

Konstanten

| | | | |
|--|-------------------|--|------------------|
| $N_A = 6,022 \cdot 10^{23} \text{ mol}^{-1}$ | (Avogadro) | $k_B = 1,381 \cdot 10^{-23} \text{ J/K}$ | (Boltzman) |
| $c = 2,998 \cdot 10^8 \frac{\text{m}}{\text{s}}$ | (lichtgeschv.) | $R = 8,314 \frac{\text{J}}{\text{mol} \cdot \text{K}}$ | (Gas-Konst) |
| $\epsilon_0 = 8,859 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$ | (E-Feld Konst) | $h = 6,626 \cdot 10^{-34} \text{ Js}$ | (Plank Konst) |
| $\mu_0 = 12,566 \cdot 10^{-7} \frac{\text{Vm}}{\text{C}}$ | (M-Feld Konst) | $\hbar = 1,055 \cdot 10^{-34} \text{ Js}$ | (Plank Konst) |
| $e = 1,602 \cdot 10^{-19} \text{ C}$ | (Elementarladung) | $\text{eV} = 1,602 \cdot 10^{-19} \text{ J}$ | (Elektronenvolt) |
| $m_e = 9,109 \cdot 10^{-31} \text{ kg}$ | (Masse Elektron) | | |
| $m_p = 1,673 \cdot 10^{-27} \text{ kg}$ | (Masse Proton) | | |
| $m_n = 1,675 \cdot 10^{-27} \text{ kg}$ | (Masse Neutron) | | |

Konsätze

| | |
|-------------------------|----------------------|
| $m : 10^{-3}$ (mili) | $K : 10^3$ (kilo) |
| $\mu : 10^{-6}$ (micro) | $M : 10^6$ (mega) |
| $n : 10^{-9}$ (nano) | $G : 10^9$ (giga) |
| $p : 10^{-12}$ (pico) | $T : 10^{12}$ (tera) |
| $f : 10^{-15}$ (femto) | $P : 10^{15}$ (peta) |
| $a : 10^{-18}$ (atto) | $E : 10^{18}$ (exa) |

Integration

$$\begin{aligned} dV &= dx dy dz && (\text{Karth.}) \\ dV &= r dr d\vartheta dz && (\text{zyl.}) \\ dV &= r^2 \sin(\vartheta) dr d\vartheta d\theta && (\text{sph.}) \\ r \cdot e^{i\varphi} &= r \cdot (\cos(\varphi) + i \cdot \sin(\varphi)) \end{aligned}$$

Geometrie

$$V = \frac{1}{3} A_G \cdot h$$

$$O = A_G + A_M$$



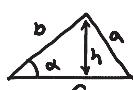
$$V = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3$$

$$O = 4\pi r^2 = \pi d^2$$



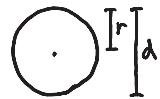
$$A = \frac{1}{2} ch = \frac{1}{2} cb \sin(a)$$

$$U = a + b + c$$



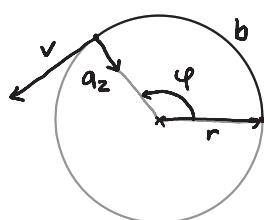
$$A = \pi r^2 = \frac{1}{4} \pi d^2$$

$$U = 2\pi r = \pi d$$



Kreisbewegung

$$\begin{aligned} \varphi &= \frac{b}{r} && [\text{rad}] \\ \omega &= \frac{\Delta \varphi}{\Delta t} = 2\pi f && [\text{rad/s}] \\ f &= \frac{1}{T} = \frac{\omega}{2\pi} && [\text{Hz}] \\ T &= \frac{1}{f} = \frac{2\pi}{\omega} && [\text{s}] \\ v &= r\omega = 2\pi r f && [\text{m/s}] \\ a_z &= v^2/r = r\omega^2 && [\text{m/s}^2] \end{aligned}$$



$$M = J \cdot \alpha \quad (\text{D-Moment})$$

$$L = J \cdot \omega \quad (\text{D-Impuls})$$

$$E_{\text{ROT}} = \frac{1}{2} J \omega^2 \quad (\text{D-Energie})$$

$$W = M \cdot \varphi \quad (\text{Arbeit})$$

$$P = M \cdot \omega \quad (\text{Leistung})$$

$$J_s = \frac{1}{12} m l^2$$



$$J_s = \frac{1}{12} m (a^2 + b^2)$$



$$J_s = \frac{1}{2} m r^2$$



$$J_s = \frac{2}{5} m r^2$$



$$J_s = J_s + m s^2$$

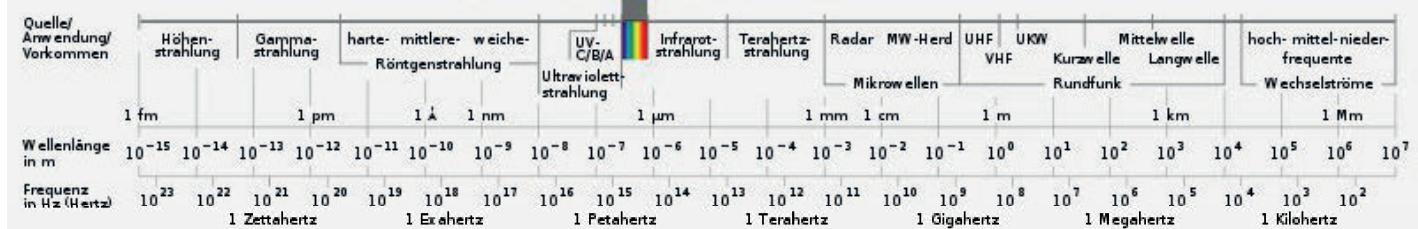


(Steiner)

Das für den Menschen sichtbare Spektrum (Licht)

Ultravioletten Infrarot

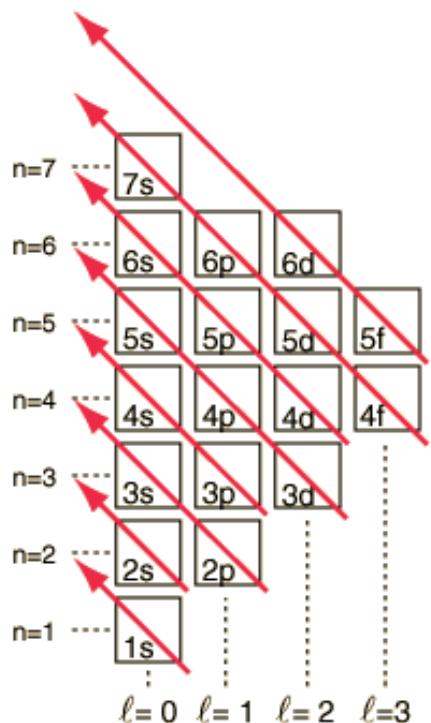
400 nm | 450 nm | 500 nm | 550 nm | 600 nm | 650 nm | 700 nm



Periodic Table of the Elements

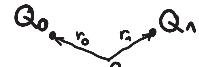
| | | | | | | | | | | | | | | | |
|---------------------------------|----------------------------------|--------------------------------|-------------------------------------|----------------------------------|-----------------------------------|----------------------------------|----------------------------------|----------------------------------|------------------------------------|------------------------------------|-----------------------------------|---------------------------------|-----------------------------------|----------------------------------|----------------------------------|
| 1 IA 1A | H Hydrogen 1.008 | 2 IIA 2A | 3 Li Lithium 6.941 | 4 Be Beryllium 9.012 | 5 B Boron 10.811 | 6 C Carbon 12.011 | 7 N Nitrogen 14.007 | 8 O Oxygen 15.999 | 9 F Fluorine 18.998 | 10 Ne Neon 20.180 | | | | | |
| 11 Na Sodium 22.990 | 12 Mg Magnesium 24.305 | 3 IIIIB 3B | 4 IVB 4B | 5 VB 5B | 6 VIB 6B | 7 VIIIB 7B | 8 | 9 | 10 | 11 IB 1B | | | | | |
| 19 K Potassium 39.098 | 20 Ca Calcium 40.078 | 21 Sc Scandium 44.956 | 22 Ti Titanium 47.867 | 23 V Vanadium 50.942 | 24 Cr Chromium 51.996 | 25 Mn Manganese 54.938 | 26 Fe Iron 55.845 | 27 Co Cobalt 58.933 | 28 Ni Nickel 58.693 | 29 Cu Copper 63.546 | | | | | |
| 37 Rb Rubidium 85.468 | 38 Sr Strontium 87.62 | 39 Y Yttrium 88.906 | 40 Zr Zirconium 91.224 | 41 Nb Niobium 92.906 | 42 Mo Molybdenum 95.95 | 43 Tc Technetium 98.907 | 44 Ru Ruthenium 101.07 | 45 Rh Rhodium 102.906 | 46 Pd Palladium 106.42 | 47 Ag Silver 107.668 | | | | | |
| 55 Cs Cesium 132.905 | 56 Ba Barium 137.328 | 57-71 | 72 Hf Hafnium 178.49 | 73 Ta Tantalum 180.948 | 74 W Tungsten 183.84 | 75 Re Rhenium 186.207 | 76 Os Osmium 190.23 | 77 Ir Iridium 192.217 | 78 Pt Platinum 195.085 | 79 Au Gold 196.967 | 80 Hg Mercury 200.592 | | | | |
| 87 Fr Francium 223.020 | 88 Ra Radium 226.025 | 89-103 | 104 Rf Rutherfordium [261] | 105 Db Dubnium [262] | 106 Sg Seaborgium [266] | 107 Bh Bohrium [264] | 108 Hs Hassium [269] | 109 Mt Meitnerium [278] | 110 Ds Darmstadtium [281] | 111 Rg Roentgenium [280] | 112 Cn Copernicium [286] | | | | |
| Lanthanide Series | 57 La Lanthanum 138.905 | 58 Ce Cerium 140.116 | 59 Pr Praseodymium 140.908 | 60 Nd Neodymium 144.243 | 61 Pm Promethium 144.913 | 62 Sm Samarium 150.36 | 63 Eu Europium 151.964 | 64 Gd Gadolinium 157.25 | 65 Tb Terbium 158.925 | 66 Dy Dysprosium 162.500 | 67 Ho Holmium 164.930 | 68 Er Erbium 167.259 | 69 Tm Thulium 168.934 | 70 Yb Ytterbium 173.055 | 71 Lu Lutetium 174.967 |
| Actinide Series | 89 Ac Actinium 227.028 | 90 Th Thorium 232.038 | 91 Pa Protactinium 231.036 | 92 U Uranium 238.029 | 93 Np Neptunium 237.048 | 94 Pu Plutonium 244.064 | 95 Am Americium 243.061 | 96 Cm Curium 247.070 | 97 Bk Berkelium 247.070 | 98 Cf Californium 251.080 | 99 Es Einsteinium [254] | 100 Fm Fermium 257.095 | 101 Md Mendelevium 258.1 | 102 No Nobelium 259.101 | 103 Lr Lawrencium [262] |

