Potentialströmungen

Helmholtz-Zerlegung (Zerlegt $\underline{u}$ in "Quellenanteil" und "Wirbelanteil")

$\underline{u} = \nabla\Phi + \underline{v}$

Φ: Potential ; $rot(\nabla\Phi)=0 \Rightarrow div(\underline{u}) = div(\nabla\Phi) = \Delta\Phi$

$\underline{v}$: Rotation ; $div(\underline{v})=0 \Rightarrow rot(\underline{u}) = rot(\underline{v}) = \underline{\omega}$

Drehungsfrei + Inkompressibel

$\nabla\times\underline{u}=0$ $(\rightarrow \underline{u}=\nabla\Phi)$ $\quad \nabla\underline{u}=0 \quad \Rightarrow \nabla(\nabla\Phi) = \Delta\Phi = 0$ (Laplace Gl.)

Bernoulli für Potentialstr. ($\underline{\omega}=\nabla\times\underline{u}=0$, $\frac{d\rho}{d[\cdot]}=0$)

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}\underbrace{\nabla\Phi\cdot\nabla\Phi}_{\underline{u}\cdot\underline{u}} + \frac{p}{\rho} + U = \text{Konst.}$$

(Schwerkraft: $U = g\cdot z$
Fliehkraft: $U = -\frac{\omega^2}{2}\cdot r^2$)

+ Stationär $\Rightarrow p(\underline{x}) = p_0 + \frac{\rho}{2}(\underline{u}_0^2 - \underline{u}(\underline{x})^2) + \rho(U_0 + U(\underline{x}))$; $c_p(\underline{x}) = \frac{p(\underline{x})-p_\infty}{\rho/2\cdot u_\infty^2} = 1 - \left(\frac{\underline{u}(\underline{x})}{\underline{u}_\infty}\right)^2$

Stromfunktion Ψ (2D, $u=\nabla\Phi$, $\frac{\partial\rho}{\partial[\cdot]}=0$)

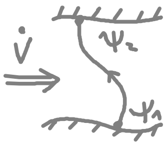
$$\underline{u} = \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix} = \nabla\times\begin{pmatrix} 0 \\ 0 \\ \Psi \end{pmatrix} = \begin{pmatrix} \partial\Psi/\partial y \\ -\partial\Psi/\partial x \\ 0 \end{pmatrix}$$

(Ψ: Stromfunktion)

$$\underline{u} = \begin{pmatrix} u_r \\ 0 \\ u_z \end{pmatrix} = \begin{pmatrix} -1/r\cdot\partial\Psi/\partial z \\ 0 \\ 1/r\cdot\partial\Psi/\partial r \end{pmatrix}$$

↳ Stromlinie: $(\nabla\Psi)\cdot\underline{u} = 0 \Rightarrow \nabla\Psi \perp \underline{u} \Rightarrow \Psi = $ Konst auf Stromlinie

↳ Volumenstr: $\dot{V}/dz = \int_{\Psi_1}^{\Psi_2} d\Psi = \Psi_2 - \Psi_1$



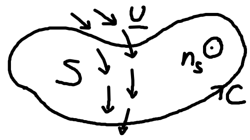
Strom- und Potentiallinien

Stromlinie: $d\Psi = \partial\Psi/\partial x\cdot dx + \partial\Psi/\partial y\cdot dy = -u_y\,dx + u_x\,dy \doteq 0$

Pot. linie: $d\Phi = \partial\Phi/\partial x\cdot dx + \partial\Phi/\partial y\cdot dy = u_x\,dx + u_y\,dy \doteq 0$

$\Rightarrow \Phi \perp \Psi$

Zirkulation / Quellenstärke



$$\Gamma_c = \oint_c \underline{u}\cdot d\underline{x} = \int_S rot(\underline{u})\cdot \underline{n}_s\, dS$$
$$= \sum \Gamma_{ins}$$

$$Q_c = \oint_c \underline{u}\cdot \underline{n}_s\, ds = \int_S div(\underline{u})\, dS$$
$$= \sum Q_{ins}$$

Komplexe Darstellung (2D !)

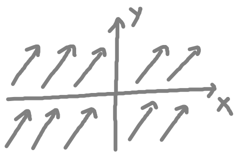
$F(z) = \Phi(x,y) + i\cdot\Psi(x,y)$ (komplexes Potential ; $z = x + iy = r\cdot e^{i\varphi}$)

$w(z) = {}^{dF(z)}/_{dz} = u_x - i\cdot u_y = (u_r - iu_\theta)e^{-i\theta}$ (komplexe Geschw.)

$"\Gamma_c" = \oint_c w\, dz = \Gamma_c + i\cdot Q_c$ (komplexe Zirkulation ; Q_c = "Quellenstärke")

Elementarlösungen

• **Parallelstr.:** $F(z) = (u_{x\infty} - i u_{y\infty})\cdot z$



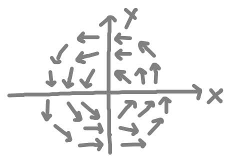
↳ $\Phi = u_{x\infty} x + u_{y\infty} y$

↳ $\Psi = u_{x\infty} y - u_{y\infty} x$

$w(z) = u_{x\infty} - i u_{y\infty}$

$u_x = u_{x\infty}$; $u_y = u_{y\infty}$

• **Pot. Wirbel:** $F(z) = -{}^{i\Gamma}/_{2\pi}\cdot \ln(z)$



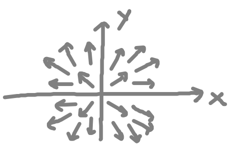
↳ $\Phi = {}^{\Gamma}/_{2\pi}\cdot\theta$

↳ $\Psi = -{}^{\Gamma}/_{2\pi}\cdot \ln(r)$

$w(z) = -{}^{i\Gamma}/_{2\pi z}$

$u_r = 0$; $u_\theta = {}^{\Gamma}/_{2\pi r}$

• **Quellen/Senken:** $F(z) = {}^{Q}/_{2\pi}\cdot \ln(z)$



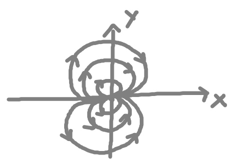
↳ $\Phi = {}^{Q}/_{2\pi} \ln(r)$

↳ $\Psi = {}^{Q}/_{2\pi}\cdot\theta$

$w(z) = {}^{Q}/_{2\pi z}$

$u_r = {}^{Q}/_{2\pi r}$; $u_\theta = 0$

• **Dipol:** $F(z) = {}^{m}/_{z}$



↳ $\Phi = {}^{mx}/_{x^2+y^2}$

↳ $\Psi = {}^{-my}/_{x^2+y^2}$

$w(z) = -{}^{m}/_{z^2}$

- **Keilströmung:** $F(z) = C \cdot z^n$ $\quad (C, n \in \mathbb{R}\ ;\ C > 0\ \ n > 1/2 \quad \Rightarrow \alpha = \pi/n)$

$\hookrightarrow \Phi = C r^n \cos(n\theta)$

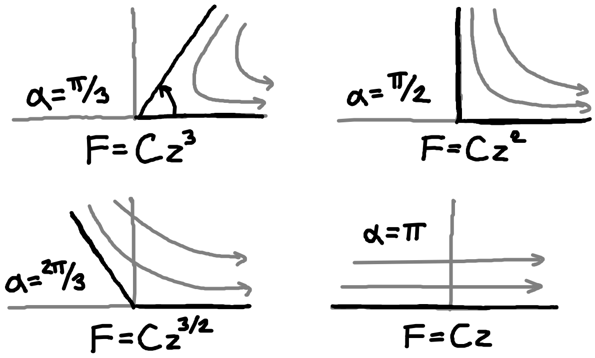
$\hookrightarrow \Psi = C r^n \sin(n\theta)$

$w(z) = C n z^{n-1} = C n r^{n-1} e^{in\theta} e^{-i\theta}$

$\hookrightarrow u_r = C n r^{n-1} \cos(n\theta)$

$\hookrightarrow u_\theta = -C n r^{n-1} \sin(n\theta)$

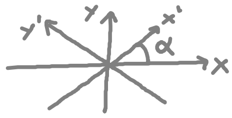
$|u| = |w| = C n r^{n-1}$



Transformation Elementarlsg.

- **Translation:** $F_t(z) = F(z - (a + ib))$

- **Rotation:** $F_r(z) = F(z \cdot e^{-i\alpha})$



Wandbedingung / Spiegelung

Überlagerungen

- **Parallelstr. + Dipol:** (Kreis)

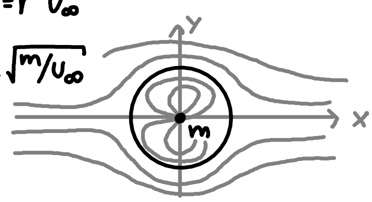
$F(z) = U_\infty z + \frac{m}{z}$

$w(z) = U_\infty + \frac{m}{z^2}$

$p_w(\theta) = p_\infty + \frac{\rho}{2} U_\infty^2 (1 - 4\sin^2(\theta))$

$m = r^2 U_\infty$

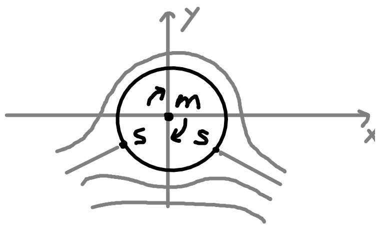
$r = \sqrt{m/U_\infty}$



- **Kreis. + Wirbel:** (Kreis dreht)

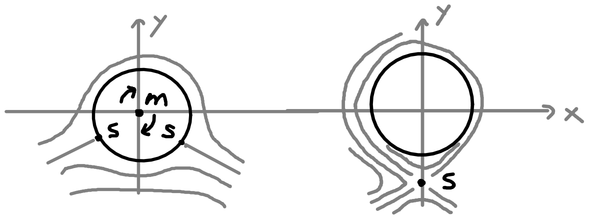
$F(z) = U_\infty \left(z + \frac{r_0^2}{z}\right) - \frac{i\Gamma}{2\pi} \ln(z)$

$w(z) = U_\infty \left(1 - \frac{r_0^2}{z^2}\right) - i\Gamma/2\pi z$



Auftrieb (Strömung → Druck → Auftrieb)

Bsp.: $F(z) = U_\infty\left(z + \frac{r_0^2}{z}\right) - \frac{i\Gamma}{2\pi}\ln(z)$



Druckbeiwert: $c_p = \frac{p_w - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - \frac{|U|^2}{U_\infty^2}$

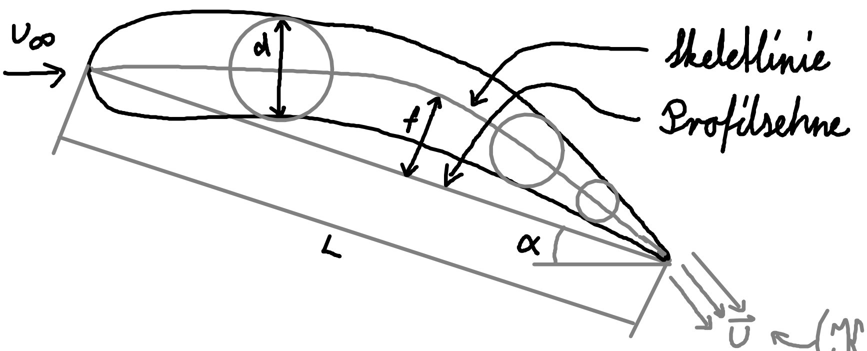
Druckverteil.: $p_w = p_\infty + \frac{\rho}{2}\left(U_\infty^2 - |U|^2\right)$

Auftrieb: $\frac{F}{dz} = -\oint_c p_w \underline{n}\, ds$

$F(z) \to \Phi(\underline{x}) \to \Phi(r,\theta) \to \overset{\text{polar!}}{U = \nabla\Phi} \to U_\theta(\theta)\big|_{r=r_0} \to |U(\theta)|^2 \to p(\theta) \to F$ $\left(\underline{n} = \binom{\cos\theta}{\sin\theta}\right)$

$\Rightarrow F_y/dz = -\Gamma \rho U_\infty \qquad F_x/dz = 0$

Ebene Profilumströmung



← (Kutta-Bedingung)

$c_W = F_x / bL\frac{\rho}{2}U_\infty^2$

$c_a = F_y / bL\frac{\rho}{2}U_\infty^2$

$\frac{c_a}{c_W} = L_{HOR}/L_{VERT}$ = Gleitzahl

$C_A \approx 2\pi\left(\alpha + 2\, f_{MAX}/L\right)$ $\left(\alpha \leq \alpha_{Abriss}\ ;\ f_{MAX}/L \ll 1\right)$

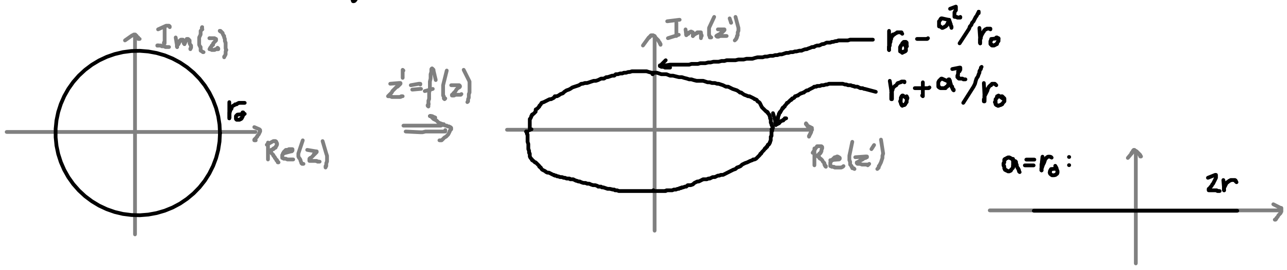
- Singularitätenverfahren (Panel-Verfahren)
  Überlagerung Quellen+Senken (=Dipol) + Wirbel auf Skelettlinie
- Numerisches Lösen der Laplace-Gl. ($\Delta\Phi = 0$ mit $\nabla\Phi \cdot \vec{n} = \vec{n}\cdot\vec{U}_{rand}$)

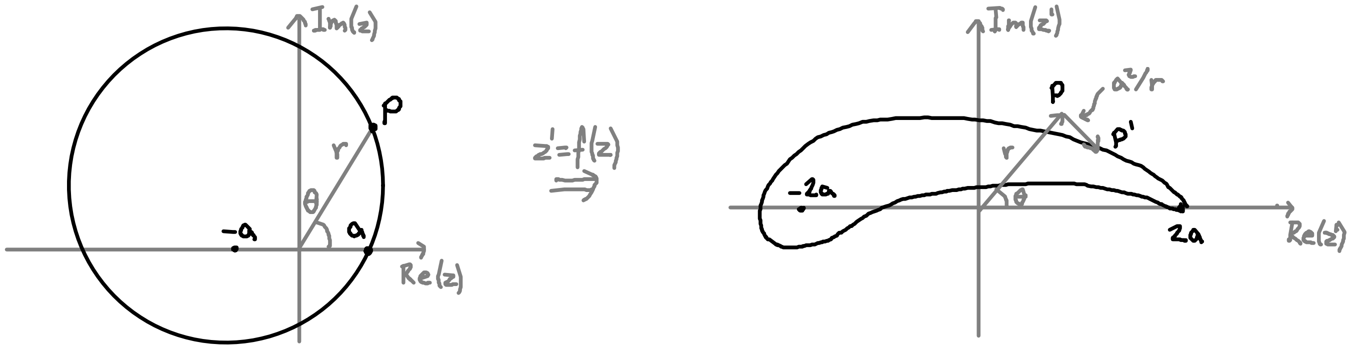
- Methode der konformen Abbildungen ($f: \mathbb{C}^2 \to \mathbb{C}^2$: reguläre funk.)

$$\underbrace{z' = f(z)}_{\text{wo } \frac{df}{dz} \neq \{0,\pm\infty\}} \Rightarrow \Phi'(z') = \Phi(f^{-1}(z')) \qquad \Rightarrow \Gamma' = \Gamma \;,\; Q' = Q \;,\; \dots$$

$$w'(z') = w(f^{-1}(z')) \cdot 1 \Big/ \frac{df(z)}{dz}\Big|_{z=f^{-1}(z')} \qquad \left(df^{-1}(z') / dz' \right)$$

Joukowski-Abbildung: $f_J(z) = z + \frac{a^2}{z}$ $\quad (0 < a \in \mathbb{R}) \quad \left(f_J(z) \xrightarrow{|z|\to\infty} z \right)$



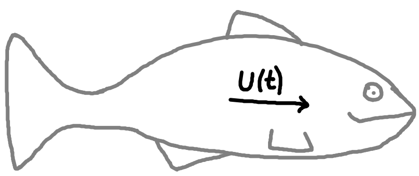


Instationäre Potentialströmung

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + \frac{p}{\rho} = C(t) \qquad \text{(instat. Bernoulli)} \qquad \left(\Delta\Phi = 0 \text{ und } (\vec{n}\cdot\nabla\Phi)\big|_R \stackrel{!}{=} \vec{n}\cdot\vec{U}_R(t) \right)$$

