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DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Regression with a perceptron

Regression Problem Motivation

Regression Problem Motivation

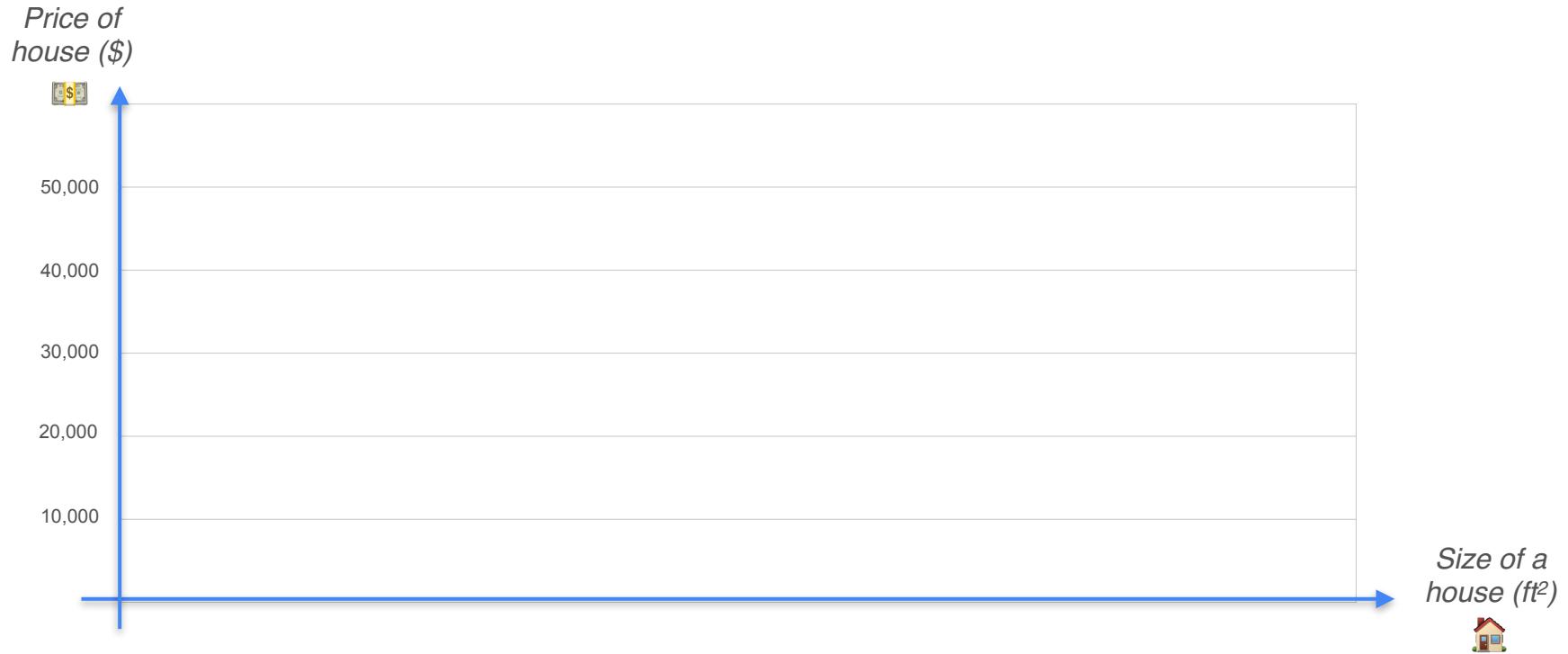
Predicting

the price of a house

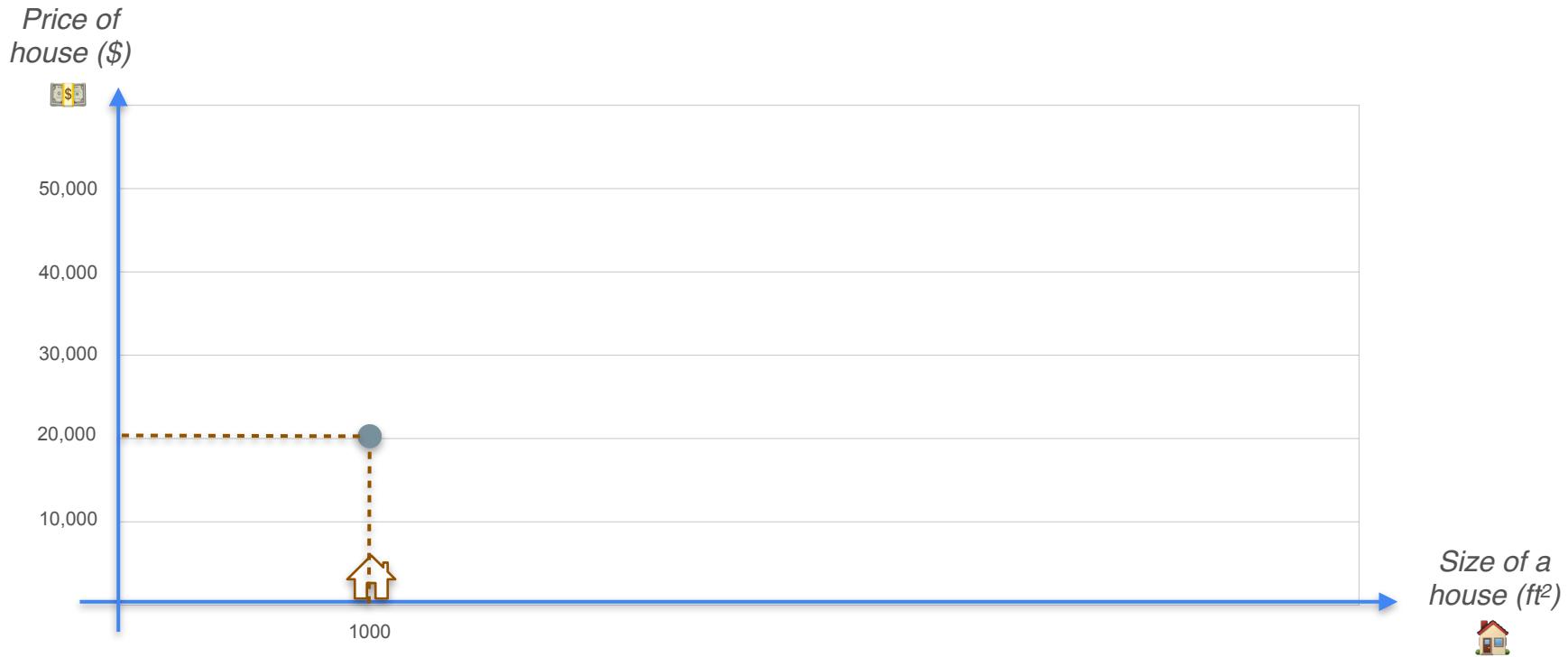
from

the size of the house

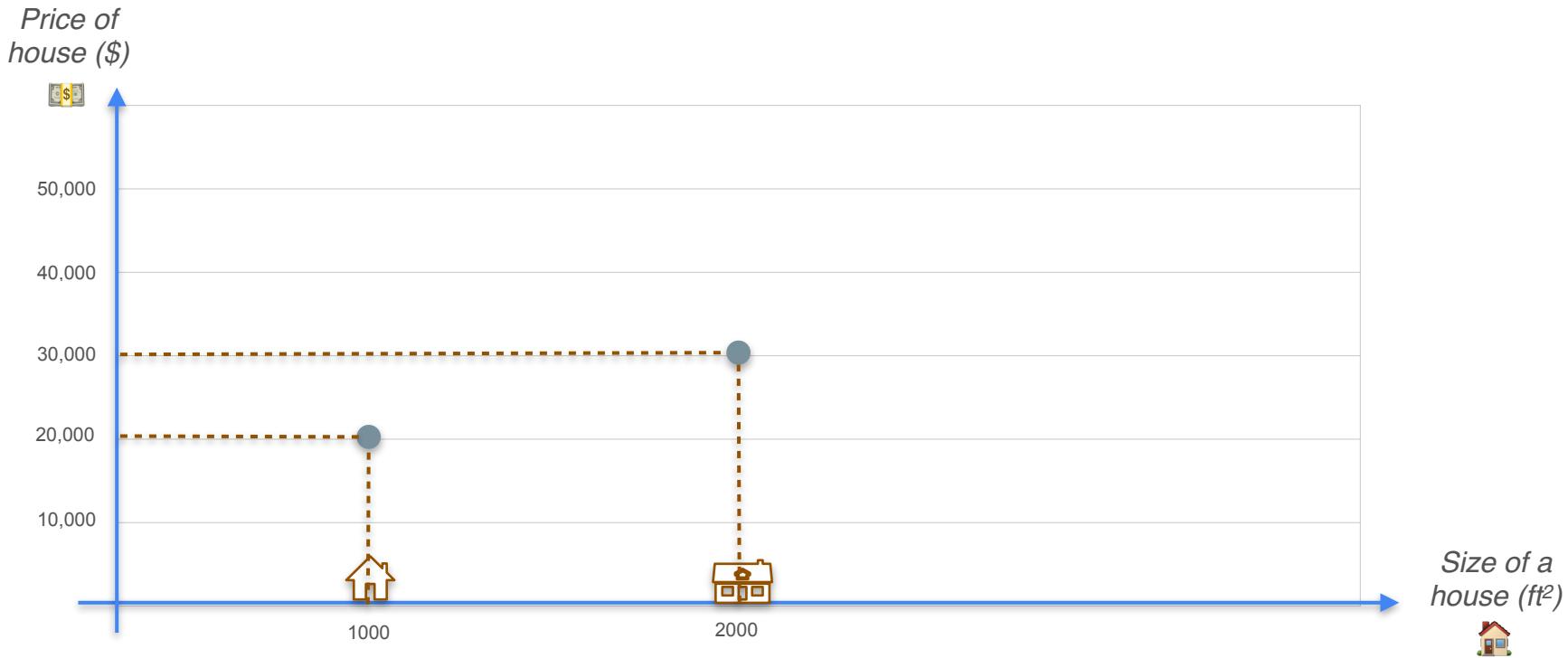
Regression Problem Motivation



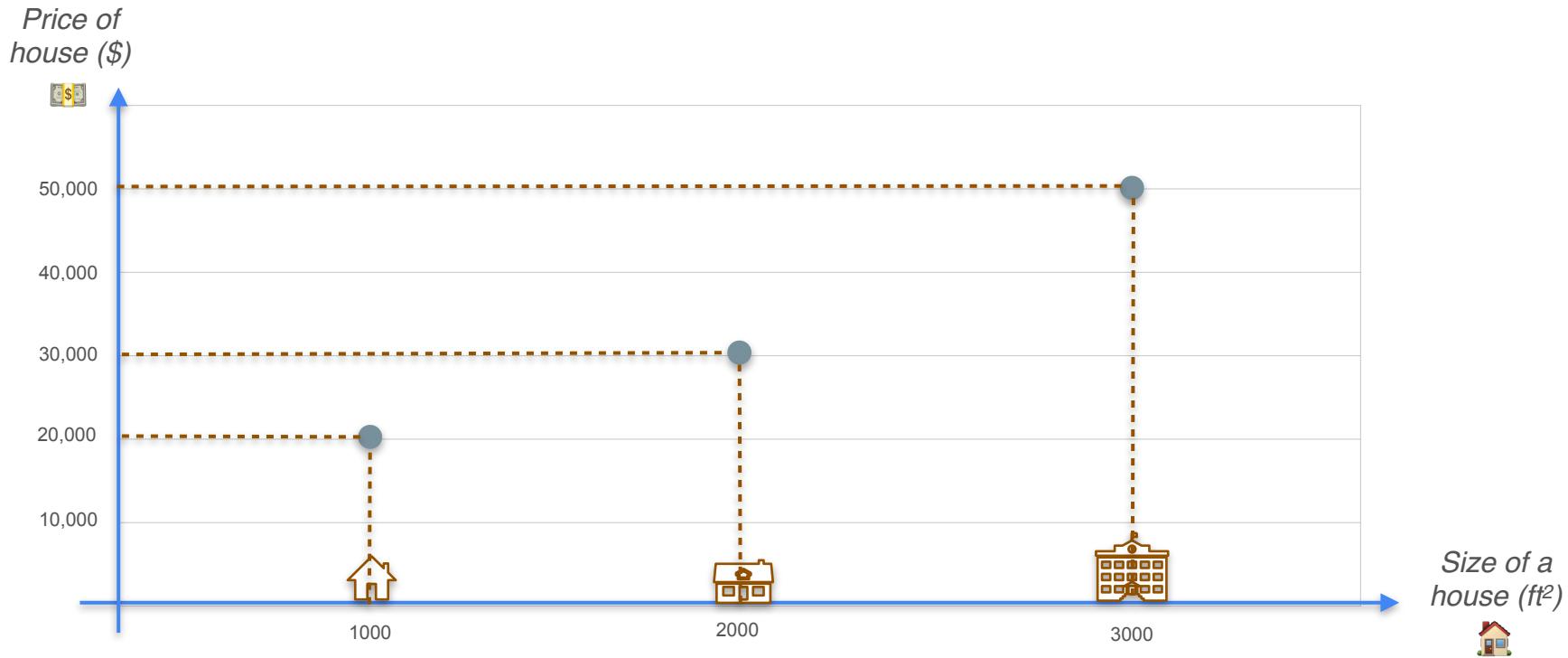
Regression Problem Motivation



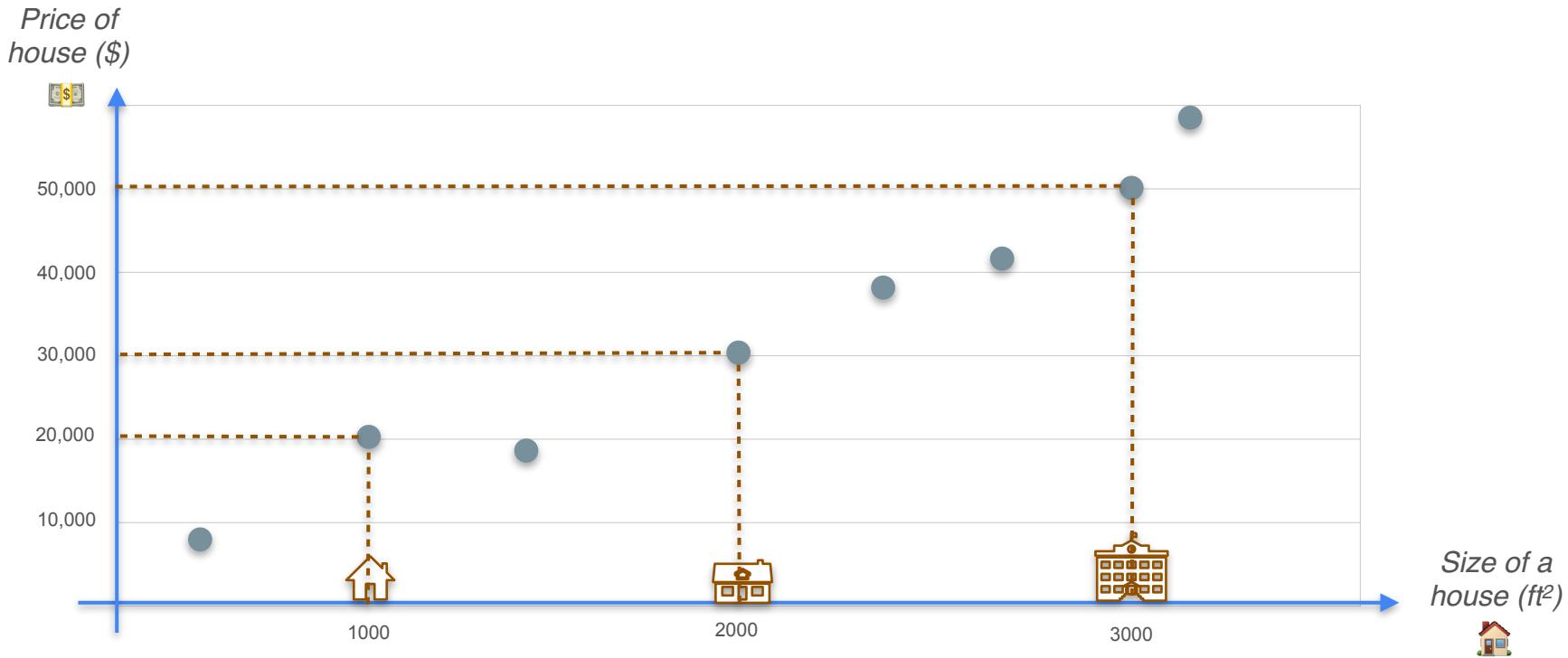
Regression Problem Motivation



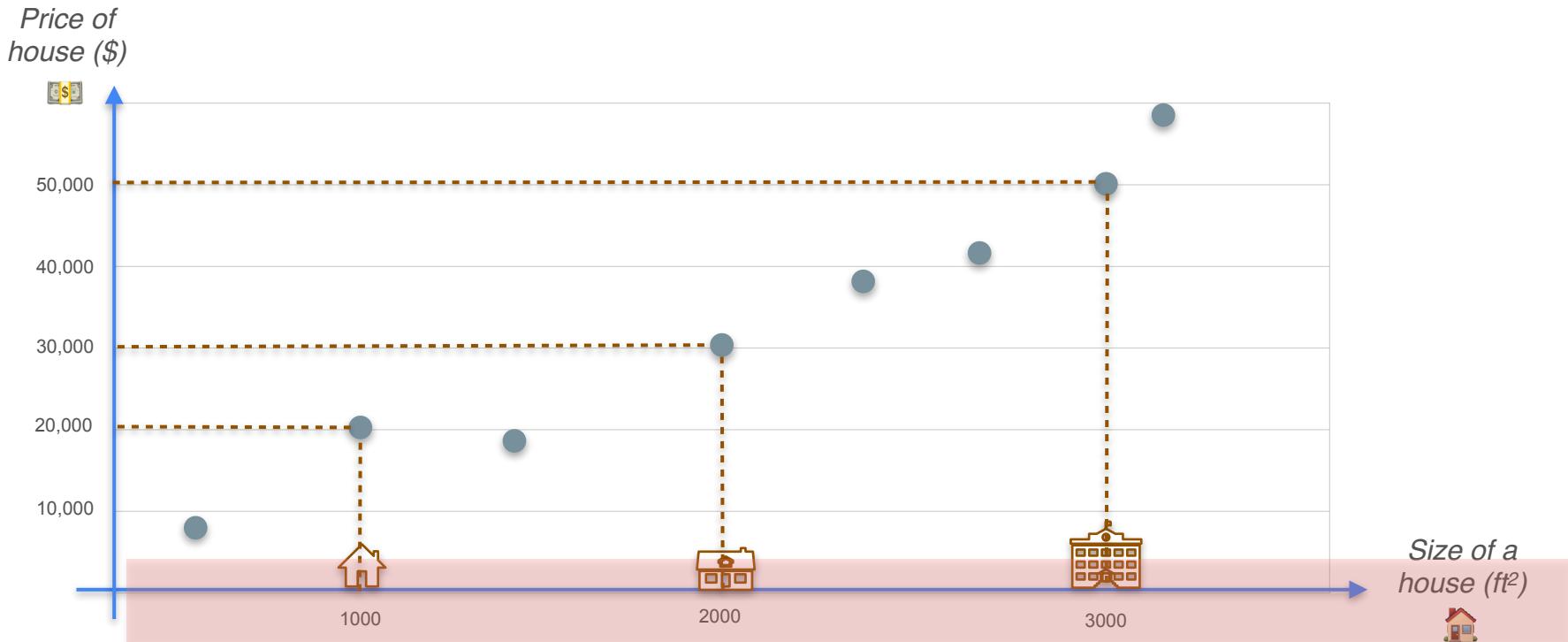
Regression Problem Motivation



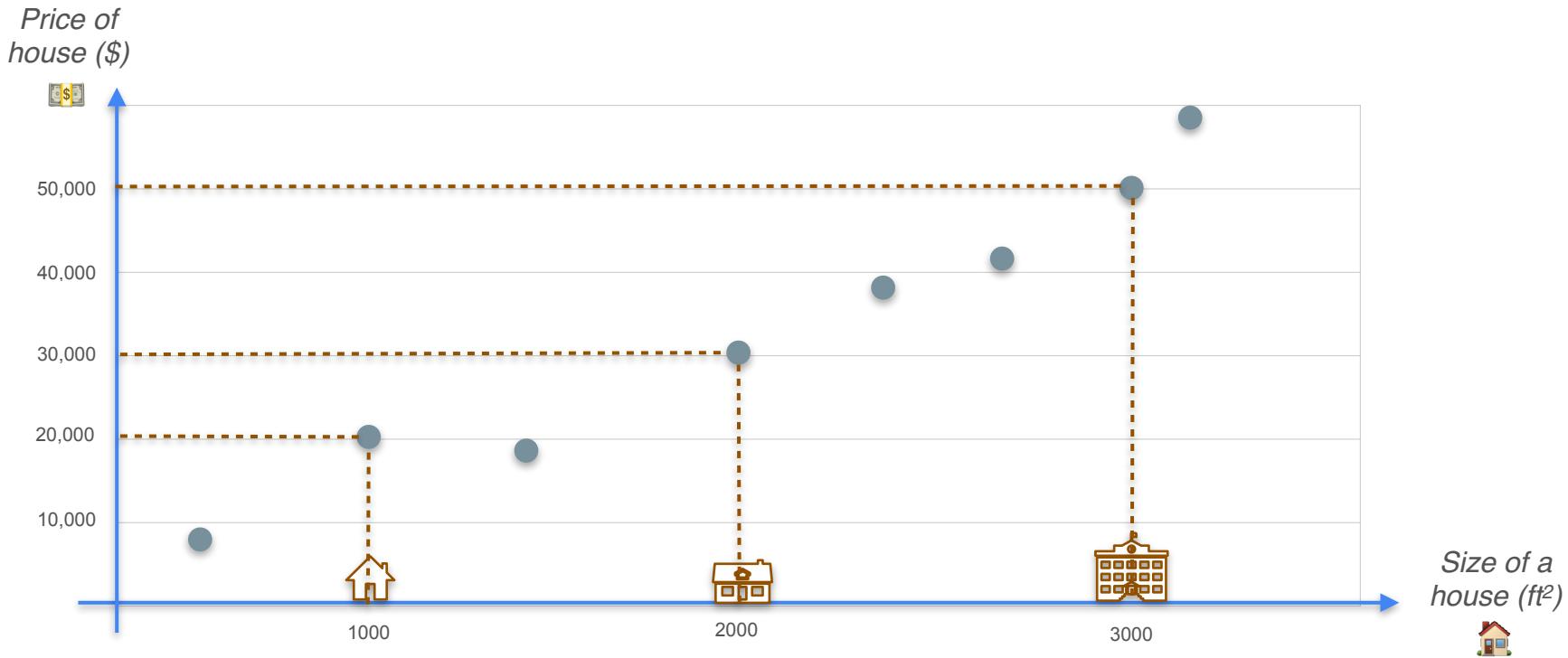
Regression Problem Motivation



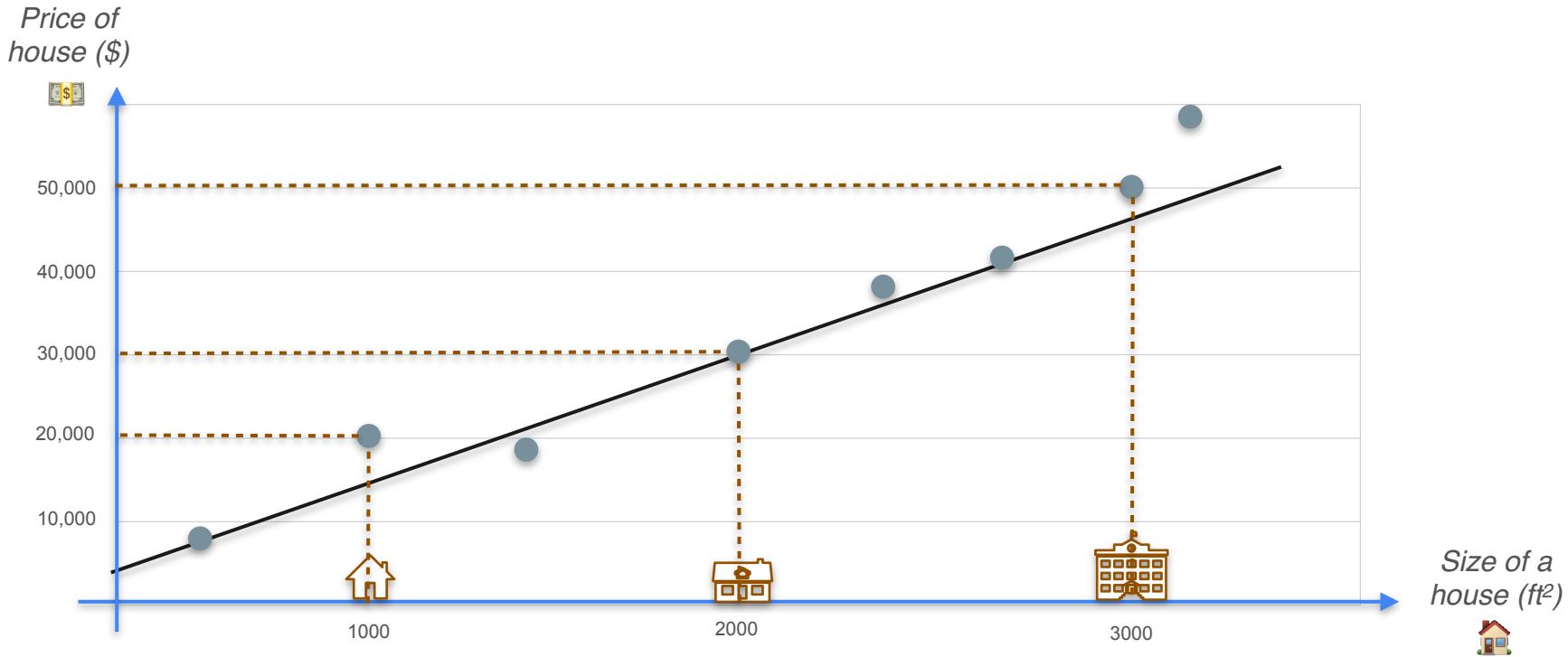
Regression Problem Motivation



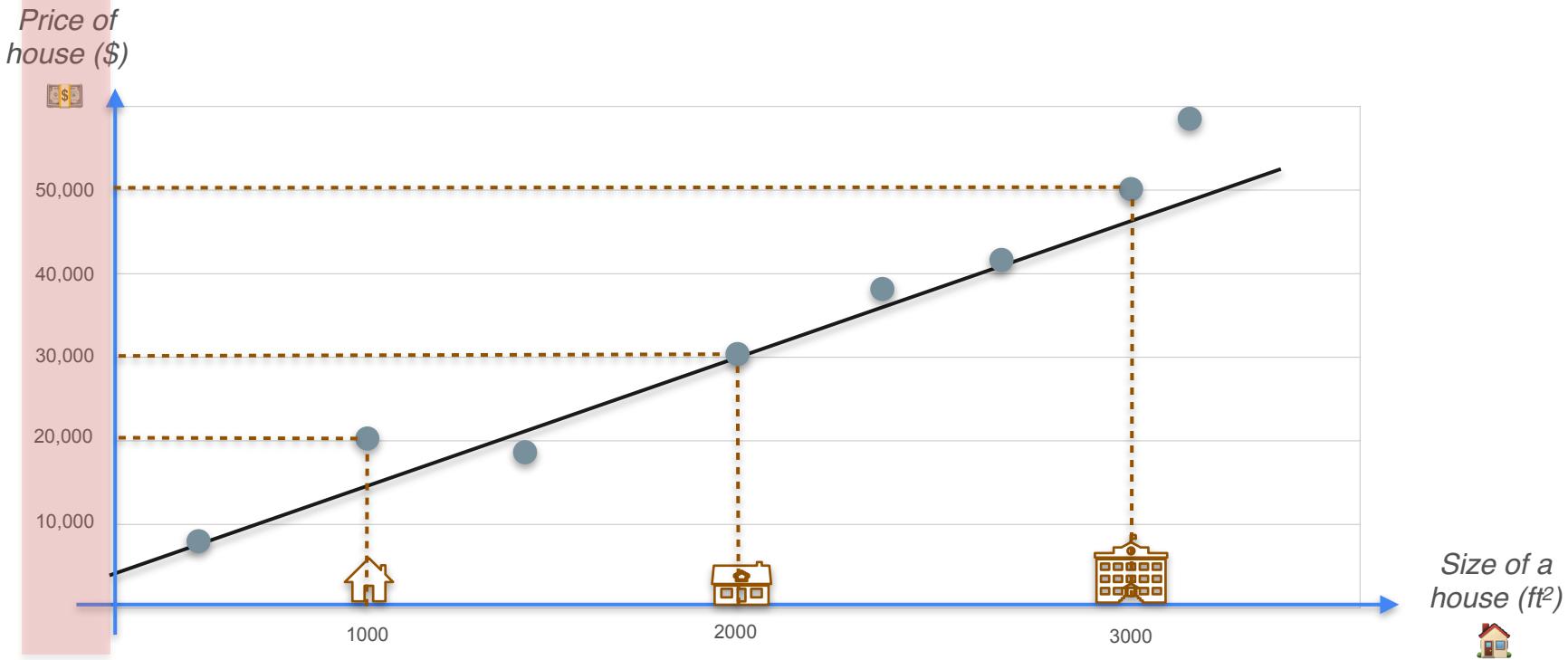
Regression Problem Motivation



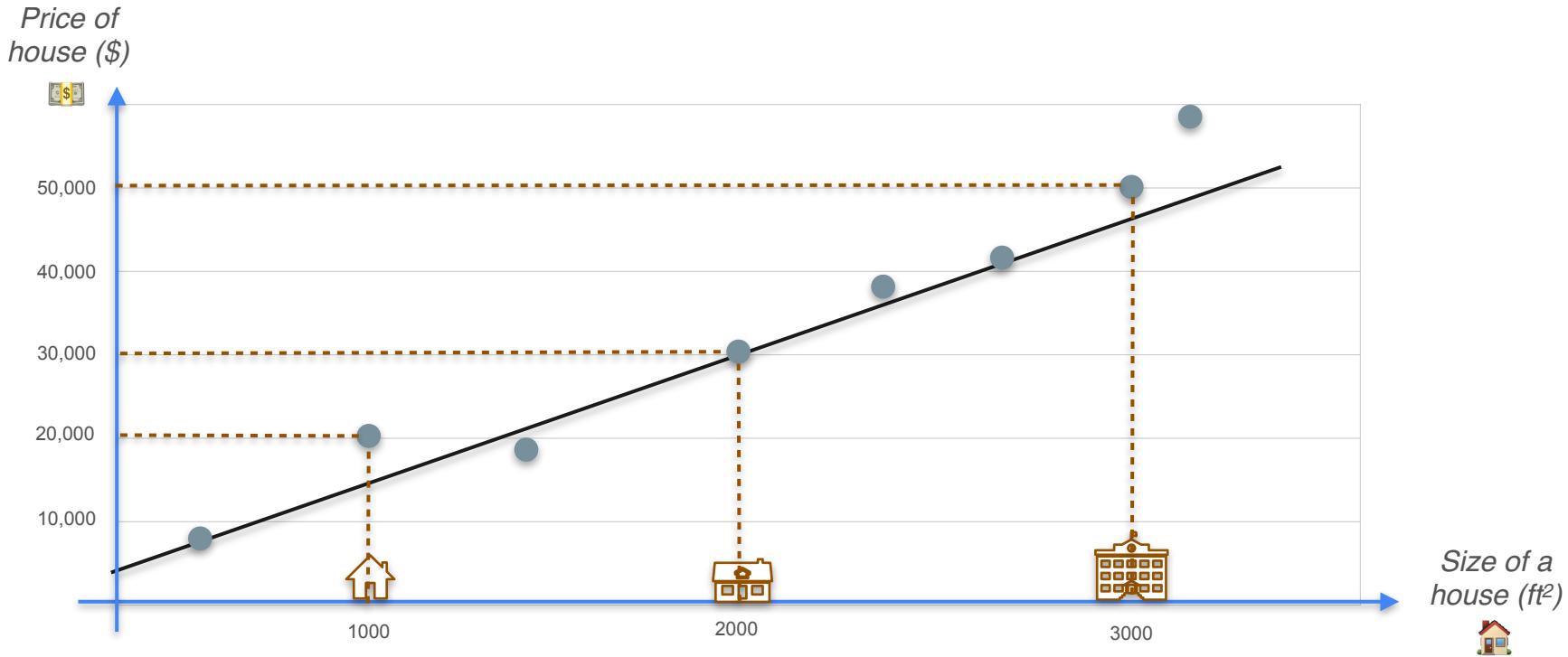
Regression Problem Motivation



Regression Problem Motivation



Regression Problem Motivation



Regression With a Perceptron

	<i>Size of a house (ft²)</i> 		<i>Price of house (\$)</i> 
			
			
			

Regression With a Perceptron

	<i>Size of a house (ft²)</i> 		<i>Price of house (\$)</i> 
	1000ft^2		\$20,000
	2000ft^2		\$30,000
	3000ft^2		\$50,000

Regression With a Perceptron

	<i>Size of a house (ft²)</i> 	<i>Number of rooms</i> 	<i>Price of house (\$)</i> 
	1000ft ²	2	\$20,000
	2000ft ²	4	\$30,000
	3000ft ²	7	\$50,000

Regression With a Perceptron

Inputs

*Size of a
house (ft²)*



*Number of
rooms*

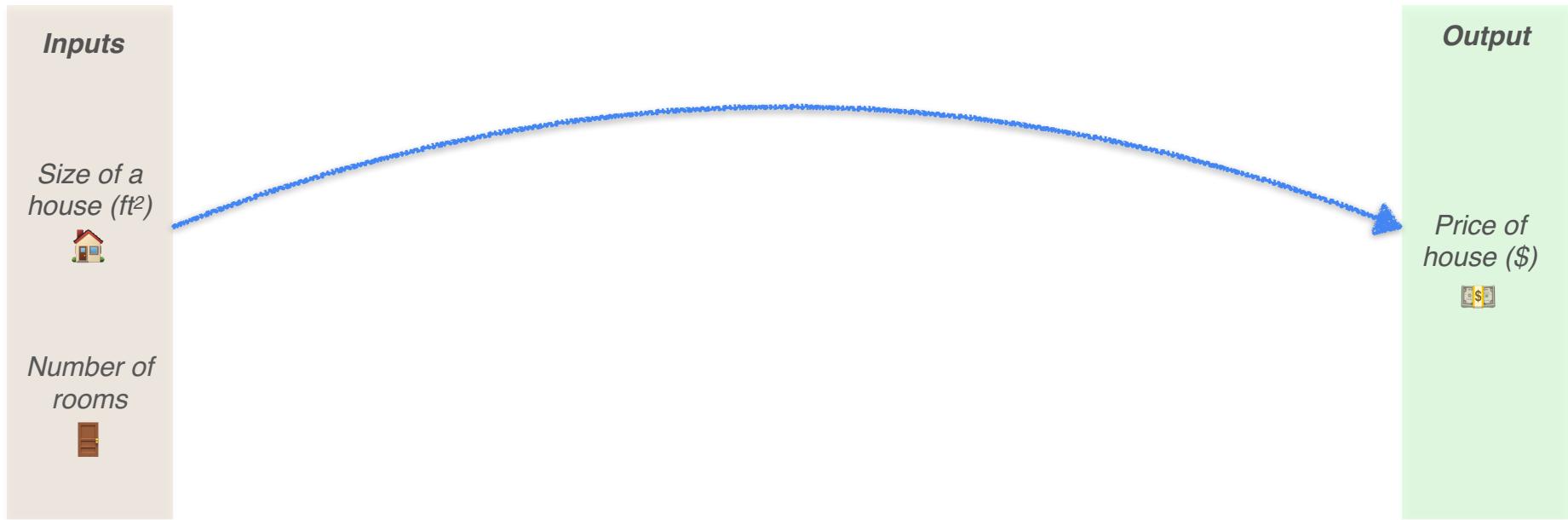


Output

*Price of
house (\$)*



Regression With a Perceptron



Regression With a Perceptron

Single Layer Neural Network Perceptron

Inputs

Size of a house (ft²)



Number of rooms



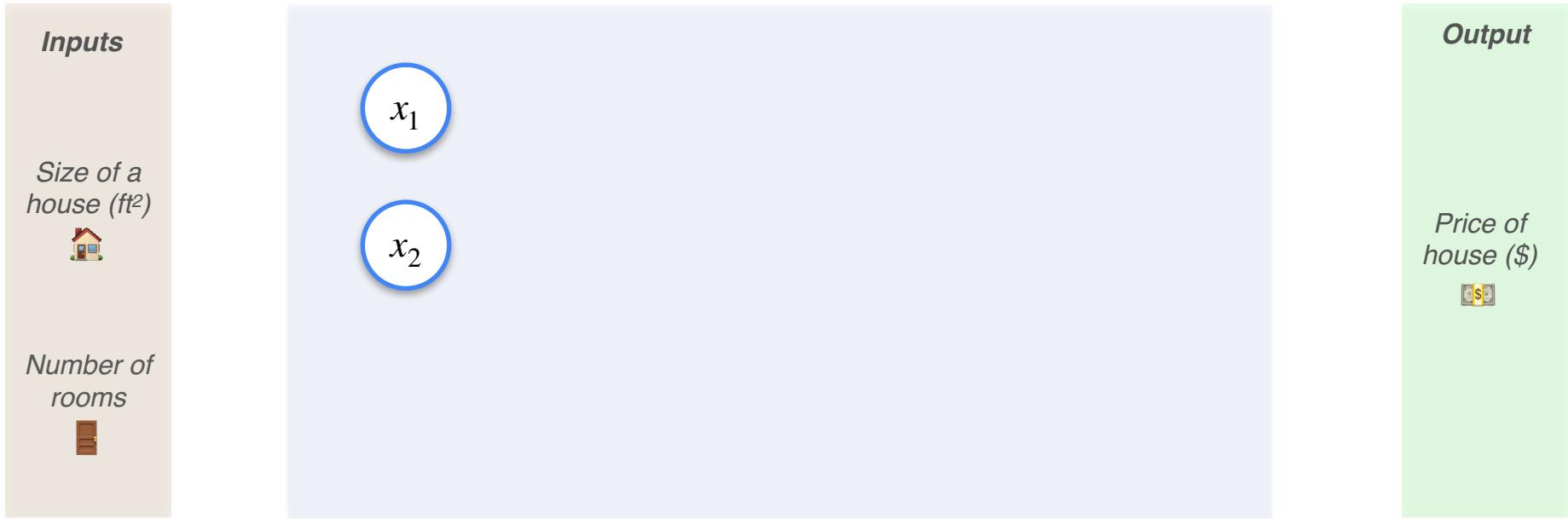
Output

Price of house (\$)



