IRBM 40 (2019) 235–243

Contents lists available at ScienceDirect

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Original Article

A Modified DWT-SVD Algorithm for T1-w Brain MR Images Contrast Enhancement

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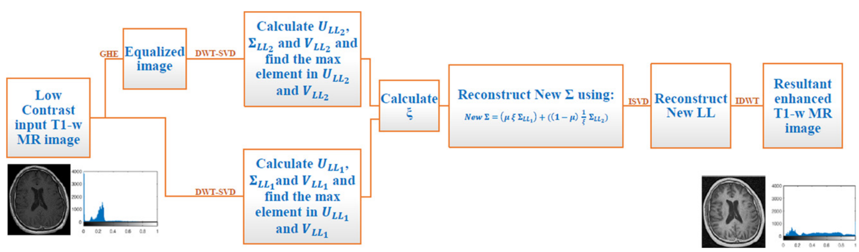
HIGHLIGHTS

- In this paper, a modified DWT-SVD method is proposed.
- Proposed method is used for the enhancement of low contrast T1-w brain MR images.
- The use of an adjustable singular value matrix equation proves better results.
- Proposed method performs very efficiently in all types of low contrast images.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 5 December 2017
Received in revised form 5 January 2019
Accepted 26 April 2019
Available online 9 July 2019

Keywords:
Low contrast brain MRI
T1-w modality
Contrast enhancement
DWT-SVD

ABSTRACT

Background: Image contrast enhancement is considered as the most useful technique permitting a better appearance of the low contrast images. This paper presents a modified Discret Wavelet Transform - Singular Value Decomposition (DWT-SVD) approach for the enhancement of low contrast Brain MR Images used for brain tissues exploration.

Methods: The proposed technique is processed as follows: first of all, we consider low contrast T1-weighted MRI slices as input images (A1) on which we apply General Histogram Equalization (GHE) algorithm to have equalized images referred as (A2) having zero as mean and one as variance. Secondly, by using the Discrete Wavelet Transform (DWT) algorithm, both (A1) and (A2) are divided into low and high frequency sub-bands. On the low frequency (LL1) and (LL2) sub-bands, SVD is processed in order to generate three matrix factorization (U, V and Σ) where the maximums of (U) and (V) matrix are used for the estimation of a correction coefficient (ξ). Our contribution in this paper is to estimate the new singular value matrix (New Σ) using a weighted sum of both original and equalized singular value matrix thanks to an adjustable parameter (μ) for the targeted low contrast images. This parameter, ranged between 0.05 and 0.95, is determined empirically according to the input low contrast image. Finally, the enhanced resulting image is easily reconstructed using the Inverse SVD (ISVD) and the Inverse DWT (IDWT) processes.

Results: The database considered in our research consisted of 120 MR brain images where T1-weighted MR brain modality are selected for the contrast enhancement process. Considering the qualitative results, our proposed contrast enhancement method have shown better distinction between brain tissues and have preserved all White Matter (WM), Gray Matter (GM) and Cerebro-Spinal Fluid (CSF) pixel edges. In fact, histogram plots of images enhanced by proposed method covered all the gray level intensities. For the quantitative results, proposed method gives the highest PSNR, QRCM, SSIM, FSIM and EME values and the lowest AMBE values for (μ) equal to 0.65 as comparing to the rest of methods. These results signifies that proposed contrast enhancement method have provided greater image quality with preservation of image structure, feature and brightness.

Conclusion: Proposed method improved performance of contrast enhancement image without creating unwanted artifact and without destroying image edge information or affecting the specificities of brain

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https://doi.org/10.1016/j.irbm.2019.04.007

tissues. This is due to the use of an empirically (μ) parameter adjustable according to the input MR images. Hence, the proposed approach is appropriate for enhancing contrast of huge type of low contrast images.

1. Introduction

Brain Magnetic Resonance Imaging (MRI) is considered as a helpful and secure medical imaging technique permitting efficient brain diagnosis and exploration by discovering brain maturity and abnormalities [1]. These high-resolution images are frequently affected by noise, artefacts, speckles, poor quality [2], etc. Low contrast, as well, is one of the main problems that disturb the production of medical images [3]. During the past few years, a lot of research in the area of medical image contrast enhancement were investigated in purpose of increasing images' contrast and brightness and such clinical need was in fact specifically for MRI modalities and Computerized Tomography CT images [4–6]. Most of the contrast enhancement research works could be classified on two classes of methods: The spatial domain enhancement techniques, which operate directly on images' pixels and the frequency domain enhancement techniques operating on images' Fourier Transform. Global Histogram Equalization (GHE) method was therefore one of the spatial domain method utilizing the linear cumulative histogram of an input image and distributes its pixel values over its dynamic intensity range [7]. The Local Histogram Equalization (LHE) method can enhance locally the overall image contrast, but it is more complex [8,9]. However, both GHE and LHE techniques are not well suited for all types of images because they over-enhance the output image, leading to an unnatural look. To deal with this over-enhancement, Yang et al. proposed the Local Bi-Histogram Equalization (LBHE) technique [5] in addition to Vidaurrazaga et al. who used the noisy wavelet coefficients based enhancement approach for enhancing medical images [6]. To preserves the mean brightness and enhance the contrast, Yeong-Taeg used the Brightness preserving Bi-Histogram Equalization (BBHE) [10], however Ibrahim et al. as well as Sun et al. used the Brightness preserving dynamic histogram equalization (BPDHE) which smooth the histogram using the local maximum [11,12]. Among time-frequency domain, wavelet transform domain like Singular Value Decomposition (SVD) are appeared to overcome the limitations associated with the HE methods [13–15], and [16]. Bhandari et al. [17,21] as well as Kumar et al. [18] proposed the DWT–SVD and DCT-SVD contrast enhancement techniques. Those methods are based on updating the singular value matrix, which contains intensity information of the input image, on the low sub-band input image. In these recent works, SVD was used to obtain the ratio of highest singular value of the created equalized matrix, generated by GHE, over a normalized input image. To alleviate the weakness existed in SVD conventional techniques, Atta et al. [19] uses a weighted sum of the singular value matrix of the input and equalized images in order to generate the new singular value matrix of the equalized image.

