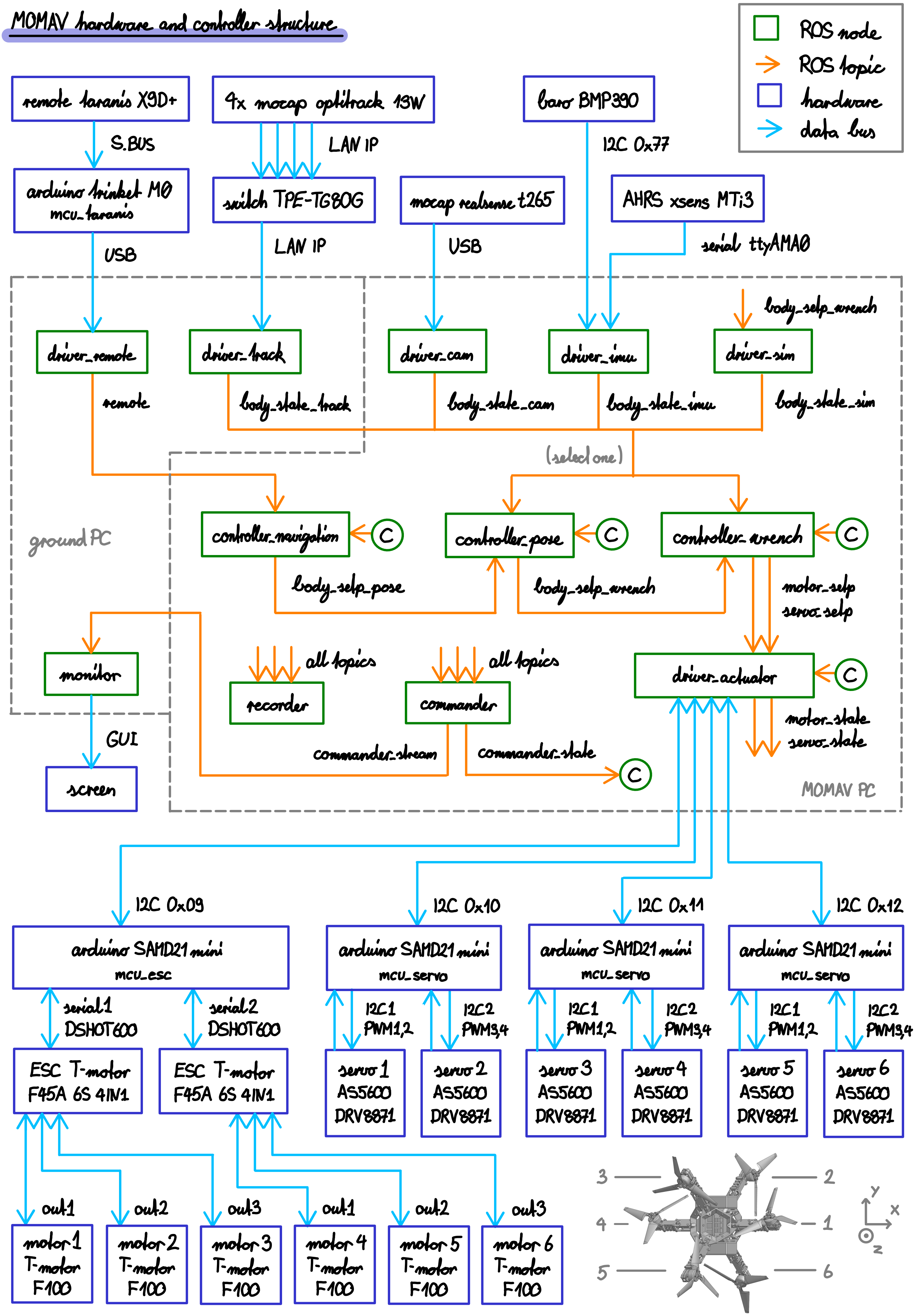
MOMAV hardware and controller structure



MOMAV controllers

wrench control

r : arm center (mm)

x : rotation axis ($\|\cdot\|=1$)

z : zero rot. direction ($\|\cdot\|=1$)

a : rotation angle (output) (rad)

n : desired arm direction ($\|\cdot\|=1$)

u : motor throttle (output) ($[0,1]$)