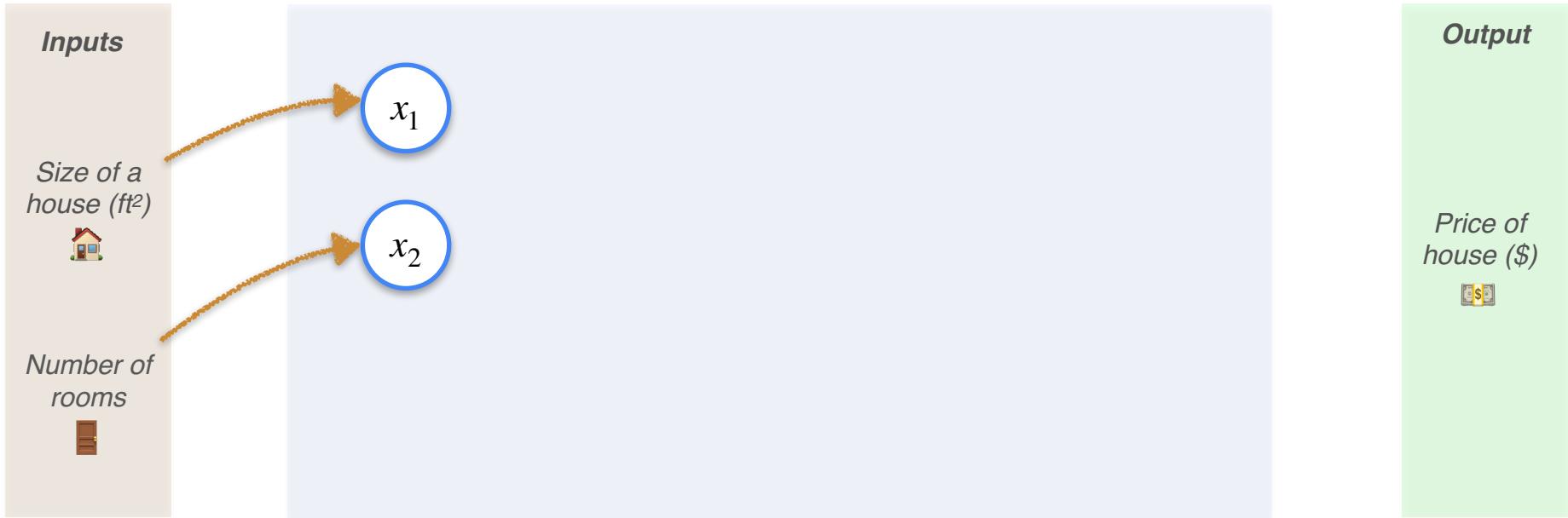
Regression With a Perceptron

Single Layer Neural Network Perceptron



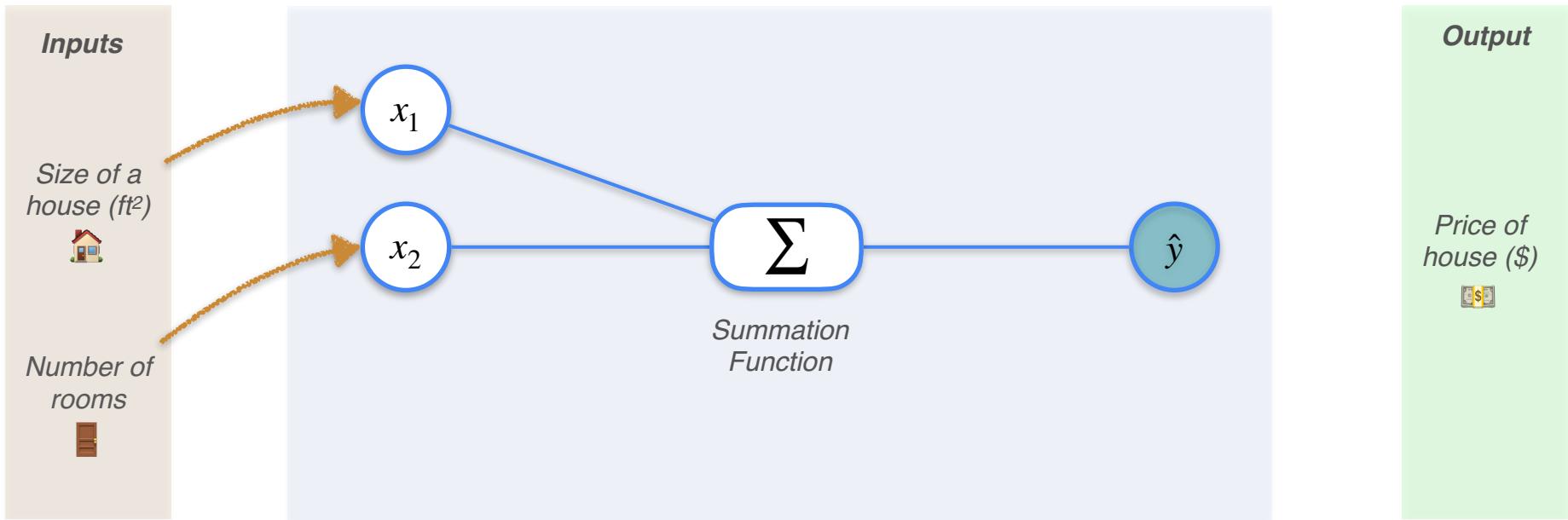
Regression With a Perceptron

Single Layer Neural Network Perceptron



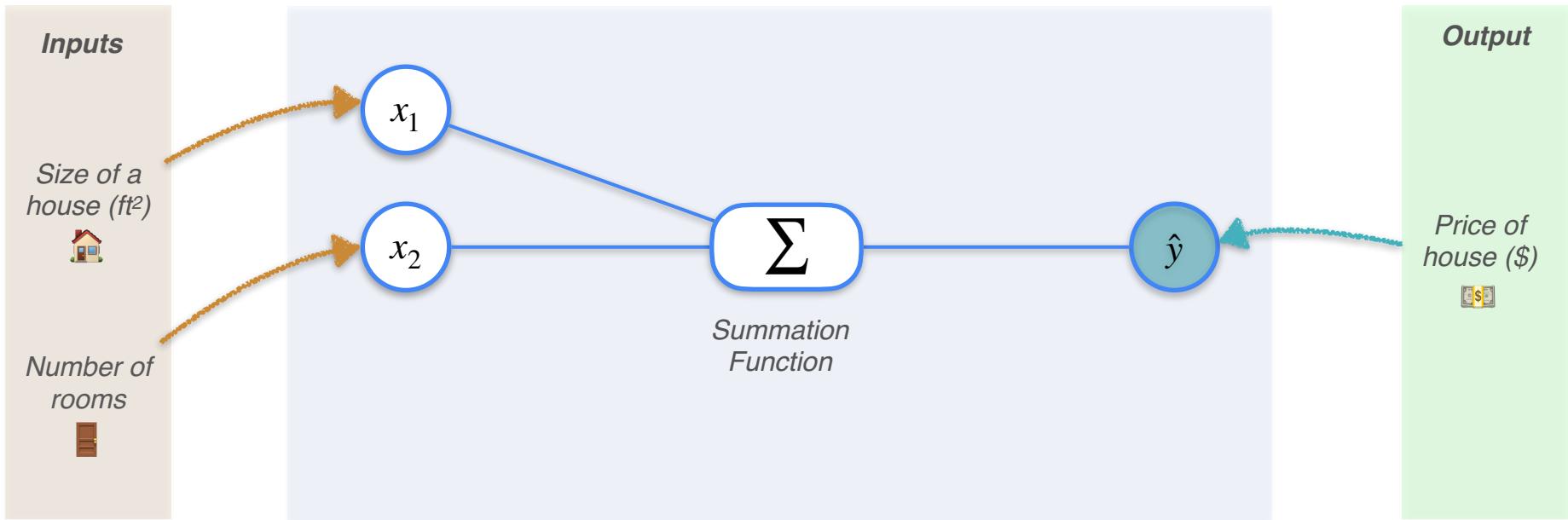
Regression With a Perceptron

Single Layer Neural Network Perceptron



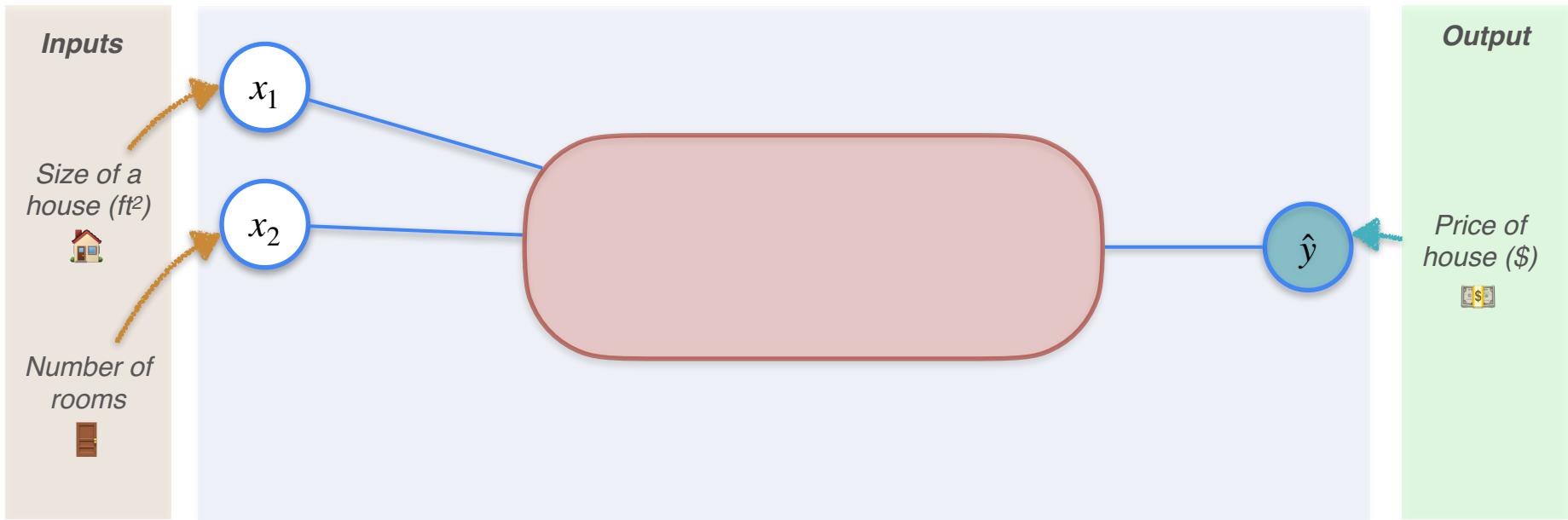
Regression With a Perceptron

Single Layer Neural Network Perceptron



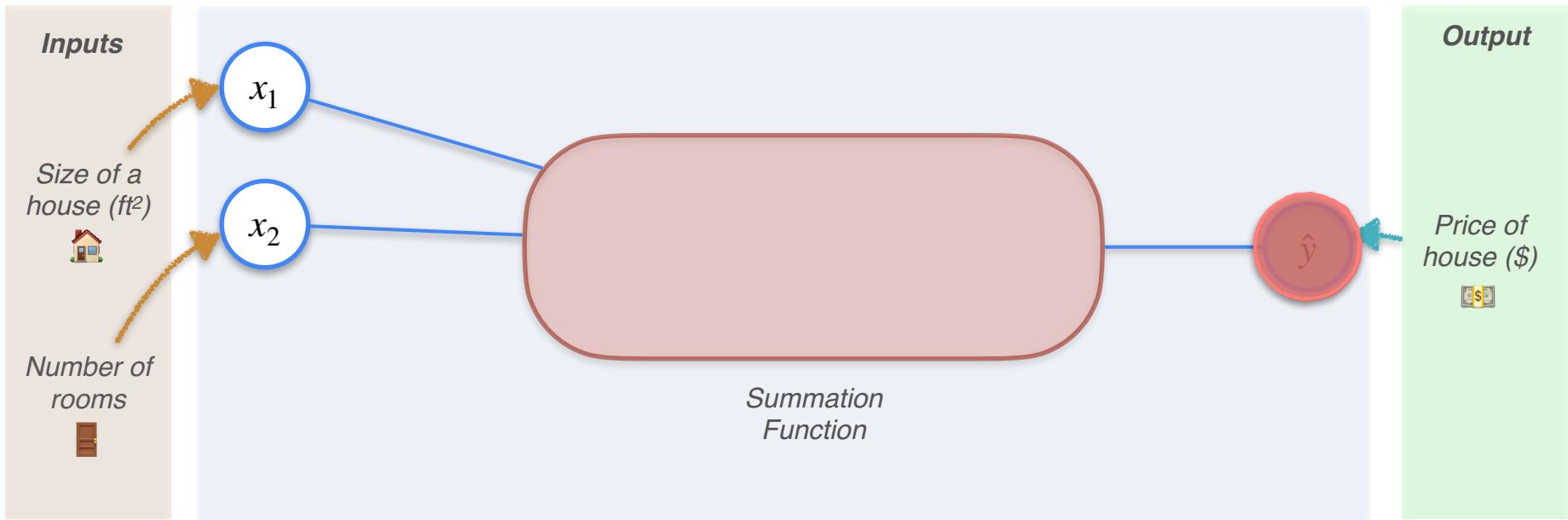
Regression With a Perceptron

Single Layer Neural Network Perceptron



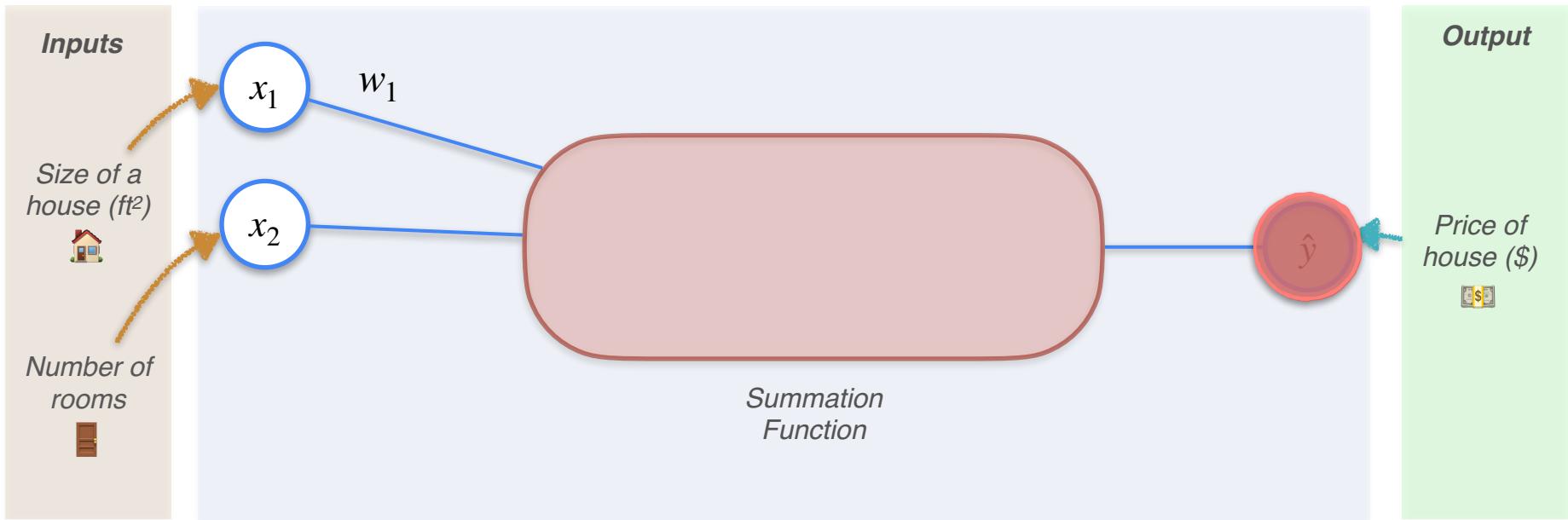
Regression With a Perceptron

Single Layer Neural Network Perceptron



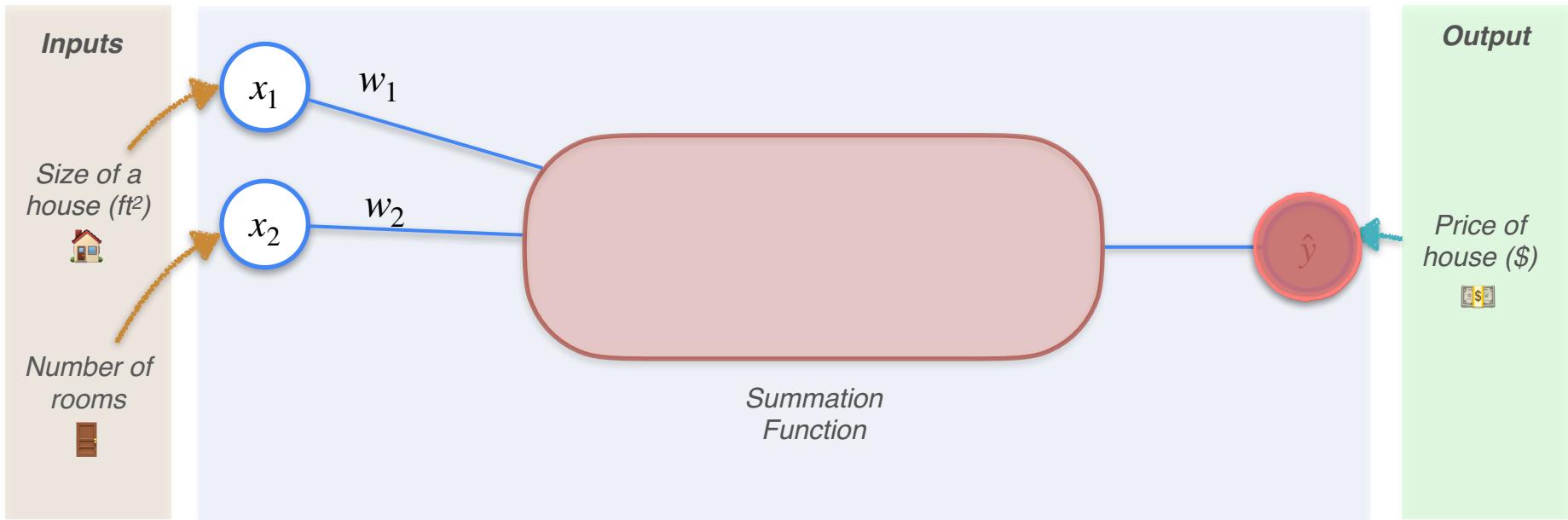
Regression With a Perceptron

Single Layer Neural Network Perceptron



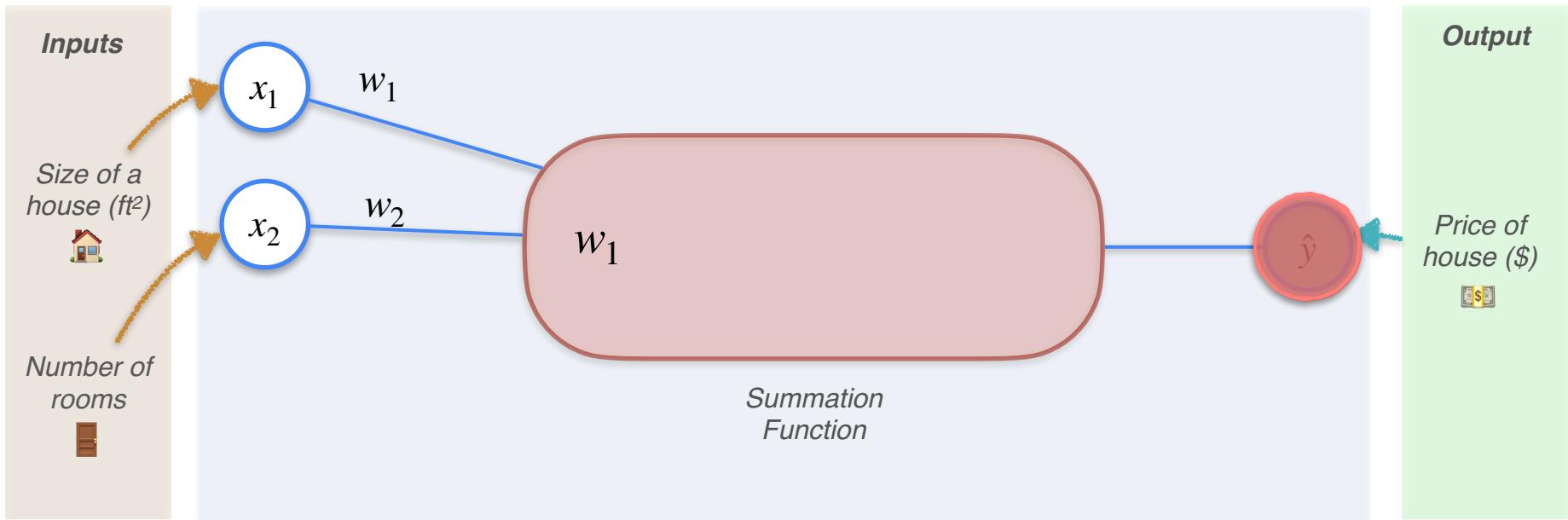
Regression With a Perceptron

Single Layer Neural Network Perceptron



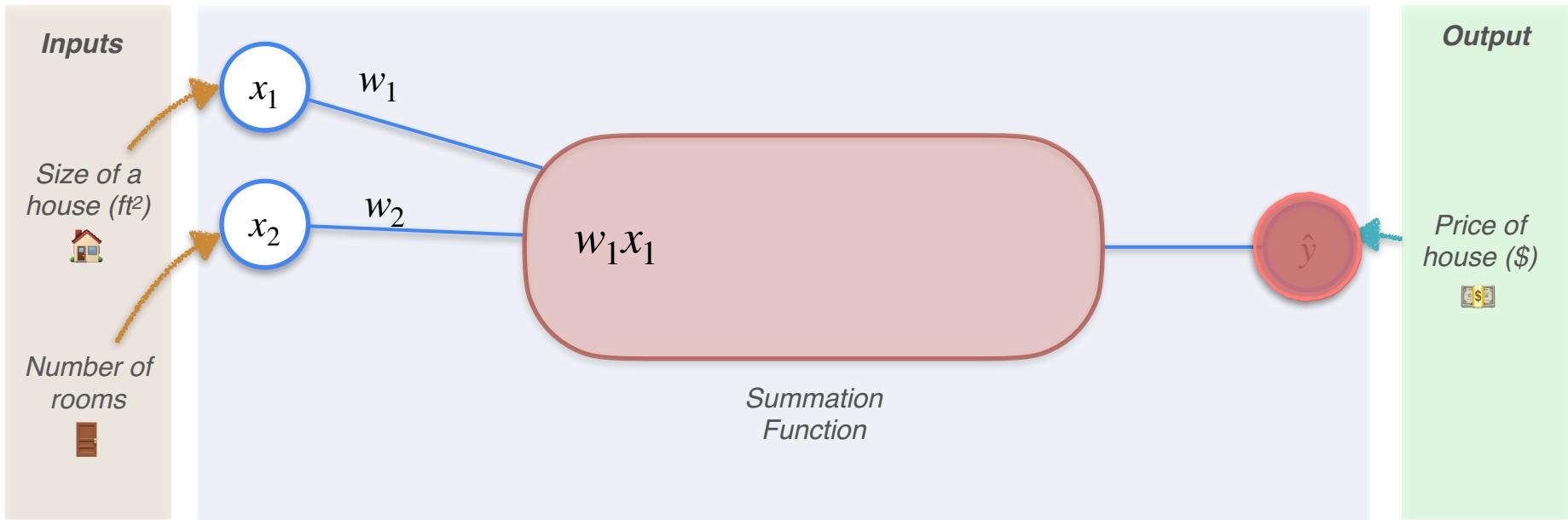
Regression With a Perceptron

Single Layer Neural Network Perceptron



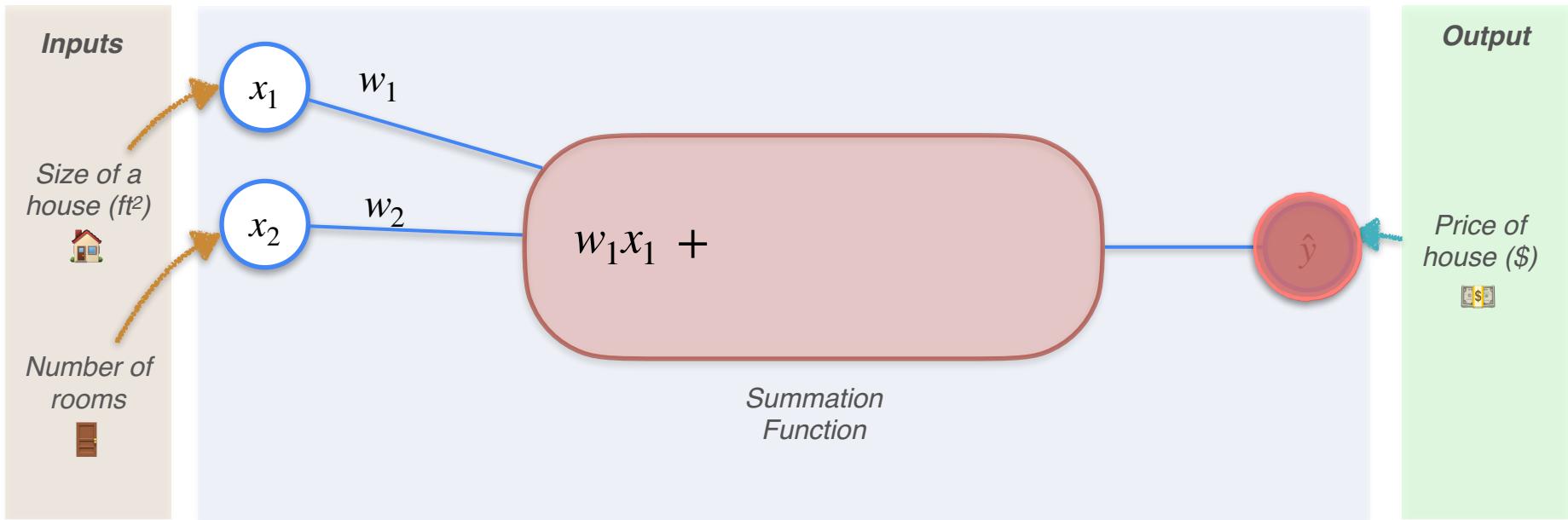
Regression With a Perceptron

Single Layer Neural Network Perceptron



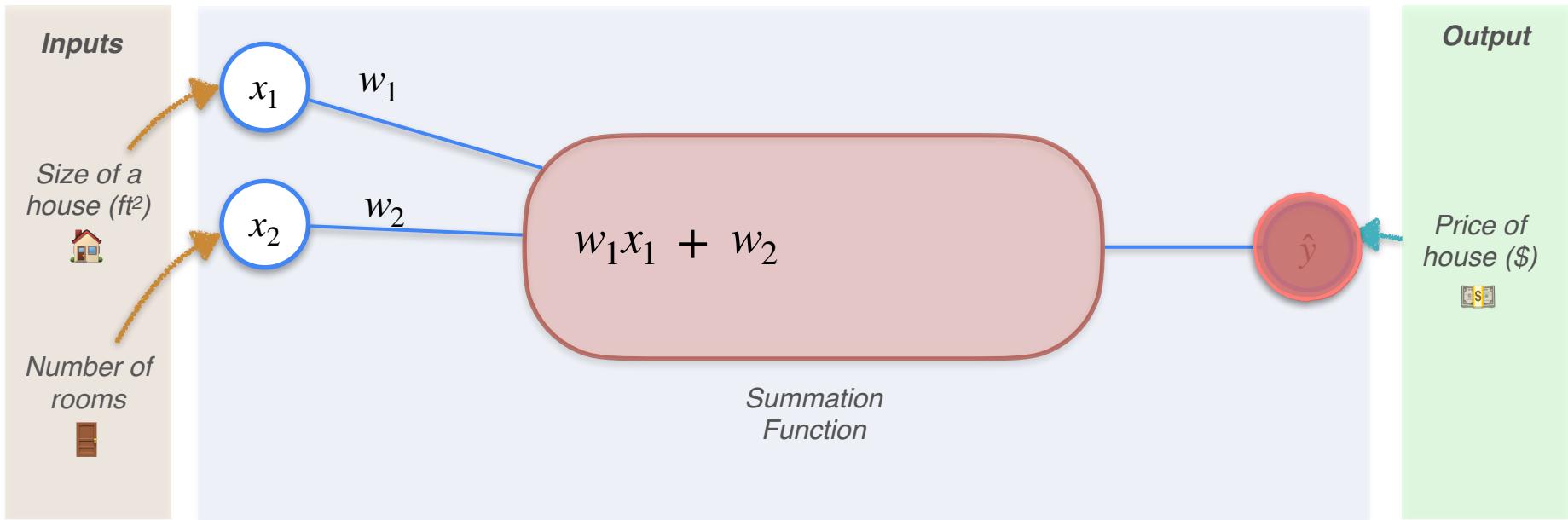
Regression With a Perceptron

Single Layer Neural Network Perceptron



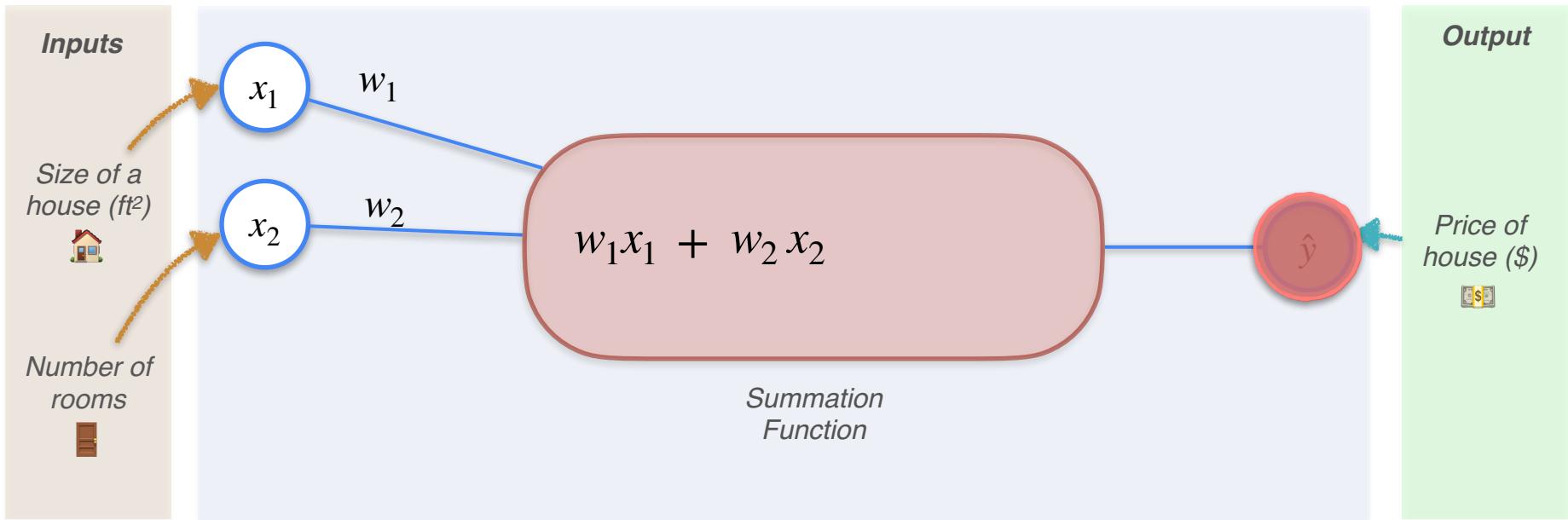
Regression With a Perceptron

Single Layer Neural Network Perceptron



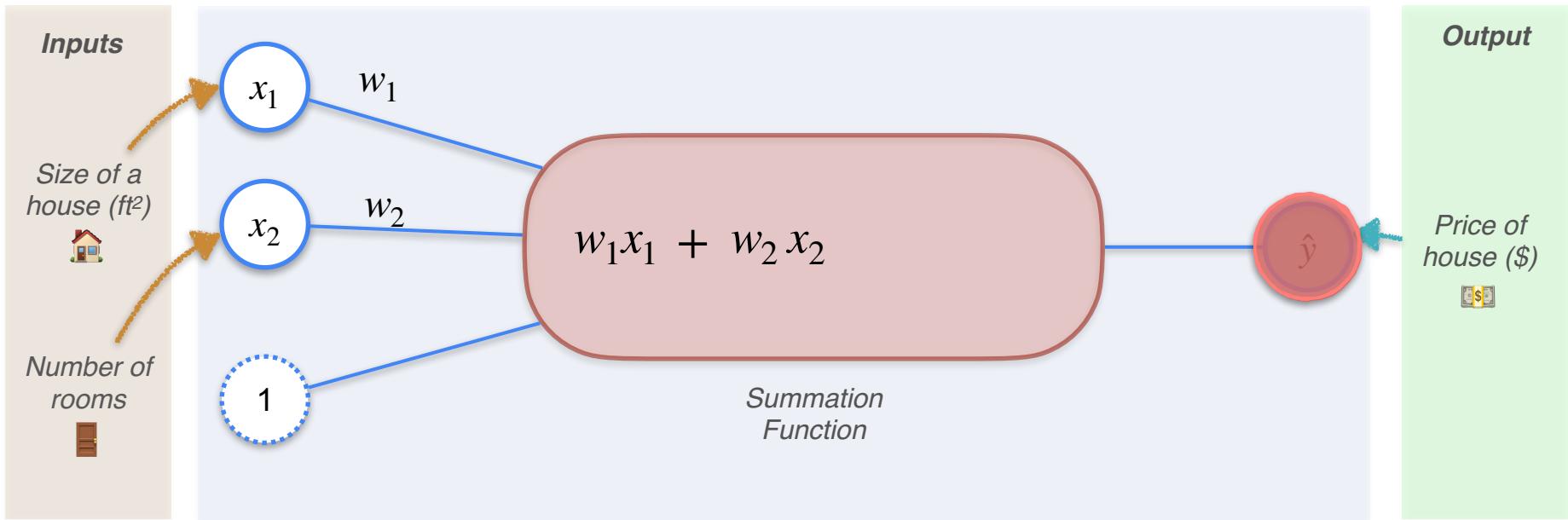
Regression With a Perceptron

Single Layer Neural Network Perceptron



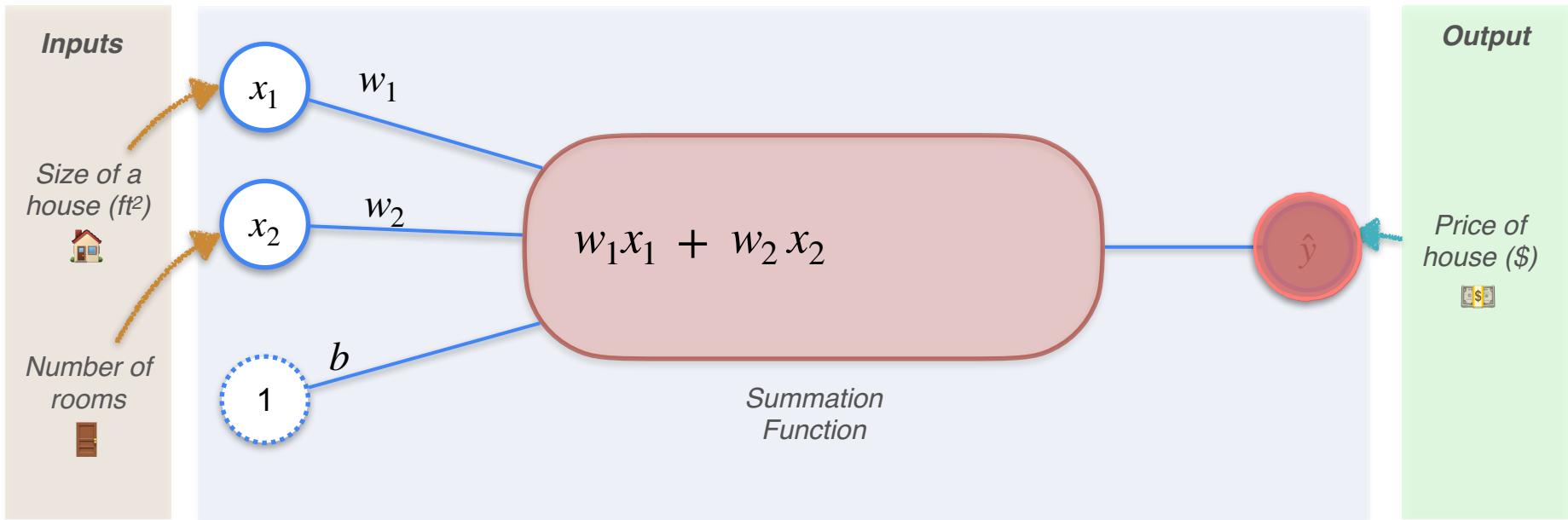
Regression With a Perceptron

Single Layer Neural Network Perceptron



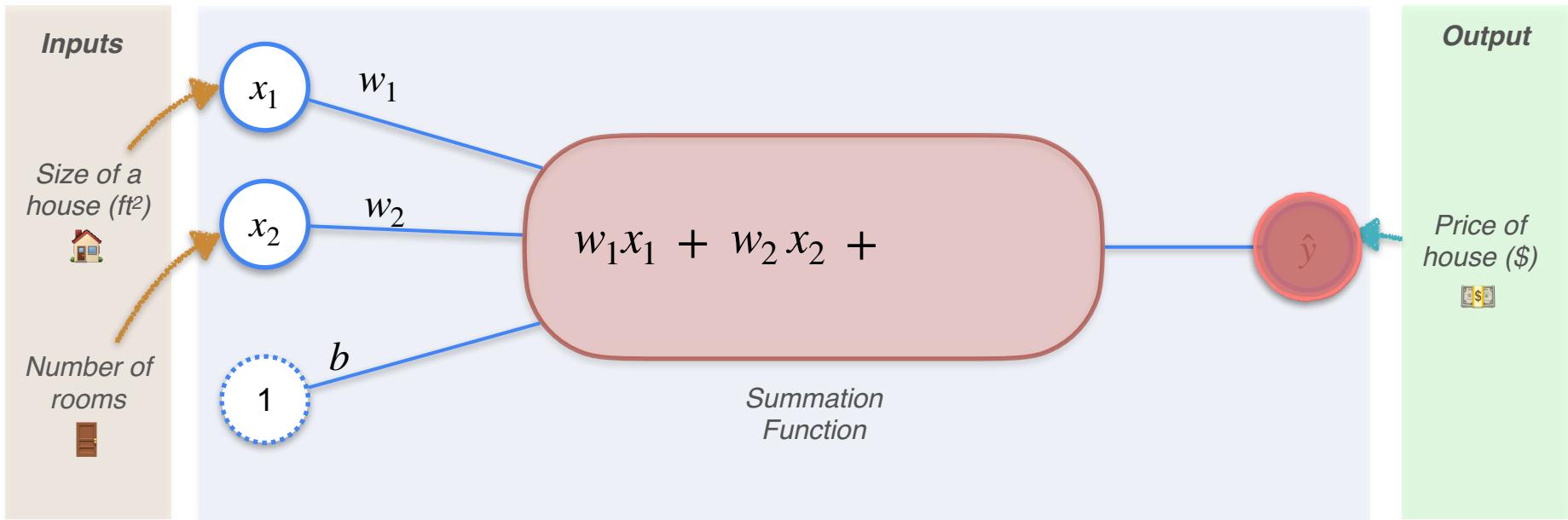
Regression With a Perceptron

Single Layer Neural Network Perceptron



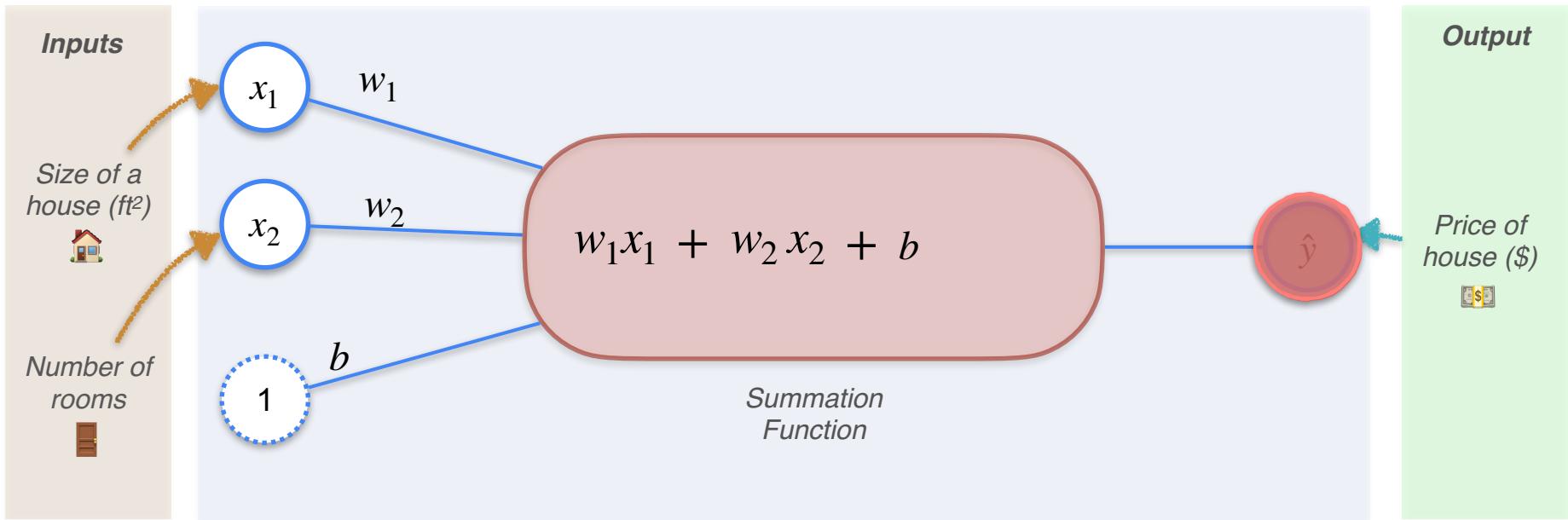
Regression With a Perceptron

Single Layer Neural Network Perceptron



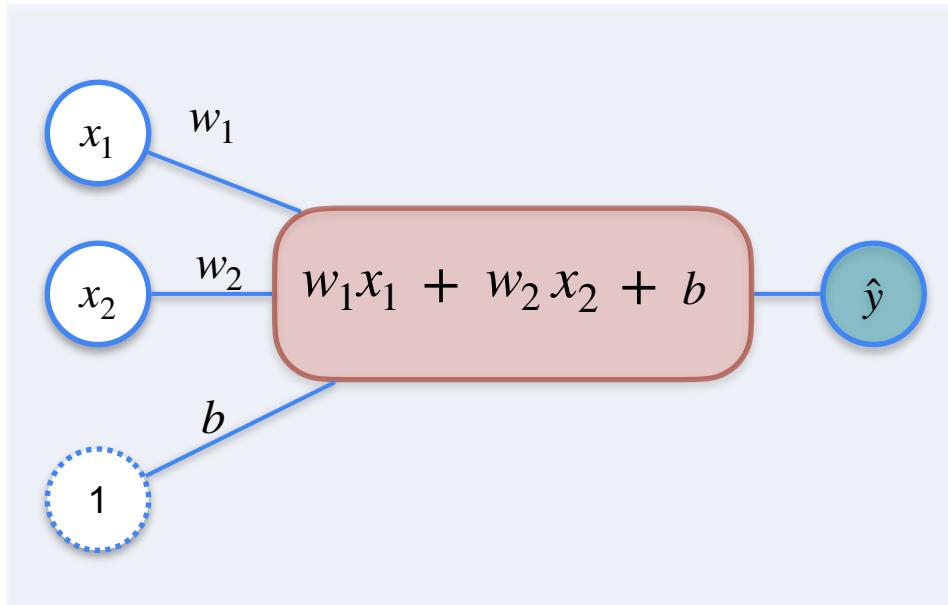
Regression With a Perceptron

Single Layer Neural Network Perceptron



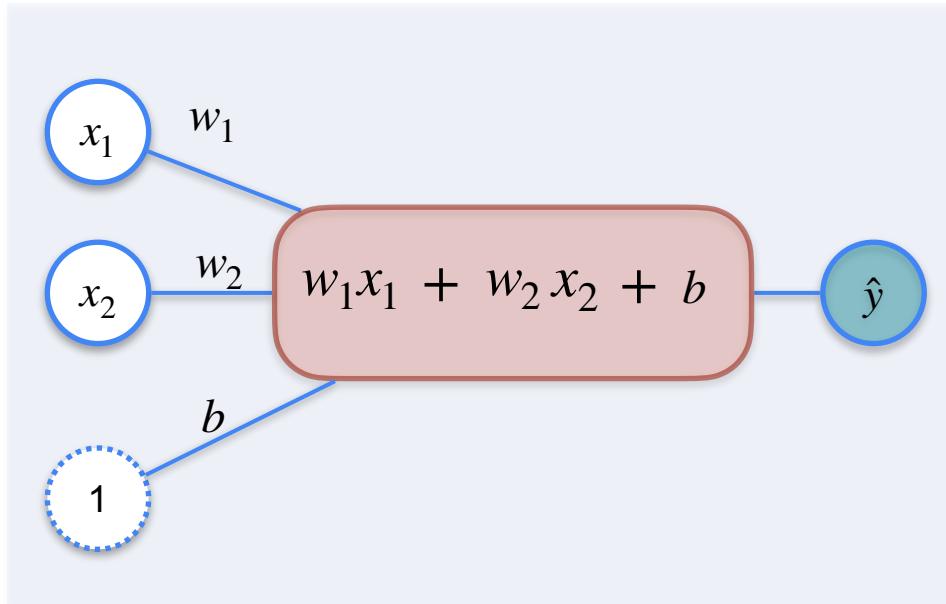
Regression With a Perceptron

Single Layer Neural Network Perceptron



Regression With a Perceptron

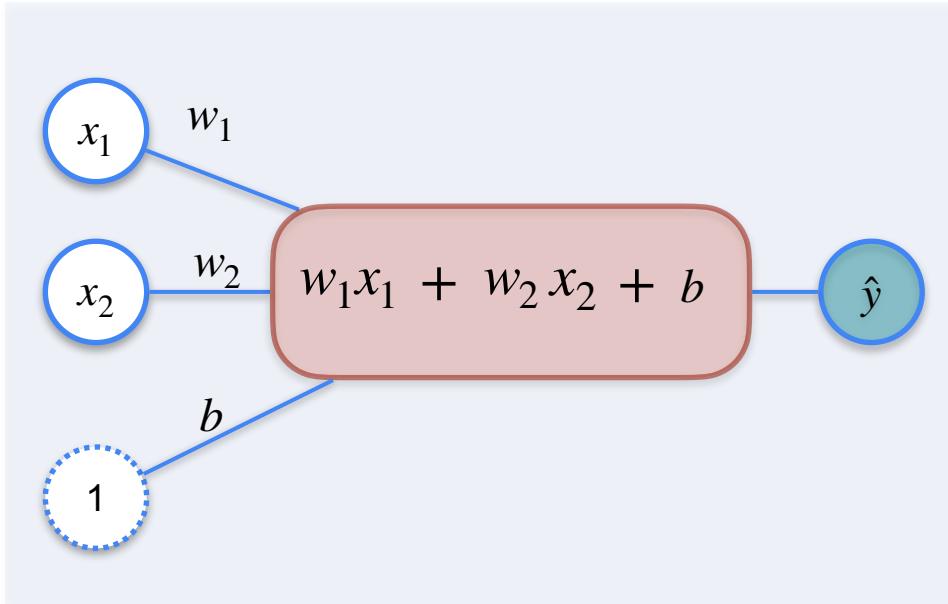
Single Layer Neural Network Perceptron



\hat{y}

Regression With a Perceptron

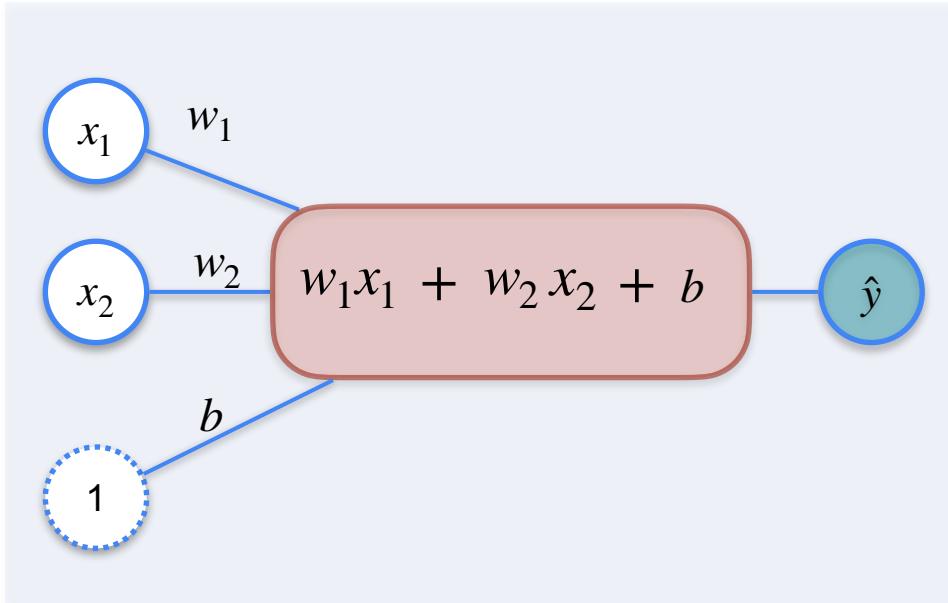
Single Layer Neural Network Perceptron



$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

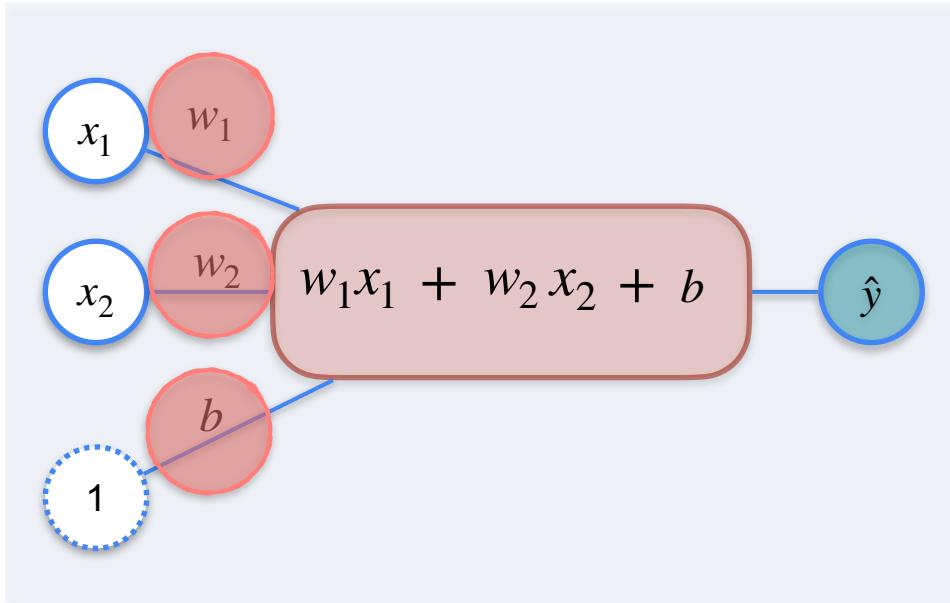


$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron

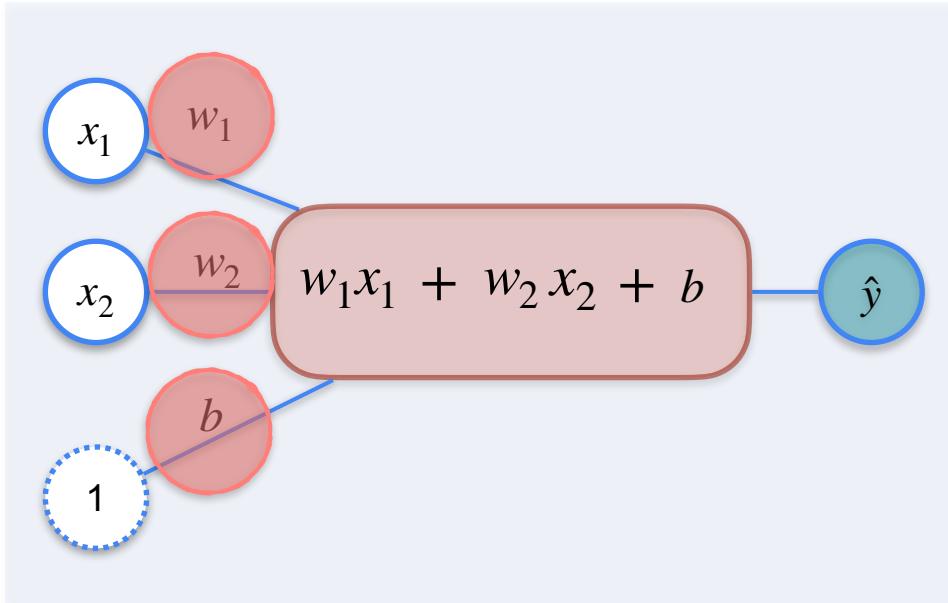


$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron



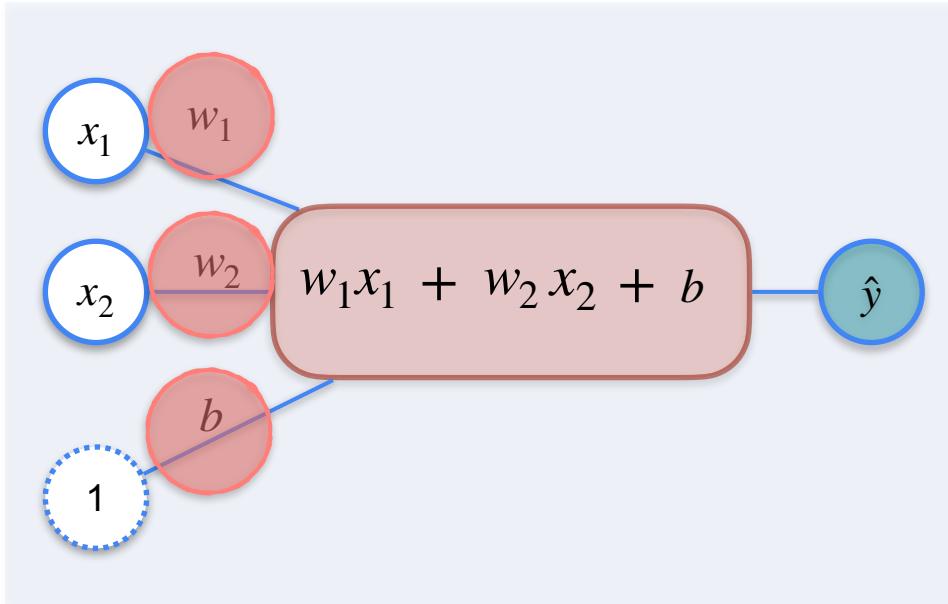
$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

Find weights and bias that will optimise the predictions.