In this sense, because of brain MR images suffer from low contrast that can disturb during clinical image analysis, contrast enhancement scheme will give way to better separation in terms of intensity between different brain tissues, which are the Gray Matter (GM), White Matter (WM) and the cerebrospinal fluid (CSF).

In this paper, we are interested in developing a novel approach for enhancing the low contrast for the T1-w brain MRI modality using a modified DWT-SVD technique. More specifically, we first perform GHE of original image to have equalized image having zero as mean and one as variance. Both original and equalized images are decomposed, using Discret Wavelet Transform (DWT), into four sub-band components, known as Low-Low (LL), Low-High (LH), High-Low (HL), and High-High (HH) frequencies. SVD is applied on LL sub-bands of both original and equalized image to generate three-matrix factorization (U, V and Σ), for both of them, useful for the calculation of the correction coefficient (ξ). The fact of scaling up the singular value matrix, using an adequate correction factor (ξ), will enhance the low contrast T1-w MR Images. Our contribution is this work is to propose a new equation to compute the new enhanced singular value matrix image by the use of an empirically adjustable parameter (μ). As a final step, the resultant enhanced T1-w MR image was reconstructed using Inverse DWT (IDWT) processes. Therefore, the proposed technique could enhance, brighten and differentiate between different brain tissues with the promise of preserving unique edges and intensities information.

The rest of the paper is organized as follows: Section 2 details the proposed enhancement technique, section 3 discusses the qualitative results of the proposed method supported by some evaluation metrics. Finally, conclusion is given in Section 4.

2. Proposed method

Histogram plots offers useful information about image contrast variation. In medical images, this variation suffers from low contrast problem that is usually caused by pixels gray levels concentration. Therefore, SVD can overcome this problem and can ensure image intensities' adjustment in order to enhance such low contrast images. Hence, we consider low contrast T1-w brain MR images as input images referred as (A_1) on which we apply the GHE method to have equalized images referred as (A_2) (having zero as mean and one as variance). By using the DWT [20] for the original and the processed equalized images; (A_1) and (A_2), we obtain four frequency sub-bands labelled as (LL_1), (LH_1), (HL_1), (HH_1) and (LL_2), (LH_2), (HL_2), (HH_2) respectively as it is shown in Fig. 1. Among these sub-bands, SVD would be applied on the (LL_1) and (LL_2) of the original and the equalized images respectively. The reason behind using (LL) sub-band was in fact to protect edge information presented in the high frequencies and modify the intensity information concentrated in low frequency.

SVD approach can lead to a matrix factorization in the form of:

$$A = U.\Sigma.V^{\top} \tag{1}$$

Where U and V are m^*m and n^*n orthogonal matrix respectively and Σ is m^*n diagonal matrix containing singular values on its diagonal.

To equalize intensity information of the image, we apply SVD method which uses the ratio of the largest singular value of the generated equalized matrix, over a normalized input image [19] to generate a correction coefficient (ξ) formulated as:

$$\xi = \frac{\max(\Sigma_{2\ (\mu=0, var=1)})}{\max(\Sigma_1)} \tag{2}$$

where ($\Sigma_{2\ (\mu=0,\ \mathrm{var}=1)}$) is the singular value matrix of the equalized image and (Σ_1) is the singular value matrix of the input image.

Different methods have been developed in the literature to calculate the correction coefficient (ξ) in order to estimate the singular value matrix (Σ), [21,17,7], and [19]. Atta et al. [19] scales up

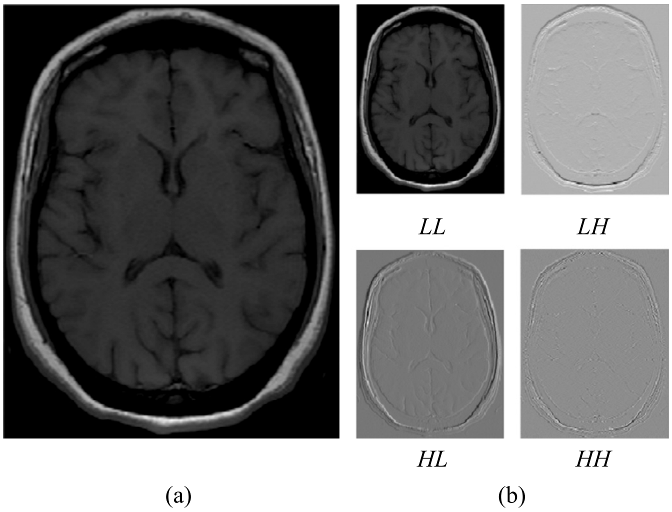

Fig. 1. Image DWT decomposition; (a) Original T1-w MR image, (b) LL, LH, HL and HH sub-band images.

the singular values of the input image (Σ_{LL_2}) using the correction coefficient (ξ) which is obtained as:

$$\xi = \frac{\max(\Sigma_{LL_2})}{\max(\Sigma_{LL_1})} \tag{3}$$

Where (Σ_{LL_2}) is the singular matrix of the equalized sub-band image using the GHE.

