**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

***μ* = 45 minutes and *σ* = 8 minutes**

***Let P be Pr(X > 50)***

**Pr(X > 50) = 1 - Pr(X ≤ 50).**

**Z = (X - 45)/8.0**

**Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%**

**Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**a)Probabilty of employees greater than age of 44= Pr(X>44)**

**> pnorm(44,38,6)**

**[1] 0.8413447**

**Probabilty that the employee will be greater than age of 44**

**1-0.8413447**

**[1] 0.1586553**

**So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5**

**=84.1345-0.5**

**= 34.1345%**

**Therefore the statement is TRUE.**

**b) Probabilty of employees less than age of 30 = Pr(X<30).**

**> pnorm(30,38,6)**

**[1] 0.09121122**

**the number of employees with probability 0.912 of them was being under age 30 = 0.0912\*400=36.48(or 36 employees).**

**Therefore the statement B of the question is also TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**iX ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then**

**X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) & X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .**

**if Z = aX + bY , where X and Y are as defined above,**

**i.e Z is linear combination of X and Y**

**then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).**

**Therefore in the question**

**2X1~ N(2 u,4 σ^2) and**

**X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Probability value between a & b = 0.99.**

**Probability outside the a and b area is**

**=1-0.99**

**=0.01**

**The Probability towards left from a = -0.005**

**The Probability towards right from b = +0.005**

**> qnorm(0.005)**

**[1] -2.575829**

**Z \* σ + μ = X**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

**D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?