# Thermo III part 1

## Radiation Heat Transfer

**Black Body**
- Perfect absorber (can't reflect or transmit energy)
- Perfect emitter (emits (more than real) radiation energy in all directions uniformly)

**Constants**

$C_1 = hc_0^2 = 5{,}9552197 \cdot 10^{-17}$ $W m^2$

$C_2 = hc_0/k = 1{,}438769 \cdot 10^{-2}$ $mK$

$C_3 = 2{,}8978 \cdot 10^{-3}$ $mK$

$C_4 = 4{,}09579 \cdot 10^{-6}$ $W/(m^3K)$

$\sigma = \frac{2C_1\pi^5}{15C_2^4} = 5{,}67051 \cdot 10^{-8}$ $W/(m^2K^4)$

## Properties

- Black Body
  - Intensity
    - spectral : $i'_{\lambda b}(\lambda,T) = {}^{2C_1}/_{\lambda^5(e^{C_2/\lambda T}-1)} = {}^{Q}/_{d\lambda d\omega dA_p} = [{}^{W}/_{m^3}]$
    - total : $i'_b(T) = \int_0^\infty i'_{\lambda b}(\lambda,T)\, d\lambda = \frac{\sigma}{\pi} T^4 = {}^{Q}/_{d\omega dA_p} = [{}^{W}/_{m^2}]$
  - Emissive power
    - directional spectral : $e'_{\lambda b}(\lambda,T,\theta) = i'_{\lambda b}(\lambda,T)\cdot \cos(\theta) = {}^{Q}/_{d\lambda d\omega dA} = [{}^{W}/_{m^3}]$
    - hemisph. spectral : $e_{\lambda b}(\lambda,T) = \int_\omega e'_{\lambda b}(\lambda,T,\theta)\, d\omega = \pi\, i'_{\lambda b}(\lambda,T)$

      $= {}^{2\pi C_1}/_{\lambda^5(e^{C_2/\lambda T}-1)} = {}^{Q}/_{d\lambda dA} = [{}^{W}/_{m^2}]$

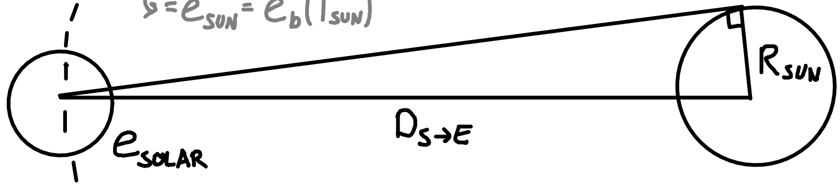
      for T fixe: $e_{\lambda_{MAX} b} = C_4 \pi T^5 \qquad \lambda_{MAX} = C_3/T$
    - hemisph. total : $e_b(T) = \int_0^\infty e_{\lambda b}(\lambda,T)\, d\lambda = \pi\, i'_b(T) = \sigma T^4 = {}^{Q}/_{dA} = [{}^{W}/_{m^2}]$
    - hemisph. interval : $e_{(\lambda_1\to\lambda_2)b} = \int_{\lambda_1}^{\lambda_2} e_{\lambda b}(\lambda,T)\, d\lambda = (F_{0-\lambda_2 T} - F_{0-\lambda_1 T})\, e_b(T) = [{}^{W}/_{m^2}]$
- Real Body
  - Emissivity
    - directional spectral : $\varepsilon'_\lambda(\lambda,T,\theta)$ $\quad (e'_\lambda = \varepsilon'_\lambda \cdot e'_{\lambda b})$
    - hemisph. spectral : $\varepsilon_\lambda(\lambda,T) = \int_\omega \varepsilon'_\lambda \cdot e'_{\lambda b}\, d\omega \,/\, e_{\lambda b}$ $\quad (e_\lambda = \varepsilon_\lambda \cdot e_{\lambda b})$
    - hemisph. total : $\varepsilon(T) = \int_0^\infty \varepsilon_\lambda \cdot e_{\lambda b}\, d\lambda \,/\, e_b$ $\quad (e = \varepsilon \cdot e_b)$
  - Absorptivity
    - directional spectral : $\alpha'_\lambda(\lambda,T,\theta)$ $\quad (e'_{\lambda A} = \alpha'_\lambda \cdot e'_{\lambda\, SUN})$
    - hemisph. spectral : $\alpha_\lambda(\lambda,T) = \int_\omega \alpha'_\lambda \cdot e'_{\lambda\, SUN}\, d\omega \,/\, e_{\lambda\, SUN}$ $\quad (e_{\lambda A} = \alpha_\lambda \cdot e_{\lambda\, SUN})$
    - hemisph. total : $\alpha(T) = \int_0^\infty \alpha_\lambda \cdot e_{\lambda\, SUN}\, d\lambda \,/\, e_{SUN}$ $\quad (e_A = \alpha \cdot e_{SUN})$
  - Reflectivity $\rho$ (analog to $\alpha$)
  - Transmissivity $t$ (here always 0)