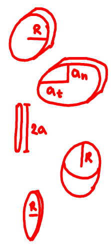
1) $F(z,t) \simeq F(z - z_0(t))$
2) $\Phi(\underline{x},t) = \mathrm{Re}(F(z,t))$
3) instat. Bernoulli: $p(\underline{x},t) = \dots$
4) ersetz $x^* = x + x_0(t)$ $\; y^* = y + y_0(t)$
5) $\oint_C p(\underline{x},t)\,ds = F_{\text{FORCE}}$ (siehe Auftrieb)

$z_0(t) = x_0(t) + i y_0(t)$
$\dot{x}_0(t) = U_x(t)$; $\dot{y}_0(t) = U_y(t)$
Achtung dipol! $m(t) = -r^2 \cdot U(t)$



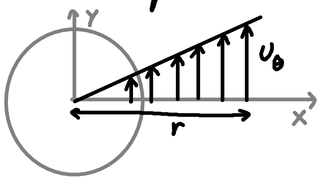
$$F_{\text{FORCE}} = -(m_K + m^*)\, dU/dt \qquad m^* = f\, V_K\, \rho_F$$
$$m_K = V_K\, \rho_K$$

Kreis:	$f = 1$	$m^* = \pi R^2 b \rho_F$
Ellyps:	$f = a_n/a_t$	$m^* = \pi a_n^2 b \rho_F$
Platte:	$f = ?$	$m^* = \pi a^2 b \rho_F$
Kugel:	$f = 1/2$	$m^* = 2/3 \pi R^3 \rho_F$
Kreisch.:	$f = ?$	$m^* = 8/3 R^3 \rho_F$



Drehungsbehaftete Strömungen

Starrkörper Wirbel

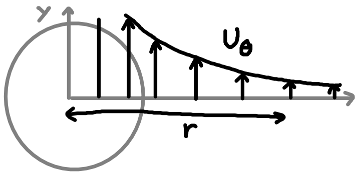


$$\Omega = u_\theta / r = 2\pi / T$$

$$\underline{u}(\underline{x}) = \begin{pmatrix} -y\Omega \\ x\Omega \\ 0 \end{pmatrix} \qquad \underline{u}_p = \begin{pmatrix} 0 \\ \Omega r \\ 0 \end{pmatrix}_p$$

$$\underline{\omega} = \mathrm{rot}\, \underline{u} = \begin{pmatrix} 0 \\ 0 \\ 2\Omega \end{pmatrix}$$

Potential Wirbel

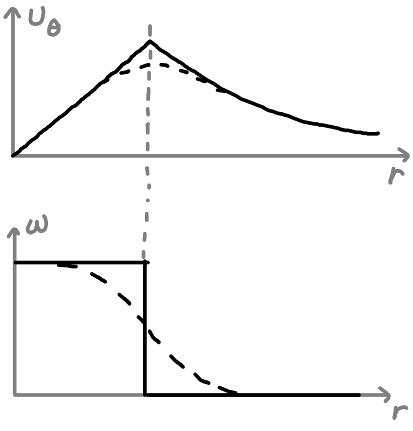


$$\Gamma = 2\pi c$$

$$\underline{u}(\underline{x}) = c \begin{pmatrix} -y/r^2 \\ x/r^2 \\ 0 \end{pmatrix} \qquad \underline{u}_p = \begin{pmatrix} 0 \\ c/r \\ 0 \end{pmatrix}_p$$

$$\underline{\omega} = \mathrm{rot}\, \underline{u} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (\underline{\omega}(\underline{0}) = \infty)$$

Rankine Wirbel



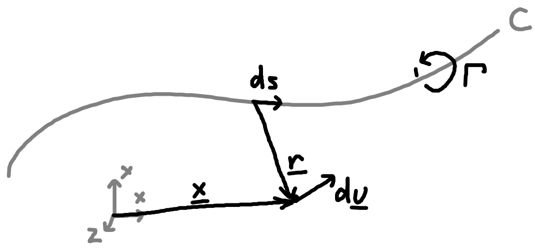
Wirbellinie, Wirbelröhre, Wirbelfaden

$\mathrm{div}\, \omega = 0$		$\mathrm{div}\, \underline{u} = 0$ (inkompressibel.)
Galilei invariant	$\omega = \mathrm{rot}(\underline{u}) = \mathrm{rot}(\underline{u} + \underline{\mathrm{const.}})$	nicht Galilei invariant
Wirbellinien	$dx/\omega_x = dy/\omega_y = dz/\omega_z$	Stromlinien
Wirbelröhren	röhre aus Wirbellinie	Stromröhre
Wirbelfaden	∞ dunne Wirbelröhre	Stromfaden
Zirkulation	$\Gamma = \int_S \underline{\omega} \cdot \underline{n}\, dS =$ Wirb. fluss	Volumenstrom $\dot{V}$

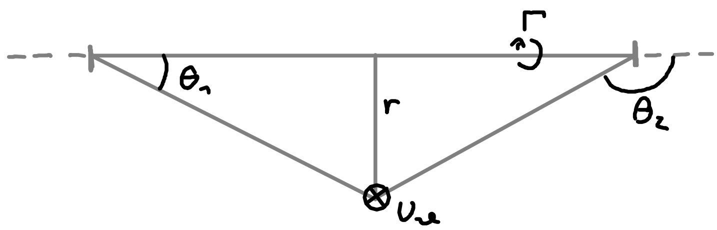
Gesetz von Biot-Savart

$$\underline{u}(\underline{x}) = \frac{\Gamma}{4\pi} \int_C \frac{d\underline{s} \times \underline{r}}{|\underline{r}|^3} \qquad \left(u_\theta = \Gamma / 2\pi r\right)$$

$$\underline{u}(\underline{x}) = \frac{1}{4\pi} \int_V \frac{\omega \times r}{|r|^3}\, dV \qquad \left(\begin{array}{l} \underline{r} = \underline{x} - \underline{r}_\omega \\ dV = dr_{\omega_1} dr_{\omega_2} dr_{\omega_3} \end{array}\right)$$



$$u_\vartheta = \frac{\Gamma}{4\pi r} \left(\cos(\theta_1) - \cos(\theta_2)\right)$$



Wirbeltransport Gl.

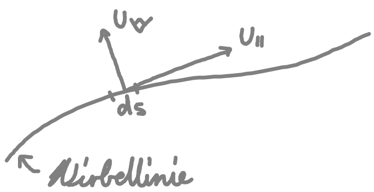
(Bedingung: $\nabla \underline{u} = 0$; $f = -\nabla U$; $\rho = \rho(p)$)

$$\frac{D\omega}{Dt} = \frac{\partial \underline{\omega}}{\partial t} + (\underline{u} \cdot \nabla)\underline{\omega} = \underbrace{(\underline{\omega} \cdot \nabla)\underline{u}}_{\text{Wirbelstreckung } \underline{W}} + \underbrace{\nu \Delta \underline{\omega}}_{\text{Diffusion}} \qquad (3D)$$

$$\frac{D\underline{\omega}}{Dt} = \frac{\partial \underline{\omega}}{\partial t} + (\underline{u}\,\nabla)\underline{\omega} = \nu \Delta \underline{\omega} \qquad (2D)$$

$$\frac{D}{Dt}\left(\frac{\underline{\omega}}{\rho}\right) = \left(\frac{\underline{\omega}}{\rho}\,\nabla\right)\underline{u} + \frac{1}{\rho^3}\left(\nabla\rho \times \nabla p\right)$$ ($\rho \neq \rho(p)$ baroklin ; $\nu = 0$ reibungsfrei)

Wirbelstreckung: $\underline{W} = (\underline{\omega}\,\nabla)\underline{u} = \underbrace{|\omega| \frac{\partial u_{\parallel}}{\partial s}}_{\text{Streckung, Stauchung}} + \underbrace{|\omega| \frac{\partial u_{\varphi}}{\partial s}}_{\text{Kippung}}$



$$\underline{W} = (\underline{\omega} + 2\underline{\Omega}) \cdot \nabla \underline{u} \qquad (\underline{\Omega}: \text{rotierendes Bezugssystem})$$

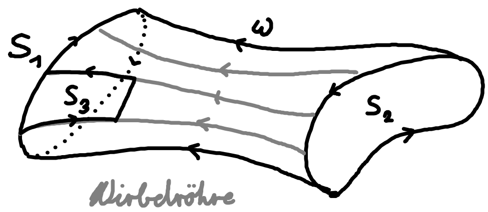
Barotrop: $\rho = \rho(p) \leftrightarrow \mathrm{rot}\left(\frac{1}{\rho}\nabla p\right) = 0 \leftrightarrow \nabla\rho \times \nabla p = 0 \leftrightarrow P(\underline{x}) = \int_{p_0}^{p(x)} 1/\rho(p)\, dp$

$\hookrightarrow \nabla P = \frac{1}{\rho}\nabla p$

Helmholtzsche Wirbelsätze

① Wirbelfrei Bereich bleibt Wirbelfrei
② Element auf Wirbellinie bleibt drauf.

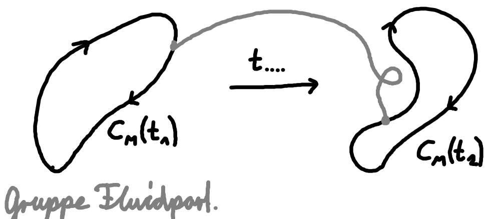
③ Γ konst. über Wirbelröhre/faden



$\Gamma_1 = \Gamma_2$
$(\Gamma_3 = 0)$

$$\begin{cases} \Gamma_1 = \oint_{S_1} \vec{u}\, d\vec{x} = \int_{S_1} \vec{\omega}\vec{n}\, ds \\ \Gamma_2 = \oint_{S_2} \vec{u}\, d\vec{x} = \int_{S_2} \vec{\omega}\vec{n}\, ds \end{cases}$$

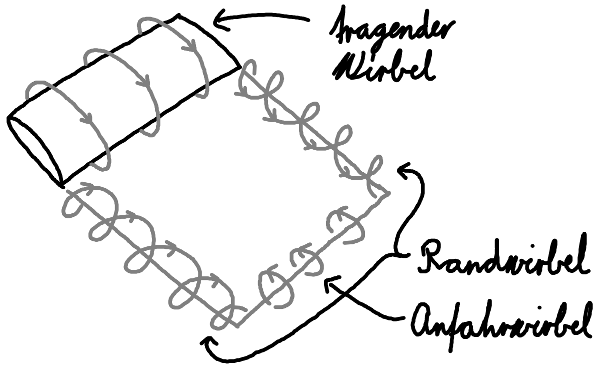
Kelvinscher Wirbelsatz



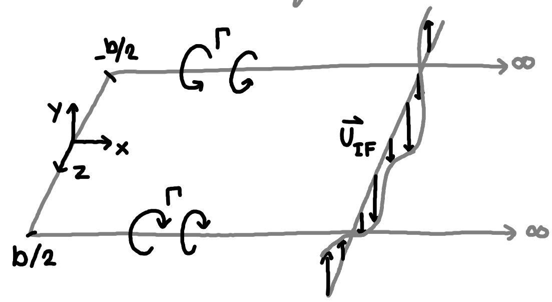
$$\frac{d\Gamma_M}{dt} = 0 \qquad (\text{für reib.frei, barotrop, konservativ } F)$$

$$\left(\Gamma_M(t) = \oint_{C_M(t)} \vec{u}\, d\vec{x} = \int_{S(t)} \mathrm{rot}(\underline{u})\, \underline{n}\, ds\right)$$

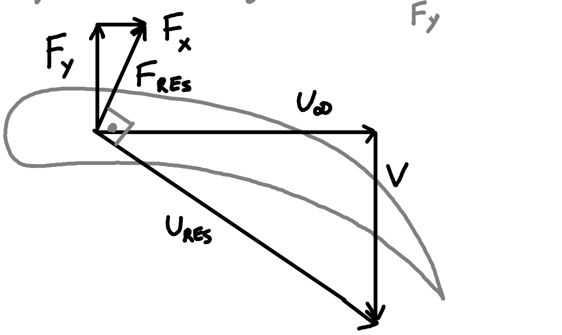
3D Strömung am endlichen Tragflügel



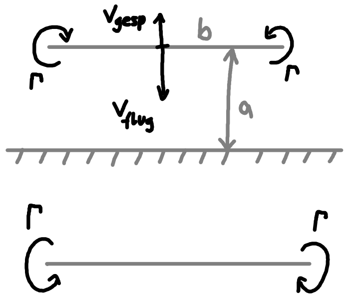
Randwirbel ∞ lang ⇒ $\vec{U}_{IF} = \frac{\Gamma}{\pi} \frac{b}{b^2-4z^2} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$



Kutta-Joukowski Flügel ⇒ $\Gamma = \underbrace{mg}_{F_y} / b \varrho U_\infty$



Bodeneffekt an Flügel (∞-lang Randwirbel)



$V = V_{flug} + V_{gesp}$

$V_{flug} = -\Gamma / \pi b$

$V_{gesp} = \Gamma b / \pi (16a^2 - b^2)$

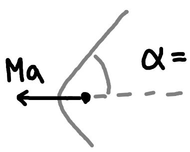
Baroklines Drehmoment $\frac{1}{\varrho^3}(\nabla \varrho \times \nabla p)$

Kompressible Strömungen (ideal gas; $c_p, c_v = \text{const.}$; quasi-1D; stationär; $v=0$; $f=0$; adiabat)

Kontinuitätsgl.: $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \underline{u} \cdot \text{grad}(\rho) = -\rho \, \text{div}(\underline{u})$

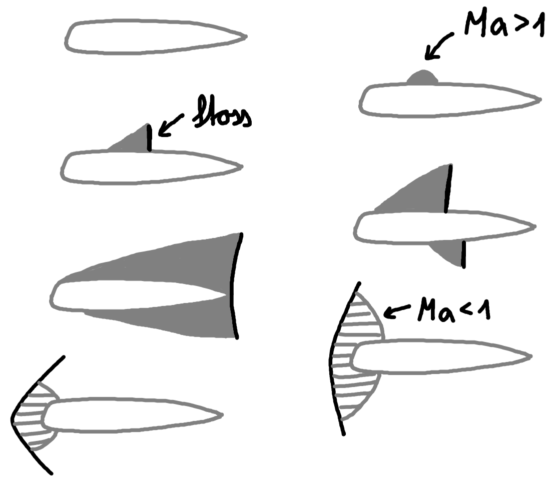
Machzahl: $Ma = \frac{u}{a} = \frac{\text{stromgeschw.}}{\text{schallgeschw.}}$ $\left(a = \sqrt{\gamma R T} = \sqrt{\gamma \, p/\rho} = \sqrt{h(1-\gamma)}\right)$

$Ma < 0.3$ → inkompressibel
$Ma < 1$ → unterschall
$Ma \approx 1$ → transsonisch
$Ma > 1$ → überschall
$Ma \geq 5$ → hyperschall



$\alpha = \sin^{-1}(1/Ma)$

$Ma = 0.6$
$Ma = 0.7$
$Ma = 0.75$
$Ma = 0.85$
$Ma = 0.95$
$Ma = 1.05$
$Ma = 1.3$



Thermo. Beziehungen für ideales Gas ($pv = RT \Leftrightarrow p = \rho R T$; $v = \frac{1}{\rho} = \frac{V}{m}$)

Spez. Wärme Kap. $c_p = \frac{\gamma}{\gamma - 1} R$; $c_v = \frac{1}{\gamma - 1} R$
Spez. Gaskonst. $R = c_p - c_v$ $\left(= \text{"}R\text{"}/M = \frac{8.314}{\text{kg/mol}}\right)$
Adiabat exp. $\gamma = c_p / c_v$
innere Energie $e = c_v \cdot T$ $(de = c_v dT)$
Enthalpie $h = c_p T = e + p/\rho = (\gamma/\gamma-1) \cdot p/\rho = (\gamma/\gamma-1) \cdot RT = a^2/\gamma-1$ $(dh = c_p dT)$
Entropie $s_2 - s_1 = c_p \cdot \ln(T_2/T_1) - R \cdot \ln(p_2/p_1)$ $(Tds = dh - vdp)$
$= c_v \cdot \ln\left((p_2/p_1) \cdot (\rho_1/\rho_2)^\gamma\right)$

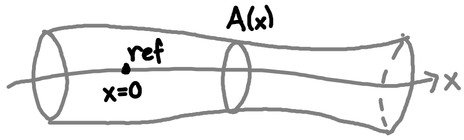
Isentropenbeziehungen (isentrop: $\Delta s = 0$)

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} \rightarrow \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}$$

Thermo Hauptsatz

$de = dq - pdv$ $\left(v = \frac{1}{\rho}\right)$
$dh = dq + dp/\rho$

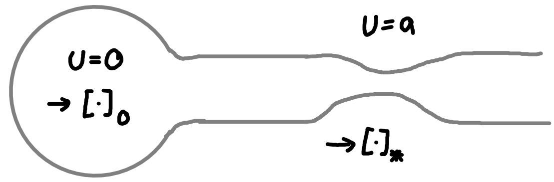
Grundgleichungen (für 1D Stromfaden)



- Konti.: $\dot{m} = \rho u A = \text{const.}$
- Impuls: $u \frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$, A: const ⇒ $\rho u^2 + p = \text{const.}$
- Energie: $h + u^2/2 = \text{const.}$
- Zustand: $p = \rho R T$

- Bernoulli: $\frac{1}{2}u(x)^2 - \frac{1}{2}u_{ref}^2 = \frac{\gamma}{\gamma-1}\frac{p_{ref}}{\rho_{ref}}\left(1 - \left(\frac{p(x)}{p_{ref}}\right)^{\gamma-1/\gamma}\right) = h_{ref} - h(x)$ (isentrop!)

