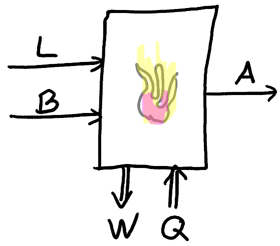
Verbrennung

$m_{[\cdot]}$: [g] $\qquad \dot{m}_{[\cdot]}$: [g/s] $\qquad [\cdot]_L$: Luft
$n_{[\cdot]}$: [mol] $\qquad \dot{n}_{[\cdot]}$: [mol/s] $\qquad [\cdot]_B$: Brennstoff
$M_{[\cdot]}$: [g/mol] $\qquad [\cdot]_A$: Abgas



$$\underbrace{C_xH_y}_{\text{Brennstoff}} + \underbrace{\nu'_{O_2} O_2 + \nu'_{N_2} N_2}_{\text{Luft}} \longrightarrow \underbrace{\sum_{[\cdot]} \nu''_{[\cdot]} \cdot [\cdot]}_{\text{Abgas}}$$ (siehe Stoffbilanz)

• Luft

$\bar{h}^\circ_{fL} = 0$
$h^\circ_{fL} = 0$
$H^\circ_{fL} = 0$

$\bar{h}_{th,L}(T) = 0{,}21(\bar{h}_{O_2}(T) - \bar{h}_{O_2}(298K)) + 0{,}79(\bar{h}_{N_2}(T) - \bar{h}_{N_2}(298K))$

$h_{th,L}(T) =$ (siehe abgas) $= c_{p,L}(T - 298K)$

$H_{th,L}(T) = n_L \cdot \bar{h}_{th,L}(T) = m_L \cdot h_{th,L}(T)$

$m_L = m_{O_2} + m_{N_2}$
$n_L = n_{O_2} + n_{N_2}$

$M_L = 28{,}84$

$X_{O_2} = 0{,}21$, $X_{N_2} = 0{,}79$
$Y_{O_2} = 0{,}233$, $Y_{N_2} = 0{,}767$

• Brennstoff:

$\bar{h}^\circ_{fB} = \bar{h}_{fB}$
$h^\circ_{fB} = \bar{h}^\circ_{fB} / M_B$
$H^\circ_{fB} = n_B \cdot \bar{h}^\circ_{fB} = m_B \cdot h^\circ_{fB}$

$\bar{h}_{th\,B}(T) = \bar{h}_B(T) - \bar{h}_B(298K)$

$h_{th\,B}(T) = \bar{h}_{th\,B} / M_B = c_{p,B}(T - 298K)$

$H_{th\,B}(T) = n_B \cdot \bar{h}_{th\,B}(T) = m_B \cdot h_{th\,B}(T)$

$M_B = (12x + y)$

• Abgas:

$\bar{h}^\circ_{fA} = \sum_i X_i \cdot \bar{h}^\circ_{fi}$
$h^\circ_{fA} = \sum_i Y_i \cdot \bar{h}^\circ_{fi} / M_i$
$H^\circ_{fA} = n_A \cdot \bar{h}^\circ_{fA} = m_A \cdot h^\circ_{fA}$

$\bar{h}_{th\,A}(T) = \sum_i X_i(\bar{h}_i(T) - \bar{h}_i(298K))$

$h_{th\,A}(T) = \sum_i Y_i(\bar{h}_i(T) - \bar{h}_i(298K))/M_i = c_{pA}(T - 298K)$

$H_{th\,A}(T) = n_A \cdot \bar{h}_{th\,A}(T) = m_A \cdot h_{th\,A}(T)$

$m_A = m_L + m_B$
$n_A \neq n_L + n_B$

$X_i = \nu_i / \sum_j \nu_j = n_i / \sum_j n_j$
$Y_i = \nu_i M_i / \sum_j \nu_j M_j = n_i M_i / \sum_j n_j M_j$

• Einh.:

[J/mol]
[J/g]
[J]

[J/mol]

[J/g]

[J]

[g/mol]

Luftüberschussfaktor

$$\lambda = \frac{m_L/m_B}{(m_L/m_B)_{ST}} = \frac{\dot{m}_L/\dot{m}_B}{(\dot{m}_L/\dot{m}_B)_{ST}} = \frac{n_L/n_B}{(n_L/n_B)_{ST}} = \frac{\dot{n}_L/\dot{n}_B}{(\dot{n}_L/\dot{n}_B)_{ST}}$$

$\left(\frac{m_L}{m_B}\right)_{ST}$ bzw. $\left(\frac{n_L}{n_B}\right)_{ST} \;\hat{=}\; \frac{m_L}{m_B}$ bzw. $\frac{n_L}{n_B}$ bei $\lambda = 1 \Rightarrow$ perfekte verbrennung

mit $$\left(\frac{m_L}{m_B}\right)_{ST} = \left(\frac{\dot{m}_L}{\dot{m}_B}\right)_{ST} = \left(x + \frac{y}{4}\right)\frac{M_{O_2} + 3{,}76 M_{N_2}}{x \cdot M_C + y \cdot M_H} = \left(x + \frac{y}{4}\right)\frac{137{,}28}{12x + y}$$

$$\left(\frac{n_L}{n_B}\right)_{ST} = \left(\frac{\dot{n}_L}{\dot{n}_B}\right)_{ST} = \left(x + \frac{y}{4}\right)$$