## Surface Types

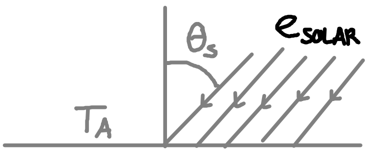
- all surfaces : $\Rightarrow \varepsilon'_\lambda = \alpha'_\lambda \;;\; \alpha'_\lambda + \rho'_\lambda = 1$
- gray surface : [Properties] = Konst. $\forall \lambda$ $\Rightarrow \varepsilon' = \alpha' \;;\; \alpha' + \rho' = 1$
- diffuse surface : [Properties] = Konst. $\forall \omega$ $\Rightarrow \varepsilon_\lambda = \alpha_\lambda \;;\; \alpha_\lambda + \rho_\lambda = 1$
- gray + diffuse : [Properties] = Konst $\forall \lambda,\omega$ $\Rightarrow \varepsilon = \alpha \;;\; \alpha + \rho = 1$

## Solar irradiation

$$Q_{SUN} = \underbrace{\sigma T_{SUN}^4}_{= e_{SUN} = e_b(T_{SUN})} \cdot (4\pi R_{SUN}^2) = e_{SOLAR} \cdot (4\pi D_{S\to E}^2) \Rightarrow e_{SOLAR} = \underbrace{(R_{SUN}/D_{S\to E})^2}_{= C} \cdot \sigma T_{SUN}^4 = 1353\ W/m^2$$

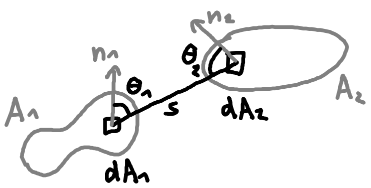



## Absorbed solar irradiation




$$Q_i/A = e_{SOLAR} \cdot \cos(\theta_s)$$
$$Q_A/A = \alpha'(\theta_s, T_A) \cdot e_{SOLAR} \cdot \cos(\theta_s)$$
$$Q_e/A = \varepsilon(T_A) \cdot \sigma T_A^4$$

## Config. Factors (gray/diffuse!)

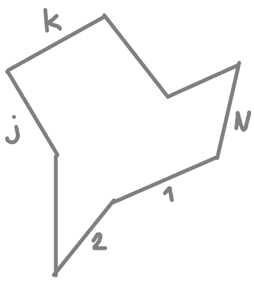



$$F_{1-2} = \frac{1}{A_1} \iint_{A_1 A_2} \left(\frac{\cos(\theta_1)\cos(\theta_2)}{\pi s^2}\right) dA_2\, dA_1$$

(Fraction of $E$ from $A_1 \to A_2$
$\hookrightarrow Q_{1-2} = F_{1-2} \cdot Q_{1TOT}$)

$$F_{1-2} \cdot A_1 = F_{2-1} \cdot A_2 \quad \text{(Reciprocity)}$$

## Enclosures $\left(\sum_{j=1}^{N} F_{k-j} = 1 \quad \forall k\right)$




$(Q_k = q_k A_k)$

$$q_k = q_{o,k} - q_{i,k} = \sigma T_k^4 - \sum_{j=1}^{N} \sigma T_j^4 \cdot F_{k-j} \quad \text{(black)}$$

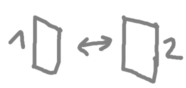
$$= \frac{\varepsilon_k}{1-\varepsilon_k} \cdot (\sigma T_k^4 - q_{o,k})$$

$$= \sum_{j=1}^{N} F_{k-j} (q_{o,k} - q_{o,j})$$

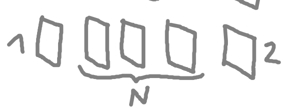
(diffuse + gray
$q_{o,k} = \varepsilon_k \sigma T^4 + \rho_k \cdot q_{i,k}$)

