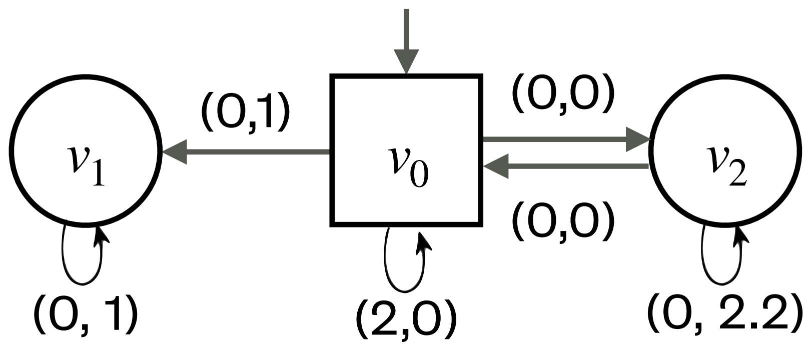
ϵ is fixed

Stackelberg Mean Payoff Games with One Epsilon-Optimal Adversarial Follower

Two Players:

Leader Follower

Mean-Payoff Game:



Sequential Move:

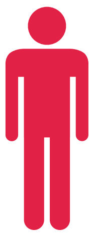
1. Leader announces her strategy

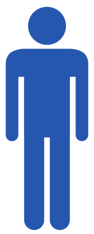
2. Follower announces his adversarial **epsilon-optimal** best response to leader's strategy

Leader

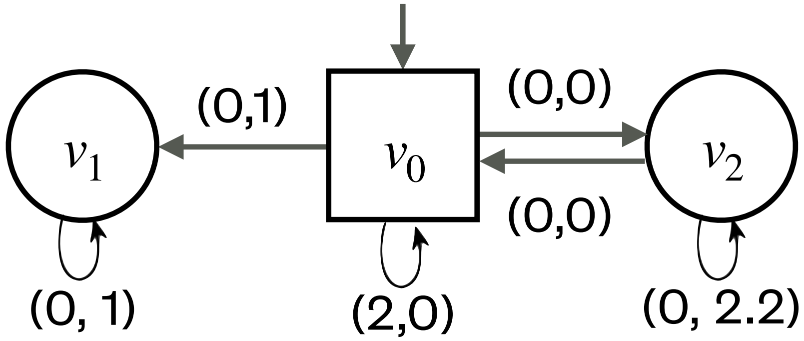
Epsilon-Optimal Adversarial Follower

Two Players:





Mean-Payoff Game:



Sequential Move:

1. Leader announces her strategy

2. Follower announces his adversarial **epsilon-optimal** best response to leader's strategy

Epsilon-Optimal Adversarial Stackelberg Value ($\mathbf{ASV}^{\epsilon}$)

$\mathbf{ASV}^{\epsilon}$ is the largest mean-payoff value the Leader can obtain when the Follower plays an **adversarial epsilon-optimal** best response.

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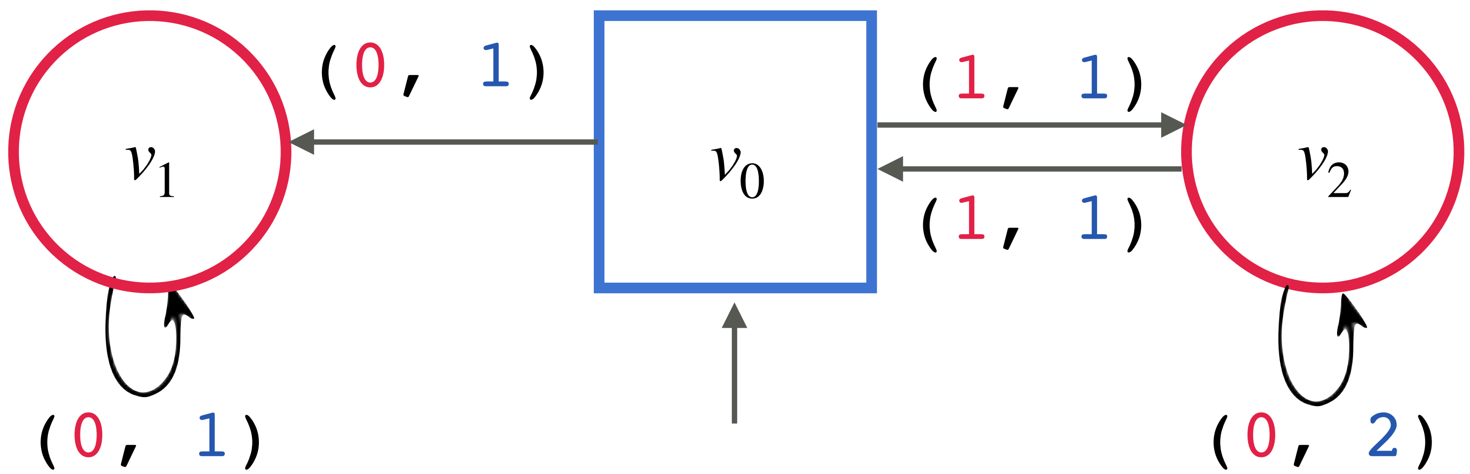
ASV May Not Be Achievable

(Filiot, Gentilini and Raskin - ICALP 2020)



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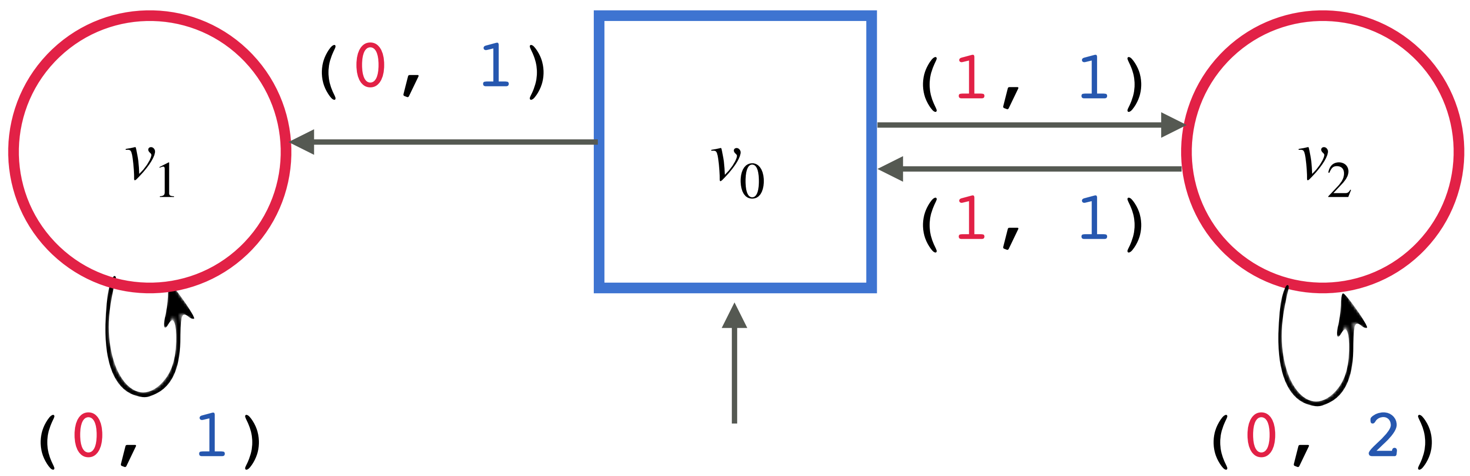

Follower must be given mean-payoff > 1
else he will play $v_0 \to v_1$

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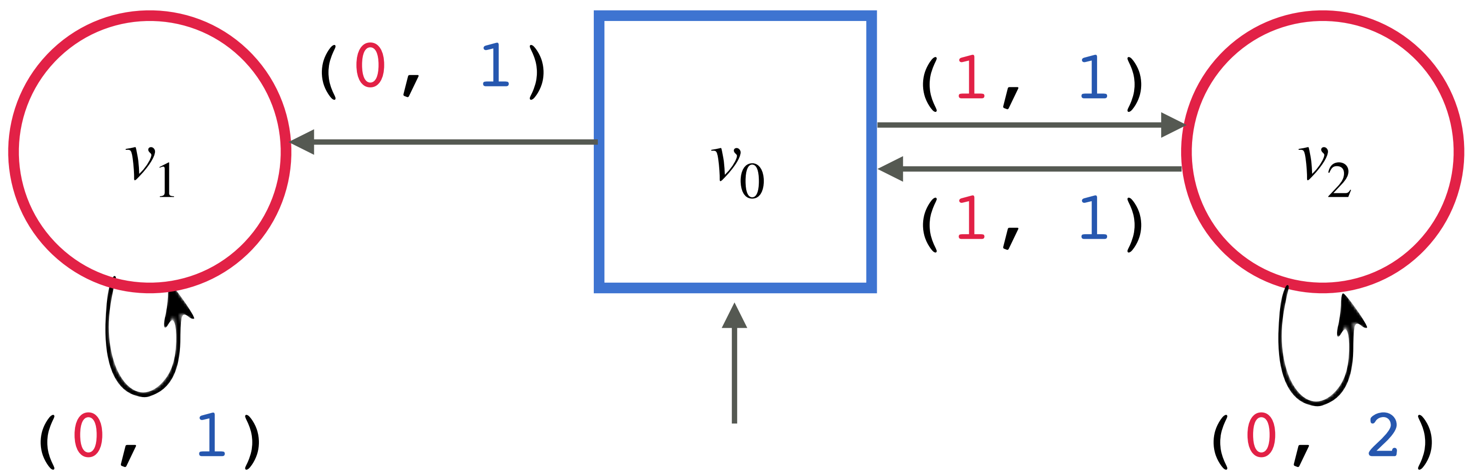

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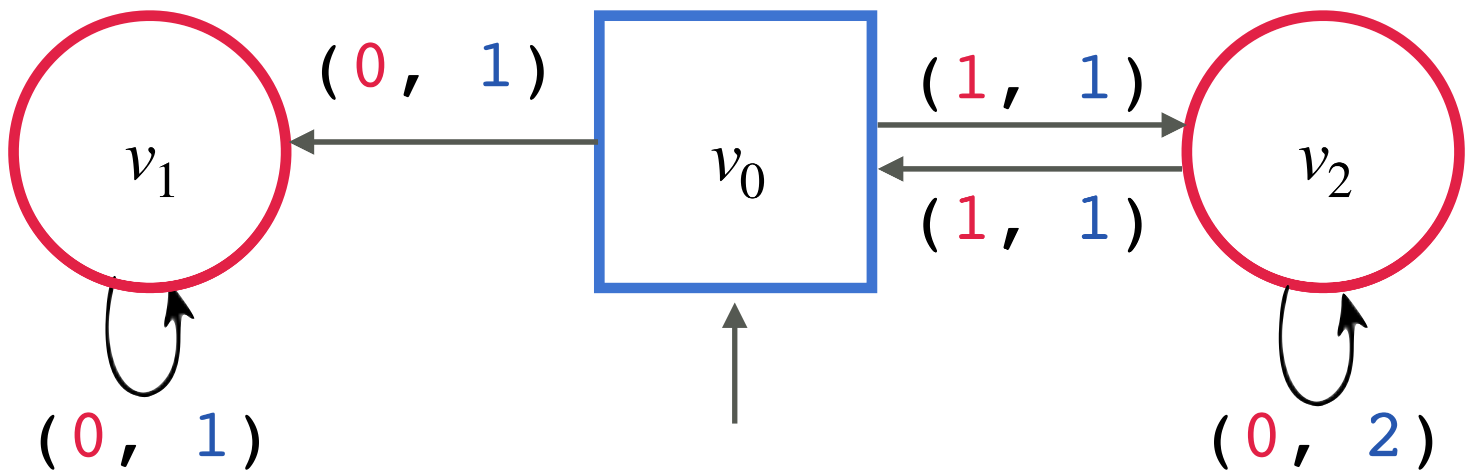

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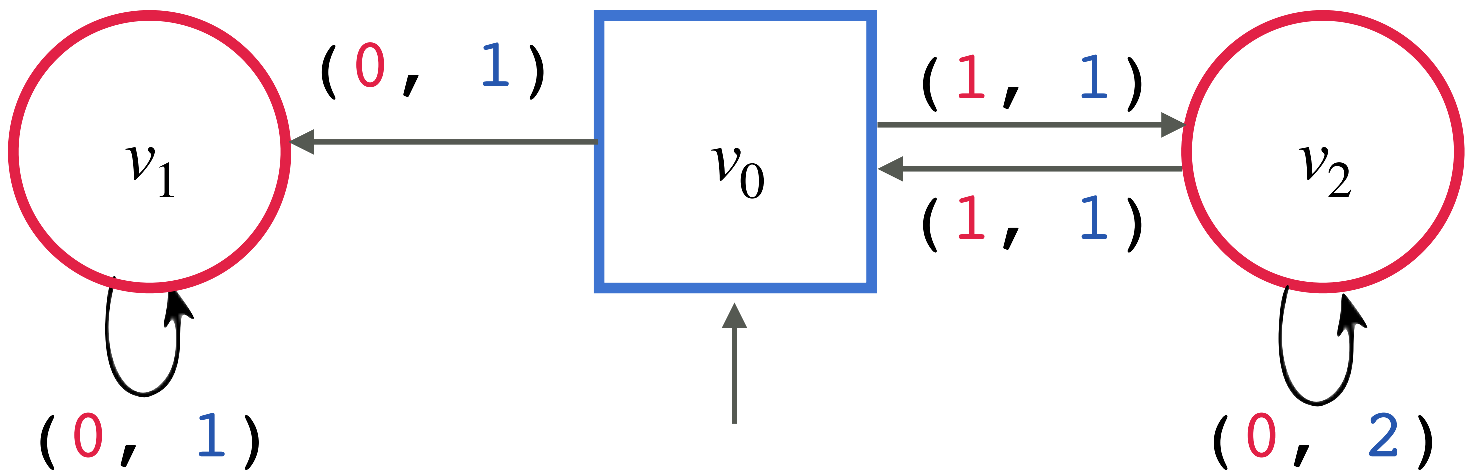
When $\delta \to 0$: Leader gets better mean-payoff $\to$ 1 (at limit)

When $\delta = 0$: Follower gets a mean-payoff = 1 and plays $v_0 \to v_1$

ASV May Not Be Achievable

(Filiot, Gentilini and Raskin - ICALP 2020)

$$\mathbf{ASV}(v_0) = 1$$



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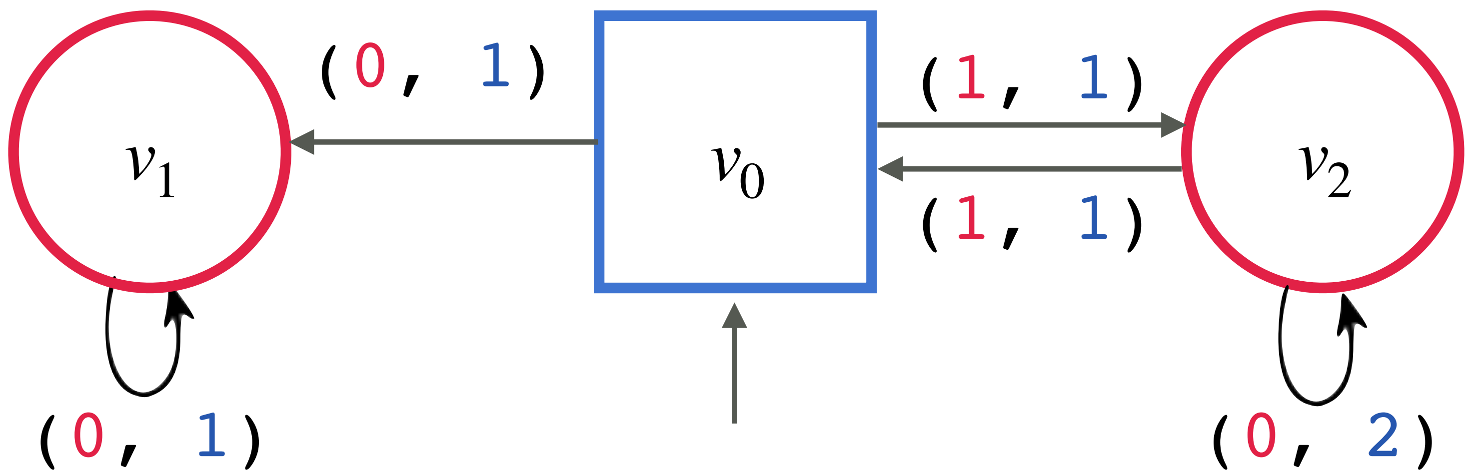