Regression With a Perceptron

Single Layer Neural Network Perceptron



$$\hat{y} = w_1x_1 + w_2x_2 + b$$

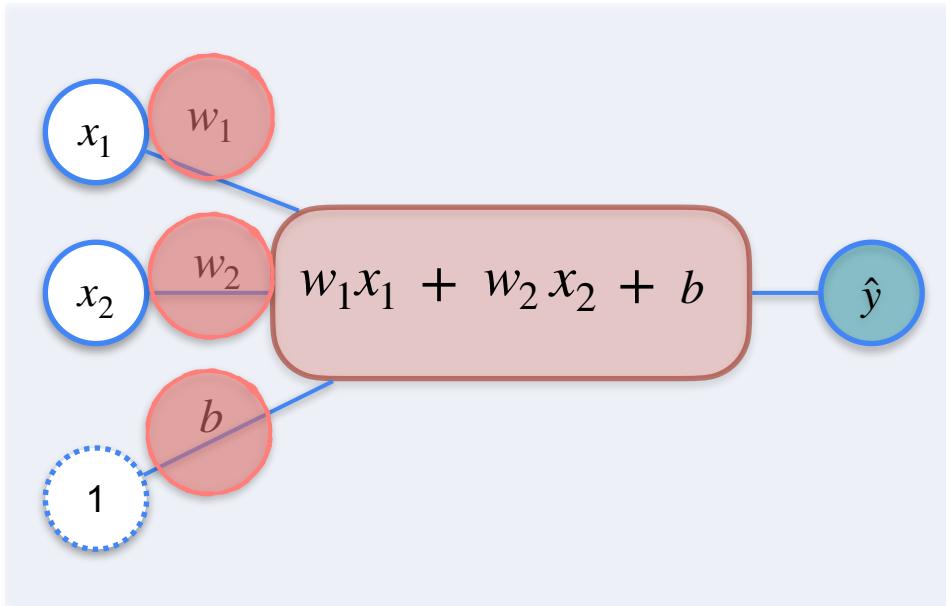
Main Goal:

Find weights and bias that will optimise the predictions.

i.e. Reduce the errors in the predictions

Regression With a Perceptron

Single Layer Neural Network Perceptron



$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

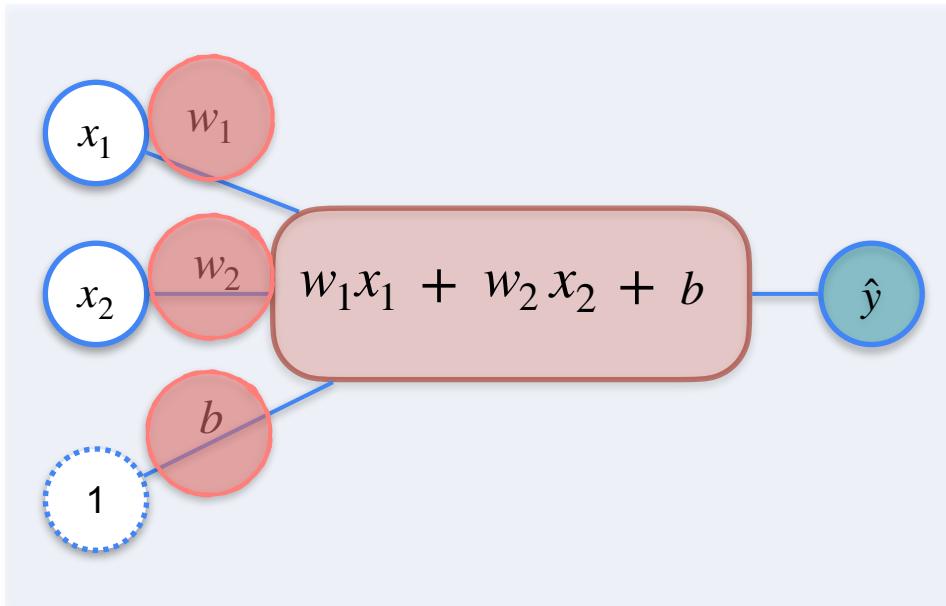
Find weights and bias that will optimise the predictions.

i.e. Reduce the errors in the predictions



Regression With a Perceptron

Single Layer Neural Network Perceptron



$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

Find weights and bias that will optimise the predictions.

i.e. Reduce the errors in the predictions



**The
Loss
Function**



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Optimization in Neural Networks and Newton's Method

Regression with a perceptron: Loss function

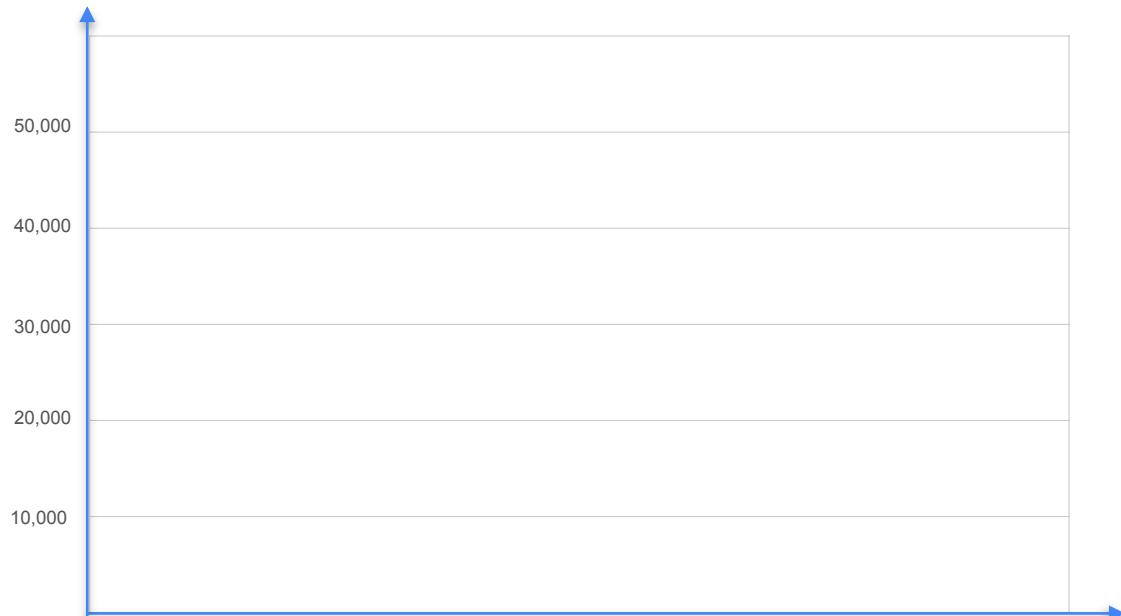
Mean Squared Error

Mean Squared Error

	y		
	\$20,000		
	\$30,000		
	\$50,000		

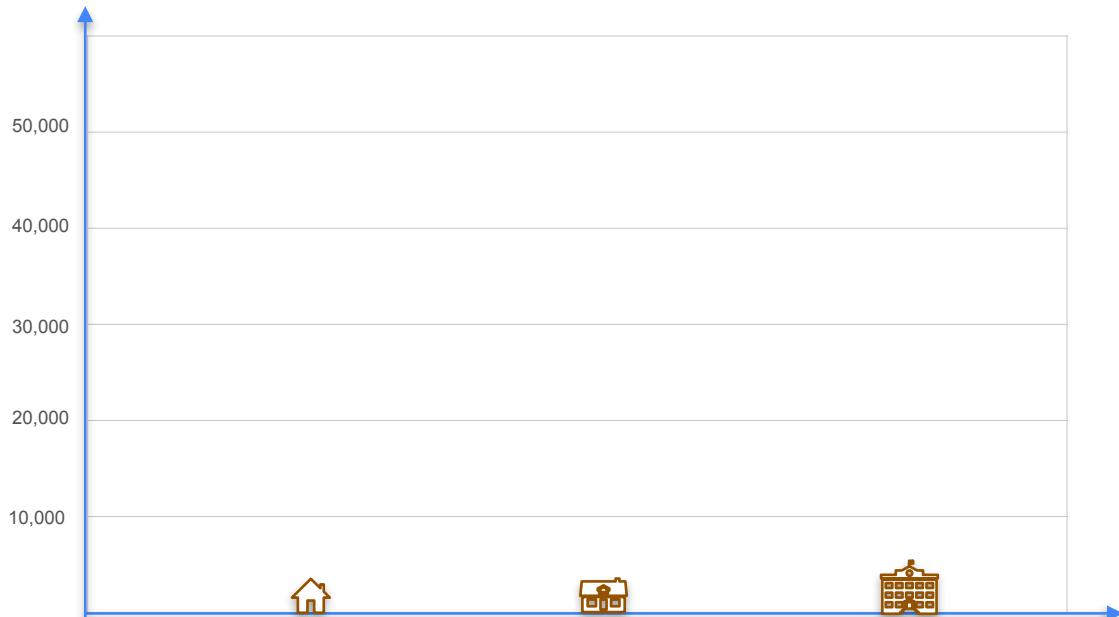
Mean Squared Error

	y		
	\$20,000		
	\$30,000		
	\$50,000		



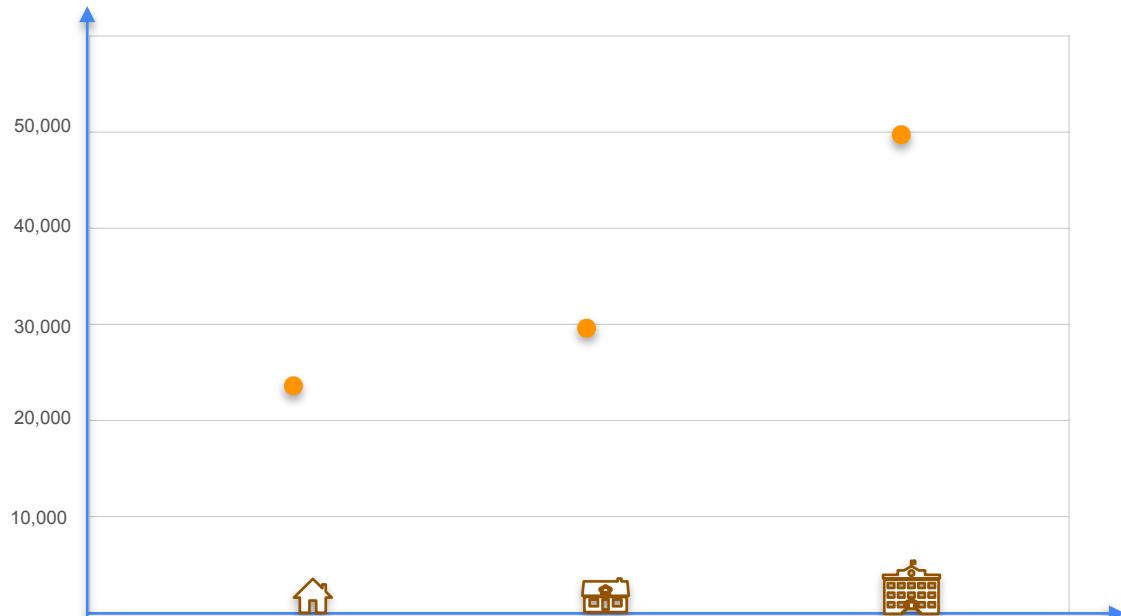
Mean Squared Error

	y		
	\$20,000		
	\$30,000		
	\$50,000		



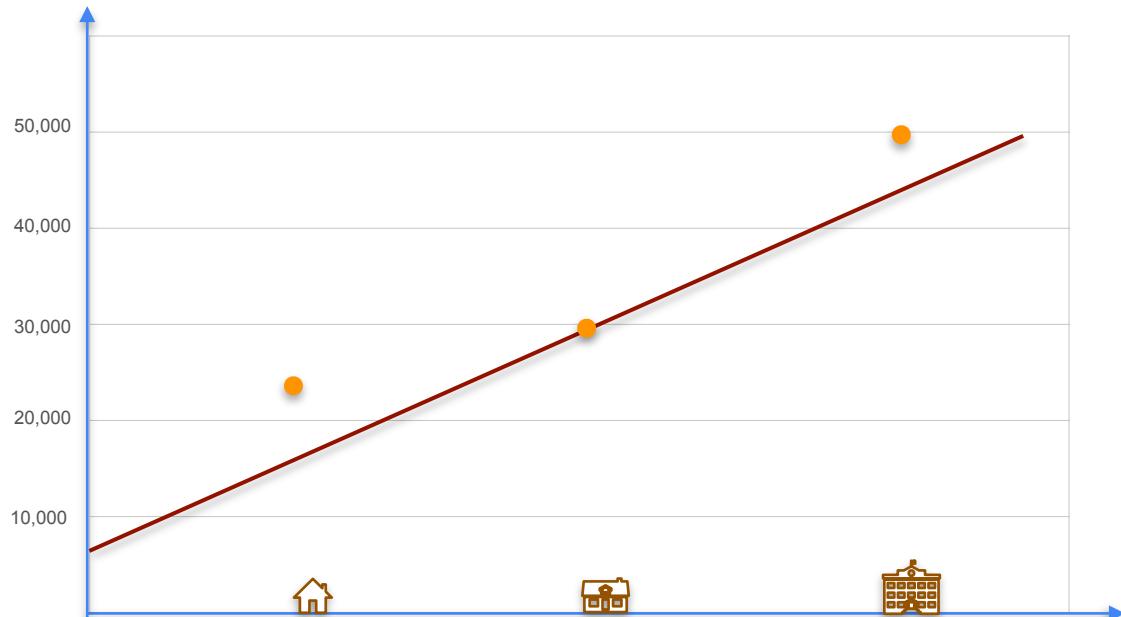
Mean Squared Error

	y		
	\$20,000		
	\$30,000		
	\$50,000		



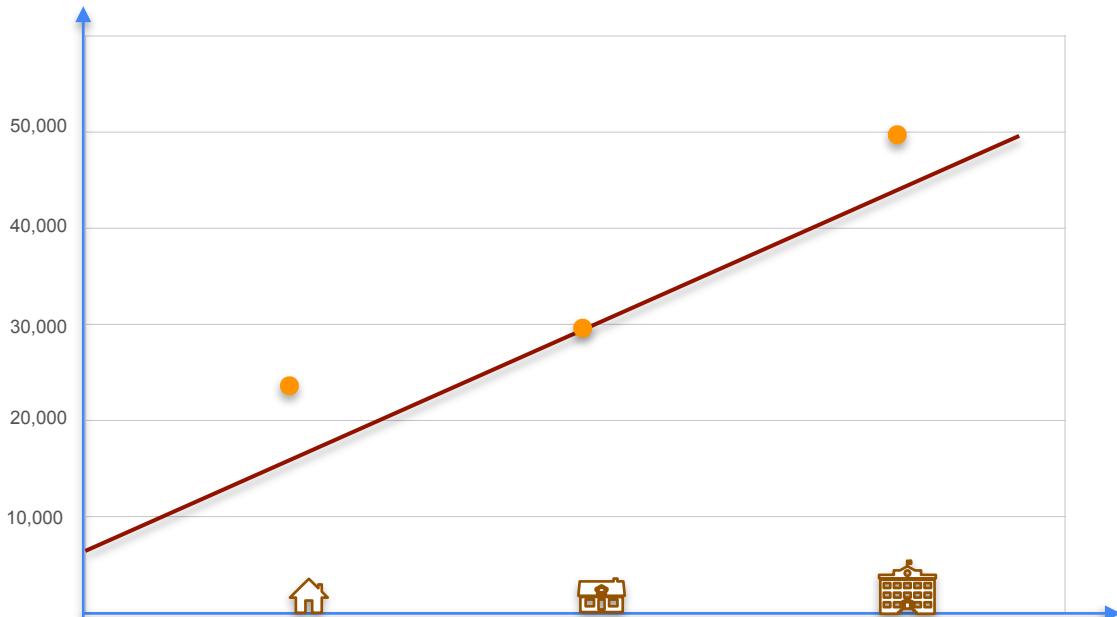
Mean Squared Error

	y		
	\$20,000		
	\$30,000		
	\$50,000		



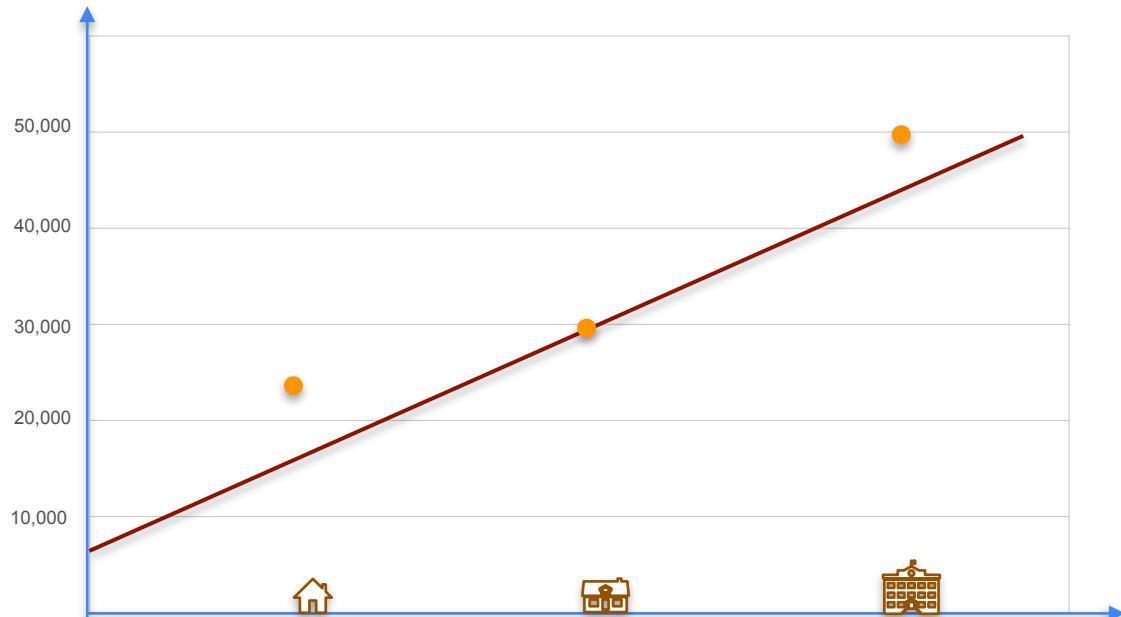
Mean Squared Error

	y	\hat{y}	
	\$20,000	\$15,000	
	\$30,000	\$30,000	
	\$50,000	\$45,000	



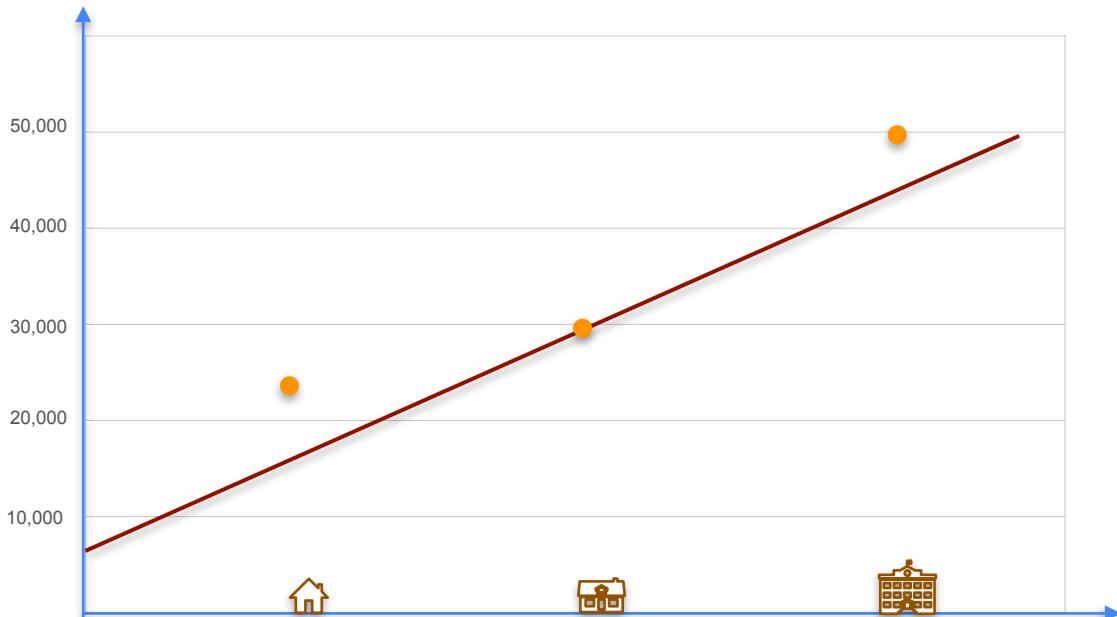
Mean Squared Error

	y	\hat{y}	$y - \hat{y}$
	\$20,000	\$15,000	
	\$30,000	\$30,000	
	\$50,000	\$45,000	



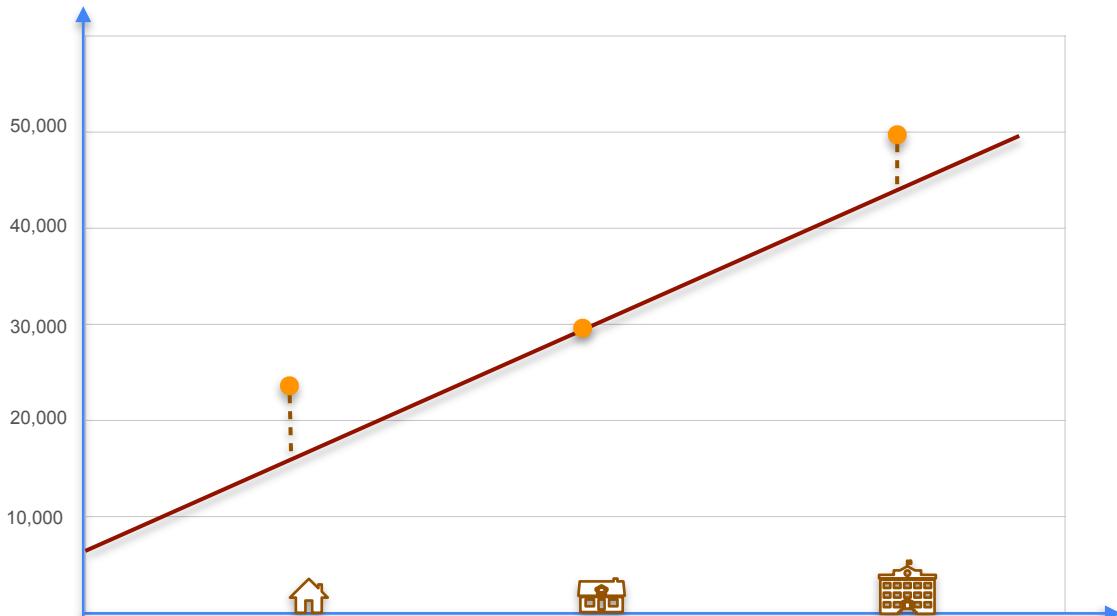
Mean Squared Error

	y	\hat{y}	$y - \hat{y}$
	\$20,000	\$15,000	Error
	\$30,000	\$30,000	Error
	\$50,000	\$45,000	Error



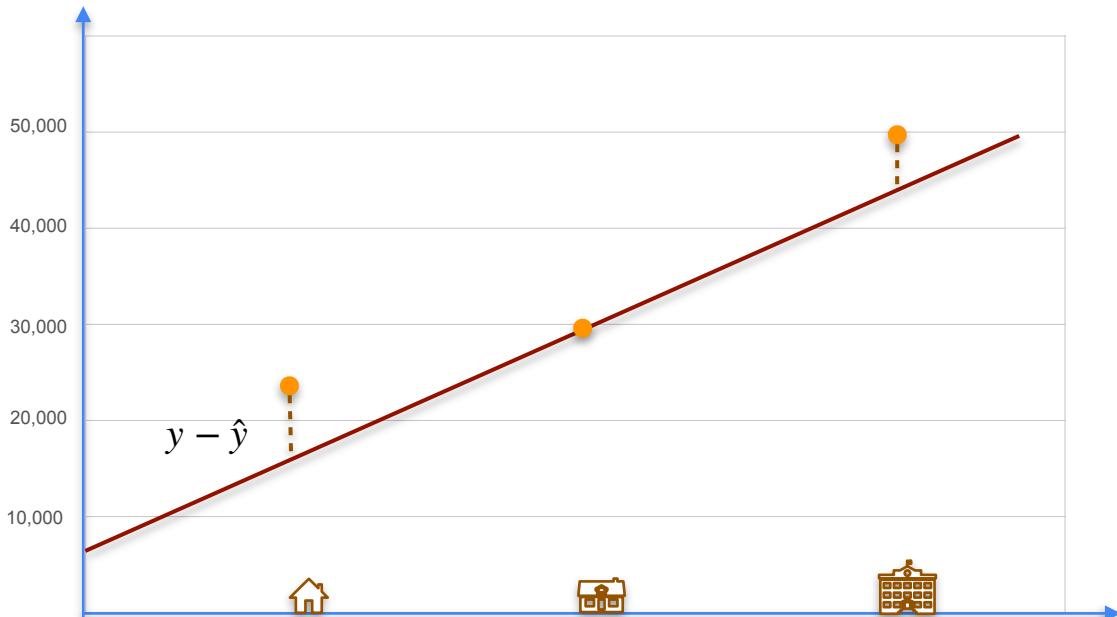
Mean Squared Error

	y	\hat{y}	$y - \hat{y}$
	\$20,000	\$15,000	Error
	\$30,000	\$30,000	Error
	\$50,000	\$45,000	Error



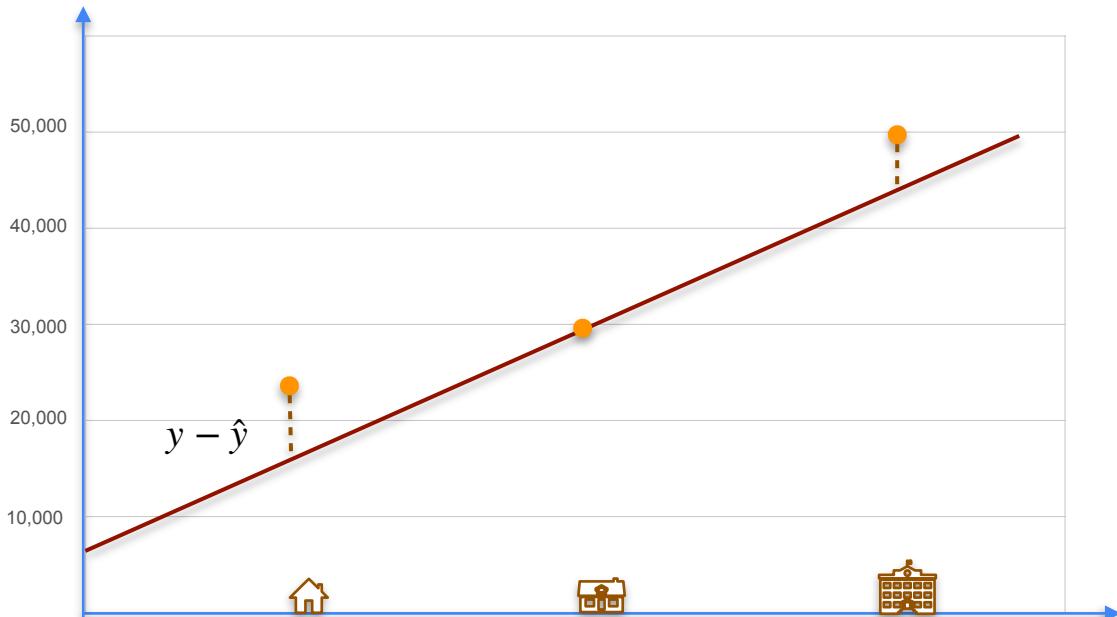
Mean Squared Error

	y	\hat{y}	$y - \hat{y}$
	\$20,000	\$15,000	Error
	\$30,000	\$30,000	Error
	\$50,000	\$45,000	Error



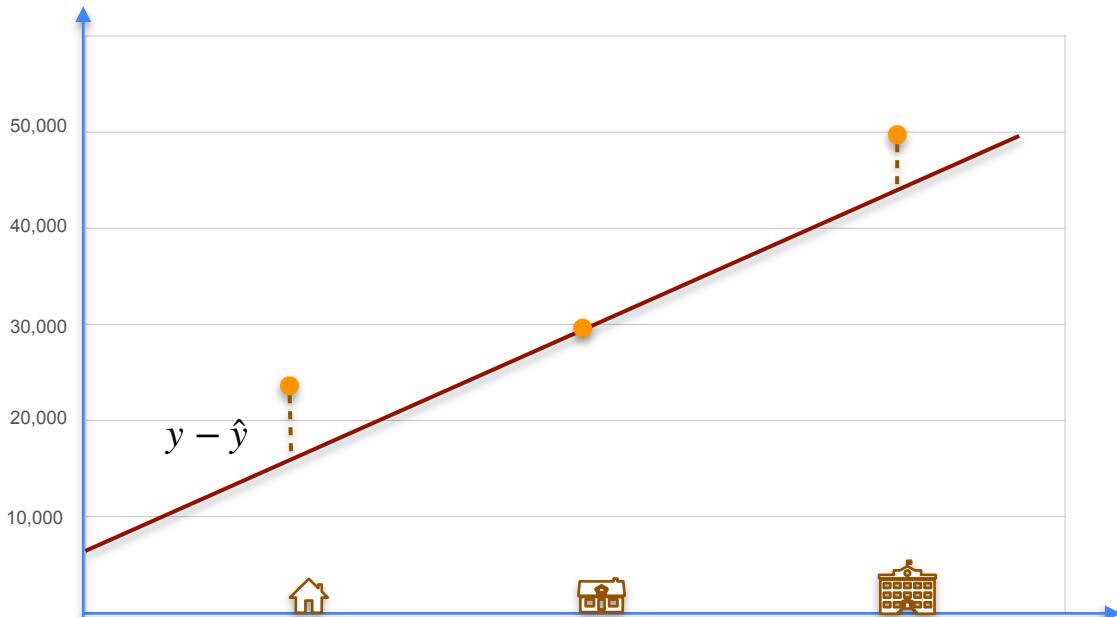
Mean Squared Error

	y	\hat{y}	$(y - \hat{y})^2$
	\$20,000	\$15,000	Error
	\$30,000	\$30,000	Error
	\$50,000	\$45,000	Error



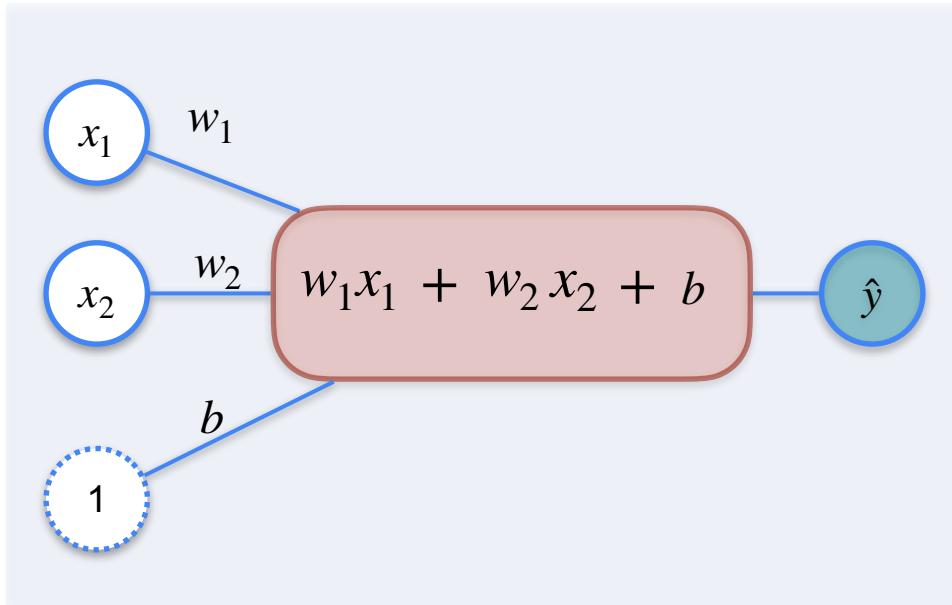
Mean Squared Error

	y	\hat{y}	$\frac{1}{2}(y - \hat{y})^2$
	\$20,000	\$15,000	Error
	\$30,000	\$30,000	Error
	\$50,000	\$45,000	Error



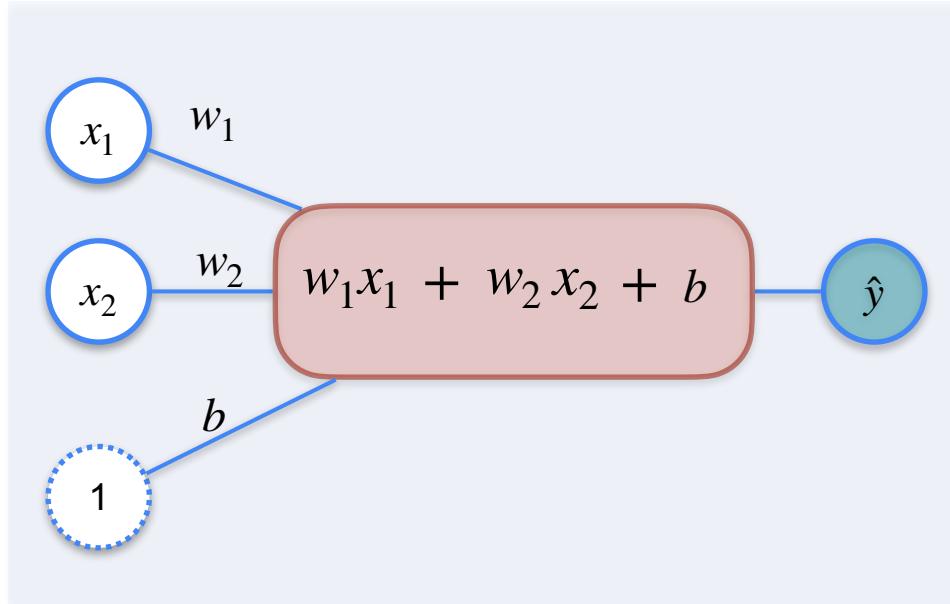
Regression With a Perceptron

Single Layer Neural Network Perceptron



Regression With a Perceptron

Single Layer Neural Network Perceptron

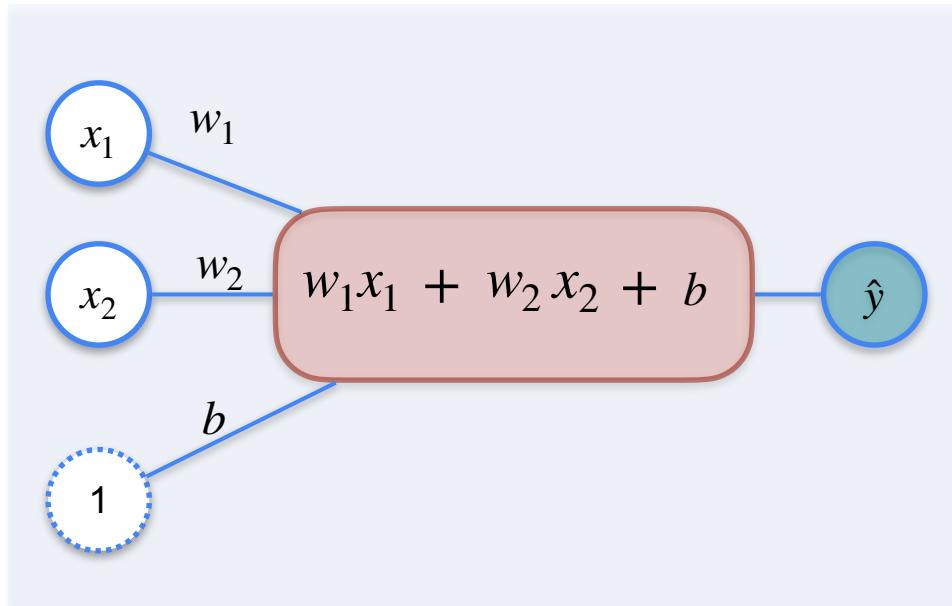


Prediction Function:

$$\hat{y}$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

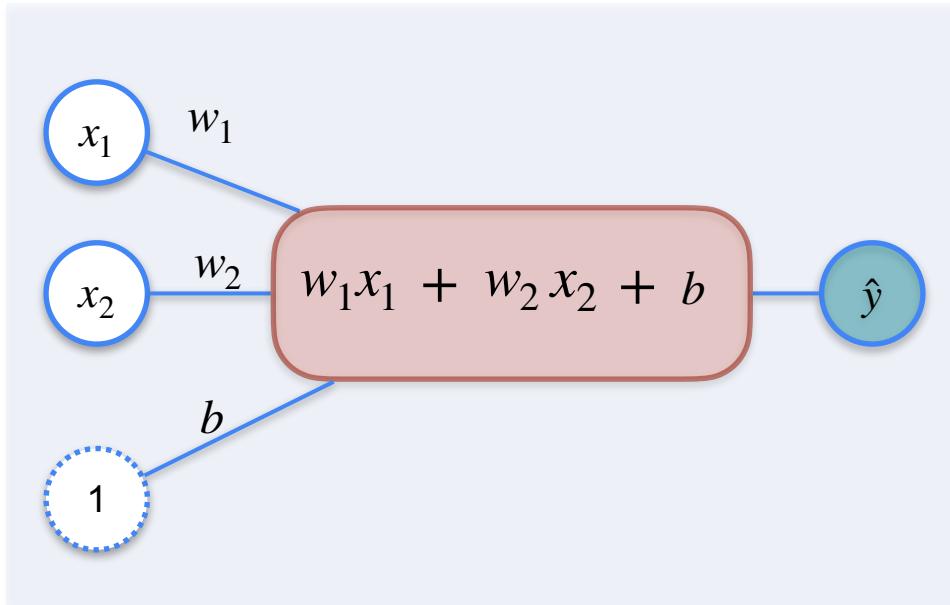


Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Regression With a Perceptron

Single Layer Neural Network Perceptron



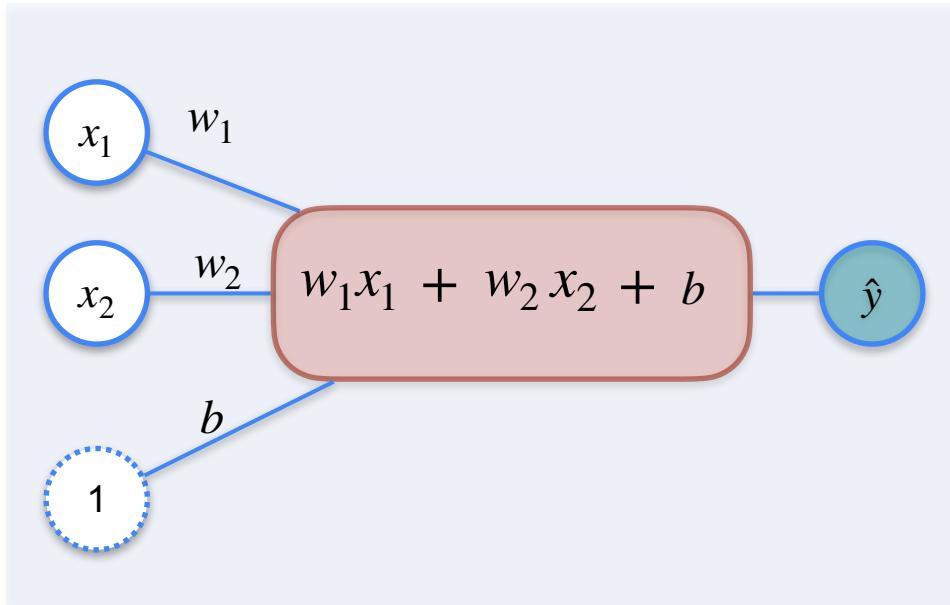
Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

Regression With a Perceptron

Single Layer Neural Network Perceptron



Prediction Function:

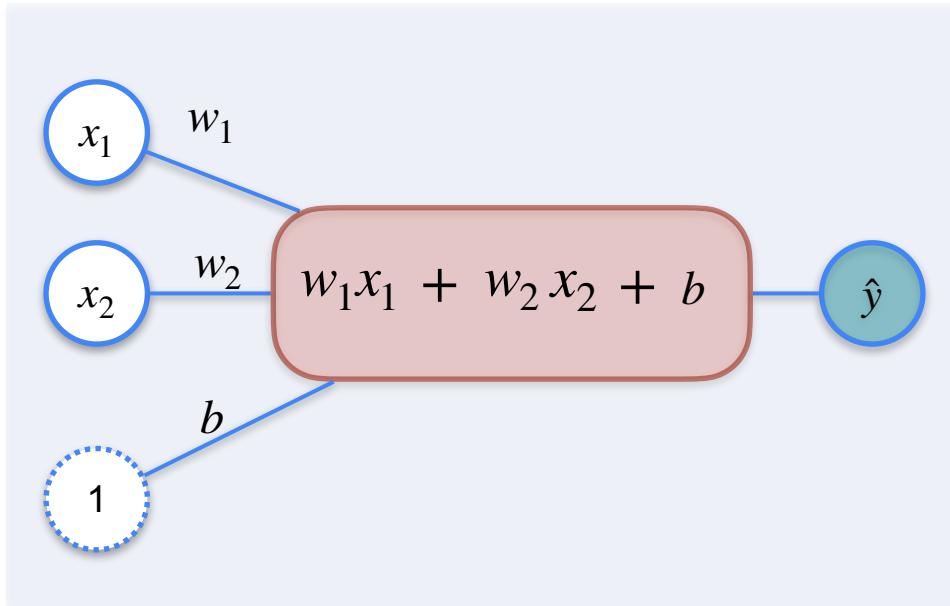
$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$= \frac{1}{2}(y - \hat{y})^2$$

Regression With a Perceptron

Single Layer Neural Network Perceptron



Prediction Function:

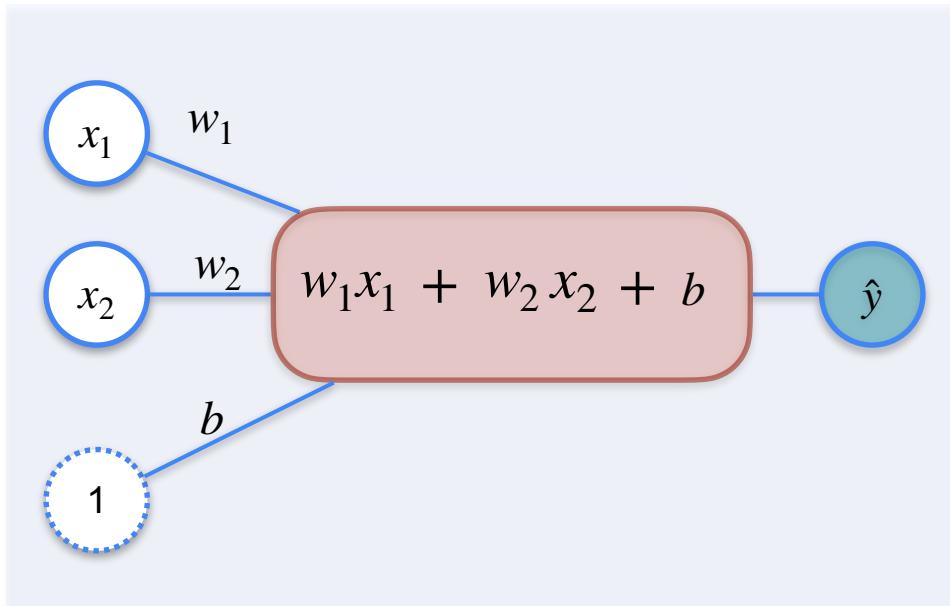
$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Regression With a Perceptron

Single Layer Neural Network Perceptron



Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

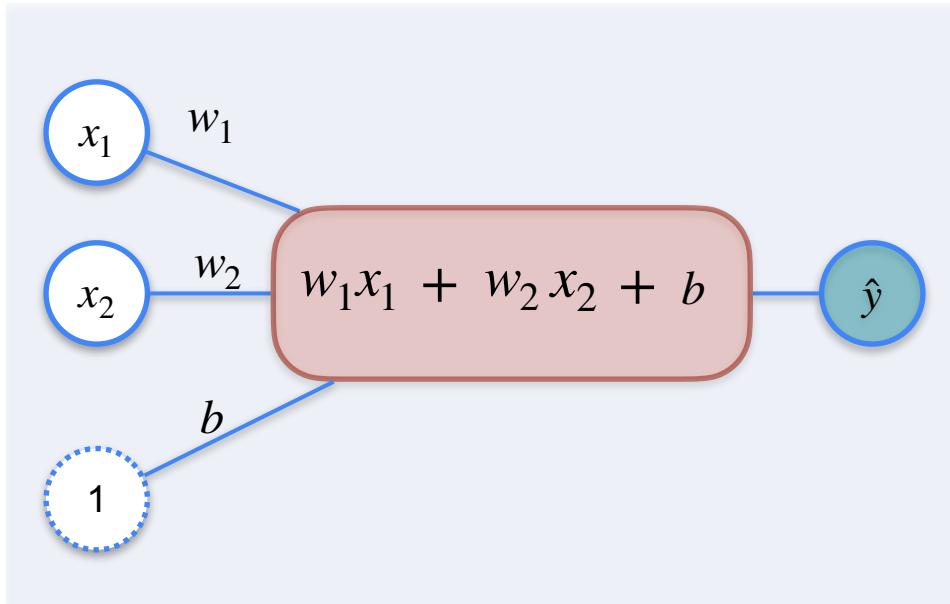
Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron



Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error



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Optimization in Neural Networks and Newton's Method

Regression with a perceptron: Gradient Descent

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

To find optimal values for:

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

To find optimal values for:

$$w_1, w_2, b$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

To find optimal values for:

$$w_1, w_2, b$$

You need gradient descent

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

To find optimal values for:

$$w_1, w_2, b$$

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

To find optimal values for:

$$w_1, w_2, b$$

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

To find optimal values for:

$$w_1, w_2, b$$

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Regression With a Perceptron

Prediction Function:

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$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

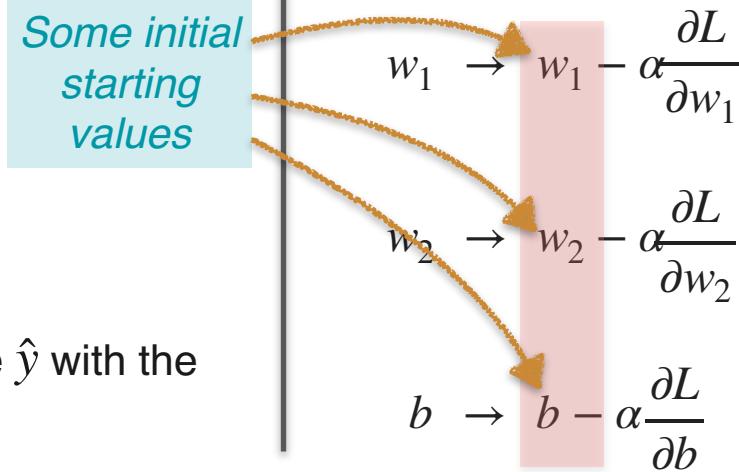
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Regression With a Perceptron

Prediction Function:

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Main Goal:

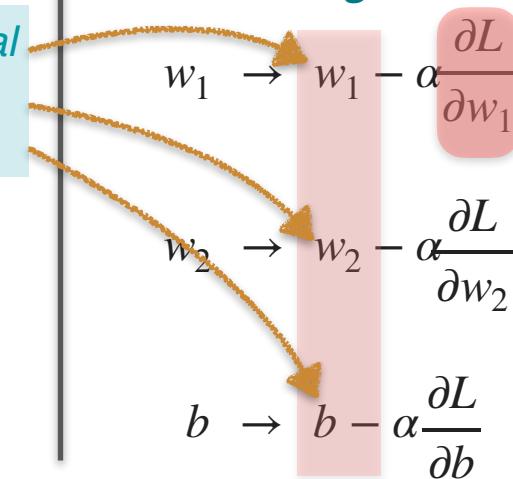
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You need gradient descent

Some initial starting values



Regression With a Perceptron

Prediction Function:

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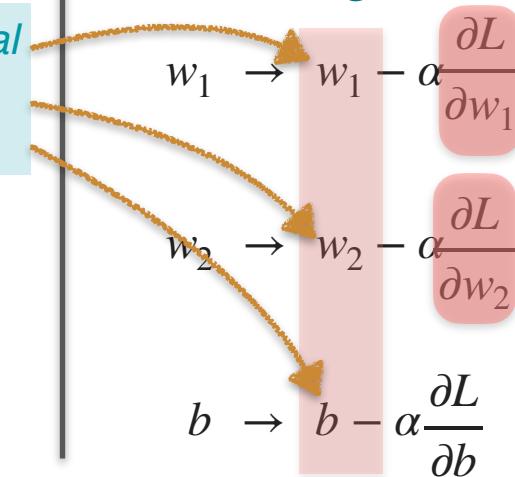
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Regression With a Perceptron

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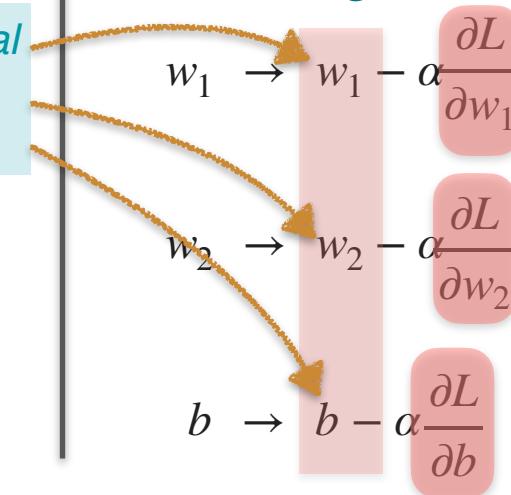
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Regression With a Perceptron

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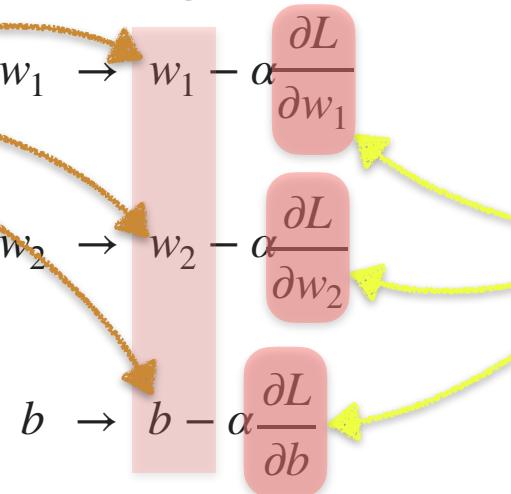
Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

Some initial starting values

To find optimal values for:
 w_1, w_2, b

You need gradient descent



SUB-TASK
Find the following partial derivatives

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

$$\frac{\partial L}{\partial b}$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} =$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} =$$

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Prediction Function:

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Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot$$

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$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	
$\frac{\partial \hat{y}}{\partial b}$	
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

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$\frac{\partial L}{\partial \hat{y}}$	
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Prediction Function:

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Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= (y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

Regression With Perceptron

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Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= (y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

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$\frac{\partial \hat{y}}{\partial w_1}$	
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$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	$= 1$
$\frac{\partial \hat{y}}{\partial w_1}$	
$\frac{\partial \hat{y}}{\partial w_2}$	

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
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$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
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Regression With Perceptron

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$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	$= 1$
$\frac{\partial \hat{y}}{\partial w_1}$	$= x_1$
$\frac{\partial \hat{y}}{\partial w_2}$	

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$\frac{\partial L}{\partial \hat{y}}$	$= -(y - \hat{y})$
$\frac{\partial \hat{y}}{\partial b}$	$= 1$
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Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

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Perform Gradient Descent

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

ie. optimal values for:

w_1 , w_2 , b

$$w_1 = w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Perform Gradient Descent

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

$$w_1 = w_1 - \alpha$$

ie. optimal values for:

w_1 , w_2 , b

Perform Gradient Descent

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

ie. optimal values for:

w_1 , w_2 , b

Perform Gradient Descent

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give \hat{y} with the least error

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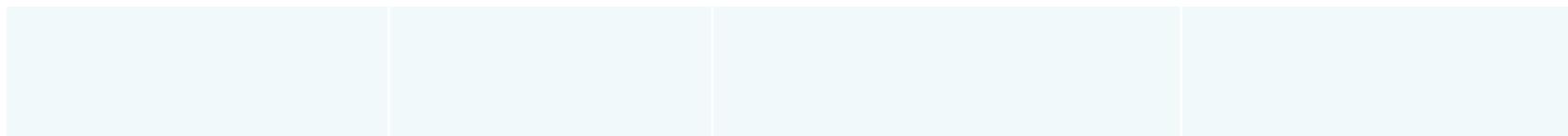
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a perceptron

Classification Problem Motivation

Classification Problem Motivation



Classification Problem Motivation

<i>Sentence</i>			

Classification Problem Motivation

<i>Sentence</i>			
<i>Aack aack aack!</i>			

Classification Problem Motivation

<i>Sentence</i>			
<i>Aack aack aack!</i>			

Beep beep!

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Classification Problem Motivation

<i>Sentence</i>			
<i>Aack aack aack!</i>			
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Classification Problem Motivation

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Classification Problem Motivation

<i>Sentence</i>			<i>Mood</i>
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Beep beep!

