

# Lab assignment 2

## Optimization in ML (CSL4010)

**Dr. Md Abu Talhamainuddin Ansary**

Compute  $\nabla f(x^0)$  and  $\nabla^2 f(x^0)$  of the following function. Check whether  $\nabla^2 f(x^0)$  is positive definite or not. Solve the quadratic programming problem

$$\min_{d \in \mathbb{R}^n} \nabla f(x^0)^T d + \frac{1}{2} d^T (\nabla^2 f(x^0) + \lambda I_n) d$$

$$s.t. \quad lb \leq x^0 + d \leq ub$$

where  $\lambda = \begin{cases} 0 & \text{if } eig_{\min} > 0.01 \\ 0.01 - eig_{\min} & \text{otherwise} \end{cases}$

1.  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $f(x) = (x_1 - r)^2 + (x_2 - r)^2$ ,  $-2r \leq x_1, x_2 \leq 2r$ ,  $x^0 = (-0.5r, 1.5r)^T$  where  $r$  is the last digit of your roll no. Use  $r = 1.5$  if last digit of your roll no is 0.

For next problems chose  $x^0$  uniformly distributed in  $(lb, ub)$

2.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $n = 5, 10, 30$

$$I_1 = \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 1\}$$

$$I_2 = \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 0\};$$

$$f_1(x) = x_1 + \frac{2}{|I_1|} \sum_{i \in I_1} (x_i - \sin(6\pi x_1 + i\pi/n))^2$$

$$f_2(x) = 1 - \sqrt{x_1} + \frac{2}{|I_2|} \sum_{i \in I_2} (x_i - \sin(6\pi x_1 + i\pi/n))^2$$

$$0.001 \leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, \dots, n.$$

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3.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 5, 10, 30$

$$\begin{aligned}
I_1 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 1\} \\
I_2 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 0\} \\
\text{if } i \in I_1 \text{ then } y_i &= x_i - (0.3 * x_1^2 \cos(24\pi x_1 + 4i\pi/n) + 0.6x_1) \cos(6\pi x_1 + i\pi/n) \\
\text{otherwise } y_i &= x_i - (0.3 * x_1^2 \cos(24\pi x_1 + 4i\pi/n) + 0.6x_1) \sin(6\pi x_1 + i\pi/n) \\
f_1(x) &= x_1 + \frac{2}{|I_1|} \sum_{i \in I_1} y_i^2 \\
f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|} \sum_{i \in I_2} y_i^2 \\
0.001 &\leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, \dots, n.
\end{aligned}$$

4.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 5, 10, 30$

$$\begin{aligned}
I_1 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 1\} \\
I_2 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 0\} \\
y_i &= x_i - x_1^{0.5(1+3\frac{i-2}{n-2})}, \quad i = 2, 3, \dots, 30 \\
f_1(x) &= x_1 + \frac{2}{|I_1|} \left( 4 \sum_{i \in I_1} y_i^2 - 2 \prod_{i \in I_1} \cos(20y_i\pi/\sqrt{i}) + 2 \right) \\
f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|} \left( 4 \sum_{i \in I_2} y_i^2 - 2 \prod_{i \in I_2} \cos(20y_i\pi/\sqrt{i}) + 2 \right) \\
0.001 &\leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, \dots, n.
\end{aligned}$$

5.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 30$

$$\begin{aligned}
I_1 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 1\} \\
I_2 &= \{i \in \{2, 3, \dots, n\} \mid i \bmod 2 = 0\} \\
y_i &= x_i - \sin(6\pi x_1 + i\pi/n), \quad i = 2, 3, \dots, n \\
f_1(x) &= x_1^{0.2} + \frac{2}{|I_1|} \sum_{i \in I_1} y_i^2 \\
f_2(x) &= 1 - x_1^{0.2} + \frac{2}{|I_2|} \sum_{i \in I_2} y_i^2 \\
0.001 &\leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, \dots, n.
\end{aligned}$$

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6.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 30$

$$\begin{aligned}
I_1 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 1\} \\
I_2 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 2\} \\
I_3 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 0\} \\
f_1(x) &= \cos(0.5x_1\pi) \cos(0.5x_2\pi) + \frac{2}{|I_1|} \sum_{i \in I_1} (x_i - 2x_2 \sin(2\pi x_1 + i\pi/n))^2 \\
f_2(x) &= \cos(0.5x_1\pi) \sin(0.5x_2\pi) + \frac{2}{|I_2|} \sum_{i \in I_2} (x_i - 2x_2 \sin(2\pi x_1 + i\pi/n))^2 \\
f_3(x) &= \sin(0.5x_1\pi) + \frac{2}{|I_3|} \sum_{i \in I_3} x_i - 2x_2 \sin(2\pi x_1 + i\pi/n)^2 \\
0 &\leq x_1, x_2 \leq 1, \quad -2 \leq x_i \leq 2, \quad i = 3, 4, \dots, n.
\end{aligned}$$

