ENPM 662

Introduction to Robot Modeling.

Final Exam. (117074109)

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Objective: Designing a 15 DoF robot to achieve task of lifting an object while squating.

Approach: The approach here is to, instead of designing a robot with entire kinematic chaing making calculations complete chaos, we will Create a base frame & calculate five different links from the base frame which will be preferabely from either neck or chest.

Joint Movements: All the joints are revolute joints with one DoF at each joint.

Head : Head Yaw

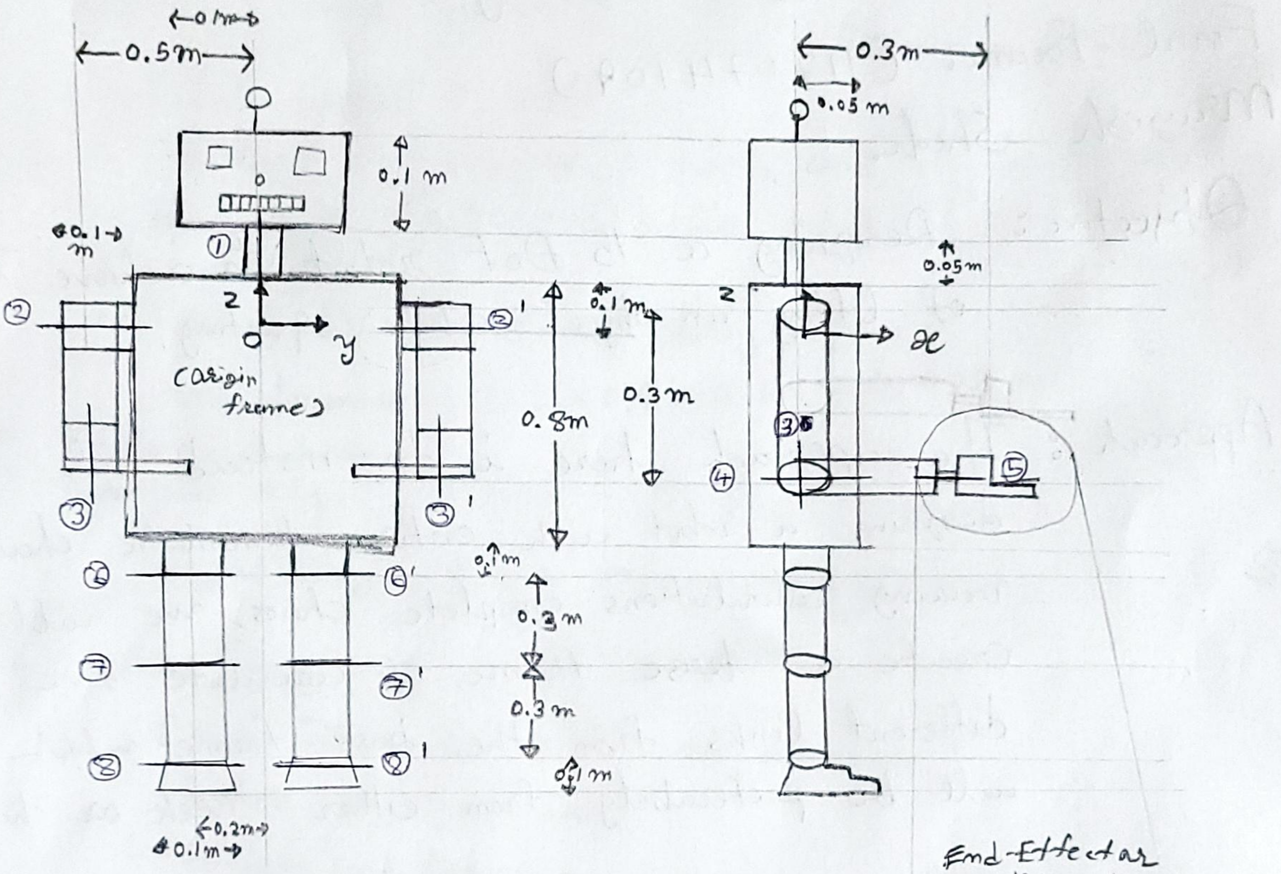
Right Arm: Shoulder Pitch, Elbow Roll, Elbow Yaw, wrist Yaw.