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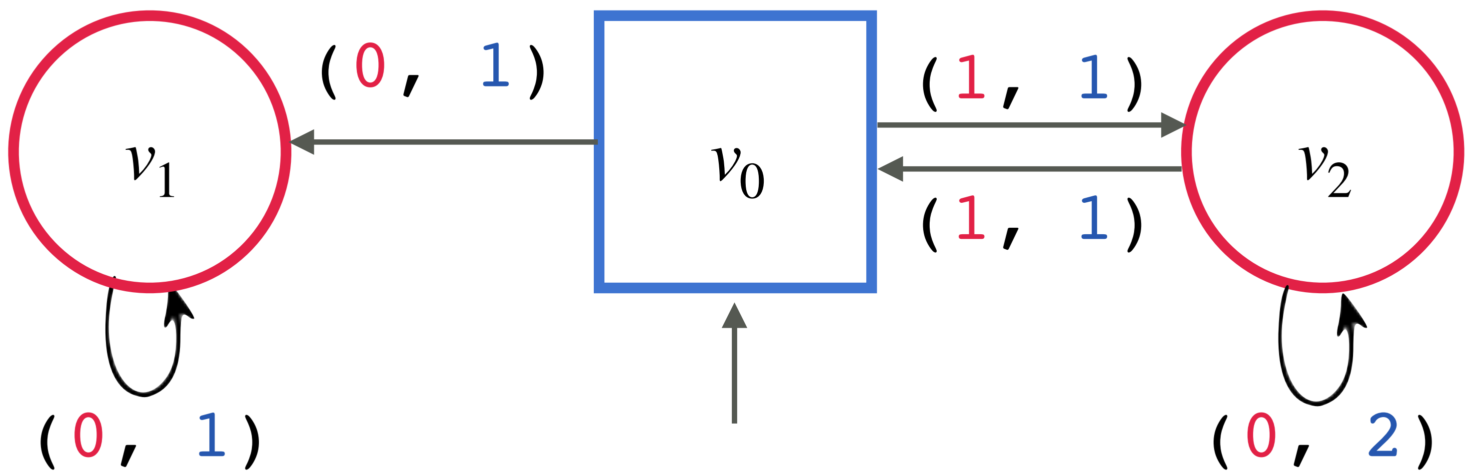

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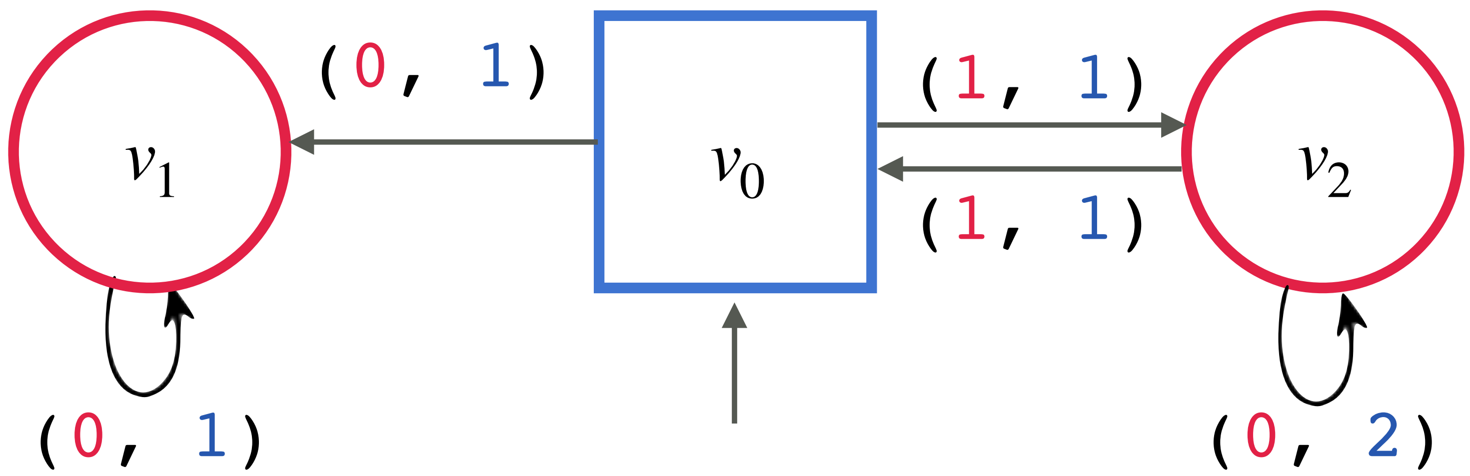

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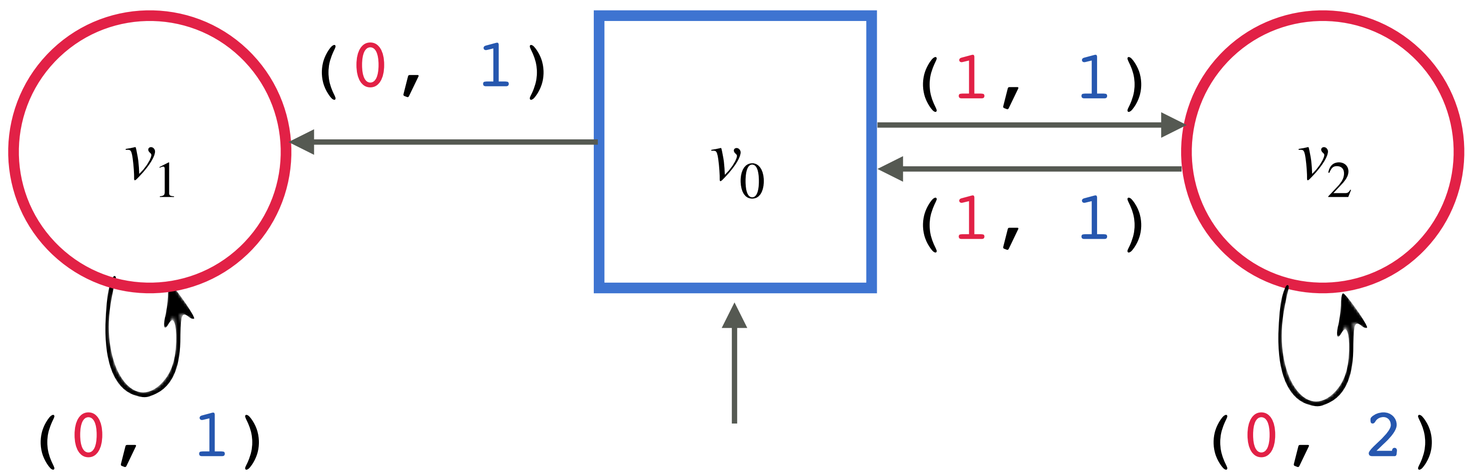

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If Follower is given a mean-payoff $< 1 + \epsilon$

playing $v_0 \rightarrow v_1$ is an epsilon-optimal best response

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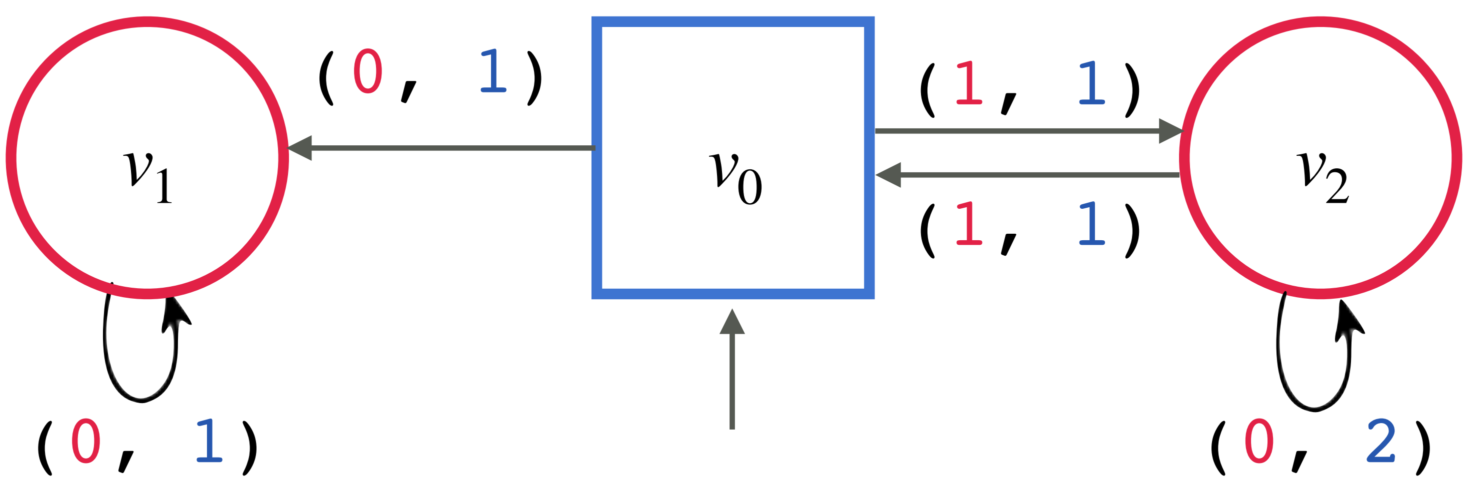

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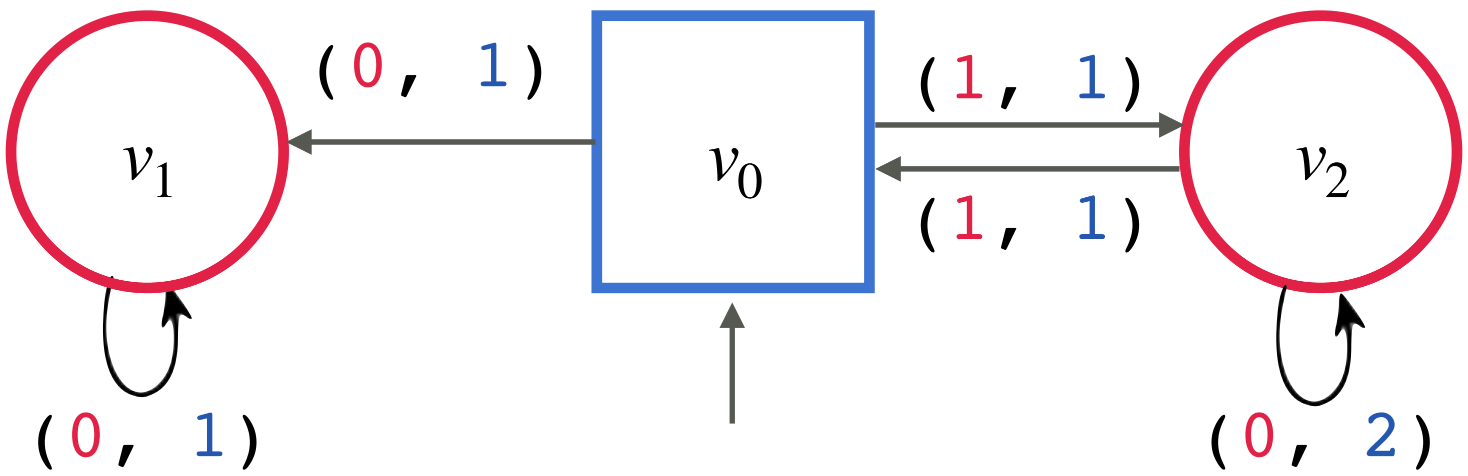
$$\mathbf{ASV}^{\epsilon}(v_0) = 1 - \epsilon$$



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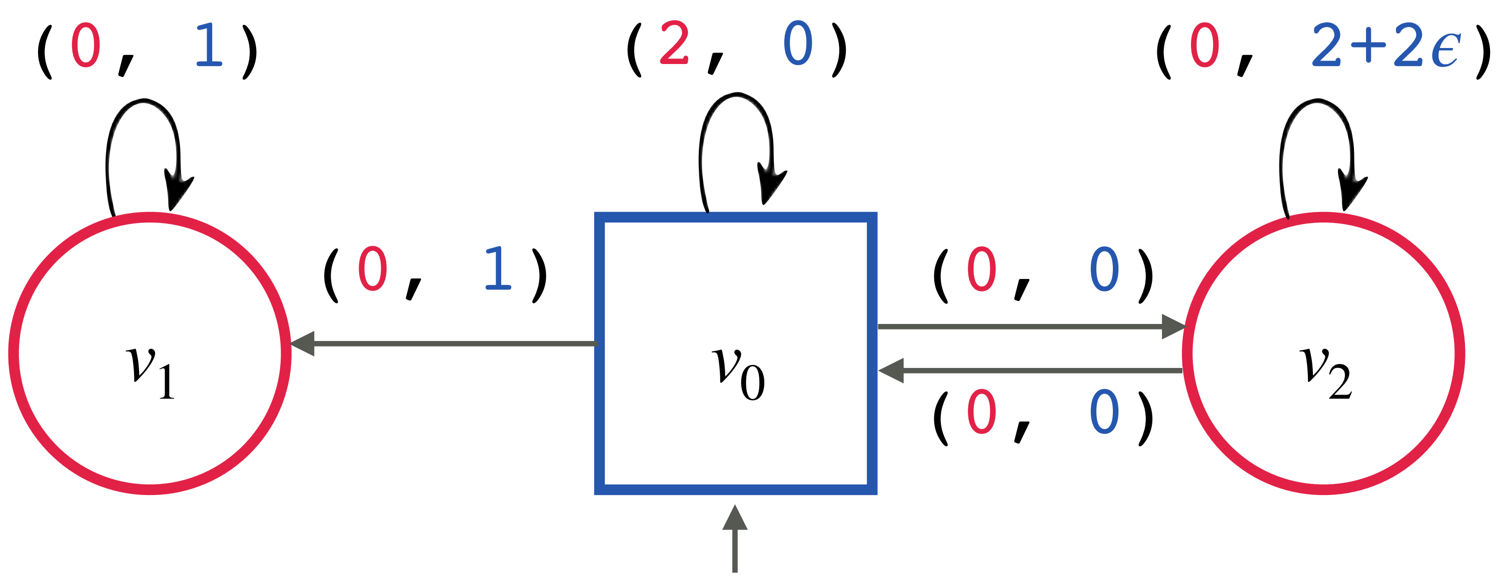
RESULT 1:

$\mathbf{ASV}^{\epsilon}$ is always achievable

Memory Requirements

Memory Requirements for Players



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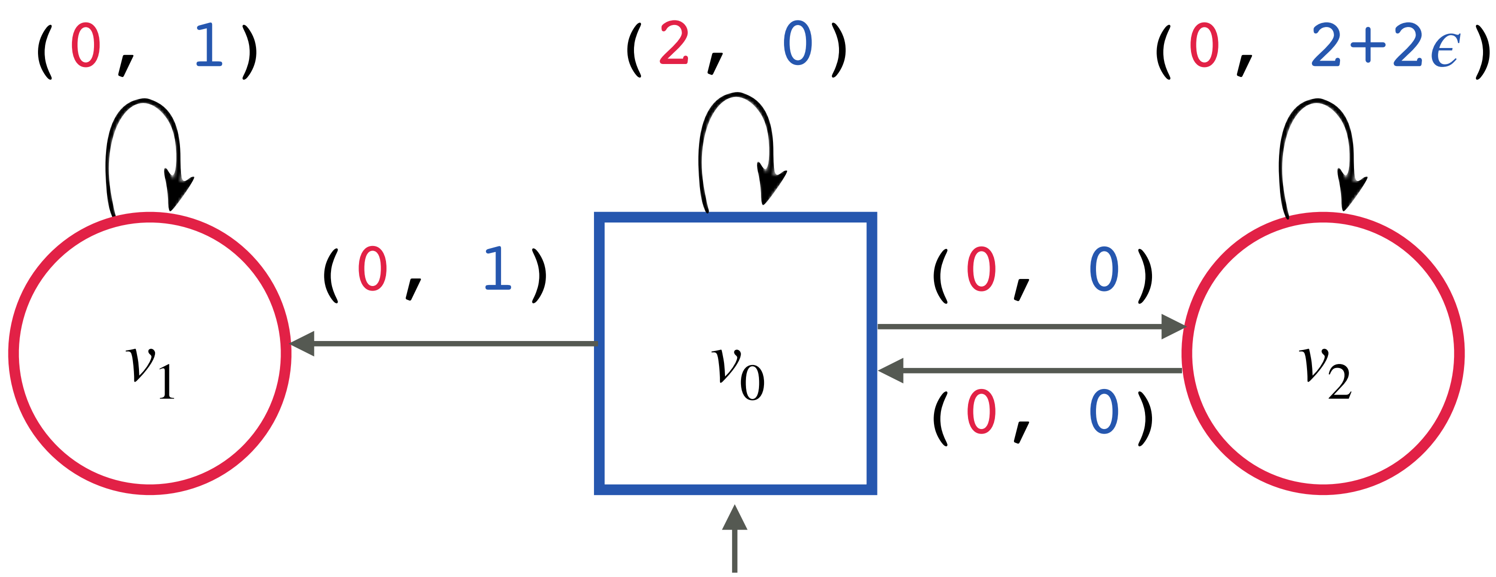

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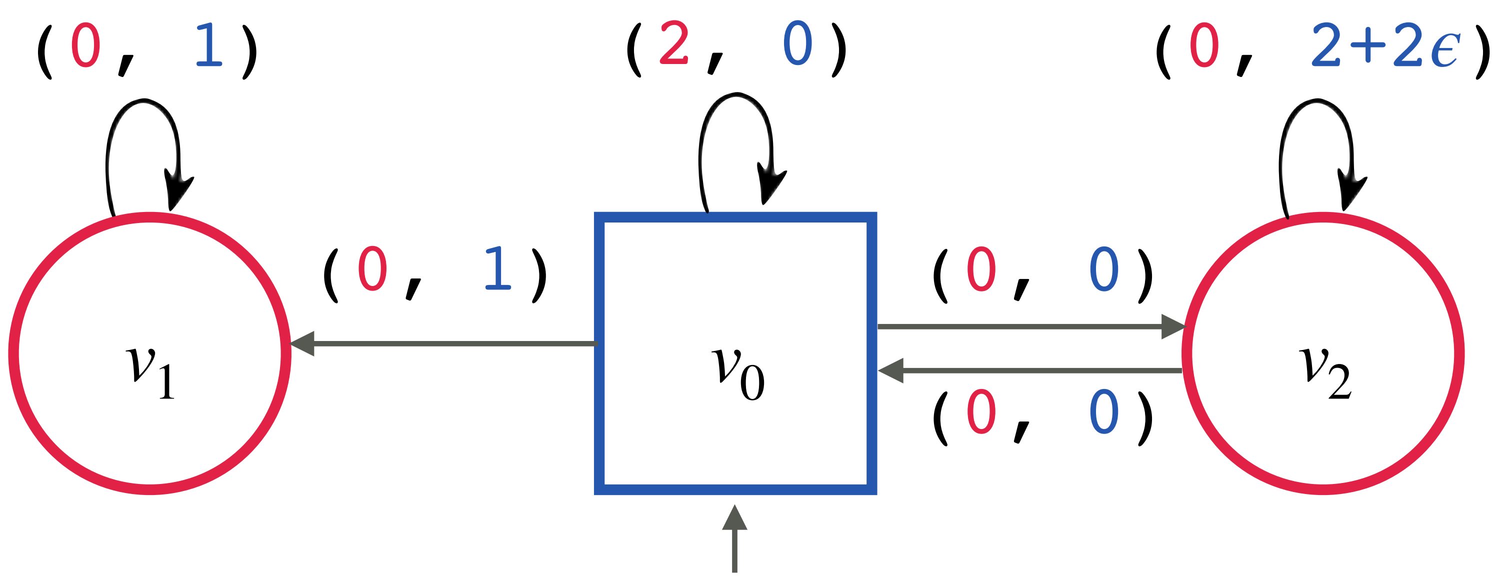

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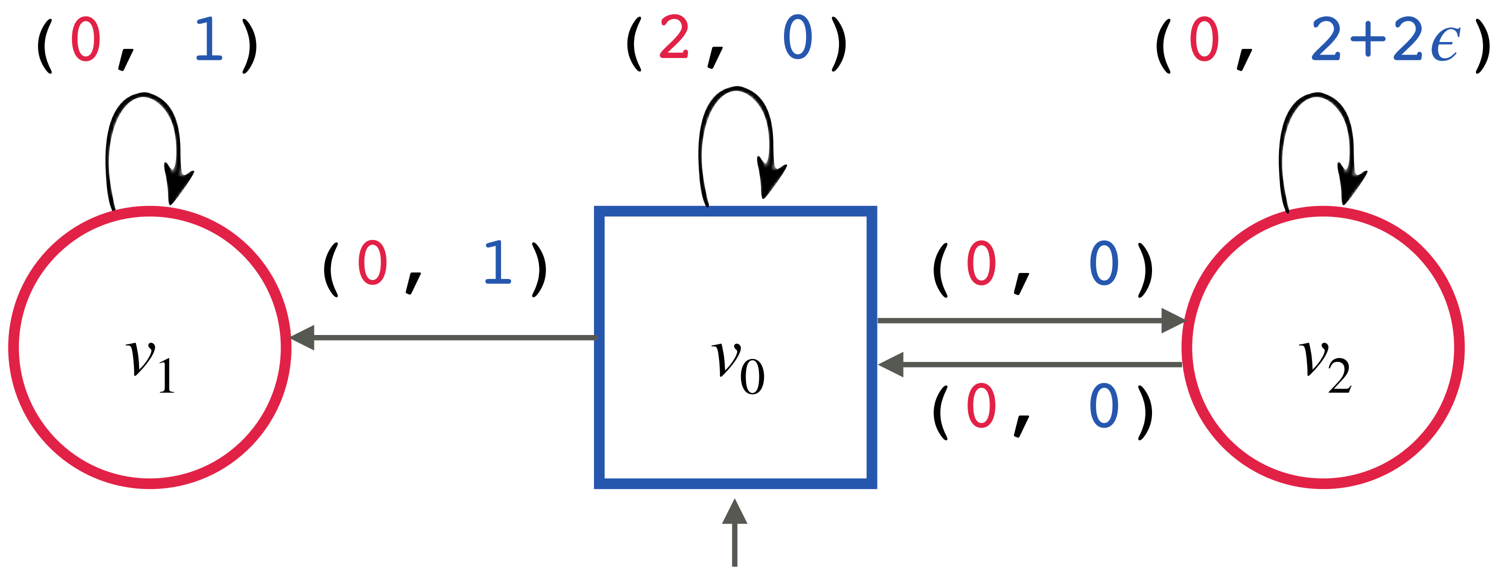

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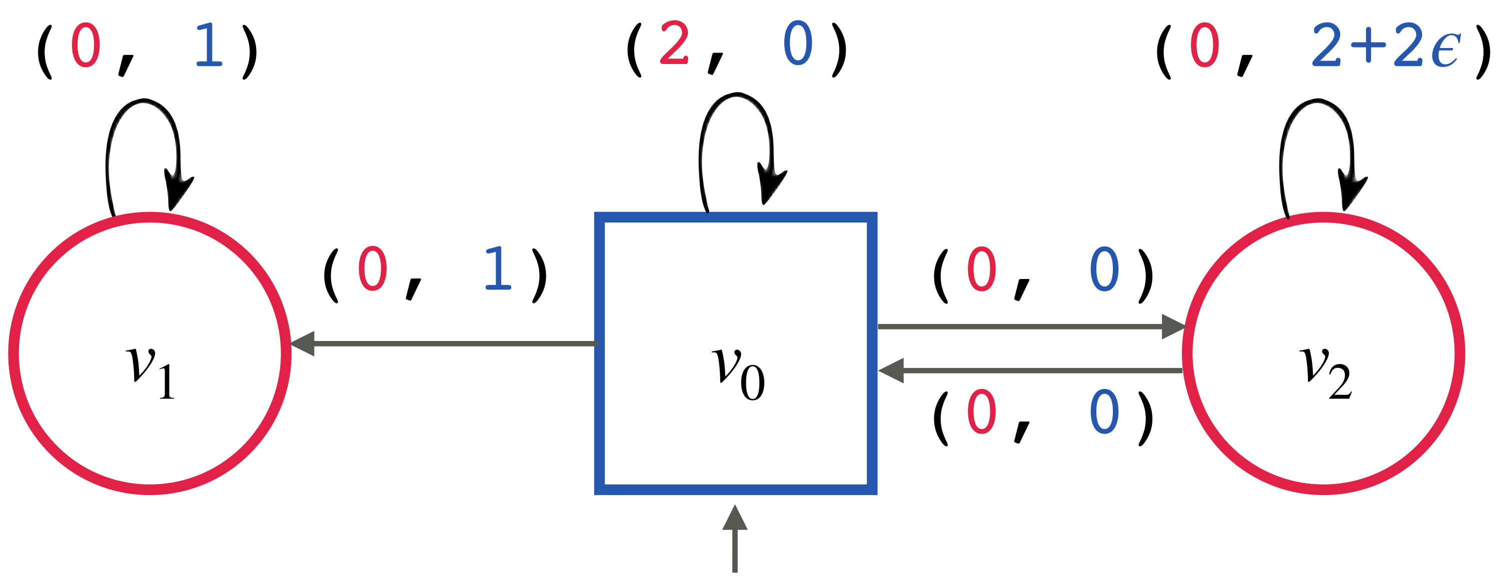

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If any deviation, then play $v_2 \to v_0$

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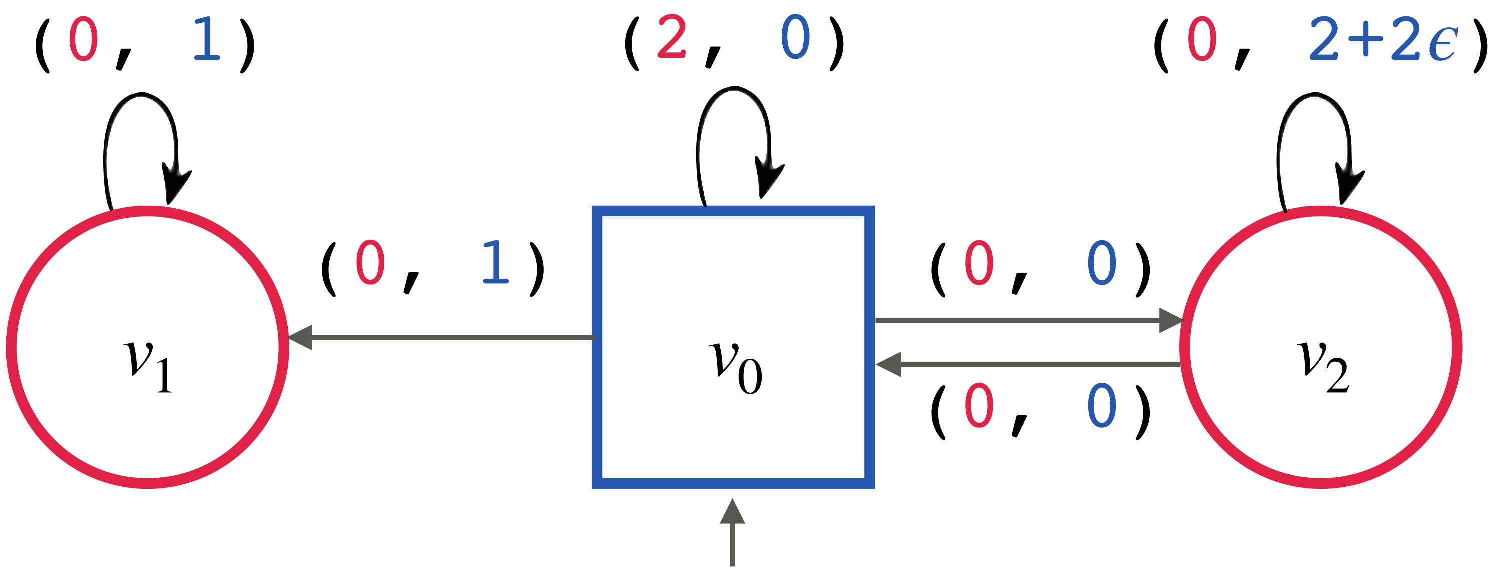

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$v_0 \cdot v_2 \cdot v_0 v_0 \cdot v_2 v_2 \cdot v_0 v_0 v_0 \cdot v_2 v_2 v_2 \ldots$

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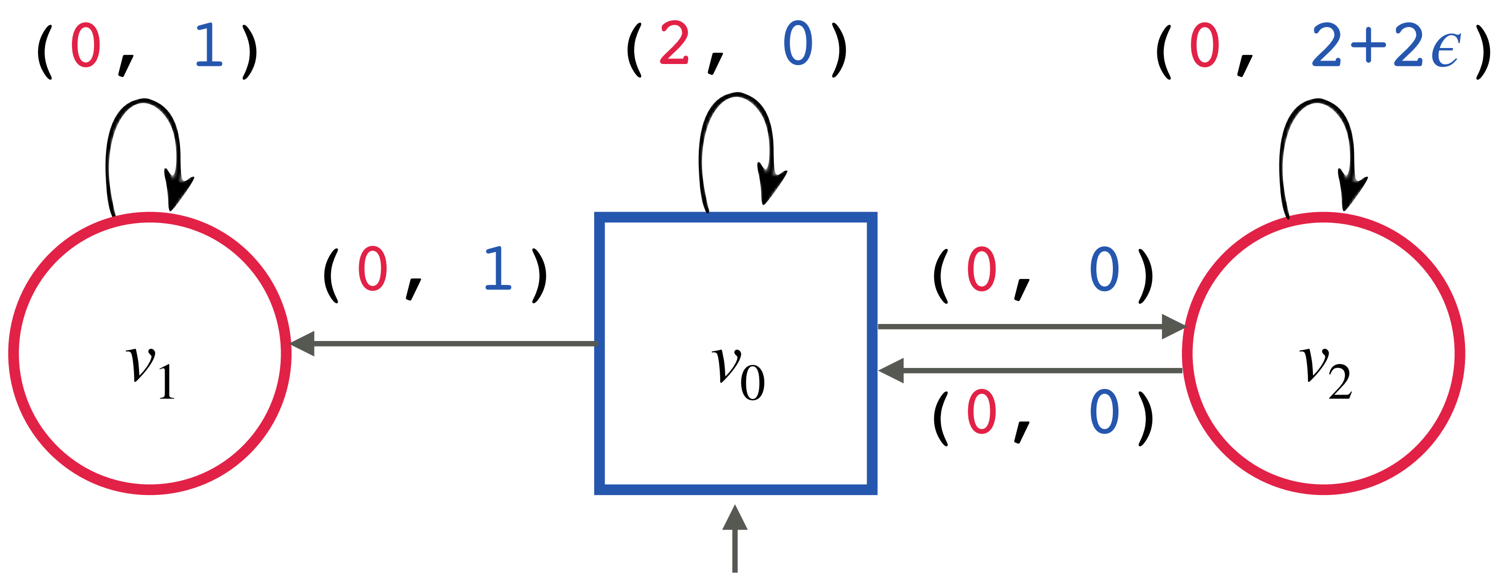