7.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 30$

$$\begin{aligned}
I_1 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 1\} \\
I_2 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 2\} \\
I_3 &= \{i \in \{3, 4, \dots, n\} \mid i \bmod 3 = 0\} \\
y_i &= x_i - 2x_2 \sin(2\pi x_1 + i\pi/n), \quad i = 3, 4, \dots, 30 \\
f_1(x) &= \cos(0.5x_1\pi) \cos(0.5x_2\pi) + \frac{2}{|I_1|} \sum_{i \in I_1} 4y_i^2 - \cos(8\pi y_i) + 1 \\
f_2(x) &= \cos(0.5x_1\pi) \sin(0.5x_2\pi) + \frac{2}{|I_2|} \sum_{i \in I_2} 4y_i^2 - \cos(8\pi y_i) + 1 \\
f_3(x) &= \sin(0.5x_1\pi) + \frac{2}{|I_3|} \sum_{i \in I_3} 4y_i^2 - \cos(8\pi y_i) + 1 \\
0 &\leq x_1, x_2 \leq 1, \quad -2 \leq x_i \leq 2, \quad i = 3, 4, \dots, n.
\end{aligned}$$

8.  $f_1, f_2 : \mathbb{R}^4 \rightarrow \mathbb{R}$

$$\begin{aligned}
F &= 10, \quad E = 2 \times 10^5, \quad L = 200, \quad \sigma = 10 \\
f_1(x) &= L(2x_1 + \sqrt{2}x_2 + \sqrt{x_3 + x_4}) \\
f_2(x) &= FL/E \left( \frac{2}{x_1} + \frac{2\sqrt{2}}{x_2} - \frac{2\sqrt{2}}{x_3} + \frac{2}{x_4} \right) \\
\frac{F}{\sigma} &\leq x_1, x_4 \leq \frac{3F}{\sigma}, \quad \frac{F}{\sqrt{2}\sigma} \leq x_2, x_3 \leq \frac{3F}{\sigma}
\end{aligned}$$

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9.  $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 2$ .

$$\begin{aligned} gx &= 2 - \exp(-(\frac{x_2 - 0.2}{0.004})^2) - 0.8 \exp(-(\frac{x_2 - 0.6}{0.4})^2) \\ f_1(x) &= x_1 \\ f_2(x) &= \frac{gx}{x_1} \\ 0.1 &\leq x_1 \leq 1, \quad 0 \leq x_2 \leq 1. \end{aligned}$$

10.  $f_1, f_2 : \mathbb{R}^4 \rightarrow \mathbb{R}, n = 2$ .

$$\begin{aligned} \beta &= 1, \quad \alpha = 2, \quad q = 4 \\ gx &= 1 + 10x_2 \\ h &= 1 - (\frac{x_1}{gx})^\alpha - (\frac{x_1}{gx}) \sin(2\pi qx_1) \\ f_1(x) &= x_1 \\ f_2(x) &= gx * h \\ 0 &\leq x_1, x_2 \leq 1 \end{aligned}$$

11.  $f_1, f_2 : \mathbb{R}^4 \rightarrow \mathbb{R}, n = 2$ .

$$\begin{aligned} \gamma &= 0.25 \\ gx &= 1 + x_2^\gamma \\ h &= 1 - (\frac{x_1}{gx})^2 \\ f_1(x) &= x_1 \\ f_2(x) &= gx * h \\ 0 &\leq x_1 \leq 1, \quad 0.001 \leq x_2 \leq 1 \end{aligned}$$

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12.  $f_1, f_2 : \mathbb{R}^4 \rightarrow \mathbb{R}, n = 2.$

$$\begin{aligned}
\gamma &= 1 \\
gx &= 1 + x_2^\gamma \\
h &= 1 - \left(\frac{x_1}{gx}\right)^2 \\
f_1(x) &= x_1 \\
f_2(x) &= gx * h \\
0 &\leq x_1, x_2 \leq 1
\end{aligned}$$

13.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 7)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 100(k + \sum_{i=m}^n (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))) \\
f_1(x) &= 0.5(1 + gx) \prod_{i=1,2,\dots,m-1} x_i \\
f_j(x) &= 0.5(1 + gx) \prod_{i=1,2,\dots,m-j} x_i (1 - x_{m-j+1}), \quad j = 2, 3, \dots, m \\
0 &\leq x_i \leq 1 \quad i = 1, 2, \dots, n.
\end{aligned}$$

14.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 10)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= \sum_{i=m}^n (x_i - 0.5)^2 \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\dots,m-1} \cos(0.5\pi x_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\dots,m-j} \cos(0.5\pi x_i) \sin(0.5\pi x_{m-i+1}), \quad j = 2, \dots, m \\
0 &\leq x_i \leq 1 \quad i = 1, \dots, n.
\end{aligned}$$