Strömungsgrössen (1D-Stromfaden, isentrop)



Mach-Zahl: $Ma(x) = u(x)/a(x)$

Laval-Zahl: $La(x) = u(x)/a_*$

Ausflussformel: $u(x) = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_0}{\rho_0}\left(1 - \frac{\rho(x)}{\rho_0}\right)^{\gamma-1}} = \sqrt{\frac{2\gamma}{\gamma-1}RT_0\left(1 - \frac{T(x)}{T_0}\right)}$ → $\begin{cases} u_{MAX} = \sqrt{2/\gamma-1}\cdot a_0 \\ La_{MAX} = \frac{u_{MAX}}{a_*} = \sqrt{\frac{\gamma+1}{\gamma-1}} \\ Ma_{MAX} = \frac{u_{MAX}}{a_{MAX}} \to \infty \end{cases}$

Energie-Ellipse: $(u/u_{MAX})^2 + (a/a_0)^2 = 1$

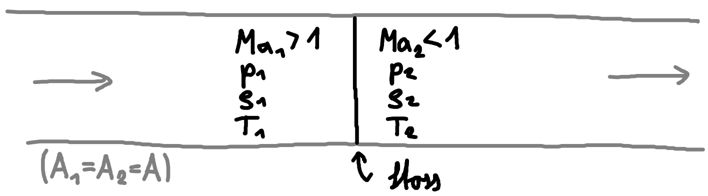
Inkomp. Nährung: $\frac{\rho_0 - \rho}{\rho_0} \approx \frac{1}{2}Ma^2$ (für $Ma \leq 0{,}3$)

Bez. Ma ↔ La: $La = \sqrt{\gamma+1/\frac{2}{Ma^2}+(\gamma-1)}$; $Ma = \sqrt{2/\frac{\gamma+1}{La^2}-(\gamma-1)}$

$\frac{T_0}{T} = \left(1 + \frac{\gamma-1}{2}Ma^2\right)$	$\frac{a_0}{a} = \left(1 + \frac{\gamma-1}{2}Ma^2\right)^{1/2}$	$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}Ma^2\right)^{\gamma/\gamma-1}$	$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}Ma^2\right)^{1/\gamma-1}$
$\frac{T}{T_0} = \left(1 - \frac{\gamma-1}{\gamma+1}La^2\right)$	$\frac{a}{a_0} = \left(1 - \frac{\gamma-1}{\gamma+1}La^2\right)^{1/2}$	$\frac{p}{p_0} = \left(1 - \frac{\gamma-1}{\gamma+1}La^2\right)^{\gamma/\gamma-1}$	$\frac{\rho}{\rho_0} = \left(1 - \frac{\gamma-1}{\gamma+1}La^2\right)^{1/\gamma-1}$
$\frac{T}{T_0} = \left(1 - \frac{u^2}{u_{MAX}^2}\right)$	$\frac{a}{a_0} = \left(1 - \frac{u^2}{u_{MAX}^2}\right)^{1/2}$	$\frac{p}{p_0} = \left(1 - \frac{u^2}{u_{MAX}^2}\right)^{\gamma/\gamma-1}$	$\frac{\rho}{\rho_0} = \left(1 - \frac{u^2}{u_{MAX}^2}\right)^{1/\gamma-1}$
$\frac{T_*}{T_0} = \left(\frac{2}{\gamma+1}\right)$	$\frac{a_*}{a_0} = \left(\frac{2}{\gamma+1}\right)^{1/2}$	$\frac{p_*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}$	$\frac{\rho_*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{1/\gamma-1}$

Senkrechter Verdichtungsstoss

(1D-Stromfaden, NICHT isentrop)



Konti: $\rho_1 u_1 = \rho_2 u_2$

Impuls: $\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$

Energie: $h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2$

Str. gr.: $T_{1*} = T_{2*}$ $\quad a_{1*} = a_{2*}$

$T_{01} = T_{02}$ $\quad a_{01} = a_{02}$

$\rho_{01} > \rho_{02}$ $\quad p_{01} > p_{02}$

$T_1 < T_2$ $\quad u_1 > u_2$

$\rho_1 < \rho_2$ $\quad p_1 < p_2$

$a_1 < a_2$

Geschw.: $u_1 \cdot u_2 = a_*^2$; [ ] ; $La_1 \cdot La_2 = 1$

Dichte: $\rho_2/\rho_1 = u_1/u_2 = La_1^2 = \frac{2}{\gamma+1} \cdot \frac{1}{Ma_1^2} + \frac{\gamma-1}{\gamma+1}$

Druck: $p_2/p_1 = 1 + \frac{2\gamma}{\gamma+1}(Ma_1^2 - 1)$

Temp.: $T_2/T_1 = \frac{p_2 \cdot \rho_1}{p_1 \cdot \rho_2} = \left(\frac{2\gamma}{\gamma+1} \cdot Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right) \cdot \left(\frac{2}{\gamma+1} \cdot \frac{1}{Ma_1^2} + \frac{\gamma-1}{\gamma+1}\right)$

Entropie: $s_2 - s_1 = c_v \cdot \ln\left(\left(\frac{2\gamma}{\gamma+1} \cdot Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right) \cdot \left(\frac{2}{\gamma+1} \cdot \frac{1}{Ma_1^2} + \frac{\gamma-1}{\gamma+1}\right)^{\gamma}\right)$

Mach: $Ma_2 = \sqrt{\left(\frac{u_2}{u_1}\right)^2 \cdot \left(\frac{u_1}{a_1}\right)^2 \cdot \left(\frac{a_1}{a_2}\right)^2} = \sqrt{1 - \frac{Ma_1^2 - 1}{1 + 2\gamma/\gamma+1 (Ma_1^2 - 1)}}$

Ruhe: $\frac{\rho_{02}}{\rho_{01}} = \frac{p_{02}}{p_{01}} = \left(1 + \frac{2\gamma}{\gamma+1}(Ma_1^2 - 1)\right)^{-1/\gamma-1} \cdot \left(\frac{(\gamma+1) Ma_1^2}{2 + (\gamma-1) Ma_1^2}\right)^{\gamma/\gamma-1}$

Strömung bei veränderlichem Querschnitt (1D-Stromfaden, A=A(x))

Konti: $\dot{m} = \rho \cdot u \cdot A = \text{const.}$

Euler: $d\rho/\rho = -Ma^2 \cdot du/u$

$du/u = \frac{1}{Ma^2 - 1} \cdot dA/A$

Ma>1 : dA>1 (erweiterung) → du>0 (beschl.)

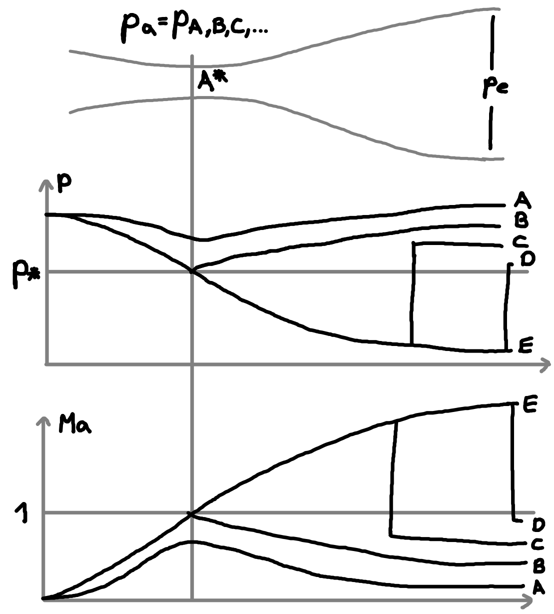
Ma<1 : dA<1 (verängung) → du>0 (beschl.)

Laval-Düse

$$\frac{A}{A^*} = \frac{1}{Ma} \cdot \left(1 + \frac{\gamma-1}{\gamma+1} \cdot (Ma^2 - 1)\right)^{\gamma+1/2(\gamma-1)}$$

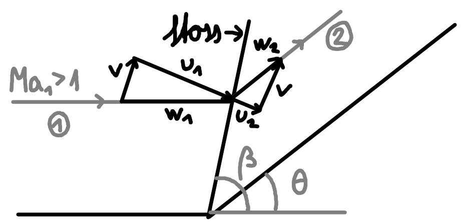
$$= \frac{1}{La}\left(1 - \frac{\gamma-1}{2}(La^2 - 1)\right)^{-1/\gamma-1}$$

$$= \frac{\rho_* \cdot a}{\rho \cdot u}$$



Schiefer Verdichtungsstoss

Konkave Rampe:



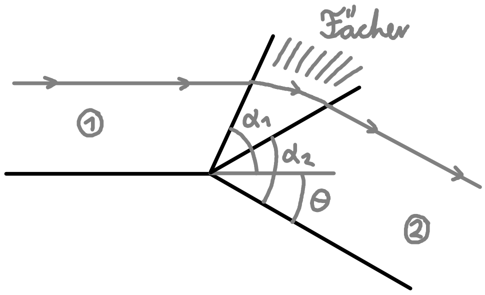
$$Ma_1 = \frac{w_1}{a_1} \quad ; \quad Ma_{n1} = \frac{u_1}{a_1} = Ma_1 \cdot \sin(\beta)$$

$$Ma_2 = \frac{w_2}{a_2} \quad ; \quad Ma_{n2} = \frac{u_2}{a_2} = Ma_2 \cdot \sin(\beta - \theta)$$

$\frac{u_2}{u_1}, \frac{\rho_2}{\rho_1}, \frac{p_2}{p_1}, \frac{T_2}{T_1} \hat{=}$ senkr. Stoss mit $Ma_1 \rightarrow Ma_{n1}$

$$\tan(\theta) = 2 \cdot \frac{Ma_1^2 \cdot \sin^2(\beta) - 1}{Ma_1^2(\gamma + \cos(2\beta)) + 2} \cdot \cot(\beta) \quad \left(\begin{array}{c}\theta \rightarrow \beta \\ \text{Tabelle!}\end{array}\right)$$

Konvexer Ecken:



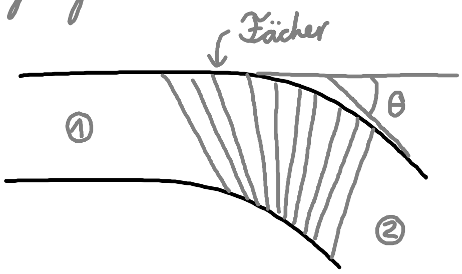
$$\sin(\alpha_1) = 1/Ma_1 \quad ; \quad \sin(\alpha_2) = 1/Ma_2$$

$\frac{u_2}{u_1}, \frac{\rho_2}{\rho_1}, \frac{p_2}{p_1}, \frac{T_2}{T_1} \hat{=}$ isentrope Strömung

$$\theta = \nu(Ma_2) - \nu(Ma_1) \quad \rightarrow \quad \nu_{MAX} = \nu(\infty) = \frac{\pi}{2}\left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right)$$

$$\nu(Ma) = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \arctan\sqrt{\frac{\gamma-1}{\gamma+1}(Ma^2 - 1)} - \arctan\sqrt{Ma^2 - 1}$$

Übergang:



Gleich wie konvexer Ecken!

$\frac{u_2}{u_1}, \frac{\rho_2}{\rho_1}, \frac{p_2}{p_1}, \frac{T_2}{T_1} \hat{=}$ isentrope Strömung

$$\theta = \nu(Ma_2) - \nu(Ma_1)$$

Anhang A

Grundlagen der Vektor- und Tensoralgebra

A.1 Einsteinsche Summenkonvention

Komponenten der Vektoren werden mit Indizes geschrieben, wobei gilt, daß über einen Index, der in einem Term zweimal vorkommt, summiert werden muss.
Beispiele:

- $u_{ii} = u_{11} + u_{22} + u_{33}$
- Laplace-Operator:

$$\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_i}a = \frac{\partial^2 a}{\partial x_1^2} + \frac{\partial^2 a}{\partial x_2^2} + \frac{\partial^2 a}{\partial x_3^2} = \Delta a$$

- Vektorprodukt:

$$\underline{u} \times \underline{v} = \epsilon_{ijk} u_j v_k$$

  wobei ϵ_{ijk} die folgenden Eigenschaften hat:

$$\epsilon_{ijk} = \begin{cases} 1 & \text{falls} \quad ijk = 123, 231 \text{ oder } 312 \\ 0 & \text{falls} \quad \text{zwei Indizes identisch sind} \\ -1 & \text{falls} \quad ijk = 321, 213 \text{ oder } 132 \end{cases}$$

- $\delta_{ii} = 3$, wobei gilt (Kronecker-δ)

$$\delta_{ij} = \begin{cases} 1 & \text{falls} \quad i = j \\ 0 & \text{falls} \quad i \neq j \end{cases}$$

A.2 Differentialoperatoren

Differential-Vektoroperator

In kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\underline{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

In Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\underline{\nabla} = \begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial x} \end{pmatrix}_p$$

Divergenz

$$\text{div } \underline{u} \equiv \underline{\nabla} \cdot \underline{u}$$

In kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\underline{\nabla} \cdot \underline{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

In Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\underline{\nabla} \cdot \underline{u} = \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_x}{\partial x}$$

Gradient

$$\text{grad } a \equiv \underline{\nabla}\ a$$

In kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\underline{\nabla}\ a = \begin{pmatrix} \frac{\partial a}{\partial x} \\ \frac{\partial a}{\partial y} \\ \frac{\partial a}{\partial z} \end{pmatrix}$$

In Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\underline{\nabla}\ a = \begin{pmatrix} \frac{\partial a}{\partial r} \\ \frac{1}{r}\frac{\partial a}{\partial \theta} \\ \frac{\partial a}{\partial x} \end{pmatrix}_p$$

Rotation

$$\text{rot } \underline{u} \equiv \underline{\nabla} \times \underline{u}$$

In kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\underline{\nabla} \times \underline{u} = \begin{pmatrix} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \end{pmatrix}$$

In Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\underline{\nabla} \times \underline{u} = \begin{pmatrix} \frac{1}{r}\frac{\partial u_x}{\partial \theta} - \frac{\partial u_\theta}{\partial x} \\ \frac{\partial u_r}{\partial x} - \frac{\partial u_x}{\partial r} \\ \frac{1}{r}\left[\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right] \end{pmatrix}_p$$

Dyadisches Produkt

In kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\begin{aligned}
\underline{\underline{\tau}} : \text{grad}\ \underline{u} &= \tau_{xx}\frac{\partial u}{\partial x} + \tau_{xy}\frac{\partial v}{\partial x} + \tau_{xz}\frac{\partial w}{\partial x} \\
&+ \tau_{yx}\frac{\partial u}{\partial y} + \tau_{yy}\frac{\partial v}{\partial y} + \tau_{yz}\frac{\partial w}{\partial y} \\
&+ \tau_{zx}\frac{\partial u}{\partial z} + \tau_{zy}\frac{\partial v}{\partial z} + \tau_{zz}\frac{\partial w}{\partial z}
\end{aligned}$$

In Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\begin{aligned}
\underline{\underline{\tau}} : \text{grad}\ \underline{u} &= \tau_{rr}\frac{\partial u_r}{\partial r} + \tau_{r\theta} r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \tau_{rx}\frac{\partial u_x}{\partial r} \\
&+ \tau_{\theta r}\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) + \tau_{\theta x}\frac{1}{r}\frac{\partial u_x}{\partial \theta} \\
&+ \tau_{xr}\frac{\partial u_r}{\partial x} + \tau_{x\theta}\frac{\partial u_\theta}{\partial x} + \tau_{xx}\frac{\partial u_x}{\partial x}
\end{aligned}$$

A.3 Integralsätze

Satz von Gauß

$$\int_V \text{div}\ \underline{u}\ \mathrm{d}V = \int_S \underline{u} \cdot \underline{n}\ \mathrm{d}S$$

Satz von Stokes

$$\iint_S \text{rot}\ \underline{u} \cdot \underline{n}\ \mathrm{d}S = \oint_K \underline{u}\ \mathrm{d}\underline{l}$$

Anhang B

Grundgleichungen

B.1 Massenerhaltung

Lagrange-Darstellung (materielles Kontrollvolumen)

$$\frac{\mathrm{D}M}{\mathrm{D}t} = 0 \,, \quad M = \int_V \rho \,\mathrm{d}V$$

Euler-Darstellung (bewegtes oder raumfestes Kontrollvolumen)

$$\int_V \frac{\partial \rho}{\partial t} \,\mathrm{d}V + \int_S \rho \,(\underline{u} \cdot \underline{n}) \,\mathrm{d}S = 0$$

Kontinuitätsgleichung (differentiell, raumfest)

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{u}) = \frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho \,(\underline{\nabla} \cdot \underline{u}) = 0$$