Stoffbilanz

bei 1 mol Reaktion: $n_B = 1\,mol$ $\quad m_B = n_B \cdot (x \cdot M_C + y \cdot M_H)$

$n_L = \lambda(x+\frac{y}{4})\,mol$ $\quad m_L = n_L \cdot (M_{O_2} + 3{,}76\, M_{N_2})$

$n_A = \dots$ $\quad m_A = m_B + m_L$

$$C_xH_y + \lambda(x+\tfrac{y}{4})(O_2 + 3{,}76 N_2) \rightarrow$$

- $\lambda \geq 1$ (mager): $\rightarrow x \cdot CO_2 + \frac{y}{2} H_2O + (\lambda-1)(x+\frac{y}{4})O_2 + 3{,}76 \cdot \lambda \cdot (x+\frac{y}{4}) N_2$
- $\lambda < 1$ (fett): $\rightarrow (1-\lambda) C_xH_y + \lambda \cdot x \cdot CO_2 + \lambda \frac{y}{2} H_2O + 3{,}76 \cdot \lambda \cdot (x+\frac{y}{4}) N_2$

$$C_xH_yO_z + \lambda(x+\tfrac{y}{4}-\tfrac{z}{2})(O_2 + 3{,}76 N_2) \rightarrow$$

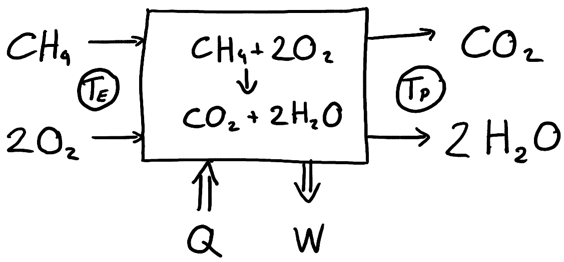
- $\lambda \geq 1$ (mager): $\rightarrow x\, CO_2 + \frac{y}{2} H_2O + (\lambda-1)(x+\frac{y}{4}-\frac{z}{2}) O_2 + 3{,}76 \cdot \lambda \cdot (x+\frac{y}{4}-\frac{z}{2}) N_2$
- $\lambda < 1$ (fett): $\rightarrow (1-\lambda) C_xH_yO_z + \lambda \cdot x \cdot CO_2 + \lambda \frac{y}{2} H_2O + 3{,}76 \cdot \lambda \cdot (x+\frac{y}{4}-\frac{z}{2}) N_2$

Bildungsenthalpie

$$\tilde{h}_{[\cdot]}(T) = \underbrace{h^\circ_{f\,[\cdot]}}_{\text{Bildungs Enthalpie}} + \underbrace{h_{th\,[\cdot]}(T,p)}_{\text{Thermische Enthalpie}} \quad [J/mol]$$

$h_{th\,[\cdot]} = h_{[\cdot]}(T) - h_{[\cdot]}(298K)$

$h_{th\,[\cdot]} = \int_{298K}^{T} c_{p[\cdot]}(T')\,dT' = c_{p[\cdot]}(T - 298K)$



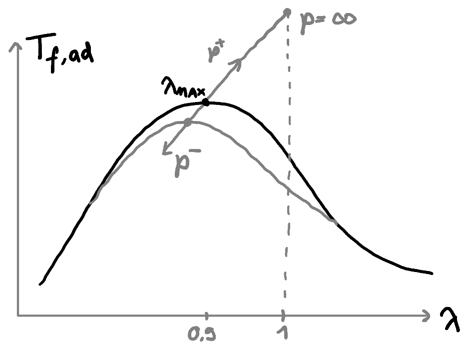
$$\Delta H_R = \Delta Q - \Delta W$$

$$\Delta H_R = \sum^{prod} \nu_i'' \tilde{h}_i(T_P) - \sum^{edu} \nu_i' \tilde{h}_i(T_E) = \underbrace{\sum^{all} (\nu_i'' - \nu_i') \cdot h^\circ_{f_i}}_{\Delta H_R|_{T_{ref}}} + \sum^{prod} \nu_i'' h_i(T_P) - \sum^{edu} \nu_i' h_i(T_E)$$

$$\left(\underbrace{1}_{\nu'_{H_2}} H_2 + \underbrace{\tfrac{1}{2}}_{\nu'_{O_2}} O_2 + \underbrace{0}_{\nu'_{H_2O}} H_2O \xleftrightarrow[\underbrace{\quad}_{n_{O_2},\, n_{H_2},\, n_{H_2O}\ [mol]}]{K_p} \underbrace{0}_{\nu''_{H_2}} H_2 + \underbrace{0}_{\nu''_{O_2}} O_2 + \underbrace{1}_{\nu''_{H_2O}} H_2O \right)$$

Adiabate Flammtemperatur

$(T_E = 298K,\ Q=0,\ W=0 \quad \Rightarrow T_{f,ad} = T_P = ?)$



@ $p = 1\,atm \rightarrow \lambda_{MAX} \in (0{,}8;0{,}9)$ $\quad T_{f,ad} \approx 2300\,K$

@ $p \rightarrow \infty\,atm \rightarrow \lambda_{MAX} = 1$ $\quad T_{f,ad} \approx$???