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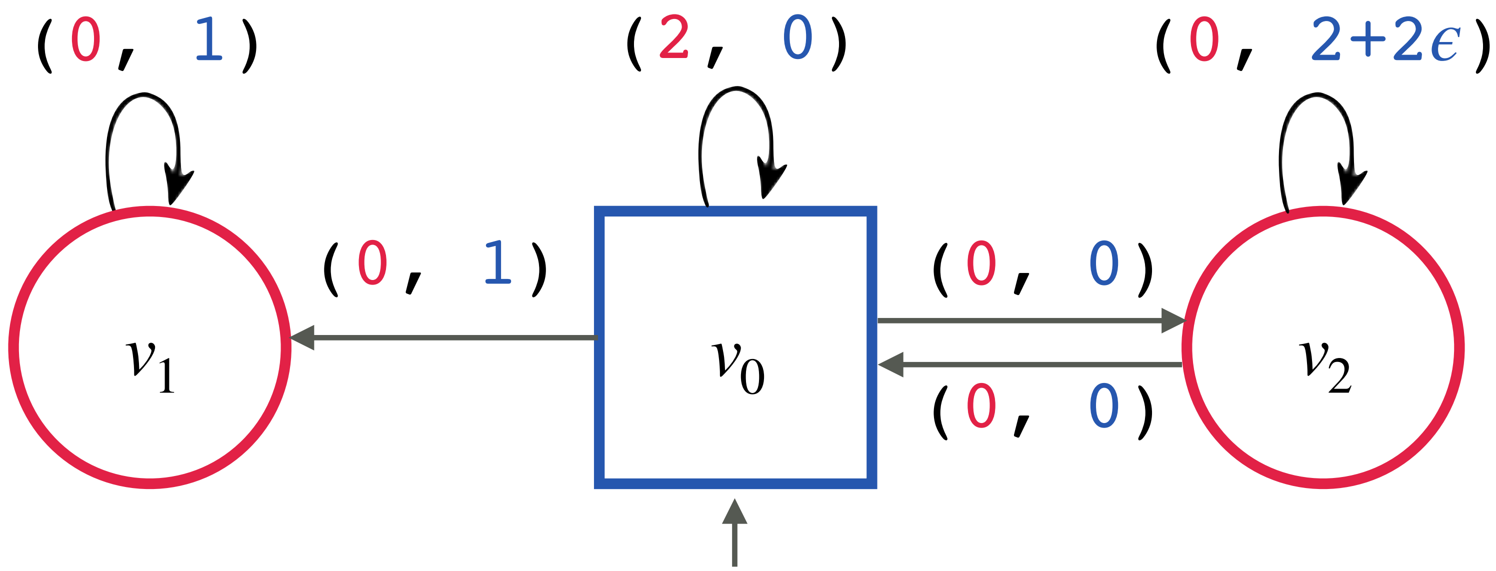

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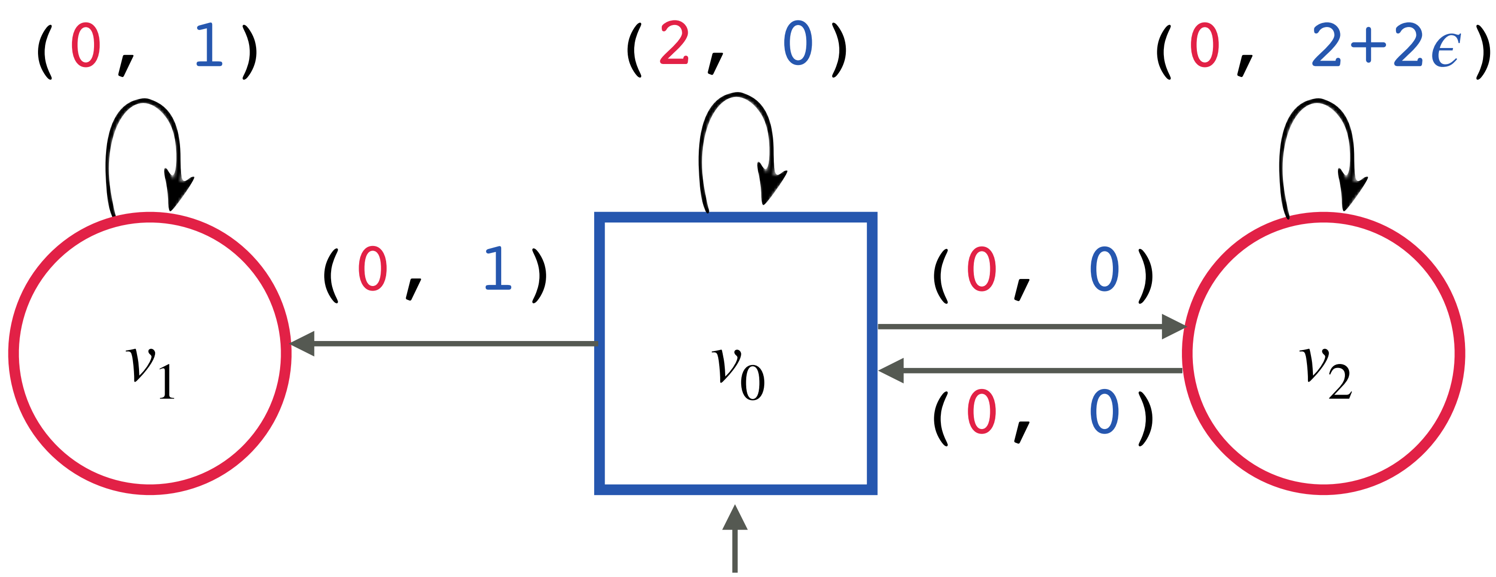
If Follower follows path, he gets mean-payoff of $1{+}\epsilon$

and Leader gets a mean-payoff of 1

Memory Requirements for Players



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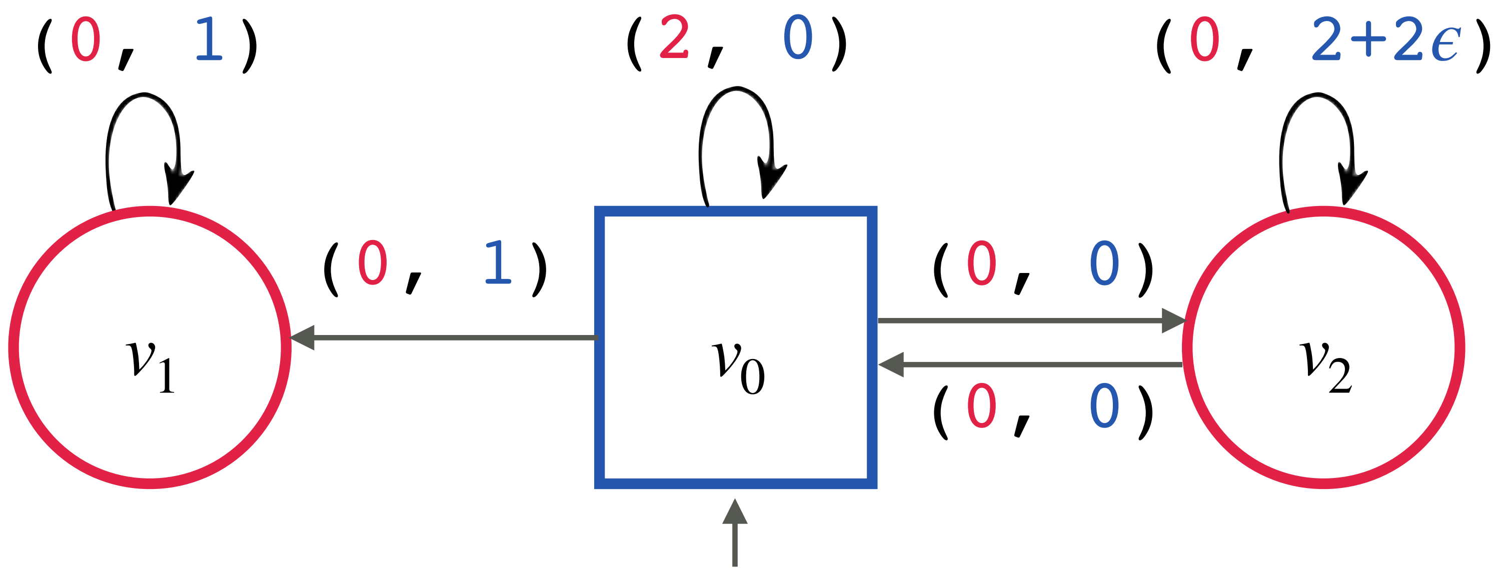
If Follower deviates from path,

the maximum mean-payoff he can get is 1

Infinite Memory Required for Follower



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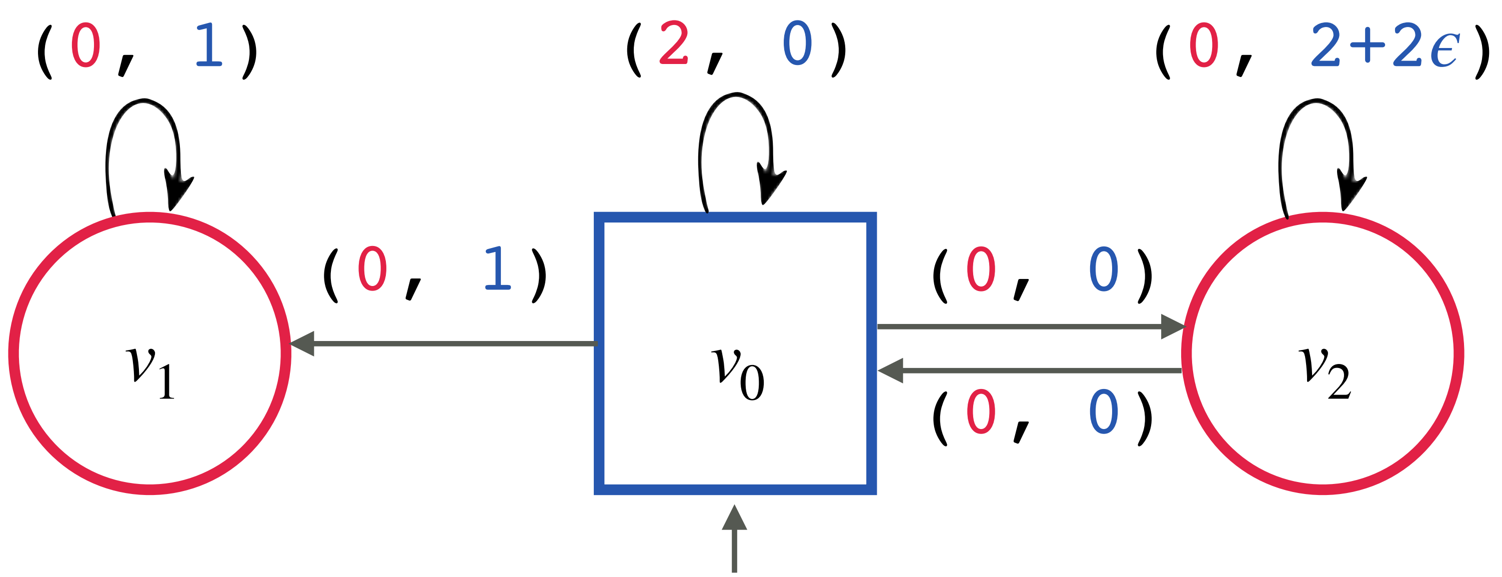

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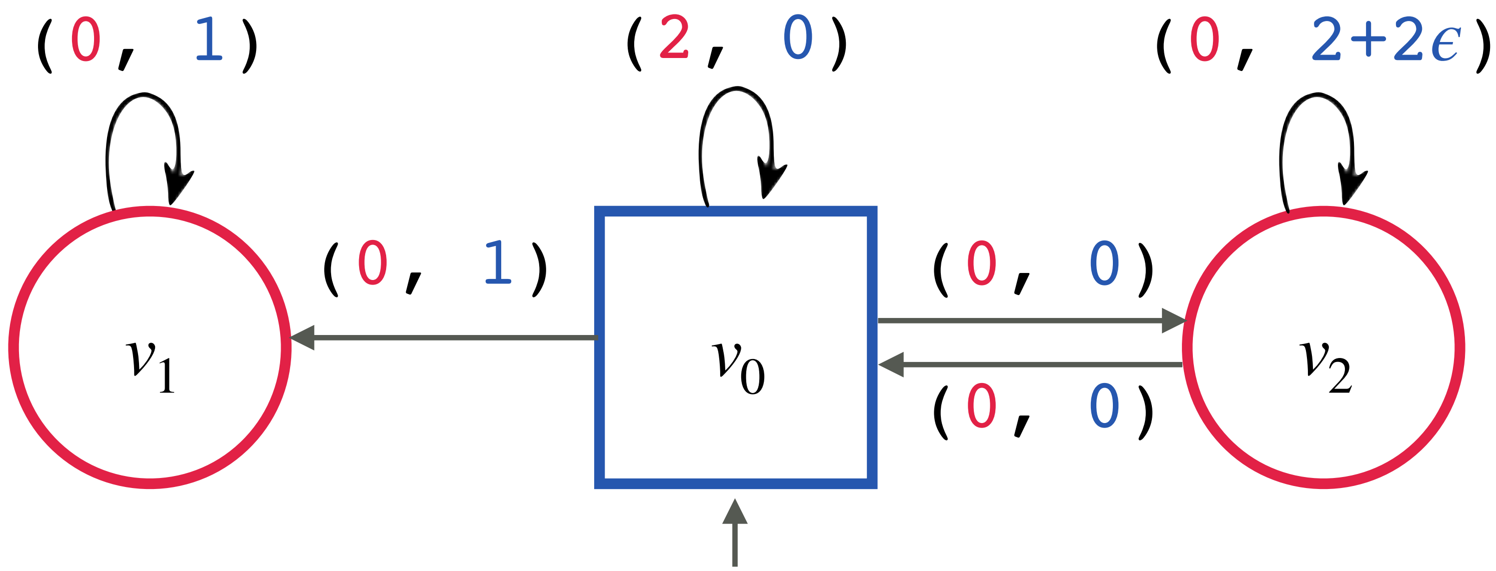
$$\mathbf{ASV}^{\epsilon}(\text{Leader Strategy})(v_0) = 1$$

