Hierarchical Methods of Moments

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Background: methods of moments

Methods of moments:
a unified framework to learn latent variable models.

Task: From iid samples $\rightarrow$ model parameters $(\mu_1, ..., \mu_k, \omega)$

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$$M_2 = \sum_i \omega_i \mu_i \otimes \mu_i \in \mathbb{R}^{d \times d}$$

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2. Parameters via tensor decomposition:

$$\mathcal{TD}(M_1, M_2, M_3, k) \rightarrow (\mu_i, ..., \mu_k, \omega)$$

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Examples:

- Single Topic Model (μ_i: topics)
- Mixture models (μ_i: centers)
- HMM (μ_i: emissions probs.)
- Many others..

Motivation of this paper

Methods of moments are appealing:

- Run in polynomial time.
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- Provable guarantees of optimality **in the realizable setting**.

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But in the real world..

- Data never comes from a specific model.
- Unknown/too big number of latent states.

SIDIWO: a new method of moments

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Simultaneous **Di**agonalization based on **W**hitening and **O**ptimization.

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The optimal l-states model approximating the model generating the data.

$\tilde{\mu}_1, \dots, \tilde{\mu}_l \in \mathbb{R}^d$: **pseudocenters.**

Each pseudocenter synthetically represents some true centers.

Application: hierarchical method of moment

The pseudocenters approximate the original centers of the model.

$$\Downarrow$$

Hierarchical learning of latent variable models.

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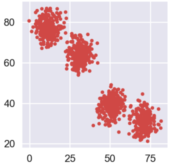

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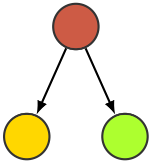

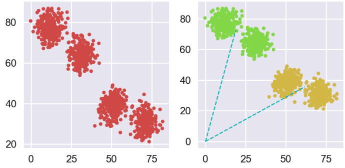

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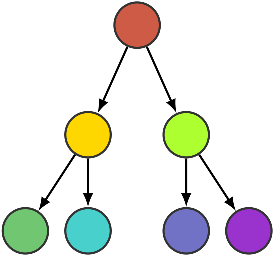
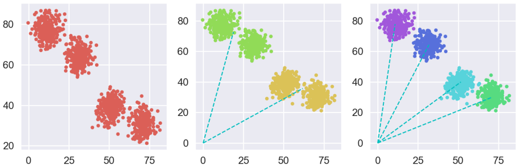
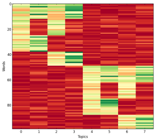
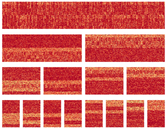




Experiments

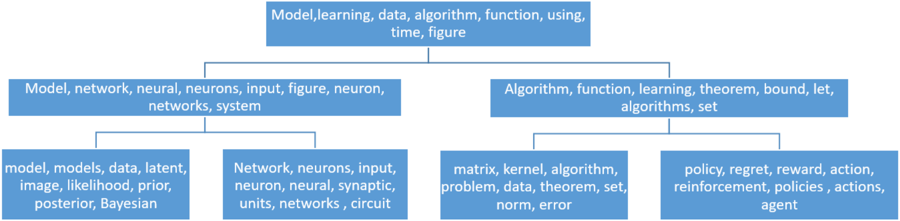
Synthetic Data





Method	Adj. Rand Idx Mean	St. dev.	Run. Time
TPM	0.93	0.06	1.2 sec.
SVD	0.52	0.13	0.1 sec.
Rand. Proj.	0.72	0.06	16 min.
SIDIWO	0.98	0.01	0.4 sec.

Real Data: NIPS papers 1987-2015



See you at the conference!

We are at poster #49 @Pacific Ballroom