## Special Geometries (diffuse + gray)

- infinite parallel flat plates: $q_1 = -q_2 = \sigma(T_1^4 - T_2^4) \,/\, (1/\varepsilon_1 + 1/\varepsilon_2 - 1)$




- " with shields ($\varepsilon$: Konst): $q_{SHIELD} = q_{NO\ SHIELD} \,/\, (N+1)$




- infinite concentric cylinders: $Q = \ldots$
- … Formelsammlung

# Heat exchangers

- Compactness: $\beta = \frac{\text{heat transfer Area}}{\text{Volume}}$ $\quad(\beta > 700 \Rightarrow \text{compact})$
- heat transfer coefficient $U$ = "k from Thermo II"
- Fouling: $R_{FOUL} = R_f'' / A$

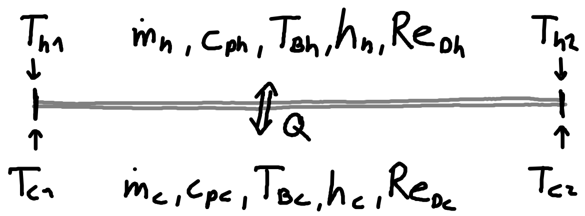
## Energy Balance

Assumptions: steady state, const. coeff., E pot/kin const., $Q_{AXIAL} = 0$, Insulated

$$Q = \dot{m}_h c_{ph} (T_{h1} - T_{h2}) = \dot{m}_c c_{pc} (T_{c1} - T_{c2})$$

$$Q = U \cdot A \cdot (T_{Bh} - T_{Bc})$$

$$U = 1 / (1/h_c + 1/h_h)$$




$$Re_{D_{h,c}} = \frac{\rho v D}{\mu} = \frac{\dot{m} D}{A \mu} = \frac{4 \dot{m}}{\pi D \mu} \quad \text{(for tubes)}$$

$$Nu_{D_{h,c}} = 0{,}023\, Re_D^{0.8} \cdot Pr^{0.4} \quad \text{(Dittus-Boelter)} \quad \left(Pr = \frac{c_p \mu}{\lambda_{FL}}\right)$$

$$h_{h,c} = Nu_D \cdot k/D \quad (k \mathrel{\hat{=}} \lambda,\ h \mathrel{\hat{=}} \alpha)$$

## Effectiveness-NTU Method

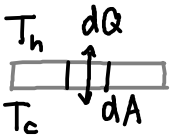
$$\varepsilon = Q / Q_{MAX}$$ ($\varepsilon$: Effectiveness ; $Q$: real Heatexchange ; $Q_{MAX}$: $\infty$-length + counter-flow)

$$Q_{MAX} = C_{MIN} \cdot (T_{h,in} - T_{c,in})$$ ($C_{MIN}$: smaller of $C_h = \dot{m}_h c_{ph}$, $C_c = \dot{m}_c c_{pc}$)

$$Q = C_c \cdot (T_{c,out} - T_{c,in}) = C_h \cdot (T_{h,in} - T_{h,out})$$

$$\varepsilon = f\left(NTU = U \cdot A / C_{MIN},\ C_r = \frac{C_{MIN}}{C_{MAX}}\right)$$ ($f(..)$: Table)

# Log-Mean temperature difference Method

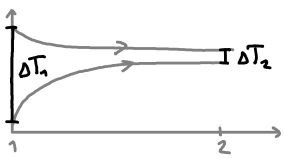



$$dQ = \underbrace{U \Delta T \, dA}_{} = -C_h dT_h = C_c dT_c \qquad (C_{[\cdot]} = \dot m_{[\cdot]} \cdot c_{p[\cdot]})$$

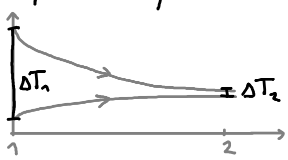
$$d(\Delta T) = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \qquad (d(\Delta T) = dT_h - dT_c)$$

$$\Rightarrow \quad Q = UA \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} \quad ; \quad \Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} \qquad (Q = UA \, \Delta T_{lm})$$