Left Arm : shoulder Pitch, Elbow Roll, Elbow Yaw, wrist Yaw.

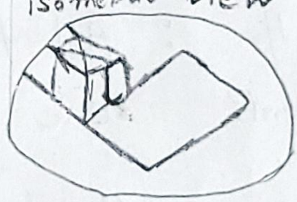
Left Leg : Hip Pitch, knee Pitch, Ankle Pitch

Right Leg : Hip pitch, knee Pitch, Ankle Pitch

(A) Sketch robot w/ dimensions & DoFs.



End-Effector isometric view



① → Head yaw

② & ②' → Shoulder Pitch

③ & ③' → Elbow Roll

④ → Elbow Yaw x 2

⑤ → wrist Yaw x 2

⑥ & ⑥' → Hip Pitch

⑦ & ⑦' → knee pitch

⑧ & ⑧' → Ankle pitch

Total DoFs = 15.

All links have
Same length of 0.3 m
Except neck has 0.05 m

(B) Estimation of Mass of Each link & Total mass of robot. [Motor weight = 1 lb]

Material = Aluminium 6061 [$\rho = 2700 \frac{kg}{m^3}$]

(i) Total weight of motors =

as each DoF require 1 motor, and we have 15 DoF, thus, total weight of motors = $\boxed{15 \text{ lb.}}$ = $\boxed{}$

(ii) Head & it's link to main body :

Head dimension = $0.2 \times 0.1 \times 0.1 = 0.002 \ m^3$

[assuming features weight 0 kg.] & Link $\cong 0 \ m^3$

weight in kg = $\rho \cdot V = 2700 \times 0.002 = \boxed{5.4 \text{ kg}}$

(iii) Arm [Each] :

[Each link in arm is of same dimensions] & there are two links.

↳ Volume each link = $0.1 \times 0.3 \times 0.1 = \frac{8.1}{2700} \ m^3$

There are total four links in both arms

⇒ weigh of Each link = $\underline{8.1 \text{ kg}}$

Total weigh for BOTH arms = $8.1 \times 4 = \boxed{32.4 \text{ kg}}$

(iv) End Effector weight :

assuming it to be a 0.02 m thick plate of 0.05 × 0.05 m² area.

weigh will be, = $2700 (0.05 \times 0.05 \times 0.02) = \boxed{0.135 \text{ kg}}$

(v) Legs :

There are two links & 1 foot, where link [Arm] = link [leg].

The foot weight = $\rho \times V = 2700 \times (0.1 \times 0.1 \times 0.1) = 2.7$ kg [assuming it's cube]

Total leg weight = $2 \times (2.7) + 4 (8.1) = \boxed{37.8 \text{ kg}}$

(vi) Main frame weight :

it's a block of (0.8m × 0.3m × 0.1m) dimensions.

The weight $= 2700 \times (0.8 \times 0.3 \times 0.1)$ kg

$= \boxed{64 \text{ kg}}$

~~Total Weight of the Robot will be :~~

~~$= [15 + 5.4 + 32.4 + 0.135 + 37.8 + 64]$ kg~~

~~$\boxed{T.W. = 154.735 \text{ kg}}$~~

Total weight of the Robot will be :