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Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			
<i>Aack beep beep beep!</i>			
<i>Aack beep aack!</i>			

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			😔
<i>Aack beep beep beep!</i>			
<i>Aack beep aack!</i>			

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			
<i>Aack beep aack!</i>			

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			😔
<i>Aack beep aack!</i>			

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			😊

Classification Problem Motivation

Sentence			Mood
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			<i>Happy</i> 😊

Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>			<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			<i>Happy</i> 😊

Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3		<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			<i>Happy</i> 😊

Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	<i>Happy</i> 😊
<i>Beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			<i>Happy</i> 😊

Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	<i>Happy</i> 😊
<i>Beep beep!</i>	0		<i>Sad</i> 😞
<i>Aack beep beep beep!</i>			<i>Sad</i> 😞
<i>Aack beep aack!</i>			<i>Happy</i> 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
<i>Aack aack aack!</i>	3	0	Happy 😊
<i>Beep beep!</i>	0	2	Sad 😞
<i>Aack beep beep beep!</i>			Sad 😞
<i>Aack beep aack!</i>			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
<i>Aack aack aack!</i>	3	0	Happy 😊
<i>Beep beep!</i>	0	2	Sad 😞
<i>Aack beep beep beep!</i>	1		Sad 😞
<i>Aack beep aack!</i>			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
<i>Aack aack aack!</i>	3	0	Happy 😊
<i>Beep beep!</i>	0	2	Sad 😞
<i>Aack beep beep beep!</i>	1	3	Sad 😞
<i>Aack beep aack!</i>			Happy 😊

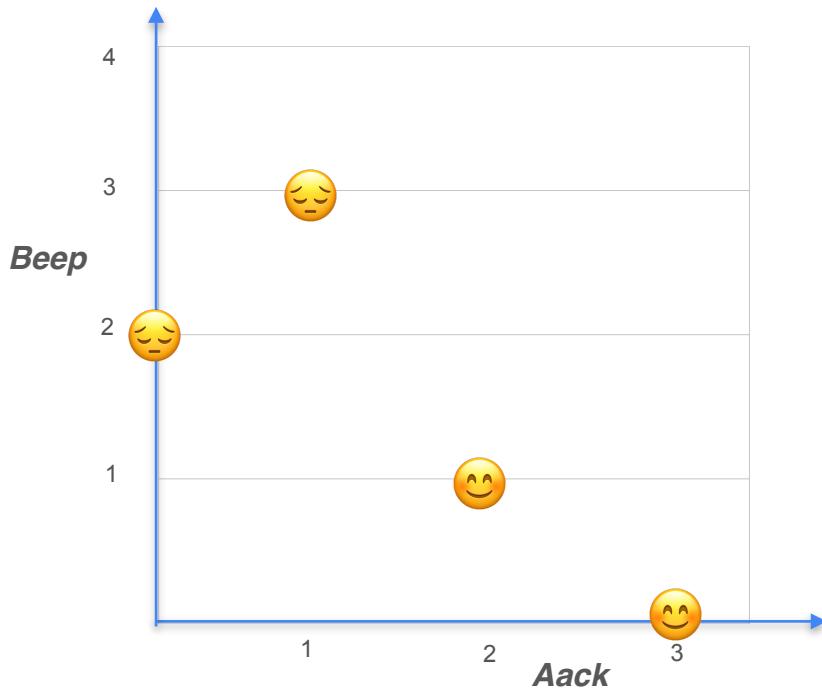
Classification Problem Motivation

Sentence	Aack	Beep	Mood
<i>Aack aack aack!</i>	3	0	Happy 😊
<i>Beep beep!</i>	0	2	Sad 😞
<i>Aack beep beep beep!</i>	1	3	Sad 😞
<i>Aack beep aack!</i>	2		Happy 😊

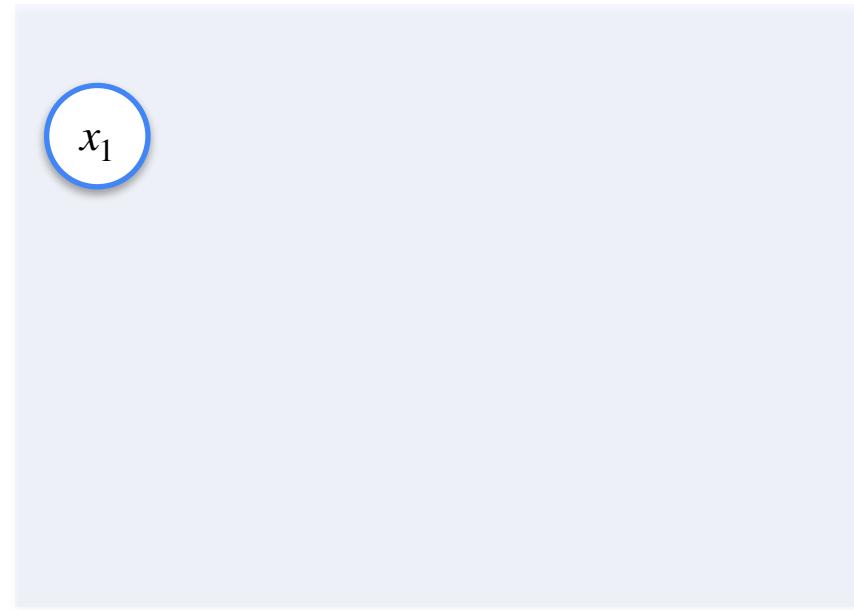
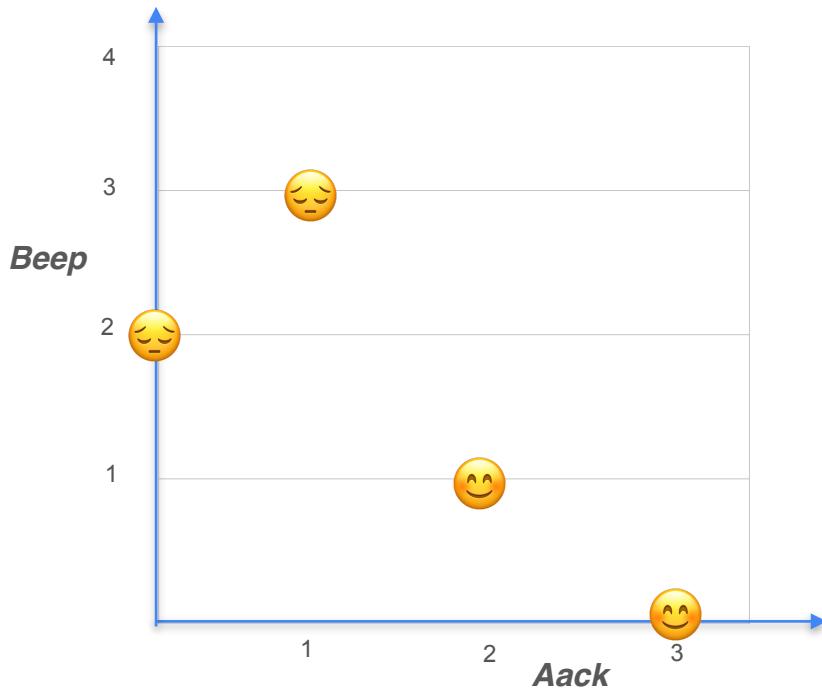
Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	<i>Happy</i> 😊
<i>Beep beep!</i>	0	2	<i>Sad</i> 😞
<i>Aack beep beep beep!</i>	1	3	<i>Sad</i> 😞
<i>Aack beep aack!</i>	2	1	<i>Happy</i> 😊

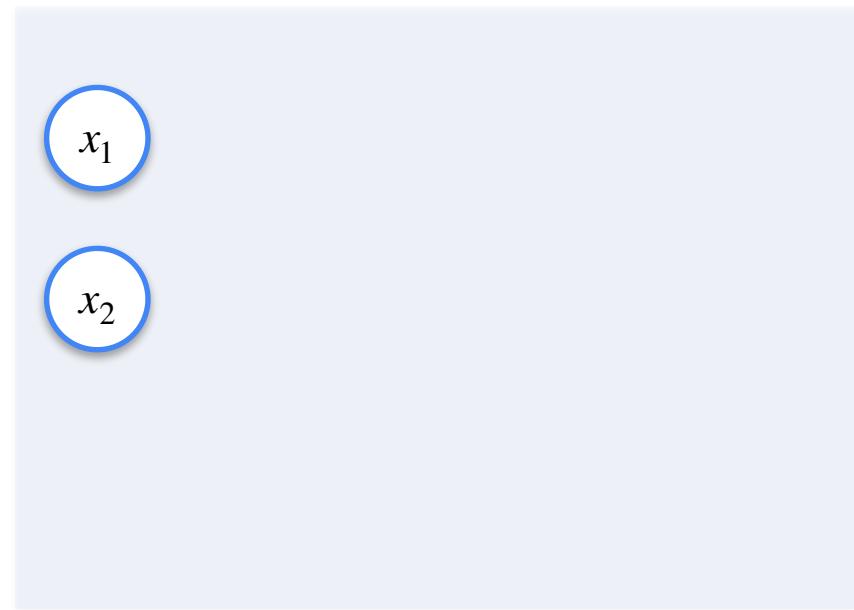
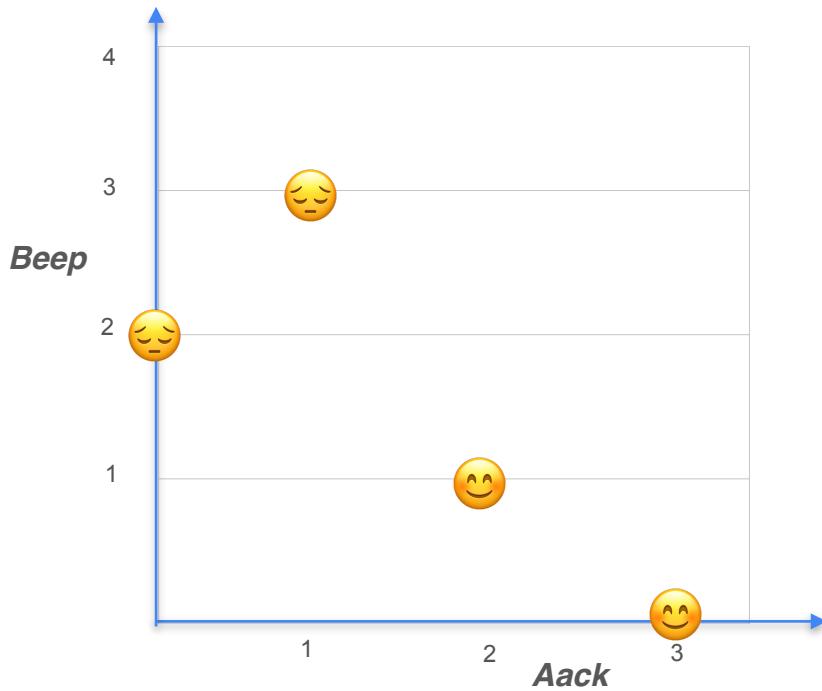
Classification Problem Motivation



