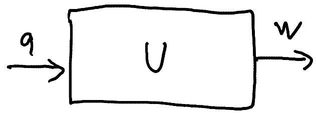
Thermodynamik
10 Januar 2017

A, B, C	total
a, b, c	pro masse
$\bar{a}, \bar{b}, \bar{c}$	pro mol

$U = q - w$ U: Innere Energie. q: wärme w: arbeit



$w = \int_{1}^{2} p\, dV$

$H = U + PV$ H: Enthalpie

$\Delta Q = m\, c\, \Delta T$

$c_v = \left(\frac{\partial u(p,T)}{\partial T}\right)_V$ const. V

$c_p = \left(\frac{\partial h(p,T)}{\partial T}\right)_p$ const. P

$k = \frac{c_p}{c_v}$

Nassdampf:

$u(x) = u_f + x(u_G - u_f) = (1-x)\,u_f + x\, u_G$

$s(x,T) = (1-x)\,s_f + x \cdot s_g$

$h(x) = h_f + x(h_G - h_f) = (1-x)\,h_f + x\, h_G$

$(h_{fg} = T \cdot s_{fg})$

$h_{fG} = h_g - h_f$ Verampfungsenthalpe / Verdampfungswärme

Flüssig: (inkompressibel)

$v(T,p) = v_f(T)$ $u(T,p) = u_f(T)$ $h(T,p) \approx h_f(T)$ $s(T,p) \approx s_f(T)$

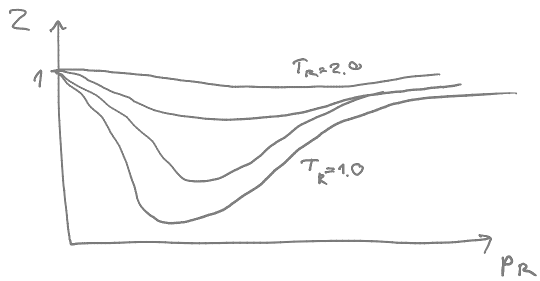
$c = c_p = c_v = \frac{du}{dT}$ $\Delta u = c \cdot \Delta T$ $\Delta h = c \cdot \Delta T + v\,\Delta p$ $\Delta s = c \cdot \ln\left(\frac{T_2}{T_1}\right)$

Gasse:

$\bar{R} = \lim_{p \to 0} \frac{p\bar{v}}{T} = 8.314\ \mathrm{J/mol\,K}$

$Z(p,T) = \frac{p\bar{v}}{RT} = \frac{pv}{RT}$ (Realgass-Faktor)

$pV = Z(p,T)\,RT$



$p_R = p/p_c$ $T_R = T/T_c$ $V_R' = \bar{V} / \bar{R}(T_c/p_c)$ (c: Kritischer Punkt)

Ideale Gase: $(Z=1)$

$pV = mRT \qquad p\bar{v} = \bar{R}T \qquad pV = n\bar{R}T$

$[R] = \frac{bar\, m^3}{kg\, K} \qquad [V] = m^3$

$[\bar{R}] = \frac{bar\, m^3}{mol\, K} \qquad [\bar{v}] = m^3/mol$

$h = h(T) \qquad u = u(T)$

$c_v(T) = \frac{du}{dT} \qquad c_p(T) = \frac{dh}{dT} \qquad \bar{c}_p - \bar{c}_v = \bar{R} \qquad \kappa = \frac{\bar{c}_p}{\bar{c}_v} = \frac{c_p}{c_v}$

$\Delta u = c_v \Delta T \qquad \Delta h = c_p \Delta T$

$$\Delta s = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_1}{V_2}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) = s_2^\circ - s_1^\circ - \bar{R} \ln\left(\frac{p_2}{p_1}\right)$$

$pV^n = \text{Konst.}$ (polytrope Zustandsänderung) $\qquad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$

- $n=0 \rightarrow p$ Konst. (Isobar)
- $n=1 \rightarrow T$ Konst. (Isotherm)
- $n=\infty \rightarrow V$ Konst. (Isochor)
- $n=\kappa \rightarrow S$ Konst. (Isentrop) (Adiabat reversibel)