(abweichung von $\lambda = 1$ wegen dissozation)
$CO_2 \rightarrow CO + O$

Gibs freie Energie / Gleichgewicht

$G = H - TS$ $\qquad g_{[i]}(T) = \tilde{h}_{[i]}(T) - T \cdot s_{[i]}(T)$

$G(T,p,n_i) = \sum_i n_i \cdot g_i$ $\qquad g_i(T,p_i) = \underbrace{g^\circ_{f,i}(T)}_{g_i(T,p_0),\ p_0 = 1\,atm} + RT \cdot \ln\left(\frac{p_i}{p_0}\right)$ $\quad [J/mol]$ $\qquad \left(p_i = \frac{n_i}{\sum_j n_j} \cdot p\right)$

$$\left(\underbrace{1}_{\nu'_{H_2}} H_2 + \underbrace{1/2}_{\nu'_{O_2}} O_2 + \underbrace{0}_{\nu'_{H_2O}} H_2O \underset{n_{O_2},\, n_{H_2},\, n_{H_2O}\ [mol]}{\xleftrightarrow{K_p}} \underbrace{0}_{\nu''_{H_2}} H_2 + \underbrace{0}_{\nu''_{O_2}} O_2 + \underbrace{1}_{\nu''_{H_2O}} H_2O \right)$$

Gleichgewicht: $\quad dG = 0 \Rightarrow \ln(K_p(T)) = -\Delta G(T)/RT$

$$\Delta G(T) = \sum_i (\nu''_i - \nu'_i) \cdot g^\circ_{f,i}(T) = \sum_i (\nu''_i - \nu'_i) \cdot (\tilde{h}_i(T) - T \cdot s_i(T))$$

$$K_p(T) = \frac{p_1^{\nu''_1} \cdot p_2^{\nu''_2} \cdot \ldots \cdot p_k^{\nu''_k}}{p_1^{\nu'_1} \cdot p_2^{\nu'_2} \cdot \ldots \cdot p_k^{\nu'_k}} \cdot \left(\frac{1}{p_0}\right)^{\sum_i (\nu''_i - \nu'_i)} = \frac{n_1^{\nu''_1} \cdot n_2^{\nu''_2} \cdot \ldots \cdot n_k^{\nu''_k}}{n_1^{\nu'_1} \cdot n_2^{\nu'_2} \cdot \ldots \cdot n_k^{\nu'_k}} \cdot \left(\frac{p}{n_{tot} \cdot p_0}\right)^{\sum_i (\nu''_i - \nu'_i)}$$

Van't Hoff Eq.: $\quad \frac{d}{dT} \ln(K_p(T)) = \Delta H_R(T)/RT^2$ $\qquad \left(\Delta H_R(T) = \sum_i (\nu''_i - \nu'_i)\, \tilde{h}_i(T)\right)$

$$\ln\left(K_p(T_2)/K_p(T_1)\right) \approx \Delta H_R/R \cdot \left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad (\text{wenn } \Delta H_R \approx \text{Konst. für } T)$$

Gleichzeit. Reak.: $\quad 1\,CO_2 + \frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftarrows a \cdot CO + b \cdot NO + c \cdot CO_2 + d \cdot O_2 + e \cdot N_2$

$\downarrow$

a, b fix: $\quad c = 1 - a \quad d = (1 + a - b)/2 \quad e = (1 - b)/2 \quad \Rightarrow n = \frac{4 + a}{2}$

① $\quad CO_2 \rightleftarrows CO + \frac{1}{2} O_2 \longrightarrow \Delta_1 = (1 + 1/2) - (1) = 1/2$

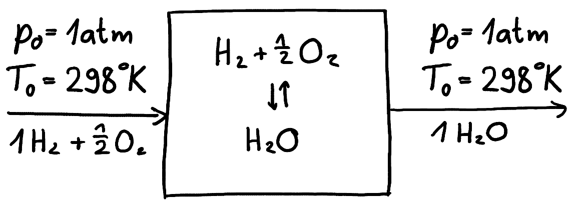
$$K_1(T) = \left(\frac{p}{n \cdot p_0}\right)^{\Delta_1} \cdot \left(\frac{[CO]\,[O_2]^{1/2}}{[CO_2]}\right) = \left(\frac{p}{n \cdot p_0}\right)^{\Delta_1} \left(\frac{a \cdot d^{1/2}}{c}\right) = \left(\frac{p/p_0}{(4+a)/2}\right)^{1/2} \cdot \frac{(a) \cdot \left(\frac{1+a-b}{2}\right)^{1/2}}{(1-a)}$$