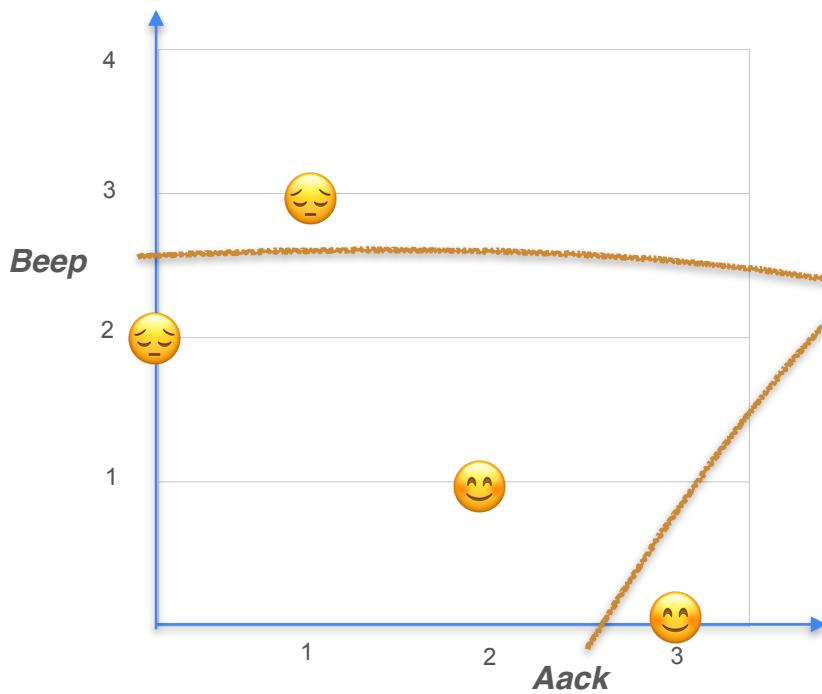
Classification Problem Motivation



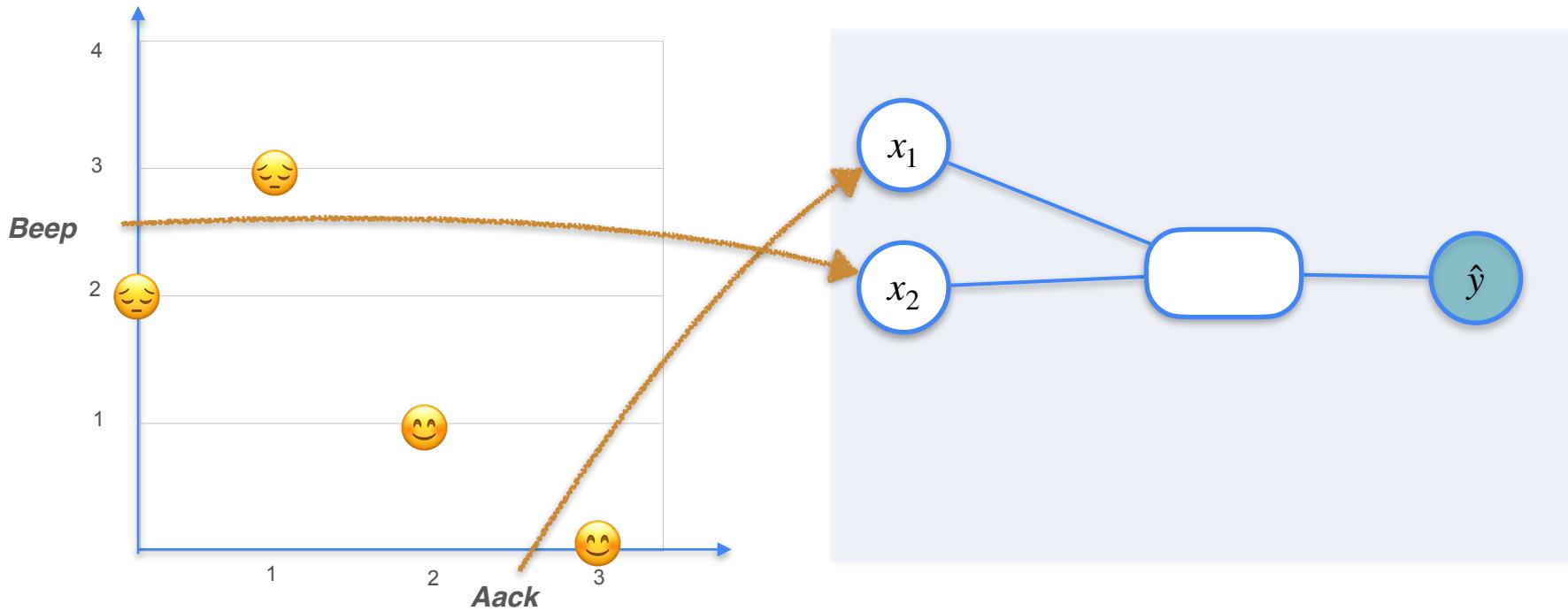
Classification Problem Motivation



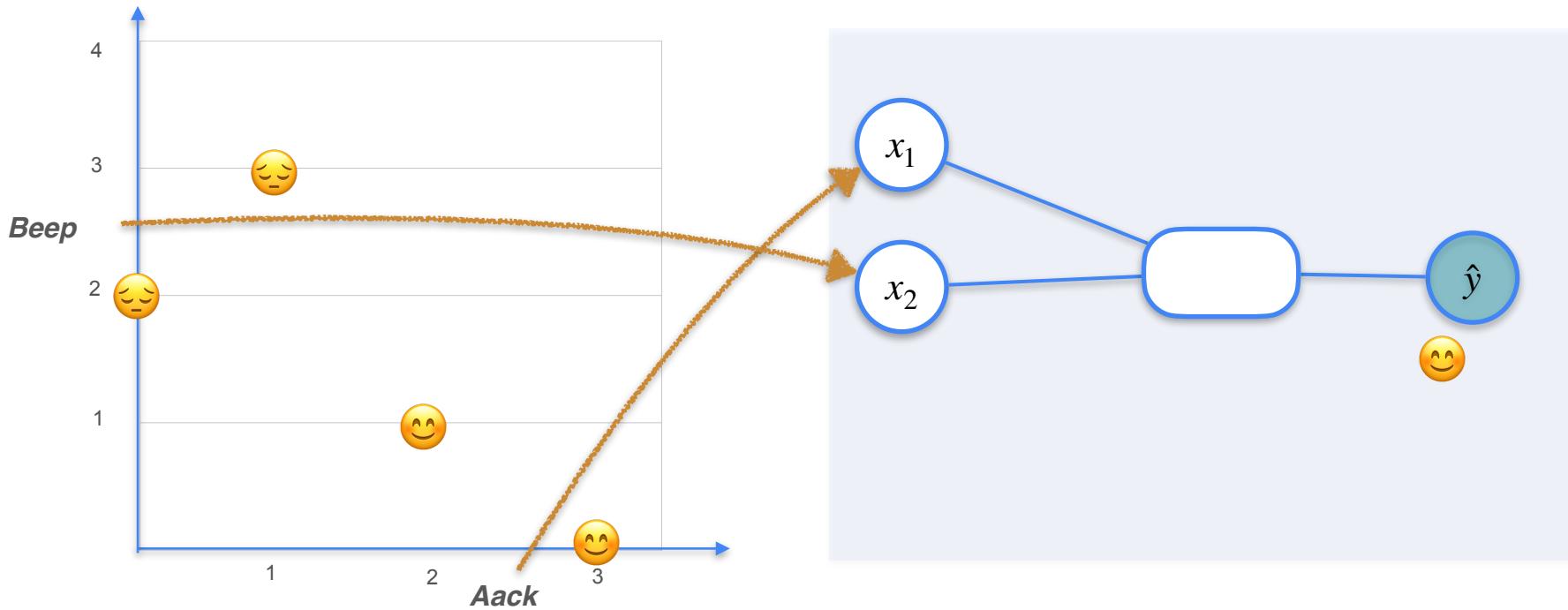
Classification Problem Motivation



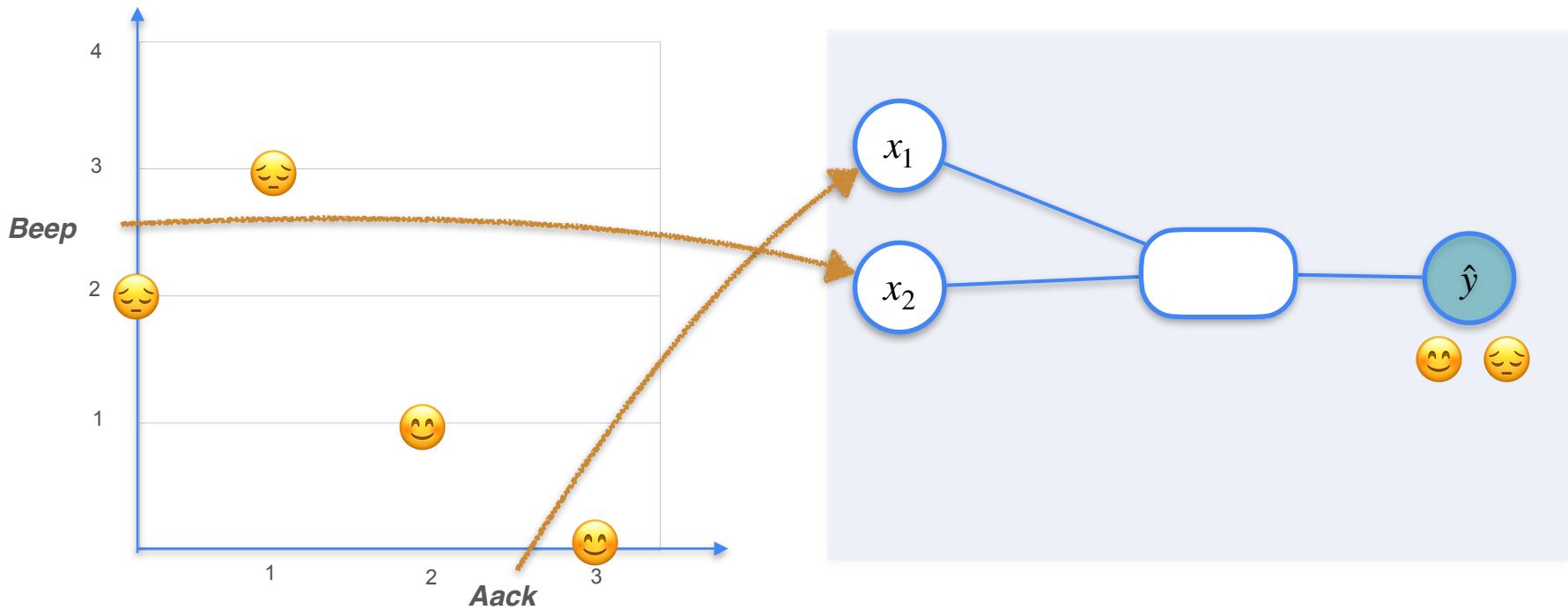
Classification Problem Motivation



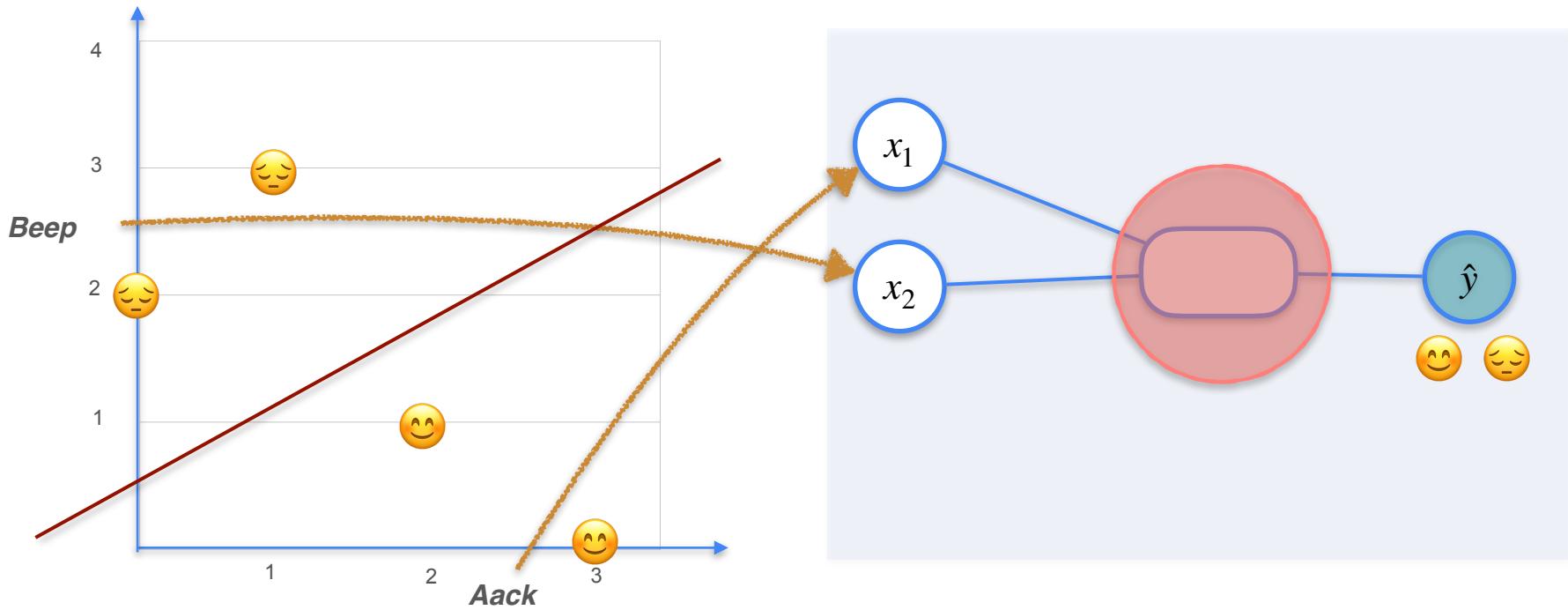
Classification Problem Motivation



Classification Problem Motivation

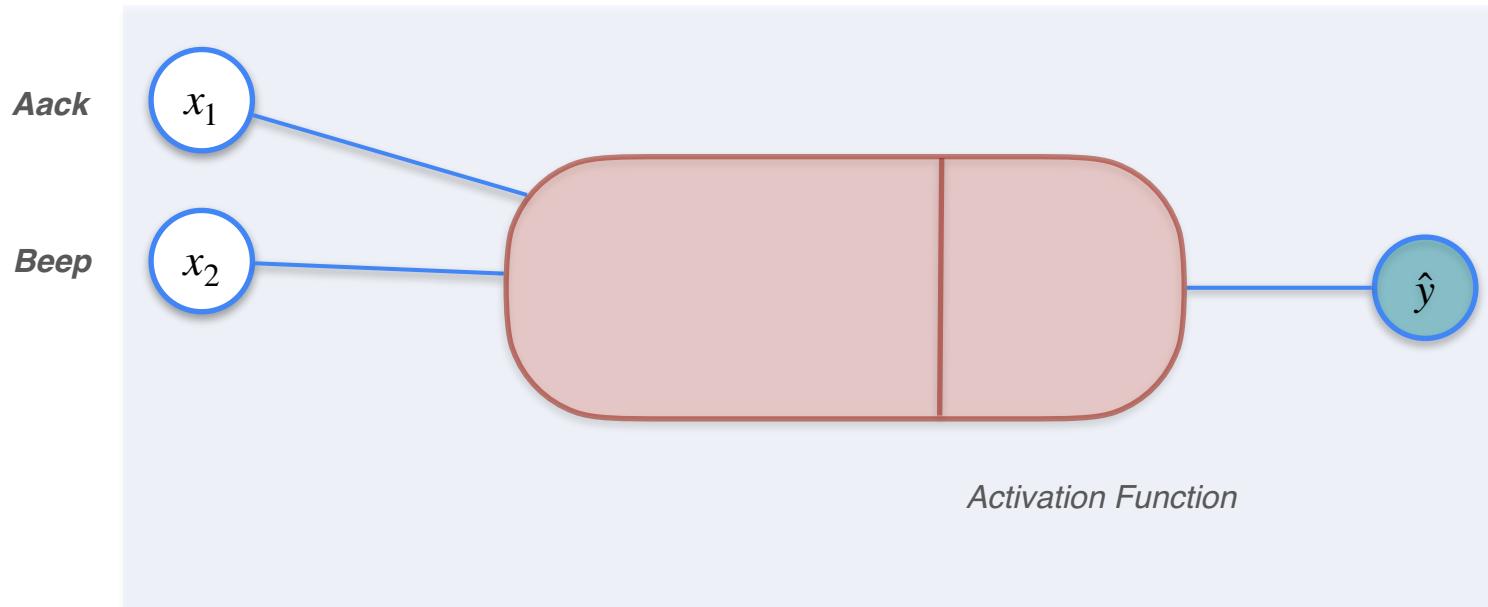


Classification Problem Motivation



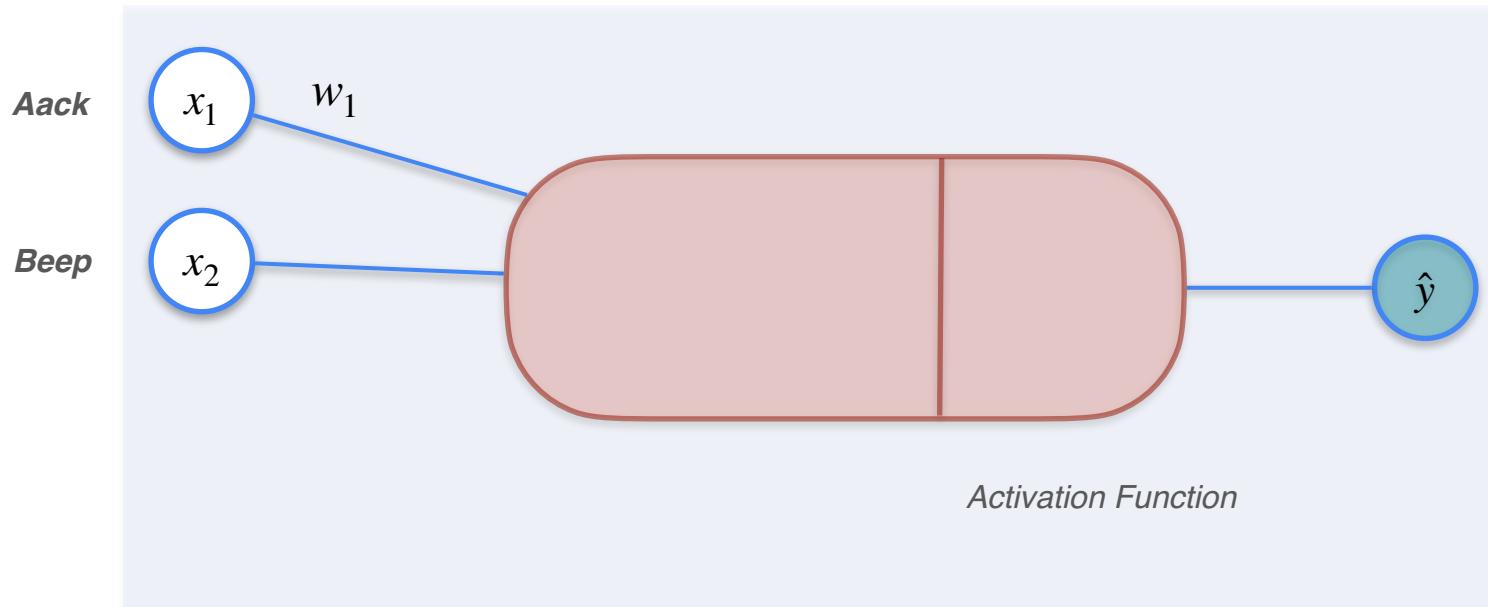
Classification With a Perceptron

Single Layer Neural Network Perceptron



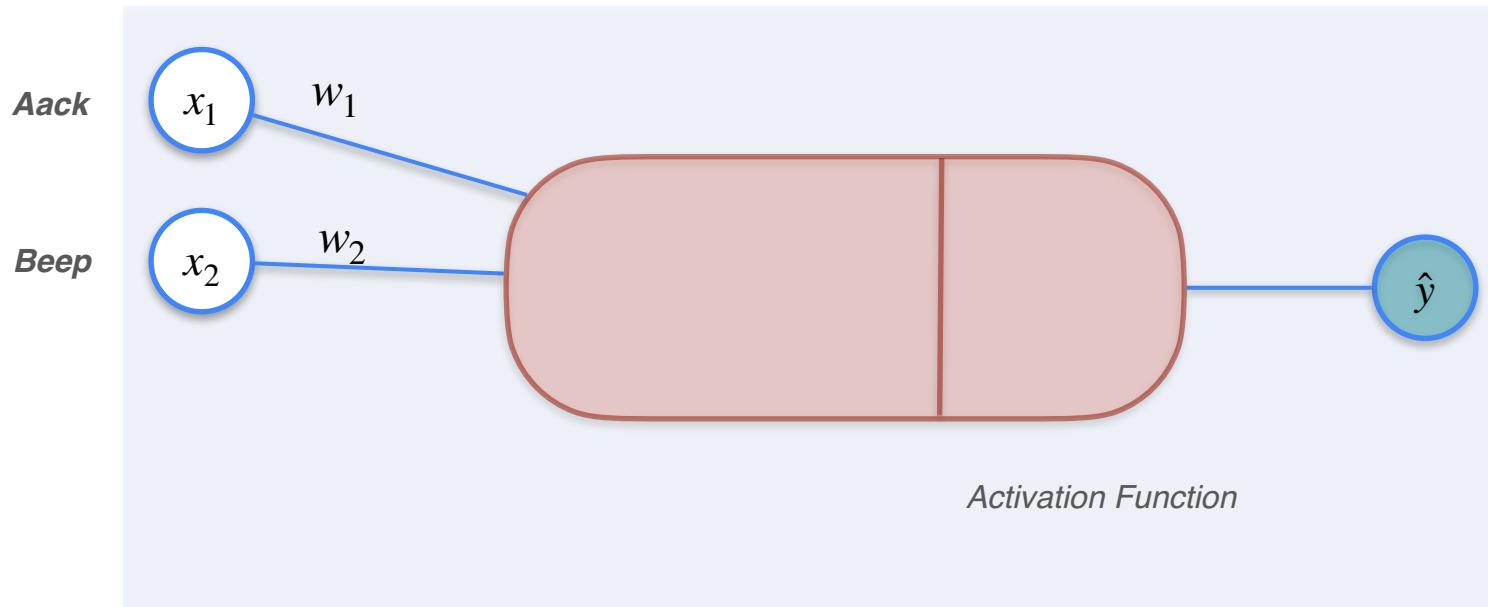
Classification With a Perceptron

Single Layer Neural Network Perceptron



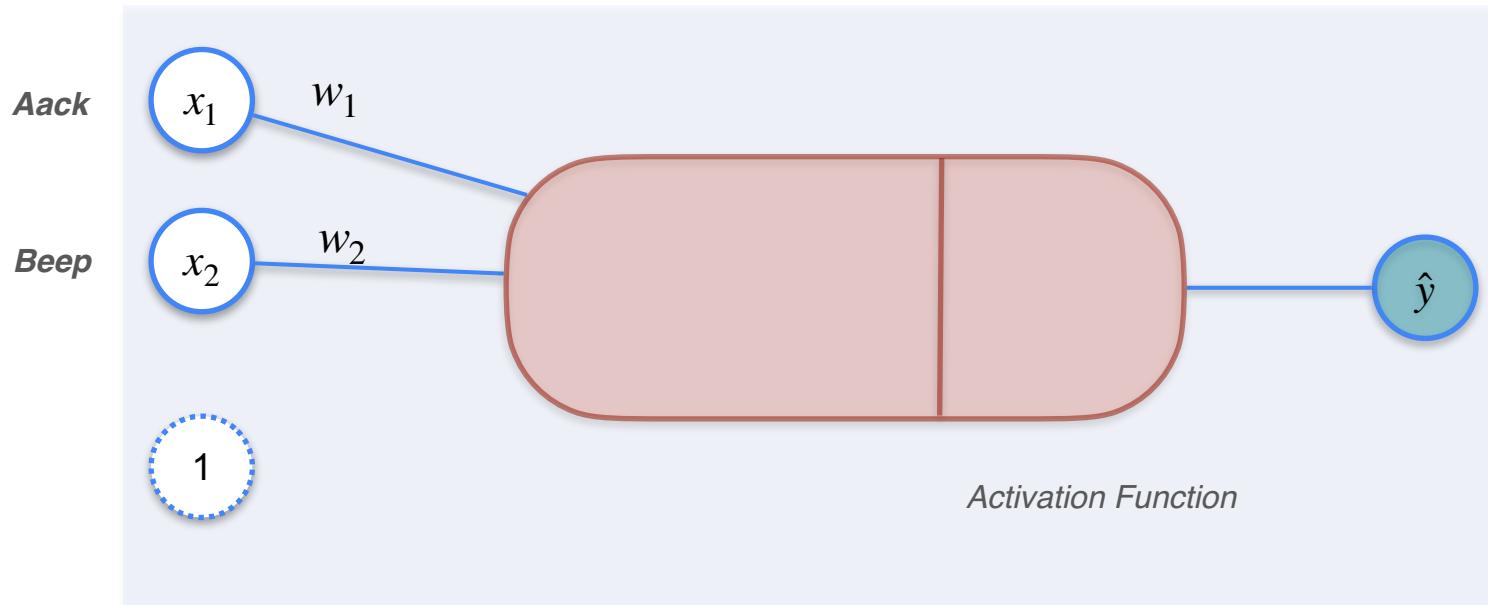
Classification With a Perceptron

Single Layer Neural Network Perceptron



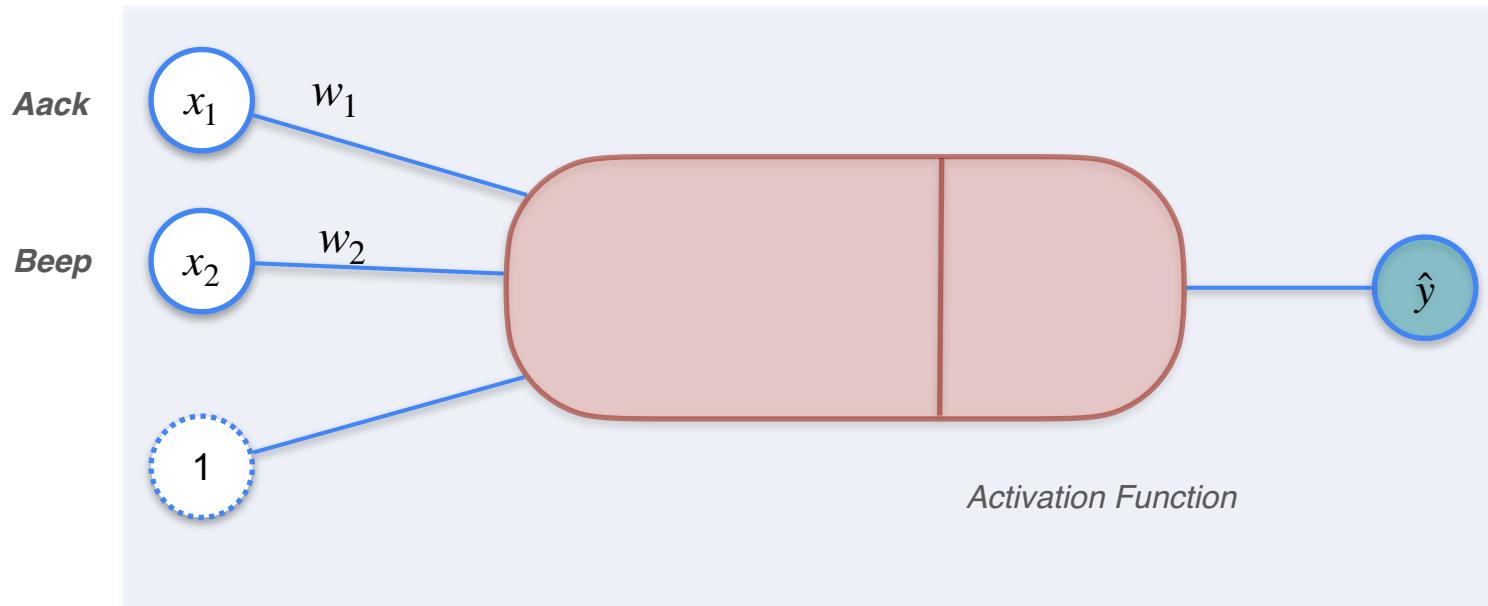
Classification With a Perceptron

Single Layer Neural Network Perceptron



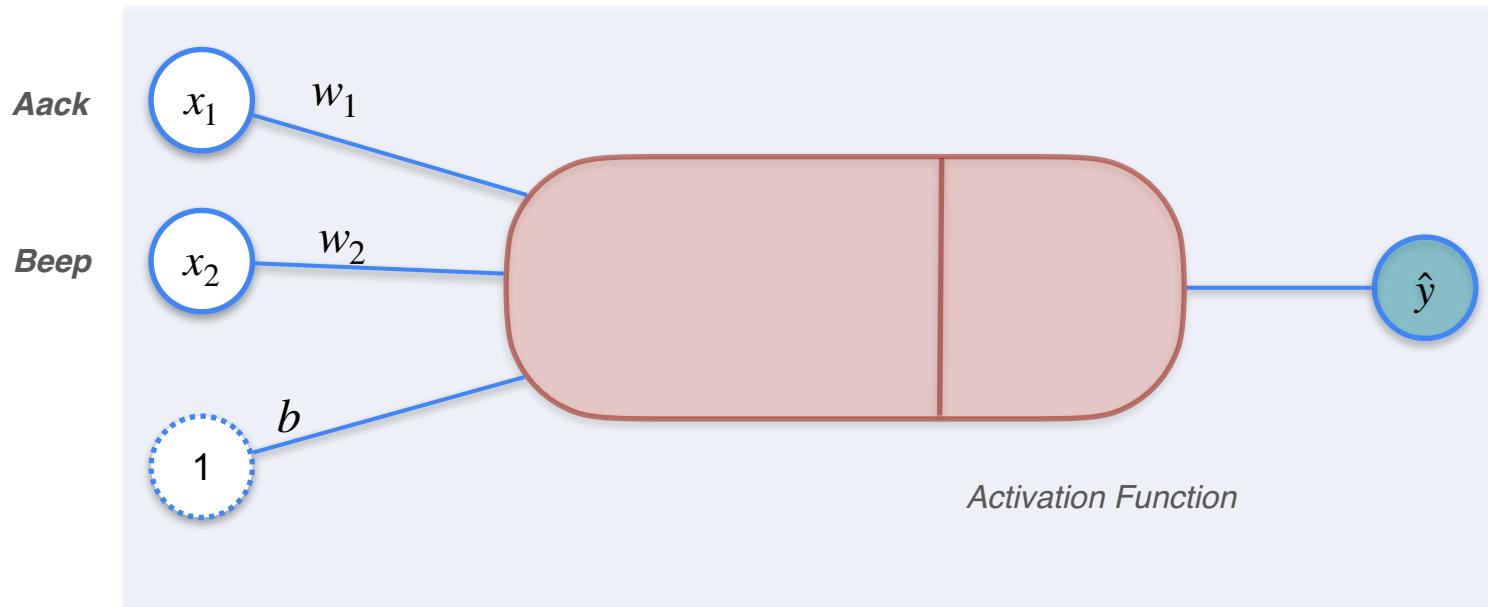
Classification With a Perceptron

Single Layer Neural Network Perceptron



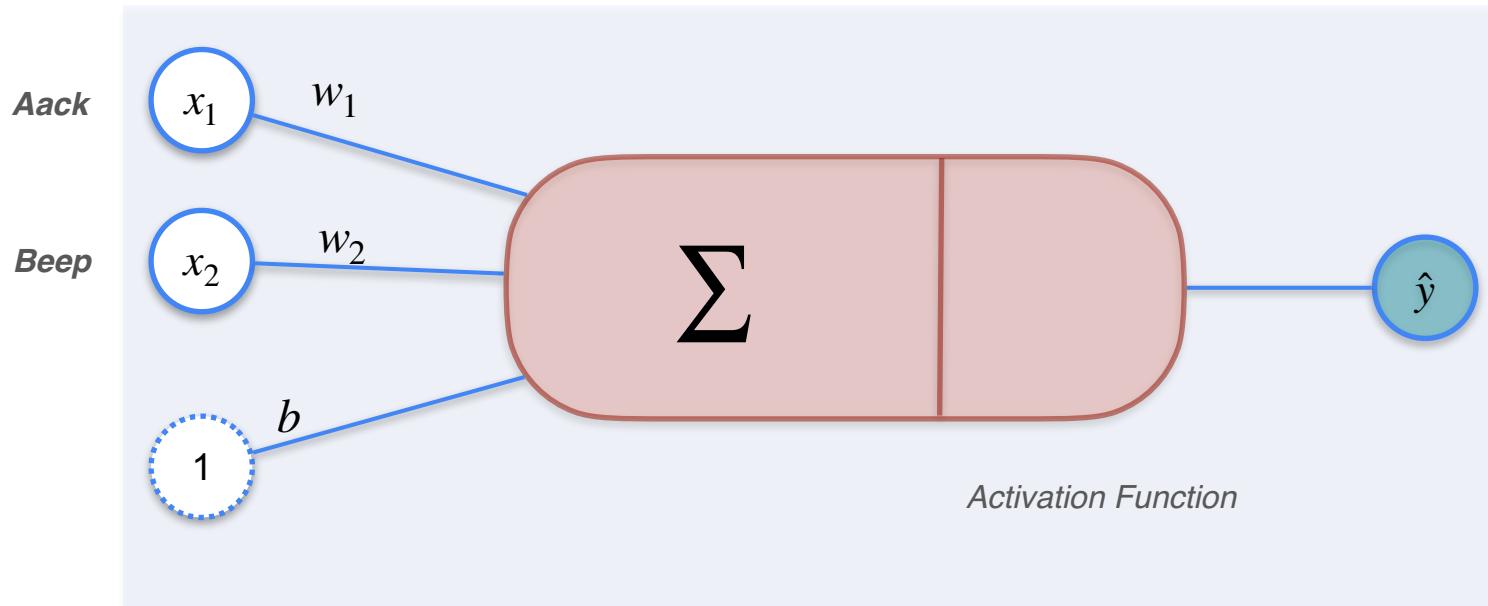
Classification With a Perceptron

Single Layer Neural Network Perceptron



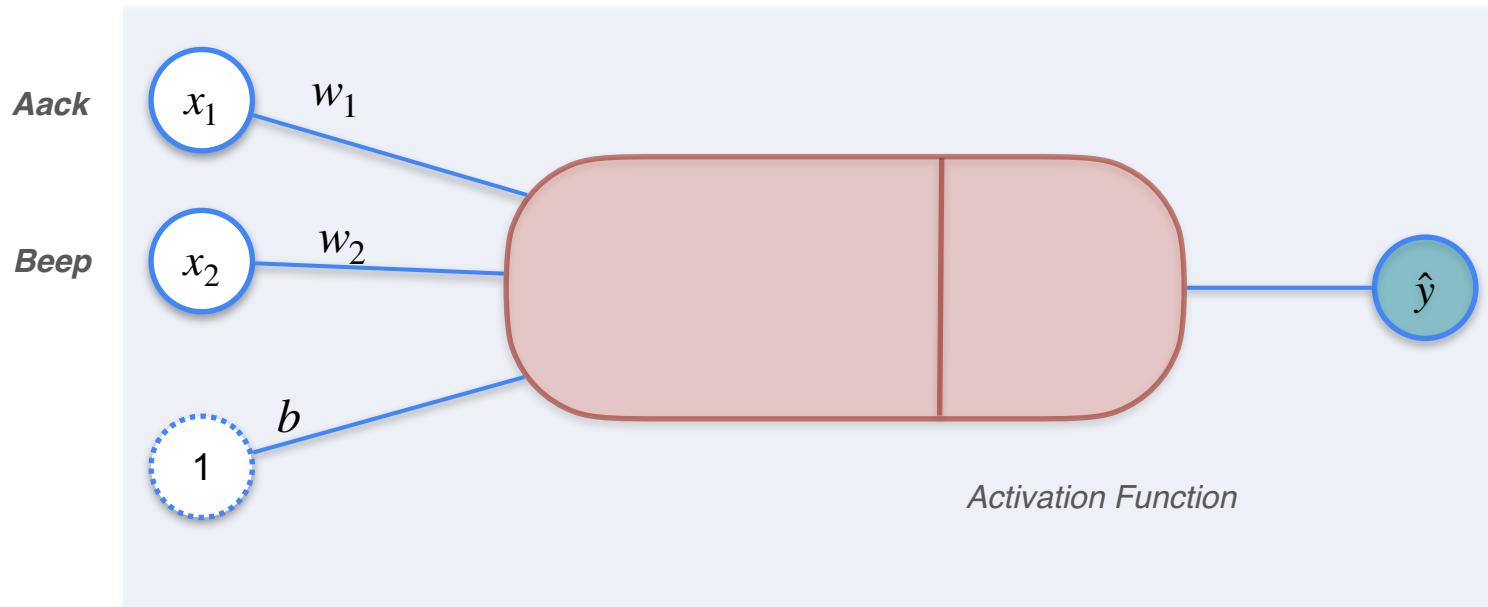
Classification With a Perceptron

Single Layer Neural Network Perceptron



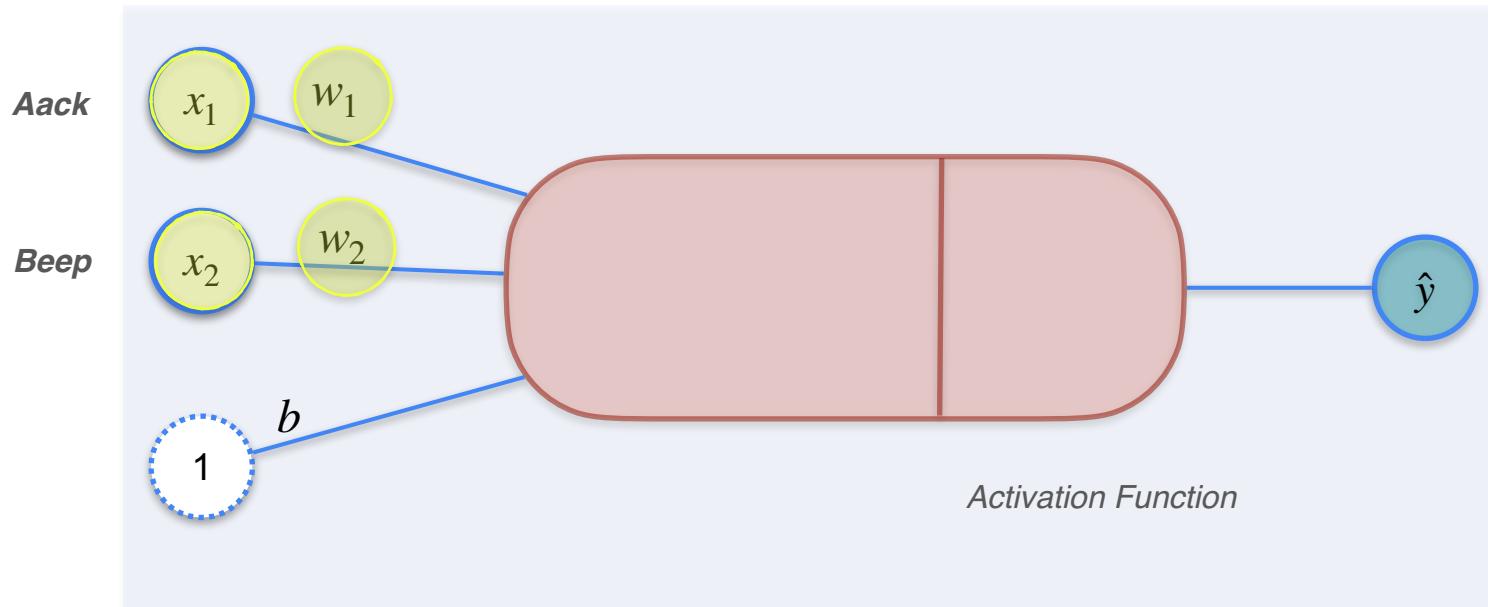
Classification With a Perceptron

Single Layer Neural Network Perceptron



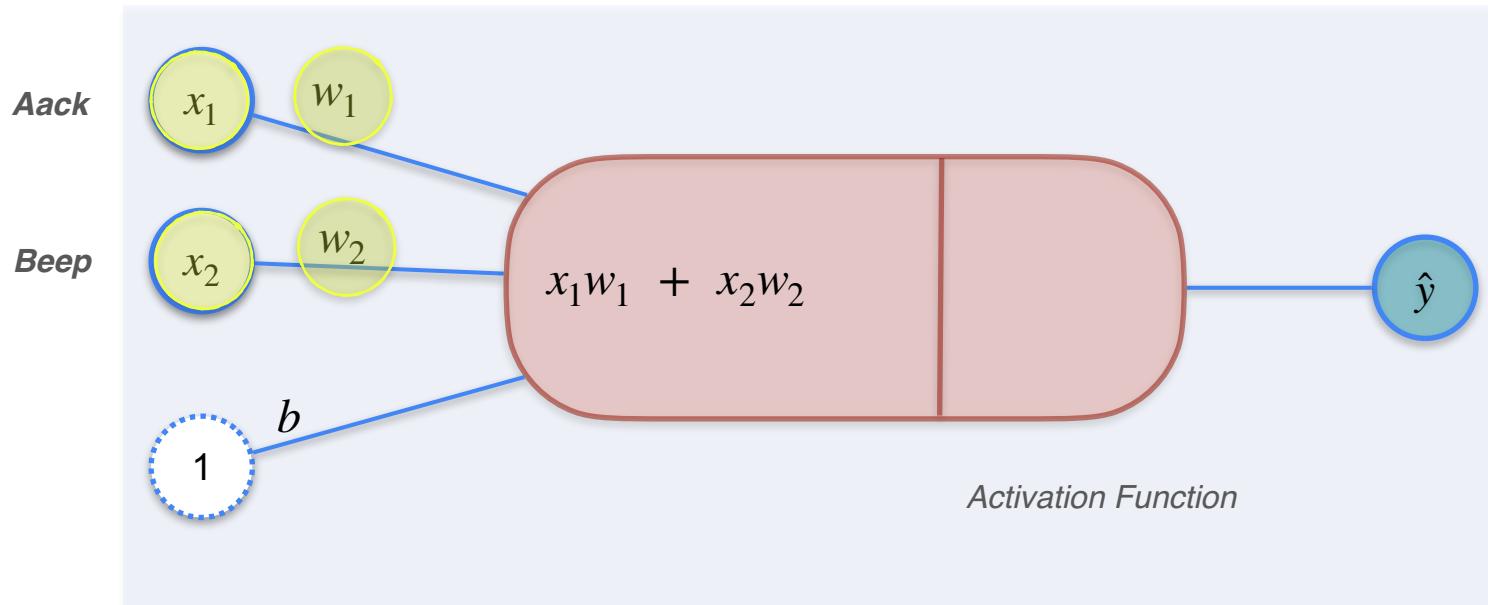
Classification With a Perceptron

Single Layer Neural Network Perceptron



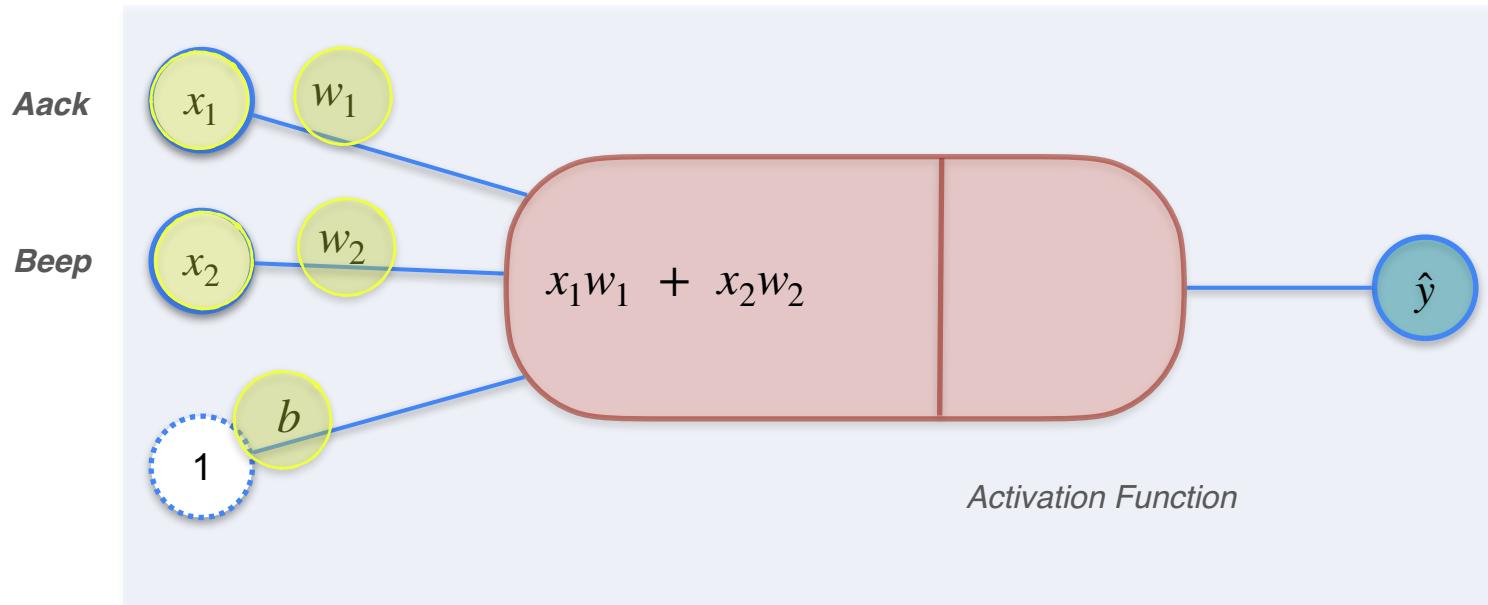
Classification With a Perceptron

Single Layer Neural Network Perceptron



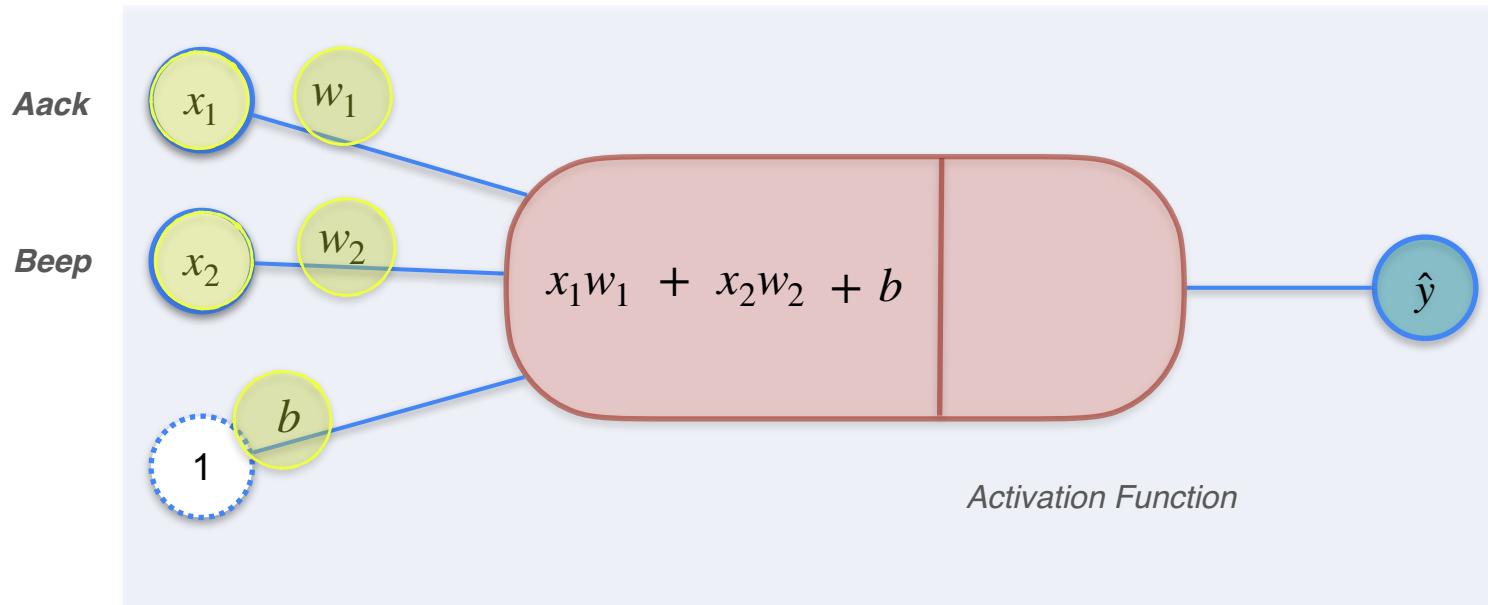
Classification With a Perceptron

Single Layer Neural Network Perceptron



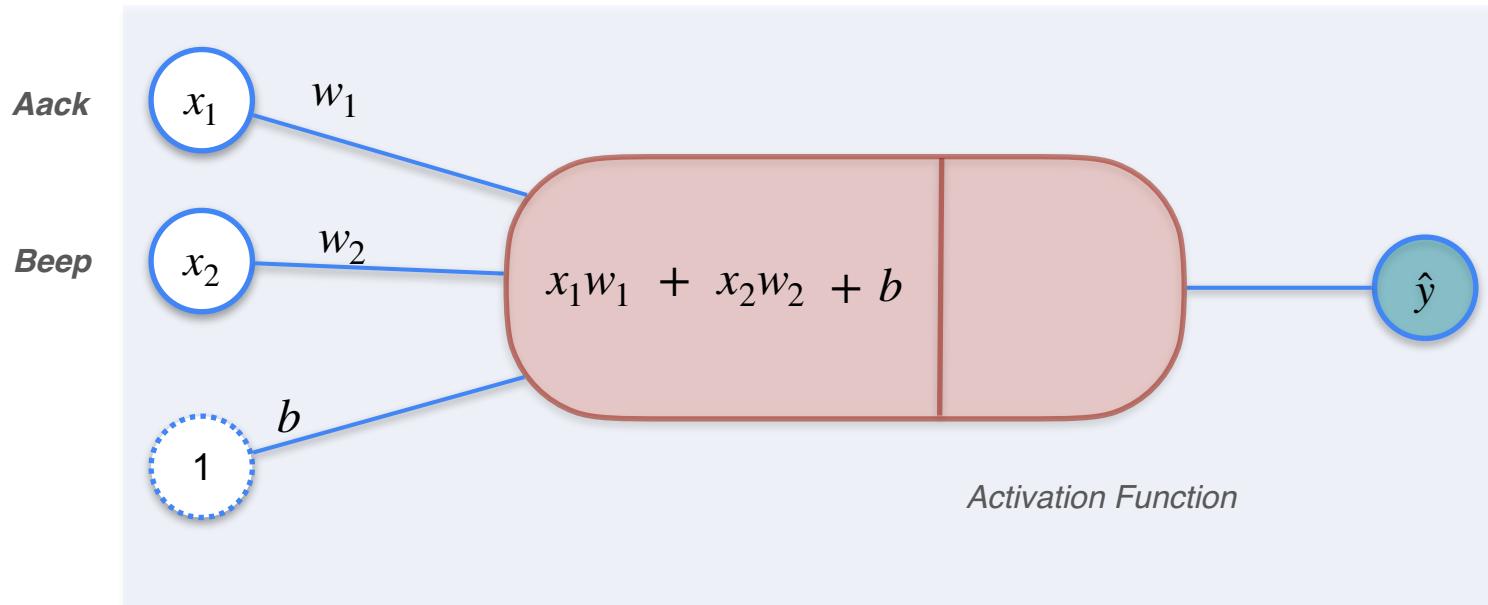
Classification With a Perceptron

Single Layer Neural Network Perceptron



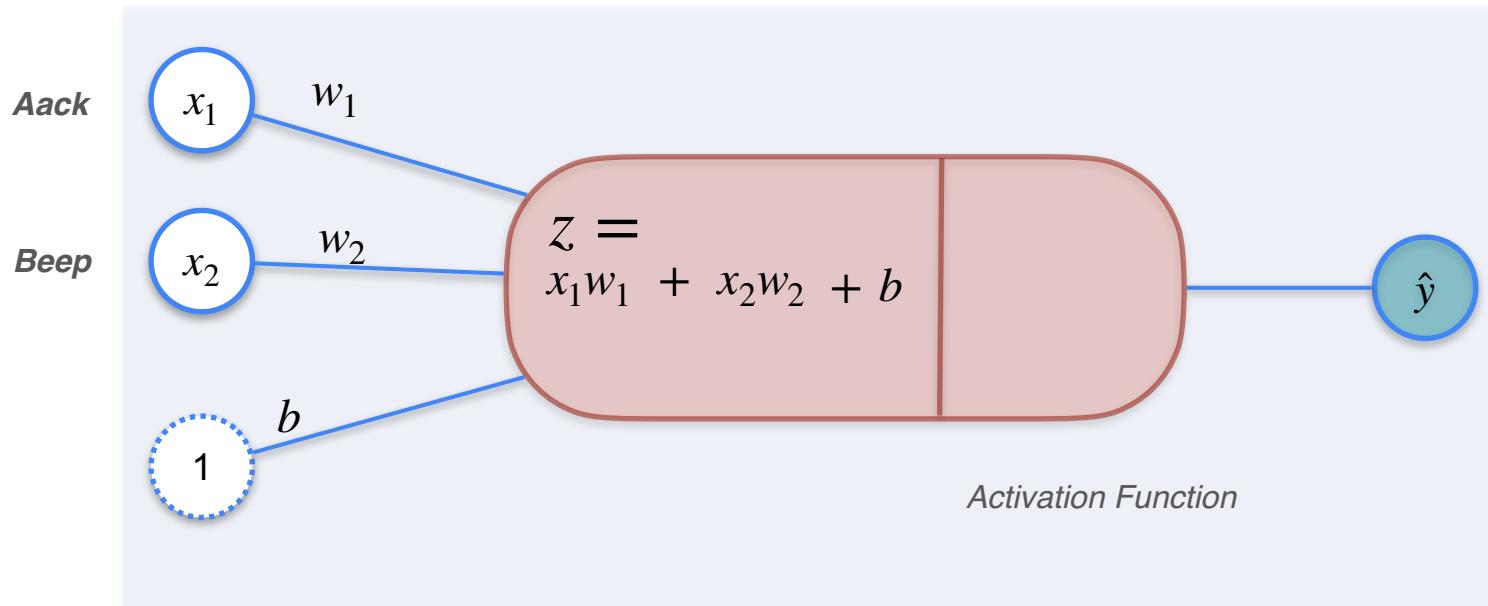
Classification With a Perceptron

Single Layer Neural Network Perceptron



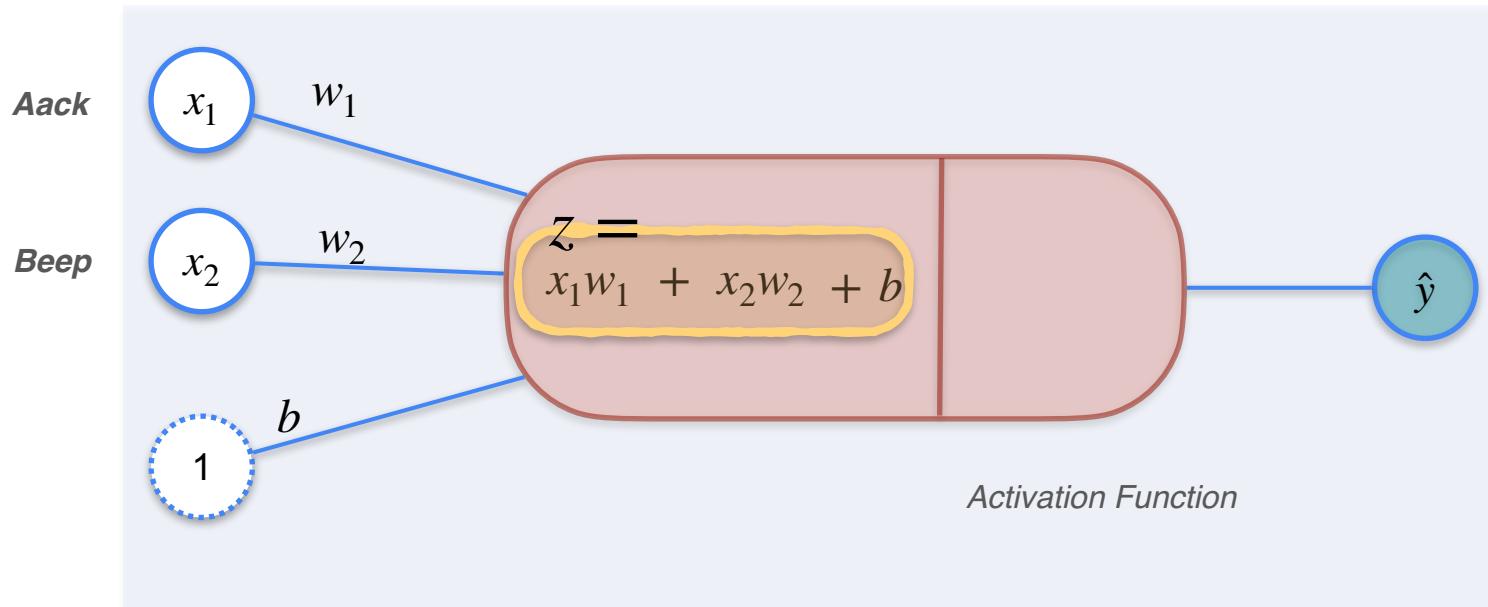
Classification With a Perceptron

Single Layer Neural Network Perceptron



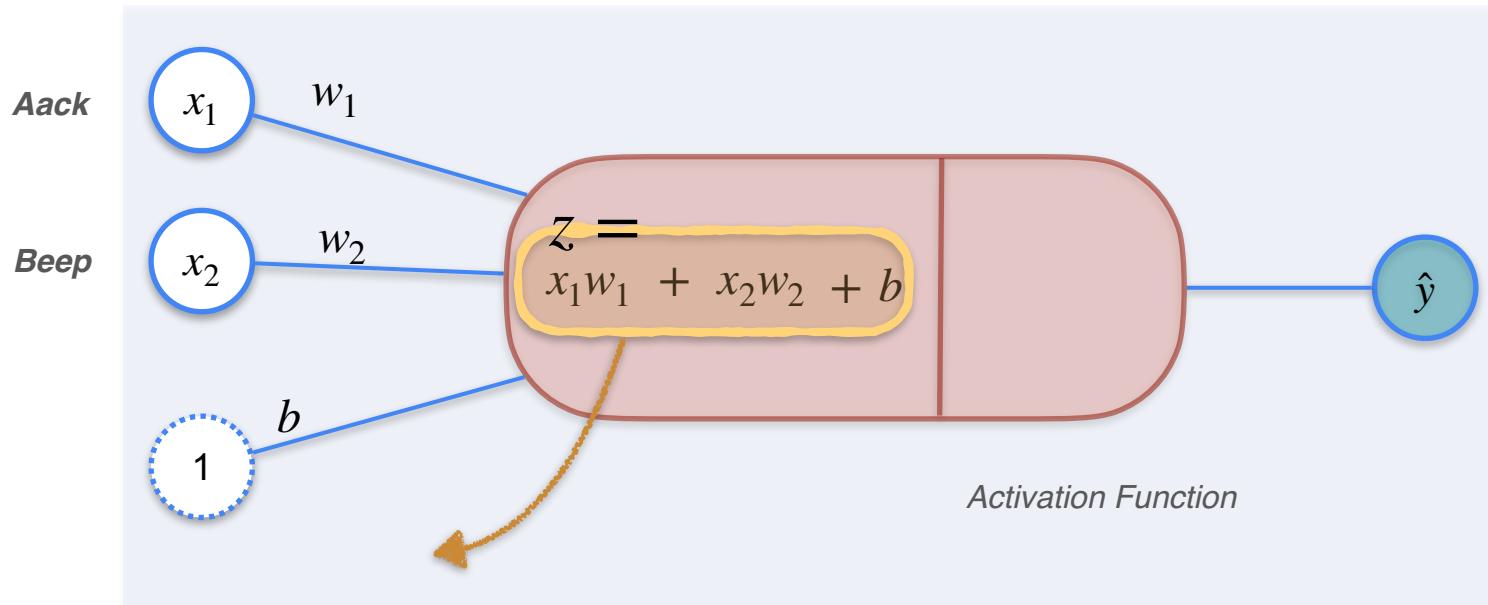
Classification With a Perceptron

Single Layer Neural Network Perceptron



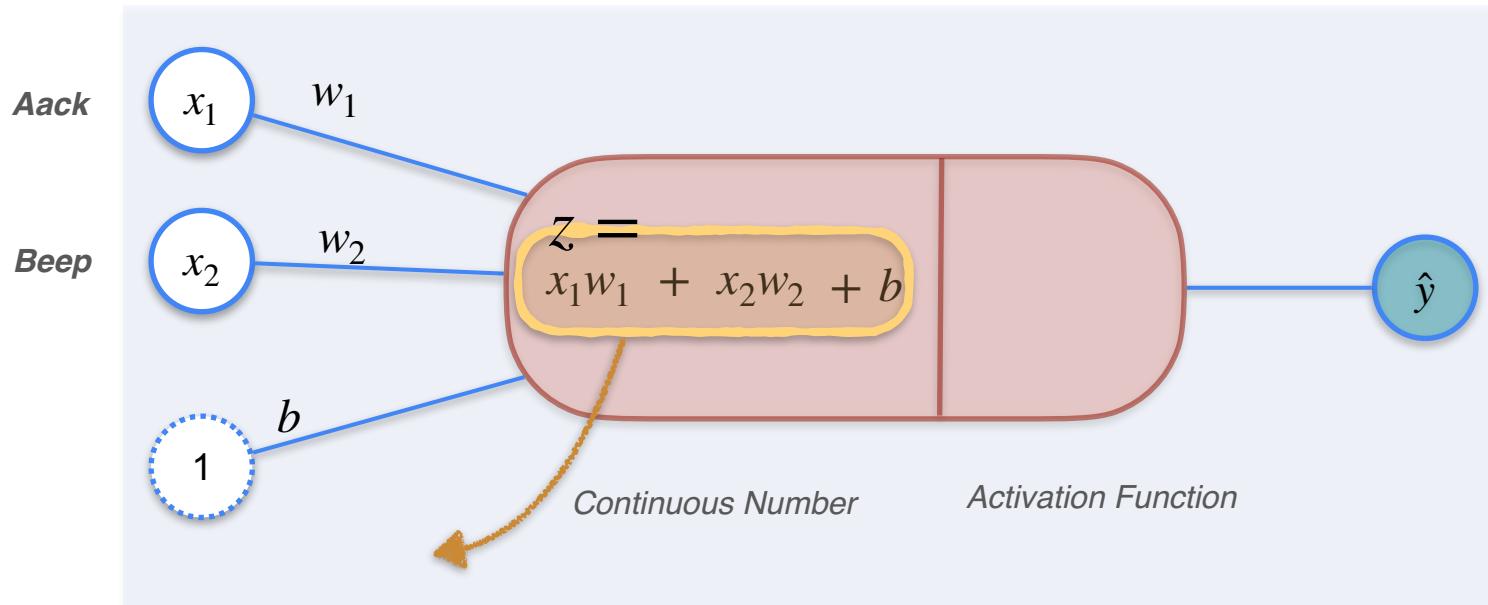
Classification With a Perceptron

Single Layer Neural Network Perceptron



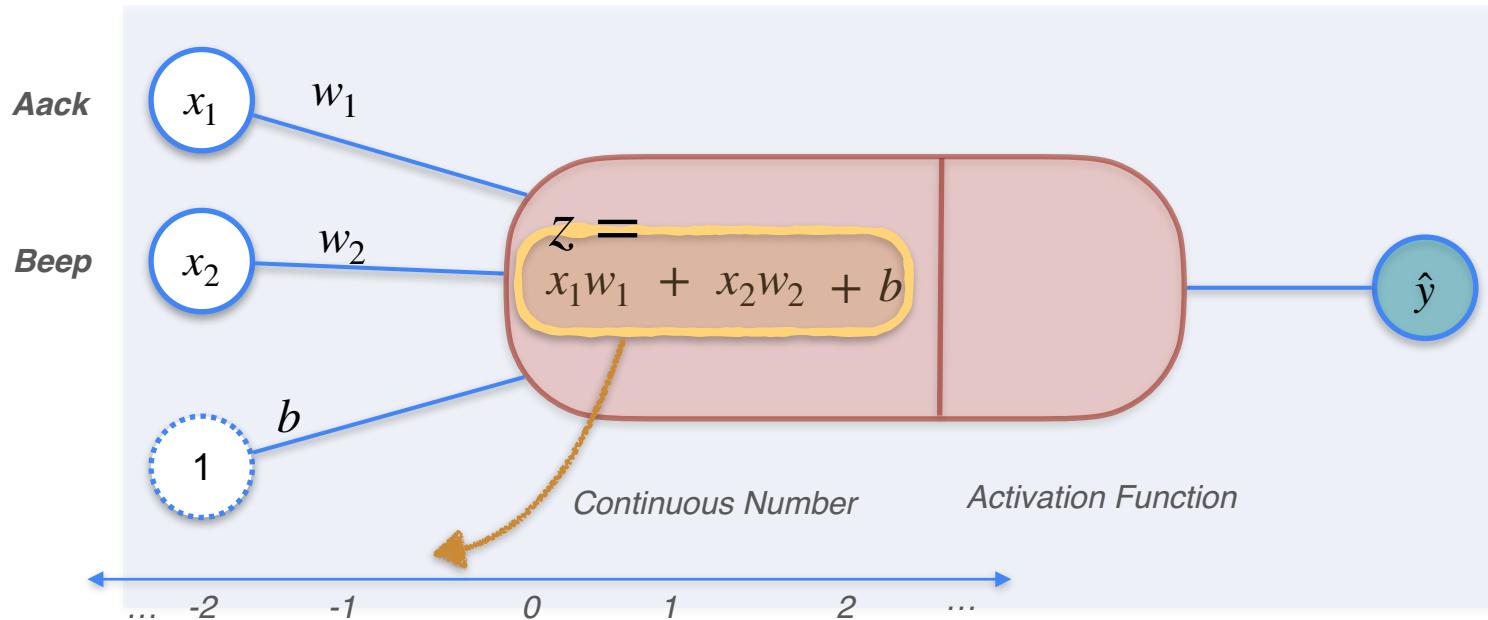
Classification With a Perceptron

Single Layer Neural Network Perceptron



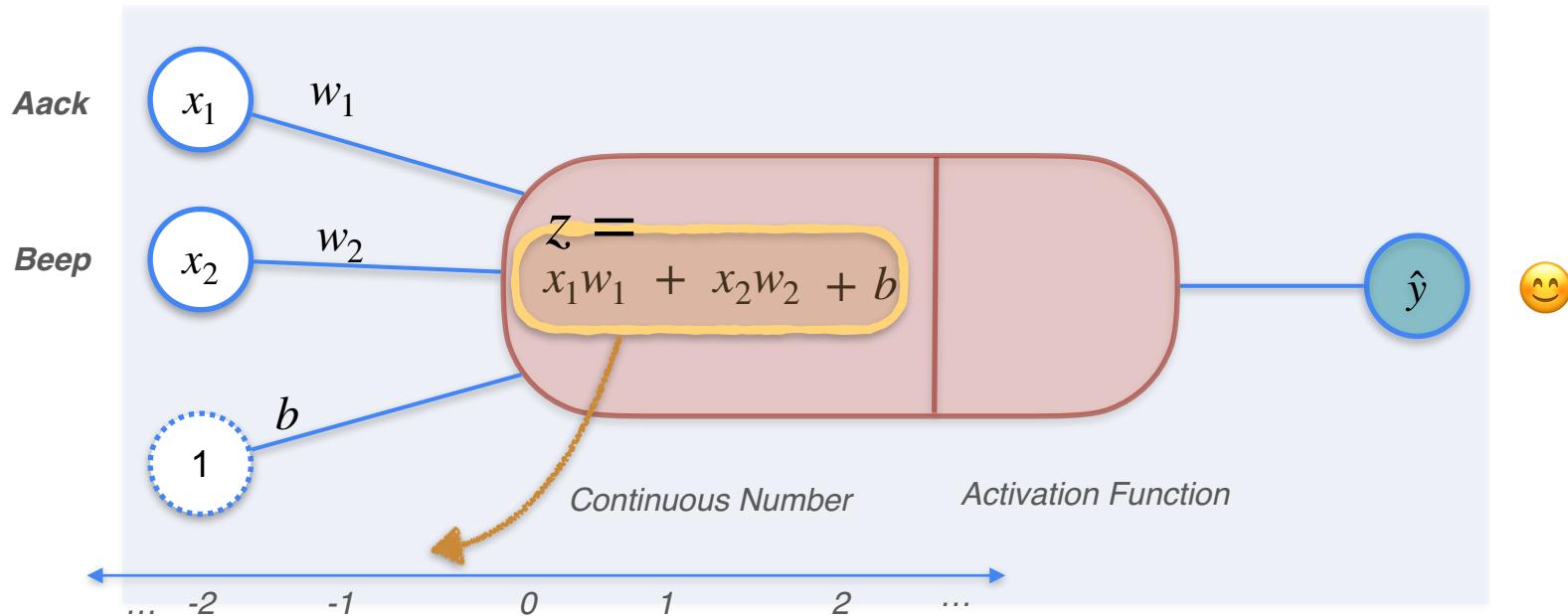
Classification With a Perceptron

Single Layer Neural Network Perceptron



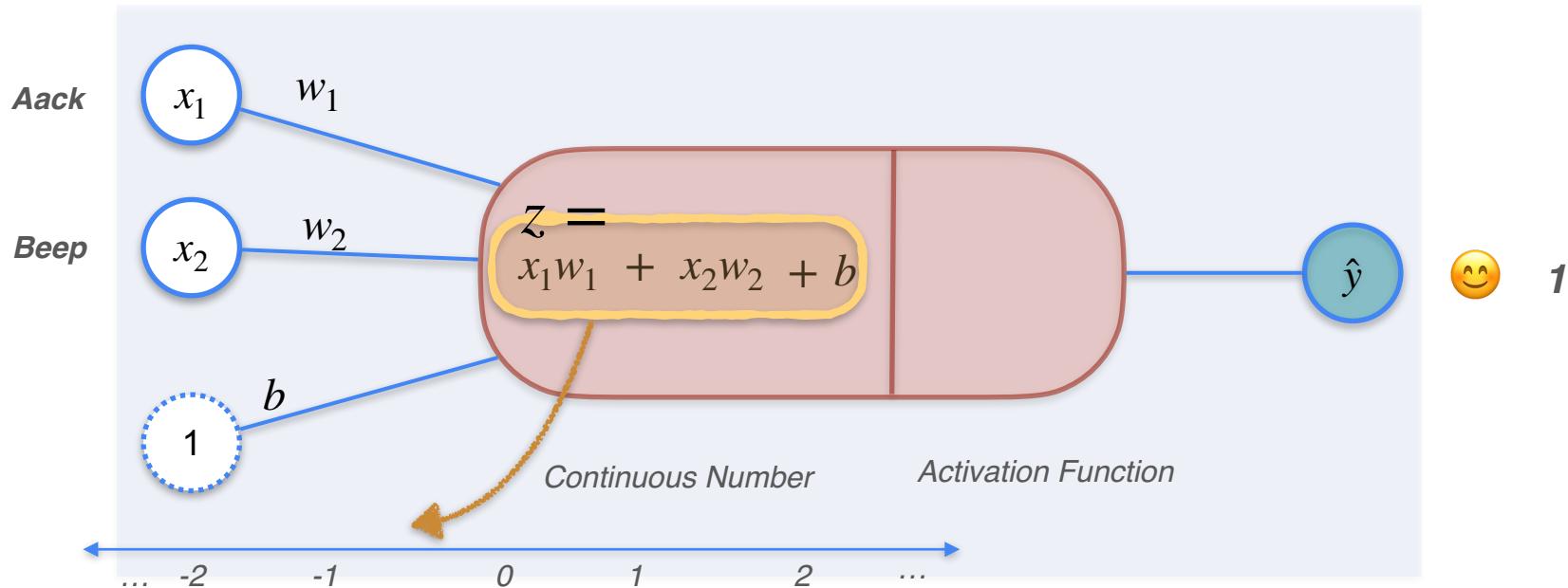
Classification With a Perceptron

Single Layer Neural Network Perceptron



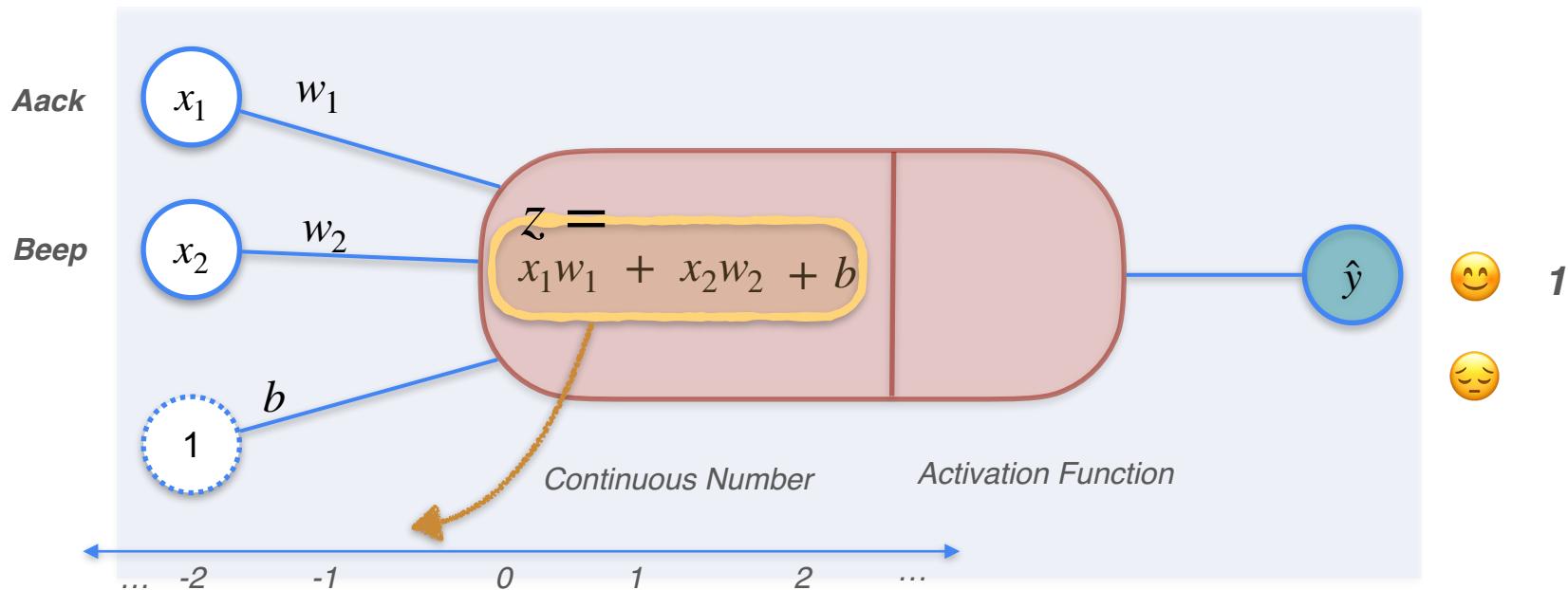
Classification With a Perceptron

Single Layer Neural Network Perceptron



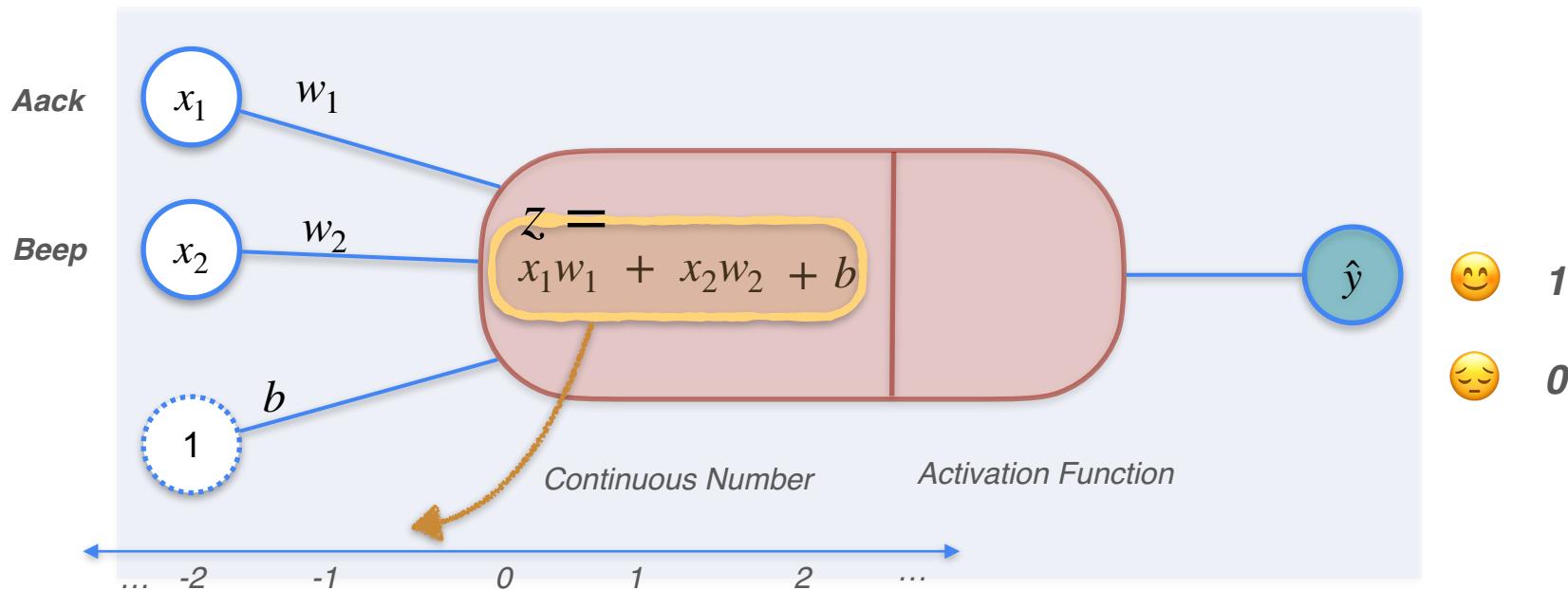
Classification With a Perceptron

Single Layer Neural Network Perceptron



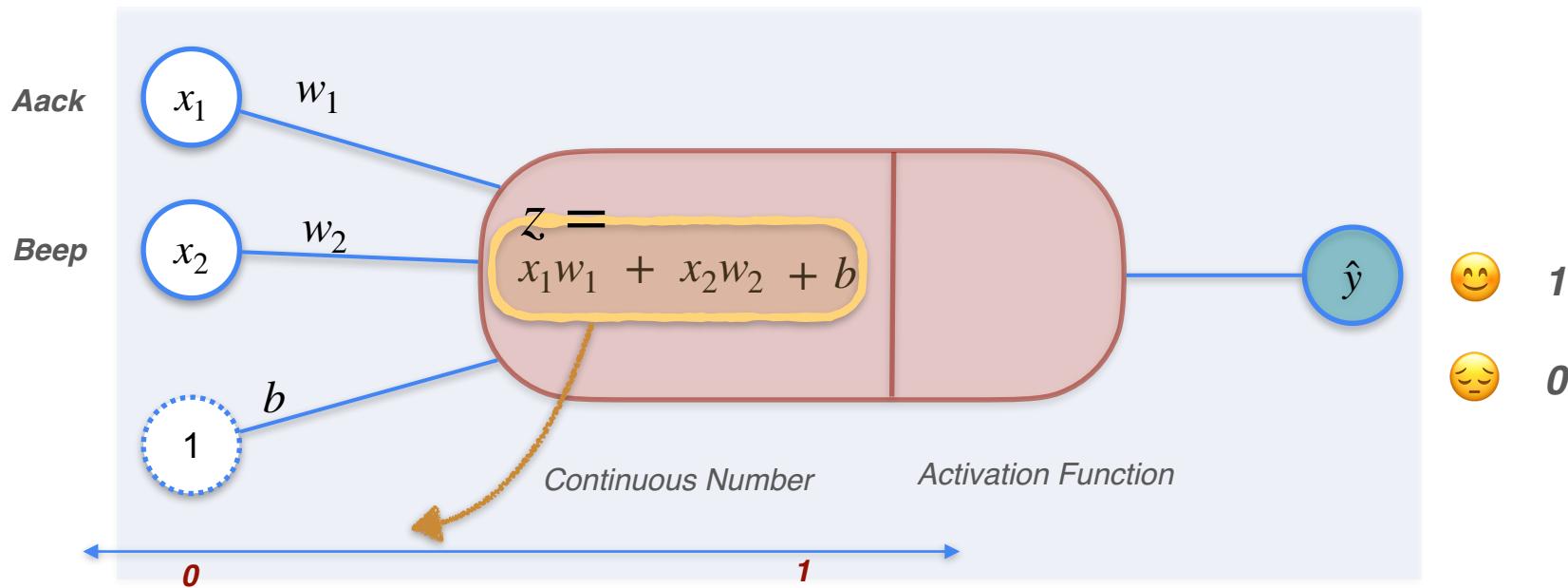
Classification With a Perceptron

Single Layer Neural Network Perceptron



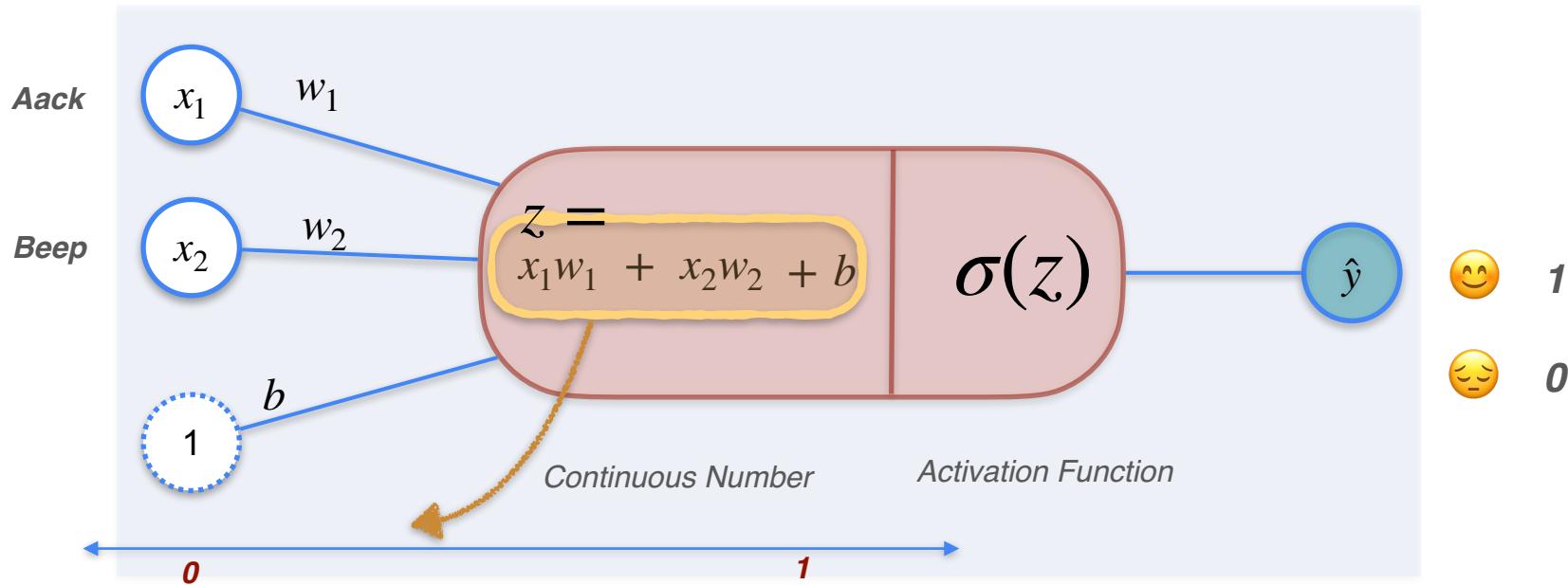
Classification With a Perceptron

Single Layer Neural Network Perceptron



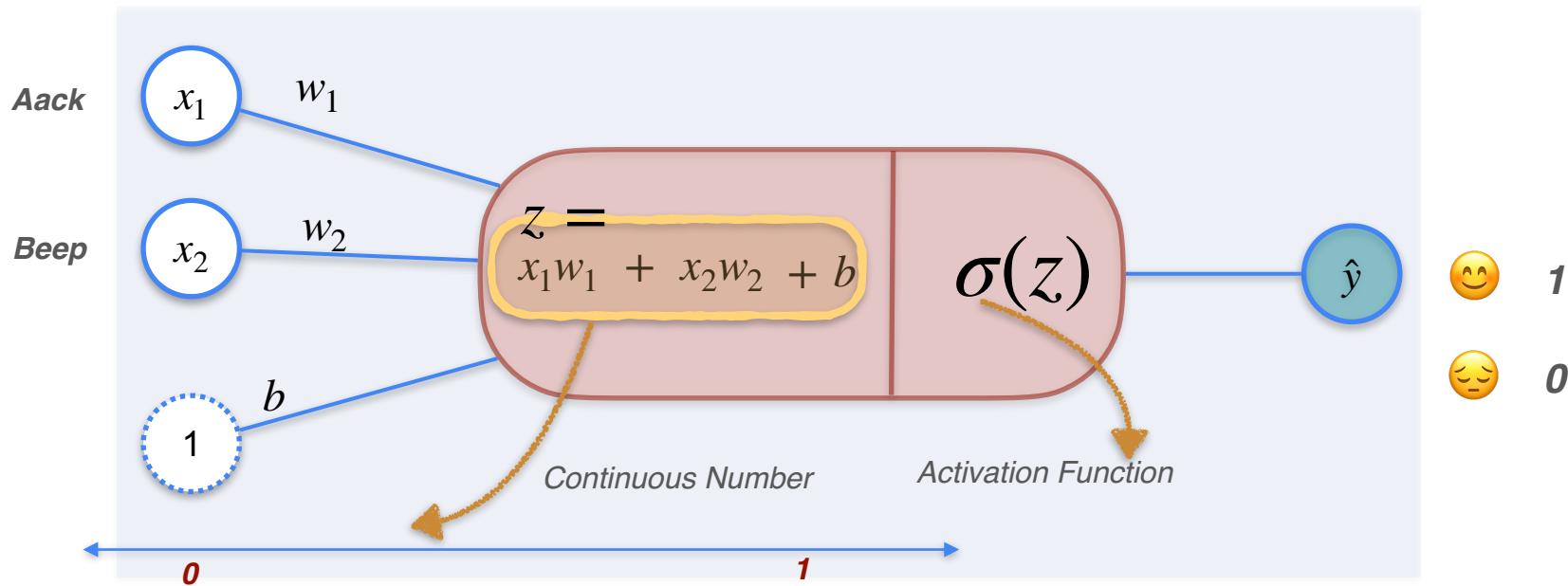
Classification With a Perceptron

Single Layer Neural Network Perceptron



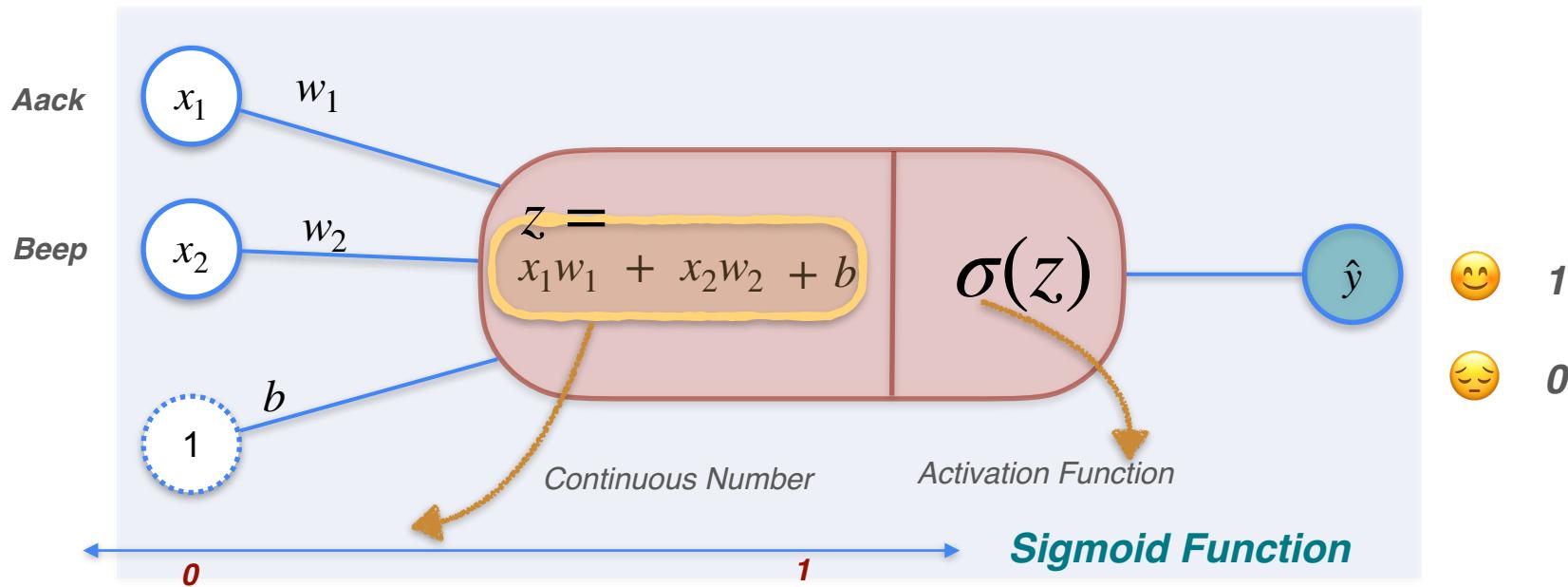
Classification With a Perceptron

Single Layer Neural Network Perceptron

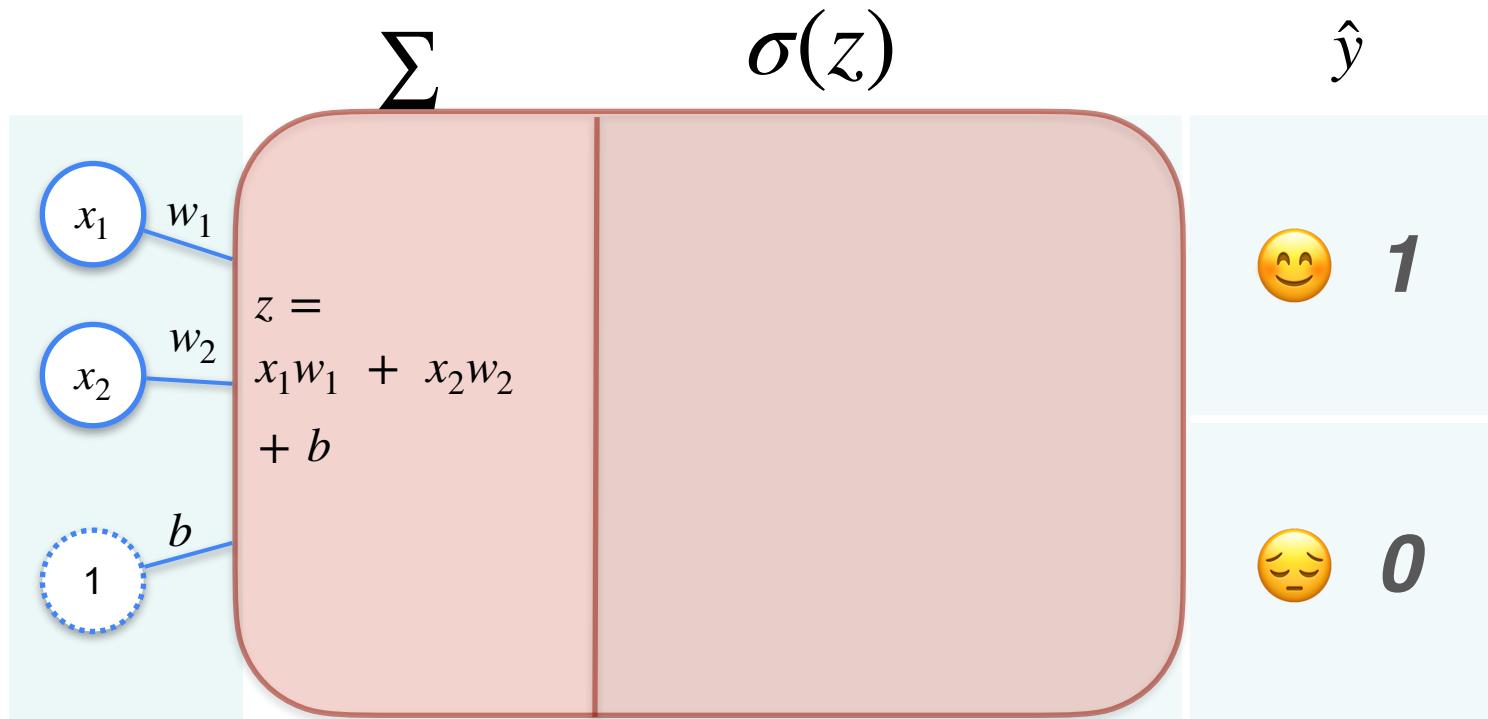


Classification With a Perceptron

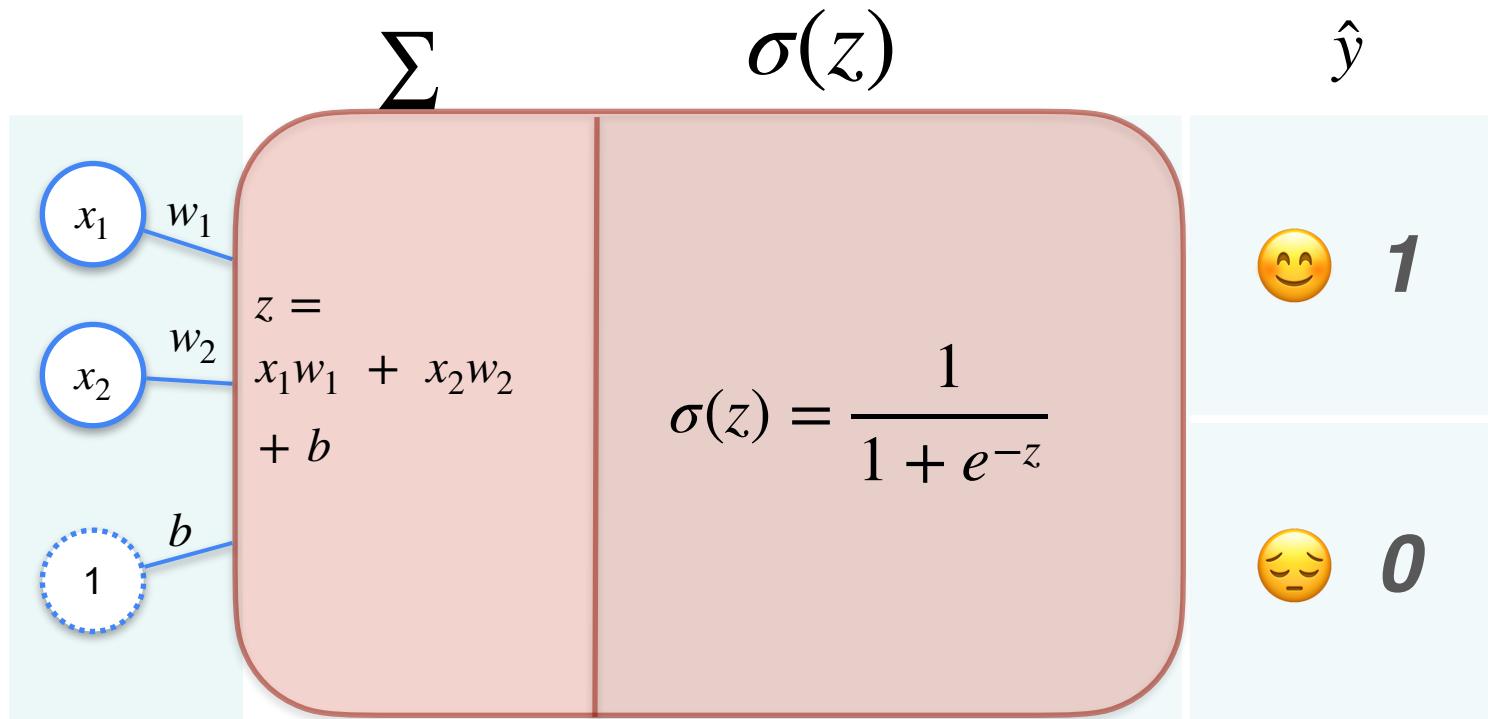
Single Layer Neural Network Perceptron



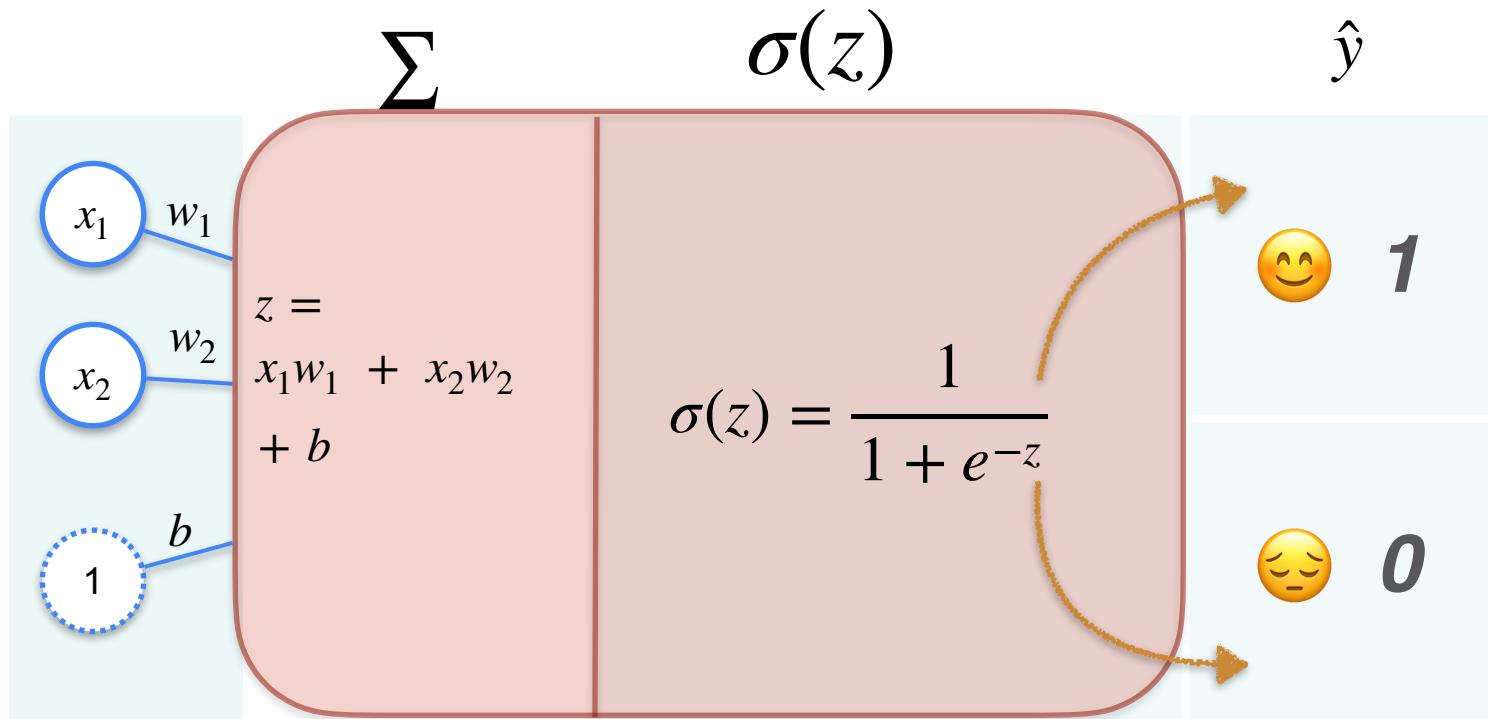
Sigmoid Function



Sigmoid Function



Sigmoid Function





DeepLearning.AI

Optimization in Neural Networks and Newton's Method

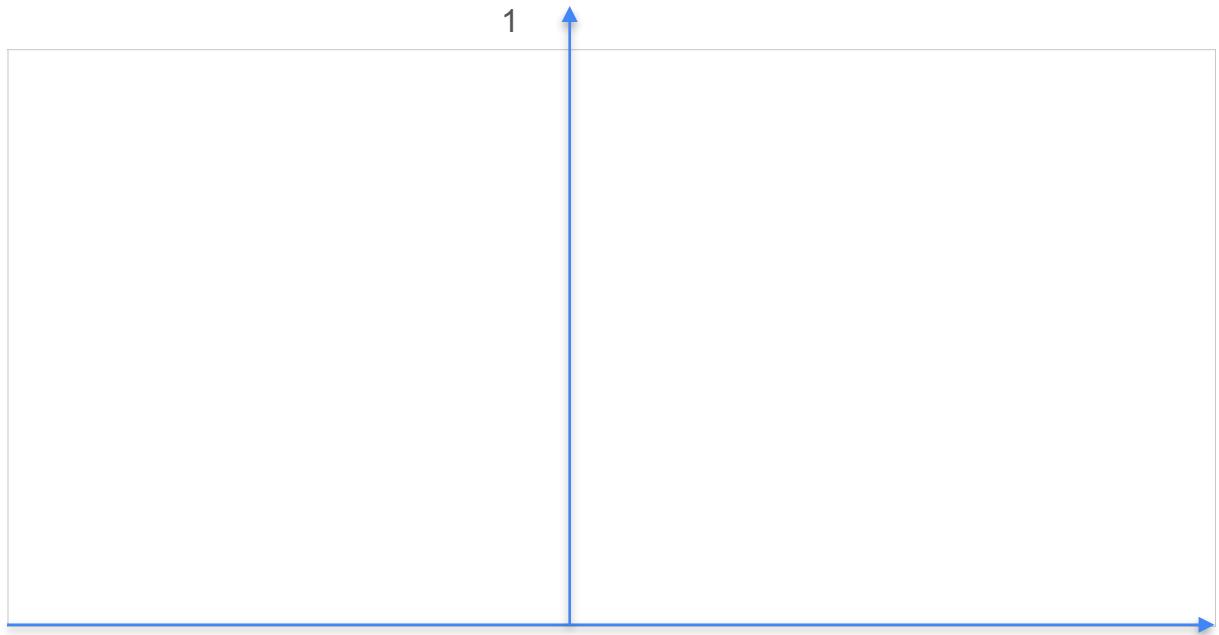
**Classification with a
perceptron:
The sigmoid function**

Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

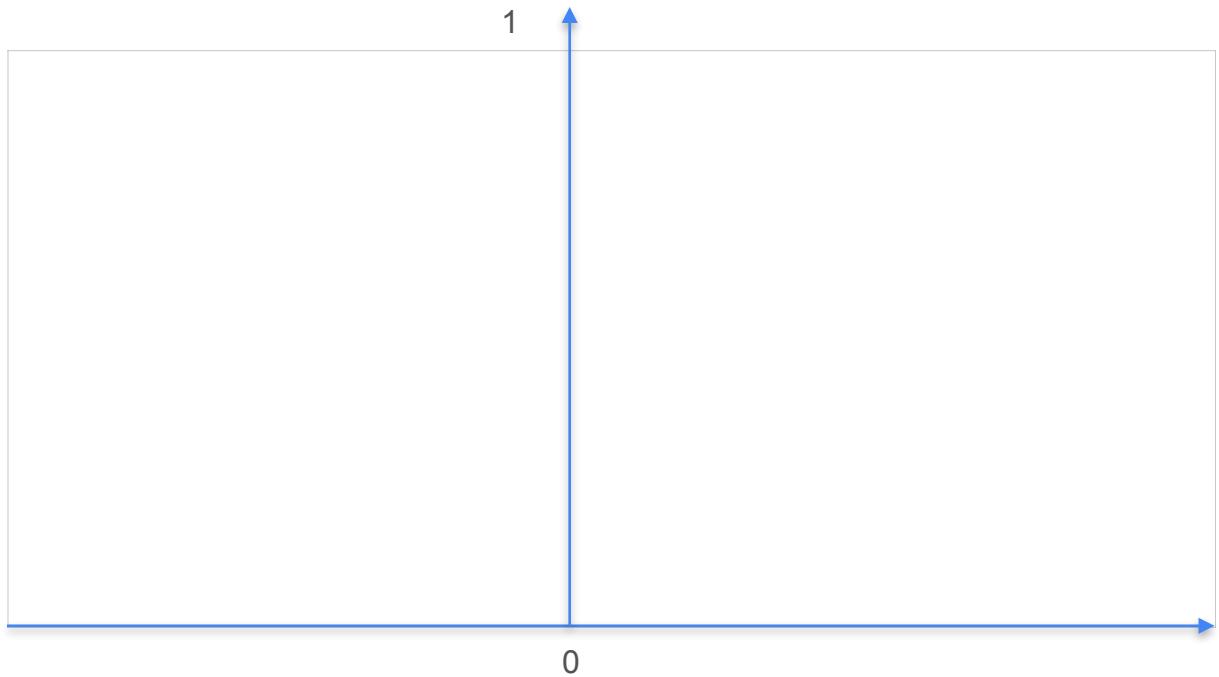
Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



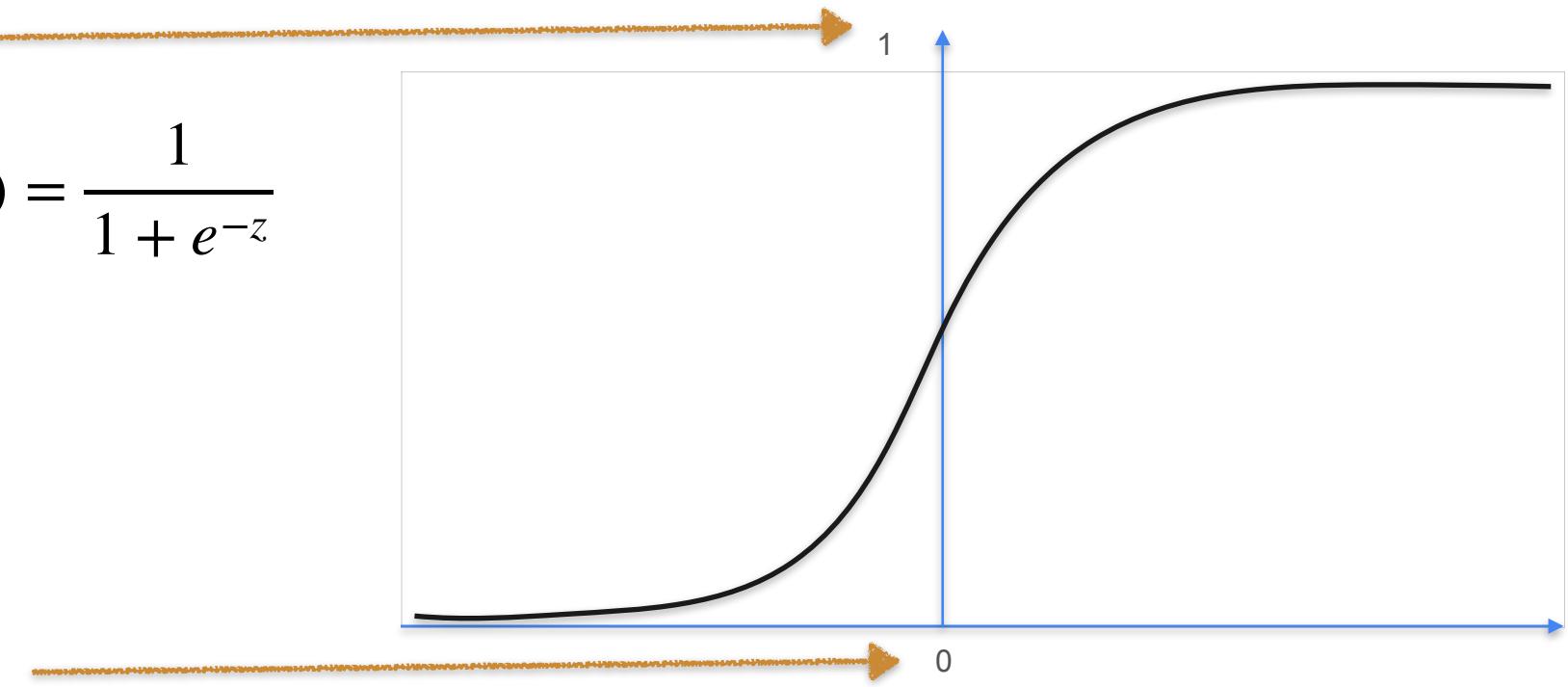
Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



Derivative of a Sigmoid Function

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz} \sigma(z)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz} \sigma(z) = \frac{d}{dz} (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$



Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{d}{dz} \sigma(z)$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz} \sigma(z) = \frac{d}{dz} (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{d}{dz} \sigma(z) = -1$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz} \sigma(z) = \frac{d}{dz} (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{d}{dz} \sigma(z) = -1 (1 + e^{-z})^{-1-1}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz} \sigma(z) = \frac{d}{dz} (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{d}{dz} \sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz} (1 + e^{-z}) \right)$$

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Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

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$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz}(1 + e^{-z})\right)$$

$$= -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz}(1 + e^{-z})\right)$$

$$= -1 (1 + e^{-z})^{-2}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{d}{dz} \sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz} (1 + e^{-z}) \right)$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$= -1 (1 + e^{-z})^{-2} \left(\frac{d}{dz} (1) \right)$$

$$\frac{d}{dz} \sigma(z) = \frac{d}{dz} (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= -1 \ (1 + e^{-z})^{-1-1} \ (\frac{d}{dz}(1 + e^{-z})) \\ &= -1 \ (1 + e^{-z})^{-2} \ (\frac{d}{dz}(1) + \frac{d}{dz}(e^{-z}))\end{aligned}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

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Derivative of a Sigmoid Function

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$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

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Derivative of a Sigmoid Function