However, in 2014, Bhandari et al. [17,21] have calculated the correction coefficient (ξ) by using the maximum element in (U_{LL_2}) and (U_{LL_1}) calculated from equalized and input low frequency sub-band images respectively according to Eq. (4) given as:

$$\xi = \frac{\max(U_{LL_2})}{\max(U_{LL_1})} \tag{4}$$

In 2015, Bhandari et al. [7] have modified the previous equation to include the maximum elements in (U_{LL_1}), (V_{LL_1}) and (U_{LL_2}), (V_{LL_2}) derived from (LL1) and (LL2) sub-band images respectively, which is written as:

$$\xi = \frac{\max\left(U_{LL_2}\right) + \max(V_{LL_2})}{\max\left(U_{LL_1}\right) + \max(V_{LL_1})} \tag{5}$$

Most commonly, the new singular matrix (New Σ) is obtained by scaling up the singular matrix of the equalized sub-band image (Σ_{LL_2}) using the correction coefficient (ξ) [21,17,7] according to Eq. (6):

$$New\ \Sigma = \xi.\Sigma_{LL_2} \tag{6}$$

The calculation of the new singular matrix (New Σ) of Eq. (6) using the correction coefficient of Eq. (4) and Eq. (5) is referred in this paper as "Bhandari et al., 2014 method" and "Bhandari et al., 2015 method" respectively.

In 2015, Atta et al. [19] has modified the (New Σ) by a weighted sum of the singular matrix of the input and the equalized image respectively multiplied by 0.5 and is given in Eq. (7). In this equation, the correction coefficient (ξ) is calculated using Eq. (3). This method is referred in this paper as "Atta et al., 2015 method".

$$New\ \Sigma = 0.5(\xi.\Sigma_{LL_1} + \frac{1}{\xi}.\Sigma_{LL_2}) \tag{7}$$

However, this method cannot be applicable for all images because multiplying the singular matrix of both input and equalized images by a constant and fixed factor (0.5) is not applicable for all types of input image. To overcome this limitation, in our present development, we propose a new adjustable method to calculate the new singular matrix (New Σ). This new Equation uses an empirically adjustable parameter (μ) and the modified singular matrix (New Σ) is computed according to Eq. (8):

$$New\ \Sigma = (\mu.\xi.\Sigma_{LL_1}) + ((1-\mu).\frac{1}{\xi}.\Sigma_{LL_2}) \tag{8}$$

In our case, the coefficient (ξ) is calculated according to Eq. (5) and the values of (μ) is ranged between 0.05 and 0.95. We note here that this parameter will affect both singular matrix of the input sub-band image (Σ_{LL_1}) and singular matrix of the equalized sub-band image (Σ_{LL_2}). The role of (μ) is to adjust the contrast of low contrast images with a greater flexibility: It can be modified at each step of contrast enhancement process, and the optimum value of (μ) is concluded depending on each input image. The last step is to reconstruct the (New LL) sub-band image using ISVD according to Eq. (9), and computing the resultant enhanced image using IDWT:

$$New\ LL = U_{LL_2}.New\ \Sigma.V_{LL_2}^{\top} \tag{9}$$

The following steps as well as Fig. 2 are therefore attentively conceived to execute main computational process of the proposed algorithm for enhancing the low contrast brain T1-w MRI images:

Step 1: *In the first step, a low contrast T1-w MRI image has been selected for the contrast enhancement approach.*

Step 2: *Equalizing T1-w MRI image* ($A1$) *using GHE technique in order to generate equalized image* ($A2$) *with zero as mean and one as variance.*

Step 3: *After equalization, computing DWT decomposition of both original image* ($A1$) *and equalized image* ($A2$) *in order to divide each image into four frequency sub-band images which are (LL1), (LH1), (HL1), (HH1) and (LL2), (LH2), (HL2), (HH2) respectively.*

Step 4: *After getting DWT frequency sub-band images, SVD is applied on both low frequency components (LL1) and (LL2) leading to three-matrix factorization U, V and* Σ.

Step 5: *Hereafter, calculating the maximum of* ($U1$) *and* ($V1$) *from (LL1) and the maximum of* ($U2$) *and* ($V2$) *from (LL2).*

Step 6: *After getting maximum values, calculating the correction factor* (ξ) *using Eq. (5).*

Step 7: *Estimating the (New* Σ*) using Eq. (8), which is a weighted sum of the singular value matrix of the original* (Σ_{LL_1}) *and the equalized sub-band images* (Σ_{LL_2}) *multiplied by an adjustable parameter* (μ) *empirically determined between the range of* 0.05 and 0.95.

Step 8: *Applying Inverse SVD using Eq. (9) to generate (New LL).*

Step 9: *Finally, applying Inverse DWT to obtain enhanced resultant T1-w MRI image.*

3. Results

3.1. Materials and database

In these experiments, we used 120 low contrast T1-w MR brain images for the contrast enhancement process. Our processed method was implemented in a MatLab environment and during the entire simulation process, we have considered 256 × 256 as pixel resolutions and with five as slice thickness. As for the DWT process, we have choose the 9/7 tap Daubechies wavelet function as the mother wavelet that performed in fact convenient results.