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Coulombkraft:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_0 \cdot Q_1}{|r_0 - r_1|^2}$$



$$\epsilon_0 = 8.854 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

$$\vec{F} = \vec{E} \cdot Q$$

$$\vec{F} = \frac{Q_0}{4\pi\epsilon_0} \int g(\vec{r}) \frac{\vec{r}_0 - \vec{r}}{|\vec{r}_0 - \vec{r}|^3} dV \quad (g \cdot dV = dQ)$$

Elektrisches Feld:

$$\vec{E} = \frac{\vec{F}(\vec{r})}{Q_0} = \frac{1}{4\pi\epsilon_0} \sum_{i=0}^n Q_i \cdot \frac{\vec{r}_0 - \vec{r}_i}{|\vec{r}_0 - \vec{r}_i|^3} = \frac{1}{4\pi\epsilon_0} \int_S g \cdot \frac{\vec{r}_0 - \vec{r}'}{|\vec{r}_0 - \vec{r}'|^3} dV'$$

$$\vec{E} = \frac{Q_0 \vec{r}}{4\pi\epsilon_0 |r|^3} \quad (\text{Punktladung / Kugel})$$

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{Platte } \sigma = C/m^2)$$

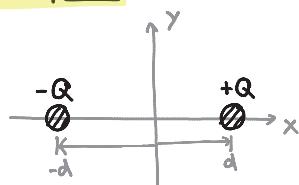
$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{Draht } \lambda = C/m)$$

Dipol:

$$\vec{E}(x) \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{2\vec{p}}{|x|^3}$$

$$\vec{E}(y) \approx -\frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p}}{|y|^3}$$

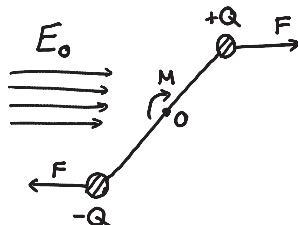
($|x, y| \gg d$)



$$\vec{E}(\vec{r}) \approx \frac{1}{4\pi\epsilon_0} \left(\frac{3(\vec{p} \cdot \vec{r}) \vec{r}}{|\vec{r}|^5} - \frac{\vec{p}}{|\vec{r}|^3} \right)$$

$$\vec{p} = Q \cdot 2\vec{d}$$

(Elektrisches Dipolmoment)



$$\vec{M} = \vec{p} \times \vec{E}_0 \quad (\text{Moment})$$

$$E_{EL} = -\vec{p} \cdot \vec{E}_0 \quad (\text{Energie})$$

Elektrischer Fluss:



$$\Phi_E = \int_A \vec{E}(\vec{r}) dA'$$

Gesetz von Gauß:



$$\oint_A \vec{E}(\vec{r}') dA' = \frac{Q}{\epsilon_0} = \Phi_E$$

$$\vec{\nabla} \cdot \vec{E} = S/\epsilon_0$$

$$\underline{\text{Energie}}: E_{EL} = - \int_{r_1}^{r_2} \vec{F}(\vec{r}') d\vec{r}' = - \int_{r_1}^{r_2} Q \cdot \vec{E}(\vec{r}') d\vec{r}' = Q (\Phi(r_2) - \Phi(r_1))$$

$$\underline{\text{Energiedichte}}: \omega = \frac{1}{2} \epsilon_0 |\vec{E}|^2$$

$$\underline{\text{Spannung}}: U = \frac{E_{EL}}{Q} = - \int_{r_1}^{r_2} \vec{E}(\vec{r}') d\vec{r}' = \Phi(r_2) - \Phi(r_1) = "E \cdot L"$$

Potential: $\Phi(\vec{r}) = - \int_{\infty}^{\vec{r}} \vec{E}(\vec{r}') d\vec{r}'$

$$\Phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} \quad (\text{Punktladung})$$

$$\Phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3} \quad (\text{Dipol})$$

$$\Phi = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \quad (\text{allg. Ladungsvert. } \rho = C/m^3)$$

Feldgleichung: $\vec{E}(\vec{r}) = -\nabla \Phi(\vec{r}) \quad ; \quad \Phi(\vec{r}) = -\vec{\nabla} \cdot \vec{E}$

Poisson-Gleichung: $\frac{\rho(\vec{r})}{\epsilon_0} = -\nabla^2 \Phi(\vec{r})$

leitende Körper:

- $\vec{n} \cdot \vec{E}(\vec{r}) = \frac{\sigma(\vec{r})}{\epsilon_0}$

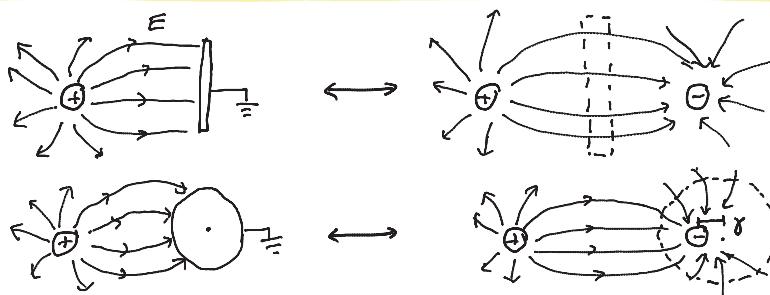


- $\vec{n} \times \vec{E}(\vec{r}) = 0$



- $\Phi(\vec{r}) = \text{Const} \forall \vec{r} \in A$

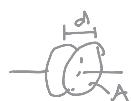
Spiegelladung:



Kondensator: $C = Q/U \quad (\text{Kapazität})$

$$E_{EL} = \frac{1}{2} U^2 C = \frac{1}{2} Q^2 / C \quad (\text{Energie})$$

Plattenkondensator: $C = \frac{A\epsilon}{d} \quad E_{EL} = \frac{Q^2 d}{2 A \epsilon} = \frac{U^2 A \epsilon}{2 d} \quad U = \frac{Qd}{A\epsilon}$



Zylinderkondensator: $C = 2\pi\epsilon \frac{L}{\ln(r_2/r_1)} \quad (= Q / \int_{r_1}^{r_2} \vec{E}(r) dr)$

Dielektrizitätszahl: $\epsilon_r = \frac{|E_{\text{ext}}|}{|E|} \quad \epsilon = \epsilon_r \epsilon_0$

Verschiebungspolarisation: Elektronenorbitale verzerrt.)

Orientierungspolarisation: Moleküle gedreht. $\epsilon_r = ...$

Ele. Feld Dielektrum: $|\vec{E}_p| = \chi_{\epsilon} |\vec{E}| \quad (\chi_{\epsilon} = \epsilon_r - 1 \text{ ele. Suszeptibilität})$

Strom: $I = \frac{dQ}{dt}$

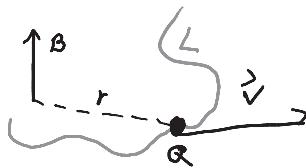
Driftgeschw.: $\vec{v}_d = -\vec{I}/n \cdot e \cdot A = -\vec{j}/en$ $\left(j: \text{Stromdichte } I/A \right)$
 $e: \text{elektron Ladung} n: \frac{\# \text{elektronen}}{\text{Volumen}}$

Widerstand: $R = V/I$ $\left(= \rho \cdot \frac{L}{A} \right)$  $\rho: \text{spez. Wld.}$

Leistung: $P = U \cdot I = R I^2 = U^2/R$

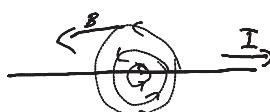
Biot Savart Gesetz: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{Q \cdot \vec{r} \times \vec{r}}{|\vec{r}|^3}$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_L I dL' \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

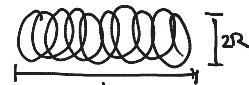


Magnetische Feldkonst.: $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$

Magnetfeld Draht: $B(r) = \frac{\mu_0 \cdot I}{2\pi \cdot r}$



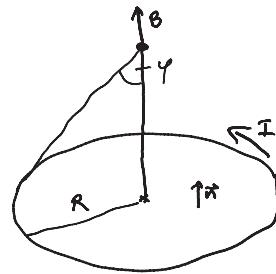
Magnetfeld Spule: $B = \frac{N}{L} \mu_0 I \quad (L \gg R)$



Magnetfeld Leiterschleife: $\vec{B}(z) = \mu_0 I R^2 / (2(R^2 + z^2)^{3/2})$

$$\vec{B}(0) = \frac{\mu_0 I}{2R}$$

$$\vec{B}(z \gg R) = \frac{\mu_0 I R^2}{2z^3}$$



Magnetischer Dipol: $\vec{m} = I \vec{A} = I \pi R^2 \vec{n}$ (aus Leiterschleife)

$$\vec{M} = \vec{m} \times \vec{B}_0 \quad (\text{Drehmoment}) \quad E_{\text{pot}} = -\vec{m} \cdot \vec{B}_0 \quad \vec{F} = \vec{\nabla} \cdot (\vec{m} \vec{B}_0)$$

Magnetischer Fluss: $\Phi_M = \int_A \vec{B}(\vec{r}') dA'$ $[\Phi_M] = T m^2 = Wb = \text{Weber}$