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$$\begin{aligned}\frac{d}{dz}\sigma(z) &= -1 \ (1 + e^{-z})^{-1-1} \ (\frac{d}{dz}(1 + e^{-z})) \\ &= -1 \ (1 + e^{-z})^{-2} \ (\frac{d}{dz}(1) + \frac{d}{dz}(e^{-z})) \\ &= -1 \ (1 + e^{-z})^{-2} \ (0\end{aligned}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz}(1 + e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} \left(\frac{d}{dz}(1) + \frac{d}{dz}(e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz}(1 + e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} \left(\frac{d}{dz}(1) + \frac{d}{dz}(e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

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$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1 (1 + e^{-z})^{-2}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

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$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1 (1 + e^{-z})^{-2} (e^{-z})$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

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$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1 (1 + e^{-z})^{-2} (e^{-z}) (-1)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1 (1 + e^{-z})^{-1-1} \left(\frac{d}{dz}(1 + e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} \left(\frac{d}{dz}(1) + \frac{d}{dz}(e^{-z}) \right)$$

$$= -1 (1 + e^{-z})^{-2} (0 + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1 (1 + e^{-z})^{-2} (e^{-z}) (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = -1 \cdot (1 + e^{-z})^{-2} \cdot (e^{-z}) \cdot (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \cancel{1} (1 + e^{-z})^{-2} (e^{-z}) (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \cancel{1} (1 + e^{-z})^{-2} (e^{-z}) \cancel{(-1)}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \cancel{1} (1 + e^{-z})^{-2} \ (e^{-z}) \cancel{(-1)} \\ &= (1 + e^{-z})^{-2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \cancel{1} (1 + e^{-z})^{-2} \ (e^{-z}) \cancel{(-1)} \\ &= (1 + e^{-z})^{-2} \ (e^{-z})\end{aligned}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \cancel{1} (1 + e^{-z})^{-2} (e^{-z}) \cancel{(-1)}$$

$$= (1 + e^{-z})^{-2} (e^{-z})$$

$$= \frac{1}{(1 + e^{-z})^2}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \cancel{1} (1 + e^{-z})^{-2} (e^{-z}) \cancel{(-1)}$$

$$= (1 + e^{-z})^{-2} (e^{-z})$$

$$= \frac{1}{(1 + e^{-z})^2} (e^{-z})$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \cancel{1} (1 + e^{-z})^{-2} (e^{-z}) \cancel{(-1)}$$

$$= (1 + e^{-z})^{-2} (e^{-z})$$

$$= \frac{1}{(1 + e^{-z})^2} (e^{-z})$$

$$= \frac{e^{-z}}{(1 + e^{-z})^2}$$

Derivative of a Sigmoid Function

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z}}{(1 + e^{-z})^2}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1$$

Derivative of a Sigmoid Function

$$\frac{d}{dz} \sigma(z) = \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z}}{(1 + e^{-z})^2}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z}}{(1 + e^{-z})^2} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z}}{(1 + e^{-z})^2} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz} \sigma(z) &= \frac{e^{-z}}{(1 + e^{-z})^2} + 1 - 1 \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\frac{d}{dz}\sigma(z)$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\frac{d}{dz}\sigma(z) = \frac{1}{(1 + e^{-z})}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2}$$

$$= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2}$$

$$= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}$$

$$\frac{d}{dz}\sigma(z) = \frac{1}{(1 + e^{-z})} -$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2}$$

$$= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2}$$

$$= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}$$

$$\frac{d}{dz}\sigma(z) = \frac{1}{(1 + e^{-z})} -$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2}$$

$$= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2}$$

$$= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2}$$

$$= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}$$

$$\frac{d}{dz}\sigma(z) = \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})} \right)$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\frac{d}{dz}\sigma(z) = \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right)$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\ &= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\ &= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\ &= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\ &= \frac{1}{(1 + e^{-z})}\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(1 + e^{-z})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Recall that:

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Recall that: $\sigma(z) = \frac{1}{1 + e^{-z}}$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Recall that: $\sigma(z) = \frac{1}{1 + e^{-z}}$

$$\frac{d}{dz}\sigma(z)$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Recall that: $\sigma(z) = \frac{1}{1 + e^{-z}}$

$$\frac{d}{dz}\sigma(z) = \sigma(z)$$

Derivative of a Sigmoid Function

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{e^{-z} + 1 - 1}{(1 + e^{-z})^2} \\&= \frac{1 + e^{-z} - 1}{(1 + e^{-z})^2} \\&= \frac{\cancel{1 + e^{-z}}}{(\cancel{1 + e^{-z}})^2} - \frac{1}{(1 + e^{-z})^2} \\&= \frac{1}{(1 + e^{-z})} - \frac{1}{(1 + e^{-z})^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dz}\sigma(z) &= \frac{1}{(1 + e^{-z})} - \left(\frac{1}{(1 + e^{-z})}\right)\left(\frac{1}{(1 + e^{-z})}\right) \\&= \frac{1}{(1 + e^{-z})} \left(1 - \frac{1}{(1 + e^{-z})}\right)\end{aligned}$$

Recall that: $\sigma(z) = \frac{1}{1 + e^{-z}}$

$$\frac{d}{dz}\sigma(z) = \sigma(z) (1 - \sigma(z))$$



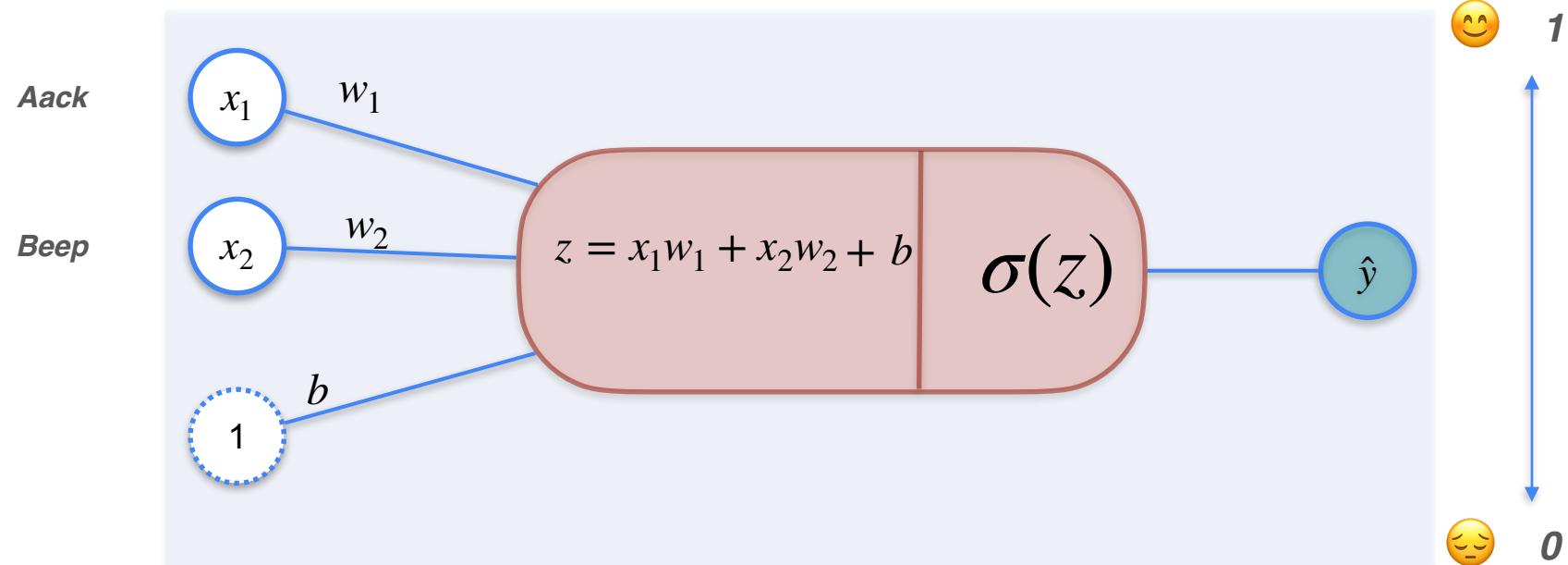
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

**Classification with a
perceptron:
Gradient Descent**

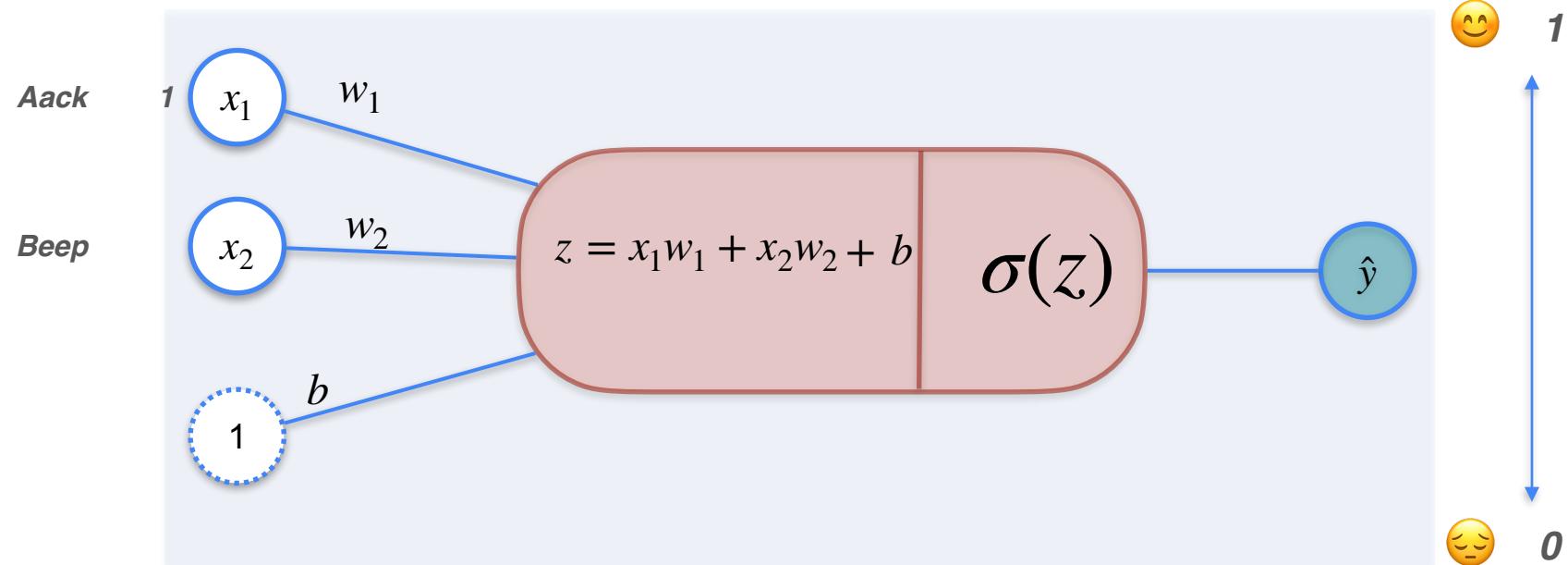
Classification With a Perceptron

Aack beep beep beep



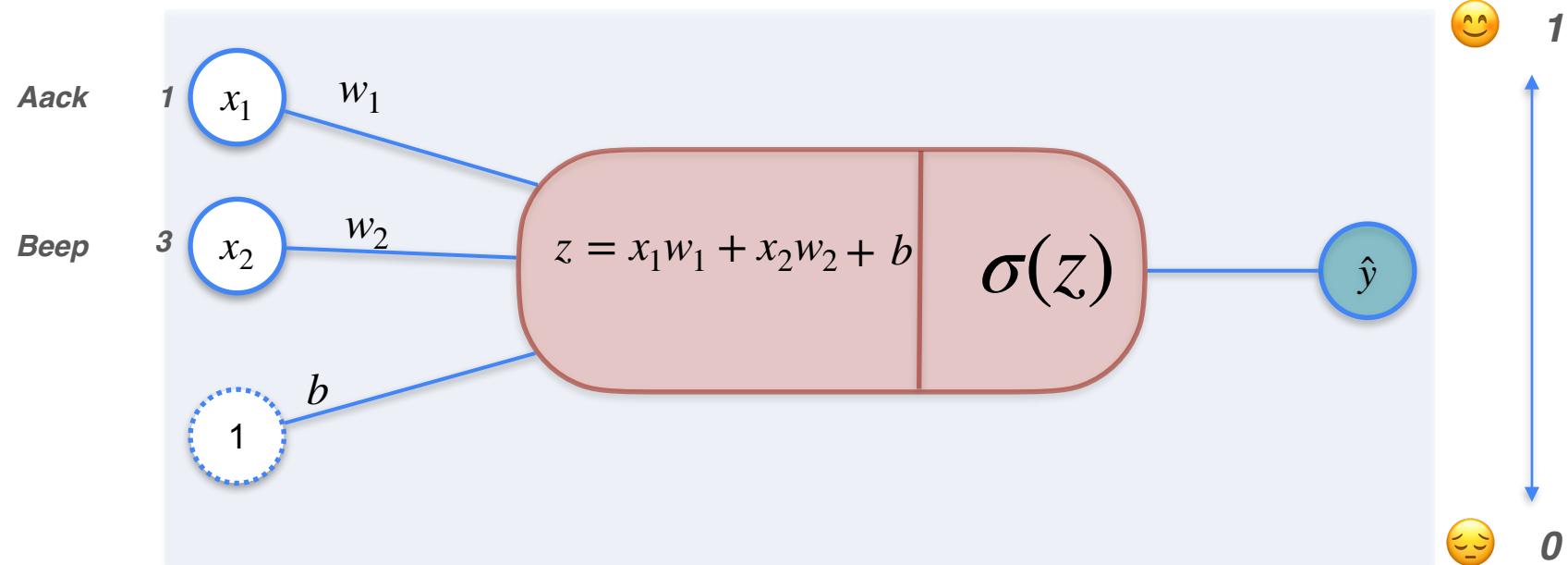
Classification With a Perceptron

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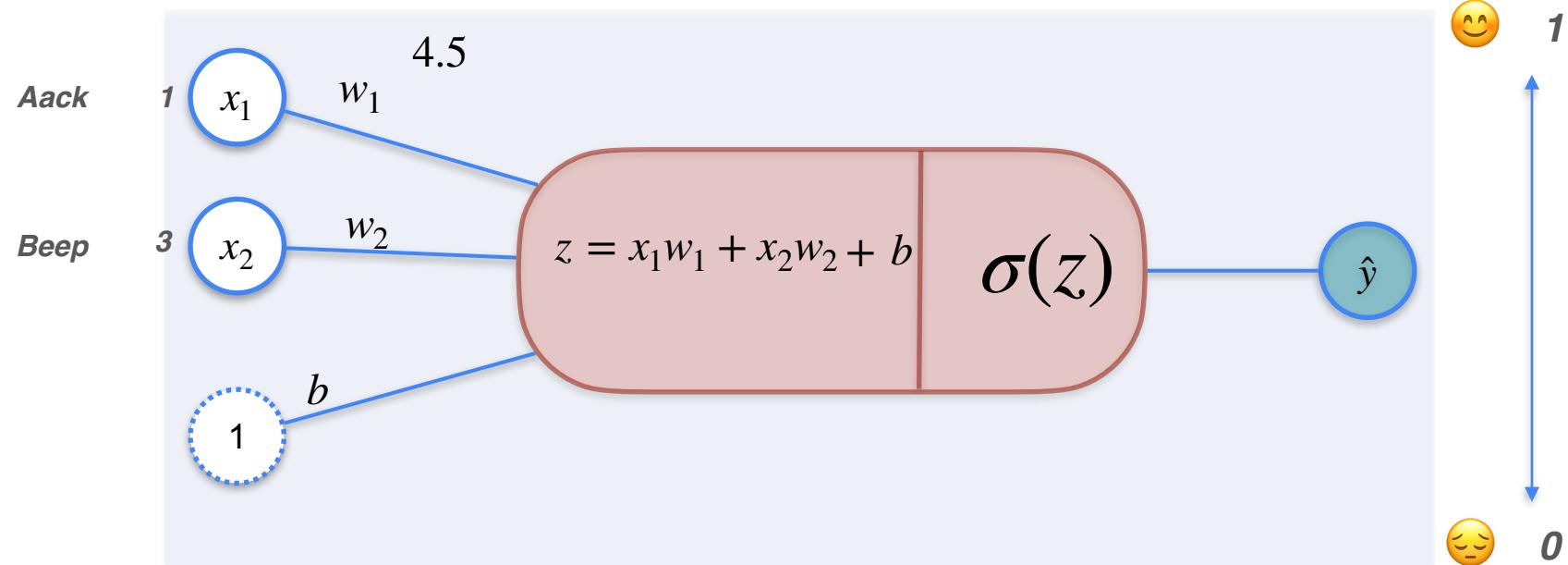
Classification With a Perceptron

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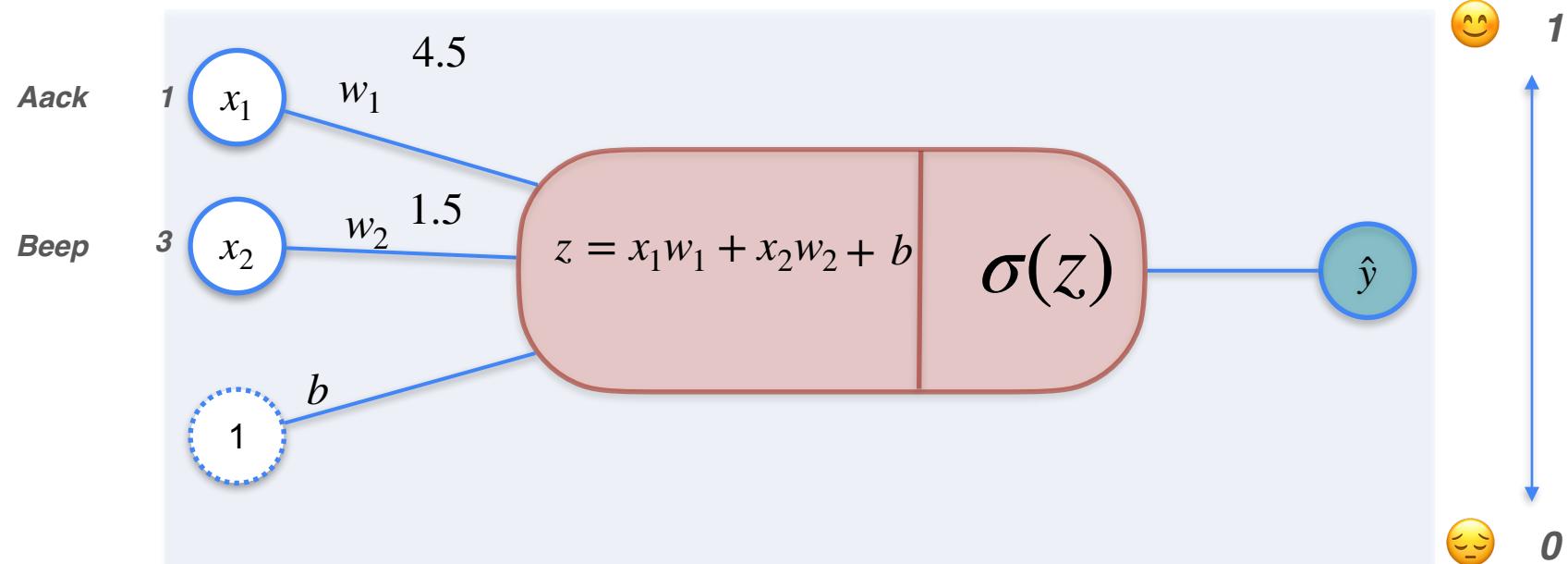
Classification With a Perceptron

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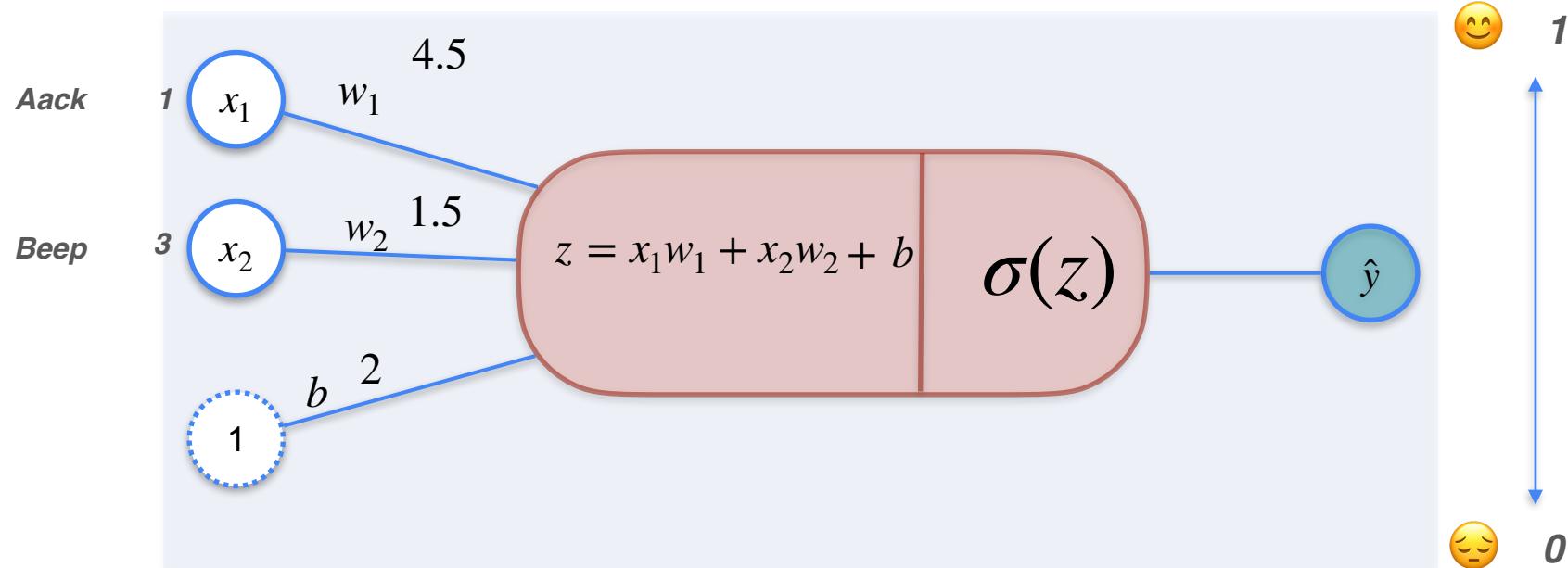
Classification With a Perceptron

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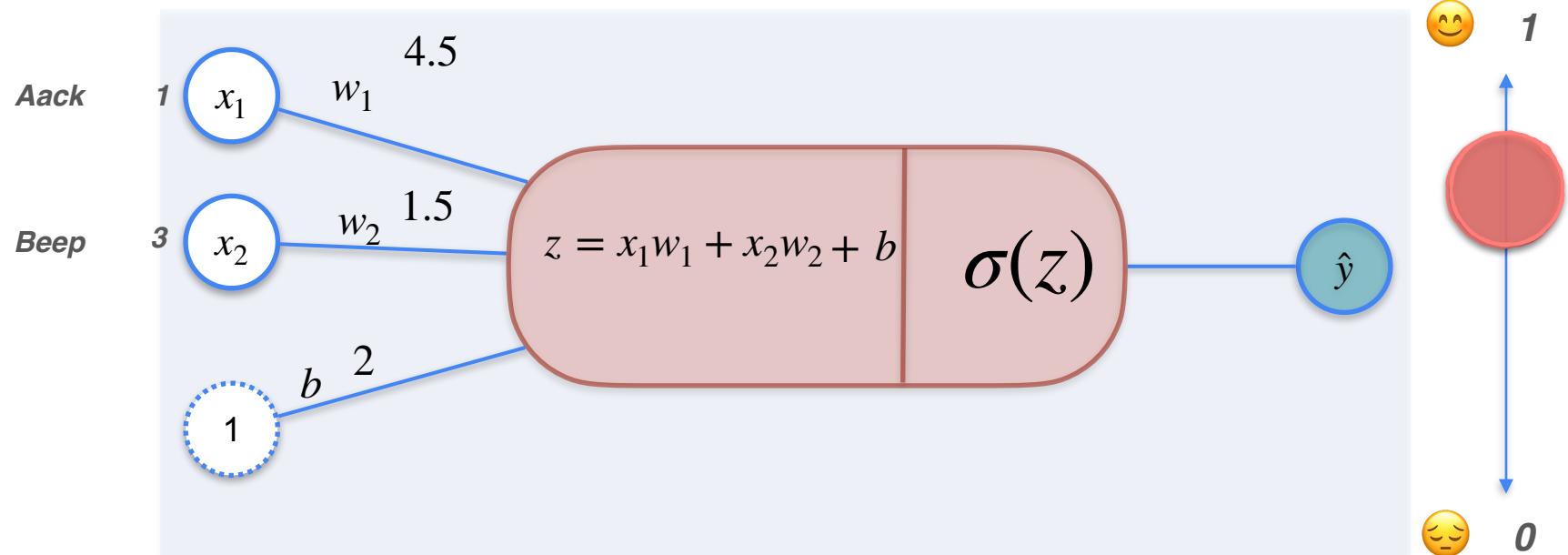
Classification With a Perceptron

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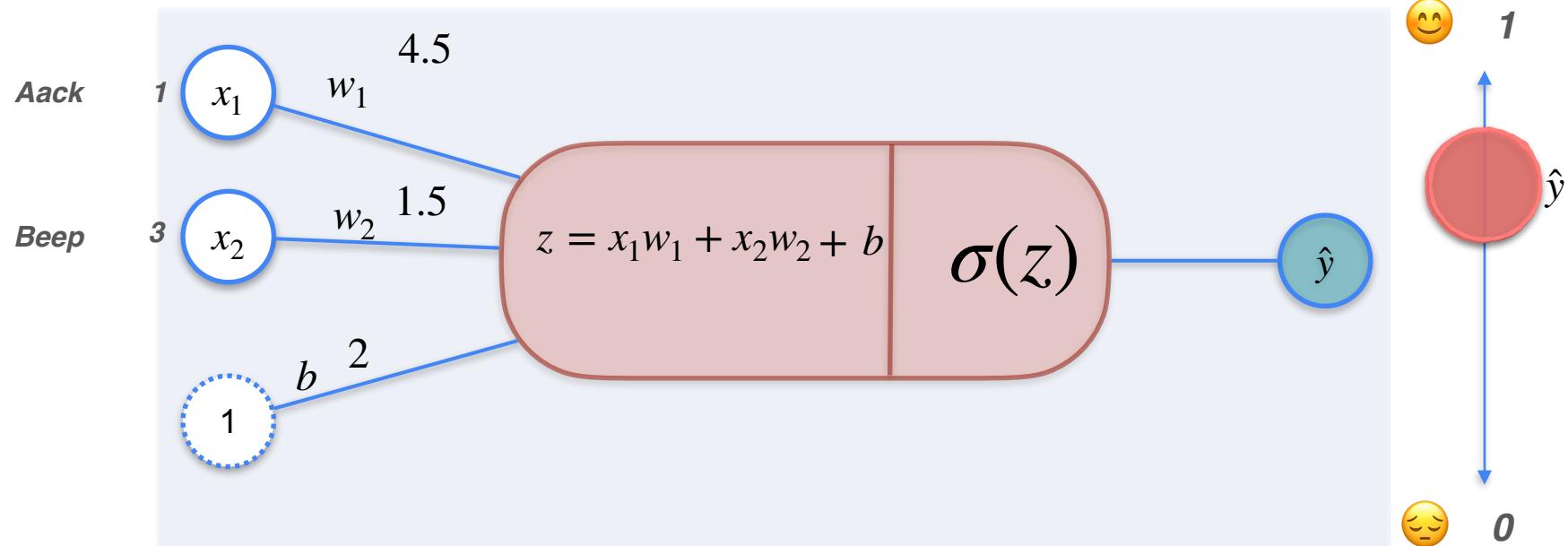
Classification With a Perceptron

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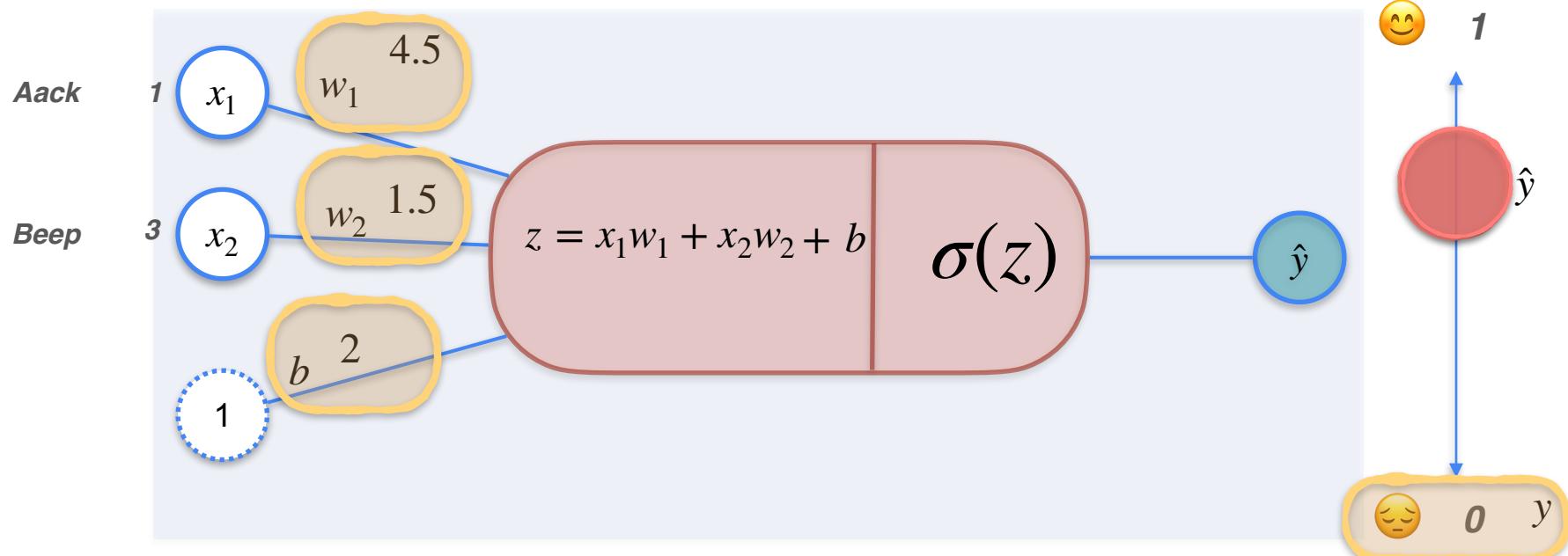
Classification With a Perceptron