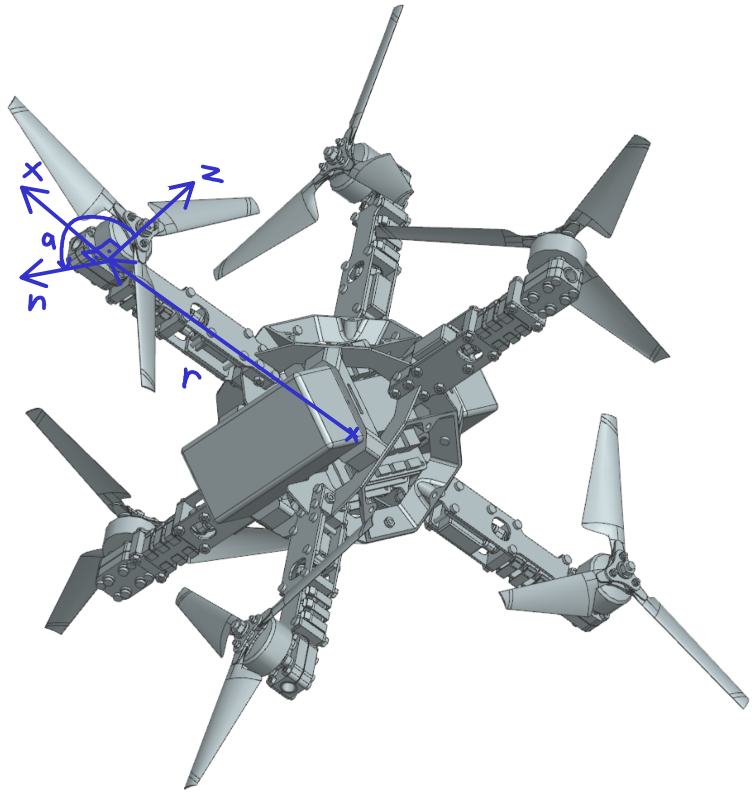
$\mu(u)$: motor force mapping (N)

$\tau(u)$: motor torque mapping (Nm)

q_B : body orientation (quaternion)

F : desired force (input) (N)

M : desired torque (input) (Nm)



geometry & forces:

$n^i = \begin{bmatrix}\cos(a^i/2)\\ x^i\cdot\sin(a^i/2)\end{bmatrix}\cdot z^i$ ← quaternion ; $n^i_a = dn^i/da^i = x^i\times n^i$; $n^i_{aa} = d^2n^i/da^{i2} = x^i\times(x^i\times n^i)$

↳ (see "transport theorem" for derivation, with $\left(\frac{d}{dt}\right)_r n = 0,\ \Omega = x\frac{da}{dt} = 1,\ t = a$)

$\mu(u), \tau(u) =$ e.g. polynomials from calibration (τ sign depends on motor rotation direction !)

$f^i = \mu(u^i)n^i$; $f^i_u = \mu_u(u^i)n^i$; $f^i_a = \mu(u^i)n^i_a$; $f^i_{uu} = \mu_{uu}(u^i)n^i$; $f^i_{aa} = \mu(u^i)n^i_{aa}$; $f^i_{ua} = \mu_u(u^i)n^i_a$; $f^i_{au} = f^i_{ua}$

$m^i = \mu(u^i)(r^i\times n^i) + \tau(u^i)n^i$; $m^i_u = \mu_u(u^i)(r^i\times n^i) + \tau_u(u^i)n^i$; $m^i_{uu} = \mu_{uu}(u^i)(r^i\times n^i) + \tau_{uu}(u^i)n^i$