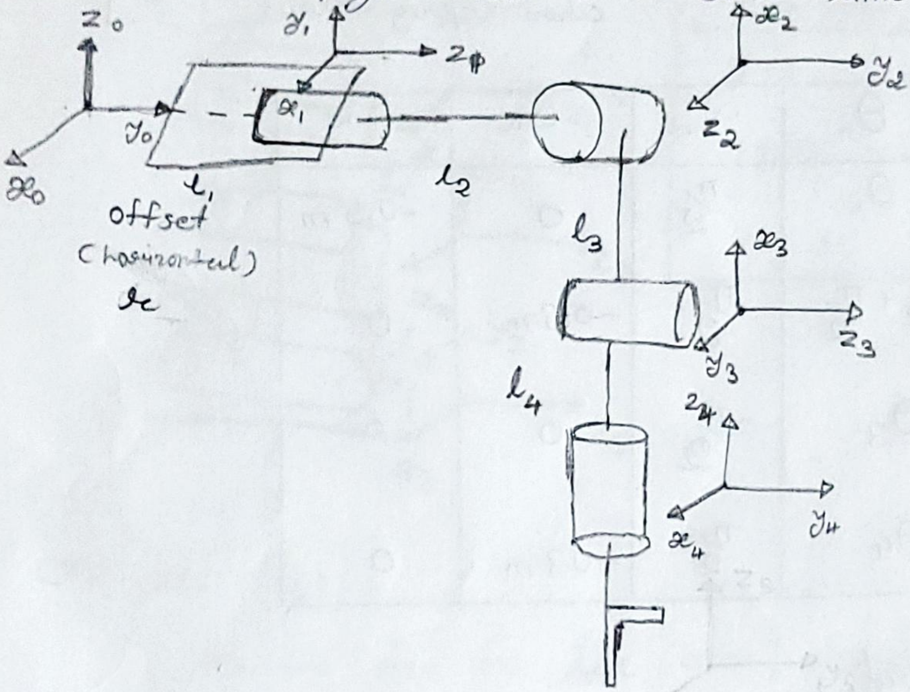
$= [6.8 + 5.4 + 32.4 + 0.135 + 37.8 + 64]$ kg

$\boxed{T.W. = 146.5 \text{ kg}}$

(C) Forward & Inverse kinematics of the robot.

↳ ① Forward kinematics.

(i) Right arm from base link:



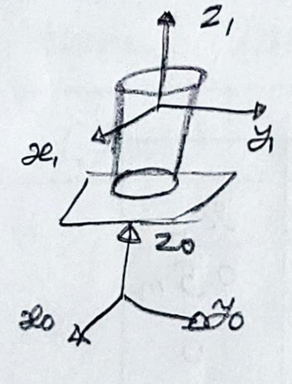
Dh-table for right Arm:

Frame (Joint)	θ	α_n	d_n	r_n
0 - 1	θ_1	$-\pi/2$	0	0.5m
1 - 2	$\theta_2 - \frac{\pi}{2}$	$\frac{\pi}{2}$	0.3m	0
2 - 3	θ_3	$-\pi/2$	0	0
3 - 4	θ_4	$\pi/2$	0.3m	0

(ii) Similarly for left hand the DH parameter table will be [referenced from changes in DH parameter when flipping arms]

Frame (Joint)	θ_n	α_n	d_n	r_n
0 - 1	θ_1	$-\pi/2$	0	$-0.5m$
1 - 2	$\theta_2 + \pi/2$	$\pi/2$	$-0.3m$	0
2 - 3	θ_3	$-\pi/2$	0	0
3 - 4	θ_4	$\pi/2$	$-0.3m$	0