3.2. Qualitative analysis

The efficiency of our proposed method was compared with three recent state of the art detailed above [7,17,19] where the

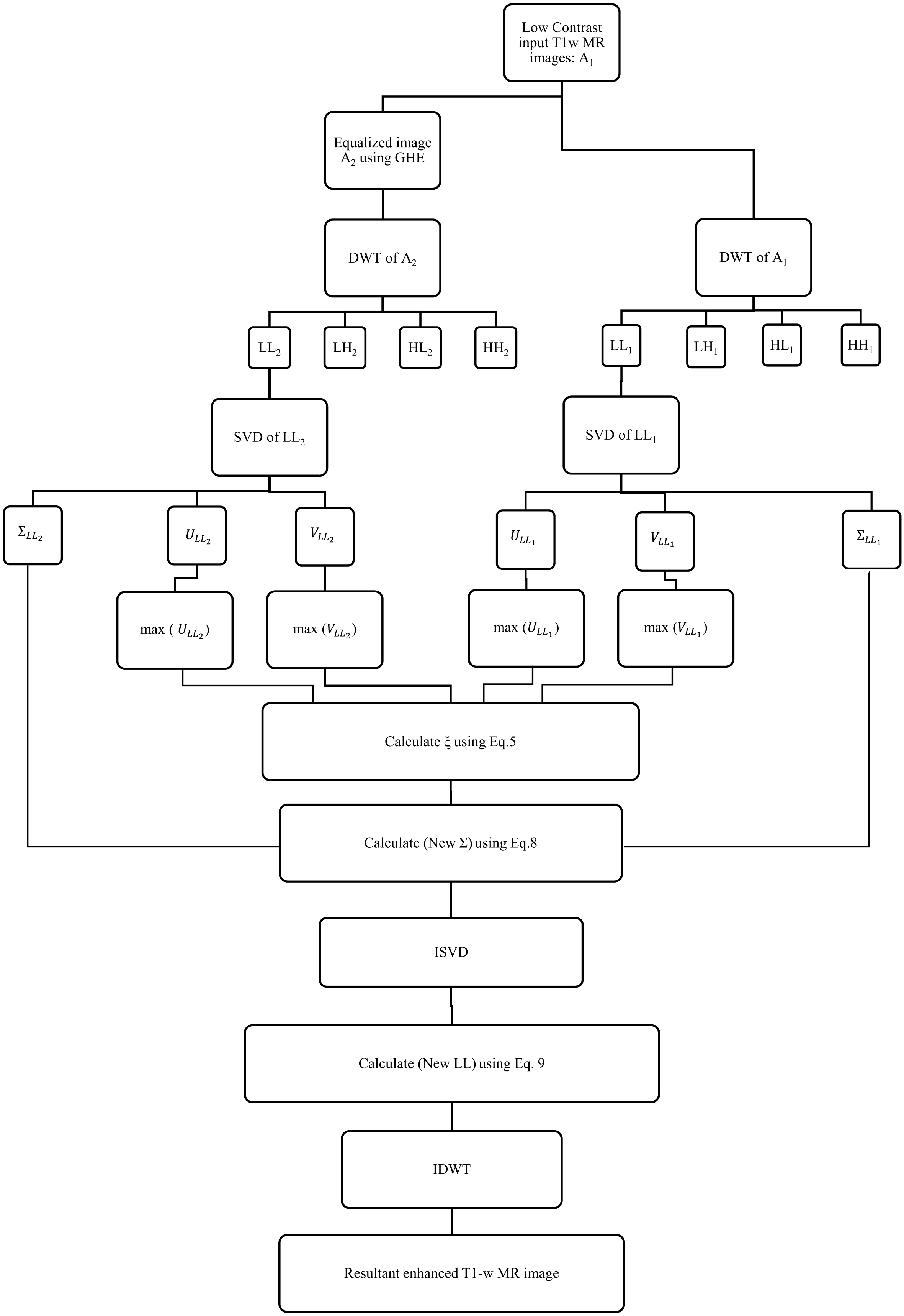

Fig. 2. Global block diagram of the proposed enhancement algorithm.