$m^i_a = \mu(u^i)(r^i\times n^i_a) + \tau(u^i)n^i_a$; $m^i_{aa} = \mu(u^i)(r^i\times n^i_{aa}) + \tau(u^i)n^i_{aa}$; $m^i_{ua} = \mu_u(u^i)(r^i\times n^i_a) + \tau_u(u^i)n^i_a$; $m^i_{au} = m^i_{ua}$

constraints (dynamics):

$G(u,a,F,M,q_B) = \begin{bmatrix}\sum_{i=1}^{6}[f^i] - q_B^{-1}F\\ \sum_{i=1}^{6}[m^i] - q_B^{-1}M\end{bmatrix}$; $G_u = [G_{u^1},\dots,G_{u^6}] = \begin{bmatrix}f^1_u,\dots,f^6_u\\ m^1_u,\dots,m^6_u\end{bmatrix}$; $G_{uu}\in\mathbb{R}^{6\times6\times6} \leftarrow G_{u^iu^i} = \begin{bmatrix}f^i_{uu}\\ m^i_{uu}\end{bmatrix}, G_{u^iu^j} = 0$

$G_a = [G_{a^1},\dots,G_{a^6}] = \begin{bmatrix}f^1_a,\dots,f^6_a\\ m^1_a,\dots,m^6_a\end{bmatrix}$; $G_{aa}\in\mathbb{R}^{6\times6\times6} \leftarrow G_{a^ia^i} = \begin{bmatrix}f^i_{aa}\\ m^i_{aa}\end{bmatrix}, G_{a^ia^j} = 0$; $G_{ua}\in\mathbb{R}^{6\times6\times6} \leftarrow G_{u^ia^i} = \begin{bmatrix}f^i_{ua}\\ m^i_{ua}\end{bmatrix}, G_{u^ia^j} = 0$

objective: $O(u,a) = \sum_{i=1}^{6}$

$W_u\cdot(u^i)^2$ minimize throttle

$+\ W_{\dot a}\cdot\left(\frac{a^i-a^{iP}}{\Delta t}\right)^2$ minimize arm velocity

$+\ W_{up}\cdot(u^i-u_{PL})^2$ if $u<u_{PL}$

$+\ W_{up}\cdot(u^i-u_{PH})^2$ if $u>u_{PH}$

$+\ W_{\dot ap}\cdot\left(\frac{a^i-a^{iP}}{\Delta t}-\dot a_{PH}\right)^2$ if $\frac{a^i-a^{iP}}{\Delta t}>\dot a_{PH}$

$+\ W_{\dot ap}\cdot\left(\frac{a^i-a^{iP}}{\Delta t}+\dot a_{PH}\right)^2$ if $\frac{a^i-a^{iP}}{\Delta t}<-\dot a_{PH}$

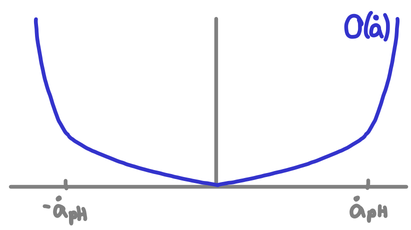
$W_u, W_{\dot a}$: objective weight

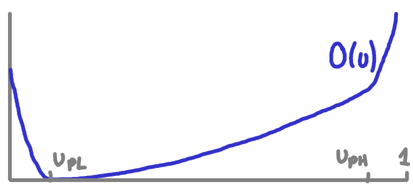
$W_{up}, W_{\dot ap}$: penalty weight

$u_{PL}, u_{PH}, \dot a_{PH}$: penalty limits

Δt : controller timestep

a^P : previous arm angle





$$O_u = 2w_u \begin{bmatrix} u^1 \\ \vdots \\ u^6 \end{bmatrix} + 2w_{up}\left(\begin{bmatrix} u^1 \\ \vdots \\ u^6 \end{bmatrix} - u_{pL}\right) + 2w_{up}\left(\begin{bmatrix} u^1 \\ \vdots \\ u^6 \end{bmatrix} - u_{pH}\right) \quad ; \quad O_{uu} = 2w_u \mathbb{I} + 2w_{up}\mathbb{I} + 2w_{up}\mathbb{I} \quad ; \quad O_{ua} = \emptyset$$