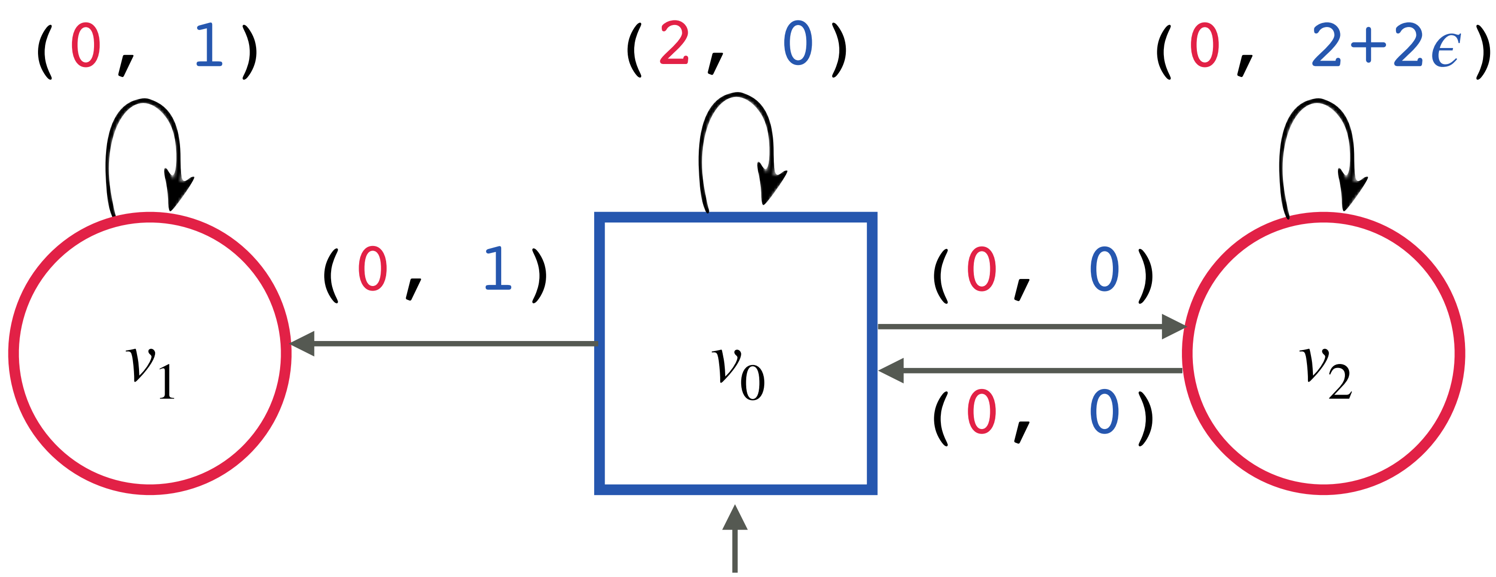
Infinite Memory Required for Leader



Leader strategy: Follow the path $\left((v_0 \to v_0)^k \cdot (v_2 \to v_2)^{k+\delta}\right)^\omega$

(Finite Memory) If any deviation, then play $v_2 \to v_0$

Infinite Memory Required for Leader



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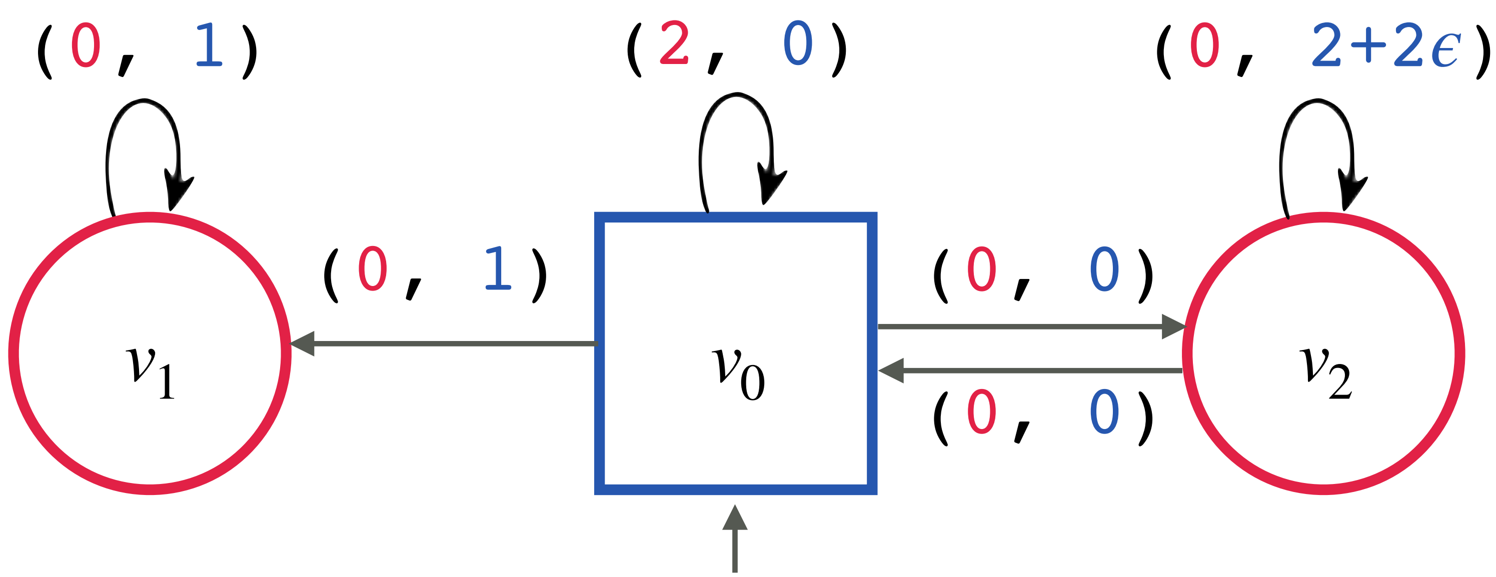
The effects of edges (0, 0) become non-negligible and decrease Leader's mean-payoff

Infinite Memory Required for Leader



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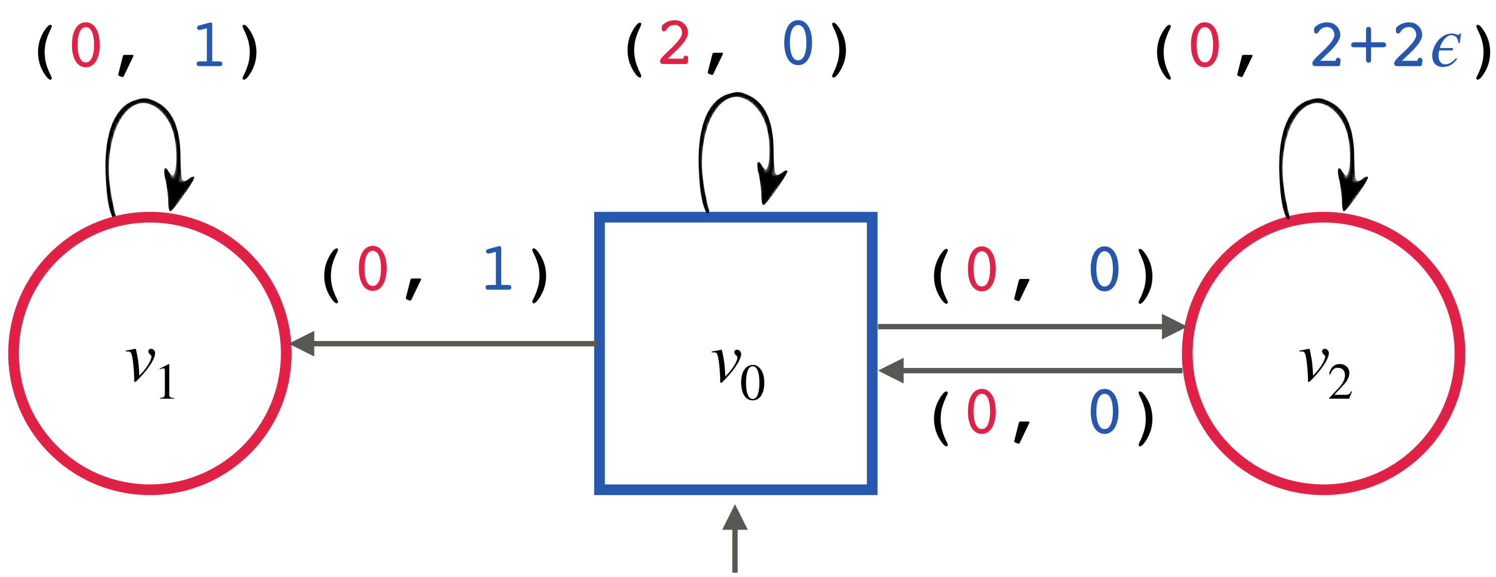
$$\mathbf{ASV}^{\epsilon}(\text{Leader Strategy})(v_0) < 1$$

Infinite Memory Required for Leader



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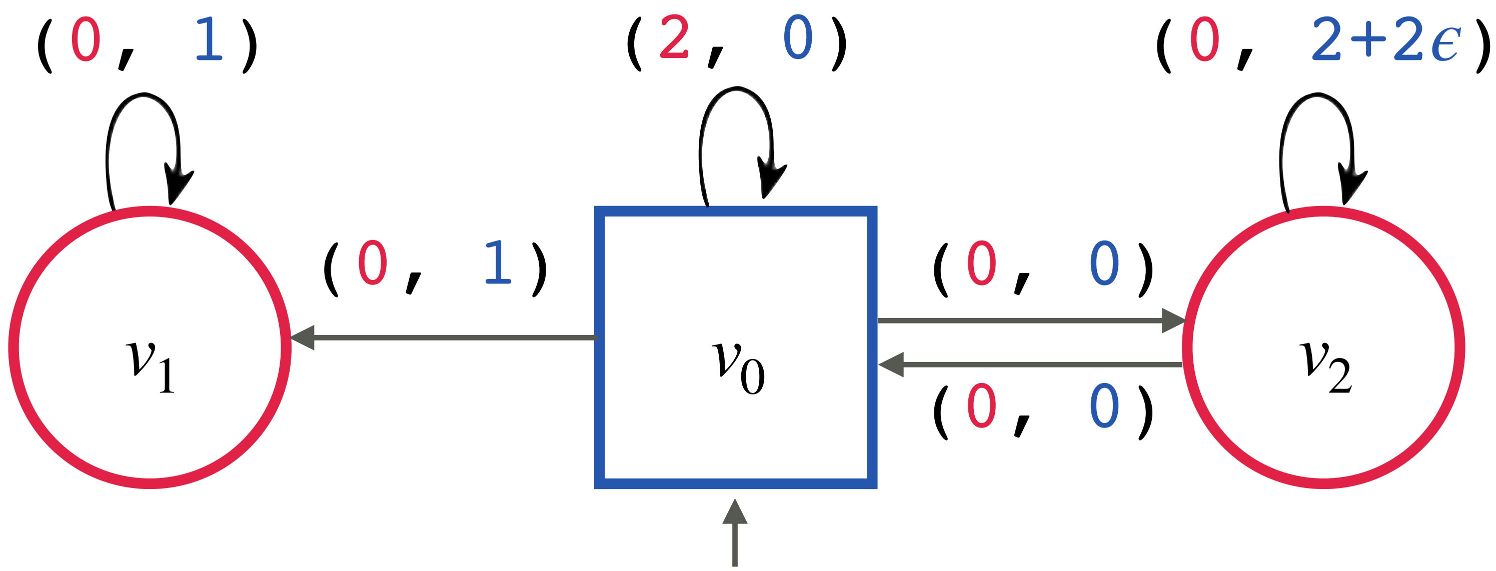

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$$\mathbf{ASV}^{\epsilon}(v_0) = 1$$

RESULT 2:

Infinite memory is might be required for Leader to achieve the $\mathbf{ASV}^{\epsilon}$

<u>RESULT 3:</u>

Infinite memory might be required for the Follower to play an epsilon-optimal best-response

Threshold Problem:

Is $\mathbf{ASV}^{\epsilon} > c$?

Witnesses and Bad Vertices

$$\Lambda^{\epsilon}(v) = \left\{ (c, d) \in \mathbb{R}^2 \mid \begin{array}{l} \text{From vertex } v\text{, the Follower can ensure that} \\ \text{Leader's payoff} \leqslant c \text{ and Follower's payoff} > d - \epsilon \end{array} \right\}$$

Witnesses and Bad Vertices

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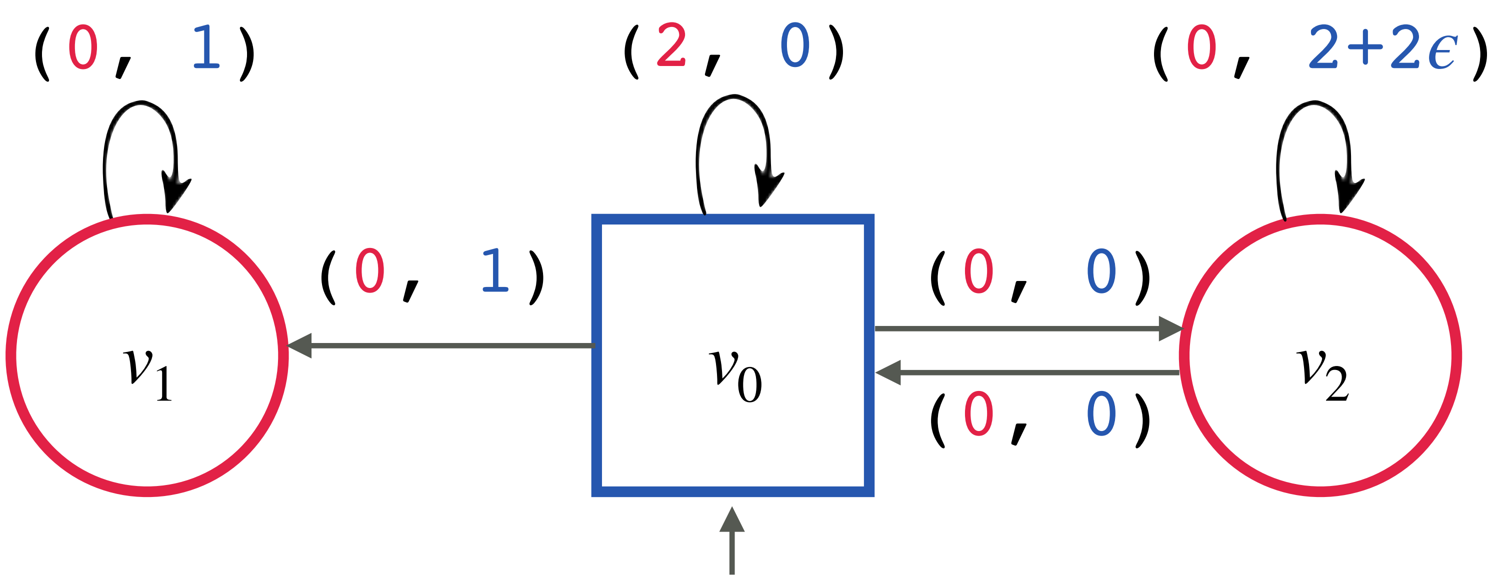