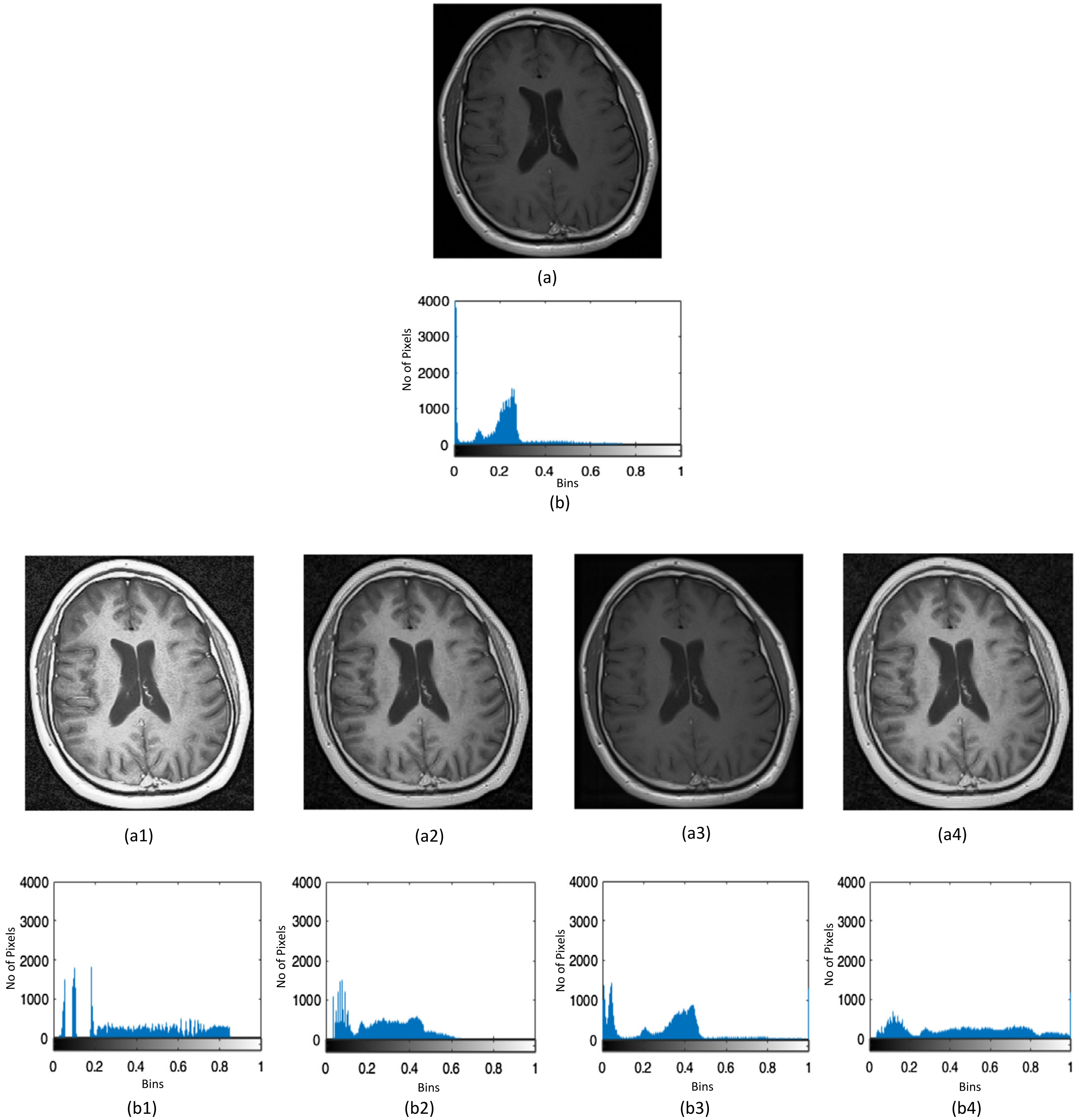

Fig. 3. Visual qualities of original and enhanced images using various enhancement methods (Subject 007): (a) Input low-contrast T1-w image, (b) Histogram of input image, (a1), (a2), (a3) and (a4) Enhanced image using (Bhandari et al., 2014) method [17], (Bhandari et al., 2015) method [7], (Atta et al., 2015) method [19] and proposed method respectively. (b1), (b2), (b3) and (b4) Histograms of corresponding enhanced images. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

subjective qualities of the used techniques are shown in Fig. 3 and 4. Fig. 3-a) and Fig. 4-a) show the input low contrast T1-w images for two different subjects. Their enhanced images using different contrast enhancement methods are shown in Fig. 3-a1), 3-a2), 3-a3) and 3-a4) for subject (007) and Fig. 4-a1), 4-a2), 4-a3) and 4-a4) for subject (081). Histogram graphs accompanies each original and enhanced images.

Our input images are considered as low contrast images because their histogram plots has limited dynamic range as it is shown in Fig. 3-b) and Fig. 4-b). In the histogram plot of Fig. 3-b1) and Fig. 4-b1), we remark that (Bhandari et al., 2014) method [17] has stretched pixel value intensity of the input image to cover a great parts of the gray scale range. However, images resulting from this contrast enhancement method look too brighten especially at the skull region. This unnatural look is due to the over-enhancement. Moreover, contrast of images processed by (Bhandari et al., 2015) method [7] are enhanced partially. On the other hand,