Komponentenschreibweise in kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Komponentenschreibweise in Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial x}(\rho u_x) = 0$$

Kontinuitätsgleichung - inkompressibles Medium

$$\rho = konst. \Longrightarrow \underline{\nabla} \cdot \underline{u} = 0$$

Kontinuitätsgleichung - inkompressible Strömung

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = 0 \Longrightarrow \underline{\nabla} \cdot \underline{u} = 0$$

B.2 Impulserhaltung

Lagrange-Darstellung (materielles Kontrollvolumen)

$$\frac{\mathrm{D}\underline{P}}{\mathrm{D}t} = \sum_i \underline{F}_i \ , \quad \underline{P} = \int_V \rho\underline{u} \ \mathrm{d}V$$

Impulssatz: Euler-Darstellung (bewegtes oder raumfestes Kontrollvolumen)

$$\int_V \frac{\partial}{\partial t}(\rho\underline{u}) \ \mathrm{d}V + \int_S \rho\underline{u}\,(\underline{u}\cdot\underline{n}) \ \mathrm{d}S = \int_V \rho\underline{f} \ \mathrm{d}V - \int_S p\underline{n} \ \mathrm{d}S + \int_S \underline{\underline{\tau}}\cdot\underline{n} \ \mathrm{d}S + \underline{F}_{ext}$$

Impulssatz (differentiell, raumfest)

$$\rho\frac{\mathrm{D}\underline{u}}{\mathrm{D}t} = -\underline{\nabla}p + \underline{\nabla}\cdot\underline{\underline{\tau}} + \rho\underline{f}$$

Komponentenschreibweise in kartesischen Koordinaten $\underline{x} = (x,y,z)^T$

$$(x) \quad : \qquad \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

$$(y) \quad : \qquad \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho f_y$$

$$(z) \quad : \qquad \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z$$

Komponentenschreibweise in Zylinderkoordinaten $\underline{x} = (r,\theta,x)^T$

$$(r): \qquad \rho\left[\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_x\frac{\partial u_r}{\partial x}\right] =$$

$$-\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rx}}{\partial x} + \rho f_r$$

$$(\theta): \qquad \rho\left[\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta u_r}{r} + u_x\frac{\partial u_\theta}{\partial x}\right] =$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta x}}{\partial x} + \rho f_\theta$$

$$(x): \qquad \rho\left[\frac{\partial u_x}{\partial t} + u_r\frac{\partial u_x}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_x}{\partial \theta} + u_x\frac{\partial u_x}{\partial x}\right] =$$

$$-\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) + \frac{1}{r}\frac{\partial \tau_{\theta x}}{\partial \theta} + \frac{\partial \tau_{xx}}{\partial x} + \rho f_x$$

Die Schubspannungen eines *Newtonschen Fluids* lassen sich schreiben

- in kartesischen Koordinaten als

$$\tau_{ij} \quad = \quad \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + (\mu' - \frac{2}{3}\mu)\delta_{ij}\nabla\cdot\underline{u} \ ,$$

  mit der *Volumenviskosität* μ', der üblichen Definition des Divergenz-Operators

$$\nabla\cdot\underline{u} \quad = \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

sowie der Kroneker Delta-Funktion

$$\begin{aligned}\delta_{ij} &= 1 \quad \text{für } i = j \\ &= 0 \quad \text{sonst .}\end{aligned}$$

Für eine inkompressible Strömung gilt $\nabla \cdot \underline{u} = 0$, und somit folgt

$$\begin{aligned}
\tau_{xx} &= 2\mu\frac{\partial u}{\partial x} \\
\tau_{yy} &= 2\mu\frac{\partial v}{\partial y} \\
\tau_{zz} &= 2\mu\frac{\partial w}{\partial z} \\
\tau_{xy} &= \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\
\tau_{xz} &= \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\
\tau_{yz} &= \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)
\end{aligned}$$

.

- bzw. in Zylinderkoordinaten als

$$\begin{aligned}
\tau_{rr} &= 2\mu\left[\frac{\partial u_r}{\partial r}\right] + (\mu' - \frac{2}{3}\mu)\nabla \cdot \underline{u} \\
\tau_{\theta\theta} &= 2\mu\left[\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right] + (\mu' - \frac{2}{3}\mu)\nabla \cdot \underline{u} \\
\tau_{xx} &= 2\mu\left[\frac{\partial u_x}{\partial x}\right] + (\mu' - \frac{2}{3}\mu)\nabla \cdot \underline{u} \\
\tau_{r\theta} &= \tau_{\theta r} = 2\mu\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\frac{u_\theta}{r}\right)\right] \\
\tau_{\theta x} &= \tau_{x\theta} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_\theta}{\partial x} + \frac{1}{r}\frac{\partial u_x}{\partial \theta}\right)\right] \\
\tau_{xr} &= \tau_{rx} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x}\right)\right]
\end{aligned}$$

wiederum mit der Volumenviskosität μ' und dem Divergenz-Operator

$$\nabla \cdot \underline{u} = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_x}{\partial x} \; .$$

Für eine inkompressible Strömung folgt wie im kartesischen Fall

$$\begin{aligned}
\tau_{rr} &= 2\mu\left[\frac{\partial u_r}{\partial r}\right] \\
\tau_{\theta\theta} &= 2\mu\left[\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right] \\
\tau_{xx} &= 2\mu\left[\frac{\partial u_x}{\partial x}\right] \\
\tau_{r\theta} &= \tau_{\theta r} = 2\mu\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\frac{u_\theta}{r}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\tau_{\theta x} &= \tau_{x\theta} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_\theta}{\partial x} + \frac{1}{r}\frac{\partial u_x}{\partial \theta}\right)\right] \\
\tau_{xr} &= \tau_{rx} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x}\right)\right]
\end{aligned}$$

.

Euler-Gleichung (Impulssatz, reibungsfrei)

$$\rho\frac{\mathrm{D}\underline{u}}{\mathrm{D}t} = -\underline{\nabla} p + \rho \underline{f}$$

Bernoulli-Gleichung (konservatives Kraftfeld, Euler-Gleichung entlang einer Stromlinie oder für wirbelfreie Strömungen ($\underline{\nabla} \times \underline{u} = 0$) im gesamten Feld)

$$\int_1^2 \frac{\partial \underline{u}}{\partial t}\cdot \mathrm{d}\underline{s} + \left[\frac{p}{\rho} + \frac{1}{2}|\underline{u}|^2 + U\right]_1^2 = 0$$

Navier-Stokes-Gleichung (Impulssatz, reibungsbehaftet, Newtonsches Medium, inkompressibel, $\mu =$ *konst.*)

$$\rho\frac{\mathrm{D}\underline{u}}{\mathrm{D}t} = -\underline{\nabla} p + \mu\nabla^2\underline{u} + \rho \underline{f}$$

Komponentenschreibweise in kartesischen Koordinaten $\underline{x} = (x, y, z)^T$

$$\begin{aligned}
(x) &: \quad \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \rho f_x \\
(y) &: \quad \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] + \rho f_y \\
(z) &: \quad \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] + \rho f_z
\end{aligned}$$

Komponentenschreibweise in Zylinderkoordinaten $\underline{x} = (r, \theta, x)^T$

$$\begin{aligned}
(r): &\quad \rho\left[\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_x\frac{\partial u_r}{\partial x}\right] = \\
&-\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial x^2}\right] + \rho f_r \\
(\theta): &\quad \rho\left[\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta u_r}{r} + u_x\frac{\partial u_\theta}{\partial x}\right] = \\
&-\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial x^2}\right] + \rho f_\theta \\
(x): &\quad \rho\left[\frac{\partial u_x}{\partial t} + u_r\frac{\partial u_x}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_x}{\partial \theta} + u_x\frac{\partial u_x}{\partial x}\right] = \\
&-\frac{\partial p}{\partial x} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_x}{\partial \theta^2} + \frac{\partial^2 u_x}{\partial x^2}\right] + \rho f_x
\end{aligned}$$

Schleichströmung $Re \ll 1$

$$\underline{\nabla} p = \mu \nabla^2 \underline{u}$$

Grenzschichtgleichungen $Re \gg 1$, 2D, stationär

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

Reynolds-gemittelte Navier-Stokes Gleichung - turbulente Strömungen

$$\frac{\partial}{\partial x_j}\left(\rho\overline{u}_i\overline{u}_j\right) = \frac{\partial\overline{p}}{\partial x_i} + \frac{\partial\overline{\tau}_{ij}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\rho\overline{u_i' u_j'}\right)$$

Wirbeltransportgleichung (keine Volumenkräfte, Newtonsches Medium)

$$\frac{\mathrm{D}\underline{\omega}}{\mathrm{D}t} = \underline{\omega}\cdot\underline{\nabla u} + \nu\nabla^2\underline{\omega}$$

B.3 Energieerhaltung

Lagrange-Darstellung (materielles Kontrollvolumen)

$$\frac{\mathrm{D}E}{\mathrm{D}t} = \sum_i \underline{F}_i\cdot\underline{u} + \sum_i \dot{Q}_i\ , \quad E = \int_V \rho\left[e + \frac{1}{2}|\underline{u}|^2\right]\,\mathrm{d}V$$

Euler-Darstellung (bewegtes oder raumfestes Kontrollvolumen)

$$\int_V \frac{\partial}{\partial t}\left(\rho\left[e+\frac{1}{2}|\underline{u}|^2\right]\right)\,\mathrm{d}V + \int_S\left(\rho\left[e+\frac{1}{2}|\underline{u}|^2\right]\right)\underline{u}\cdot\underline{n}\,\mathrm{d}S =$$

$$= \int_V \rho\underline{f}\cdot\underline{u}\,\mathrm{d}V - \int_S p\underline{u}\cdot\underline{n}\,\mathrm{d}S + \int_S\left(\underline{\underline{\tau}}\cdot\underline{u}\right)\cdot\underline{n}\,\mathrm{d}S + \int_V \rho q_V\,\mathrm{d}V - \int_S \underline{q}\cdot\underline{n}\,\mathrm{d}S$$

Energiegleichung (differentiell, raumfest)

Gleichung der Gesamtenergie $\rho[e + |\underline{u}|^2/2]$

$$\frac{\partial}{\partial t}\left(\rho\left[e+\frac{1}{2}|\underline{u}|^2\right]\right) + \frac{\partial}{\partial x_i}\left(\rho u_i\left[e+\frac{1}{2}|\underline{u}|^2\right]\right) = \rho f_i u_i + \frac{\partial}{\partial x_i}\left(\sigma_{ij}u_j\right) - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

Gleichung der kinetischen Energie $\rho|\underline{u}|^2/2$

$$\frac{\partial}{\partial t}\left(\frac{\rho}{2}|\underline{u}|^2\right) + \frac{\partial}{\partial x_i}\left(u_i\frac{\rho}{2}|\underline{u}|^2\right) = \rho f_i u_i + u_j\frac{\partial\sigma_{ij}}{\partial x_i}$$

Gleichung der inneren Energie ρe

$$\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x_i}(\rho u_i e) = \sigma_{ij}\frac{\partial u_j}{\partial x_i} - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

Enthalpiegleichung (differentiell)

Gleichung der Enthalpie $h = e + p/\rho$

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_i}(\rho u_i h) = \frac{\partial p}{\partial t} + u_i\frac{\partial p}{\partial x_i} + \tau_{ij}\frac{\partial u_j}{\partial x_i} - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

mit der Dissipationsfunktion $\Phi = \tau_{ij}\partial u_j/\partial x_i$

$$\rho T\frac{\mathrm{D}h}{\mathrm{D}t} = \frac{\mathrm{D}p}{\mathrm{D}t} + \Phi - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

Gleichung der Gesamtenthalpie $h + |\underline{u}|^2/2$

$$\frac{\partial}{\partial t}\left(\rho\left[h + \frac{1}{2}|\underline{u}|^2\right]\right) + \frac{\partial}{\partial x_i}\left(\rho u_i\left[h + \frac{1}{2}|\underline{u}|^2\right]\right) = \frac{\partial p}{\partial t} + \rho f_i u_i + \frac{\partial}{\partial x_i}(\tau_{ij}u_j) - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

Entropiegleichung

$$T\mathrm{d}s = \mathrm{d}h - \frac{\mathrm{d}p}{\rho} \quad \Longrightarrow \quad \rho T\frac{\mathrm{D}s}{\mathrm{D}t} = \Phi - \frac{\partial q_i}{\partial x_i} + \rho q_V$$

reibungsfrei, adiabat

$$\frac{\mathrm{D}s}{\mathrm{D}t} = 0\ , \quad \rho\frac{\mathrm{D}h}{\mathrm{D}t} = \frac{\mathrm{D}p}{\mathrm{D}t}$$

Anhang C

Fluiddynamik II: Komplexe Zahlen und Funktionen

C.1 Komplexe Zahlen

Die Menge der **komplexen Zahlen** wird bezeichnet mit

$$\mathbb{C} = \{x + iy \mid x, y \in \mathbb{R}\}\ , \quad \mathrm{i}^2 = -1\ .$$

x heisst **Realteil**, y **Imaginärteil** der komplexen Zahl z,

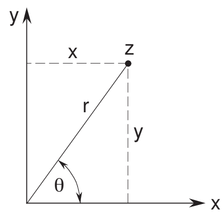
$$\boxed{z = z_r + iz_i = x + iy\ .}$$

Häufig verwendet man auch die **Polardarstellung**

$$\boxed{z = r \cdot e^{i\theta}}$$

mit

$$\begin{aligned} r^2 &:= |z|^2 = x^2 + y^2\ , \\ \theta &:= \arg(z)\ , \\ x &= r \cdot \cos\theta\ , \\ y &= r \cdot \sin\theta\ . \end{aligned}$$



Es gilt

$$\begin{aligned} e^{i\theta} &\equiv \cos\theta + i\sin\theta \\ \left|e^{i\theta}\right|^2 &= \cos^2\theta + \sin^2\theta = 1 \quad \text{falls} \quad \theta \in \mathbb{R}\ . \end{aligned}$$

Die zu z **konjugiert komplexe Zahl** wird bezeichnet als

$$\begin{aligned} z^* &= x - iy \\ &= re^{-i\theta} \end{aligned}$$

Als alternative Notation wird auch $\overline{z} = z^*$ benutzt.

Die **Multiplikation** von $z_1 = x_1 + iy_1$ mit $z_2 = x_2 + iy_2$ ergibt

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = x_1x_2 - y_1y_2 + i\,(x_2y_1 + x_1y_2)$$

Damit gilt

$$z \cdot z^* = |z|^2 = r^2\ .$$

Für die **Division** zweier komplexer Zahlen erhält man

$$\frac{z_1}{z_2} = \frac{z_1z_2^*}{z_2z_2^*} = \frac{1}{|z_2|^2} \cdot z_1z_2^*\ .$$

C.2 Differenzierbarkeit, reguläre komplexwertige Funktion

Eine **komplexwertige Funktion**

$$\boxed{F(z) = F_r(x,y) + iF_i(x,y), \quad z = x + iy}$$

heisst im Punkt z_0 **(komplex) differenzierbar**, wenn der Grenzwert

$$\lim_{\zeta \to 0,\, \zeta \in \mathbb{C}} \frac{F(z_0 + \zeta) - F(z_0)}{\zeta}$$

existiert und unabhängig ist von der Wahl von $\zeta \in \mathbb{C},\ \zeta \to 0$. Dieser Grenzwert heisst die **Ableitung** von $F(z)$ in z_0,

$$\left.\frac{\mathrm{d}F}{\mathrm{d}z}\right|_{z_0} .$$

Ist $F(z)$ in jedem Punkt eines Gebietes $G \subset \mathbb{C}$ differenzierbar, so heisst $F(z)$ in G **regulär** oder **holomorph**. Regularität einer komplexen Funktion ist eine sehr einschneidende Eigenschaft. Ist eine Funktion einmal differenzierbar, so ist sie auch unendlich oft differenzierbar. Eine reguläre Funktion, deren Ableitung in einem Gebiet G nicht verschwindet, ist **konform**, d.h. sie ist winkeltreu und erhält den Drehsinn. Reguläre Funktionen spielen bei der Beschreibung ebener Potentialströmungen eine wichtige Rolle (s. Kap. 9.2 ff).