② $\quad \frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftarrows NO \longrightarrow \Delta_2 = (1) - (1/2 + 1/2) = 0$

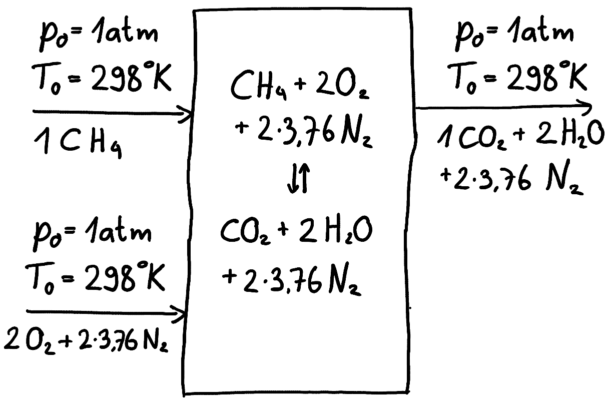
$$K_2(T) = \left(\frac{p}{n \cdot p_0}\right)^{\Delta_2} \left(\frac{[NO]}{[O_2]^{1/2} [N_2]^{1/2}}\right) = \left(\frac{p}{n \cdot p_0}\right)^{\Delta_2} \left(\frac{b}{d^{1/2} \cdot e^{1/2}}\right) = \left(\frac{p/p_0}{(4+a)/2}\right)^{0} \cdot \frac{b}{\left(\frac{1+a-b}{2}\right)^{1/2} \cdot \left(\frac{1-b}{2}\right)^{1/2}}$$

$\Rightarrow \quad K_1(T), K_2(T)$ aus Tabelle $\rightarrow a, b$

Exergy



$$E_x = -\Delta G = \sum_i (\nu_i'' - \nu_i')\, g^\circ_{i,f}(T_0) \quad [J/mol]$$



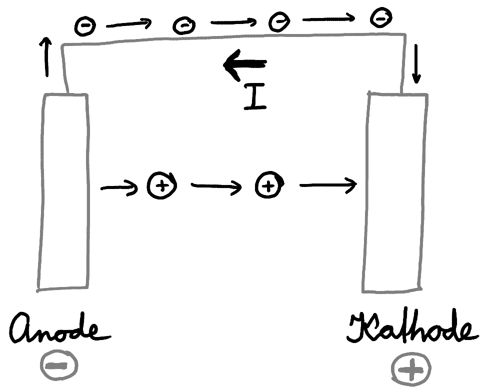
$$E_x = -\Delta G + RT_0 \ln\left(\frac{(p_{O_2}/p_0)^2 \cdot (p_{CH_4}/p_0)^1 \cdot (p_{N_2}/p_0)^{2\cdot3.76}}{(p_{CO_2}/p_0)^1 \cdot (p_{H_2O}/p_0)^2 \cdot (p_{N_2}/p_0)^{2\cdot3.76}}\right)$$

If input/output = "air":

$p_{O_2} = 0{,}2035 \cdot p_0$ $\qquad p_{H_2O} = 0{,}0312 \cdot p_0$

$p_{N_2} = 0{,}7567 \cdot p_0$ $\qquad p_{CO_2} = 0{,}003 \cdot p_0$

Brennstoffzelle



Total: $CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$

Anode: $CH_4 + O_2 \rightarrow 4H^+ + 4e^- + CO_2$

Kathode: $O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$

(Formeln auf LAV Zusammenfassung)

$\boxed{°K = °C + 273{,}15}$

Methods of heat transfer

Conduction:

$\left(\dot q'': \quad \lambda: \left[\frac{W}{m°K}\right]\right)$

3D: $\dot q'' = -\lambda \nabla T$

1D: $\dot q'' = -\lambda \frac{dT}{dn}$

LIN: $\dot q'' = -\lambda \frac{T_H - T_c}{d}$

Convection:

$\left(\dot q'': \left[\frac{W}{m^2}\right] \quad \alpha: \left[\frac{W}{m^2 °K}\right]\right)$

$\dot q'' = \alpha \cdot (T_H - T_c)$

Radiation:

$\left(\dot q'': \left[\frac{W}{m^2}\right] \quad \sigma: \left[\frac{W}{m^2 °K^4}\right]\right)$

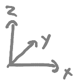
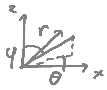
$\dot q'' = \sigma \left(T_H^4 - T_c^4\right) \cdot \underbrace{\varepsilon}_{\text{deviation from black body rad.}}$

Heat conduction Eq.