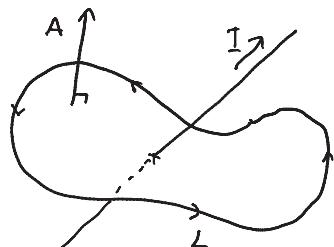
-ges. Gauß für Mag.: $\oint_A \vec{B}(\vec{r}') dA' = 0 \quad (\Rightarrow \text{immer Quellfrei})$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Durchflutungsgesetz: $\oint_L \vec{B}(\vec{r}') dL' = \mu_0 I =$

$$\hookrightarrow \mu_0 \oint_A \vec{j}(\vec{r}') dA' = \oint_A (\vec{\nabla} \times \vec{B}(\vec{r}')) dA'$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{j}(\vec{r})$$



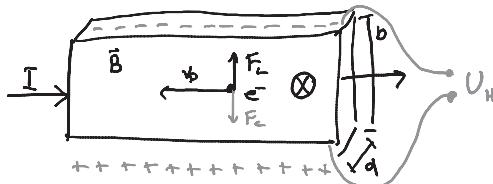
Drehmoment: $\vec{F}(\vec{r}) = Q \vec{v} \times \vec{B}(\vec{r})$

$$\vec{F} = \int_L I d\vec{l} \times \vec{B}(\vec{r})$$

$$F = \frac{\mu_0}{4\pi} \frac{Q_1 \vec{v}_1 \times (Q_2 \vec{v}_2 \times (\vec{r}_1 - \vec{r}_2))}{|\vec{r}_1 - \vec{r}_2|^3}$$

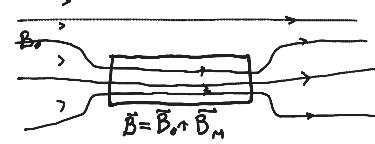
$$\vec{F} = \frac{\mu_0}{4\pi} \iint_{L_1 L_2} \frac{I_1 d\vec{l}_1 \times (I_2 d\vec{l}_2 \times (\vec{r}_1 - \vec{r}_2))}{|\vec{r}_1 - \vec{r}_2|^3}$$

Hall-Effekt: $U_H = \frac{IB}{\text{Leiterdichte}}$



$$(F_L = F_c)$$

Relative Mag. Feldkonst.: $I(\vec{B}) = \mu_r |\vec{B}_0| \quad \mu = \mu_r \mu_0$



Mag. Suszeptibilität: $|\vec{B}_m| = \chi_m |\vec{B}_0| \quad \chi_m = \mu_r - 1$

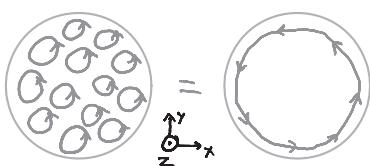
Mag. Erregung: $\vec{H} = \vec{B}_0 / \mu_0 \quad \text{Magnetisierung } \vec{M} = \vec{B}_m / \mu_0 = \frac{\vec{m}}{V} = j_A \times \vec{n}$ (Oberflächen-Kreistrom j_A / m)

Diamagnetismus: bei allen Materialien, schwach richtung geringe Mag. Feld. dichte gezogen. $\mu_r < 1 \quad \chi_m < 0 \quad \chi_m(T) = \chi_{\text{konst.}}$

Paramagnetismus: bei ungepaarten e-. Stark richtung hohe Mag. Feld. dichte gezogen. $\mu_r > 1 \quad \chi_m > 0 \quad T \rightarrow \chi_m \downarrow$

Ferromagnetismus: wie Paramagnetismus nur stärker $\mu_r \gg 1 \quad \chi_m > 0 \quad T \uparrow \chi_m \downarrow \quad T_c \rightarrow \chi_m = 0$ Curie-Temp.

Oberflächenstrom:



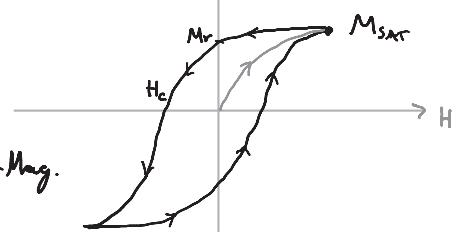
$$\begin{aligned} dB &= \frac{\mu_0 r^2}{2(r^2 + (z-z')^2)^{3/2}} dI \\ dI &= j_A dz' \\ dI &= j_A R(y) dy \end{aligned}$$

$d\psi = dz'/r$

Magnetische Abschirmung:

$$\begin{aligned} \mu_{rel} &= 1 & B_{ext} &= B_{int} \\ B_{2z} &= B_{1z} & B_{2x} &\approx \mu_{rel} B_{1x} \end{aligned}$$

Hysterese (Ferro-Mag.): $M_r = \text{Remanenz-Mag.} \quad M_{SAT} = \text{Sättigungs-Mag.}$
 $H_c = \text{Koerzitivfeld}$



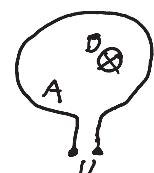
Anisotropie: Geometrie X N ✓ ; Kristallsstruktur X ✓

Faradaysches Induktionsgesetz:

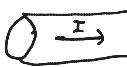
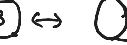
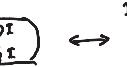
$$U = -\frac{d}{dt} \int_A B(\vec{r}) d\vec{A} = -\frac{d\Phi_B}{dt}$$

$$(U = \oint_A E(\vec{r}) d\vec{L})$$

$$\hookrightarrow \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$



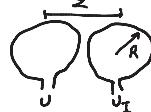
Lenzsche Regel: Induzierte Spannung, Strom wirken Mosche entgegen

Stromeffekt:  \leftrightarrow  \leftrightarrow  \leftrightarrow  Bei Wechselstrom

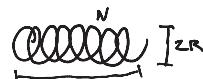
Induktivität: $L = \frac{\Phi_m}{I}$ $[L] = Tm^2/A \cdot H$ (Henry)

$$U = -\frac{d\Phi_m}{dt} = -L \frac{dI}{dt}$$

Gegeninduktion Leiterschleifen: $L = \frac{\mu_0 \pi R^4}{2z^2}$



Selbstinduktion Spule: $L = N^2 \mu_0 \pi R^2 / l$



Energie Magnettfeld: $E_{MAG} = \int_0^T U(t) I(t) dt = \frac{1}{2} L I_0^2$



Energiedichte Mag. Feld: $w = E_{MAG} / V = \frac{1}{2} \mu_0 |B|^2$

Maxwell'sche Gleichungen:

$$\vec{F}(\vec{r}) = Q_0 (\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r}))$$

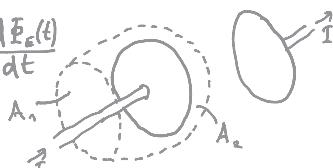
$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = g(\vec{r}) / \epsilon_0 \quad \leftrightarrow \quad \oint_A \vec{E}(\vec{r}) d\vec{A} = Q / \epsilon_0$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}) = 0 \quad \leftrightarrow \quad \oint_A \vec{B}(\vec{r}) d\vec{A} = 0$$

$$\vec{\nabla} \times \vec{E}(\vec{r}) = - \frac{d\vec{B}(\vec{r})}{dt} \quad \leftrightarrow \quad \oint_L \vec{E}(\vec{r}') d\vec{l}' = - \frac{d}{dt} \oint_A \vec{B}(\vec{r}') d\vec{A}'$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{j} + \mu_0 \epsilon_0 \underbrace{\frac{d\vec{E}(t)}{dt}}_{\vec{D}(t)} \quad \leftrightarrow \quad \oint_L \vec{B}(\vec{r}) d\vec{l}' = \mu_0 I + \frac{d}{dt} \oint_A \vec{E}(\vec{r}') d\vec{A}'$$

Verschiebungsstrom $I_v = \epsilon_0 \frac{d\vec{E}_e(t)}{dt}$



Wellengleichung: $\frac{d^2\psi(x,t)}{dt^2} = c^2 \frac{d^2\psi(x,t)}{dx^2}$ ($\psi(x,t) = \psi(x-ct, 0)$)

Seilwelle: $c = \sqrt{T/\mu}$ $E_{kin} = \frac{1}{2} M \omega x \left(\frac{d\psi}{dt}\right)^2$ $P = \frac{1}{2} \mu c \left(\frac{d\psi}{dt}\right)^2$ $Z = \frac{T}{c} = \sqrt{\mu T}$

$$T = \frac{\text{Kraft}}{\text{masse}} = \frac{\text{F}}{\text{m}}$$

Harmonische Wellen: $\psi(x,t) = \text{Re}(\psi_0 e^{i(\omega t - kx + \varphi)}) = \psi_0 \cos(\omega t - kx + \varphi)$

ψ_0 : Amplitude

φ : Phase

$$c: \text{Wellengeschw.} = \frac{\omega}{k} = \lambda f$$

ω : Kreisfrequenz

$$\tilde{\nu}: \text{Wellenzahl} = \frac{k}{2\pi}$$

$$T: \text{Periode} = \frac{1}{f} = \frac{2\pi}{\omega}$$

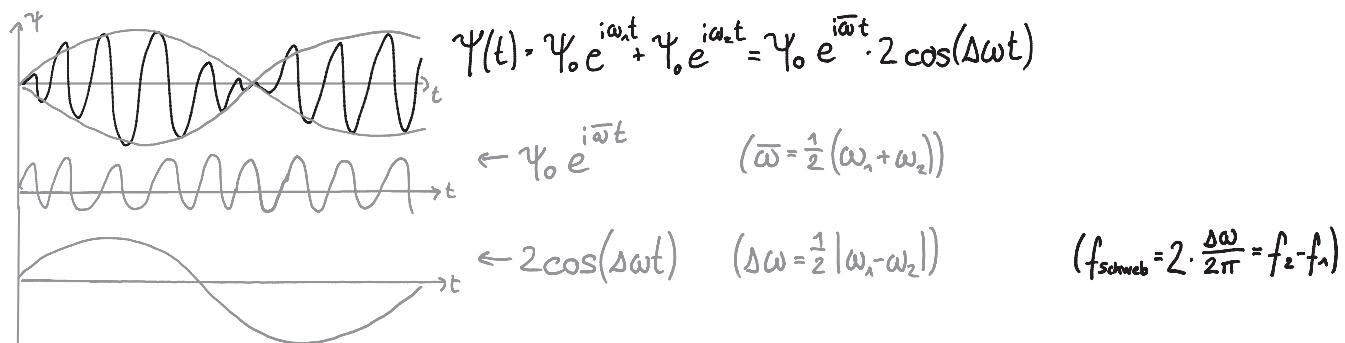
k : Wellenvektor

$$\lambda: \text{Wellenlänge} = \frac{1}{\tilde{\nu}} = \frac{2\pi}{k}$$

$$f: \text{Frequenz} = \frac{\omega}{2\pi}$$

Daufrichtung: (für harmonische) $\text{sign}(\omega) = \text{sign}(k)$

Schwingung: (für harmonische $(\omega_1, \psi_0), (\omega_2, \psi_0)$; $\psi=0, x=0$)