Wegen der freien Wählbarkeit von $\zeta = \xi + i\eta$ beim Grenzübergang zur Bildung der Ableitung $F'(z)$ kann ζ insbesondere rein reell ($\zeta = \xi \to 0$), rein imaginär ($\zeta = i\eta \to 0$) oder rein radial ($\zeta = r \cdot e^{i\theta}, r \to 0, \theta = \text{const}$) gewählt werden. Es gilt also

$$\begin{aligned} \frac{\mathrm{d}F}{\mathrm{d}z} &= \frac{\partial F_r}{\partial x} + i\frac{\partial F_i}{\partial x} && (\zeta = \xi \quad \text{reell}) \\ &= \frac{1}{i}\frac{\partial F_r}{\partial y} + \frac{i}{i}\frac{\partial F_i}{\partial y} = \frac{\partial F_i}{\partial y} - i\frac{\partial F_r}{\partial y} && (\zeta = i\eta \quad \text{imaginär}) . \end{aligned}$$

Daraus folgen die **Cauchy-Riemannschen Differentialgleichungen**:

$$\boxed{\frac{\partial F_r}{\partial x} = \frac{\partial F_i}{\partial y} , \quad \frac{\partial F_r}{\partial y} = -\frac{\partial F_i}{\partial x} .}$$

In der Polardarstellung sei

$$F(z) = G_r(r,\theta) + iG_i(r,\theta) .$$

Dann ist

$$\frac{\mathrm{d}F}{\mathrm{d}z} = \frac{1}{e^{i\theta}}\left(\frac{\partial G_r}{\partial r} + i\frac{\partial G_i}{\partial r}\right) \qquad (\zeta = r \cdot e^{i\theta} , \quad \theta = \text{const}, \quad \text{radiale Ableitung}) .$$

Durch wiederholte Anwendung der Cauchy-Riemannschen Differentialgleichungen folgt

$$\frac{\partial^2 F_r}{\partial x^2} = \frac{\partial^2 F_i}{\partial x \partial y} , \quad \frac{\partial^2 F_r}{\partial y^2} = -\frac{\partial^2 F_i}{\partial x \partial y}$$

$$\Longrightarrow \frac{\partial^2 F_r}{\partial x^2} + \frac{\partial^2 F_r}{\partial y^2} = 0$$

$$\frac{\partial^2 F_i}{\partial x^2} = -\frac{\partial^2 F_r}{\partial x \partial y} \ , \quad \frac{\partial^2 F_i}{\partial y^2} = \frac{\partial^2 F_r}{\partial x \partial y}$$
$$\Longrightarrow \frac{\partial^2 F_i}{\partial x^2} + \frac{\partial^2 F_i}{\partial y^2} = 0 \ .$$

Es folgt also: Realteil F_r und Imaginärteil F_i einer regulären Funktion $F(z) = F_r + iF_i$ erfüllen die Laplace-Gleichung

$$\boxed{\Delta F_r = 0 \ , \quad \Delta F_i = 0 \ .}$$

Folgende **elementare Funktionen** im Komplexen werden häufig benötigt:

$$\boxed{\begin{aligned} & e^z = e^{x+iy} = e^x \cdot e^{iy} \ , \quad |e^z| = e^x \\ & \ln(z) = \ln\left(re^{i\theta}\right) = \ln(r) + i\theta \\ & a^z = e^{z \cdot \ln a} \ , \quad z^a = e^{a \cdot \ln z} \end{aligned}}$$

Weitere elementare Funktionen (z.B. sinh, cosh, tanh, sin, cos, tan, cot) werden wie im Reellen gebildet. Eventuelle Mehrdeutigkeit ist zu berücksichtigen. Die vom Reellen bekannten Potenzreihenentwicklungen übertragen sich in die komplexe Ebene.

Es gelten die vom Reellen her gewohnten **Differentiationsregeln**, z.B.

$$\frac{\mathrm{d}}{\mathrm{d}z} \ln z = \frac{1}{z} \ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}z} e^z = e^z \ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}z} z^b = b \cdot z^{b-1} \ .$$

Anhang D

Übersicht Potentialströmungen

D.1 Darstellung ebener, inkompressibler Potentialströmungen

$$\underbrace{\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix},\ \underline{u} = \begin{pmatrix} u \\ v \end{pmatrix}}_{\text{Ebene,}} \qquad \underbrace{\underbrace{\operatorname{div}\underline{u} = 0}_{\text{inkompressible}}\ ,\qquad \underbrace{\underline{u} = \nabla\Phi}_{\text{Potentialströmung}}}_{\Delta\Phi = 0}\ , \qquad \operatorname{rot}\underline{u} = 0$$

Potential Φ, Stromfunktion Ψ

Zirkulation: $\Gamma_C = \Gamma(C, \underline{u}) := \oint_C \underline{u} \cdot \mathrm{d}\underline{x} = \oint_C \nabla\Phi\, \mathrm{d}\underline{x} = \oint_C \mathrm{d}\Phi,$

$\Gamma_C \neq 0$ wenn Φ mehrdeutig; Wirbel

$\Gamma_C \overset{St}{=} \int_S (\operatorname{rot}\underline{u}) \cdot \underline{n}\ \mathrm{d}S$

(geschlossene Kurve C, berandet Fläche S,

$\overset{St}{=}$ gilt unter den Voraussetzungen des Satzes von Stokes)

Quellstärke: $Q_C = Q(C, \underline{u}) := \oint_C \underline{u} \cdot \underline{n}\, \mathrm{d}s = \oint_C \nabla\Psi \cdot \mathrm{d}\underline{x} = \oint_C \mathrm{d}\Psi,$

$Q_C \neq 0$ wenn Ψ mehrdeutig, Quelle

$Q_C \overset{G}{=} \int_S \operatorname{div}\underline{u}\, \mathrm{d}S$

($\overset{G}{=}$ gilt unter den Voraussetzungen des Satzes von Gauss)

komplexes Potential: $\boxed{F(z) = \Phi + i\Psi}$, $z = x + iy$

komplexe Geschwindigkeit: $\boxed{w(z) = \dfrac{\mathrm{d}F}{\mathrm{d}z}}$

komplexe Zirkulation: $\mathbf{\Gamma_C} := \oint_C w\ \mathrm{d}z = \Gamma_C + iQ_C$, also auch $\mathbf{\Gamma_C} = \oint_C \dfrac{dF}{dz} dz = \oint_C dF$

$\mathbf{\Gamma_C} \neq 0$ wenn F mehrdeutig; Singularität

	kartesisch	**polar**
Koordinaten:	$\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix},\ z = x + iy$	$\underline{x}_p = \begin{pmatrix} r \\ \vartheta \end{pmatrix}_p,\ z = re^{i\vartheta}$
Geschwindigkeit:	$\boxed{\begin{aligned} \underline{u} &= \begin{pmatrix} u \\ v \end{pmatrix} \\ w(z) &= u - iv \end{aligned}}$	$\boxed{\begin{aligned} \underline{u}_p &= \begin{pmatrix} u_r \\ u_\vartheta \end{pmatrix}_p \\ w(z) &= (u_r - iu_\vartheta)e^{-i\vartheta} \end{aligned}}$
$\underline{u} = \nabla\Phi$:	$\boxed{\begin{aligned} u &= \partial\Phi/\partial x \\ v &= \partial\Phi/\partial y \end{aligned}}$	$\boxed{\begin{aligned} u_r &= \partial\Phi/\partial r \\ u_\vartheta &= \frac{1}{r}\frac{\partial\Phi}{\partial\vartheta} \end{aligned}}$
$\underline{u} = \operatorname{rot}\begin{pmatrix} 0 \\ 0 \\ \Psi \end{pmatrix}$:	$\boxed{\begin{aligned} u &= \partial\Psi/\partial y \\ v &= -\partial\Psi/\partial x \end{aligned}}$	$\boxed{\begin{aligned} u_r &= \frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} \\ u_\vartheta &= -\partial\Psi/\partial r \end{aligned}}$
	$\begin{aligned} u &= u_r\cos\theta - u_\theta\sin\theta \\ v &= u_r\sin\theta + u_\theta\cos\theta \end{aligned}$	$\begin{aligned} u_r &= u\cos\theta + v\sin\theta \\ u_\theta &= -u\sin\theta + v\cos\theta \end{aligned}$

Druck $p(\underline{x})$: Bernoulli-Gleichung für Potentialströmung (U = Kräftepotential, $\underline{f} = -\nabla U$)

stationär: $$p(\underline{x}) + \frac{\rho}{2} \mid \underline{u}(\underline{x}) \mid^2 + \rho U(\underline{x}) = p_0 + \frac{\rho}{2} \mid \underline{u}_0 \mid^2 + \rho U_0$$
(Werte in Referenzpunkt $\underline{x}_0$)

beachte: $$\mid \underline{u} \mid^2 \;=\; \mid \nabla\Phi \mid^2 \;=\; u^2 + v^2 \;=\; u_r^2 + u_\vartheta^2 \;=\; \mid w \mid^2$$

Stromlinien: $\Psi = \mathrm{const}$

Potentiallinien: $\Phi = \mathrm{const}$, orthogonal zu Stromlinien

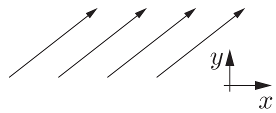
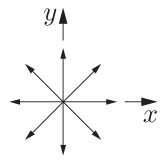
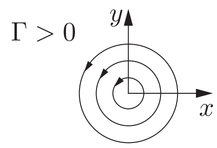
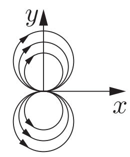
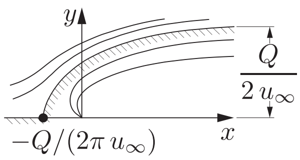
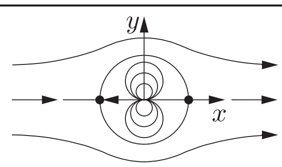
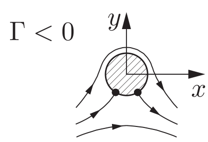
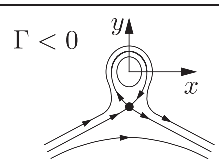
Staupunkt $\underline{x}_s$: $\underline{u}(\underline{x}_s) = 0$ bzw. $w(z_s) = 0,\ z_s = x_s + iy_s$

Staustromlinien: $\Psi(x,y) = \Psi(\underline{x}_s)$

D.2 Beispiele ebener Potentialströmungen*

komplexes Potential $F(z) = \Phi + i\,\Psi$	**Potential** $\Phi(x,y)$	**Stromfunktion** $\Psi(x,y)$
$(u_\infty - i\,v_\infty)\,z$ Parallelströmung	$u_\infty\,x + v_\infty\,y$	$u_\infty\,y - v_\infty\,x$
$\frac{Q}{2\pi}\ln z$ Quelle $Q > 0$, Senke $Q < 0$	$\frac{Q}{2\pi}\ln r = \frac{Q}{2\pi}\ln\sqrt{x^2+y^2}$	$\frac{Q}{2\pi}\vartheta = \frac{Q}{2\pi}\arg(z)$
$-\frac{\Gamma}{2\pi}\,i\,\ln z$ Wirbel, $\Gamma > 0$ math. positiv, $\Gamma < 0$ math. negativ	$\frac{\Gamma}{2\pi}\vartheta = \frac{\Gamma}{2\pi}\arg(z)$	$-\frac{\Gamma}{2\pi}\ln\sqrt{x^2+y^2}$
$\frac{m}{z}$ Dipol	$\frac{m\,x}{x^2+y^2}$	$-\frac{m\,y}{x^2+y^2}$
$u_\infty\,z + \frac{Q}{2\pi}\ln z$ Parallelströmung + Quelle/Senke	$u_\infty\,x + \frac{Q}{2\pi}\ln r$	$u_\infty\,y + \frac{Q}{2\pi}\vartheta$
$u_\infty\left(z + \frac{R^2}{z}\right)$ Parallelströmung + Dipol = Kreiszylinderumström.	$u_\infty\,x\left(1 + \frac{R^2}{x^2+y^2}\right)$	$u_\infty\,y\left(1 - \frac{R^2}{x^2+y^2}\right)$
$u_\infty\left(z + \frac{R^2}{z}\right) - \frac{\Gamma}{2\pi}\,i\ln z$ Kreiszylinderumströmung + Wirbel	$u_\infty x\left(1 + \frac{R^2}{x^2+y^2}\right) + \frac{\Gamma}{2\pi}\vartheta$	$u_\infty y\left(1 - \frac{R^2}{x^2+y^2}\right) - \frac{\Gamma}{2\pi}\ln r$
$u_\infty z - \frac{\Gamma}{2\pi}\,i\ln z$ Parallelströmung + Wirbel	$u_\infty\,x + \frac{\Gamma}{2\pi}\vartheta$	$u_\infty\,y - \frac{\Gamma}{2\pi}\ln r$

* (nach Zierep, Grundzüge der Strömungslehre, Springer 1997)

Geschwindigkeit $w = \frac{\mathrm{d}F}{\mathrm{d}z} = u - i\,v$			**Stromlinien**
u	v	$\lvert \underline{u} \rvert$	$\Psi = \text{konst}$
u_∞	v_∞	$\sqrt{u_\infty^2 + v_\infty^2}$	
$\frac{Q}{2\pi}\,\frac{x}{x^2+y^2}$	$\frac{Q}{2\pi}\,\frac{y}{x^2+y^2}$	$\frac{\lvert Q \rvert}{2\pi\, r}$	
$-\frac{\Gamma}{2\pi}\,\frac{y}{x^2+y^2}$	$\frac{\Gamma}{2\pi}\,\frac{x}{x^2+y^2}$	$\frac{\lvert \Gamma \rvert}{2\pi\, r}$	
$m\,\frac{y^2-x^2}{(x^2+y^2)^2}$	$-m\,\frac{2\,x\,y}{(x^2+y^2)^2}$	$\frac{\lvert m \rvert}{r^2}$	
$u_\infty + \frac{Q}{2\pi}\,\frac{x}{x^2+y^2}$	$\frac{Q}{2\pi}\,\frac{y}{x^2+y^2}$		
auf dem Zylinder:			
$2\,u_\infty \sin^2\vartheta$	$-2\,u_\infty \sin\vartheta\cos\vartheta$	$2\,u_\infty\,\lvert\sin\vartheta\rvert$	
auf dem Zylinder:			
$2\,u_\infty \sin^2\vartheta - \frac{\Gamma}{2\pi\,R}\sin\vartheta$	$-2\,u_\infty \sin\vartheta\cos\vartheta + \frac{\Gamma}{2\pi\,R}\cos\vartheta$	$\left\lvert 2\,u_\infty \sin\vartheta - \frac{\Gamma}{2\pi\,R}\right\rvert$	
$u_\infty - \frac{\Gamma}{2\pi}\,\frac{y}{x^2+y^2}$	$\frac{\Gamma}{2\pi}\,\frac{x}{x^2+y^2}$		

Anhang E

Kompressibler Stromfaden - Formeln und Tabellen

Isentrope Strömung

Abhängigkeit von Ruhegrößen und Mach-Zahl

$$
\begin{aligned}
\frac{T_0}{T} &= \left(1+\frac{\gamma-1}{2}Ma^2\right) \\
\frac{a_0}{a} &= \left(1+\frac{\gamma-1}{2}Ma^2\right)^{\frac{1}{2}} \\
\frac{p_0}{p} &= \left(1+\frac{\gamma-1}{2}Ma^2\right)^{\frac{\gamma}{\gamma-1}} \\
\frac{\rho_0}{\rho} &= \left(1+\frac{\gamma-1}{2}Ma^2\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

Zusammenhang zwischen kritischen und Ruhegrößen ($\gamma = 1.4$)

$$
\begin{aligned}
\frac{T_*}{T_0} &= \left(\frac{2}{\gamma+1}\right) \qquad (= \ 0.833) \\
\frac{a_*}{a_0} &= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}} \qquad (= \ 0.913) \\
\frac{p_*}{p_0} &= \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (= \ 0.528) \\
\frac{\rho_*}{\rho_0} &= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \qquad (= \ 0.634)
\end{aligned}
$$

Abhängigkeit von der Laval-Zahl

$$
\begin{aligned}
\frac{T}{T_0} &= \left(1-\frac{\gamma-1}{\gamma+1}La^2\right) \\
\frac{a}{a_0} &= \left(1-\frac{\gamma-1}{\gamma+1}La^2\right)^{\frac{1}{2}} \\
\frac{p}{p_0} &= \left(1-\frac{\gamma-1}{\gamma+1}La^2\right)^{\frac{\gamma}{\gamma-1}} \\
\frac{\rho}{\rho_0} &= \left(1-\frac{\gamma-1}{\gamma+1}La^2\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

Abhängigkeit von der Maximalgeschwindigkeit

$$
\frac{T}{T_0} = \left(1-\frac{u^2}{u_{\max}^2}\right)
$$

$$\frac{a}{a_0} = \left(1 - \frac{u^2}{u_{\max}^2}\right)^{\frac{1}{2}}$$

$$\frac{p}{p_0} = \left(1 - \frac{u^2}{u_{\max}^2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_0} = \left(1 - \frac{u^2}{u_{\max}^2}\right)^{\frac{1}{\gamma-1}}$$

Senkrechter Verdichtungsstoss

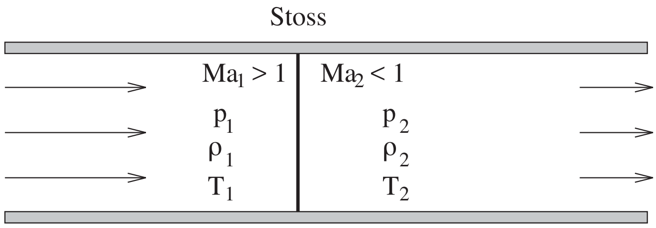


Abbildung E.1: Senkrechter Verdichtungsstoß.