$\left(\lambda: \left[\frac{W}{m°K}\right], \rho: \left[\frac{kg}{m^3}\right], c: \left[\frac{J}{°K}\right], \dot Q''': \left[\frac{W}{m^3}\right]\right)$

$$\frac{\partial(T \rho c)}{\partial t} = \vec\nabla(\lambda \cdot \vec\nabla T) + \dot Q'''_{Source} \qquad (\text{General})$$

$$\frac{1}{a}\frac{\partial T}{\partial t} = \vec\nabla^2 T + \frac{1}{\lambda}\dot Q'''_{Source} \qquad \left(\lambda, \rho, c \text{ const.}, \; a = \frac{\lambda}{\rho c}\right)$$

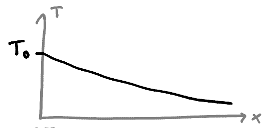
Cart:  Zyl:  Sph: 

$$\vec\nabla \Phi = \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial y} + \frac{\partial \Phi}{\partial z} = \frac{\partial \Phi}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial \theta} \qquad \frac{\partial \Phi}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial \varphi} + \frac{1}{r \cdot \sin(\varphi)}\frac{\partial \Phi}{\partial \theta}$$

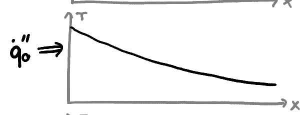
$$\vec\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin(\varphi)}\frac{\partial}{\partial \varphi}\left(\sin(\varphi)\frac{\partial \Phi}{\partial \varphi}\right) + \frac{1}{r^2 \sin^2(\varphi)}\frac{\partial^2 \Phi}{\partial \theta^2}$$

Boundary conditions (1D)

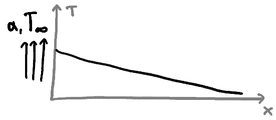
First kind: $T(x,t)\big|_{x=0} = T_0(t)$



Second kind: $\frac{\partial T(x,t)}{\partial x}\Big|_{x=0} = -\frac{1}{\lambda}\dot q_0''(t)$



Third kind: $\frac{\partial T(x,t)}{\partial x}\Big|_{x=0} = -\frac{\alpha}{\lambda}\left(T(0,t) - T_\infty\right)$

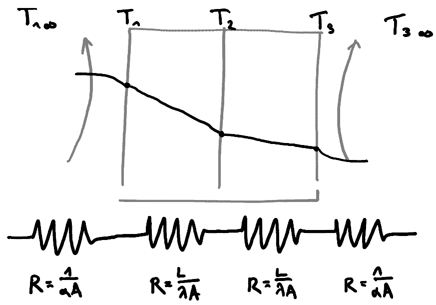


Thermal resistance / Thermal transmittance $\left(1D,\ \frac{dT}{dt}=0,\ \dot{Q}'''_{SOURCE}=0,\ 1st\ Const.\right)$

Plane wall:

$\Delta T = R_{th} \cdot \dot{Q}_x$ "$U = R \cdot I$"

$\dot{Q}_x = k \cdot A \cdot \Delta T$ $\left(k = \frac{1}{R_{th} A}\right)$



Conduction: $R_{th} = L/A\lambda$ $k = \lambda/L$

Convection: $R_{th} = 1/A\cdot\alpha$ $k = \alpha$

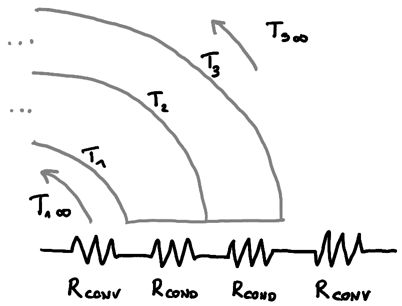
$R_{th\,TOT} = \frac{1}{\alpha_H A} + \sum_{i=1}^{n} \frac{L_i}{\lambda_i A} + \frac{1}{\alpha_C A}$ (add like resistors!) $k_{TOT} = \frac{1}{\frac{1}{\alpha_H} + \sum_i^n \frac{L_i}{\lambda_i} + \frac{1}{\alpha_C}}$

Biot-Number: $Bi = \frac{\alpha L}{\lambda} = \frac{k_{CONV}}{k_{COND}}$ $Bi \gg 1$:  $Bi \ll 1$: 

Tube:

$\Delta T = R_{th} \cdot \dot{Q}_r$ "$U = R \cdot I$"

$\dot{Q}_r = k_L \cdot L \cdot \Delta T$ $\left(k_L = \frac{1}{R_{th} \cdot L}\right)$