Gauss Wellenpaket: $\psi(t) = \underbrace{\psi_0 e^{i\omega t}}_{\text{Nelle}} \cdot \underbrace{e^{-t^2/2(\Delta\omega)^2}}_{\text{Gaus-Glocke}}$ $\rightarrow \psi(\omega) = \frac{\psi_0}{\Delta\omega} \underbrace{e^{-\frac{(\omega-\bar{\omega})^2}{2(\Delta\omega)^2}}}_{\text{Fourier Gaus-Glocke}}$ $\begin{pmatrix} \psi(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt \\ \psi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(\omega) e^{i\omega t} d\omega \end{pmatrix}$

Intensität: $I = P(1D)$ $I = P/A(3D)$ $I = \frac{1}{T} \int_0^T \psi^2 dt = \frac{1}{2} \psi_0^2$ (manchmal)

Mittlere Amplitude: $\psi_{\text{RMS}} = \frac{1}{\sqrt{2}} |\psi_0|$

Dispersion: $\frac{\omega}{k(\omega)} \neq \text{Konst. } \forall \omega$ (\rightarrow Wellengleichung nicht erfüllt)

| | | |
|--|-------------------------------------|---|
| $v_p = \frac{\omega}{k}$ $v_g = \frac{d\omega}{dk} = v_p + K \frac{dv_p}{dk}$ | (Phasengeschw.) (Gruppengeschw.) | (Geschw. WellenMax.) (Geschw. Einhüllende) |
|--|-------------------------------------|---|

Absorption: $\psi(x,t) = \psi_w(x,t) \cdot e^{-\alpha x/c} = \psi_w(x,t) \cdot e^{-\alpha x/\delta_e} = \psi_w(x,t) \cdot e^{-\alpha x/\alpha}$ ($I(\psi) = I(\psi_w) \cdot e^{-\alpha x}$)

| | | |
|-------------------------------------|--|--|
| $c\tau$ \uparrow Abhängzeit | $=$ δ_e \uparrow Eindringtiefe | $=$ $2/\alpha$ \uparrow AbsorptionsKoeff. |
|-------------------------------------|--|--|

3D Wellen: $\Psi(\vec{r}, t) = \Psi_0 e^{i(\omega t - \vec{k} \cdot \vec{r} + \varphi)}$



(Ebene Welle)

$\Psi(\vec{r}, t) = \frac{\Psi_0}{\sqrt{4\pi r}} e^{i(\omega t - k|\vec{r}| + \varphi)}$



(Kugel Welle)

Amplitude als Vektor: $\vec{\Psi} \parallel \vec{k}$



(Längitudinalwellen) (z.B. Schall)

$\vec{\Psi} \not\parallel \vec{k}$



(Transversalwellen) (z.B. E-Mag.)

Reflexion / Transmission:

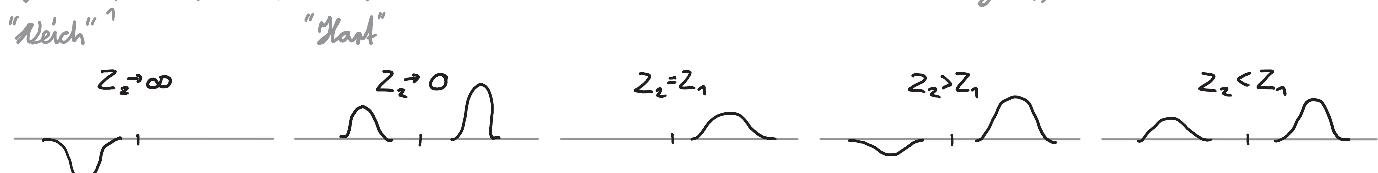


$$r = \frac{\Psi_{0B}}{\Psi_{0A}} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad (\text{Reflektions-Koeff.})$$

$$t = \frac{\Psi_{0C}}{\Psi_{0A}} = \frac{2Z_1}{Z_1 + Z_2} = 1 + r \quad (\text{Transmission-Koeff.})$$

$$R = \frac{P_B}{P_A} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2 = r^2 = 1 - T \quad (\text{Reflektionsgrad})$$

$$T = \frac{P_C}{P_A} = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2} = \frac{Z_2}{Z_1} t^2 = 1 - R \quad (\text{Transmissiongrad})$$



$$\begin{array}{ll} Z_2 \rightarrow \infty & Z_2 \rightarrow 0 \\ \Psi_B = -\Psi_A & \Psi_B = \Psi_A \\ P_B = P_A & P_B = P_A \\ \Psi_c = 0 & \Psi_c = 2\Psi_A \\ P_c = 0 & P_c = 0 \\ \Psi_c = 2\Psi_A & \Psi_c = \Psi_A \\ P_c = 0 & P_c = P_A \\ \Psi_B = 0 & \Psi_c = 0 \\ P_B = 0 & P_c = P_A \\ \Psi_c = 0 & \Psi_c = 0 \\ \Psi_B < 0 & \Psi_c > 0 \\ \Psi_c > 0 & \Psi_c > 0 \end{array}$$

Nehmende Wellen: $\Psi(x, t) = 2\Psi_0 \sin(kx) \sin(\omega t + \varphi)$

- Wellenlänge: $\lambda_n = \frac{2L}{n}$ (gerade)
 $\lambda_n = \frac{4L}{2n-1}$ (ungerade)

- Eigenfrequenz: $f_n = \frac{c}{\lambda_n} = \frac{nc}{2L}$ (gerade)
 $f_n = \frac{c}{\lambda_n} = \left(n - \frac{1}{2}\right) \frac{c}{2L}$ (ungerade)

Akustische Wellen: $s(x, t) = s_0 \cos(\omega t - kx + \varphi)$ (Auslenkung)

$v(x, t) = v_0 \sin(\omega t - kx + \varphi) \quad (\text{Schallschnelle})$

$p(x, t) = p_0 + s p_0 \sin(\omega t - kx + \varphi) \quad (\text{Schalldruck})$

Akustische Wellengleichung $\frac{d^2 p}{dx^2} = \frac{K}{\rho} \frac{d^2 s}{dt^2} \quad (K: \text{Kompressionsmodul}, \rho: \text{Dichte})$

Schallgeschwindigkeit $c = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\gamma R T}{M}} \quad (\gamma: \text{Isentropenkonstante})$

Akustische Impedanz: $Z = \rho c = \sqrt{K_S} = \frac{\Delta p_0}{v_0}$ (Feldimpedanz)

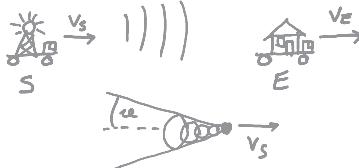
$Z_A = \frac{Z}{A}$ (Flussimpedanz) (wenn $A_1 \neq A_2$ z.B. Rohr)

Schallintensität: $I = \frac{1}{2} \rho c v_0^2 = \frac{1}{2} \frac{\Delta p_0^2}{Z}$

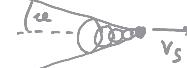
Schallleistung: $P = I \cdot A = \frac{1}{2} \rho A c v_0^2$

Schallpegel: $L = 10 \cdot \log_{10} \left(\frac{I}{I_0} \right) = 20 \cdot \log_{10} \left(\frac{\Delta p_0}{p_0} \right)$ ($I_0 = 10^{-12} \text{ W/m}^2$ $p_0 = 2 \cdot 10^6 \text{ Pa}$)

Doppler-Effekt: $f_E = f_s \left(\frac{c - v_E}{c - v_s} \right)$

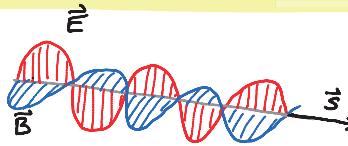


Machscher Regel: $\sin(\alpha) = \frac{c}{v_s}$



Elektromagnetische Wellen

$$\left. \begin{aligned} \vec{\nabla} \times \vec{E} &= \frac{d\vec{B}}{dt} \\ \vec{\nabla} \times \vec{B} &\approx \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} \end{aligned} \right\} \rightarrow \begin{aligned} \frac{d^2 \vec{B}}{dt^2} &= \frac{1}{\mu_0 \epsilon_0} \vec{\nabla}^2 \vec{B} \\ \frac{d^2 \vec{E}}{dt^2} &= \frac{1}{\mu_0 \epsilon_0} \vec{\nabla}^2 \vec{E} \end{aligned} \xrightarrow{\text{Wellegleichungen}} \begin{aligned} E(x, t) &= \vec{E}_0 \cdot e^{i(\omega t - kx + \varphi)} \\ B(x, t) &= \vec{B}_0 \cdot e^{i(\omega t - kx + \varphi)} \end{aligned}$$



Licht Geschw. $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{|E|}{|B|} = \frac{E_0}{B_0}$ ($= 2,997 \cdot 10^8 \frac{\text{m}}{\text{s}}$) $C = \frac{c_0}{\sqrt{\epsilon_r \mu_r}} = \frac{c_0}{n}$

Poynting Vektor: $\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) = \vec{E} \times \vec{H}$ ($\mu_r = 1$)

Wellenimpedanz: $Z_0 = \frac{E_0}{H_0} = \mu_0 c_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ ($= 376,73 \Omega$) $Z = \sqrt{\frac{\mu_r}{\epsilon_r}} Z_0 \approx \frac{Z_0}{n}$