(iii) For the head,

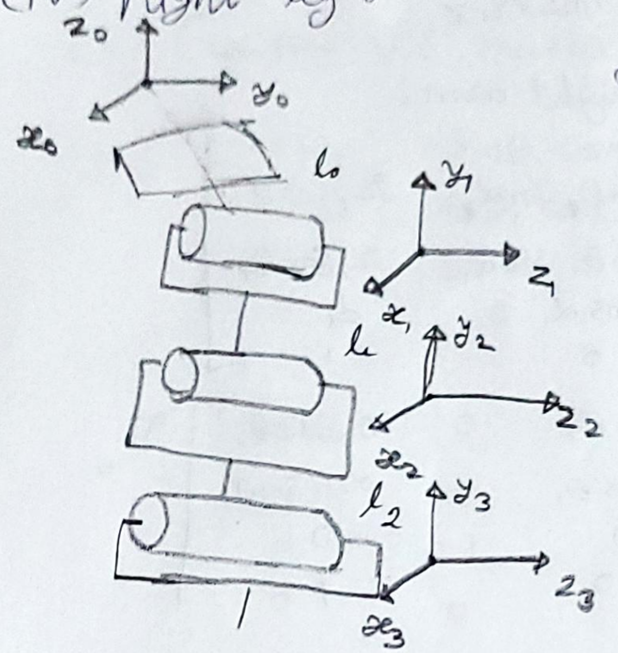


Frame (Joint 0-1)	θ_n	α_n	d_n	r_n
	θ_1	0	0	$0.15m$

(iv) For Right leg,

[to be continued...]

(iv) Right leg :



DH- table :

Frame (Joint)	θ	α_n	d_n	r_n
0 - 1	θ_1	$\pi/2$	0.1m	-0.8 m
1 - 2	θ_2	0	0	-0.3 m
2 - 3	θ_3	0	0	-0.3 m

(v) Left leg :

As we did for the arm, changing / flipping the leg will change DH-parameters to followings.

Frame (Joint)	θ	α_n	d_n	r_n
0 - 1	θ_1	$\pi/2$	-0.1m	-0.8m
1 - 2	θ_2	0	0	-0.3m
2 - 3	θ_3	0	0	-0.3m

Now creating transformation matrix.

(i) Creating t.f. matrix for right arm.

$$
{}_0T^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos\alpha_1 & \sin\theta_1\sin\alpha_1 & r_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos\alpha_1 & -\cos\theta_1\sin\alpha_1 & r_1\sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_0T^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0.5\cos\theta_1 \\ \sin\theta_1 & 0 & +\cos\theta_1 & 0.5\sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_1T^2 = \begin{bmatrix} -\sin\theta_2 & 0 & +\cos\theta_2 & \overset{0}{\cancel{0.5\sin\theta_2}} \\ +\cos\theta_2 & 0 & +\sin\theta_2 & -\overset{0}{\cancel{0.5\cos\theta_2}} \\ 0 & 1 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$[\because \cos(\theta_2 - \pi/2) = -\sin\theta_2]$

$$
{}_2T^3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_3 & 0 & +\cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_3T^4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_0T^4 = {}_0T^1\,{}_1T^2\,{}_2T^3\,{}_3T^4
$$

[matrix multiplication will give us the final transform]

(ii) Creating t.f. matrix for left arm.

$$ {}_0T^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \cos\alpha_1 & \sin\theta_1 \sin\alpha_1 & r_1 \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 \cos\alpha_1 & -\cos\theta_1 \sin\alpha_1 & r_1 \sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_0T^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & -0.5\cos\theta_1 \\ \sin\theta_1 & 0 & +\cos\theta_1 & -0.5\sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_1T^2 = \begin{bmatrix} \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ -\cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & -0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_2T^3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & +\cos\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_3T^4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & -0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_0T^4 = {}_0T^1 \cdot {}_1T^2 \cdot {}_2T^3 \cdot {}_3T^4 $$

(iii) T.f. matrix for head:

$$
{}_0T^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0.15\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0.15\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(iv) T.f. matrix for right leg;

$$
{}_0T^1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & -0.8\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & -0.8\sin\theta_1 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_1T^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & -0.3\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & -0.3\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_2T^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & -0.3\cos\theta_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & -0.3\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_0T^3 = {}_0T^1 \, {}_1T^2 \, {}_2T^3.
$$

(v) T.f. matrix for left leg:

$$
{}_0T^1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & -0.8\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & -0.8\sin\theta_1 \\ 0 & 1 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_1T^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & -0.3\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & -0.3\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