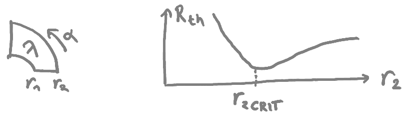


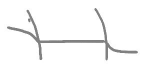
conduction: $R_{th} = \frac{\ln(r_2/r_1)}{2\pi\lambda L}$ $k_L = \frac{2\pi\lambda}{\ln(r_2/r_1)}$

convection: $R_{th} = \frac{1}{2\pi r L \alpha}$ $k_L = 2\pi r \alpha$

$R_{th\ TOT} = \frac{1}{2\pi r_H L \alpha_H} + \sum_{i=1}^{n} \frac{\ln(r_{i+1}/r_i)}{2\pi L \lambda_i} + \frac{1}{2\pi r_C L \alpha_C}$ $k_{L\ TOT} = \frac{1}{\frac{1}{2\pi r_H \alpha_H} + \sum_{i=1}^{n} \frac{\ln(r_{i+1}/r_i)}{2\pi\lambda_i} + \frac{1}{2\pi r_C \alpha_C}}$

Critical radius: $r_{2\,crit} = \frac{\lambda}{\alpha}$



Biot-Number: $Bi = \frac{2r\alpha}{\lambda}$ $Bi \gg 1$:  $Bi \ll 1$: 

Heat conduction with source $\left(1D,\ \frac{dT}{dt}=0,\ \dot{Q}'''_{SOURCE}=Const.,\ 1st\ Constr.\right)$

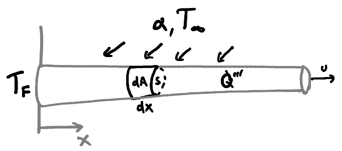
Plain wall: $T(x) = -\frac{\dot{Q}'''_{SOURCE}}{2\lambda} \cdot x^2 + \left(\frac{T_2-T_1}{L} + \frac{\dot{Q}'''_{SOURCE}\cdot L}{2\lambda}\right) \cdot x + T_1$ $\left(x_{MAX} = \frac{L}{2} + \frac{T_2-T_1}{L} \cdot \frac{\lambda}{\dot{Q}'''_{SOURCE}}\right)$

$\dot{q}''(x) = -\lambda \frac{dT}{dx} = \dot{Q}'''_{SOURCE} \cdot x - \lambda \frac{T_2-T_1}{L} - \frac{\dot{Q}'''_{SOURCE}\cdot L}{2}$

Tube: $T(r) = \frac{\dot{Q}'''_{SOURCE}\cdot r_0^2}{4\lambda} \cdot \left(1 - \frac{r^2}{r_0^2}\right) + T_0$ $(r_{MAX} = 0)$

$\dot{q}''(r_0) = -\lambda \frac{dT}{dr} = \frac{\dot{Q}'''_{SOURCE} \cdot r_0}{2}$

Heat conduction in fins $(1D, \frac{dT}{dt}=0)$



$P(x) = 2\pi r = 2\cdot(w+h)$ (Circumference of fin)
$S(x) = \pi r^2 = w\cdot h$ (Cross-Section-Area of fin)
$m = \sqrt{\alpha P / \lambda S}$ (Fin-Parameter)
$\theta = T - T_\infty$ (Temperature difference)

$$\rho\cdot c\cdot\frac{\partial}{\partial t}\Big(\underbrace{\frac{dV}{dx}}_{S}\cdot T\Big) = \lambda\cdot\frac{\partial}{\partial x}\Big(S\cdot\frac{\partial T}{\partial x}\Big) - \rho\cdot c\cdot\frac{\partial}{\partial x}\big(S\cdot u\cdot T\big) - \underbrace{\frac{\partial A}{\partial x}}_{P}\cdot\alpha\,(T-T_\infty) + \dot{Q}'''_{SOURCE}\cdot\underbrace{\frac{dV}{dx}}_{S} - \dot{Q}''_{RAD}\cdot\underbrace{\frac{dA}{dx}}_{P}$$

simplified fin: $(S(x)=\text{Const},\ u=0,\ \frac{dT}{dt}=0,\ \dot{Q}'''_{source}=0,\ \dot{Q}''_{RAD}=0)$

$$\frac{d^2\theta}{dx} - m^2\cdot\theta = 0 \Rightarrow \theta(x) = C_1\cdot e^{mx} + C_2\cdot e^{-mx}$$ $(+\ \dot{Q}'''/\lambda m^2$ with source)

$$\dot{Q}_F = -\lambda\cdot S\cdot\frac{\partial T}{\partial x}\Big|_{x=0}$$

- Bound 1: $\theta(0) = C_1 + C_2 = T_F - T_\infty$
- Bound 2a: $d\theta/dx|_{x=L} = C_1\cdot m\cdot e^{mL} - C_2\cdot m\, e^{-mL} = 0$ (end isolated)
- Bound 2b: $C_1 m e^{mL} - C_2 m e^{-mL} = -\frac{\alpha}{\lambda}(C_1 e^{mL} - C_2 e^{-mL})$ (end with convection)
- Bound 2c: $C_1 e^{mL} + C_2 e^{-mL} = T_K - T_\infty$ (end has temp. T_K)
- Bound 2d: $C_1 e^{m(L\to\infty)} = 0 \Rightarrow C_1 = 0$ $(L\to\infty)$

more simplified fin: (simplified fin: Bound 1 + 2a)

$$\theta(x) = \theta_F\,\frac{\cosh(m(L-x))}{\cosh(mL)} \qquad \dot{Q}_F = \lambda\cdot S\cdot\theta_F\cdot m\cdot\tanh(mL)$$ $\left(L_{OPT} < \frac{1}{m} = \sqrt{\frac{\lambda S}{\alpha P}}\right)$