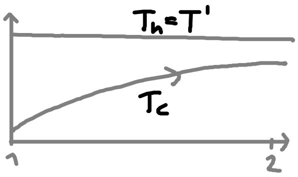
- parallel flow $(C_h > C_c)$



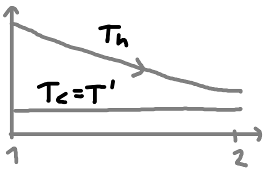

- parallel flow $(C_c > C_h)$




- condenser $(C_h \to \infty)$

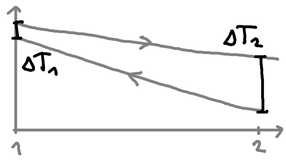



- boiler $(C_c \to \infty)$

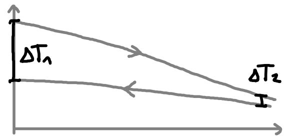



$$T(x) = T' - (T' - T_{IN})\, e^{\wedge}(-hA/\dot m c_p)$$

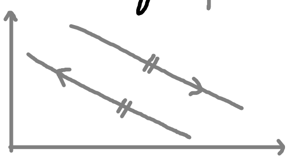
- counter flow $(C_h > C_c)$



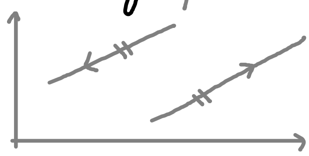

- counter flow $(C_c > C_h)$




- cooling $(q = \text{Const.})$



- heating $(q = \text{Const.})$



$$q = \text{Const.} \to \Delta T = \text{Const.} \to \Delta T_1 = \Delta T_2 \to C_h = C_c$$

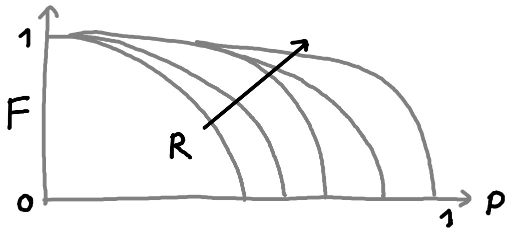
## Correction Factors F: (Table!)

$$Q = U \cdot A \cdot F \, \Delta T_{lm,CF}$$ (F: correction from counterflow to crossflow / shell+tube.)

- shell + tube:

$$R = T_{in} - T_{OUT} / t_{OUT} - t_{IN}$$
$$P = t_{OUT} - t_{IN} / T_{IN} - t_{IN}$$

($T_{IN,OUT}$: shell temp; $t_{IN,OUT}$: tube temp) $\Rightarrow$



# Ideal Gas Mixtures + Psychrometry


## Mixture of j components

Avg. molecular weight: $M = m/n = \sum_j m_j / \sum_j n_j$

Molecular weight: $M_i = \frac{m_i}{n_i}$

Total molecular weight: $m = m_1 + \ldots + m_j = \sum_j m_j$

mass fraction: $mf_i = m_i/m = m_i / \sum_j m_j$

Total number of moles: $n = n_1 + \ldots + n_j = \sum_j n_j$

mole fraction: $y_i = n_i/n = n_i / \sum_j n_j$

Total pressure: $p = p_1 + \ldots + p_j = \sum_j p_j$

pressure frac.: $y_i = p_i/p = p_i / \sum_j p_j$

Energy, Enthalpy, Entropy: $u = \sum_i y_i u_i \qquad h = \sum_i y_i h_i \qquad s = \sum_i y_i s_i$

Specific heats: $c_v = \sum_i y_i c_{vi} \qquad c_p = \sum_i y_i c_{pi}$

## State changes A → B


$u_B - u_A = \sum_i y_i \overbrace{(u_i(T_B) - u_i(T_A))}^{\Delta u_i}$

$\Delta u_i = c_{vi} (T_B - T_A)$

$h_B - h_A = \sum_i y_i \overbrace{(h_i(T_B) - h_i(T_A))}^{\Delta h_i}$

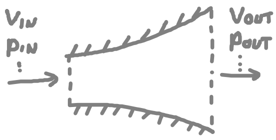
$\Delta h_i = c_{pi} (T_B - T_A)$

$s_A - s_B = \sum_i y_i \overbrace{(s_i(T_B) - s_i(T_A))}^{\Delta s_i}$

$\Delta s_i = s_i^\circ(T_B) - s_i^\circ(T_A) - R \cdot \ln\left(\frac{p_B}{p_A}\right)$