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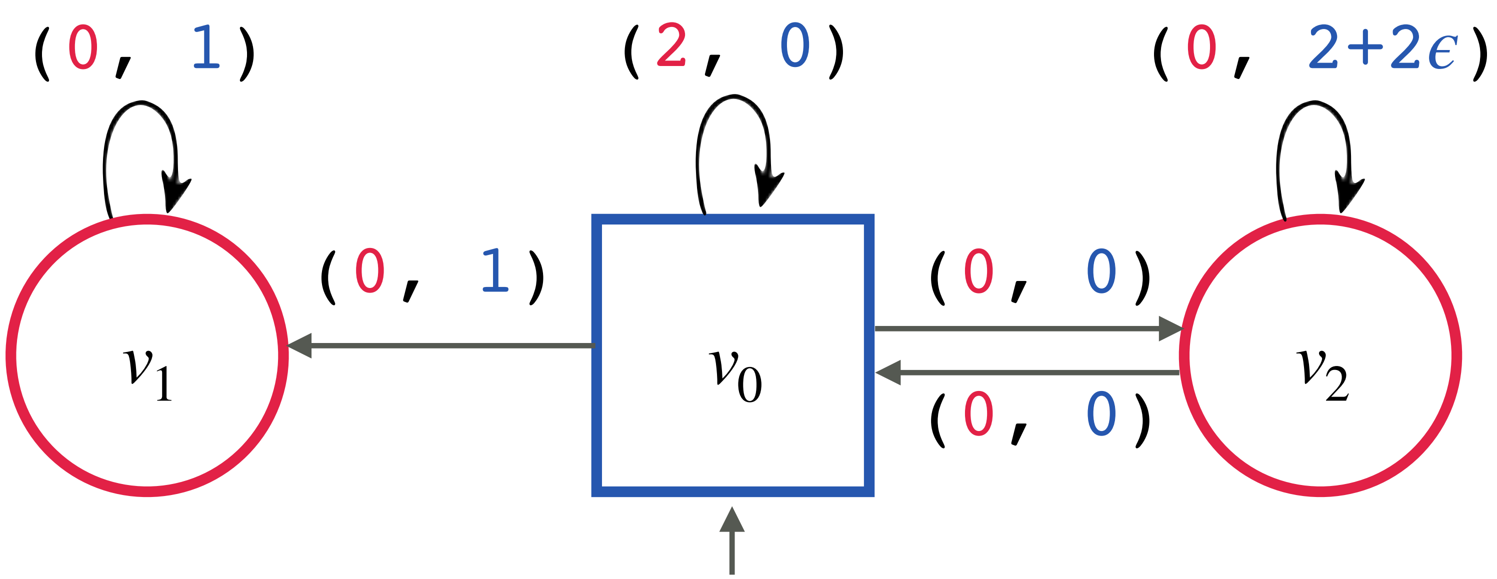

From v_0, Follower can ensure a payoff of (0, 1)

For all, $0 \leqslant c < \infty$ and $-\infty < d < 1+\epsilon$,

$(c, d) \in \Lambda^{\epsilon}(v_0)$

Witnesses and Bad Vertices

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A vertex v is $(c, d)^{\epsilon}$-bad if $(c, d) \in \Lambda^{\epsilon}(v)$

Witnesses and Bad Vertices

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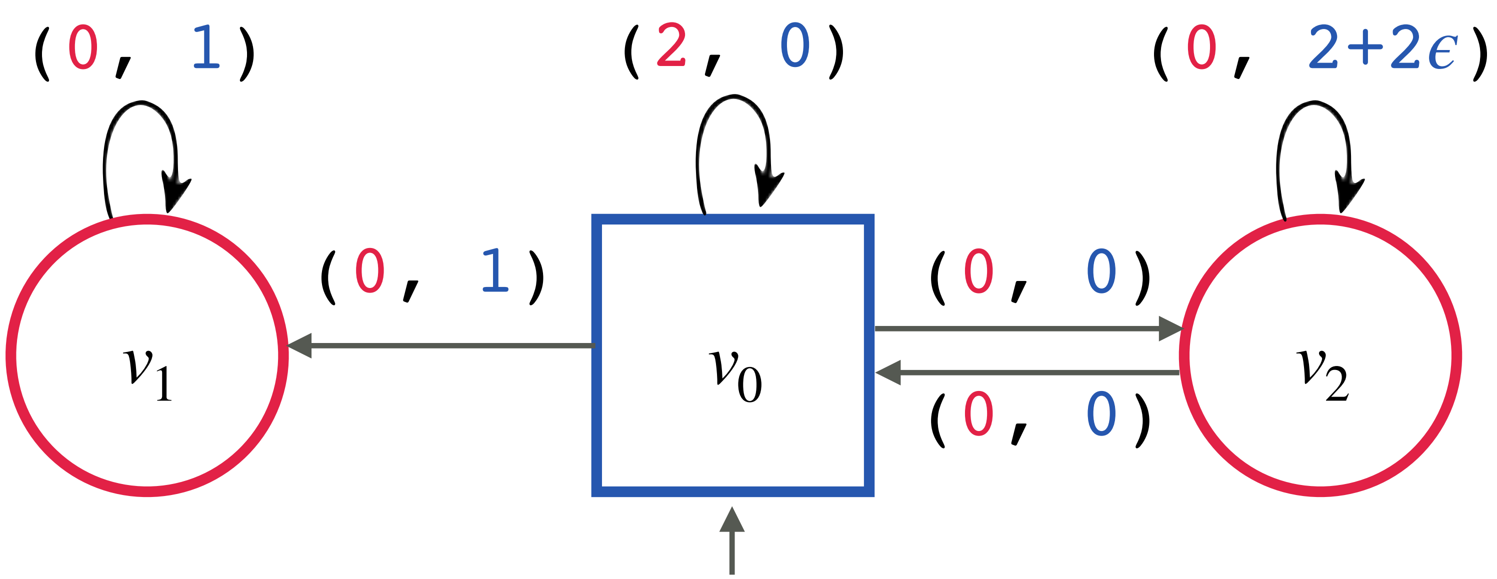

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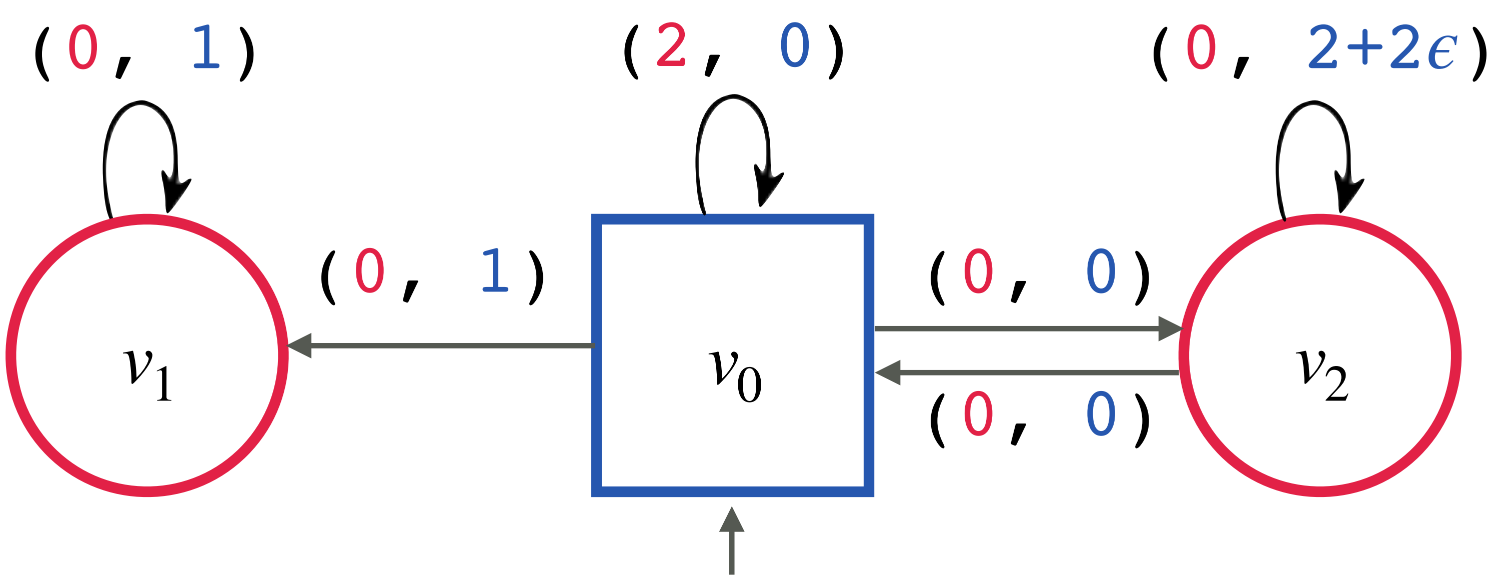

For all $0 \leqslant c < \infty$ and $-\infty < d < 1+\epsilon$,

vertex v_0 is $(c, d)^{\epsilon}$-bad.

Witnesses and Bad Vertices

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A path π is a witness for $\mathbf{ASV}^{\epsilon} > c$ if

Witnesses and Bad Vertices

$\Lambda^{\epsilon}(v) = \Big\{$ (c, d) $\in \mathbb{R}^2$ | From vertex v, the Follower can ensure that Leader's payoff $\leqslant$ c and Follower's payoff $>$ d - ϵ $\Big\}$

A vertex v is $(c, d)^{\epsilon}$-bad if $(c, d) \in \Lambda^{\epsilon}(v)$

A path π is a witness for $\mathbf{ASV}^{\epsilon} > c$ if

Mean-Payoff of π is (c′, d), where c′ > c

Witnesses and Bad Vertices

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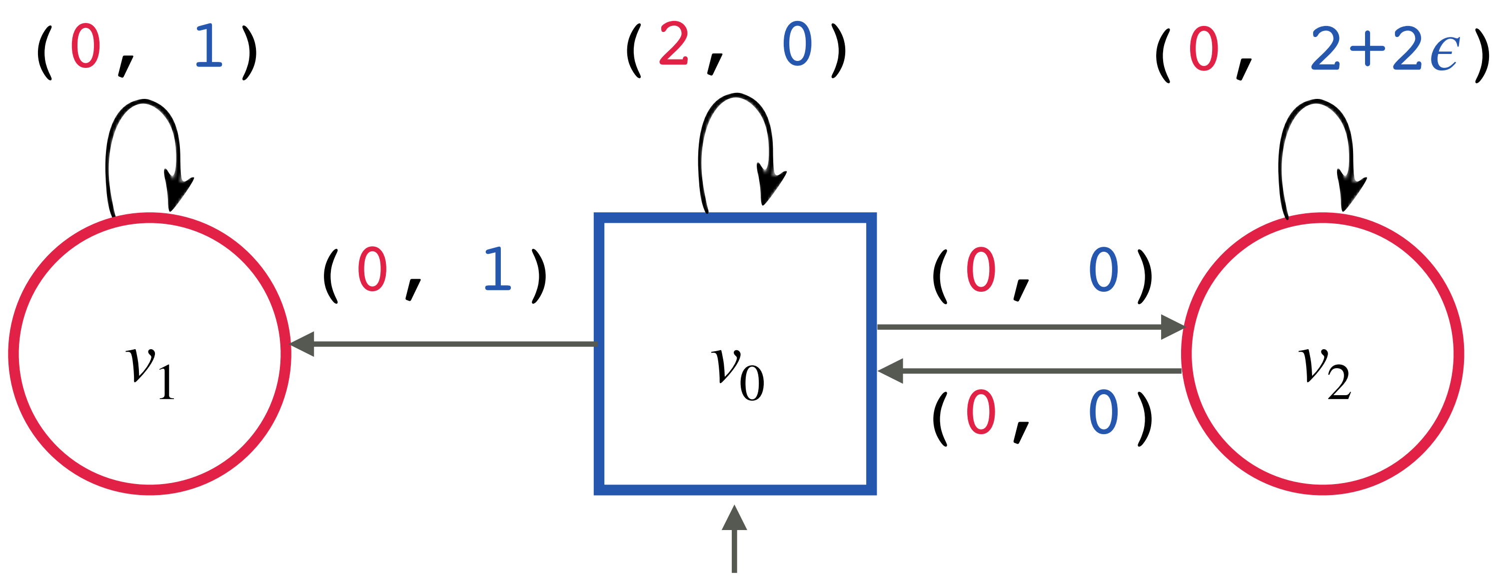
A path π is a witness for $\mathbf{ASV}^{\epsilon} > c$ if

Mean-Payoff of π is (c', d), where $c' > c$

and π does not cross a $(c, d)^{\epsilon}$-bad vertex.

Witnesses and Bad Vertices



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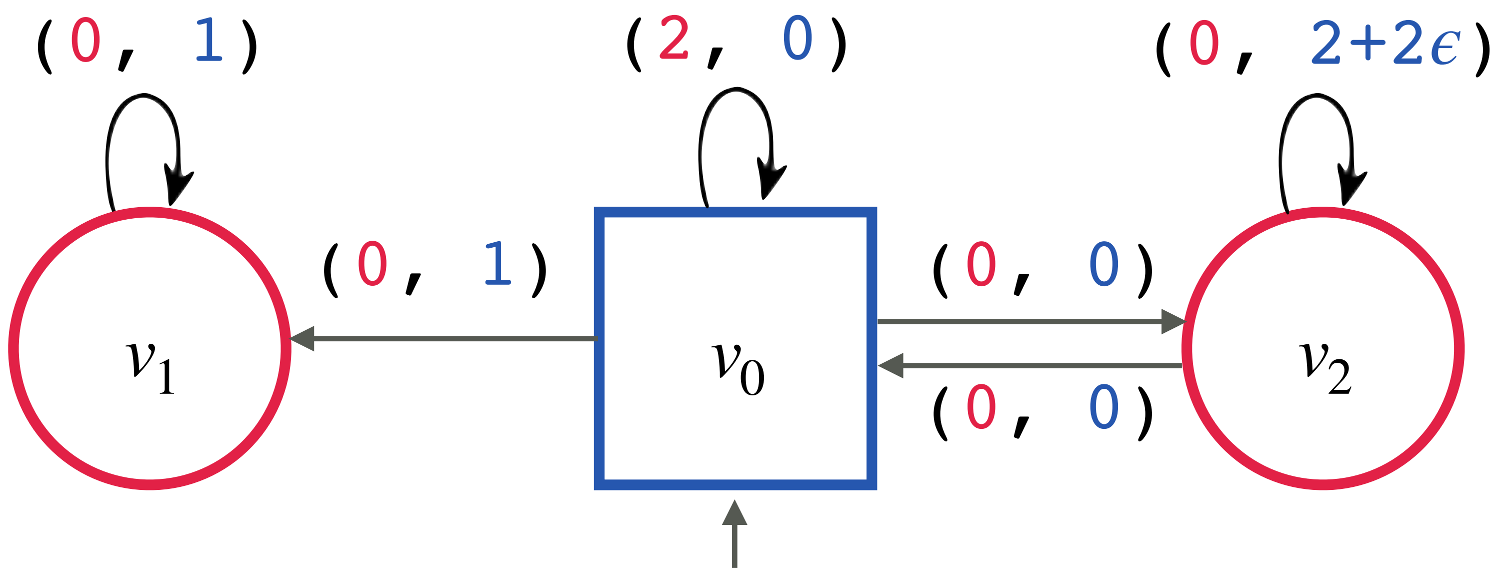