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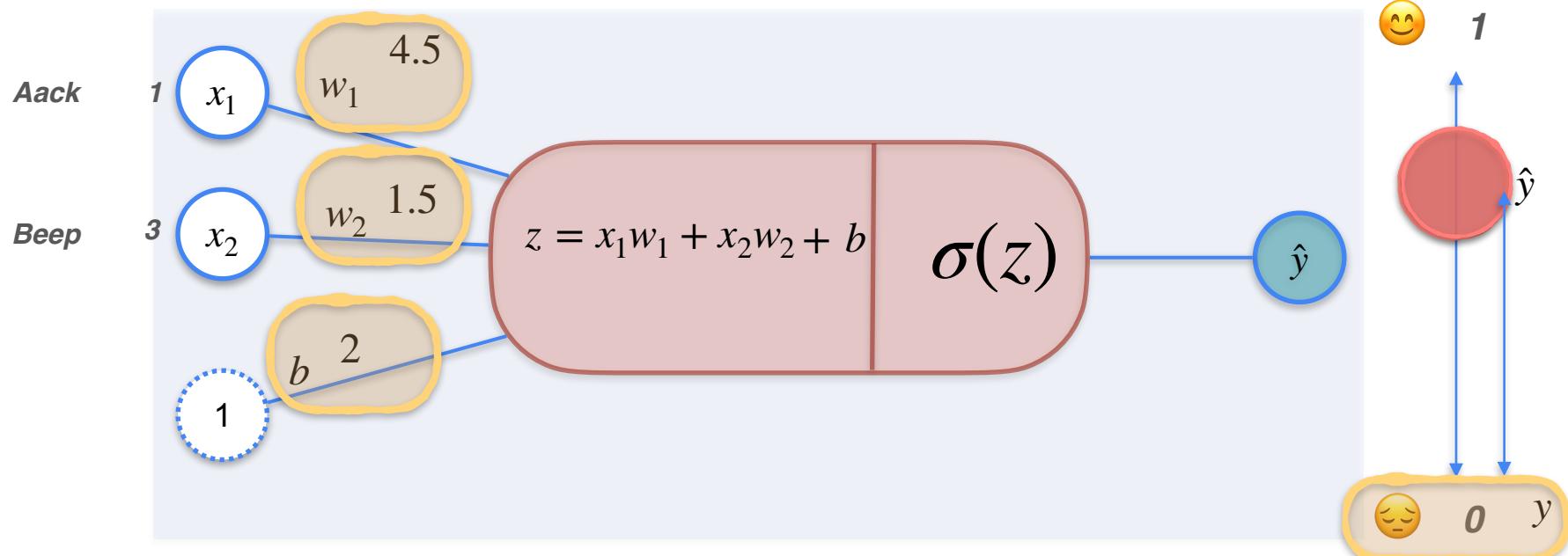
Classification With a Perceptron

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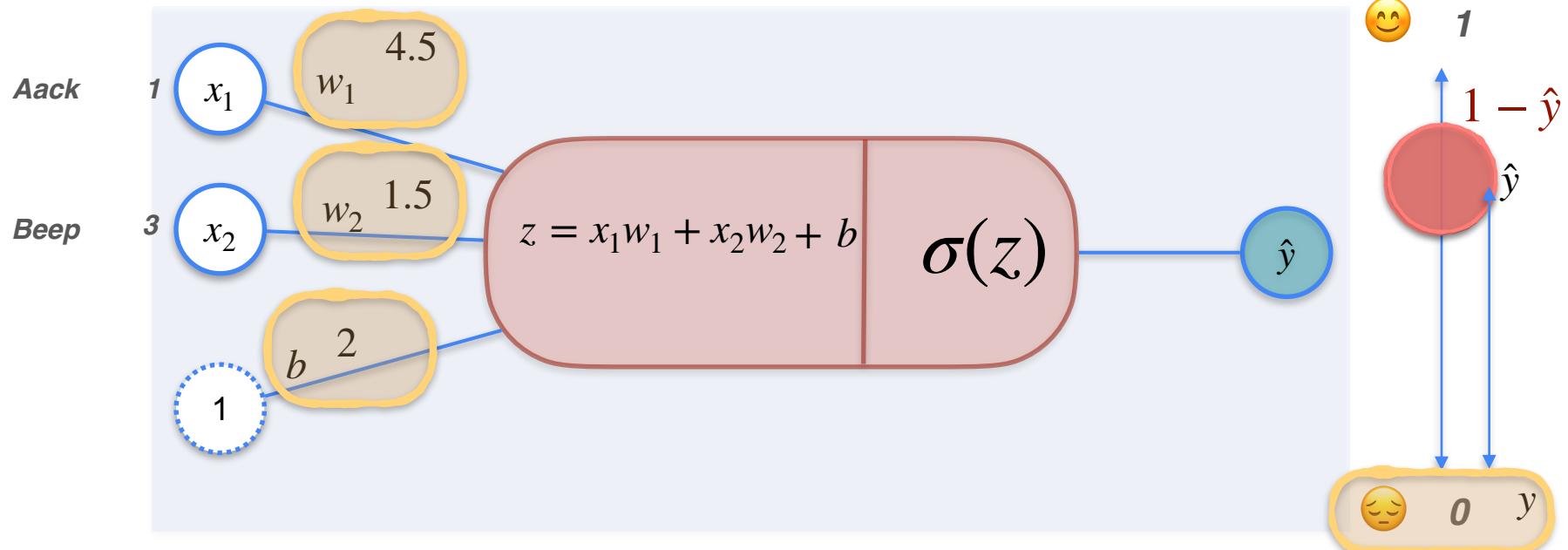
Classification With a Perceptron

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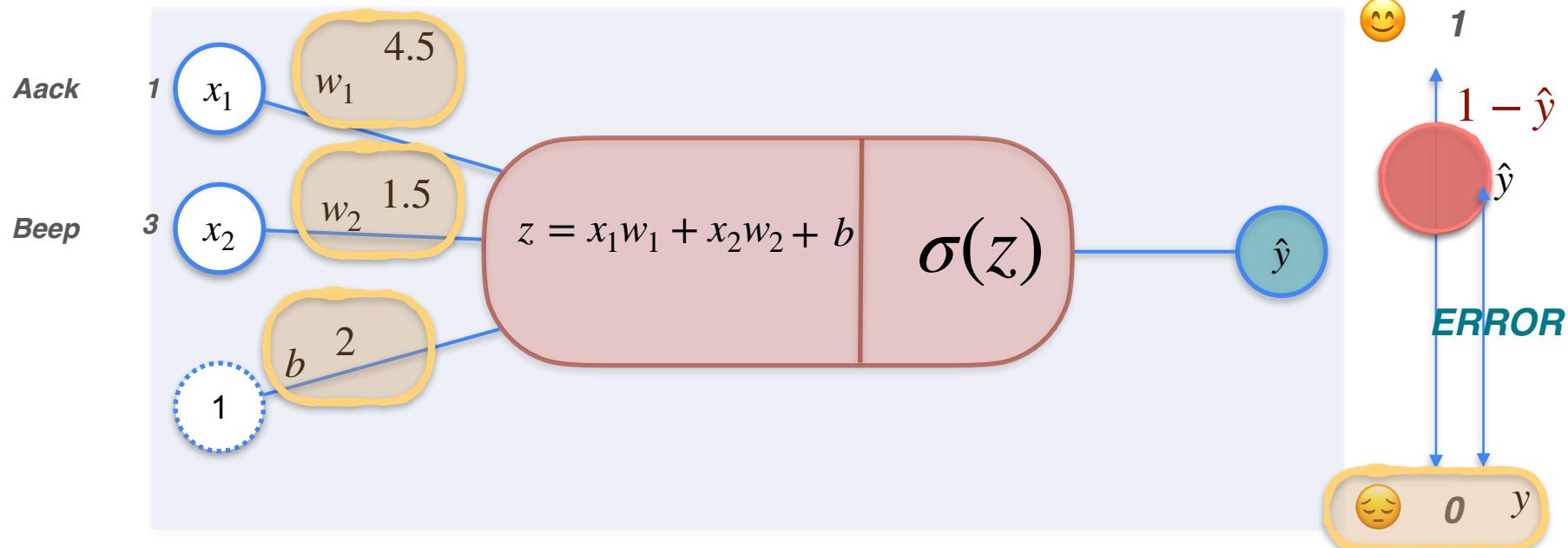
Classification With a Perceptron

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Classification With a Perceptron

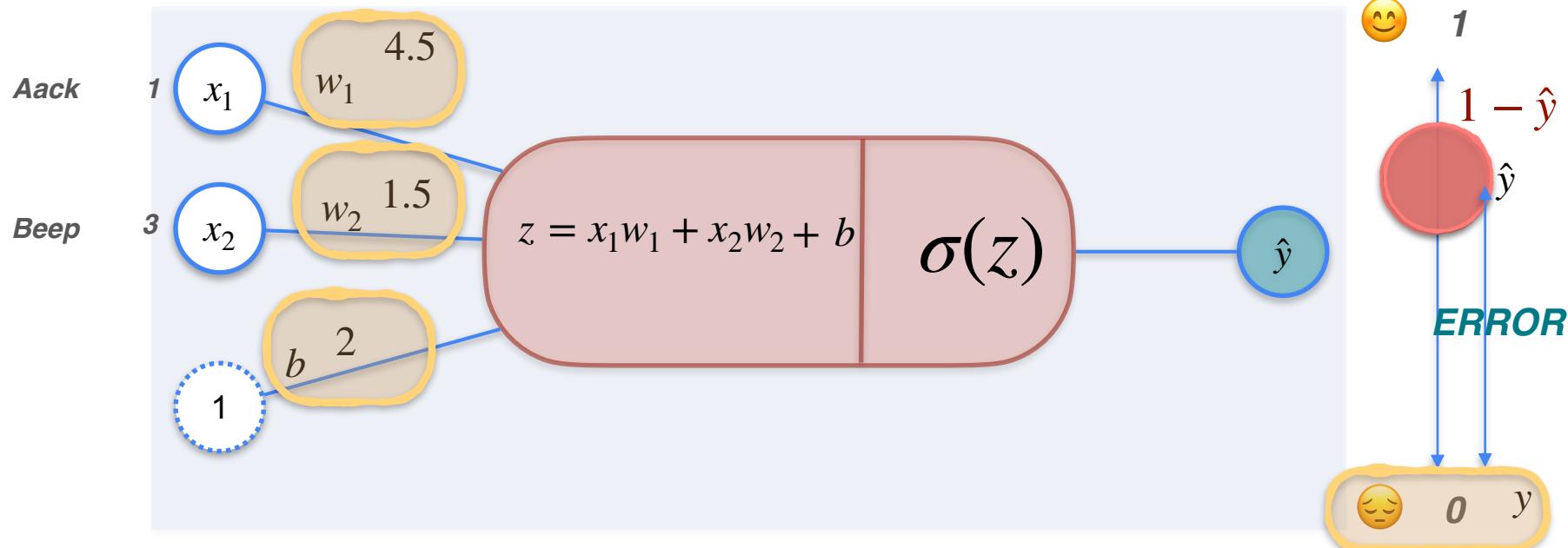
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Classification With a Perceptron

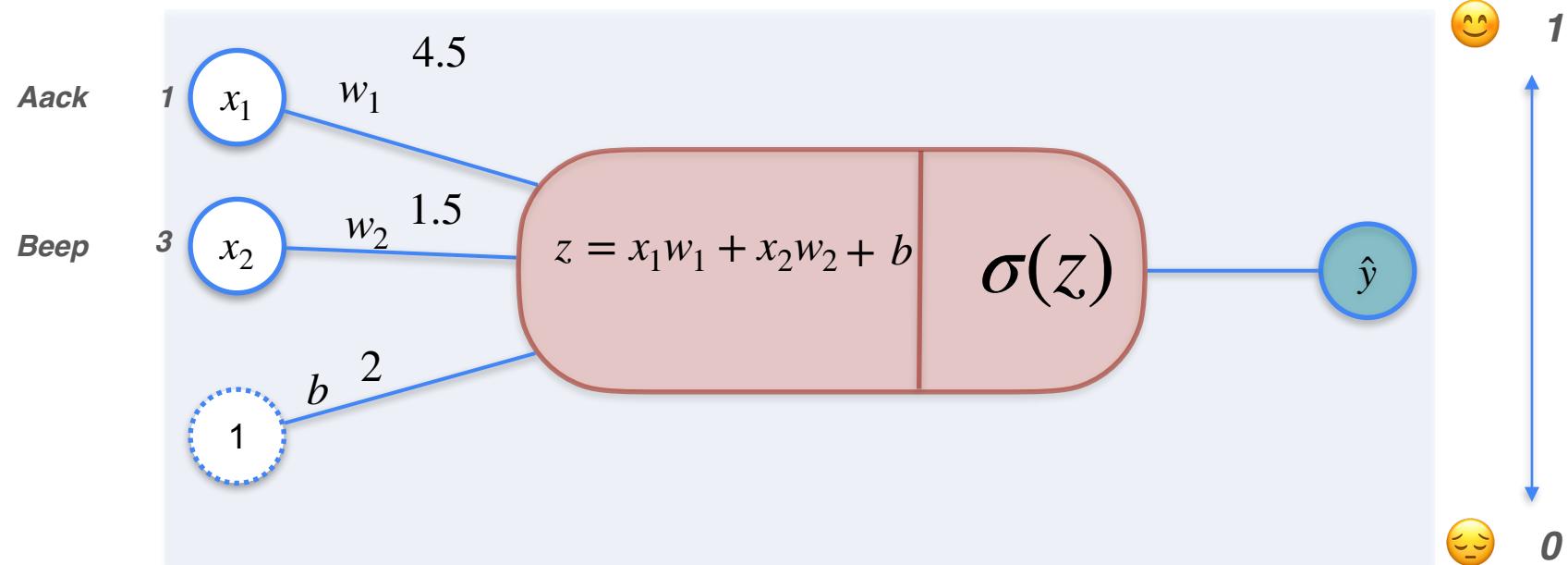
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LOG LOSS



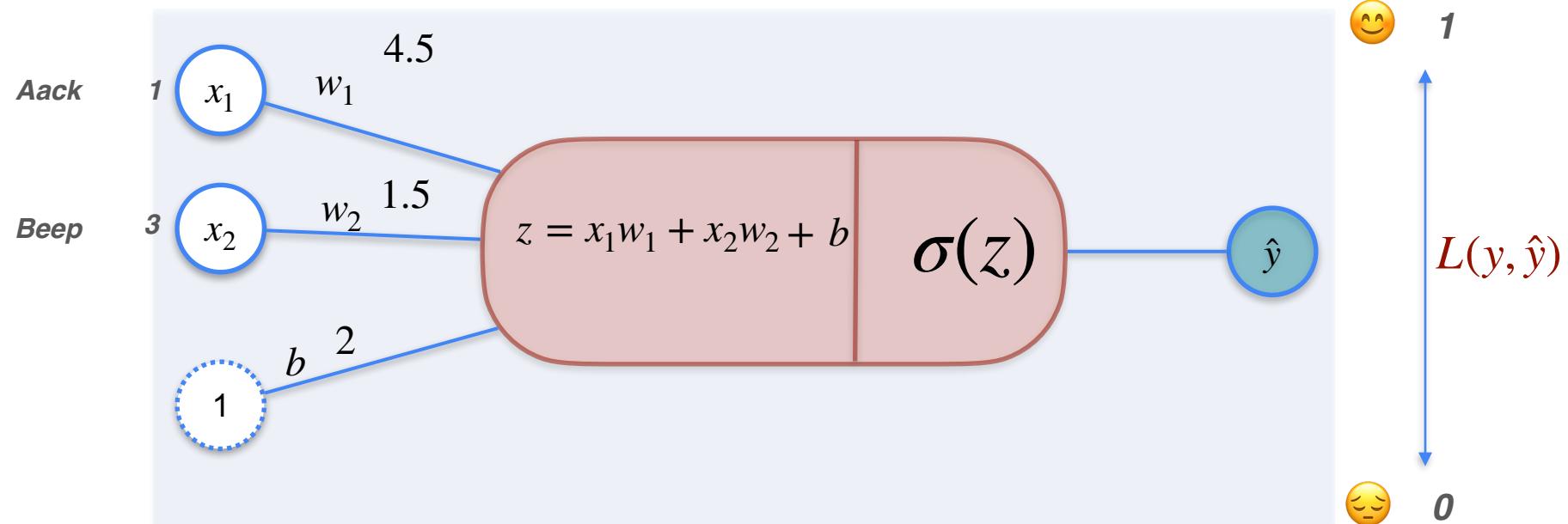
Classification With a Perceptron

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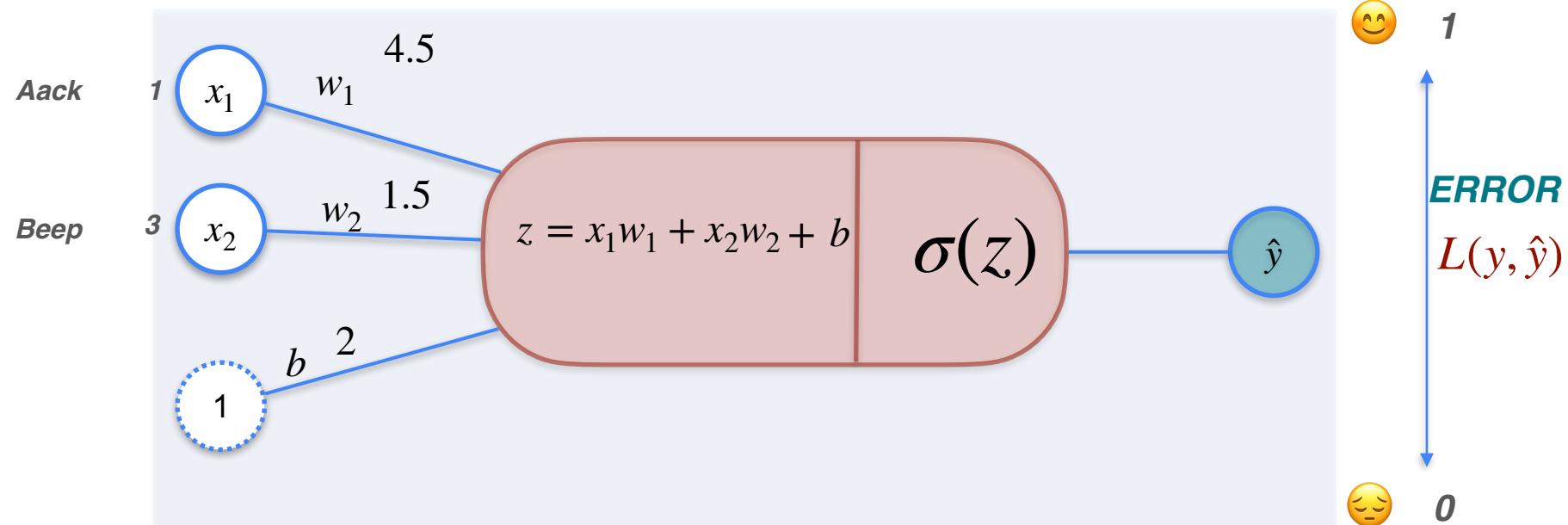
Classification With a Perceptron

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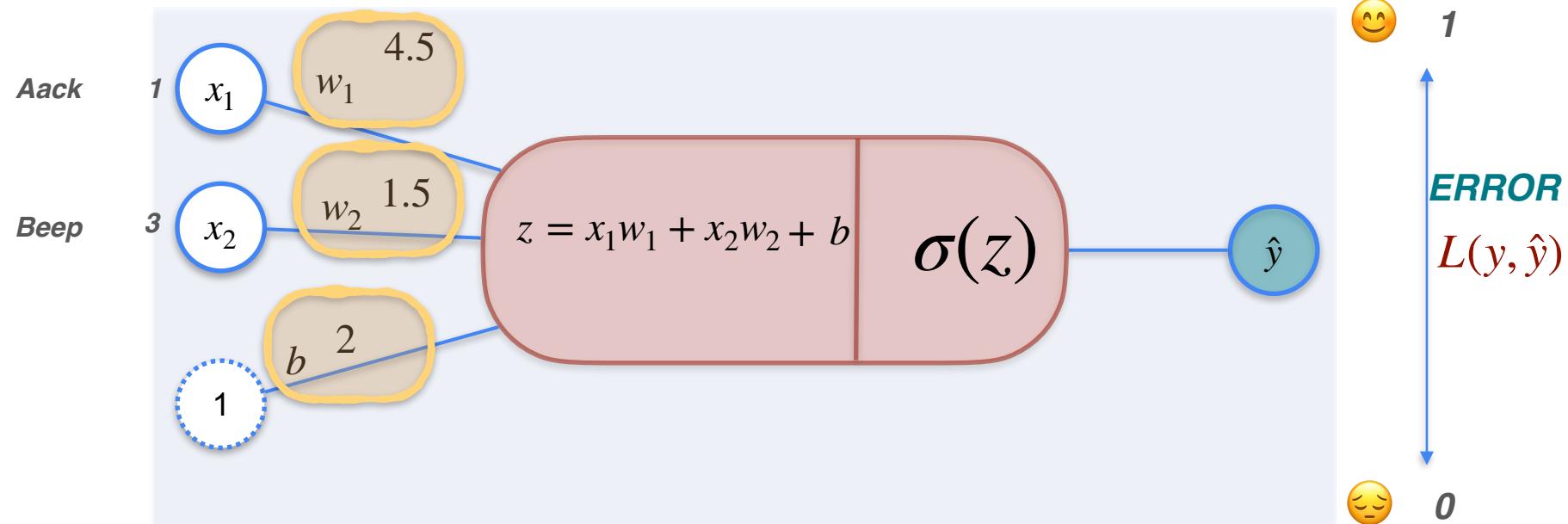
Classification With a Perceptron

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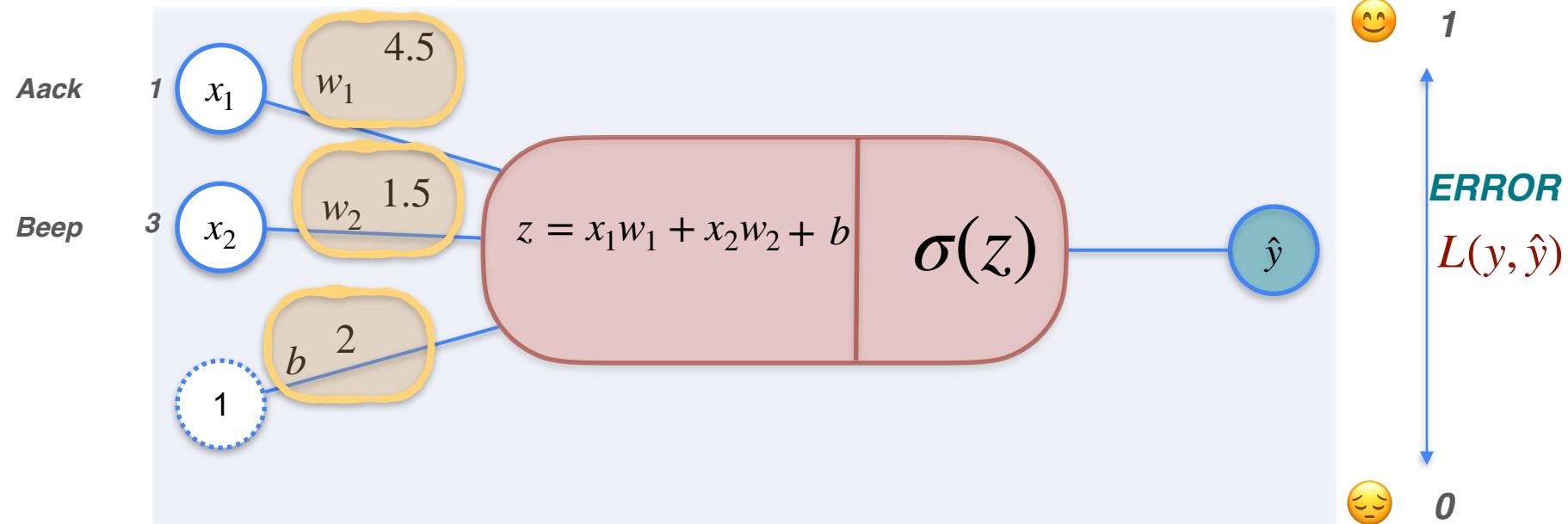
Classification With a Perceptron

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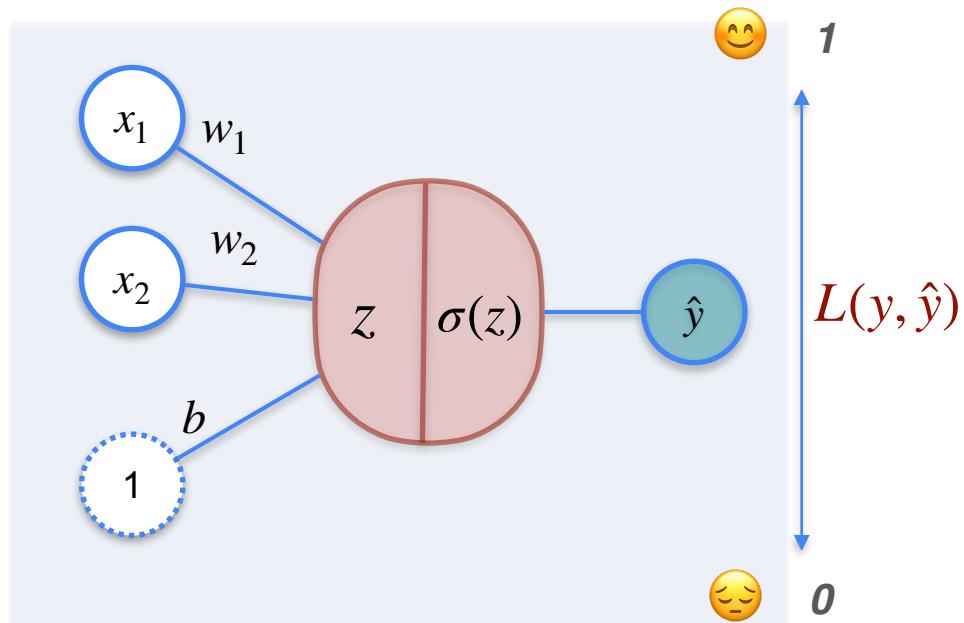


Classification With a Perceptron

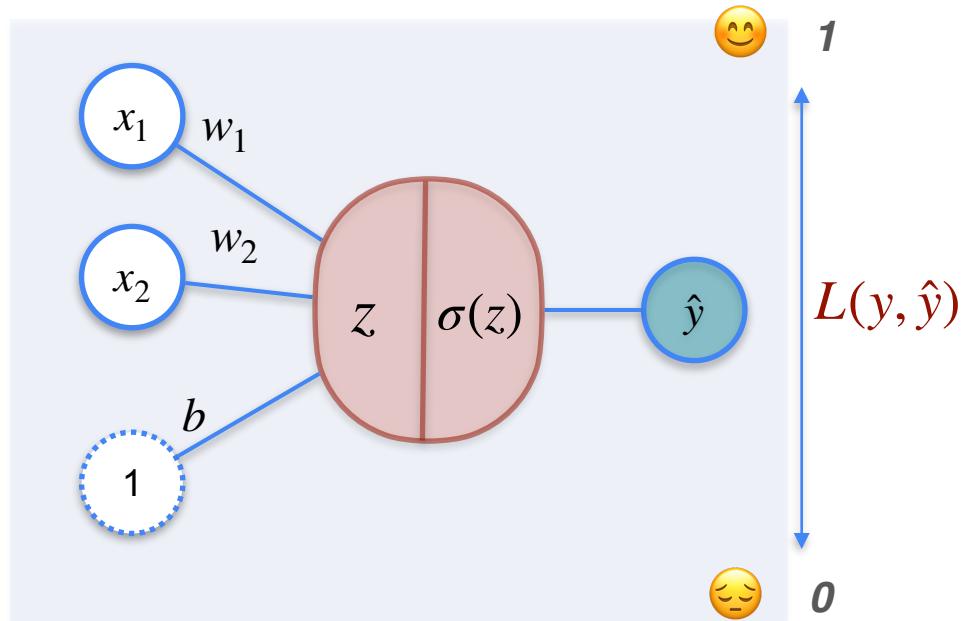
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Classification With a Perceptron



Classification With a Perceptron

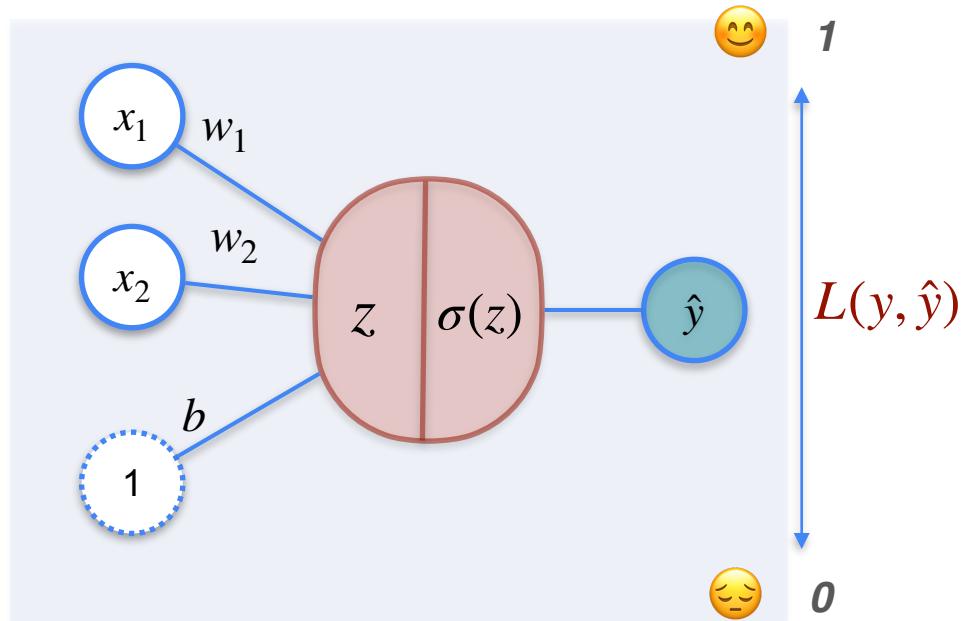


Prediction Function:

$$\hat{y}$$

$$L(y, \hat{y})$$

Classification With a Perceptron

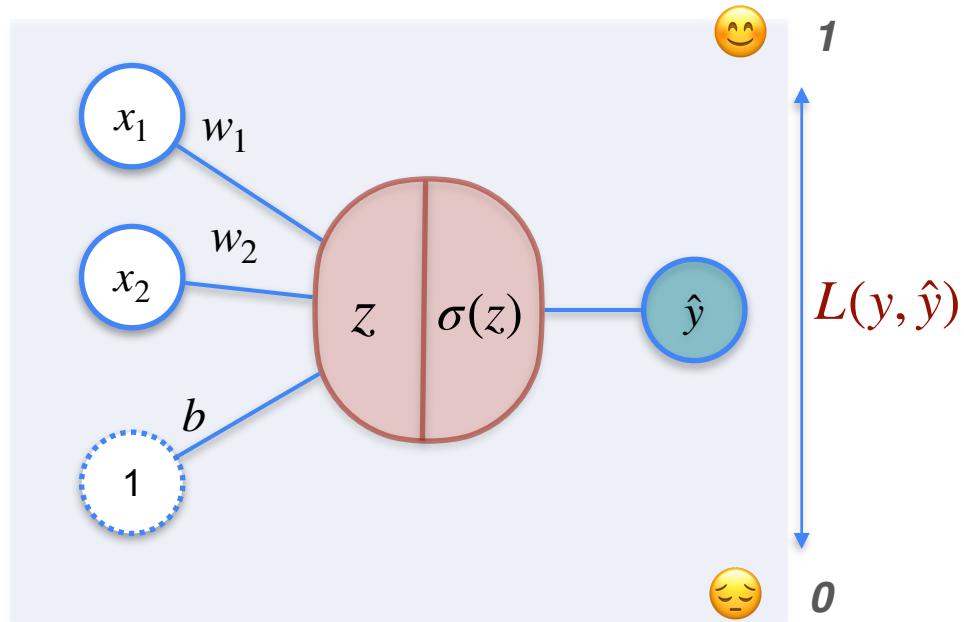


Prediction Function:

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

$$L(y, \hat{y})$$

Classification With a Perceptron



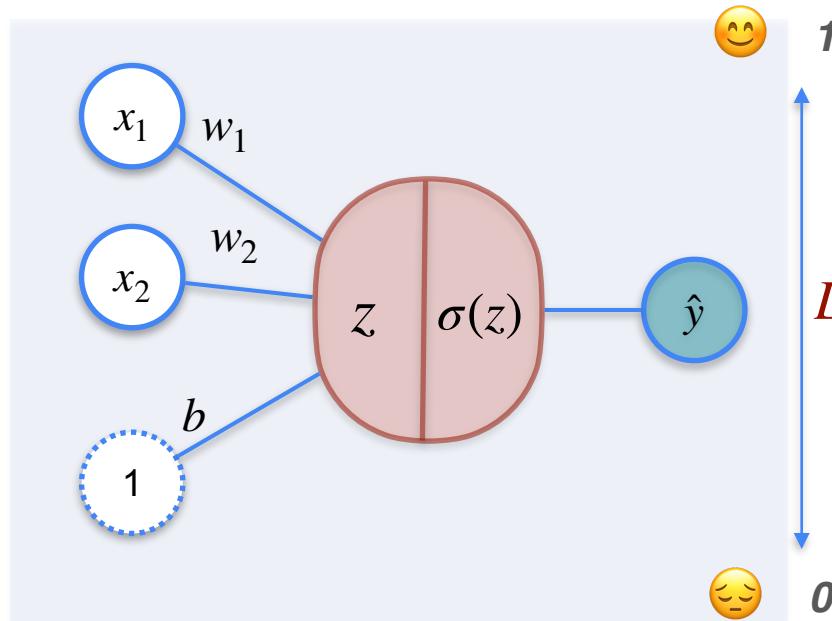
Prediction Function:

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

Loss Function:

$$L(y, \hat{y})$$

Classification With a Perceptron



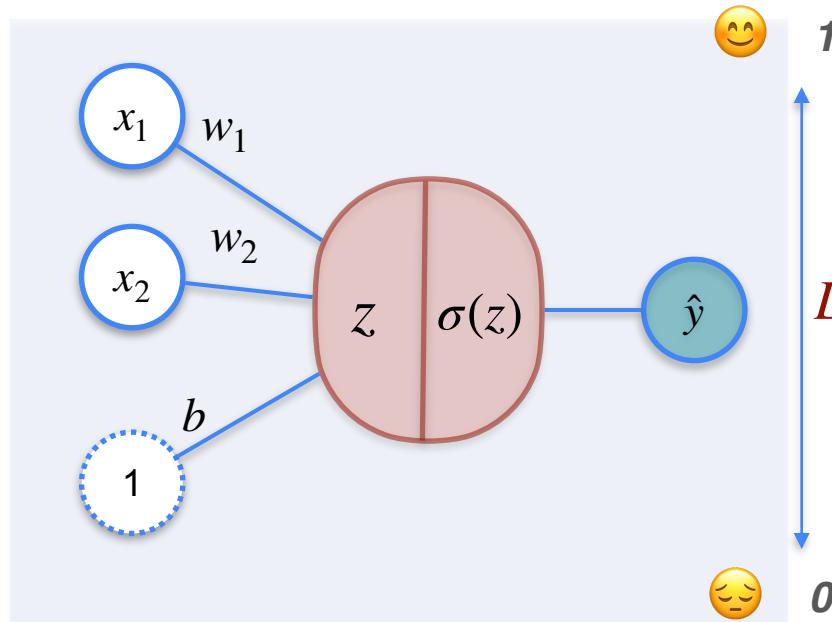
Prediction Function:

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

Loss Function:

$$L(y, \hat{y}) \quad L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

Classification With a Perceptron



Prediction Function:

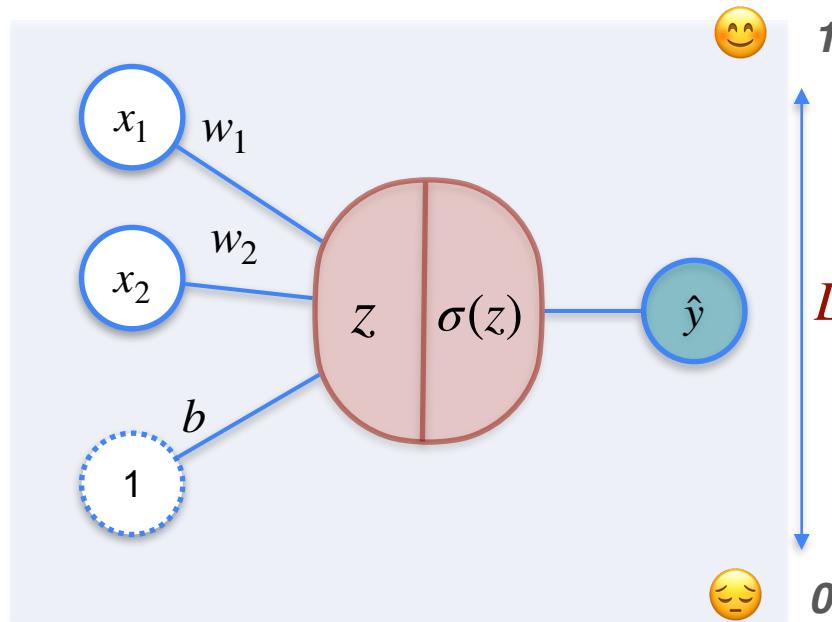
$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

Loss Function:

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

Main Goal:

Classification With a Perceptron



Prediction Function:

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

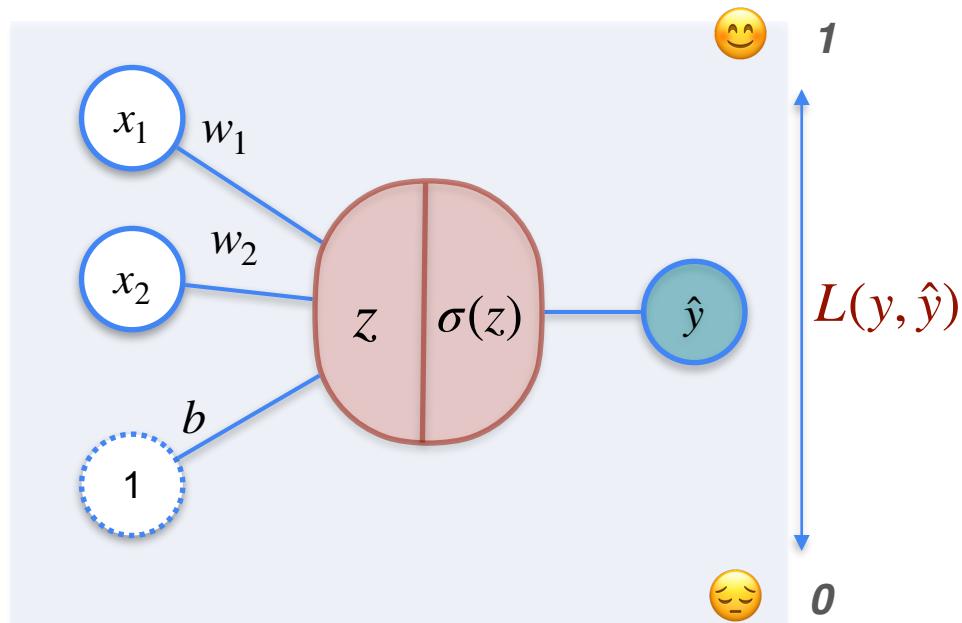
Loss Function:

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

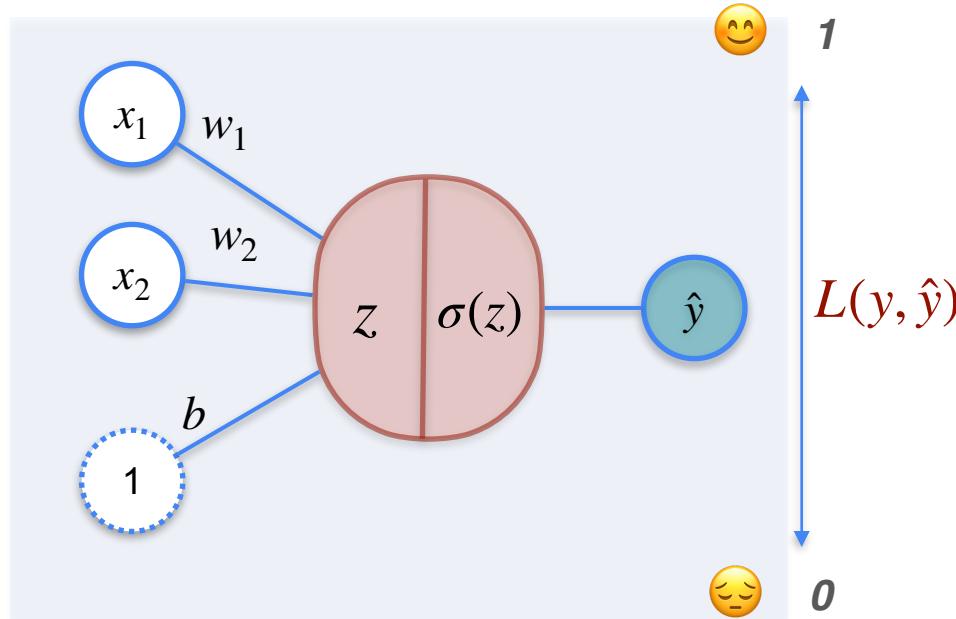
Main Goal:

Find w_1, w_2, b that give \hat{y} with the least error

Classification With a Perceptron

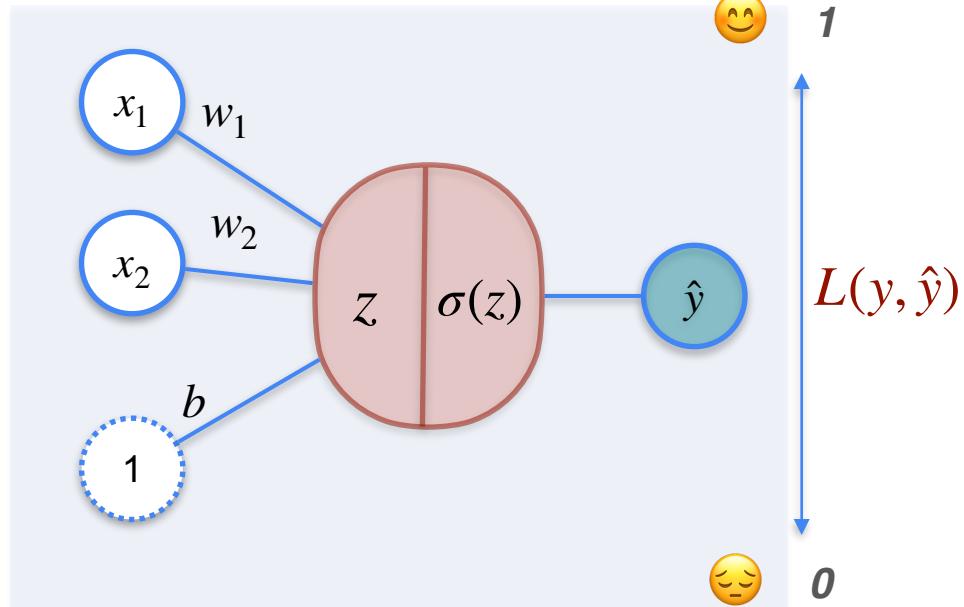


Classification With a Perceptron



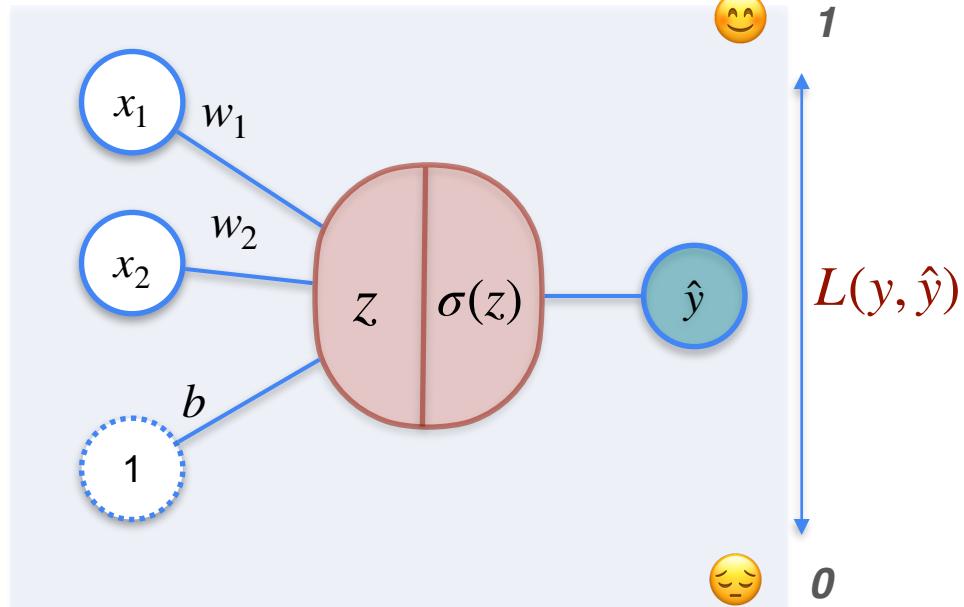
To find optimal values for:

Classification With a Perceptron



To find optimal values for:
 w_1 , w_2 , b

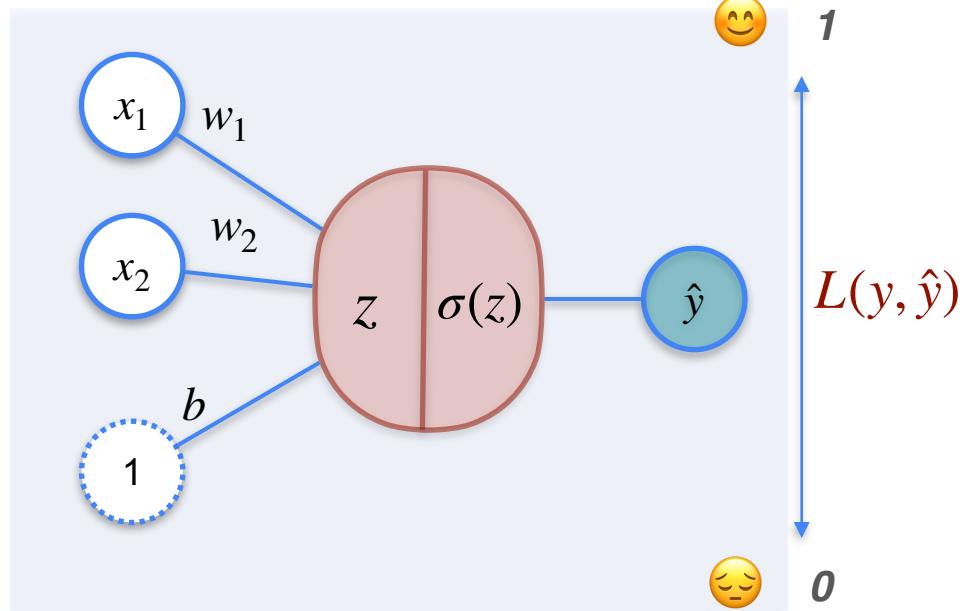
Classification With a Perceptron



To find optimal values for:
 w_1 , w_2 , b

You need gradient descent

Classification With a Perceptron

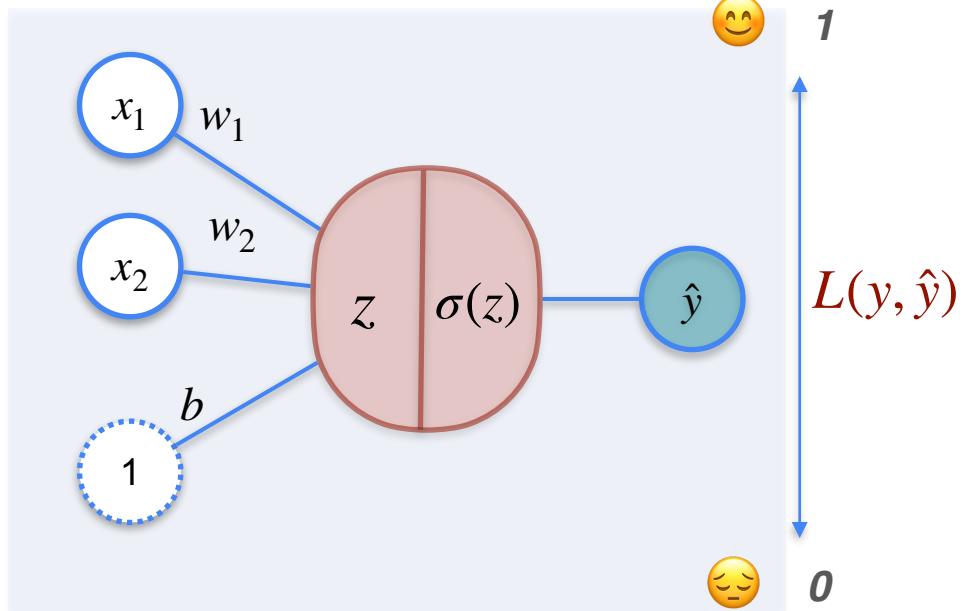


To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Classification With a Perceptron



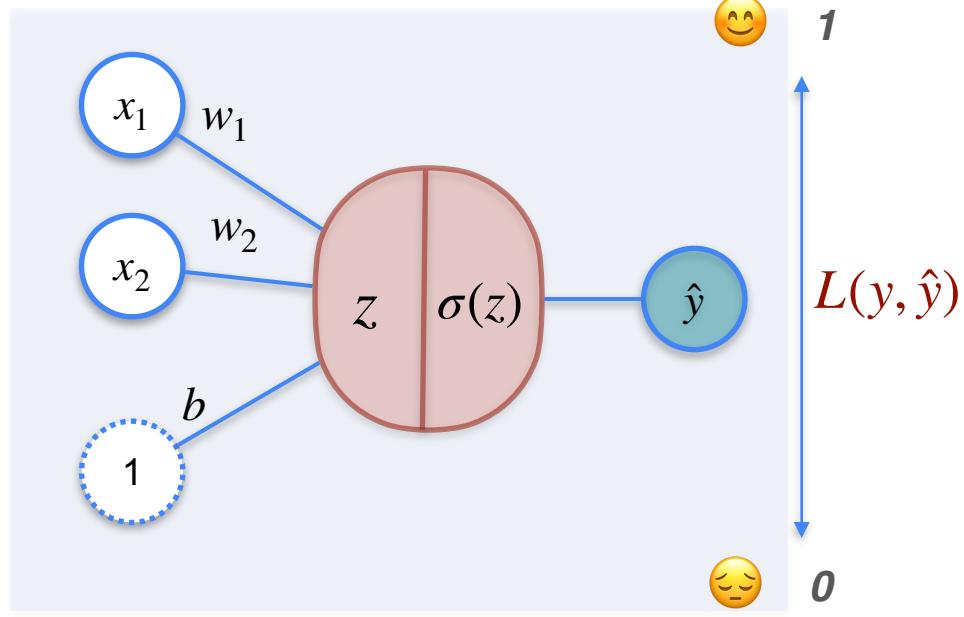
To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Classification With a Perceptron