$= c_{pi} \ln\left(\frac{T_B}{T_A}\right) - R \cdot \ln\left(\frac{p_B}{p_A}\right)$

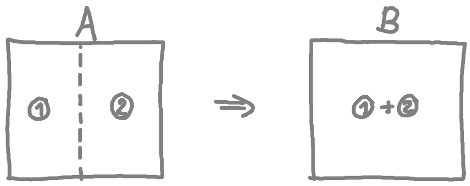
### Example Turbine




Isentropic → $\Delta s = 0$

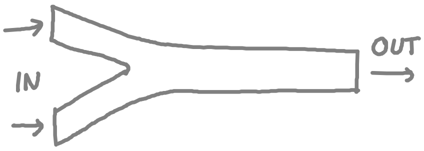
Energy Balance → $1/M (h_{out} - h_{in}) + 1/2 (v_{out}^2 - v_{in}^2) = 0$

### Example Mixing



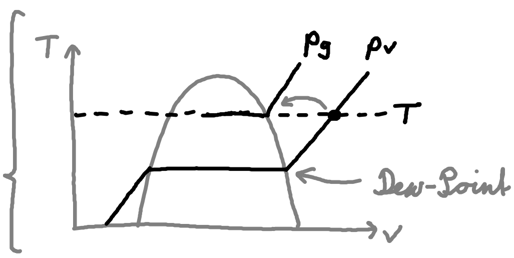

Energy Balance → $\Delta U = U_B - U_A = Q - W = 0$

### Example Mixing + Tube




Energy Balance → $\Delta H = H_{OUT} - H_{IN} = 0$

## Psychrometry




humidity ratio → $\omega = \frac{m_v}{m_A} = \frac{m \text{ vapour}}{m \text{ air}} = 0{,}622 \; p_v / p - p_v$

relative humidity → $\phi = y_v / y_{v,sat} = p_v / p_g \qquad (h_v \approx h_g)$

# Thermo III part 2

A,B,C : total  a,b,c : pro kg  $\bar{a},\bar{b},\bar{c}$ : pro mol  $\dot{A},\dot{B},\dot{C}$ : pro sec

$$dU = dQ - \underbrace{pdV}_{dW}$$

$$dH = d(U+pV) = dU + pdV + Vdp = dQ + Vdp$$

U: Innere Energie  H: Enthalpie
Q: Wärme  W: Arbeit

## Flüssigkeit (inkompressibel $\Rightarrow v = v_f(T)$)

$\boxed{c = c_p = c_v}$

| | | | |
|---|---|---|---|
| $u = u_f(T)$ | $h = h_f(T) + v_f(T)\cdot(p - p_f(T))$ | $s =$ ? | (TABLE) |
| $\Delta u = c\cdot\Delta T$ | $\Delta h = c\cdot\Delta T + v\cdot\Delta p$ | $\Delta s = c\cdot\ln(T_2/T_1)$ | (CALC) |

## Idealgas (Z=1)

$\boxed{pv = RT}$

$$\boxed{\begin{array}{ll} R = c_p - c_v & c_p = R/(1 - 1/\gamma) \\ \gamma = c_p/c_v & c_v = R/(\gamma - 1) \end{array}}$$

| | | | |
|---|---|---|---|
| $u = u(T,p)$ | $h = h(T,p)$ | $s = s(T,p)$ | (TABLE) |
| $\Delta u = c_v\,\Delta T$ | $\Delta h = c_p\,\Delta T$ | | (CALC) |

$$\Delta s = c_v\cdot\ln(T_2/T_1) + R\cdot\ln(V_1/V_2) = c_p\cdot\ln(T_2/T_1) - R\cdot\ln(p_2/p_1) = s_2^\circ - s_1^\circ - R\cdot\ln(p_2/p_1)$$

$w_s = w \pm$ in/out_w

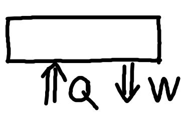
| | | wärme | geschl. w | offen + stat. ? |
|---|---|---|---|---|
| Isochor: | $\Delta v = 0$ | $\Delta q = c_v\,\Delta T = \Delta u$ | $\Delta w = 0$ | $\Delta w_s = R\,\Delta T$ |
| Isobar: | $\Delta p = 0$ | $\Delta q = c_p\,\Delta T = \Delta h$ | $\Delta w = R\,\Delta T$ | $\Delta w_s = 0$ |
| Isotherm: | $\Delta T = 0$ | $\Delta q = RT\cdot\ln(V_1/V_2)$ | $\Delta w = \Delta q$ | $\Delta w_s = \Delta q$ |
| Isentrop: | $\Delta s = 0$ | $\Delta q = 0$ | $\Delta w = c_v\cdot\Delta T = \Delta u$ | $\Delta w_s = c_p\,\Delta T = \Delta h$ |
| Isenthalp: | $\Delta h = 0$ | → same as Isotherm. ? | | |