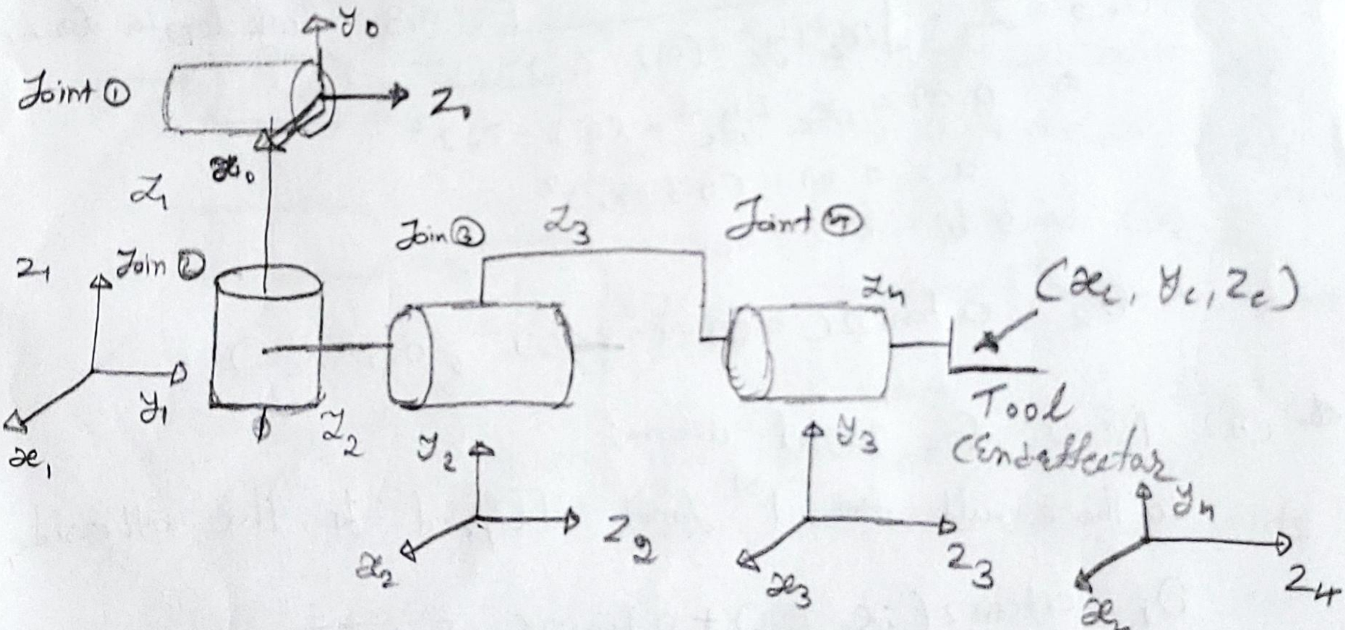
$$
{}^{3}_{2}T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & -0.3\cos\theta_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & -0.3\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{0}T = {}^{1}_{0}T \, {}^{2}_{1}T \, {}^{3}_{2}T .
$$

↳ (2) Inverse kinematics

(i) ~~Right~~ Left Arm from base link,

for invers kinematics, our goal is to find θ_n values for given (X_c, y_c, z_c) which are end effector location. Thus, we can make robot use only two DOF in each hand to make movements in 3-D.



- Now, we know that Joint ③ & Joint ④ are revolute joints that will be reason for end-effector's orientation, not position.

- Thus, Finding θ_1 in terms of x_c & ~~y~~ z_c

↳ For a Revolute Joint w/ offset ;

* $\theta_1 = atan2(x_c, z_c) - atan2(\sqrt{x_c^2 + z_c^2 - 0.25}, +0.5)$

That's inverse kinematics for a revolute joint with offset d to reach at projected point on 2-D plane which θ_1 joint can control.,

* $\theta_2 \Rightarrow$ for that, joint 2 & link length has to be included -,

$\theta_2 = atan2(a, b)$ $\quad a = \sqrt{x_c^2 + y_c^2}$

& $b = 0.3 - z_c$

[$\because$ 0.3 = link length for L_1]

$0.3 = L_3 = \sqrt{x_c^2 + y_c^2 + (0.3 - z_c)^2}$

$\Rightarrow 0.09 = x_c^2 + y_c^2 + (0.3 - z_c)^2$

$a = 0.09 - (0.3 - z_c)^2$

& $b = 0.3 - z_c$

$\theta_2 = atan2(0.09 - (0.3 - z_c)^2, 0.3 - z_c)$.