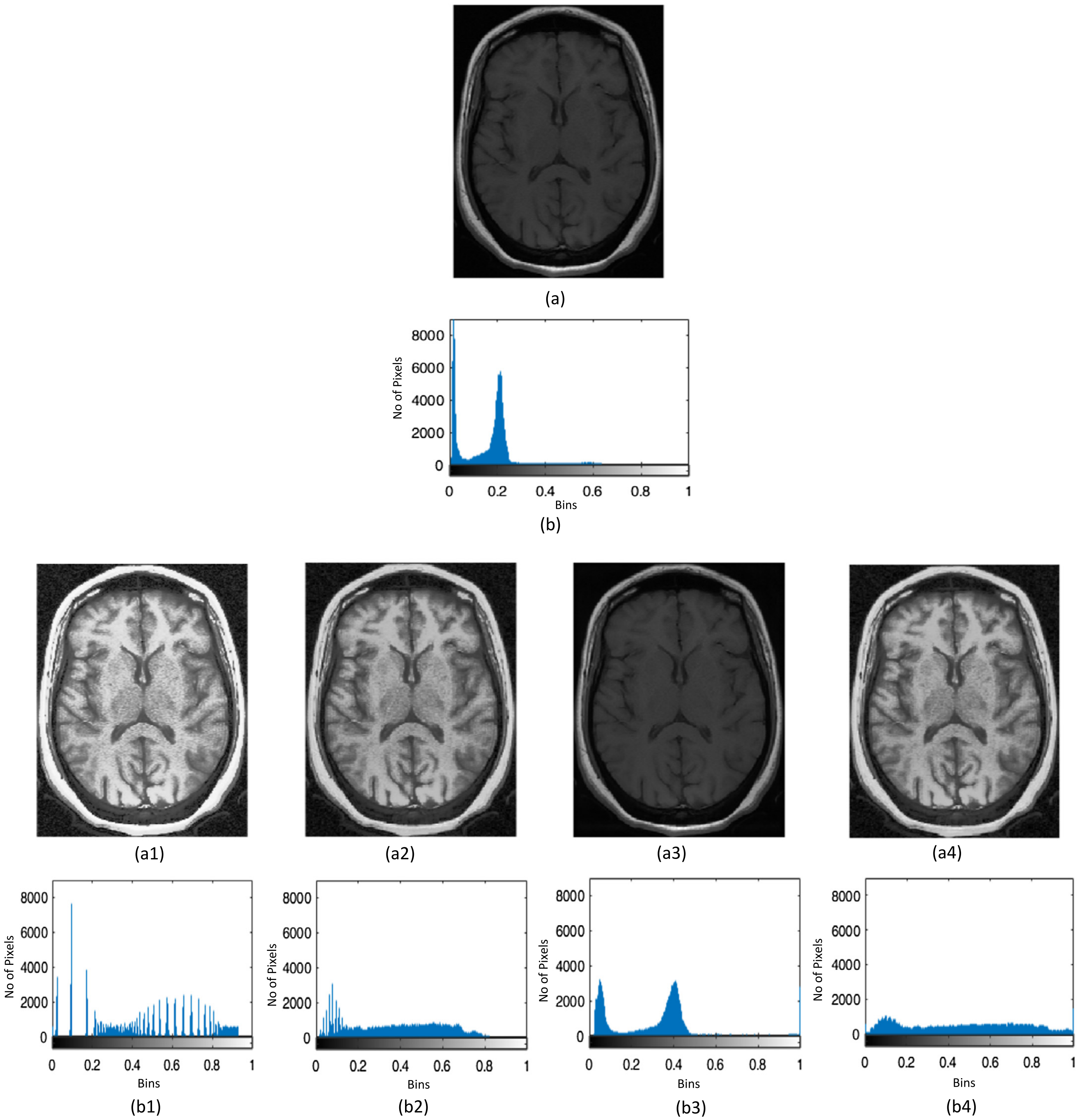

Fig. 4. Visual qualities of original and enhanced images using various enhancement methods (Subject 081): (a) Input low-contrast T1-w image, (b) Histogram of input image, (a1), (a2), (a3) and (a4) Enhanced image using (Bhandari et al., 2014) method [17], (Bhandari et al., 2015) method [7], (Atta et al., 2015) method [19] and proposed method respectively. (b1), (b2), (b3) and (b4) Histograms of corresponding enhanced images.

(Atta et al., 2015) method [19] has not made a huge change to the contrast as comparing to the input image.

Unlike that, proposed method has provided significantly better contrast and details for human visual perception than the rest of methods under the influence of the histogram graph which has occupied all gray values intensities. As a consequence, proposed contrast enhancement method have shown better distinction between brain tissues and have preserved all (WM), (GM) and (CSF) pixel edges which is clearly shown in Fig. 3-a4) and Fig. 4-a4). Thus, the success of the proposed method was in fact due to the use of an adjustable (μ) parameter with a weighted sum of the input and the equalized singular value matrix.

3.3. Quantitative analysis

In order to support the qualitative analysis presented in the last section, a quantitative analysis is therefore important. For this purpose, different metrics like PSNR, QRCM, SSIM, FSIM, AMBE and EME are considered. All the 120 T1-w brain low contrast MR images are processed with the previously discussed methods and the