Fin efficiency: $\eta_{FIN} = \frac{\dot{Q}_F}{\dot{Q}_{MAX}} = \frac{\tanh(mL)}{mL}$ ($\dot{Q}_{MAX}$ is $\dot{Q}$ if $\theta(x)=\theta_F\ \forall x$)

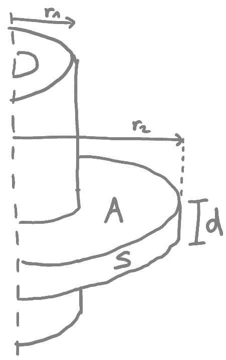
more simplified fin: (simplified fin: Bound 1 + 2d)

$$\theta(x) = \theta_F\cdot e^{-mx} \qquad \dot{Q}_F = \lambda\cdot S\cdot\theta_F\cdot m$$

fins on a tube: $(d=\text{Const},\ u=0,\ \frac{dT}{dt}=0,\ \dot{Q}'''_{source}=0,\ \dot{Q}''_{RAD}=0)$

$S(r) = 2\pi\cdot r\cdot d \qquad A(r) = 2\pi(r^2 - r_1^2) \qquad P(r) = 4\pi\cdot r$

$$\theta(r) = \theta_F\,\frac{I_0(m\cdot r)\cdot K_1(m\cdot r_2) + I_1(m r_2)\cdot K_0(m r)}{I_0(m\cdot r_1)\cdot K_1(m\cdot r_2) + I_1(m r_2)\cdot K_0(m r_1)}$$ $\left(m^2 = \frac{2\alpha}{\lambda d}\right)$



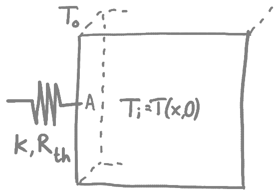
(I_n, K_n: Bessel functions)

Instationary heat exchange

$$Bi = \frac{\alpha \cdot L}{\lambda} \left(= \frac{T_i - T_0}{T_0 - T_\infty} \text{ at } t=0 \right) \qquad (L \Rightarrow D \text{ for cylinder or sphere})$$

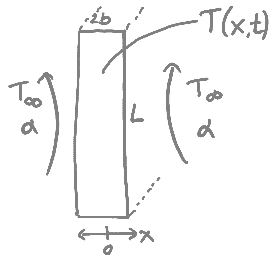
- $Bi \ll 1 \rightarrow T(x,t) = T(t)$:

  - $T_\infty(t) = Const.$:
    $$T(t) = e^{-t/\tau} \cdot (T_i - T_\infty) + T_\infty \qquad \left(\tau = \frac{M \cdot c}{k \cdot A} = M \cdot c \cdot R_{th} \right)$$
    $$Q_{TOT} = M \cdot c \cdot (T_i - T_\infty)$$



  - $T_\infty(t) = C \cdot t + B$:
    $$T_\infty(t) - T(t) \overset{t\to\infty}{=} C \cdot \tau$$

- $Bi \nless 1,\ L \gg b$:



$$\theta(x,t) = e^{-a \cdot \beta^2 \cdot t} \left(A \cdot \sin(\beta x) + B \cdot \cos(\beta x) \right) \qquad \left(a = \frac{\lambda}{\rho \cdot c} \right)$$

↳ A, B, β from boundary conditions. e.g.:

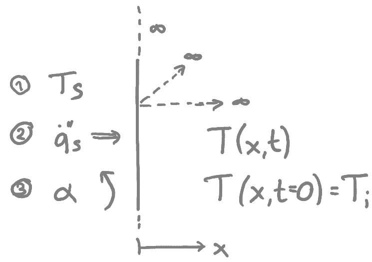
$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = 0 \,, \quad -\lambda \left.\frac{\partial \theta}{\partial x}\right|_{x=b} = \alpha \cdot \theta|_{x=b} \,, \quad \theta_0 = T(x,t=0) - T_\infty$$

$$\theta(x,t) = T(x,t) - T_\infty = \sum_{n=1}^{\infty} B_n \cdot \cos(\beta_n \cdot x)\, e^{-a \beta_n^2 t} \qquad \left(\beta_n : n\text{-th solution of } \beta = \frac{\alpha}{\lambda} \cdot \cot(\beta b) \right)$$