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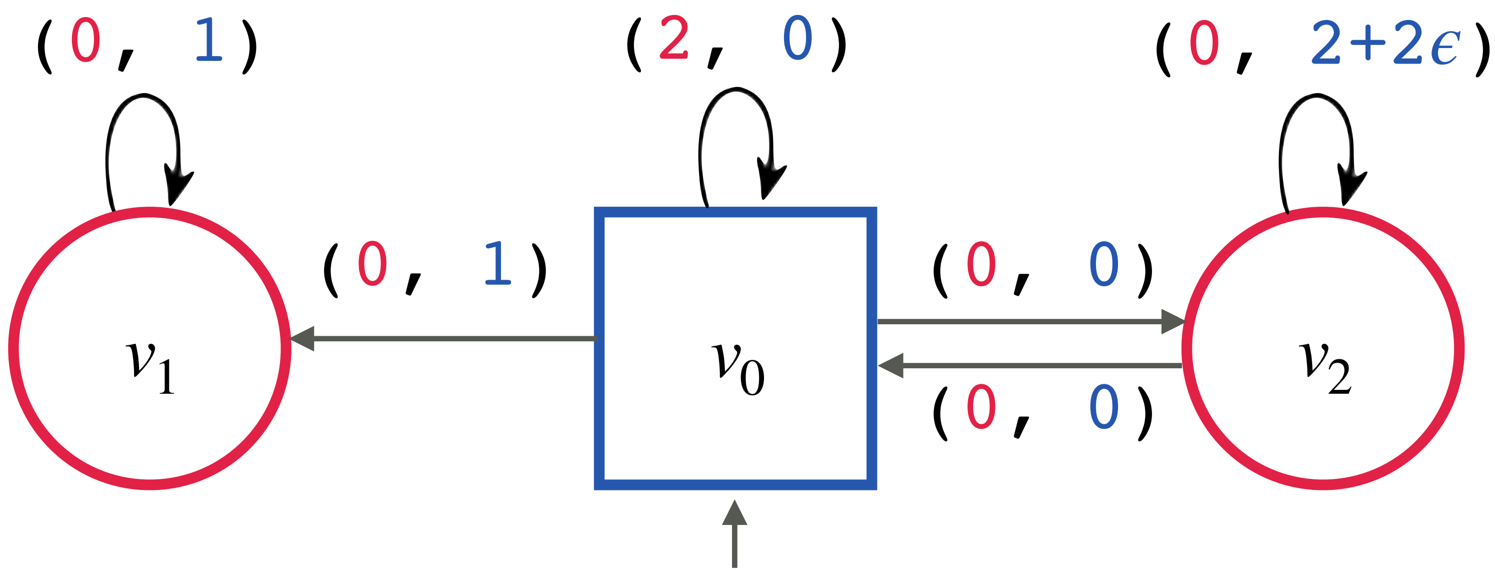

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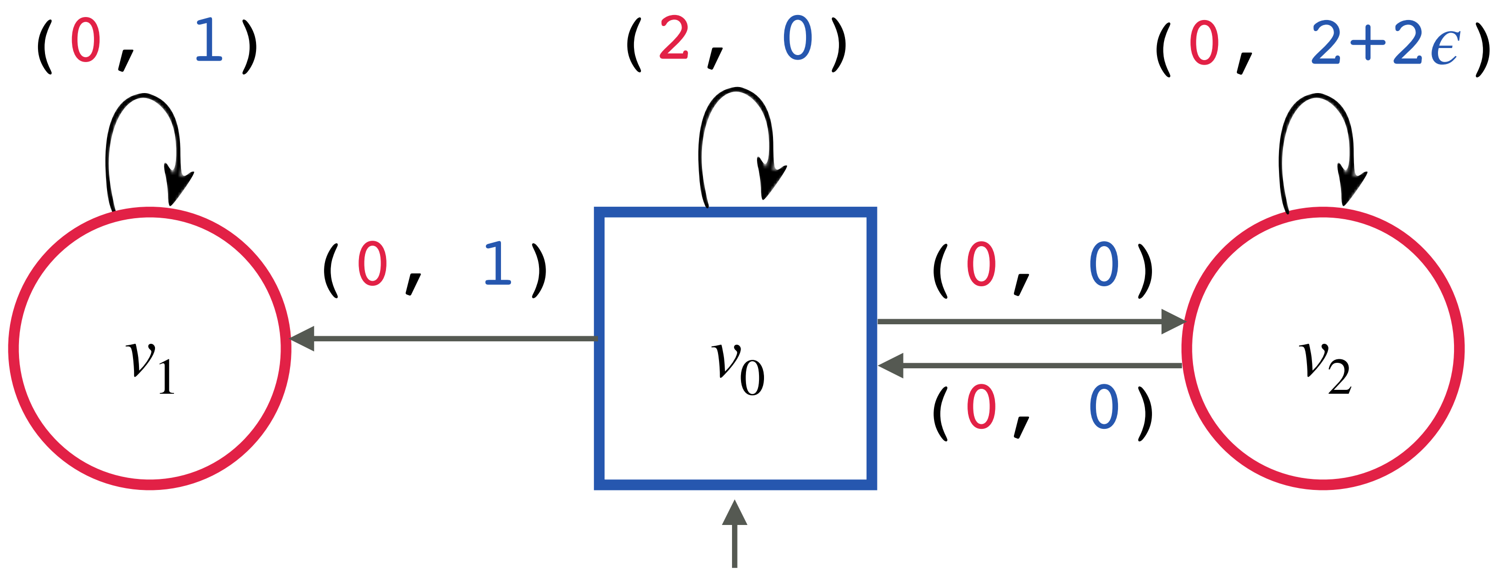

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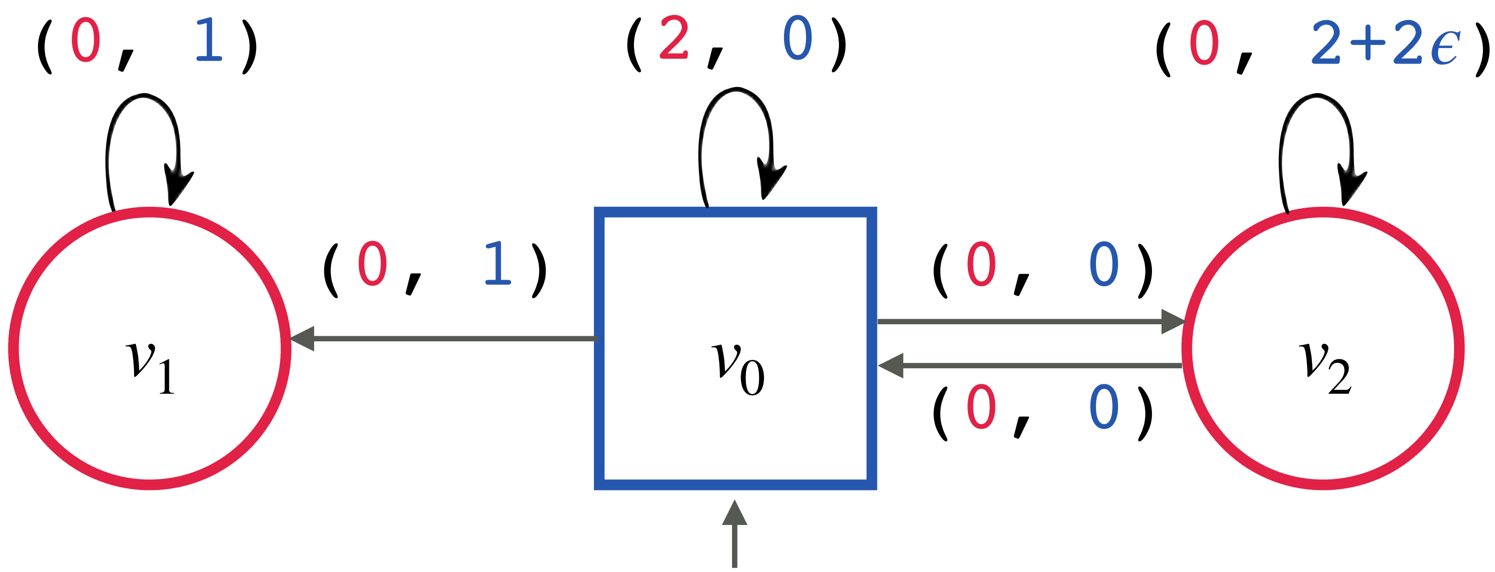

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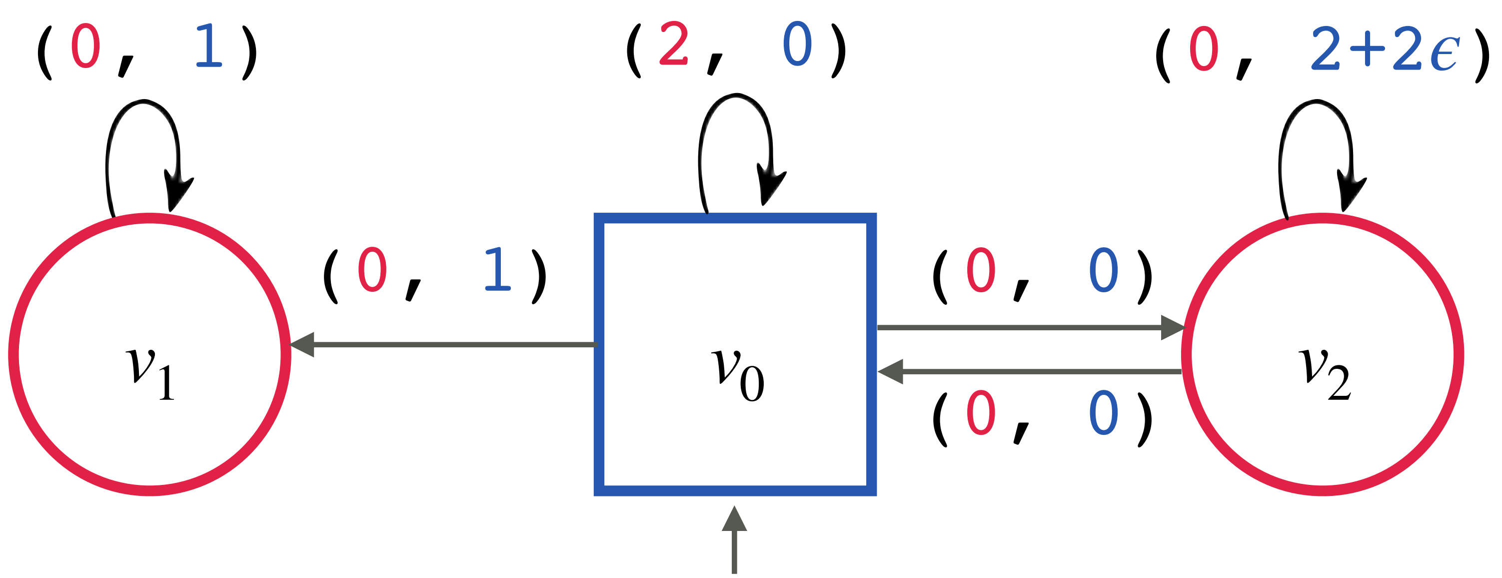

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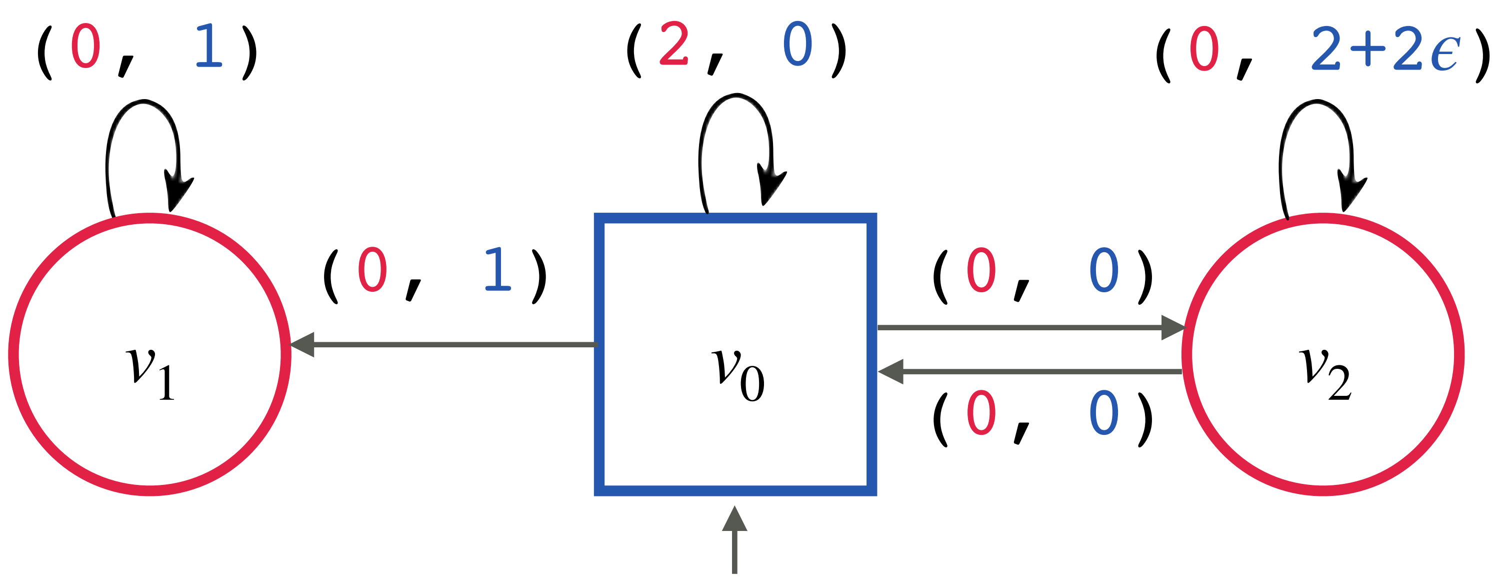

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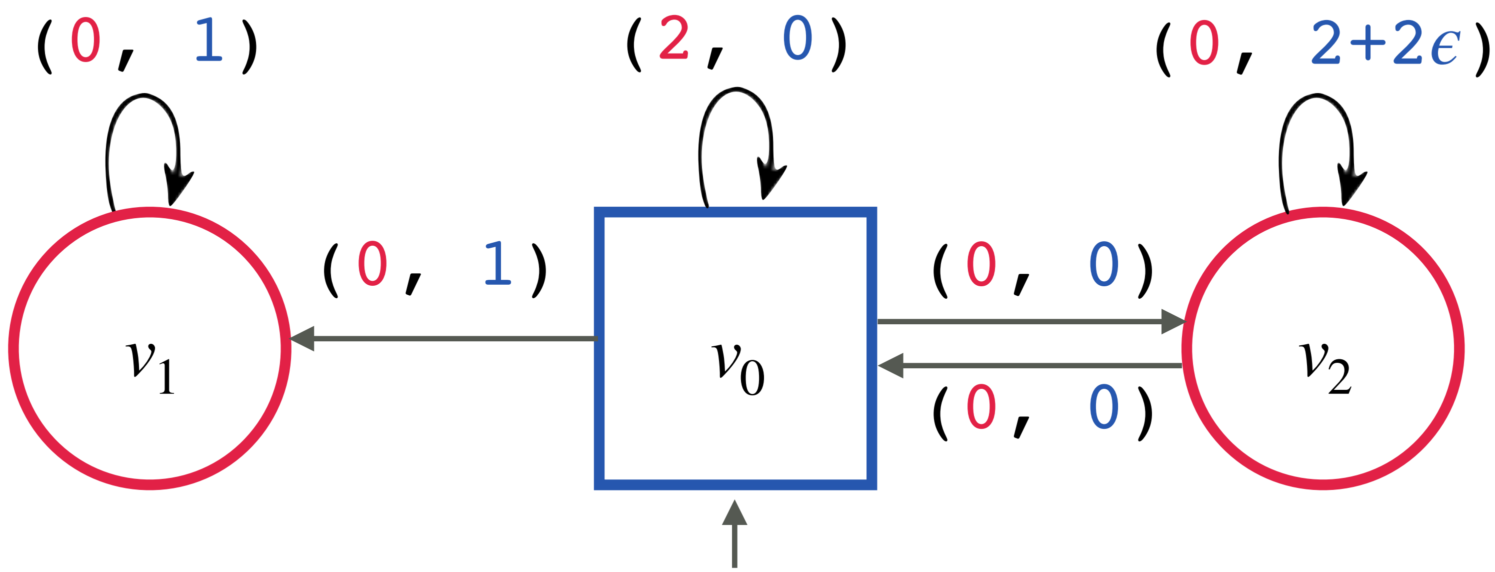