* (ii) Now, for right arm,

↳ There will be 1^{st} Joint flipped to the otherside

$\theta_1 = atan2(x_c, z_c) + atan2(-\sqrt{x_c^2 + z_c^2 - d^2}, -d)$

where d = offset which is 0.5m but orientation is important

$\theta_1 = atan2(x_c, z_c) + atan2(-\sqrt{x_c + z_c^2 - 0.25}, -0.5)$

- $\theta_2 \Rightarrow$ The Joint 2 is same for the right & Left hand, the inverse kinematics will be same for the both of them.

$$\theta_2 = atan2(0.09 - (0.3 - z_c)^2, 0.3 - z_c)$$

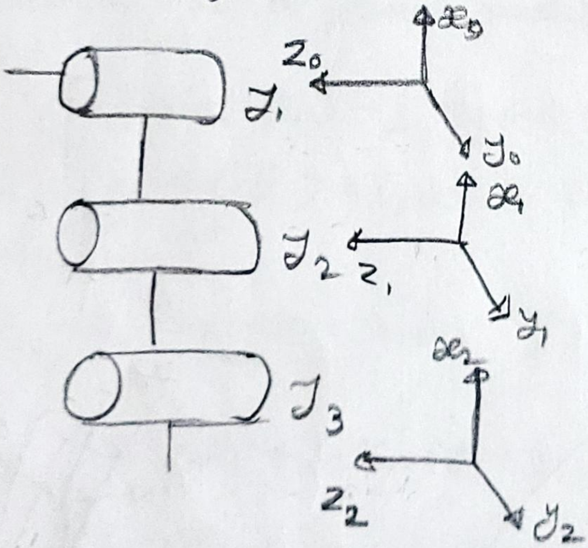
(iii) inverse kinematics for head:

it's simple as,

$$\theta_1 = atan2(x_c, y_c)$$

or $\theta_1 = atan2(x_c, y_c) + \pi$ [for flipped orientation]

(iv) Left leg Inverse kinematics,



- This leg will only be able to move in 2-D. Thus, i.k. has (x_c, z_c) elements [moves in X-Z plane]
- Assuming J_3 is fixed as moving in 2-D requires only two joints

thus, $L_1 = 0.3m$, but $L_2 = 0.6$ m.

Using cosine law, $\cos\theta_2 = \dfrac{x_c^2 + y_c^2 - L_1^2 - L_2^2}{2L_1 L_2} = D$

$$\theta_2 = atan2(D \pm \sqrt{1-D^2})/2$$

& $\theta_1 = atan2(x_c, y_c)/2$

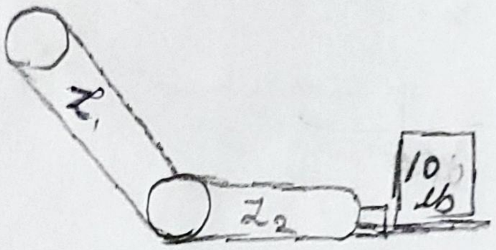
(v) Right leg I.K. :-

The righ leg will also be same as left leg. we have to have same I.K. functions.

$$\theta_2 = atan2(D \pm \sqrt{1-D^2})/2$$

$$\theta_1 = atan2(x_c, y_c)/2$$

(d) Using dynamics equation, find max torque required for each motor in arm to move against gravity.