Isentrop:

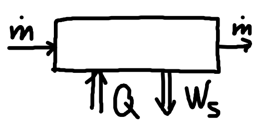
$$pv^\gamma = \text{Konst.} \Rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma \; ; \; \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

isentropen Effizienz: $\eta_{KOMP} = \frac{1}{\eta_{TURB}} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{h_{2s} - h_1}{h_2 - h_1}$ ($T_{2s}, h_{2s}$: Wert wen $s_1 = s_2$)

## Systeme: (stationär: U=0)

geschlossen: 

→ Q−W=0

offen: 

→ $Q - W_s =$ ?

## Nassdampf

$u = u_f + X(u_G - u_f)$ $\quad h = h_f + X(h_G - h_f)$ $\quad s = s_f + X(s_G - s_f)$ $\quad$ (TABLE)

## Diagrame:

| Turbine | Pumpe | Drossel | Kühler | Erhitzer | Mixer |
|---|---|---|---|---|---|

| $\Delta s = 0$ | $\Delta s = 0$ | $\Delta h = 0$ | $\Delta p = 0$ | $\Delta p = 0$ | $\Sigma h_{IN} = \Sigma h_{OUT}$ |
|---|---|---|---|---|---|

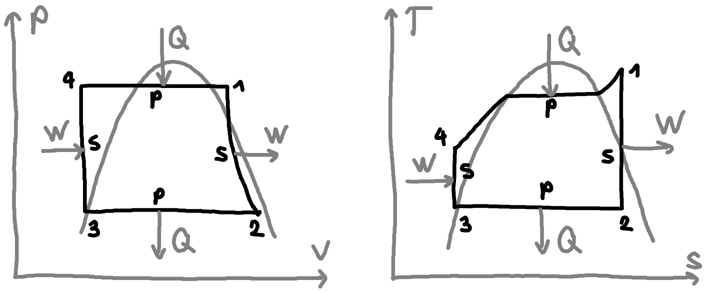
## Effizienz

$$\eta = \dot{W}_{OUT} - \dot{W}_{IN} / \dot{Q}_{IN} = \dot{Q}_{IN} - \dot{Q}_{OUT} / \dot{Q}_{IN}$$