Beziehungen zwischen den Zuständen vor und hinter dem Stoß

$$\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{2}{\gamma+1}\frac{1}{Ma_1^2} + \frac{\gamma-1}{\gamma+1}$$

$$\frac{\Delta p}{p_1} = \frac{p_2 - p_1}{p_1} = \frac{2\gamma}{\gamma+1}\left(Ma_1^2 - 1\right)$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}Ma_1^2 - \frac{\gamma-1}{\gamma+1}$$

$$\frac{T_2}{T_1} = \left(\frac{2\gamma}{\gamma+1}Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right)\left(\frac{2}{\gamma+1}\frac{1}{Ma_1^2} + \frac{\gamma-1}{\gamma+1}\right)$$

$$Ma_2^2 = \frac{1 + (\gamma-1)/2 \cdot Ma_1^2}{\gamma Ma_1^2 - (\gamma-1)/2} = 1 - \frac{Ma_1^2 - 1}{1 + \frac{2\gamma}{\gamma+1}\left(Ma_1^2 - 1\right)}$$

Verhalten der Ruhegrößen über den Stoß

$$T_{02} = T_{01}$$

$$\frac{\rho_{02}}{\rho_{01}} = \frac{p_{02}}{p_{01}} = \left(1 + \frac{2\gamma}{\gamma+1}\left(Ma_1^2 - 1\right)\right)^{-\frac{1}{\gamma-1}}\left(\frac{(\gamma+1)\,Ma_1^2}{2 + (\gamma-1)Ma_1^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Isentrope Unterschallströmung eines idealen Gases:
Zusammenhang zwischen Strömungsgrössen und Ruhegrössen
für $\gamma = 1.4$

Ma	p/p_0	ρ/ρ_0	T/T_0	A/A_*
0.00	1.00000	1.00000	1.00000	∞
0.02	0.99972	0.99980	0.99992	28.94213
0.04	0.99888	0.99920	0.99968	14.48149
0.06	0.99748	0.99820	0.99928	9.66591
0.08	0.99553	0.99681	0.99872	7.26161
0.10	0.99303	0.99502	0.99800	5.82183
0.12	0.98998	0.99284	0.99713	4.86432
0.14	0.98640	0.99027	0.99610	4.18240
0.16	0.98228	0.98731	0.99491	3.67274
0.18	0.97765	0.98398	0.99356	3.27793
0.20	0.97250	0.98028	0.99206	2.96352
0.22	0.96685	0.97620	0.99041	2.70760
0.24	0.96070	0.97177	0.98861	2.49556
0.26	0.95408	0.96698	0.98666	2.31729
0.28	0.94700	0.96185	0.98456	2.16555
0.30	0.93947	0.95638	0.98232	2.03507
0.32	0.93150	0.95058	0.97993	1.92185
0.34	0.92312	0.94446	0.97740	1.82288
0.36	0.91433	0.93803	0.97473	1.73578
0.38	0.90516	0.93130	0.97193	1.65870
0.40	0.89561	0.92427	0.96899	1.59014
0.42	0.88572	0.91697	0.96592	1.52890
0.44	0.87550	0.90940	0.96272	1.47401
0.46	0.86496	0.90157	0.95940	1.42463
0.48	0.85413	0.89349	0.95595	1.38010
0.50	0.84302	0.88517	0.95238	1.33984
0.52	0.83165	0.87663	0.94869	1.30339
0.54	0.82005	0.86788	0.94489	1.27032
0.56	0.80823	0.85892	0.94098	1.24029
0.58	0.79621	0.84978	0.93696	1.21301
0.60	0.78400	0.84045	0.93284	1.18820
0.62	0.77164	0.83096	0.92861	1.16565
0.64	0.75913	0.82132	0.92428	1.14515
0.66	0.74650	0.81153	0.91986	1.12654
0.68	0.73376	0.80162	0.91535	1.10965
0.70	0.72093	0.79158	0.91075	1.09437
0.72	0.70803	0.78143	0.90606	1.08057
0.74	0.69507	0.77119	0.90129	1.06814
0.76	0.68207	0.76086	0.89644	1.05700
0.78	0.66905	0.75046	0.89152	1.04705
0.80	0.65602	0.73999	0.88652	1.03823
0.82	0.64300	0.72947	0.88146	1.03046
0.84	0.63000	0.71891	0.87633	1.02370
0.86	0.61703	0.70831	0.87114	1.01787
0.88	0.60412	0.69768	0.86589	1.01294
0.90	0.59126	0.68704	0.86059	1.00886
0.92	0.57848	0.67640	0.85523	1.00560
0.94	0.56578	0.66576	0.84982	1.00311
0.96	0.55317	0.65513	0.84437	1.00136
0.98	0.54067	0.64452	0.83887	1.00034
1.00	0.52828	0.63394	0.83333	1.00000

Überschallströmung eines idealen Gases für $\gamma = 1.4$

Isentrope Strömung					senkrechter Verdichtungsstoss					
Ma	p/p_0	ρ/ρ_0	T/T_0	A/A_*	Ma_1	Ma_2	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{02}/p_{01}
1.00	0.52828	0.63394	0.83333	1.00000	1.00	1.00000	1.00000	1.00000	1.00000	1.00000
1.02	0.51602	0.62339	0.82776	1.00033	1.02	0.98052	1.04713	1.03344	1.01325	0.99999
1.04	0.50389	0.61289	0.82215	1.00131	1.04	0.96203	1.09520	1.06709	1.02634	0.99992
1.06	0.49189	0.60243	0.81651	1.00291	1.06	0.94445	1.14420	1.10092	1.03931	0.99975
1.08	0.48005	0.59203	0.81085	1.00512	1.08	0.92771	1.19413	1.13492	1.05217	0.99943
1.10	0.46835	0.58170	0.80515	1.00793	1.10	0.91177	1.24500	1.16908	1.06494	0.99893
1.12	0.45682	0.57143	0.79944	1.01131	1.12	0.89656	1.29680	1.20338	1.07763	0.99821
1.14	0.44545	0.56123	0.79370	1.01527	1.14	0.88204	1.34953	1.23779	1.09027	0.99726
1.16	0.43425	0.55112	0.78795	1.01978	1.16	0.86816	1.40320	1.27231	1.10287	0.99605
1.18	0.42322	0.54108	0.78218	1.02484	1.18	0.85488	1.45780	1.30693	1.11544	0.99457
1.20	0.41238	0.53114	0.77640	1.03044	1.20	0.84217	1.51333	1.34161	1.12799	0.99280
1.22	0.40171	0.52129	0.77061	1.03657	1.22	0.82999	1.56980	1.37636	1.14054	0.99073
1.24	0.39123	0.51154	0.76481	1.04323	1.24	0.81830	1.62720	1.41116	1.15309	0.98836
1.26	0.38093	0.50189	0.75900	1.05041	1.26	0.80709	1.68553	1.44599	1.16566	0.98568
1.28	0.37083	0.49234	0.75319	1.05810	1.28	0.79631	1.74480	1.48084	1.17825	0.98268
1.30	0.36091	0.48290	0.74738	1.06630	1.30	0.78596	1.80500	1.51570	1.19087	0.97937
1.32	0.35119	0.47357	0.74158	1.07502	1.32	0.77600	1.86613	1.55055	1.20353	0.97575
1.34	0.34166	0.46436	0.73577	1.08424	1.34	0.76641	1.92820	1.58538	1.21624	0.97182
1.36	0.33233	0.45526	0.72997	1.09396	1.36	0.75718	1.99120	1.62018	1.22900	0.96758
1.38	0.32319	0.44628	0.72418	1.10419	1.38	0.74829	2.05513	1.65494	1.24181	0.96304
1.40	0.31424	0.43742	0.71839	1.11493	1.40	0.73971	2.12000	1.68966	1.25469	0.95819
1.42	0.30549	0.42869	0.71262	1.12616	1.42	0.73144	2.18580	1.72430	1.26764	0.95306
1.44	0.29693	0.42007	0.70685	1.13790	1.44	0.72345	2.25253	1.75888	1.28066	0.94765
1.46	0.28856	0.41158	0.70110	1.15015	1.46	0.71574	2.32020	1.79337	1.29377	0.94196
1.48	0.28039	0.40322	0.69537	1.16290	1.48	0.70829	2.38880	1.82777	1.30695	0.93600
1.50	0.27240	0.39498	0.68966	1.17617	1.50	0.70109	2.45833	1.86207	1.32022	0.92979
1.52	0.26461	0.38688	0.68396	1.18994	1.52	0.69413	2.52880	1.89626	1.33357	0.92332
1.54	0.25700	0.37890	0.67828	1.20423	1.54	0.68739	2.60020	1.93033	1.34703	0.91662
1.56	0.24957	0.37105	0.67262	1.21904	1.56	0.68087	2.67253	1.96427	1.36057	0.90970
1.58	0.24233	0.36332	0.66699	1.23438	1.58	0.67455	2.74580	1.99808	1.37422	0.90255
1.60	0.23527	0.35573	0.66138	1.25024	1.60	0.66844	2.82000	2.03175	1.38797	0.89520
1.62	0.22839	0.34827	0.65579	1.26663	1.62	0.66251	2.89513	2.06526	1.40182	0.88765
1.64	0.22168	0.34093	0.65023	1.28355	1.64	0.65677	2.97120	2.09863	1.41578	0.87992
1.66	0.21515	0.33372	0.64470	1.30102	1.66	0.65119	3.04820	2.13183	1.42985	0.87201
1.68	0.20879	0.32664	0.63919	1.31904	1.68	0.64579	3.12613	2.16486	1.44403	0.86394
1.70	0.20259	0.31969	0.63371	1.33761	1.70	0.64054	3.20500	2.19772	1.45833	0.85572
1.72	0.19656	0.31287	0.62827	1.35674	1.72	0.63545	3.28480	2.23040	1.47274	0.84736
1.74	0.19070	0.30617	0.62285	1.37643	1.74	0.63051	3.36553	2.26289	1.48727	0.83886
1.76	0.18499	0.29959	0.61747	1.39670	1.76	0.62570	3.44720	2.29520	1.50192	0.83024
1.78	0.17944	0.29315	0.61211	1.41755	1.78	0.62104	3.52980	2.32731	1.51669	0.82151
1.80	0.17404	0.28682	0.60680	1.43898	1.80	0.61650	3.61333	2.35922	1.53158	0.81268
1.82	0.16879	0.28061	0.60151	1.46101	1.82	0.61209	3.69780	2.39093	1.54659	0.80376
1.84	0.16369	0.27453	0.59626	1.48365	1.84	0.60780	3.78320	2.42244	1.56173	0.79476
1.86	0.15873	0.26857	0.59104	1.50689	1.86	0.60363	3.86953	2.45373	1.57700	0.78569
1.88	0.15392	0.26272	0.58586	1.53076	1.88	0.59957	3.95680	2.48481	1.59239	0.77655
1.90	0.14924	0.25699	0.58072	1.55526	1.90	0.59562	4.04500	2.51568	1.60792	0.76736
1.92	0.14470	0.25138	0.57561	1.58039	1.92	0.59177	4.13413	2.54633	1.62357	0.75812
1.94	0.14028	0.24588	0.57054	1.60617	1.94	0.58802	4.22420	2.57675	1.63935	0.74884
1.96	0.13600	0.24049	0.56551	1.63261	1.96	0.58437	4.31520	2.60695	1.65527	0.73954
1.98	0.13184	0.23521	0.56051	1.65972	1.98	0.58082	4.40713	2.63692	1.67132	0.73021
2.00	0.12780	0.23005	0.55556	1.68750	2.00	0.57735	4.50000	2.66667	1.68750	0.72087
2.02	0.12389	0.22499	0.55064	1.71597	2.02	0.57397	4.59380	2.69618	1.70382	0.71153
2.04	0.12009	0.22004	0.54576	1.74514	2.04	0.57068	4.68853	2.72546	1.72027	0.70218
2.06	0.11640	0.21519	0.54091	1.77502	2.06	0.56747	4.78420	2.75451	1.73686	0.69284
2.08	0.11282	0.21045	0.53611	1.80561	2.08	0.56433	4.88080	2.78332	1.75359	0.68351
2.10	0.10935	0.20580	0.53135	1.83694	2.10	0.56128	4.97833	2.81190	1.77045	0.67420
2.12	0.10599	0.20126	0.52663	1.86902	2.12	0.55829	5.07680	2.84024	1.78745	0.66492
2.14	0.10273	0.19681	0.52194	1.90184	2.14	0.55538	5.17620	2.86835	1.80459	0.65567
2.16	0.09956	0.19247	0.51730	1.93544	2.16	0.55254	5.27653	2.89621	1.82188	0.64645
2.18	0.09649	0.18821	0.51269	1.96981	2.18	0.54977	5.37780	2.92383	1.83930	0.63727
2.20	0.09352	0.18405	0.50813	2.00497	2.20	0.54706	5.48000	2.95122	1.85686	0.62814
2.22	0.09064	0.17998	0.50361	2.04094	2.22	0.54441	5.58313	2.97837	1.87456	0.61905
2.24	0.08785	0.17600	0.49912	2.07773	2.24	0.54182	5.68720	3.00527	1.89241	0.61002
2.26	0.08514	0.17211	0.49468	2.11535	2.26	0.53930	5.79220	3.03194	1.91040	0.60105
2.28	0.08251	0.16830	0.49027	2.15381	2.28	0.53683	5.89813	3.05836	1.92853	0.59214
2.30	0.07997	0.16458	0.48591	2.19313	2.30	0.53441	6.00500	3.08455	1.94680	0.58329
2.32	0.07751	0.16095	0.48158	2.23332	2.32	0.53205	6.11280	3.11049	1.96522	0.57452
2.34	0.07512	0.15739	0.47730	2.27440	2.34	0.52974	6.22153	3.13620	1.98378	0.56581
2.36	0.07281	0.15391	0.47305	2.31638	2.36	0.52749	6.33120	3.16167	2.00249	0.55718
2.38	0.07057	0.15052	0.46885	2.35928	2.38	0.52528	6.44180	3.18690	2.02134	0.54862

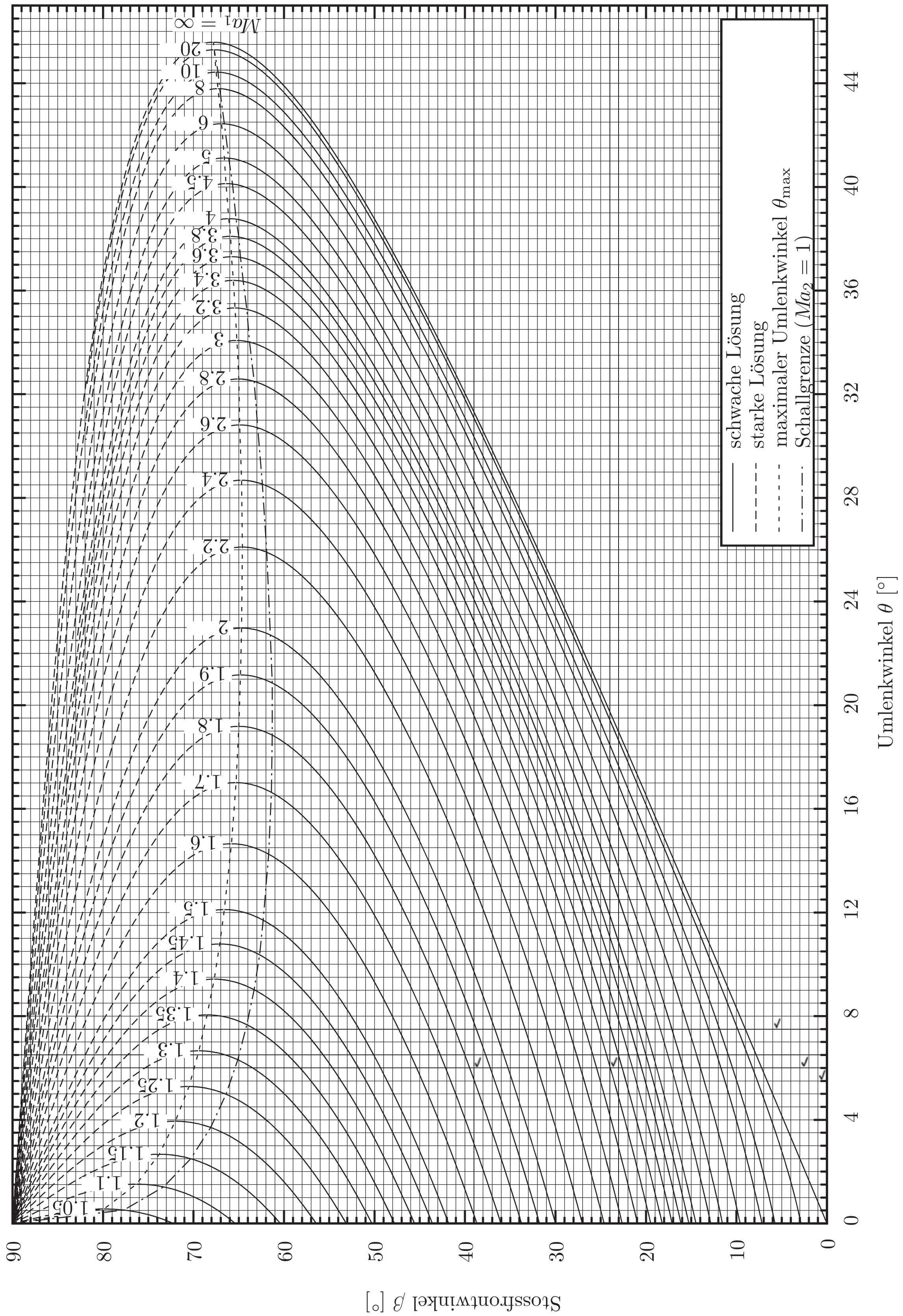
Überschallströmung eines idealen Gases für $\gamma = 1.4$