- $\theta_0 = Const.$: $\quad B_n = \theta_0 \cdot \sin(\beta_n b) \cdot \left(\frac{b \cdot \beta_n}{2} + \frac{\sin(2\beta_n b)}{4} \right)^{-1}$
- $\theta_0 = \theta_0(x)$: $\quad B_n = \int_0^b \theta_0(x) \cos(\beta_n x)\, dx \cdot \left(\frac{b \cdot \beta_n}{2} + \frac{\sin(2\beta_n b)}{4} \right)^{-1}$

$$\dot{q}''(t)|_{x=b} = \lambda \cdot \sum_{n=1}^{\infty} B_n \cdot \beta_n \sin(\beta_n b) \cdot e^{-a \beta_n^2 t}$$
$$q''|_{\substack{x=b \\ t\to\infty}} = \int_0^\infty \dot{q}''(t)|_{x=b}\, dt = \lambda \cdot \sum_{n=1}^{\infty} B_n \cdot \sin(\beta_n b) \frac{1}{\beta_n \cdot a}$$

- $Bi \nless 1 \,;\ b \rightarrow \infty$



$$\eta = \frac{x}{\sqrt{4 \cdot a \cdot t}} \rightarrow \frac{d^2 T}{d\eta^2} = -2\eta \cdot \frac{dT}{d\eta}$$

  - $T(x=0, t>0) = T_s$:
    $$T(\eta) = \frac{2(T_i - T_s)}{\sqrt{\pi}} \cdot \int_0^\eta e^{-u^2} du + T_s = (T_i - T_s) \cdot erf(\eta) + T_s$$

    Diffusion length: $\eta = 0{,}5 \rightarrow x_D = \sqrt{a \cdot t_D}$ $\quad (erf(0{,}5) = 0{,}5205)$

    $$\dot{q}''_s(t) = \frac{\lambda (T_s - T_i)}{\sqrt{\pi \cdot a \cdot t}}$$

- $\dot{q}''_s(t) = \dot{q}''_0 = \text{Const.}$

$$T(x,t) = \frac{\dot{q}''_0}{\lambda}\left(\sqrt{\frac{4\cdot a\cdot t}{\pi}}\cdot e^{\frac{-x^2}{4\cdot a\cdot t}} - x\left(1-\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right)\right)\right) + T_i$$

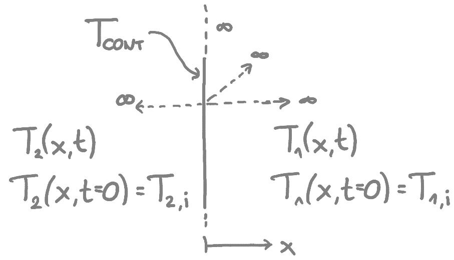
- Convection : T_∞, α

$$T(x,t) = \left[\left(1-\mathrm{erf}\left(\frac{x}{\sqrt{4at}}\right)\right) - \exp\left(\frac{\alpha\cdot x}{\lambda} + \frac{\alpha^2\cdot a\cdot t}{\lambda^2}\right)\cdot\left(1-\mathrm{erf}\left(\frac{x}{\sqrt{4at}} + \frac{\alpha\sqrt{at}}{\lambda}\right)\right)\right]\cdot(T_\infty - T_i) + T_i$$

- $Bi \nless 1$; $2\times b \to \infty$

$$T_{CONT} = \frac{\sqrt{\lambda_1\varrho_1 c_1}\,T_{1,i} + \sqrt{\lambda_2\varrho_2 c_2}\,T_{2,i}}{\sqrt{\lambda_1\varrho_1 c_1} + \sqrt{\lambda_2\varrho_2 c_2}}$$

($T_{1,2}(x,t)$ from above with $T_s = T_{CONT}$)



Convection

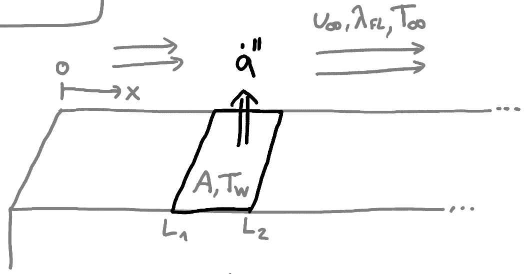
General : $Nu_x = \frac{\alpha(x)\cdot x}{\lambda_{FL}} \Rightarrow \alpha(x) = Nu_x \cdot \frac{\lambda_{FL}}{x} \Rightarrow \bar{\alpha} = \frac{1}{\Delta L}\int_{L_1}^{L_2}\alpha(x)dx$

Laminar Wall $Pr > 0{,}6$: $Nu_x = 0{,}332\cdot Re_x^{1/2}\cdot Pr^{1/3}$

$Pr < 0{,}6$: $Nu_x = 0{,}565\cdot Re_x^{1/2}\cdot Pr^{1/2}$

Turbulent Wall : $Nu_x = 0{,}0296\, Re_x^{4/5}\cdot Pr^{1/3}$

$\left(Pr = \frac{c_p\cdot\mu}{\lambda_{FL}} \qquad Re_x = \frac{\varrho\cdot U_\infty\cdot x}{\mu} = \frac{U_\infty\cdot x}{\nu}\right)$



$$\dot{q}'' = \bar{\alpha}\cdot(T_W - T_\infty)$$