$$O_a = 2\frac{w_a}{\Delta t}\dot{a} + 2\frac{w_a}{\Delta t}(\dot{a} - \dot{a}_{pH}) + 2\frac{w_a}{\Delta t}(\dot{a} + \dot{a}_{pH}) \quad ; \quad O_{aa} = 2w_a\mathbb{I} + 2w_{aH}\mathbb{I} + 2w_{aH}\mathbb{I} \quad ; \quad O_{au} = \emptyset$$

: conditional term

$$\dot{a} = \frac{1}{\Delta t}\left(\begin{bmatrix} a^1 \\ \vdots \\ a^6 \end{bmatrix} - \begin{bmatrix} a^{1p} \\ \vdots \\ a^{6p} \end{bmatrix}\right)$$

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problem: $\begin{bmatrix} u \\ a \end{bmatrix}^*(F, M, q_B) = \underset{u,a}{\operatorname{argmin}}\ O(u,a) \quad \text{subj.to:}\ G(u, a, F, M, q_B) = 0$

first order optimality: $\nabla L = 0$ (KKT condition) $\leftarrow L(u,a,\lambda) = O(u,a) + \lambda^T G(u,a)\big|_{F,M,q_B}$ (Lagrangian)

$\hookrightarrow$ *linearize:* $\nabla L(u+\delta u, a+\delta a, \lambda+\delta\lambda) = \nabla L(u,a,\lambda) + \nabla^2 L(u,a,\lambda)\cdot[\delta u, \delta a, \delta\lambda]^T = 0$ (Newton method)

$$\nabla L = \begin{bmatrix} \nabla_u L \\ \nabla_a L \\ \nabla_\lambda L \end{bmatrix} = \begin{bmatrix} O_u + G_u^T\lambda \\ O_a + G_a^T\lambda \\ G \end{bmatrix} \qquad \nabla^2 L = \begin{bmatrix} \nabla_{uu}L & \nabla_{ua}L & \nabla_{u\lambda}L \\ \nabla_{au}L & \nabla_{aa}L & \nabla_{a\lambda}L \\ \nabla_{\lambda u}L & \nabla_{\lambda a}L & \nabla_{\lambda\lambda}L \end{bmatrix} = \begin{bmatrix} O_{uu} + G_{uu}:\lambda & O_{ua} + G_{ua}:\lambda & G_u^T \\ O_{au} + G_{au}:\lambda & O_{aa} + G_{aa}:\lambda & G_a^T \\ G_u & G_a & 0 \end{bmatrix}$$

(notation: $G_{\circ\triangle} : \lambda = \nabla_\triangle G_\circ^T \lambda = \nabla_\triangle \begin{pmatrix} G_{\circ 1}^T\lambda \\ \vdots \\ G_{\circ 6}^T\lambda \end{pmatrix} = \begin{pmatrix} G_{\circ 1 \triangle 1}^T\lambda & \cdots & G_{\circ 1\triangle 6}^T\lambda \\ \vdots & & \vdots \\ G_{\circ 6\triangle 1}^T\lambda & \cdots & G_{\circ 6\triangle 6}^T\lambda \end{pmatrix} = \operatorname{diag}(G_{\circ 1\triangle 1}^T\lambda, \ldots, G_{\circ 6\triangle 6}^T\lambda)$ when $G_{\circ i\triangle j} = 0$ for $i \neq j$)