Energiedichte: $W = \frac{1}{2} \epsilon_0 E_0^2 \quad [\text{W/m}^3]$

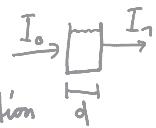
Intensität: $I = \frac{1}{2} c \epsilon_0 E_0^2 \quad [\text{W/m}^2]$

Brechungsindex: $n = \sqrt{\mu_r \epsilon_r} \geq 1$ (Vakuum: $n=1$ nicht-Mag: $n=\sqrt{\epsilon_r}$)

Refaktion / Transmission: $\mu_r \approx 1 \rightarrow r, t, R, T$ aus Akustik mit $Z_1 = \frac{Z_0}{n_1}, Z_2 = \frac{Z_0}{n_2}$

Lambert-Beer-Gesetz: $E_\lambda = \log_{10} \left(\frac{I_0}{I_n} \right) = \log_{10} (e^{-\alpha d}) \approx 0,434 \cdot \alpha d = \epsilon_\lambda c d$

\downarrow Extinktions Koeff. molarer-ext.-koeff. \uparrow Konzentration



Komplexer Wellenvektor: $\tilde{k} = k - i \frac{\alpha}{2}$ $\rightarrow E(x,t) = E_0 e^{i(\omega t - \tilde{k}x + \psi)}$ (gedämpft)

Komplexe Dielektrizitätskonstante: $\tilde{\epsilon}_r = \epsilon_{r1} + i\epsilon_{r2}$

Komplexe Brechungsindex: $\tilde{n} = n - ik$ $(k = \frac{\alpha}{2k_0} = \frac{\alpha c_0}{2\omega})$

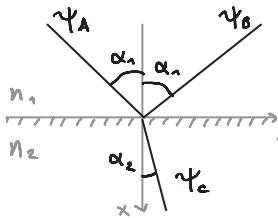
$$\tilde{n} = \sqrt{\tilde{\epsilon}_r} \quad (\mu_r = 1) \rightarrow n = \left(\frac{1}{2} \sqrt{\epsilon_{r1}^2 + \epsilon_{r2}^2} + \frac{1}{2} \epsilon_{r1} \right)^{1/2}$$

$$k = \left(\frac{1}{2} \sqrt{\epsilon_{r1}^2 + \epsilon_{r2}^2} - \frac{1}{2} \epsilon_{r1} \right)^{1/2}$$

Reflektionsgrad bei Absorption: $R = \frac{(n_1 - n_2)^2 + k_z^2}{(n_1 + n_2)^2 + k_z^2}$ $(\mu_1 = \mu_2 = 1 \quad k_1 = 0 \quad k_2 > 0)$

Reflektion an einem Leiter: $\tilde{\epsilon}_r = \epsilon_r - i \frac{\alpha}{\omega \epsilon_0}$ $k = \sqrt{\frac{\alpha}{2\omega \epsilon_0}}$ $\delta_c = \sqrt{\frac{2}{\omega \alpha \mu_0}}$

Brechung:



$$\Psi_A(x,y) = \Psi_{0A} e^{-i(k_{Ax}x + k_{Ay}y)}$$

$$\Psi_B(x,y) = \Psi_{0B} e^{-i(k_{Bx}x + k_{By}y)}$$

$$\Psi_C(x,y) = \Psi_{0C} e^{-i(k_{Cx}x + k_{Cy}y)}$$

$$\vec{k}_A = \begin{pmatrix} k_{Ax} \\ k_{Ay} \end{pmatrix}$$

$$\vec{k}_B = \begin{pmatrix} k_{Bx} \\ k_{By} \end{pmatrix}$$

$$\vec{k}_C = \begin{pmatrix} k_{Cx} \\ k_{Cy} \end{pmatrix}$$

$$\hookrightarrow k_{Ay} = k_{By} = k_{Cy}$$

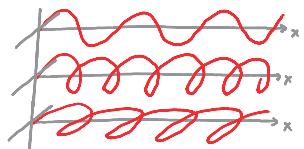
Brechungsgesetz: $n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$ $(\rightarrow \alpha_2 = \sin^{-1}\left(\frac{n_1}{n_2} \cdot \sin(\alpha_1)\right))$

Totalreflektion: $n_1 > n_2 \Rightarrow \alpha_{1,c} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ $(\alpha_1 > \alpha_{1,c} \rightarrow \nexists \alpha_2)$

Evanescens: $n_1 > n_2, \alpha_1 > \alpha_{1,c} \Rightarrow \beta = |\vec{k}_A| \cdot \sqrt{\sin^2 \alpha_1 - \left(\frac{n_2}{n_1}\right)^2}$ $(k_{Cx} = -i\beta)$
 $\delta = \lambda_1 / (2\pi \sqrt{\sin^2 \alpha_1 - (n_2/n_1)^2}) = 1/\beta$

Polarisation: $\vec{E}(x,t) = \vec{E}_{0,y} e^{i(\omega t - kx + \psi_y)} + \vec{E}_{0,z} e^{i(\omega t - kx + \psi_z)}$

- linear polarisiert: $\Delta \psi = m\pi$
- zirkular polarisiert: $\Delta \psi = (m + \frac{1}{2})\pi$ $|\vec{E}_{0,y}| = |\vec{E}_{0,z}|$
- elliptisch polarisiert: else



Polarisationsfilter: $I_1 = \frac{1}{2} I_0$

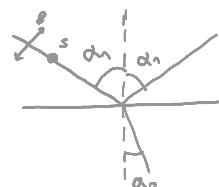


$$I_2 = I_1 \cos^2 \beta$$

Reflektion bei Winkel: $Z_{1s} = \frac{z_0}{n_1 \cos(\alpha_1)}$ $Z_{2s} = \frac{z_0}{n_2 \cos(\alpha_2)}$

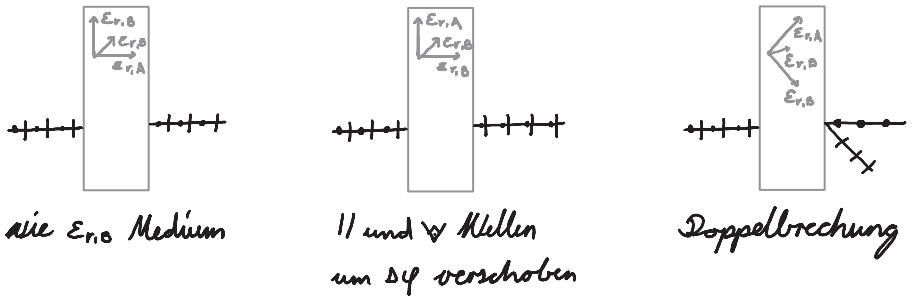
$$Z_{1p} = \frac{z_0 \cos(\alpha_1)}{n_1} \quad Z_{2p} = \frac{z_0 \cos(\alpha_2)}{n_2}$$

$$\hookrightarrow R_s, R_p = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$$



Brewsterwinkel: $\alpha_{1,B} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$ $(\alpha_1 = \alpha_{1,B} \rightarrow R_p = 0)$

Doppelbrechung:



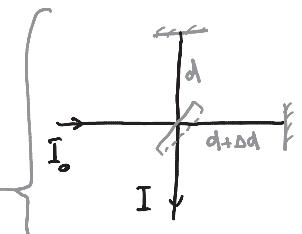
Kohärenz: $\Delta\varphi = \varphi_A - \varphi_B = \text{Konst } \forall t$ (ja falls: $\Psi_A = \Psi_{A_0} e^{i(\omega t - kx + \varphi_0)}$; $\Psi_B = \Psi_{B_0} e^{i(\omega t - kx + \varphi_0)}$)

Kohärenz-Länge/Zeit: L_c, t_c mit $\Delta\varphi(t \pm t_c) \approx \Delta\varphi(t)$ $\Delta\varphi(L \pm L_c) \approx \Delta\varphi(L)$

Interferenz:

• Konstruktiv: $\Delta\varphi = m\pi$

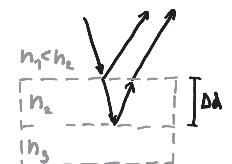
• Destruktiv: $\Delta\varphi = (m+\frac{1}{2})\pi$



Michelson-Interferometer: $I = I_0 \cos^2(k \cdot \Delta d)$

Interferenz am dünnen Film: • Konstruktiv: $\Delta d = m \frac{\lambda_2}{2}$ ($n_3 > n_2$) $\Delta d = (m+\frac{1}{2}) \frac{\lambda_2}{2}$ ($n_3 < n_2$)

• Destruktiv: $\Delta d = (m+\frac{1}{2}) \frac{\lambda_2}{2}$ ($n_3 > n_2$) $\Delta d = m \frac{\lambda_2}{2}$ ($n_3 < n_2$)



Interferenz am Doppelspalt:

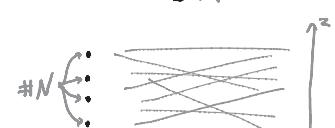
$$I(\alpha) = 4I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin(\alpha)\right)$$

$$\alpha_{\min} = \sin^{-1}\left(\frac{k \cdot \lambda}{d}\right) \quad \alpha_{\max} = \sin^{-1}\left(\frac{(k+0.5)\lambda}{d}\right)$$



Interferenz am Mehrfachspalt:

$$I(\alpha) = I_0 \frac{\sin^2(N\pi d \sin(\alpha)/\lambda)}{\sin^2(\pi d \sin(\alpha)/\lambda)}$$



Beugung am Spalt:

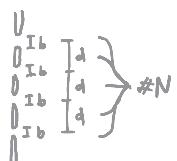
$$I(\alpha) = I_{\text{tot}} \frac{\sin^2(b\pi \sin(\alpha)/\lambda)}{(b\pi \sin(\alpha)/\lambda)^2}$$

$$\alpha_{\min} = \sin^{-1}\left(\frac{k \cdot \lambda}{d}\right) \quad \alpha_{\max} = \sin^{-1}\left(\frac{(k+0.5)\lambda}{d}\right) \quad k=1,2,\dots$$