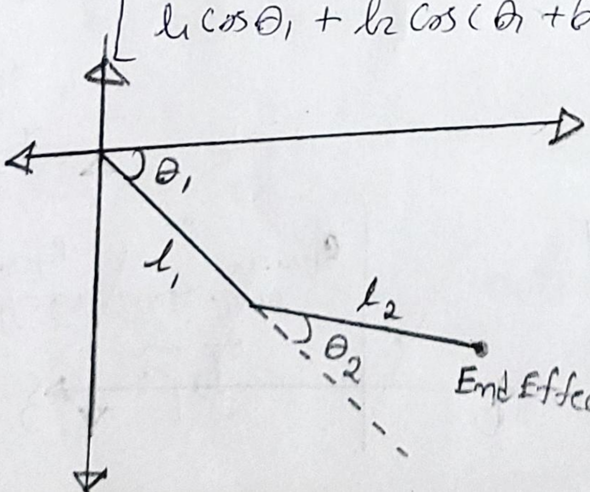
Considerations:
wrist joint (yaw) & Elbow joint (yaw) are not facing any torque against gravity as by lifting object.

- while other two joints, Shoulder (Pitch) & Elbow (yaw) does
- Since it's a planar manipulator, Jacobian for a planar manipulator is,

$$J = \begin{bmatrix} -l_1 \sin(-\theta_1) - l_2 \sin(-\theta_1 - \theta_2) & -l_2 \sin(-\theta_1 - \theta_2) \\ l_1 \cos(-\theta_1) + l_2 \cos(-\theta_1 - \theta_2) & + l_2 \cos(-\theta_1 - \theta_2) \end{bmatrix}$$

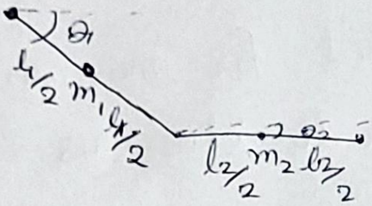
for $\sin(-\theta) = \sin\theta$ & $\cos(-\theta) = \cos\theta$.

$$J = \begin{bmatrix} l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) & l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$



By finding Jacobian, we can Calculate Torque on each joint by the force on Ende-Effector,

First we will calculate torque for lifting 10 lb (as stated in question 20 lb devided in both arms).
assuming the mass is in center of each link;



$m_1 = m_2 = 8.1\ kg$ $\quad [\because g = 10\ m/s^2]$

$\Rightarrow \tau_2 = F \times l^{\perp}$

$= (m_T \times g)\ \frac{l_2}{2} \cos(\theta)$

motor in wrist

$\cancel{\tau_2 = (8.1 \times 10)\ \frac{0.3}{2}}$ $\quad \tau_2 = (8.1 + 0.45) \times 10 \times \frac{0.3}{2}$

$\cancel{\tau_2 = 12.15}$ $\quad \tau_2 = 12.825\ Nm$

$\tau_1 \Rightarrow$ [Remember to include motors]

$$\tau_1 \Rightarrow (8.1 + 8.1 + (3 \times 0.45)) \times 10 \times \frac{0.3}{2} \times \cos(\theta_2) \cdot Nm$$

$\tau_1 = 50.27\ N.m$ $\quad$ [Considering max torque cond^n $\theta_1 = \theta_2 = 0°$]

Now finding Torque required with the object.

$$\tau = J^T F$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 0.3 \sin(\theta_1) + 0.3 \sin(\theta_1 + \theta_2) & 0.3 \cos(\theta_1 + \theta_2) + 0.3 \cos(\theta_1 + \theta_2) \\ 0.3 \sin(\theta_1 + \theta_2) & 0.3 \cos(\theta_1 + \theta_2) \end{bmatrix} \times \begin{bmatrix} 20 \times 10 \\ 0 \end{bmatrix}$$

$\theta_1 = \theta_2 = 45° = \frac{\pi}{4}$

& $F = (20 \times 10, 0)$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 0.3 \times \frac{1}{\sqrt{2}} + 0.3(1) & 0.3 \times \frac{1}{\sqrt{2}} + 0.3(0) \\ 0.3 & 0.3(0) \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 0.512 & 0.212 \\ 0.3 & 0 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix} = \begin{bmatrix} 102.4 \\ 60 \end{bmatrix}$$

$\tau_1 = 50.27 + 102.4 \ Nm$ | $\tau_2 = 12.825 + 60 \ Nm$

$\tau_1 = 152.67 \ Nm.$ | $\tau_2 = 72.82 \ Nm.$

* Since, the τ_1 & τ_2 are calculated for maximum in each condition, these values will give us ability to move in full-workspace.