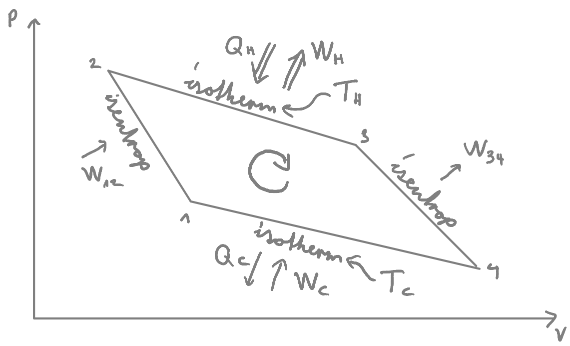
Adiabat: $\Delta Q = 0$ $\qquad$ Reversibel: $S_{erz} = 0$

$T v^{n-1} = \text{Konst} \qquad T p^{\frac{1-n}{n}} = \text{Konst.}$

$$W = \int_1^2 p\, dV = \begin{cases} n=\infty: & 0 \\ n=1: & n\bar{R}T\ln\left(\frac{V_2}{V_1}\right) \\ n\neq 1: & n\bar{R}(T_2-T_1)/(1-n) \end{cases}$$

mol !!! ?

Carnot Kreisprozess (idealgas)



$W_{23} = -Q_{23} = n\bar{R}T_H \ln\left(\frac{V_3}{V_2}\right)$ $\qquad$ (da $U(T_2) = U(T_3)$)

$W_{41} = -Q_{41} = n\bar{R}T_C \ln\left(\frac{V_1}{V_4}\right)$

$W_{12} = -W_{34} = n\bar{R}(T_2 - T_1)/(1-\kappa)$

$Q_{12} = Q_{34} = 0$

$\eta = \frac{W_{23} - W_{41}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{T_{1,4}}{T_{2,3}}$

$\left(\begin{array}{l} \varepsilon_{KM} = \frac{Q_C}{\Delta W} = \frac{T_C}{T_H - T_C} = \varepsilon_{WP} - 1 \\ \varepsilon_{WP} = \frac{Q_H}{\Delta W} = \frac{T_H}{T_H - T_C} = 1 + \varepsilon_{KM} \end{array} \right)$

Energiebilanz

- $\dot m_i = 0 \quad \forall i$: $\quad \Delta E = Q - W \qquad \frac{dE}{dt} = \dot Q - \dot W$
- $\dot m_i \neq 0 \quad \forall i$: $\quad \Delta E = Q - W + \sum_{i=1}^{n} \Delta m_{i,e}\left(h_{i,e} + \frac{V_{i,e}^2}{2} + g z_{i,e}\right) - \sum_{j=1}^{k} \Delta m_{j,a}\left(h_{j,a} + \frac{V_{j,a}^2}{2} + g z_{j,a}\right)$
- $\sum \dot m_{i,e} = \sum \dot m_{j,a}$: $\quad \frac{dE}{dt} = \dot Q - \dot W + \sum_{i=1}^{n} \dot m_{i,e}\left(h_{i,e} + \underbrace{\frac{V_{i,e}^2}{2} + g z_{i,e}}_{\text{vernachlässigbar}}\right) - \sum_{j=1}^{k} \dot m_{j,a}\left(h_{j,a} + \underbrace{\frac{V_{j,a}^2}{2} + g z_{j,a}}_{\text{vernachlässigbar}}\right)$

Entropie

$\frac{\Delta Q}{T} = \Delta S \qquad \oint \frac{dQ}{T} = -\Delta S \leq 0 \quad \left(= \frac{Q_H}{T_H} - \frac{Q_K}{T_K} = 0 \text{ für Carnot} \right)$

$TdS = \overbrace{dU + pdV}^{dQ} = dH - Vdp \qquad \left(T = \left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_p \; ; \; p = -\left(\frac{\partial U}{\partial V}\right)_S \; ; \; V = \left(\frac{\partial H}{\partial p}\right)_S \right)$

Entropiebilanz

(→ FS)

P-v, T-v, T-s, H-s Diagram