Beugung am Mehrfachspalt:

$$I(\alpha) = I_{\text{tot}} \frac{\sin^2(b\pi \sin(\alpha)/\lambda)}{(b\pi \sin(\alpha)/\lambda)^2} \cdot \frac{\sin^2(N\pi d \sin(\alpha)/\lambda)}{\sin^2(\pi d \sin(\alpha)/\lambda)}$$



Beugung an allg. Blende:

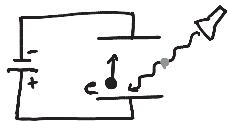
$$\Psi(y, z) = \frac{\Psi_0}{\sqrt{4\pi R}} e^{-ikR} \int_A e^{i(k_y y' + k_z z')} dy' dz'$$

Auflösungsvermögen:

$$\sin(\alpha_{\min}) = 1.22 \frac{\lambda}{D} \approx \alpha_{\min} \approx \frac{z}{L}$$



Photoelektrische Effekt:



$$\frac{1}{2} m_e v_e^2 = h\nu - W_{\text{aus}}$$

$$E_{\text{kin},e} = E_{\text{photon}} - W_{\text{austrit}}$$

Plancksche Konstante: $h = 6,626 \cdot 10^{-34} \text{ Js} = 4,136 \cdot 10^{-15} \text{ eV/Hz}$ ($\hbar = h/2\pi$)

Photon: $E = h\nu = h c/\lambda$

$$(v=c, m=0) \quad p = E/c = h/\lambda$$

$$P = p \cdot \frac{\#_{\text{phot}}}{\text{sec}} / A = F/A$$

$$I = E \cdot \frac{\#_{\text{phot}}}{\text{sec}} / A = F \cdot c / A$$

(Energie)

(Impuls)

(Strahlungsdruck N/m^2)

(Intensität W/m^2)

Photon
Absorb.
sonst 2x

Teilchen: $v_{\text{BRO}} \cdot \lambda_{\text{BRO}} \neq c$

$(v \ll c, m \neq 0)$

$$E_{\text{kin}} = h\nu_{\text{BRO}} = \frac{1}{2} m v^2$$

$$p = h/\lambda_{\text{BRO}} = m \cdot v = \sqrt{2mE_{\text{kin}}}$$

$$v_{\text{BRO}} = E_{\text{kin}}/h$$

$$\lambda_{\text{BRO}} = h/m \cdot v = h/\sqrt{2mE_{\text{kin}}}$$

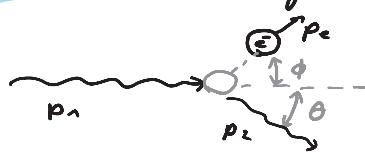
(Energie)

(Impuls)

(frequenz)

(Wellenlänge)

Compton Streuung:



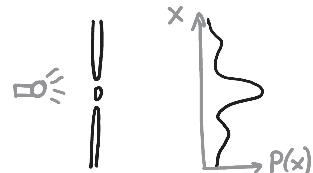
$$\frac{E_2}{E_1} = \frac{\lambda_1}{\lambda_2} = \frac{1}{1 + E_1/m_ec^2 \cdot (1 - \cos(\theta))}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_ec} \cdot (1 - \cos(\theta))$$

$$\lambda_{\text{compt}} = \Delta\lambda|_{\theta=90^\circ} = \frac{h}{m_ec}$$

Wahrscheinlichkeitsdichte: $P(x) = \Psi^*(x) \cdot \Psi(x)$

$$(\text{Normierung}: \int_{-\infty}^{\infty} P(x) dx = \int_{-\infty}^{\infty} \Psi^*(x) \Psi(x) dx = 1)$$



Erwartungswert:

$$\langle a \rangle = \int_{-\infty}^{\infty} \Psi^*(x) \cdot a(x) \cdot \Psi(x) dx$$

$\langle a \rangle$: Mittelwert von Mengengröße a.

$$\text{Ebene Welle: } \Psi(x, t_0) = \Psi_0 \cdot e^{i(kx - \omega t_0 + \phi)}; \quad \Psi^*(x, t_0) = \Psi_0^* \cdot e^{-i(\dots)}$$

$$\text{Wellenpaket: } \Psi(x, t_0) = \Psi_0 \cdot e^{-(x - ct_0)^2/4\sigma_x^2} \cdot e^{i(kx - \omega t_0 + \phi)}; \quad \Psi^*(x, t_0) = \Psi_0^* \cdot e^{-(\dots)} \cdot e^{i(\dots)}$$

$$a(x) = x : \text{Position}$$

$$a \dots : \text{Zeit}$$

$$a(x) = -i\hbar \frac{\partial}{\partial x} : \text{Impuls}$$

$$a(x) = i\hbar \frac{\partial}{\partial t} : \text{Energie}$$

Munschärfe:

$$\Delta a = \sqrt{\langle a^2 \rangle - \langle a \rangle^2}$$

Heisenbergsche Munschärfe:

$$\Delta x \cdot \Delta p \geq \hbar/2 \quad ; \quad \Delta E \cdot \Delta t \geq \hbar/2$$

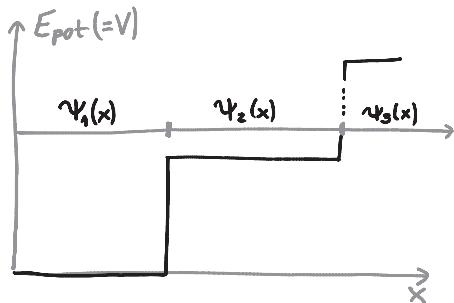
Schrödinger-Gleichung:

$$(1D) \quad -\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \cdot \Psi(x,t) + E_{pot}(x,t) \cdot \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t) \quad (1D \text{ freie Teilchen})$$

$$-\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \cdot \Psi(x) + E_{pot}(x) \cdot \Psi(x) = E \cdot \Psi(x) \quad (1D \text{ gebund. Teilchen})$$

dir. Schrödinger gebunden:

(1D)



Ansatz für Teilstück:

$$\Psi(x) = A \cdot e^{ikx} + B \cdot e^{-ikx}$$

$$(\text{bei Kästen}): \Psi(x) = A \cdot \sin(kx) + B \cdot \cos(kx)$$

$$k = \begin{cases} E > E_{pot}: \frac{\sqrt{2m(E-E_{pot})}}{\hbar} \\ E < E_{pot}: \frac{i\sqrt{2m(E_{pot}-E)}}{\hbar} \\ E_{pot}=0: \frac{\sqrt{2mE}}{\hbar} \\ E_{pot}=\infty: i \cdot \infty \rightarrow \Psi=0 \end{cases} \quad (\text{für } E_{pot} = \text{Konst auf Stück})$$

$$E_{pot}(x): \text{löse Schröd.} \quad (\text{für } E_{pot} = E_{pot}(x))$$

Aufenthalt: $|\Psi_{\text{total}}|^2$ bzw $|\Psi_{\text{transl}}|^2$

$$\text{Energie: } E = \int_{-\infty}^{\infty} \Psi^* \left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) \Psi dx = E \text{ aus Schrödinger}$$

$$\left. \begin{array}{l} \Psi_i(x_{\text{GRENZ}}) = \Psi_j(x_{\text{GRENZ}}) \\ \frac{d}{dx} \Psi_i(x_{\text{GRENZ}}) = \frac{d}{dx} \Psi_j(x_{\text{GRENZ}}) \\ \int_{-\infty}^{\infty} \Psi^*(x) \Psi(x) dx = 1 \end{array} \right\} \begin{array}{l} \text{stetig} \\ \text{außer } E_{pot} = \infty \\ \text{Normiert} \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Rand Bed.}$$

2D/3D Schrödinger: $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Rightarrow -\frac{\hbar^2}{2m} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \right)$ Ansatz bzg. $\Psi(x,y,z) = \Psi_x(x) \cdot \Psi_y(y) \cdot \Psi_z(z)$

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi_{ij}([]) + E_{pot}([]) \cdot \Psi_{ij}([]) = E_{ij} \cdot \Psi_{ij}([]) \quad \text{lösen für } [] = x, y, z \quad (E = E_x + E_y + E_z)$$

Teilchen im Kasten:

1D

2D

3D

$$\Psi_{N_x}(x) = \sqrt{\frac{2}{L_x}} \cdot \sin\left(\frac{\pi \cdot N_x \cdot x}{L_x}\right)$$

$$\Psi_{N_x N_y}(x,y) = \prod_{i=x,y} \sqrt{\frac{2}{L_i}} \sin\left(\frac{i\pi \cdot N_i}{L_i}\right)$$

$$\Psi_{N_x N_y N_z}(x,y,z) = \prod_{i=x,y,z} \sqrt{\frac{2}{L_i}} \sin\left(\frac{i\pi \cdot N_i}{L_i}\right)$$

(Wellenfunk.)

$$E_{N_x} = \frac{\hbar^2}{8m} \cdot \left(\frac{N_x^2}{L_x^2}\right)$$

$$E_{N_x N_y} = \frac{\hbar^2}{8m} \cdot \left(\frac{N_x^2}{L_x^2} + \frac{N_y^2}{L_y^2}\right)$$

$$E_{N_x N_y N_z} = \frac{\hbar^2}{8m} \cdot \left(\frac{N_x^2}{L_x^2} + \frac{N_y^2}{L_y^2} + \frac{N_z^2}{L_z^2}\right)$$

(Energie)