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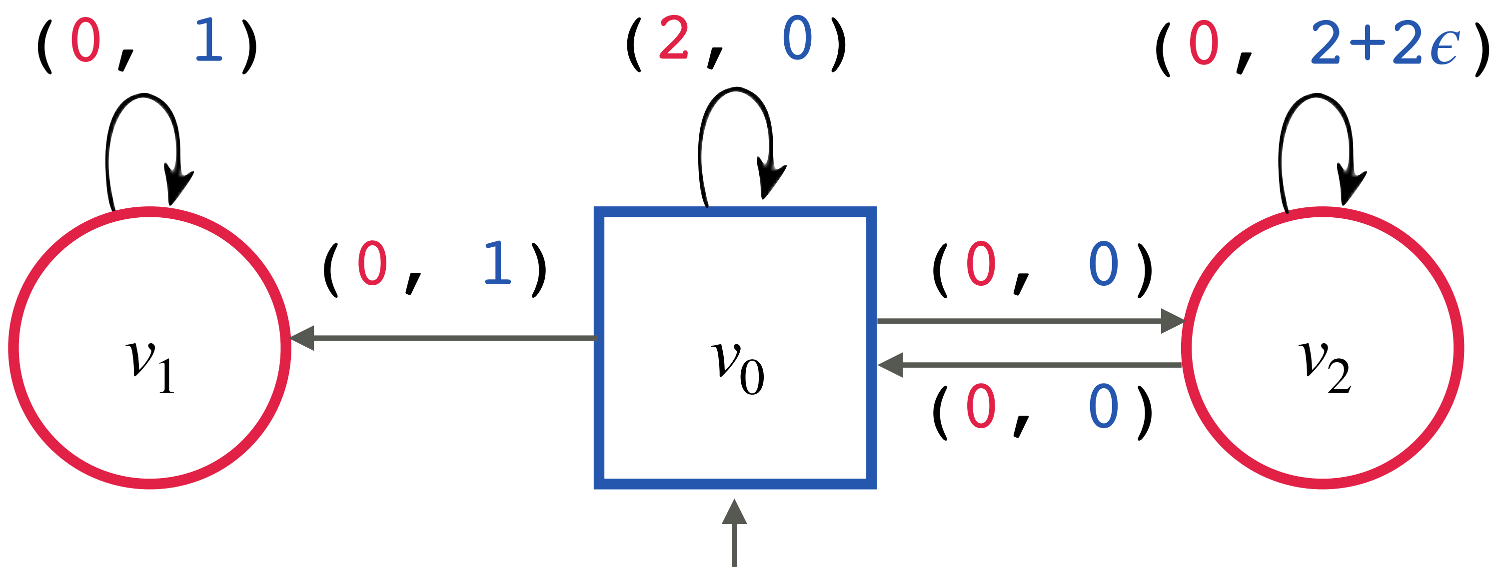

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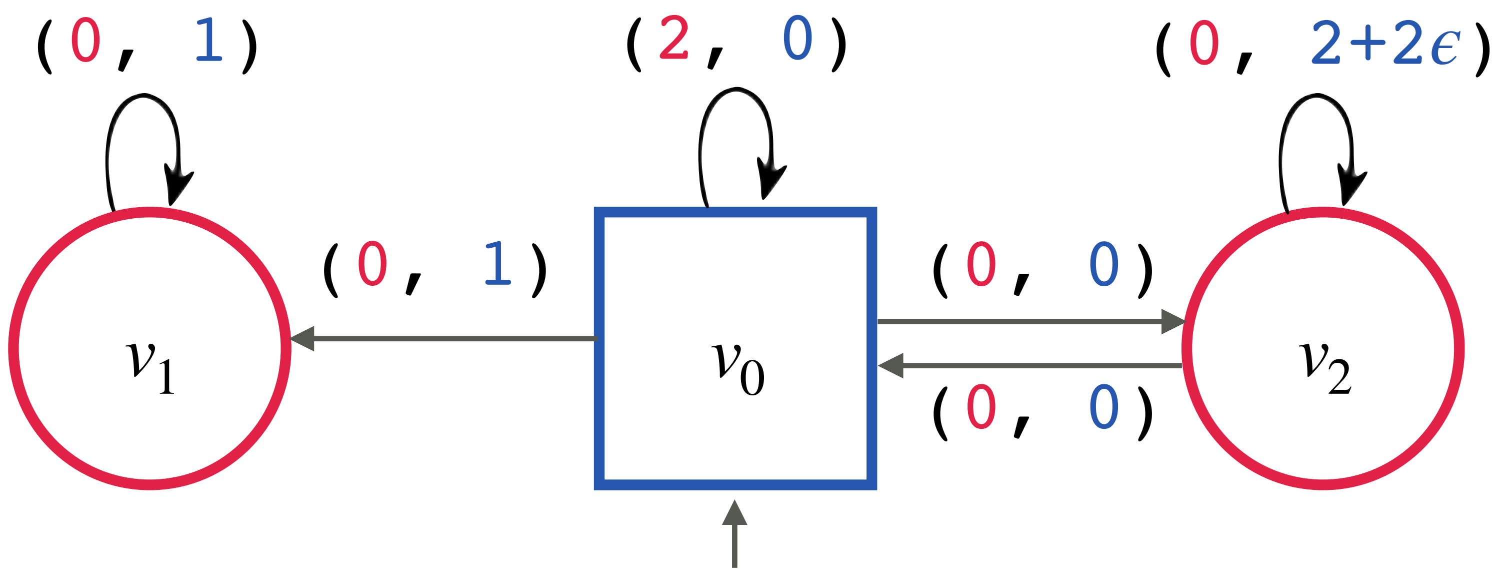

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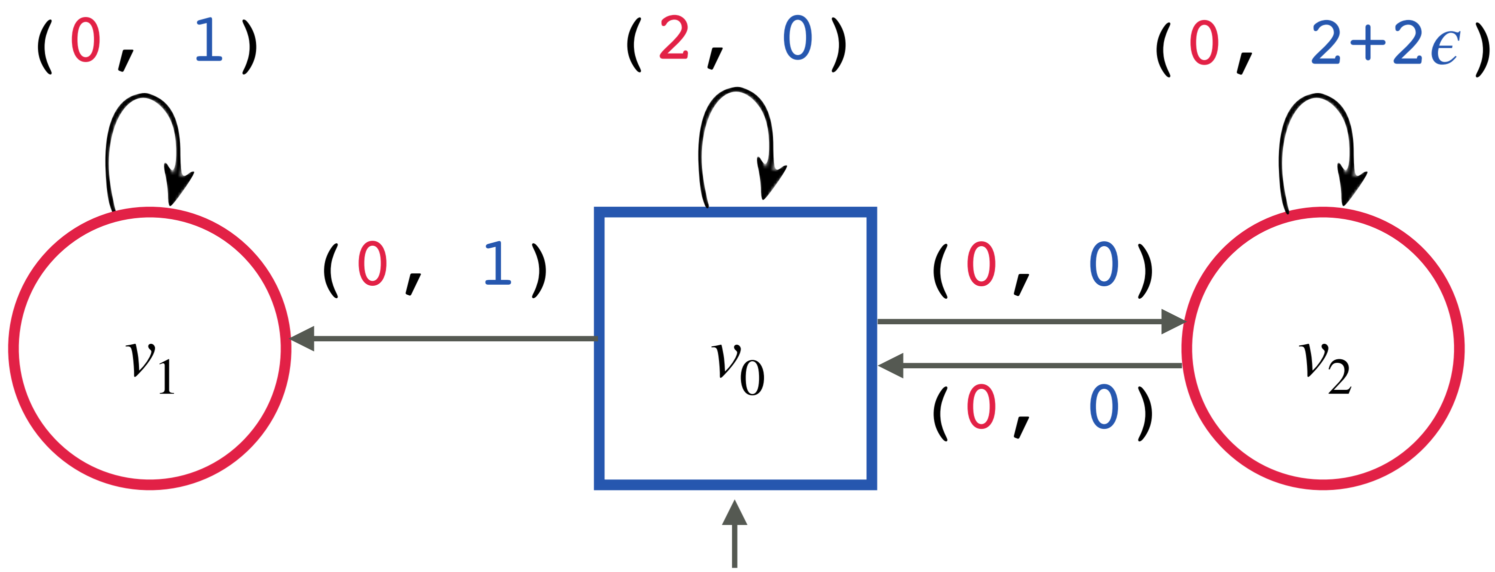

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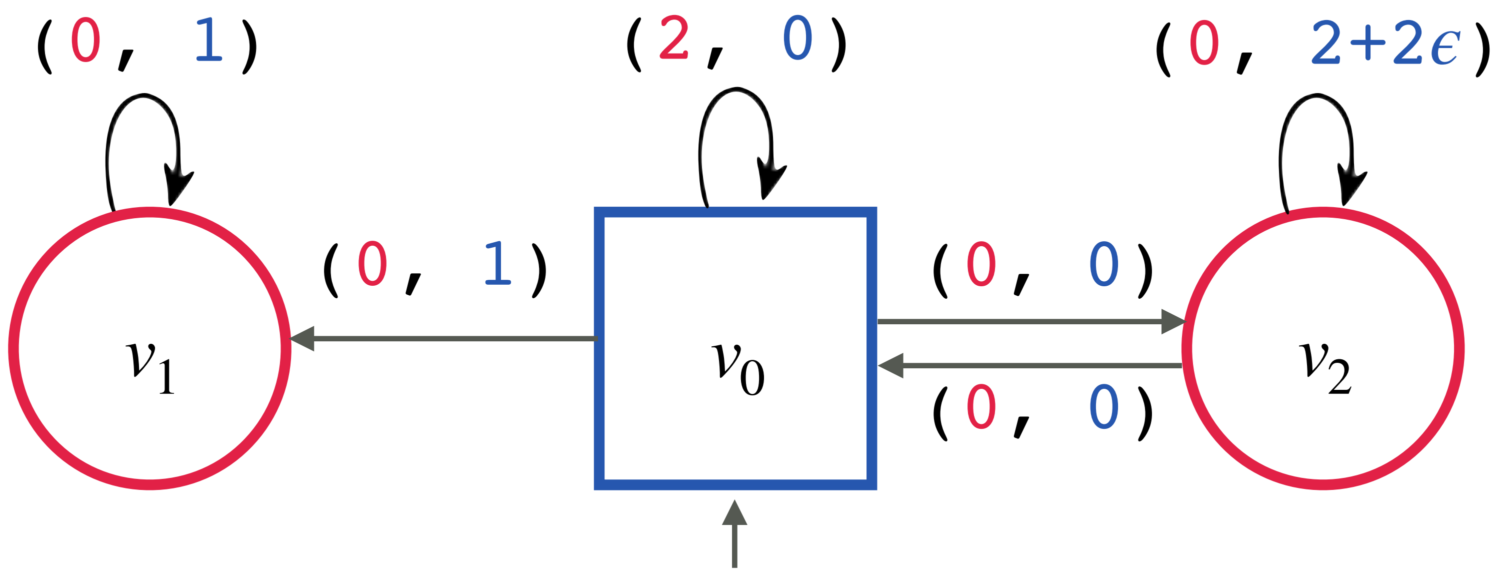

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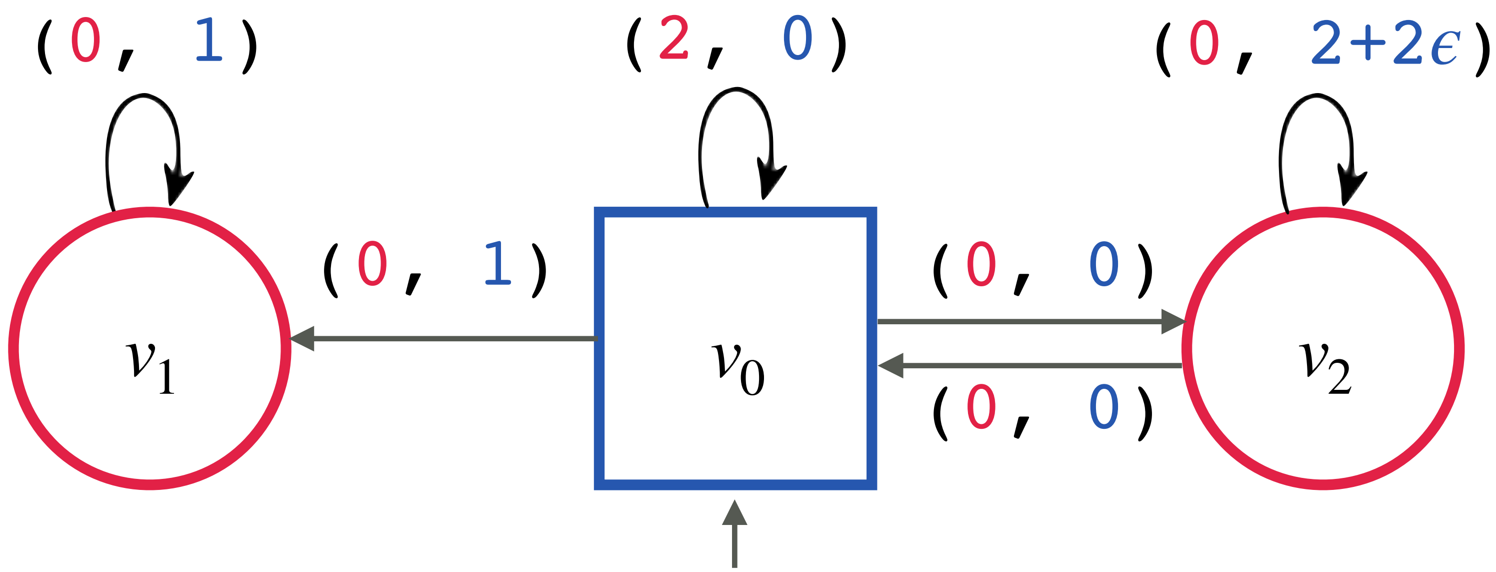

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RESULT 4:

$\mathbf{ASV}^{\epsilon} > c$ if and only if

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If $\mathbf{ASV}^{\epsilon} >$ c, we can find

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$\mathbf{ASV}^{\epsilon}_{\mathsf{FM}}$: Leader is restricted to playing Finite Memory Strategies

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We can also express $\rho(c)$ as a set of linear programs

In the linear program, we maximise **c**.

Conclusion & Future Work

Results

- Results in our work
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General Case	NP-Time Finite Memory Strategy	Theory of Reals	No
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$$\mathbf{ASV}^{\epsilon} = \mathbf{ASV}^{\epsilon}_{\mathsf{FM}}$$

Future Work

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Thank You