(c)

(e) Now we have to create two scenarios where the robot will be reaching to the object and then grasping the object.

The solution can be derived by first creating a path for the robot to move and then with the help of inverse kinematics, calculating all the joint angles. After all the joint angles are calculated, the Jacobian matrix is used to find the torque on each joint.

For the sake of simplicity, the robot is assumed to have end effector plates instead of the grasper and robot is not required to bend with lower limbs to grasp this object. So, we can compute the angles from I.K. and torque from the following equation

$$\tau = J^T F$$

Following are the figures where the plots are sketched for both the cases.

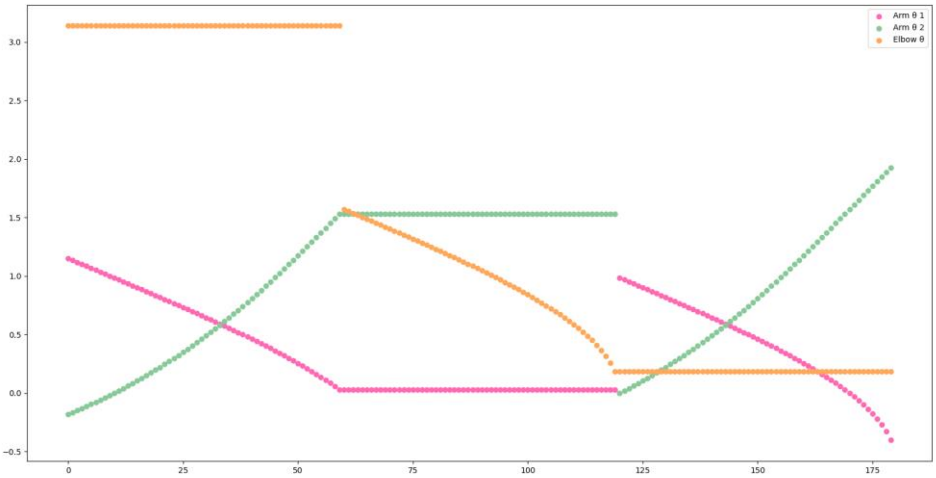


Figure 1. θ for each joint over time

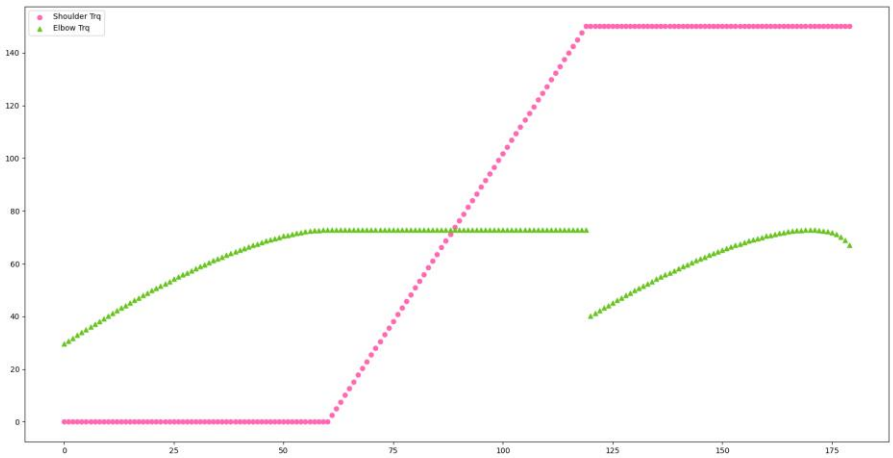


Figure 2. Torque plot on both acting joints over time