Kasten aufgefüllt mit e^- bis zu Energie E_F , ($L_x=L_y=L_z=L$):

$$E_F = E_{N_x} = \frac{\hbar^2}{8m_e} \cdot \left(\frac{n}{2}\right)^2$$

$$n = \left(\frac{32m_e}{\hbar^2} \cdot E\right)^{1/2} = \frac{M}{L}$$

$$g(E) = \left(\frac{8m_e}{\hbar^2}\right)^{1/2} \cdot E^{-1/2}$$

$$M = 2 \cdot N_x$$

$$E_F = E_{N_x N_y} = \frac{\hbar^2}{8m_e} \cdot \left(\frac{2 \cdot n}{\pi}\right)$$

$$n = \frac{4\pi m_e}{\hbar^2} \cdot E = \frac{M}{L^2}$$

$$g(E) = \frac{4\pi m_e}{\hbar^2}$$

$$M = 2 \cdot \frac{4}{3} \cdot \frac{\pi}{3} \left(N_x^2 + N_y^2 + N_z^2\right)^{1/2}$$

$$E_F = E_{N_x N_y N_z} = \frac{\hbar^2}{8m_e} \cdot \left(\frac{3n}{\pi}\right)^{2/3}$$

$$n = \frac{\pi}{3} \left(\frac{8m_e}{\hbar^2} \cdot E\right)^{3/2} = \frac{M}{L^3}$$

$$g(E) = \frac{\pi}{2} \left(\frac{8m_e}{\hbar^2}\right)^{3/2} \cdot E^{1/2}$$

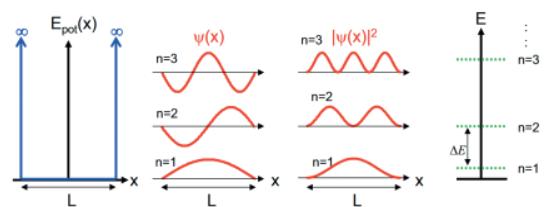
$$M = 2 \cdot \frac{4}{3} \cdot \frac{\pi}{3} \left(N_x^2 + N_y^2 + N_z^2\right)^{1/2}$$

(Fermi-Energie)

(Teilchendichte $\frac{\#e^-}{m^3}$)

(Zustandsdichte $\frac{\partial n(E)}{\partial E}$)

($\#e^-$ mit $E < E_F$)

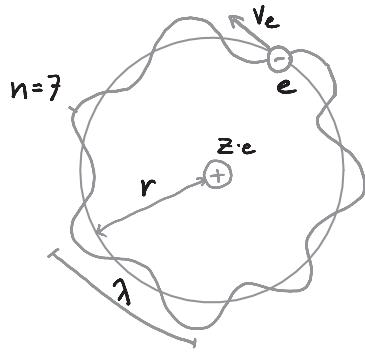


Bohr Model: $\frac{Z \cdot e^2}{4\pi\epsilon_0 r^2} = \frac{m_e v_e^2}{r}$ ($F_c = F_z$)

$$r_n = \frac{n \cdot \lambda}{2\pi} = \frac{n^2}{Z} \cdot \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} = \frac{n^2}{Z} \cdot \underbrace{0.0529 \text{ nm}}_{a_0: \text{Bohr-radius}}$$

$$E_n = -\frac{Z^2 e^2}{8\pi\epsilon_0 r_n} = -\frac{Z^2}{n^2} \cdot \frac{m_e e^4}{32\pi^2 \epsilon_0^2 \hbar^2} = -\frac{Z^2}{n^2} \cdot \underbrace{13.606 \text{ eV}}_{E_R: \text{Rydberg Energy}}$$

$$L_n = v_n \cdot m_e \cdot r_n = n \cdot \hbar \quad (\text{Drehimpuls})$$



Quantenzahlen:

z.B.: $2 \begin{smallmatrix} \leftarrow n \\ \leftarrow l \\ \leftarrow m_l \end{smallmatrix}$

n : Hauptquantenzahl $n = 1, 2, 3, \dots$

l : Nebenquantenzahl $l = 0, \dots, n-1 = s, p, d, f, g, h, i, j$

m_l : Magnetische Quantenzahl $m_l = -l, \dots, 0, \dots, l$

s : Spinquantenzahl $s = 1/2$

m_s : Mag. Spinquantenzahl $m_s = \pm 1/2$

(1 e^- existiert pro Kombi von n, l, m_l, m_s)

Quant. Mech. Modell:
(1 e^- pro Atom!)

$$\Psi_{n,l,m_l}(r, \vartheta, \varphi) = \underbrace{\sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n(n+l)!}}}_{\text{Normierung}} \cdot \underbrace{e^{-\frac{r}{na_0}}}_{\text{rad. Wellenfunk.}} \cdot \underbrace{s^l L^{2l+1}_n(g)}_{\text{ord. Wellenfunk.}} \cdot \underbrace{Y_l^{m_l}(\vartheta, \varphi)}_{\text{anis. Wellenfunk.}}$$

$g = \frac{2 \cdot z \cdot r}{n \cdot a_0}$: reduzierter Radius $L_n^k(x)$: Laguerre-Polynom

Quant. e^- -Energie:

$E_{n,l,m_l} = E_n$ in Bohr-Modell

Quant. e^- -Impuls:

$$|L| = \sqrt{l(l+1)} \hbar$$

$$|S| = \sqrt{\frac{3}{4}} \hbar$$

$$|J| = \sqrt{j(j+1)} \hbar$$

$$|L|_z = m_l \cdot \hbar$$

$$|S|_z = m_s \cdot \hbar$$

$$|J|_z = m_j \cdot \hbar$$

(Bahndrehimpuls)

(Eigendrehimpuls)

(Gesamtdrehimpuls
 $j = l + m_s$, $m_j = -j, \dots, 0, \dots, j$)

Quant. e^- -Moment:

$$\mu_{z,L} = -\mu_B \cdot m_L$$

$$\mu_{z,S} = -2 \cdot \mu_B \cdot m_s$$

$$\mu_{z,\text{tot}} = \mu_{z,L} + \mu_{z,S}$$

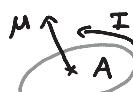
$$\mu_B = 5,788 \cdot 10^{-5} \text{ eV/T}$$

(Bahn Mag. Dipolmoment)

(Eigen Mag. Dipolmoment)

(Gesamt Mag. Dipolmoment)

(Bohrsches Magneton)



(Liebe Mag. Dipol)
mit $\vec{m} = \vec{n} \cdot \mu_{z,\text{tot}}$

Wahrscheinlichkeitsdichte: $P(r) = 4\pi r^2 |\Psi(r, \vartheta, \varphi)|^2$

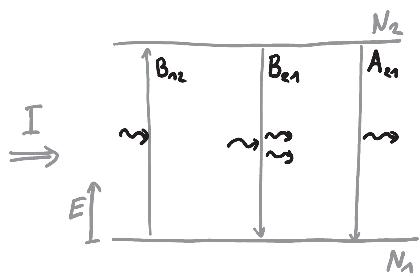
Knoten:

Jet. Knoten = $n-1$

Rad. Knoten = $n-1-l$

Mink. Knoten = L

Absorption/Emission 1-e-Atom:



$$E_2 - E_1 = E_{\text{phot}} = h\nu = hc/\lambda$$

$$\frac{\partial N_1}{\partial t} = -I \cdot B_{12} N_1 + I \cdot B_{21} N_2 + A_{21} N_2$$

$$\frac{\partial N_2}{\partial t} = +I \cdot B_{12} N_1 - I \cdot B_{21} N_2 - A_{21} N_2$$

Absorption stim. spont.
 Emission Emission

$$\frac{\partial N}{\partial t} = -2B_{12}I\Delta N - A_{21}(N + \Delta N)$$

$$\frac{\partial N}{\partial t} = 0 \rightarrow A_{21} > 0 \rightarrow N_1 = N, N_2 = 0$$

$$\rightarrow A_{21} = 0 \rightarrow N_1 = N_2 = N/2$$

$$E_n: \text{Bohr Energie}$$

$$N_{1,2}: \# e^- \text{ bei } ①, ②$$

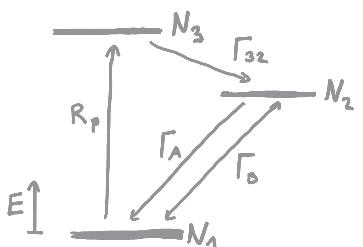
$$N = N_1 + N_2$$

$$\Delta N = N_2 - N_1$$

$$B_{12} = B_{21}$$

Rydberg-Formel: $\frac{1}{\lambda} = Z^2 \cdot R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (e^-: E_{n_1} \rightarrow E_{n_2} + \text{phot.}) \quad (R_H = 1,097 \cdot 10^7 \frac{1}{m})$

Dreiniveaulaser



$$\frac{\partial N_1}{\partial t} = +\Gamma_B N_p (N_2 - N_1) + \Gamma_A N_2 - R_p N_1$$

$$\frac{\partial N_2}{\partial t} = -\Gamma_B N_p (N_2 - N_1) - \Gamma_A N_2 + \Gamma_{32} N_3$$

$$\frac{\partial N_3}{\partial t} = R_p N_1 - \Gamma_{32} N_3$$

$$\frac{\partial \Delta N}{\partial t} = -2\Gamma_B N_p \Delta N - \Gamma_A (N + \Delta N) + R_p (N - \Delta N)$$

$$\frac{\partial \Delta N}{\partial t} = 0 \Rightarrow \frac{\Delta N}{N} = \frac{R_p - \Gamma_A}{2\Gamma_B N_p + R_p + \Gamma_A}$$

$$N = N_1 + N_2 : \text{Tot. } e^- \quad (N_3 \approx 0)$$

$$\Delta N = N_2 - N_1$$

R_p : Pumprate

N_p : # Photonen im Laser

Γ_A : A_{21} Γ_B : $B_{21} = B_{12}$

Γ_{32} : schnell, strahlunglos

Besetzungsinvolution \rightarrow Laser: $\Delta N > 0 \Leftrightarrow N_2 > N_1 \Leftrightarrow R_p > \Gamma_A$

Dreiniveaulaser mit "abzapfen"



$$\frac{\partial N_p}{\partial t} = \underbrace{\Gamma_B \Delta N \cdot N_p}_{\text{stimulierte Emission}} + \underbrace{\Gamma_A N_2}_{\text{spontane Emission}} - \underbrace{R_{\text{LEAK}} N_p}_{\text{Austritt}}$$

$$R_{\text{LEAK}} = \frac{c_0}{n \cdot 2L} (1 - R_2)$$

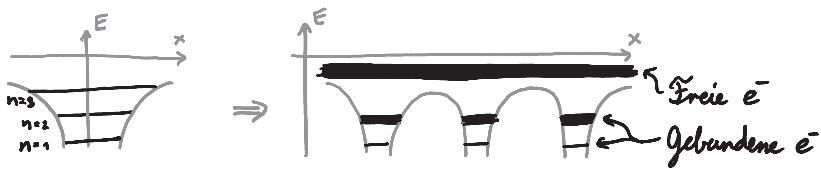
$$t_0 = 1/R_{\text{LEAK}} \quad (\text{Lebensdauer Photon})$$

$$P_{\text{out}} = R_{\text{LEAK}} \cdot N_p \cdot E_{\text{phot}} \quad (\text{Leistung Laser})$$