Isentrope Strömung					senkrechter Verdichtungsstoss					
Ma	p/p_0	ρ/ρ_0	T/T_0	A/A_*	Ma_1	Ma_2	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{02}/p_{01}
2.40	0.06840	0.14720	0.46468	2.40310	2.40	0.52312	6.55333	3.21190	2.04033	0.54014
2.42	0.06630	0.14395	0.46056	2.44787	2.42	0.52100	6.66580	3.23665	2.05947	0.53175
2.44	0.06426	0.14078	0.45647	2.49360	2.44	0.51894	6.77920	3.26117	2.07876	0.52344
2.46	0.06229	0.13768	0.45242	2.54031	2.46	0.51691	6.89353	3.28546	2.09819	0.51521
2.48	0.06038	0.13465	0.44841	2.58801	2.48	0.51493	7.00880	3.30951	2.11777	0.50707
2.50	0.05853	0.13169	0.44444	2.63672	2.50	0.51299	7.12500	3.33333	2.13750	0.49901
2.52	0.05674	0.12879	0.44051	2.68645	2.52	0.51109	7.24213	3.35692	2.15737	0.49105
2.54	0.05500	0.12597	0.43662	2.73723	2.54	0.50923	7.36020	3.38028	2.17739	0.48318
2.56	0.05332	0.12321	0.43277	2.78906	2.56	0.50741	7.47920	3.40341	2.19756	0.47540
2.58	0.05169	0.12051	0.42895	2.84197	2.58	0.50562	7.59913	3.42631	2.21788	0.46772
2.60	0.05012	0.11787	0.42517	2.89598	2.60	0.50387	7.72000	3.44898	2.23834	0.46012
2.62	0.04859	0.11530	0.42143	2.95109	2.62	0.50216	7.84180	3.47143	2.25896	0.45263
2.64	0.04711	0.11278	0.41772	3.00733	2.64	0.50048	7.96453	3.49365	2.27972	0.44522
2.66	0.04568	0.11032	0.41406	3.06472	2.66	0.49883	8.08820	3.51565	2.30063	0.43792
2.68	0.04429	0.10792	0.41043	3.12327	2.68	0.49722	8.21280	3.53743	2.32168	0.43070
2.70	0.04295	0.10557	0.40683	3.18301	2.70	0.49563	8.33833	3.55899	2.34289	0.42359
2.72	0.04165	0.10328	0.40328	3.24395	2.72	0.49408	8.46480	3.58033	2.36425	0.41657
2.74	0.04039	0.10104	0.39976	3.30611	2.74	0.49256	8.59220	3.60146	2.38576	0.40965
2.76	0.03917	0.09885	0.39627	3.36952	2.76	0.49107	8.72053	3.62237	2.40741	0.40283
2.78	0.03799	0.09671	0.39282	3.43418	2.78	0.48960	8.84980	3.64307	2.42922	0.39610
2.80	0.03685	0.09463	0.38941	3.50012	2.80	0.48817	8.98000	3.66355	2.45117	0.38946
2.82	0.03574	0.09259	0.38603	3.56737	2.82	0.48676	9.11113	3.68383	2.47328	0.38293
2.84	0.03467	0.09059	0.38268	3.63593	2.84	0.48538	9.24320	3.70389	2.49554	0.37649
2.86	0.03363	0.08865	0.37937	3.70584	2.86	0.48402	9.37620	3.72375	2.51794	0.37014
2.88	0.03263	0.08675	0.37610	3.77711	2.88	0.48269	9.51013	3.74341	2.54050	0.36389
2.90	0.03165	0.08489	0.37286	3.84977	2.90	0.48138	9.64500	3.76286	2.56321	0.35773
2.92	0.03071	0.08307	0.36965	3.92383	2.92	0.48010	9.78080	3.78211	2.58607	0.35167
2.94	0.02980	0.08130	0.36647	3.99932	2.94	0.47884	9.91753	3.80117	2.60908	0.34570
2.96	0.02891	0.07957	0.36333	4.07625	2.96	0.47760	10.05520	3.82002	2.63224	0.33982
2.98	0.02805	0.07788	0.36022	4.15466	2.98	0.47638	10.19380	3.83868	2.65555	0.33404
3.00	0.02722	0.07623	0.35714	4.23457	3.00	0.47519	10.33333	3.85714	2.67901	0.32834
3.02	0.02642	0.07461	0.35410	4.31599	3.02	0.47402	10.47380	3.87541	2.70263	0.32274
3.04	0.02564	0.07303	0.35108	4.39895	3.04	0.47287	10.61520	3.89350	2.72639	0.31723
3.06	0.02489	0.07149	0.34810	4.48347	3.06	0.47174	10.75753	3.91139	2.75031	0.31180
3.08	0.02416	0.06999	0.34515	4.56959	3.08	0.47063	10.90080	3.92909	2.77438	0.30646
3.10	0.02345	0.06852	0.34223	4.65731	3.10	0.46953	11.04500	3.94661	2.79860	0.30121
3.12	0.02276	0.06708	0.33934	4.74667	3.12	0.46846	11.19013	3.96395	2.82298	0.29605
3.14	0.02210	0.06568	0.33648	4.83769	3.14	0.46741	11.33620	3.98110	2.84750	0.29097
3.16	0.02146	0.06430	0.33365	4.93039	3.16	0.46637	11.48320	3.99808	2.87218	0.28597
3.18	0.02083	0.06296	0.33085	5.02481	3.18	0.46535	11.63113	4.01488	2.89701	0.28106
3.20	0.02023	0.06165	0.32808	5.12096	3.20	0.46435	11.78000	4.03150	2.92199	0.27623
3.22	0.01964	0.06037	0.32534	5.21887	3.22	0.46336	11.92980	4.04794	2.94713	0.27148
3.24	0.01908	0.05912	0.32263	5.31857	3.24	0.46240	12.08053	4.06422	2.97241	0.26681
3.26	0.01853	0.05790	0.31995	5.42008	3.26	0.46144	12.23220	4.08032	2.99785	0.26222
3.28	0.01799	0.05671	0.31729	5.52343	3.28	0.46051	12.38480	4.09625	3.02345	0.25771
3.30	0.01748	0.05554	0.31466	5.62865	3.30	0.45959	12.53833	4.11202	3.04919	0.25328
3.32	0.01698	0.05440	0.31206	5.73576	3.32	0.45868	12.69280	4.12762	3.07509	0.24892
3.34	0.01649	0.05329	0.30949	5.84479	3.34	0.45779	12.84820	4.14306	3.10114	0.24463
3.36	0.01602	0.05220	0.30694	5.95577	3.36	0.45691	13.00453	4.15833	3.12734	0.24043
3.38	0.01557	0.05113	0.30443	6.06873	3.38	0.45605	13.16180	4.17345	3.15370	0.23629
3.40	0.01512	0.05009	0.30193	6.18370	3.40	0.45520	13.32000	4.18841	3.18021	0.23223
3.42	0.01470	0.04908	0.29947	6.30070	3.42	0.45436	13.47913	4.20321	3.20687	0.22823
3.44	0.01428	0.04808	0.29702	6.41976	3.44	0.45354	13.63920	4.21785	3.23369	0.22431
3.46	0.01388	0.04711	0.29461	6.54092	3.46	0.45273	13.80020	4.23234	3.26065	0.22045
3.48	0.01349	0.04616	0.29222	6.66419	3.48	0.45194	13.96213	4.24668	3.28778	0.21667
3.50	0.01311	0.04523	0.28986	6.78962	3.50	0.45115	14.12500	4.26087	3.31505	0.21295
3.52	0.01274	0.04433	0.28751	6.91723	3.52	0.45038	14.28880	4.27491	3.34248	0.20929
3.54	0.01239	0.04344	0.28520	7.04705	3.54	0.44962	14.45353	4.28880	3.37006	0.20570
3.56	0.01204	0.04257	0.28291	7.17912	3.56	0.44887	14.61920	4.30255	3.39780	0.20218
3.58	0.01171	0.04172	0.28064	7.31346	3.58	0.44814	14.78580	4.31616	3.42569	0.19871
3.60	0.01138	0.04089	0.27840	7.45011	3.60	0.44741	14.95333	4.32962	3.45373	0.19531
3.62	0.01107	0.04008	0.27618	7.58910	3.62	0.44670	15.12180	4.34294	3.48192	0.19197
3.64	0.01076	0.03929	0.27398	7.73045	3.64	0.44600	15.29120	4.35613	3.51027	0.18869
3.66	0.01047	0.03852	0.27180	7.87421	3.66	0.44530	15.46153	4.36918	3.53878	0.18547
3.68	0.01018	0.03776	0.26965	8.02040	3.68	0.44462	15.63280	4.38209	3.56743	0.18230
3.70	0.00990	0.03702	0.26752	8.16907	3.70	0.44395	15.80500	4.39486	3.59624	0.17919
3.72	0.00963	0.03629	0.26542	8.32023	3.72	0.44329	15.97813	4.40751	3.62521	0.17614
3.74	0.00937	0.03558	0.26333	8.47393	3.74	0.44263	16.15220	4.42002	3.65433	0.17314
3.76	0.00912	0.03489	0.26127	8.63020	3.76	0.44199	16.32720	4.43241	3.68360	0.17020

Überschallströmung eines idealen Gases für $\gamma = 1.4$

Isentrope Strömung					senkrechter Verdichtungsstoss					
Ma	p/p_0	ρ/ρ_0	T/T_0	A/A_*	Ma_1	Ma_2	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{02}/p_{01}
3.78	0.00887	0.03421	0.25922	8.78907	3.78	0.44136	16.50313	4.44466	3.71302	0.16731
3.80	0.00863	0.03355	0.25720	8.95059	3.80	0.44073	16.68000	4.45679	3.74260	0.16447
3.82	0.00840	0.03290	0.25520	9.11477	3.82	0.44012	16.85780	4.46879	3.77234	0.16168
3.84	0.00817	0.03227	0.25322	9.28167	3.84	0.43951	17.03653	4.48067	3.80223	0.15895
3.86	0.00795	0.03165	0.25126	9.45131	3.86	0.43891	17.21620	4.49243	3.83227	0.15626
3.88	0.00774	0.03104	0.24932	9.62373	3.88	0.43832	17.39680	4.50407	3.86246	0.15362
3.90	0.00753	0.03044	0.24740	9.79897	3.90	0.43774	17.57833	4.51559	3.89281	0.15103
3.92	0.00733	0.02986	0.24550	9.97707	3.92	0.43717	17.76080	4.52699	3.92332	0.14848
3.94	0.00714	0.02929	0.24362	10.15806	3.94	0.43661	17.94420	4.53827	3.95398	0.14598
3.96	0.00695	0.02874	0.24176	10.34197	3.96	0.43605	18.12853	4.54944	3.98479	0.14353
3.98	0.00676	0.02819	0.23992	10.52886	3.98	0.43550	18.31380	4.56049	4.01575	0.14112
4.00	0.00659	0.02766	0.23810	10.71875	4.00	0.43496	18.50000	4.57143	4.04688	0.13876
4.02	0.00641	0.02714	0.23629	10.91168	4.02	0.43443	18.68713	4.58226	4.07815	0.13643
4.04	0.00624	0.02663	0.23450	11.10770	4.04	0.43390	18.87520	4.59298	4.10958	0.13415
4.06	0.00608	0.02613	0.23274	11.30684	4.06	0.43338	19.06420	4.60359	4.14116	0.13191
4.08	0.00592	0.02564	0.23099	11.50915	4.08	0.43287	19.25413	4.61409	4.17290	0.12972
4.10	0.00577	0.02516	0.22925	11.71465	4.10	0.43236	19.44500	4.62448	4.20479	0.12756
4.12	0.00562	0.02470	0.22754	11.92340	4.12	0.43186	19.63680	4.63478	4.23684	0.12544
4.14	0.00547	0.02424	0.22584	12.13543	4.14	0.43137	19.82953	4.64496	4.26904	0.12335
4.16	0.00533	0.02379	0.22416	12.35079	4.16	0.43089	20.02320	4.65505	4.30140	0.12131
4.18	0.00520	0.02335	0.22250	12.56951	4.18	0.43041	20.21780	4.66503	4.33391	0.11930
4.20	0.00506	0.02292	0.22085	12.79164	4.20	0.42994	20.41333	4.67491	4.36657	0.11733
4.22	0.00493	0.02250	0.21922	13.01722	4.22	0.42947	20.60980	4.68470	4.39939	0.11540
4.24	0.00481	0.02209	0.21760	13.24629	4.24	0.42901	20.80720	4.69438	4.43236	0.11350
4.26	0.00468	0.02169	0.21601	13.47890	4.26	0.42856	21.00553	4.70397	4.46549	0.11163
4.28	0.00457	0.02129	0.21442	13.71509	4.28	0.42811	21.20480	4.71346	4.49877	0.10980
4.30	0.00445	0.02090	0.21286	13.95490	4.30	0.42767	21.40500	4.72286	4.53221	0.10800
4.32	0.00434	0.02052	0.21131	14.19838	4.32	0.42723	21.60613	4.73217	4.56580	0.10623
4.34	0.00423	0.02015	0.20977	14.44557	4.34	0.42680	21.80820	4.74138	4.59955	0.10450
4.36	0.00412	0.01979	0.20825	14.69652	4.36	0.42638	22.01120	4.75050	4.63345	0.10280
4.38	0.00402	0.01944	0.20674	14.95127	4.38	0.42596	22.21513	4.75953	4.66750	0.10112
4.40	0.00392	0.01909	0.20525	15.20987	4.40	0.42554	22.42000	4.76847	4.70171	0.09948
4.42	0.00382	0.01875	0.20378	15.47236	4.42	0.42514	22.62580	4.77733	4.73608	0.09787
4.44	0.00372	0.01841	0.20232	15.73879	4.44	0.42473	22.83253	4.78609	4.77060	0.09628
4.46	0.00363	0.01808	0.20087	16.00921	4.46	0.42433	23.04020	4.79477	4.80527	0.09473
4.48	0.00354	0.01776	0.19944	16.28366	4.48	0.42394	23.24880	4.80337	4.84010	0.09320
4.50	0.00346	0.01745	0.19802	16.56219	4.50	0.42355	23.45833	4.81188	4.87509	0.09170
4.52	0.00337	0.01714	0.19662	16.84486	4.52	0.42317	23.66880	4.82031	4.91022	0.09022
4.54	0.00329	0.01684	0.19522	17.13170	4.54	0.42279	23.88020	4.82866	4.94552	0.08878
4.56	0.00321	0.01654	0.19385	17.42277	4.56	0.42241	24.09253	4.83692	4.98097	0.08735
4.58	0.00313	0.01625	0.19248	17.71812	4.58	0.42205	24.30580	4.84511	5.01657	0.08596
4.60	0.00305	0.01597	0.19113	18.01779	4.60	0.42168	24.52000	4.85321	5.05233	0.08459
4.62	0.00298	0.01569	0.18979	18.32185	4.62	0.42132	24.73513	4.86124	5.08824	0.08324
4.64	0.00291	0.01542	0.18847	18.63032	4.64	0.42096	24.95120	4.86919	5.12430	0.08192
4.66	0.00284	0.01515	0.18716	18.94328	4.66	0.42061	25.16820	4.87706	5.16053	0.08062
4.68	0.00277	0.01489	0.18586	19.26076	4.68	0.42026	25.38613	4.88486	5.19690	0.07934
4.70	0.00270	0.01464	0.18457	19.58283	4.70	0.41992	25.60500	4.89258	5.23343	0.07809
4.72	0.00264	0.01438	0.18330	19.90953	4.72	0.41958	25.82480	4.90023	5.27012	0.07685
4.74	0.00257	0.01414	0.18203	20.24091	4.74	0.41925	26.04553	4.90780	5.30696	0.07564
4.76	0.00251	0.01390	0.18078	20.57703	4.76	0.41891	26.26720	4.91531	5.34396	0.07445
4.78	0.00245	0.01366	0.17954	20.91795	4.78	0.41859	26.48980	4.92274	5.38111	0.07329
4.80	0.00239	0.01343	0.17832	21.26371	4.80	0.41826	26.71333	4.93010	5.41842	0.07214
4.82	0.00234	0.01320	0.17710	21.61437	4.82	0.41794	26.93780	4.93739	5.45588	0.07101
4.84	0.00228	0.01298	0.17590	21.96999	4.84	0.41763	27.16320	4.94461	5.49349	0.06991
4.86	0.00223	0.01276	0.17471	22.33061	4.86	0.41731	27.38953	4.95177	5.53126	0.06882
4.88	0.00218	0.01254	0.17352	22.69631	4.88	0.41701	27.61680	4.95885	5.56919	0.06775
4.90	0.00213	0.01233	0.17235	23.06712	4.90	0.41670	27.84500	4.96587	5.60727	0.06670
4.92	0.00208	0.01213	0.17120	23.44311	4.92	0.41640	28.07413	4.97283	5.64551	0.06567
4.94	0.00203	0.01192	0.17005	23.82434	4.94	0.41610	28.30420	4.97972	5.68390	0.06465
4.96	0.00198	0.01173	0.16891	24.21086	4.96	0.41581	28.53520	4.98654	5.72244	0.06366
4.98	0.00193	0.01153	0.16778	24.60272	4.98	0.41552	28.76713	4.99330	5.76114	0.06268
5.00	0.00189	0.01134	0.16667	25.00000	5.00	0.41523	29.00000	5.00000	5.80000	0.06172