$$\begin{bmatrix} O_{uu} + G_{uu}:\lambda & O_{ua} + G_{ua}:\lambda & G_u^T \\ O_{au} + G_{au}:\lambda & O_{aa} + G_{aa}:\lambda & G_a^T \\ G_u & G_a & 0 \end{bmatrix}\begin{bmatrix} \delta u \\ \delta a \\ \delta\lambda \end{bmatrix} = -\begin{bmatrix} O_u + G_u^T\lambda \\ O_a + G_a^T\lambda \\ G \end{bmatrix} \rightarrow \underbrace{\begin{bmatrix} O_{uu} + G_{uu}:\lambda & O_{ua} + G_{ua}:\lambda & G_u^T \\ O_{au} + G_{au}:\lambda & O_{aa} + G_{aa}:\lambda & G_a^T \\ G_u & G_a & 0 \end{bmatrix}}_{H}\begin{bmatrix} \delta u \\ \delta a \\ \lambda^+ \end{bmatrix} = -\underbrace{\begin{bmatrix} O_u \\ O_a \\ G \end{bmatrix}}_{K}$$

$\lambda^+ = \lambda + \delta\lambda$

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algorithm:

get current q_B, F, M ; set $u = u^p$, $a = a^p$, $\lambda = \lambda^p$ (p: previous)

solve $H^{-1}[\delta u, \delta a, \lambda^+]^T = -K$

update $u \leftarrow u + \alpha\delta u$, $a \leftarrow a + \alpha\delta a$, $\lambda \leftarrow \lambda + \alpha(\lambda^+ - \lambda)$

($\alpha = 1 \rightarrow$ decrease such that $\alpha\delta u < \delta u_{max}$ and $\alpha\delta a < \delta a_{max}$)

break if $|O - O^+|/O < tol$ and $|G(u,a)| < tol$

return u, a

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simflified wrench control: (fallback when above fails $\rightarrow$ minimizes $u^T u$, but ignores other weights. fast + linear in esechange!)

$u^*, a^* = \min u^T u \quad \text{subj.to:}\ G(u, a, F, M, q_B) = 0$

$c^i = u^i \cos(a^i) \qquad y^i = x^i \times z^i$

$s^i = u^i \sin(a^i) \qquad u^T u = c^T c + s^T s$

$f^i = c^i \mu^i z^i + s^i \mu^i y^i$

$m^i = c^i(\mu^i r^i \times z^i + \tau^i z^i) + s^i(\mu^i r^i \times y^i + \tau^i y^i)$

$q_B F = \sum_{i=1}^{6} f^i$

$q_B M = \sum_{i=1}^{6} m^i$

$$\underbrace{\begin{bmatrix} \mu^1 z^1 \cdots \mu^6 z^6, & \mu^1 y^1 \cdots \mu^6 y^6 \\ \tau^1 r^1 \times z^1 \cdots \tau^6 r^6 \times z^6, & \tau^1 r^1 \times y^1 \cdots \tau^6 r^6 \times y^6 \end{bmatrix}}_{K}\begin{bmatrix} c \\ s \end{bmatrix} = \begin{bmatrix} q_B F \\ q_B M \end{bmatrix}$$

$$\begin{bmatrix} c^* \\ s^* \end{bmatrix} = K^+ \begin{bmatrix} q_B F \\ q_B M \end{bmatrix} \qquad (K^+ = K^T(KK^T)^{-1} \text{ pseudoinverse})$$

$u^{*i} = \sqrt{c^{*i} c^{*i} + s^{*i} s^{*i}}$

$a^{*i} = \operatorname{atan2}(s^{*i}, c^{*i})$

pose control

q_B : current body orientation , q_S : desired body orientation

$q_E = q_S^{-1} \cdot q_B$ (quat. rot. error) $\rightarrow$ $a_E = 2 \cdot \cos^{-1}(q_{E,0})$ (angle rot. error) $\rightarrow$ $r_E = q_B \cdot \left(a_E / \sin(a_E/2) \cdot q_{E,123} \right)$ (vector rot. error)

$\rightarrow T = -K_P \cdot r_E - K_D \cdot \dot{r}_E$ (K_P K_D gains, $\dot{r}_E$ gyroscope meas.)

$\rightarrow F =$ K_I

integral, position feedback, (wrench estim.?), etc.

simulation

IMU transform