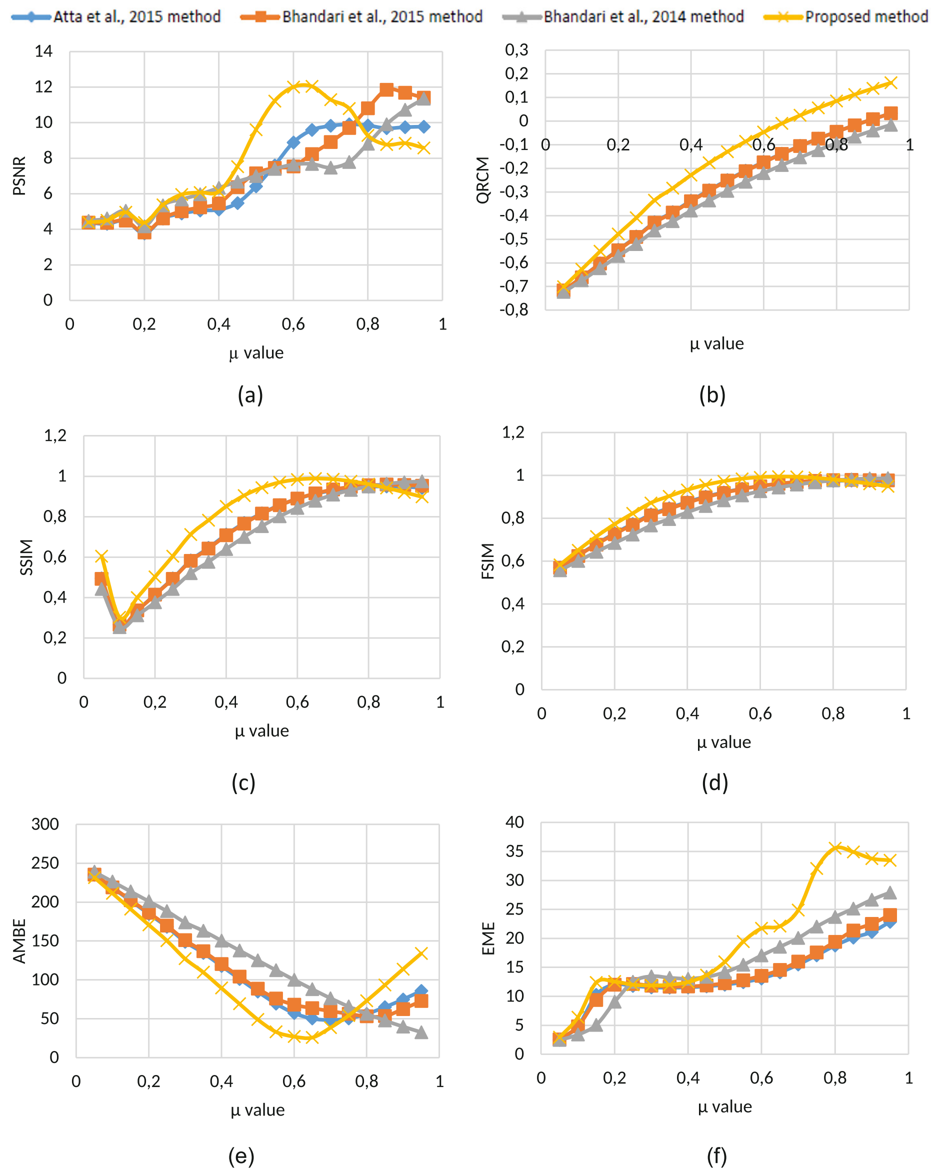

Fig. 5. Representation of PSNR, QRCM, SSIM, FSIM, AMBE and EME values for the (Bhandari, et al., 2014) method, (Bhandari et al., 2015) method, (Atta et al., 2015) method and Proposed method.

mean values for indicated quantitative metrics are computed and illustrated in Fig. 5.

- *Measure of Peak Signal-to-Noise Ratio* (*PSNR*) [22] measures the ratio between the maximum possible value of a signal and the power of distorting noise that affects the quality of its representation. In Fig. 5-a), while the value of (μ) is 0.65, PSNR value of proposed method is the highest as compared to the other methods and it is up to 12 dB. This involves better quality of the image as well as best noise reduction.
- *Measure of Quality-aware Relative Contrast Measure* (*QRCM*) [23] gives idea about contrast measure and image quality together. QRCM penalizes the contrast changes when there is a significant difference between the gradients of original and enhanced images. This is happened generally when there are visual distortions on the processed image. Thus, QRCM does not only measure the relative change of contrast but also takes the distortion introduced on the enhanced image relative to the considered original image. Negative QRCM values indicate that considered contrast enhancement algorithm distorted enhanced image as comparing to the original one. QRCM values are improved when (μ) parameter is increased, Fig. 5-b) indicates lower distortions in enhanced images and therefore better performance for contrast enhancement algorithm.
- *Structure similarity index measurement* (*SSIM*) [24] is a metric based on measuring the similarity between two images. The SSIM can take values in $[-1, 1]$ range. Higher value suggests the better preservation of image structures. In Fig. 5-c), it is clearly noticed that our proposed method gives the highest SSIM value as comparing to the rest of methods and it is about 0.988 for (μ) equal to 0.65.
- *Feature similarity index measurement* (*FSIM*) [25] guides us to determine how well has the image feature been preserved. Higher value suggests the better preservation of image features. In Fig. 5-d), FSIM values of proposed method increase with increasing (μ) values and gives the highest value equal to 0.991 for (μ) equal to 0.65 leading to better performance of proposed method.
- *Absolute Mean Brightness Error* (*AMBE*) [26,27] presents the degree of brightness preservation between mean of input and enhanced images. Better brightness preservation is concluded for a lower AMBE value. In Fig. 5-e), AMBE value of proposed

method is the lowest and is equal to 25.84. AMBE values decrease when increasing (μ) parameter until a value of 0.65 from which AMBE values increases gradually. Thus, the proposed algorithm best outperforms the image brightness.

- *Measure of enhancement by entropy* (*EME*) [28] is used to evaluate the quality of contrast enhancement. A higher value of EME suggests a better contrast. The proposed algorithm gives improved EME values. However for (μ) values less than 0.45, the change in EME is not significant as illustrated in Fig. 5-f). For values greater than 0.6, proposed method gives increased EME values as compared to other methods. Hence, the images enhanced by the proposed algorithm have natural look with great visibility of details.

To conclude, it is clearly noticed that the proposed method yield superior EME values for ($\mu = 0.65$) suggesting improvement in contrast. Important QRCM values lead to a better image quality. Brain tissues edges are conserved without added distortions justified by the highest SSIM value. For this (μ) value, the proposed method preserves image structure with the highest FSIM value. On the other hand, it preserves the image brightness with the lowest AMBE value as compared to other methods.

4. Conclusion

In this research, a novel T1-w MRI contrast enhancement technique based on a modified DWT-SVD method was proposed. Such enhancement is considered as clinically helpful for brain exploration and brain tissue differentiation. In this proposed technique, we have applied the GHE algorithm on the original image (A1) to generate equalized image. By the use of DWT, both original and equalized images are divided into four frequency sub-bands and are processed by SVD to generate three matrix factorization useful for the generation of the correction coefficient (ξ). Our contribution in this work is to include a new adjustable parameter (μ) while estimating the new enhanced singular value matrix. This parameter, ranged between 0.05 and 0.95, was determined empirically according to the input low contrast images. Finally, we reconstruct the final enhanced image using IDWT and ISVD. Our proposed technique was tested on 120 T1-w MR images. Their PSNR, QRCM, SSIM, FSIM, AMBE and EME values has revealed superiority of proposed method over conventional and state of the art techniques especially for (μ) value equal to 0.65. Thus, proposed method improves performance of contrast enhancement image without creating unwanted artifact and without destroying image and information or affecting the specificities of brain tissues. Therefore, enhanced images will have natural look with preserved edge and brightness.

Informed consent and patient details

The authors declare that this report does not contain any personal information that could lead to the identification of the patient(s).

Funding

This work did not receive any grant from funding agencies in the public, commercial, or not-for-profit sectors.