$$\varepsilon_W = \dot{Q}_{OUT} / \dot{W}_{KOMP} \quad (?)$$

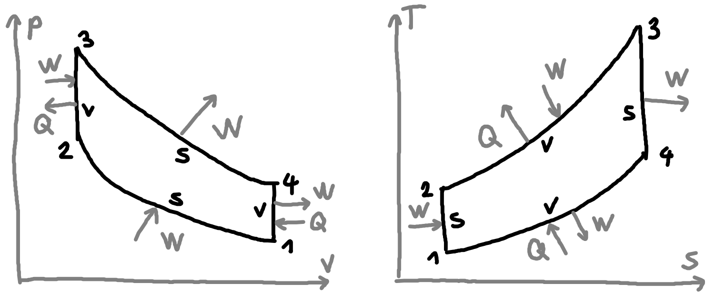
## Rankine

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$



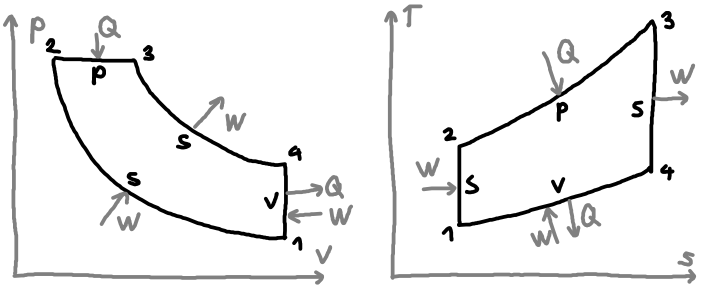

## Otto

$$\eta = 1 - 1/\varepsilon^{\gamma - 1} \quad (\varepsilon = V_1/V_2)$$



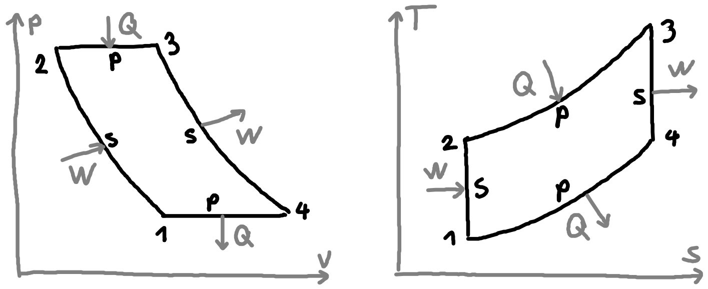

## Diesel

$$\eta = 1 - 1/\varepsilon^{\gamma-1} \cdot \frac{\varphi^{\gamma} - 1}{\gamma(\varphi - 1)} \quad \left(\varepsilon = \frac{V_1}{V_2},\ \varphi = V_3/V_2\right)$$



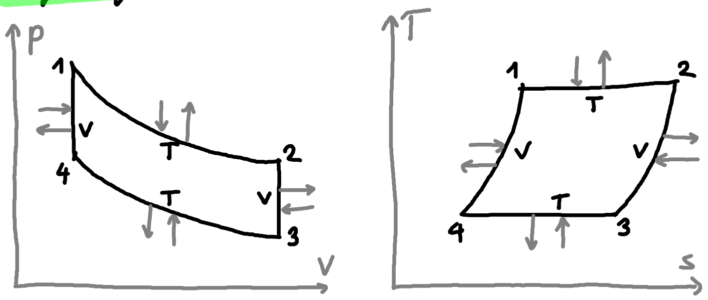

## Brayton/Joules

$$\eta = 1 - T_1/T_2$$



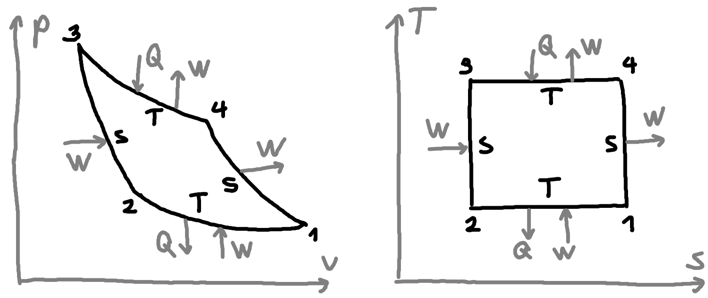

## Stirling

$$\eta = 1 - T_K/T_H$$

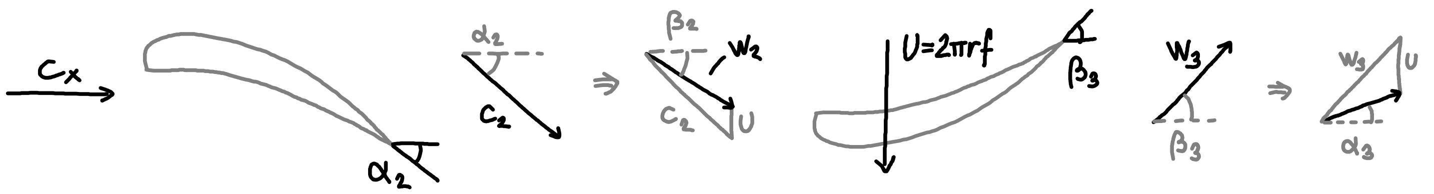


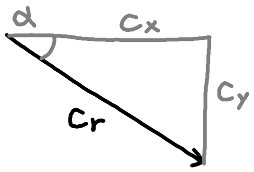

## Carnot

$$\eta = 1 - T_K/T_H$$



# Turbine / Geschw. Dreiecke




$c_r = c_x / \cos(\alpha)$

$c_y = c_x \cdot \tan(\alpha)$

$\alpha = \operatorname{atan}(c_y / c_x)$

$P = \dot{m}\, U\, (c_{2y} - c_{3y})$

$\Delta h = U\, (c_{2y} - c_{3y})$

$\Delta p \approx \Delta h \cdot \rho$