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15.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 12)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 100(k + \sum_{i=m}^n (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))) \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\dots,m-1} \cos(0.5\pi x_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\dots,m-j} \cos(0.5\pi x_i) \sin(0.5\pi x_{m-i+1}), \quad j = 2, \dots, m \\
0 &\leq x_i \leq 1 \quad i = 1, \dots, n.
\end{aligned}$$

16.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 10)$

$$\begin{aligned}
k &= n - m + 1, \quad \alpha = 100 \\
y_i &= x_i^\alpha \quad i = 1, 2, \dots, n \\
gx &= \sum_{i=m}^n (y_i - 0.5)^2 \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\dots,m-1} \cos(0.5\pi y_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\dots,m-j} \cos(0.5\pi y_i) \sin(0.5\pi y_{m-i+1}), \quad j = 2, \dots, m \\
0 &\leq x_i \leq 1 \quad i = 1, \dots, n.
\end{aligned}$$

17.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 12)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= \sum_{i=m}^n x_i^{0.1} \\
\theta_i &= \frac{\pi(1 + 2 * gx * x_i)}{4(1 + gx)} \quad i = 1, 2, \dots, n \\
f_1(x) &= (1 + gx) \cos(0.5\pi x_1) \prod_{i=1,2,\dots,m-1} \cos(\theta_i) \\
f_j(x) &= (1 + gx) \cos(0.5\pi x_1) \prod_{i=1,2,\dots,m-j} \cos(\theta_i) \sin(\theta_{m-i+1}), \quad j = 2, \dots, m \\
0.001 &\leq x_i \leq 1 \quad i = 1, \dots, n.
\end{aligned}$$

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18.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2), (3, 3), (3, 10)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 1 + \frac{9}{k} \sum_{i=m}^n x_i \\
f_j(x) &= x_j, \ j = 1, \dots, m-1 \\
f_m(x) &= (1 + gx) * \left( m - \sum_{i=1}^{m-1} \frac{x_i}{1 + gx} (1 + \sin(3\pi x_i)) \right) \\
0 &\leq x_i \leq 1 \quad i = 1, \dots, n.
\end{aligned}$$

19.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2).$

$$\begin{aligned}
f_1(x) &= x_1^2 - x_2^2 \\
f_2(x) &= \frac{x_1}{x_2} \\
-1 &\leq x_1 \leq 2, \ 1 \leq x_2 \leq 2
\end{aligned}$$

20.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2).$

$$\begin{aligned}
f_1(x) &= -2e^{15(-(x_1-0.1)^2-x_2^2)} - e^{20(-(x_1-0.6)^2-(x_2-0.6)^2)} + e^{20(-(x_1+0.6)^2-(x_2-0.6)^2)} \\
&\quad + e^{20(-(x_1-0.6)^2-(x_2+0.6)^2)} + e^{20(-(x_1+0.6)^2-(x_2+0.6)^2)} \\
f_2(x) &= 2e^{20(-x_1^2-x_2^2)} + e^{20(-(x_1-0.4)^2-(x_2-0.6)^2)} + -e^{20(-(x_1+0.5)^2-(x_2-0.7)^2)} \\
&\quad - e^{20(-(x_1-0.5)^2-(x_2+0.7)^2)} + e^{20(-(x_1+0.4)^2-(x_2+0.8)^2)} \\
-1 &\leq x_1, x_2 \leq 1
\end{aligned}$$

21.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 2).$

$$\begin{aligned}
f_1(x) &= 1 - e^{-(x_1-1)^2-(x_2+1)^2} \\
f_2(x) &= 1 - e^{-(x_1+1)^2-(x_2-1)^2} \\
-4 &\leq x_1, x_2 \leq 4
\end{aligned}$$

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22.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (2, 10).$

$$\begin{aligned} f_1(x) &= 1 - e^{-\sum_{i=1}^n (x_i - \frac{1}{\sqrt{n}})^2} \\ f_2(x) &= 1 - e^{-\sum_{i=1}^n (x_i + \frac{1}{\sqrt{(n)}})^2} \\ -4 &\leq x_i \leq 4 \quad i = 1, 2, \dots, n. \end{aligned}$$

23.  $f_1, f_2, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R} \ (m, n) = (3, 2).$

$$\begin{aligned} f_1(x) &= (1 + x_3)(x_1^3 x_2^2 - 10x_1 - 4x_2) \\ f_2(x) &= (1 + x_3)(x_1^3 x_2^2 - 10x_1 + 4x_2) \\ f_3(x) &= 3(1 + x_3)x_1^2 \\ 1 &\leq x_1 \leq 3.5, \quad -2 \leq x_2 \leq 2, \quad 0 \leq x_3 \leq 1 \end{aligned}$$