Author contributions

All authors attest that they meet the current International Committee of Medical Journal Editors (ICMJE) criteria for Authorship.

Declaration of Competing Interest

The authors declare that they have no known competing financial or personal relationships that could be viewed as influencing the work reported in this paper.

Acknowledgements

Authors would like to thank the Neurology and the Radiology departments of the CHU Habib Bourguiba Sfax, Tunisia for providing MRI images and supports for this research.

References

[1] http://kidshealth.org/parent/system/medical/mri_brain.html.
[2] Yang Y, Su Z, Sun L. Medical image enhancement algorithm based on wavelet transform. J Electron Lett 2010;46:120–1.
[3] Michailovich OV, Tannenbaum A. Despeckling of medical ultrasound images. IEEE Trans Ultrason Ferroelectr Freq Control 2006;53(1):64–78.
[4] Wang H, Nie P, Chen B, Hou F, Dong C, He F, et al. Contrast-enhanced CT findings of intravenous Leiomyomatosis. Clin Radiol 2018;73(5):503.e1–e6.
[5] Rasselet B, Larbi A, Viala P, Molinari N, Tetreau R, Faruch-Bilfeld M, et al. Prevalence and characteristics of intravertebral enhancement on contrast-enhanced CT scans in cancer patients. Eur J Radiol 2017;86:1–5.
[6] Borusewicz P, Stańczyk E, Kubiak K, Spużak J, Glińska-Suchocka K, Jankowski M, et al. Liver enhancement in healthy dogs after gadoxetic acid administration during dynamic contrast-enhanced magnetic resonance imaging. Vet J 2018;235:16–21.
[7] Bhandari AK, Kumar A, Singh GK, et al. Dark satellite image enhancement using knee transfer function and gamma correction based on DWT–SVD. Multidimens Syst Signal Process 2016;27:453.
[8] Kim TK, Paik JK, Kang BS. Contrast enhancement system using spatially adaptive histogram equalization with temporal filtering. IEEE Trans Consum Electron 1998;44(1).
[9] Kim JY, Kim LS, Hwang S. An advanced contrast enhancement using partially overlapped sub-block histogram equalization. IEEE Trans Circuits Syst Video Technol 2001;11:475–84.
[10] Kim YT. Contrast enhancement using brightness preserving bi-histogram equalization. IEEE Trans Consum Electron 1997;43(1–8).
[11] Ibrahim H, Kong NSP. Brightness preserving dynamic histogram equalization for image contrast enhancement. IEEE Trans Consum Electron 2007;53:1752–8.
[12] Sun CC, Ruan SJ, Shie MC, Pai TW. Dynamic contrast enhancement based on histogram specification. IEEE Trans Consum Electron 2005;51:1300–5.
[13] Demirel H, Anbarjafari G, Jahromi MN. Image equalization based on singular value decomposition. IEEE Int Sympos Comput Inform Sci 2008:1–5.
[14] Demirel H, Ozcinar C, Anbarjafari G. Satellite image contrast enhancement using discrete wavelet transform and singular value decomposition. IEEE Geosci Remote Sens Lett 2010;7:333–7.
[15] Bhandari AK, Kumar A, Padhy PK. Enhancement of low contrast satellite images using discrete cosine transform and singular value decomposition. World Acad Sci Eng Technol 2011;55:35–41.
[16] Atta R, Ghanbari M. Low-contrast satellite images enhancement using discrete cosine transform pyramid. IET Image Process 2013;7(5).
[17] Bhandari AK, Soni V, Kumar A, Singh GK. Artificial Bee Colony-based satellite image contrast and brightness enhancement technique using DWT-SVD. Int J Remote Sens 2014;35:1601–24.
[18] Kumar A, Bhandari AK, Padhy PK. Improved normalised difference vegetation index method based on discrete cosine transform and singular value decomposition for satellite image processing. IET Signal Process 2012;6:617–25.
[19] Atta R, Abdel-Kader RF. Brightness preserving based on singular value decomposition for image contrast enhancement. Optik Int J Light Electron Opt 2015;126(7–8):799–803.
[20] Ozcinar H, Anbarjafari G. Satellite image contrast enhancement using discrete wavelet transform and singular value decomposition. IEEE Geosci Remote Sens Lett 2010;7:333–7.
[21] Bhandari AK, Singh VK, Kumar A, Singh GK. Cuckoo search algorithm and wind driven optimization based study of satellite image segmentation for multilevel thresholding using Kapur's entropy. Expert Syst Appl 2014;41:3538–60.
[22] https://en.wikipedia.org/wiki/Peak_signal-to-noise_ratio.
[23] Celik T. Spatial mutual information and pagerank-based contrast enhancement and quality-aware relative contrast measure. IEEE Trans Image Process 2016;25(10):4719–28.
[24] Oliva D, Cuevas E, Pajares G, Zaldivar D, Osuna V. A multilevel thresholding algorithm using electromagnetism optimization. Neurocomputing 2014;139:357–81.
[25] Garg R, Mittal B, Garg S. Histogram equalization techniques for image enhancement. IJECT 2011;2(1).

[26] Kumar V, Choudhary RR. A comparative analysis of image contrast enhancement techniques based on histogram equalization for gray scale static images. Int J Comput Appl 2012;45(21).
[27] Wang Z, Bovik AC, Sheikh HR, Simoncelli EP. Image quality assessment: from error measurement to structural similarity. IEEE Trans Image Process 2004;13:87–94.
[28] Kaur A, Singh C. Contrast enhancement for cephalometric images using wavelet-based modified adaptive histogram equalization. Appl Soft Comput J 2016;51:180–91.