Stossdiagramm



Prandtl-Meyer Funktion $\nu(Ma)$ und Machscher Winkel $\alpha(Ma)$:
Ideales Gas mit $\gamma = 1.4$

Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$
1.00	0.00000	90.00000	4.00	65.78482	14.47751	7.00	90.97273	8.21321
1.02	0.12569	78.63512	4.02	66.04803	14.40392	7.02	91.07748	8.18965
1.04	0.35098	74.05763	4.04	66.30934	14.33109	7.04	91.18169	8.16623
1.06	0.63669	70.62996	4.06	66.56876	14.25899	7.06	91.28537	8.14293
1.08	0.96804	67.80839	4.08	66.82630	14.18763	7.08	91.38853	8.11978
1.10	1.33620	65.38002	4.10	67.08200	14.11698	7.10	91.49116	8.09675
1.12	1.73504	63.23450	4.12	67.33585	14.04704	7.12	91.59327	8.07385
1.14	2.15996	61.30559	4.14	67.58789	13.97780	7.14	91.69487	8.05109
1.16	2.60735	59.54969	4.16	67.83812	13.90924	7.16	91.79596	8.02845
1.18	3.07426	57.93621	4.18	68.08656	13.84136	7.18	91.89654	8.00594
1.20	3.55823	56.44269	4.20	68.33324	13.77415	7.20	91.99662	7.98356
1.22	4.05720	55.05199	4.22	68.57816	13.70759	7.22	92.09619	7.96130
1.24	4.56936	53.75068	4.24	68.82134	13.64168	7.24	92.19527	7.93916
1.26	5.09315	52.52800	4.26	69.06280	13.57640	7.26	92.29386	7.91715
1.28	5.62720	51.37517	4.28	69.30256	13.51176	7.28	92.39195	7.89526
1.30	6.17029	50.28486	4.30	69.54063	13.44773	7.30	92.48956	7.87349
1.32	6.72133	49.25095	4.32	69.77702	13.38431	7.32	92.58669	7.85185
1.34	7.27937	48.26818	4.34	70.01176	13.32149	7.34	92.68334	7.83032
1.36	7.84351	47.33207	4.36	70.24485	13.25927	7.36	92.77951	7.80891
1.38	8.41297	46.43872	4.38	70.47631	13.19762	7.38	92.87521	7.78761
1.40	8.98702	45.58469	4.40	70.70617	13.13656	7.40	92.97044	7.76643
1.42	9.56502	44.76700	4.42	70.93442	13.07606	7.42	93.06520	7.74537
1.44	10.14636	43.98296	4.44	71.16109	13.01612	7.44	93.15950	7.72442
1.46	10.73050	43.23022	4.46	71.38619	12.95674	7.46	93.25335	7.70359
1.48	11.31694	42.50664	4.48	71.60973	12.89789	7.48	93.34673	7.68287
1.50	11.90521	41.81031	4.50	71.83174	12.83959	7.50	93.43967	7.66226
1.52	12.49489	41.13951	4.52	72.05222	12.78181	7.52	93.53215	7.64176
1.54	13.08559	40.49266	4.54	72.27119	12.72456	7.54	93.62419	7.62136
1.56	13.67696	39.86834	4.56	72.48866	12.66782	7.56	93.71579	7.60108
1.58	14.26865	39.26525	4.58	72.70464	12.61159	7.58	93.80694	7.58091
1.60	14.86035	38.68219	4.60	72.91915	12.55586	7.60	93.89766	7.56084
1.62	15.45180	38.11806	4.62	73.13221	12.50062	7.62	93.98794	7.54088
1.64	16.04271	37.57187	4.64	73.34382	12.44587	7.64	94.07779	7.52103
1.66	16.63285	37.04267	4.66	73.55400	12.39161	7.66	94.16722	7.50128
1.68	17.22198	36.52961	4.68	73.76276	12.33782	7.68	94.25622	7.48163
1.70	17.80991	36.03188	4.70	73.97012	12.28449	7.70	94.34479	7.46209
1.72	18.39643	35.54874	4.72	74.17609	12.23163	7.72	94.43295	7.44265
1.74	18.98137	35.07951	4.74	74.38067	12.17923	7.74	94.52069	7.42331
1.76	19.56456	34.62354	4.76	74.58389	12.12728	7.76	94.60802	7.40407
1.78	20.14584	34.18022	4.78	74.78575	12.07577	7.78	94.69493	7.38493
1.80	20.72506	33.74899	4.80	74.98627	12.02470	7.80	94.78144	7.36589
1.82	21.30211	33.32933	4.82	75.18546	11.97406	7.82	94.86754	7.34694
1.84	21.87685	32.92073	4.84	75.38333	11.92386	7.84	94.95324	7.32810
1.86	22.44917	32.52275	4.86	75.57989	11.87407	7.86	95.03854	7.30935
1.88	23.01896	32.13493	4.88	75.77516	11.82470	7.88	95.12344	7.29070
1.90	23.58613	31.75686	4.90	75.96915	11.77574	7.90	95.20795	7.27214
1.92	24.15059	31.38817	4.92	76.16186	11.72719	7.92	95.29207	7.25368
1.94	24.71226	31.02847	4.94	76.35331	11.67905	7.94	95.37580	7.23531
1.96	25.27106	30.67742	4.96	76.54351	11.63129	7.96	95.45914	7.21703
1.98	25.82691	30.33471	4.98	76.73248	11.58393	7.98	95.54210	7.19885
2.00	26.37976	30.00000	5.00	76.92022	11.53696	8.00	95.62467	7.18076
2.02	26.92955	29.67301	5.02	77.10674	11.49037	8.02	95.70687	7.16275
2.04	27.47622	29.35347	5.04	77.29205	11.44415	8.04	95.78869	7.14484
2.06	28.01973	29.04110	5.06	77.47617	11.39831	8.06	95.87014	7.12702
2.08	28.56003	28.73565	5.08	77.65911	11.35284	8.08	95.95121	7.10929
2.10	29.09708	28.43689	5.10	77.84087	11.30773	8.10	96.03192	7.09165
2.12	29.63085	28.14458	5.12	78.02147	11.26298	8.12	96.11226	7.07409
2.14	30.16130	27.85851	5.14	78.20092	11.21858	8.14	96.19224	7.05662
2.16	30.68841	27.57847	5.16	78.37922	11.17454	8.16	96.27185	7.03924
2.18	31.21215	27.30426	5.18	78.55639	11.13084	8.18	96.35111	7.02194
2.20	31.73250	27.03569	5.20	78.73243	11.08749	8.20	96.43001	7.00473
2.22	32.24943	26.77259	5.22	78.90737	11.04447	8.22	96.50855	6.98760
2.24	32.76294	26.51477	5.24	79.08120	11.00179	8.24	96.58674	6.97055
2.26	33.27301	26.26209	5.26	79.25393	10.95944	8.26	96.66458	6.95359
2.28	33.77963	26.01437	5.28	79.42558	10.91742	8.28	96.74208	6.93671

Prandtl-Meyer Funktion $\nu(Ma)$ und Machscher Winkel $\alpha(Ma)$

Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$
2.30	34.28279	25.77146	5.30	79.59616	10.87572	8.30	96.81923	6.91992
2.32	34.78249	25.53323	5.32	79.76567	10.83433	8.32	96.89603	6.90320
2.34	35.27871	25.29953	5.34	79.93412	10.79327	8.34	96.97250	6.88657
2.36	35.77146	25.07023	5.36	80.10153	10.75252	8.36	97.04862	6.87001
2.38	36.26073	24.84520	5.38	80.26789	10.71207	8.38	97.12441	6.85354
2.40	36.74653	24.62432	5.40	80.43323	10.67193	8.40	97.19987	6.83714
2.42	37.22886	24.40747	5.42	80.59755	10.63209	8.42	97.27499	6.82082
2.44	37.70772	24.19454	5.44	80.76086	10.59255	8.44	97.34979	6.80458
2.46	38.18312	23.98541	5.46	80.92316	10.55330	8.46	97.42425	6.78842
2.48	38.65507	23.77999	5.48	81.08447	10.51435	8.48	97.49839	6.77234
2.50	39.12356	23.57818	5.50	81.24479	10.47568	8.50	97.57221	6.75633
2.52	39.58862	23.37987	5.52	81.40413	10.43730	8.52	97.64570	6.74039
2.54	40.05026	23.18497	5.54	81.56251	10.39920	8.54	97.71887	6.72454
2.56	40.50847	22.99339	5.56	81.71992	10.36138	8.56	97.79173	6.70875
2.58	40.96329	22.80505	5.58	81.87639	10.32383	8.58	97.86427	6.69304
2.60	41.41471	22.61986	5.60	82.03190	10.28656	8.60	97.93650	6.67741
2.62	41.86275	22.43775	5.62	82.18648	10.24956	8.62	98.00841	6.66184
2.64	42.30744	22.25862	5.64	82.34013	10.21282	8.64	98.08002	6.64635
2.66	42.74877	22.08241	5.66	82.49286	10.17635	8.66	98.15132	6.63093
2.68	43.18678	21.90905	5.68	82.64468	10.14014	8.68	98.22231	6.61559
2.70	43.62148	21.73846	5.70	82.79558	10.10418	8.70	98.29300	6.60031
2.72	44.05288	21.57058	5.72	82.94560	10.06848	8.72	98.36338	6.58510
2.74	44.48100	21.40534	5.74	83.09472	10.03304	8.74	98.43347	6.56997
2.76	44.90586	21.24267	5.76	83.24295	9.99784	8.76	98.50326	6.55490
2.78	45.32749	21.08252	5.78	83.39031	9.96290	8.78	98.57275	6.53991
2.80	45.74589	20.92483	5.80	83.53681	9.92819	8.80	98.64194	6.52498
2.82	46.16109	20.76954	5.82	83.68244	9.89373	8.82	98.71085	6.51012
2.84	46.57312	20.61659	5.84	83.82721	9.85951	8.84	98.77946	6.49533
2.86	46.98198	20.46593	5.86	83.97114	9.82552	8.86	98.84778	6.48060
2.88	47.38770	20.31751	5.88	84.11423	9.79177	8.88	98.91582	6.46594
2.90	47.79031	20.17127	5.90	84.25649	9.75826	8.90	98.98357	6.45135
2.92	48.18982	20.02717	5.92	84.39792	9.72497	8.92	99.05104	6.43683
2.94	48.58626	19.88516	5.94	84.53852	9.69191	8.94	99.11822	6.42236
2.96	48.97965	19.74520	5.96	84.67832	9.65907	8.96	99.18512	6.40797
2.98	49.37000	19.60723	5.98	84.81731	9.62646	8.98	99.25175	6.39364
3.00	49.75735	19.47122	6.00	84.95550	9.59407	9.00	99.31810	6.37937
3.02	50.14171	19.33712	6.02	85.09289	9.56189	9.02	99.38417	6.36517
3.04	50.52310	19.20490	6.04	85.22950	9.52994	9.04	99.44997	6.35103
3.06	50.90156	19.07450	6.06	85.36533	9.49819	9.06	99.51550	6.33695
3.08	51.27710	18.94591	6.08	85.50038	9.46666	9.08	99.58076	6.32293
3.10	51.64974	18.81906	6.10	85.63467	9.43534	9.10	99.64574	6.30898
3.12	52.01952	18.69394	6.12	85.76819	9.40422	9.12	99.71047	6.29509
3.14	52.38644	18.57050	6.14	85.90096	9.37331	9.14	99.77492	6.28126
3.16	52.75053	18.44872	6.16	86.03298	9.34261	9.16	99.83911	6.26749
3.18	53.11182	18.32854	6.18	86.16425	9.31210	9.18	99.90304	6.25378
3.20	53.47033	18.20996	6.20	86.29479	9.28180	9.20	99.96671	6.24013
3.22	53.82609	18.09292	6.22	86.42459	9.25169	9.22	100.03012	6.22654
3.24	54.17910	17.97741	6.24	86.55367	9.22178	9.24	100.09327	6.21301
3.26	54.52941	17.86339	6.26	86.68203	9.19206	9.26	100.15617	6.19954
3.28	54.87703	17.75083	6.28	86.80967	9.16253	9.28	100.21881	6.18613
3.30	55.22198	17.63970	6.30	86.93661	9.13320	9.30	100.28120	6.17277
3.32	55.56428	17.52998	6.32	87.06284	9.10405	9.32	100.34333	6.15947
3.34	55.90397	17.42164	6.34	87.18837	9.07509	9.34	100.40522	6.14623
3.36	56.24105	17.31465	6.36	87.31321	9.04631	9.36	100.46686	6.13305
3.38	56.57556	17.20899	6.38	87.43737	9.01771	9.38	100.52825	6.11992
3.40	56.90751	17.10464	6.40	87.56084	8.98930	9.40	100.58939	6.10685
3.42	57.23694	17.00156	6.42	87.68363	8.96106	9.42	100.65030	6.09384
3.44	57.56385	16.89973	6.44	87.80576	8.93301	9.44	100.71095	6.08088
3.46	57.88828	16.79913	6.46	87.92722	8.90513	9.46	100.77137	6.06797
3.48	58.21024	16.69975	6.48	88.04802	8.87742	9.48	100.83155	6.05512
3.50	58.52976	16.60155	6.50	88.16816	8.84988	9.50	100.89148	6.04233
3.52	58.84686	16.50452	6.52	88.28765	8.82252	9.52	100.95118	6.02959
3.54	59.16155	16.40863	6.54	88.40650	8.79532	9.54	101.01065	6.01690
3.56	59.47387	16.31386	6.56	88.52471	8.76830	9.56	101.06988	6.00427
3.58	59.78383	16.22020	6.58	88.64228	8.74144	9.58	101.12888	5.99169

Prandtl-Meyer Funktion $\nu(Ma)$ und Machscher Winkel $\alpha(Ma)$

Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$	Ma	$\nu(Ma)$	$\alpha(Ma)$
3.60	60.09146	16.12762	6.60	88.75922	8.71474	9.60	101.18764	5.97916
3.62	60.39677	16.03611	6.62	88.87554	8.68821	9.62	101.24618	5.96668
3.64	60.69978	15.94564	6.64	88.99123	8.66184	9.64	101.30448	5.95426
3.66	61.00052	15.85621	6.66	89.10631	8.63563	9.66	101.36256	5.94189
3.68	61.29902	15.76778	6.68	89.22078	8.60958	9.68	101.42041	5.92956
3.70	61.59527	15.68035	6.70	89.33464	8.58368	9.70	101.47804	5.91729
3.72	61.88932	15.59390	6.72	89.44789	8.55794	9.72	101.53544	5.90508
3.74	62.18118	15.50840	6.74	89.56055	8.53236	9.74	101.59262	5.89291
3.76	62.47086	15.42385	6.76	89.67262	8.50693	9.76	101.64958	5.88079
3.78	62.75840	15.34023	6.78	89.78410	8.48165	9.78	101.70631	5.86872
3.80	63.04380	15.25752	6.80	89.89499	8.45652	9.80	101.76283	5.85670
3.82	63.32709	15.17571	6.82	90.00530	8.43154	9.82	101.81913	5.84473
3.84	63.60829	15.09479	6.84	90.11504	8.40671	9.84	101.87522	5.83281
3.86	63.88741	15.01473	6.86	90.22421	8.38202	9.86	101.93109	5.82094
3.88	64.16448	14.93553	6.88	90.33281	8.35748	9.88	101.98674	5.80912
3.90	64.43952	14.85717	6.90	90.44085	8.33308	9.90	102.04219	5.79734
3.92	64.71254	14.77963	6.92	90.54832	8.30883	9.92	102.09742	5.78561
3.94	64.98356	14.70291	6.94	90.65525	8.28472	9.94	102.15244	5.77393
3.96	65.25260	14.62699	6.96	90.76162	8.26074	9.96	102.20725	5.76230
3.98	65.51968	14.55186	6.98	90.86745	8.23691	9.98	102.26186	5.75071
4.00	65.78482	14.47751	7.00	90.97273	8.21321	10.00	102.31625	5.73917

Werte für $\nu(Ma)$ und $\alpha(Ma)$ sind in Grad (°) angegeben

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Begleitend zur Vorlesung besonders empfohlen ist das Lehrbuch von Kundu und Cohen [7].
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Einige Web-Adressen zur Fluiddynamik:
www.efluids.com
www.desktopaero.com/appliedaero/welcome.html
www.grc.nasa.gov/WWW/K-12/airplane/index.html

Siehe auch Vorlesungs-Webseiten.
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