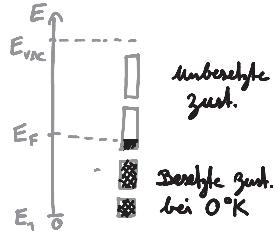
Besetzungsinvolution \rightarrow Laser: $\Delta N > \frac{R_{\text{LEAK}}}{\Gamma_B} \quad (\text{wen } \Gamma_A = 0 \text{ und photonen abgezapft})$

Festkörperphysik

Bändermodell:



Fermienergie:



$$E_F = \frac{\hbar^2 n^2}{32 m_e} \quad (1D \text{ Hartenpot.}, n = \frac{\#e^-}{m}, \text{"keine Bänder"})$$

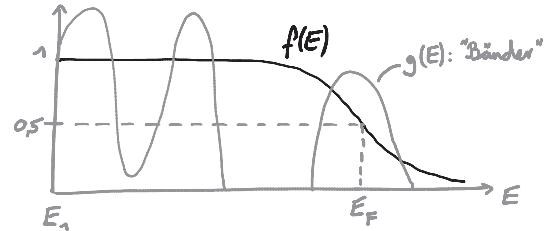
$$E_F = \frac{\hbar^2 n}{4 m_e \pi} \quad (2D \text{ Hartenpot.}, n = \frac{\#e^-}{m^2}, \text{"keine Bänder"})$$

$$E_F = \frac{\hbar^2}{8 m_e} \left(\frac{3n}{\pi} \right)^{2/3} \quad (3D \text{ Hartenpot.}, n = \frac{\#e^-}{m^3}, \text{"keine Bänder"})$$

Fermi-Dirac Verteilung: $f(E)$: Anteil e^- mit Energie $> E$ bzr. Besetzungswahr. bei E

$$\begin{cases} f(E) \cdot g(E) = \text{Anzahlhalt.} \\ \text{während bei } E \\ \int f(E) \cdot g(E) dE = \#e^- \end{cases}$$

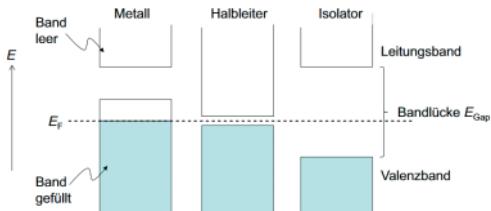
$$\begin{cases} T=0^\circ\text{K}: f(E) = \begin{cases} 1, & E \leq E_F \\ 0, & E > E_F \end{cases} \\ T>0^\circ\text{K}: f(E) = \frac{1}{1 + e^{\frac{E-E_F}{k_B T}}} \end{cases}$$



$$\text{Fermi Geschw.: } V_F = \sqrt{\frac{2 E_F}{m_e}} \quad (\text{"geschw." } e^- \text{ bei } E_F)$$

$$\text{Fermi Temperatur: } T_F = E_F / k_B \quad (\text{grenz. T für } f(E))$$

Leiter, Halbleiter, Isolator:

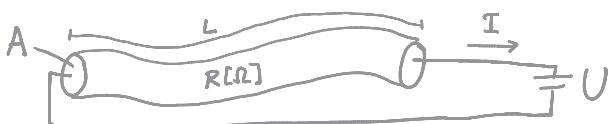


$$\begin{array}{ll} \text{Leiter:} & E_{\text{GAP}} = 0 \\ \text{Halbleiter:} & 0 < E_{\text{GAP}} < 4 \text{ eV} \\ \text{Isolator:} & 4 \text{ eV} < E_{\text{GAP}} \end{array}$$

Leitfähigkeit von Leitern: (Drude Modell)

$$V_d = \frac{j}{e \cdot n_e} = \frac{e \tau}{m_e} \cdot E = \frac{e \lambda}{m_e V_F} \cdot E$$

$$\sigma = \frac{j}{E} = \frac{L}{RA} = \frac{e n_e V_d}{E} = \frac{e^2 n_e \tau}{m_e} = \frac{e^2 n_e \lambda}{m_e V_F}$$



$n_e = \frac{\#e^-}{m^3}$ Konz. leitungs e^-

V_d : Drift geschw.

V_F : Fermi geschw.

$$\lambda = \vec{V}_F \cdot \tau \quad \tau: \text{mittlere Strezeit}$$

$$\lambda: \text{mittlere freie Weglänge}$$

σ : Leitfähigkeit

E : U/L elek. Feld

j : I/A Stromdichte

$$\tau \cdot V_d = \lambda \text{ bei Halbleiter ??}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_{\text{acc}}} + \frac{1}{\lambda_{\text{ss}}}$$

$$\lambda_{\text{osc}} = \frac{m_{\text{ion}} \cdot \omega_0^2}{n_{\text{ion}} \pi k_B T} = \frac{k_e}{n_{\text{ion}} \pi k_B T}$$

(Streuung Gitterschw.)

\Downarrow

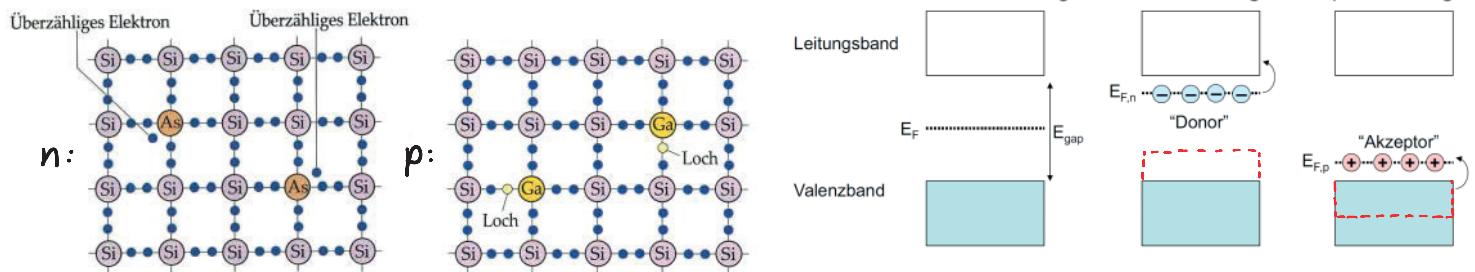
$$\lambda_{\text{ss}} = \frac{1}{n_{\text{ss}} \cdot A_{\text{ss}}} = \frac{1}{n_{\text{ss}} \pi r_{\text{ss}}^2}$$

(Streuung Fehlstelle)

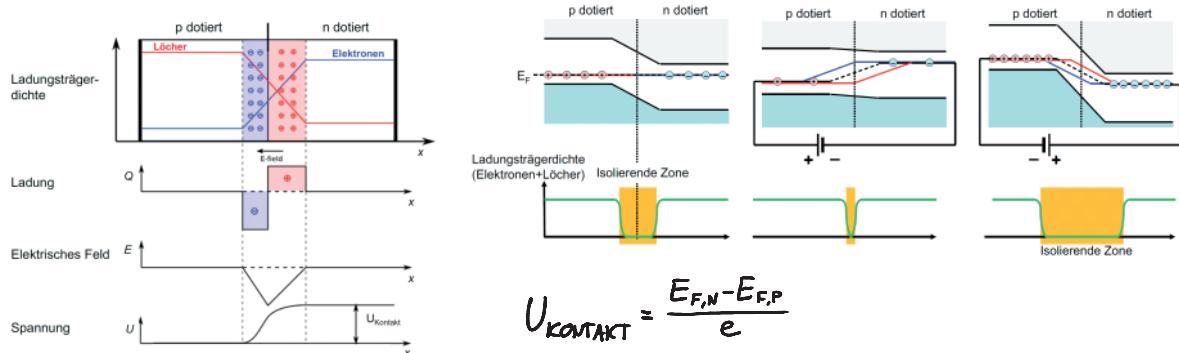
$$\sigma = \frac{e^2 \cdot n_e}{m_e V_F} \left(\frac{n_{\text{ion}} \pi \cdot k_B \cdot T}{m_{\text{ion}} \cdot \omega_0^2} + n_{\text{ss}} \cdot A_{\text{ss}} \right)^{-1}$$

$$(R = \frac{L}{A \sigma} [\Omega] \quad A \odot \overset{\text{L}}{\text{---}})$$

Dotierung von Halbleiter:



-pn-Halbleiterübergang: (Diode)



Solarzelle : (strom produzieren)

$$h \cdot v > E_{GAP}$$

Photodiode : (Lichtstärke messen)

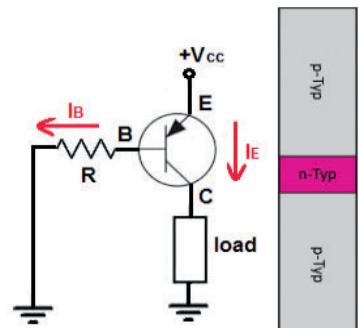
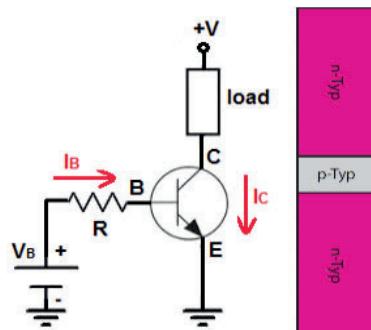
$$I \propto \# \text{phot/sec}$$

Leuchtdiode : (Licht abstrahlen)

$$E_{GAP} = h \cdot v$$

Transistor

Bipolartransistor:
(pnp, npn)



Feldeffekttransistor:
(FET)