To find optimal values for:
 w_1, w_2, b

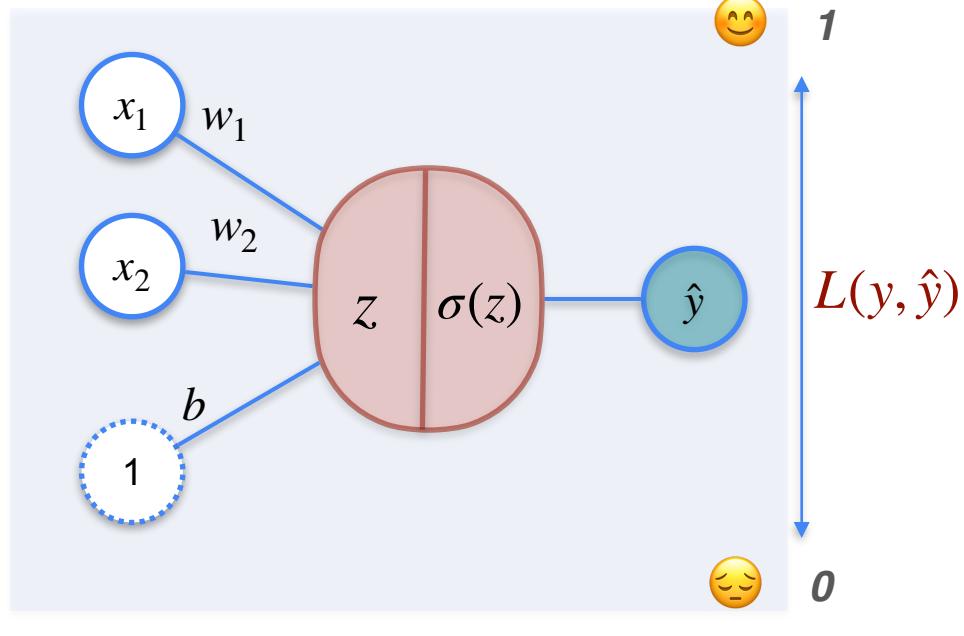
You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Classification With a Perceptron



To find optimal values for:
 w_1, w_2, b

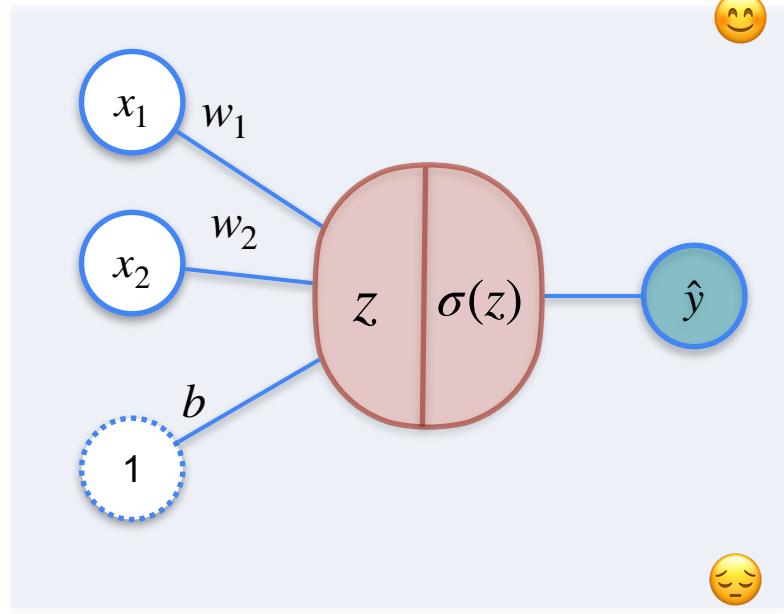
You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

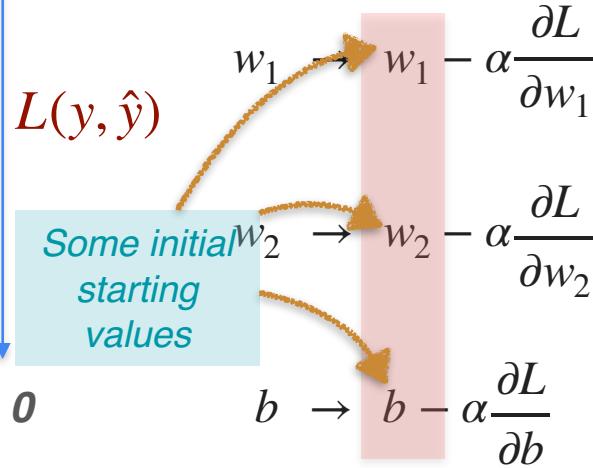
$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Classification With a Perceptron

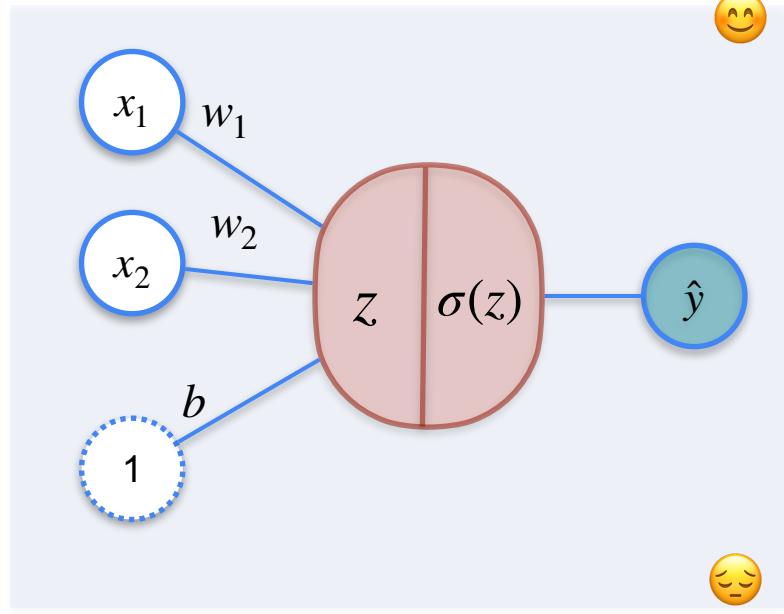


To find optimal values for:
 w_1, w_2, b

You need gradient descent

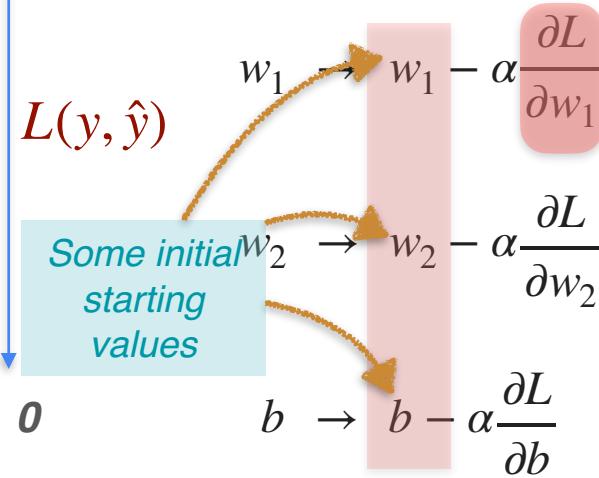


Classification With a Perceptron

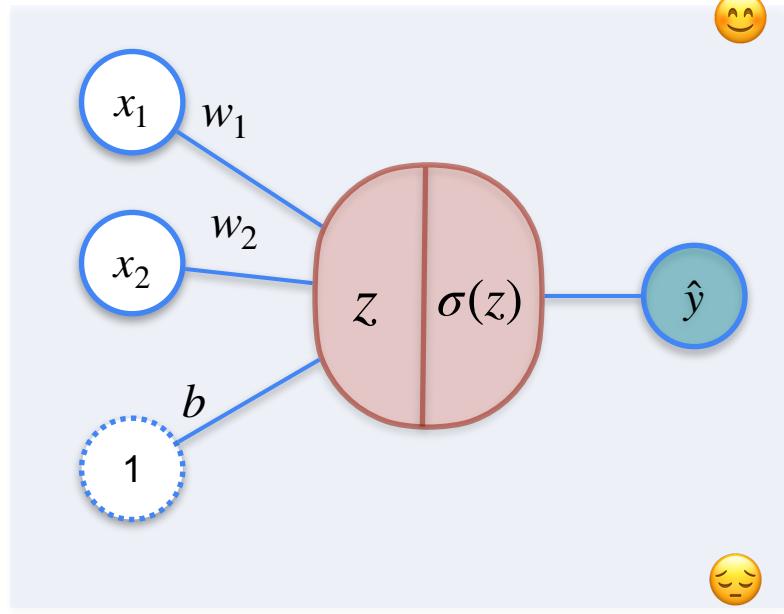


To find optimal values for:
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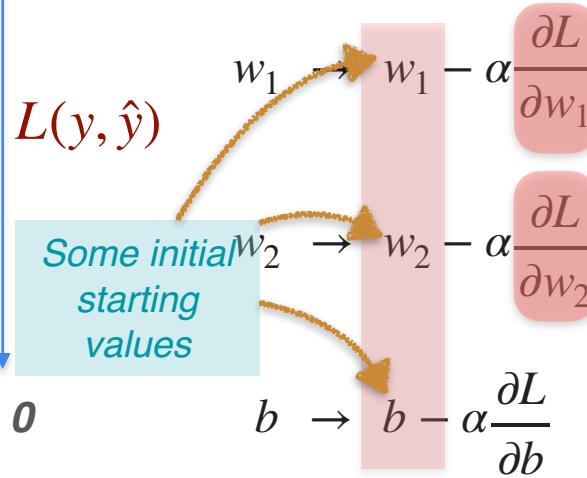


Classification With a Perceptron

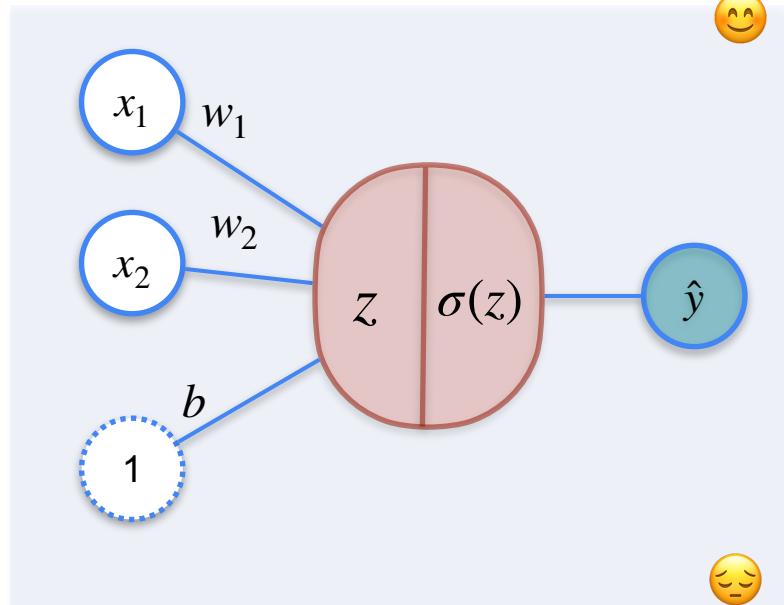


To find optimal values for:
 w_1 , w_2 , b

You need gradient descent

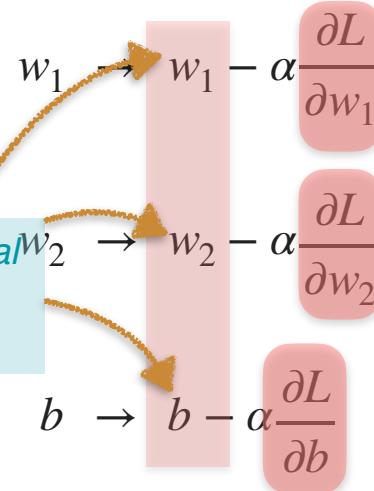


Classification With a Perceptron

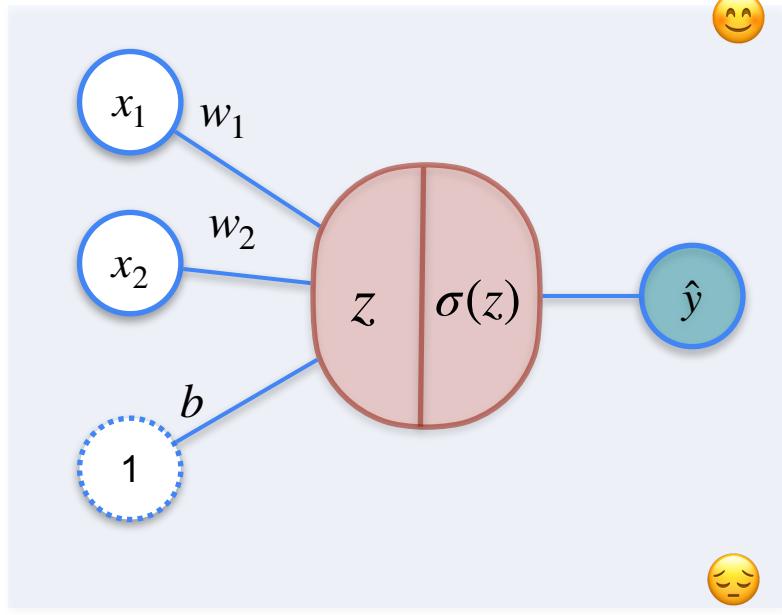


To find optimal values for:
 w_1 , w_2 , b

You need gradient descent

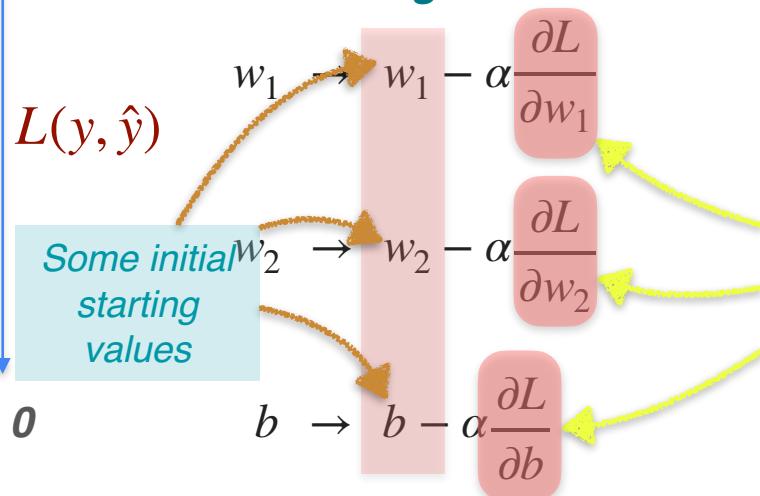


Classification With a Perceptron



To find optimal values for:
 w_1, w_2, b

You need gradient descent



SUB-TASK

Find the following partial derivatives

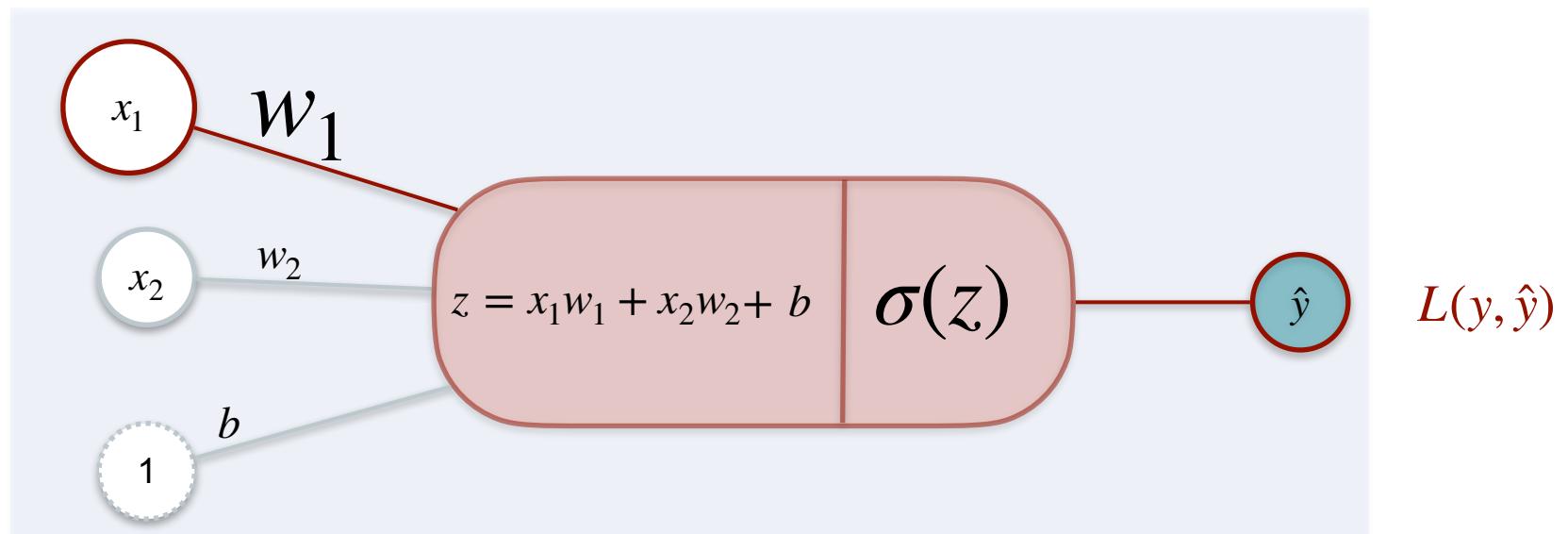


DeepLearning.AI

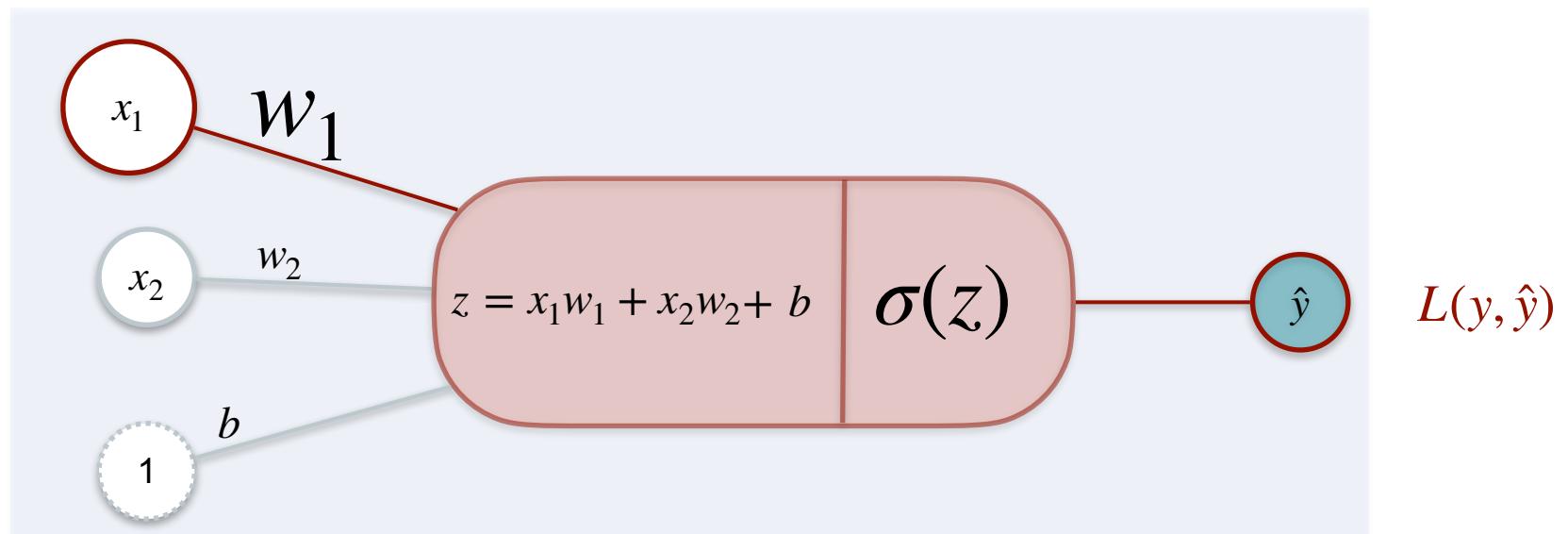
Optimization in Neural Networks and Newton's Method

**Classification with a
perceptron:
Calculating the derivatives**

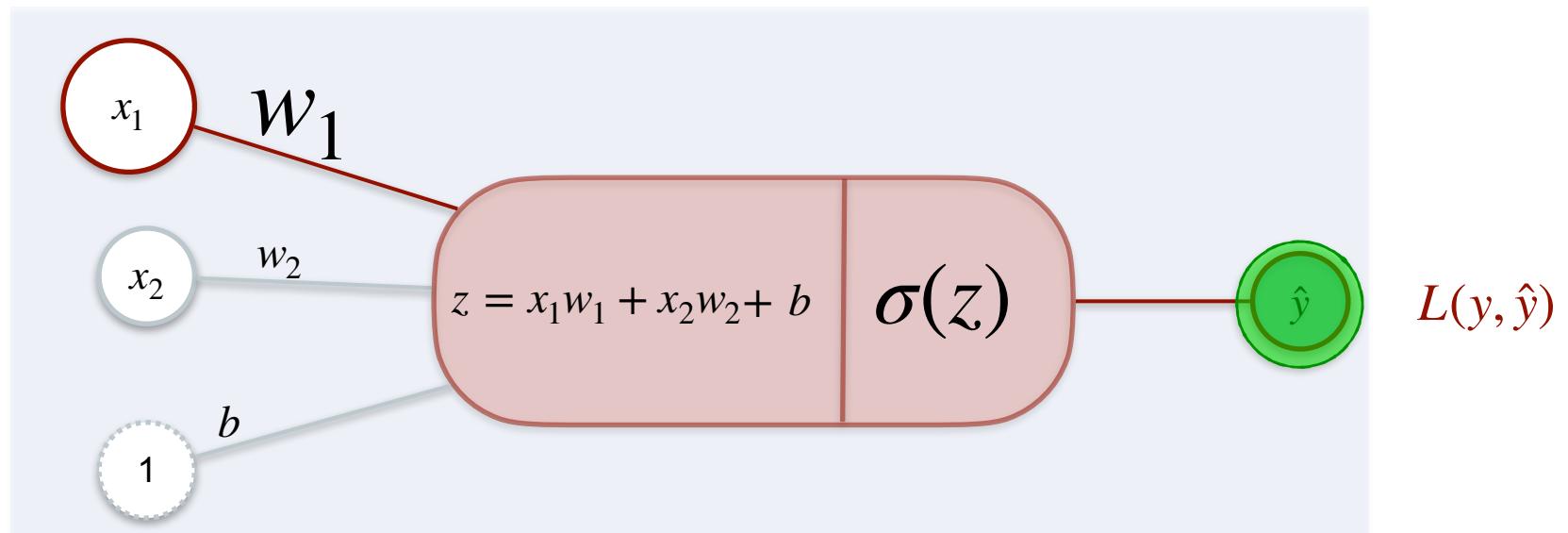
Classification With a Perceptron



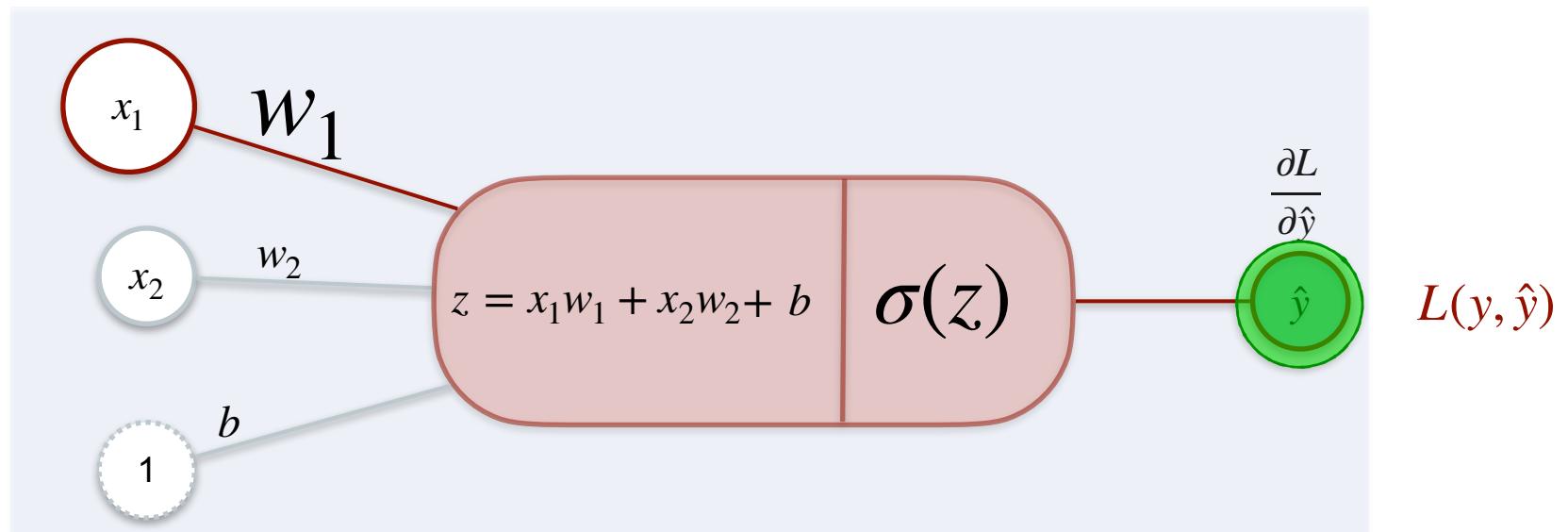
Classification With a Perceptron



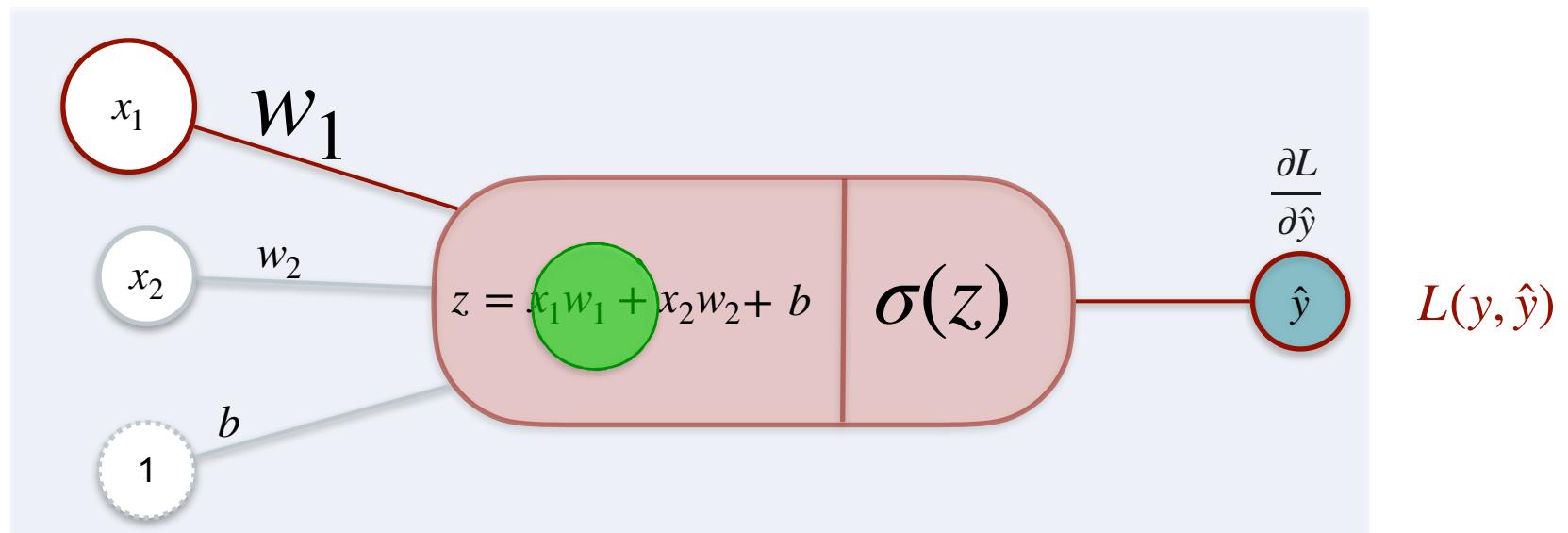
Classification With a Perceptron



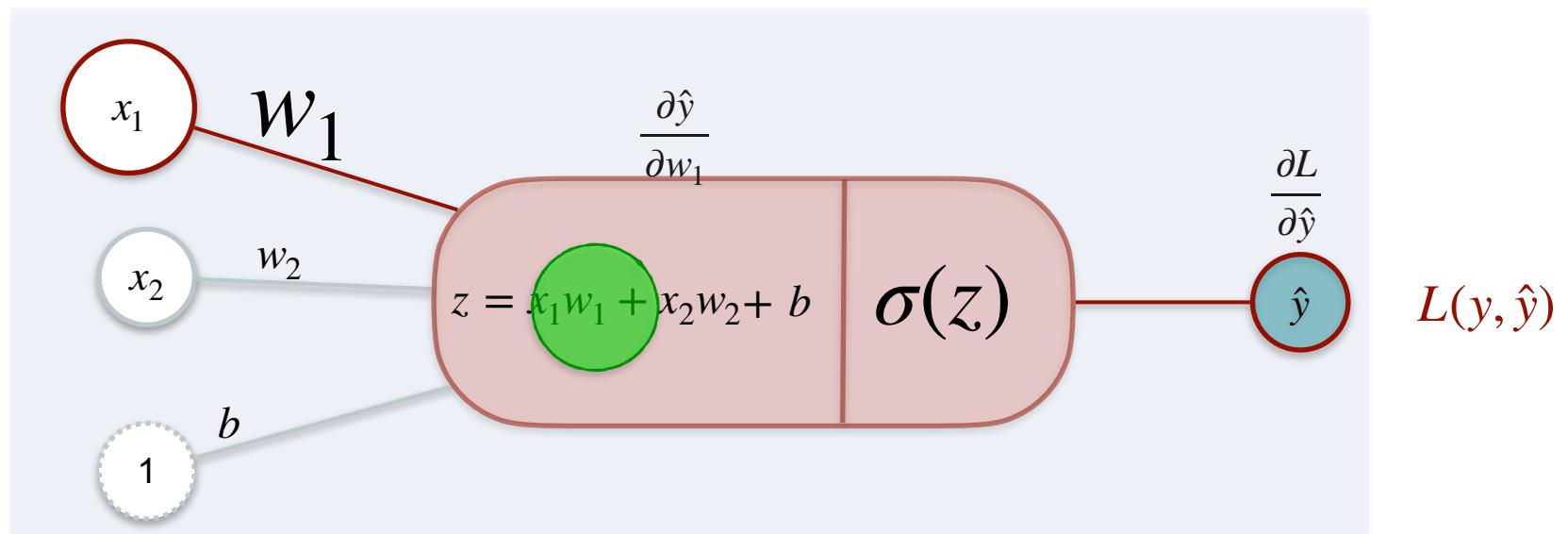
Classification With a Perceptron



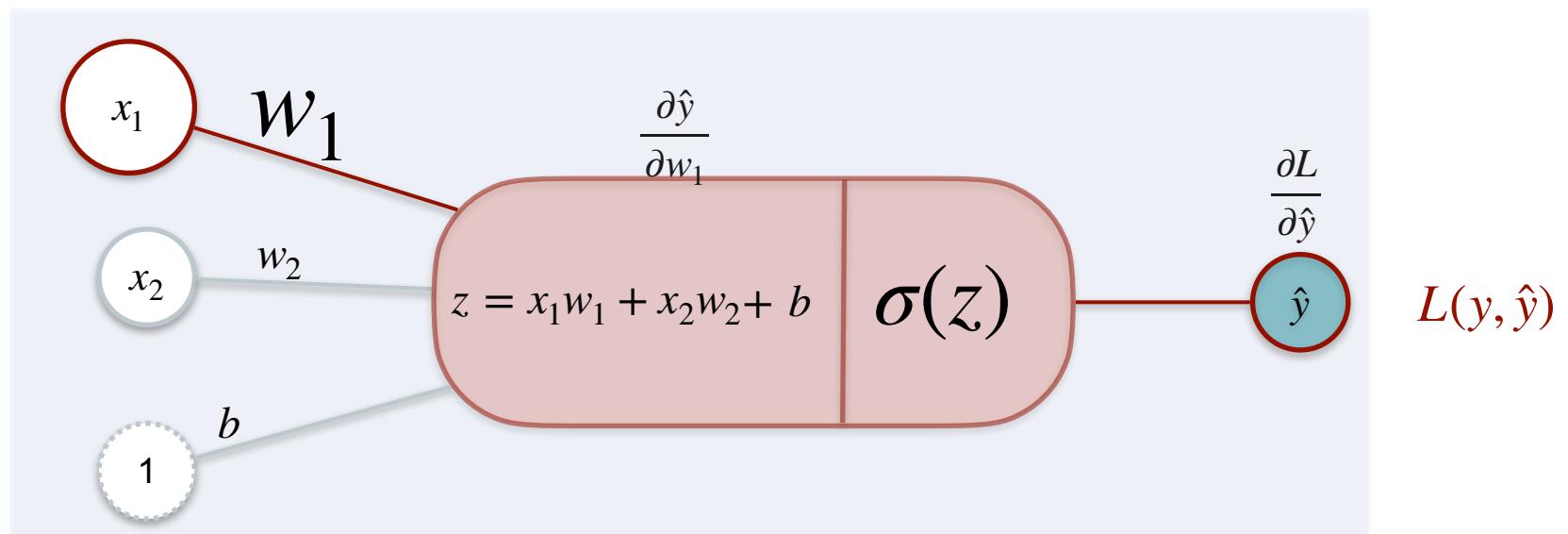
Classification With a Perceptron



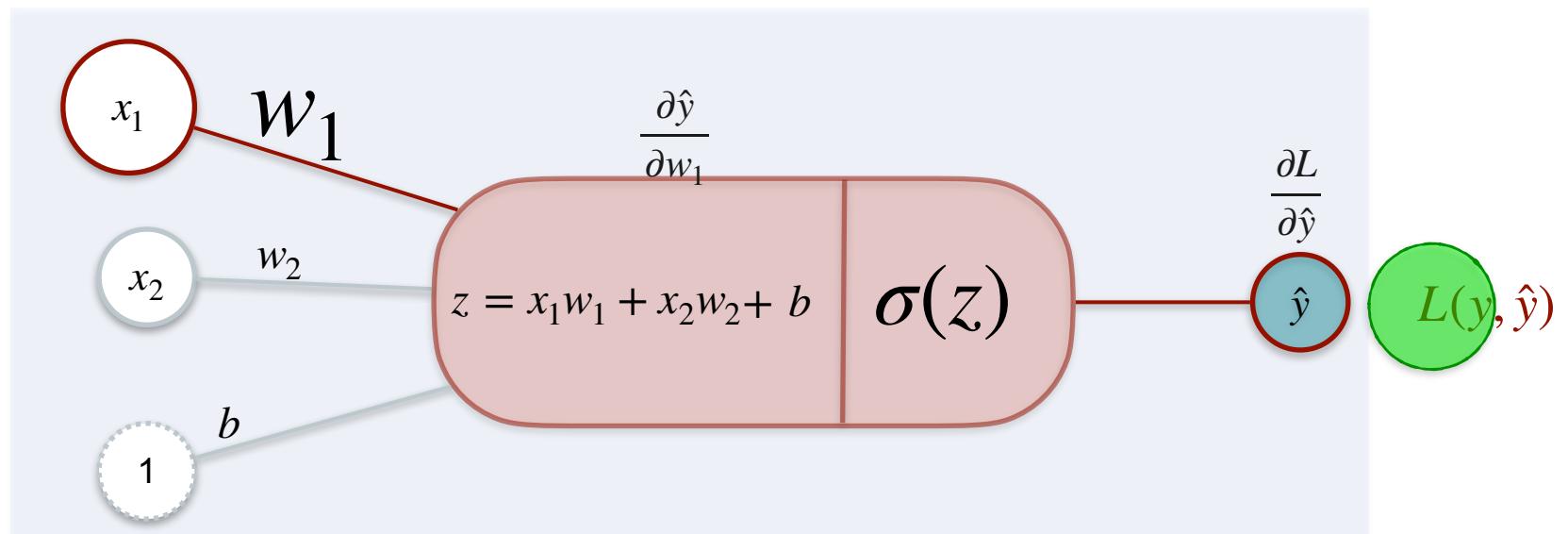
Classification With a Perceptron



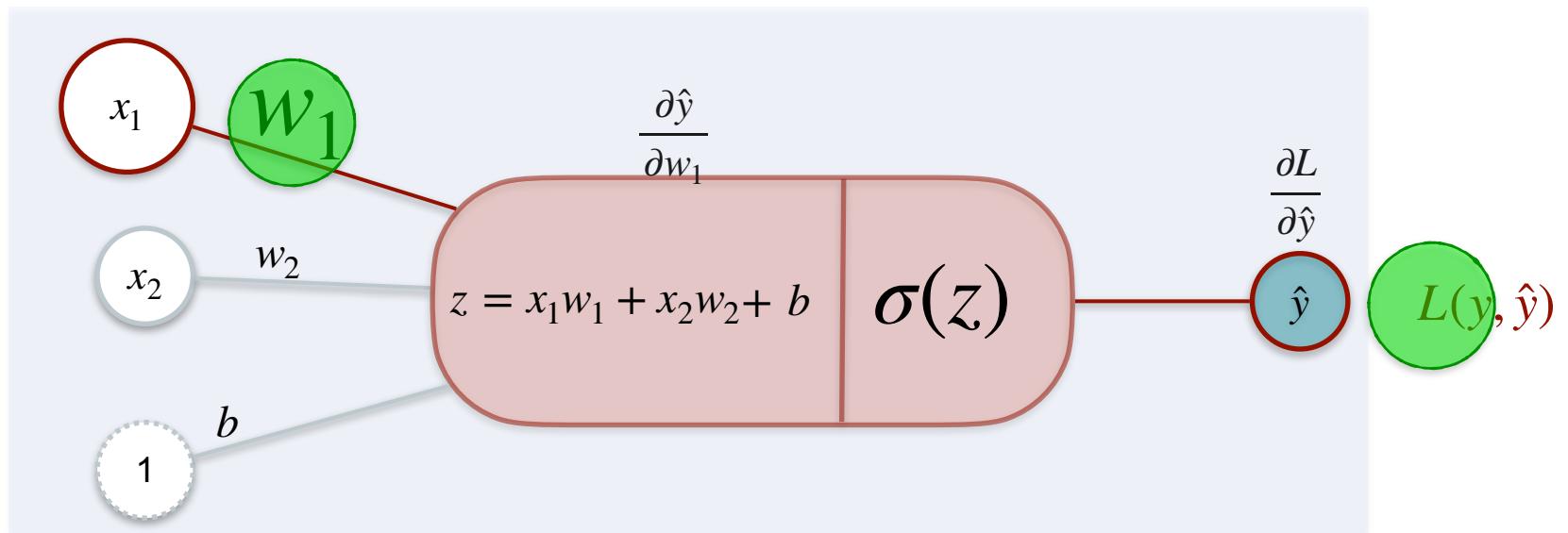
Classification With a Perceptron



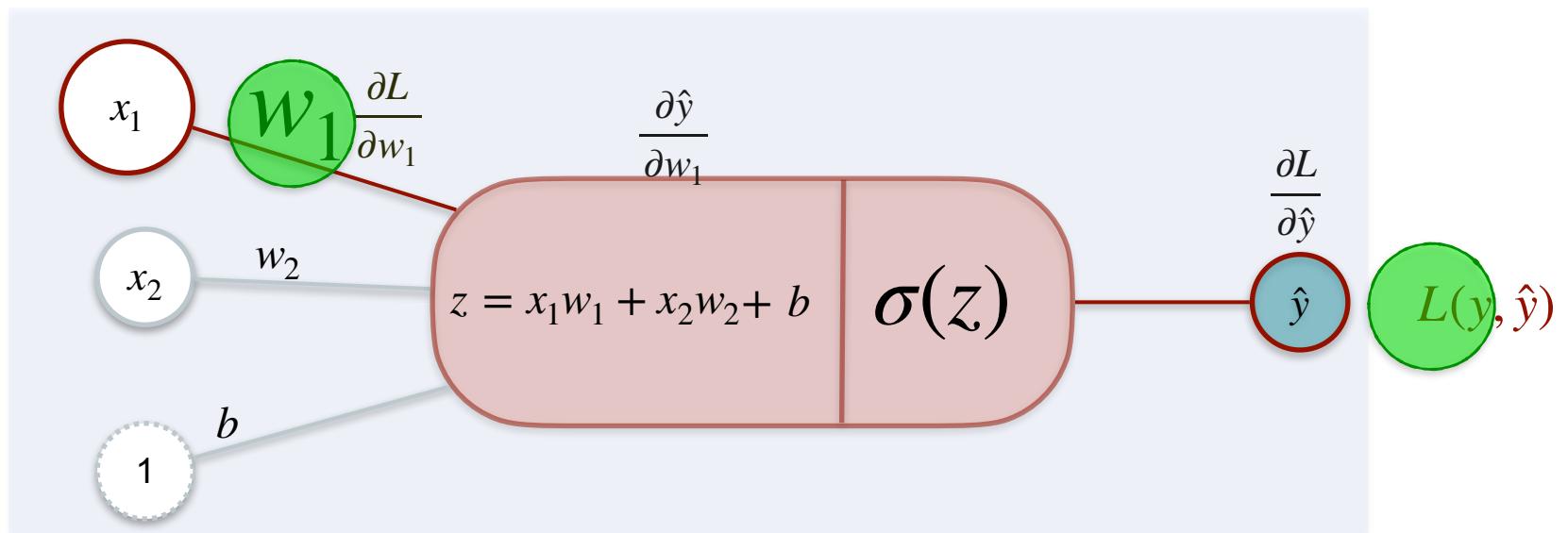
Classification With a Perceptron



Classification With a Perceptron

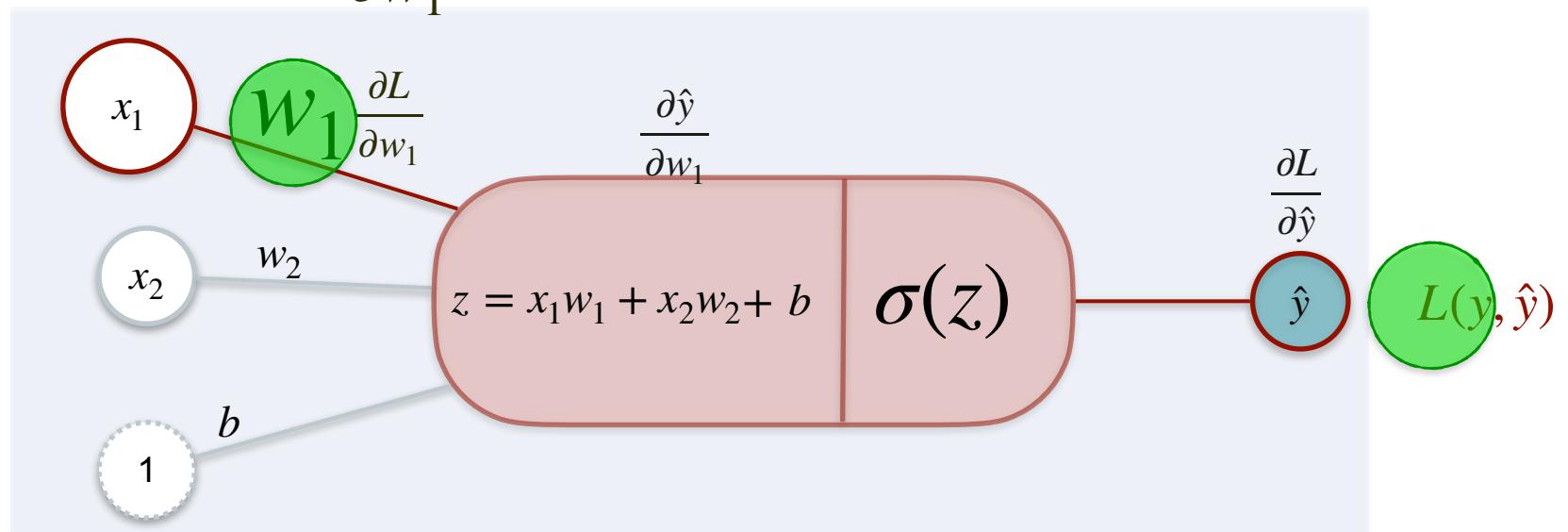


Classification With a Perceptron



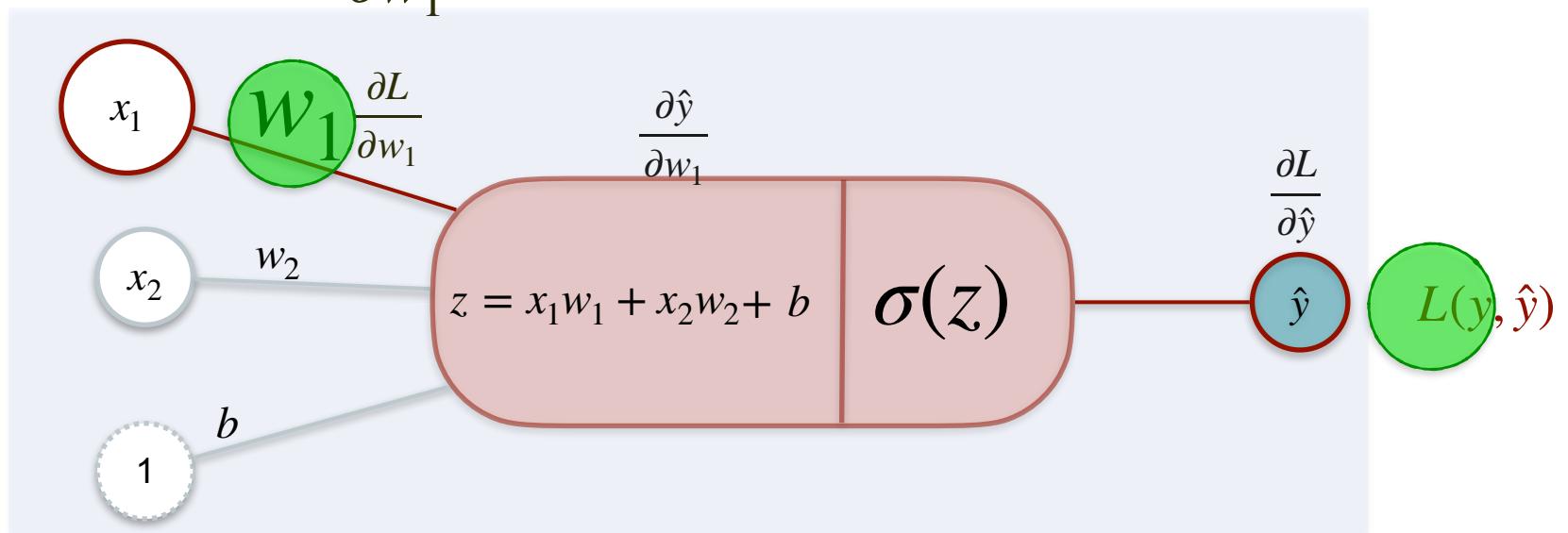
Classification With a Perceptron

$$\frac{\partial L}{\partial w_1}$$



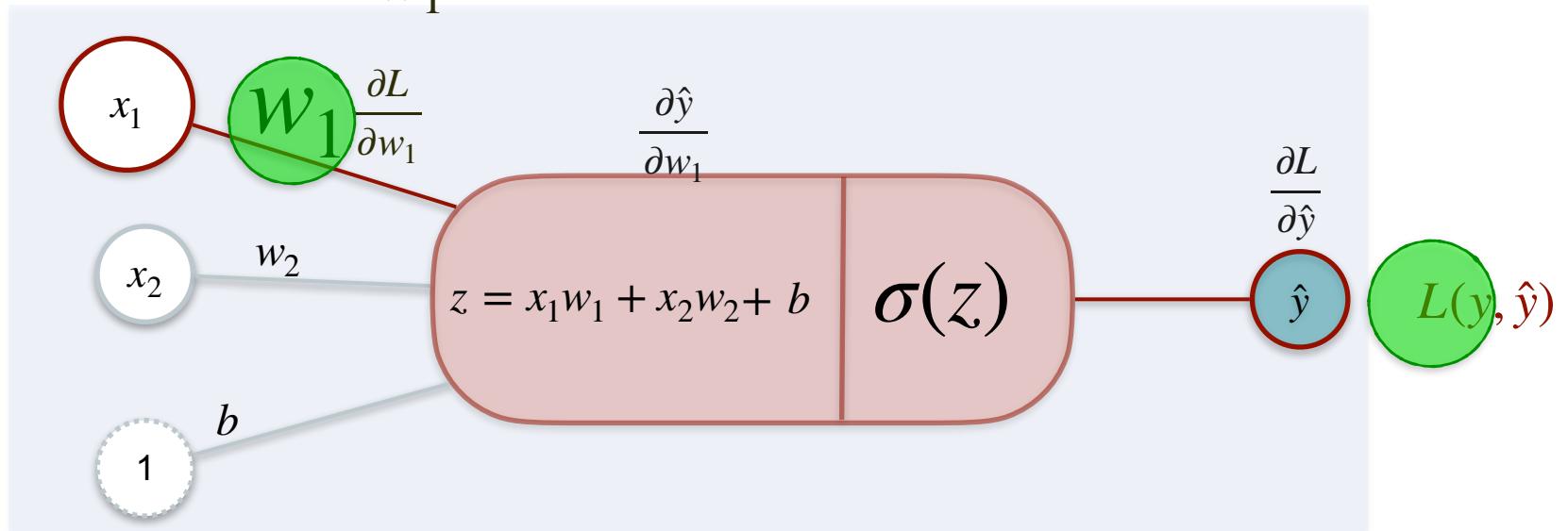
Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} =$$



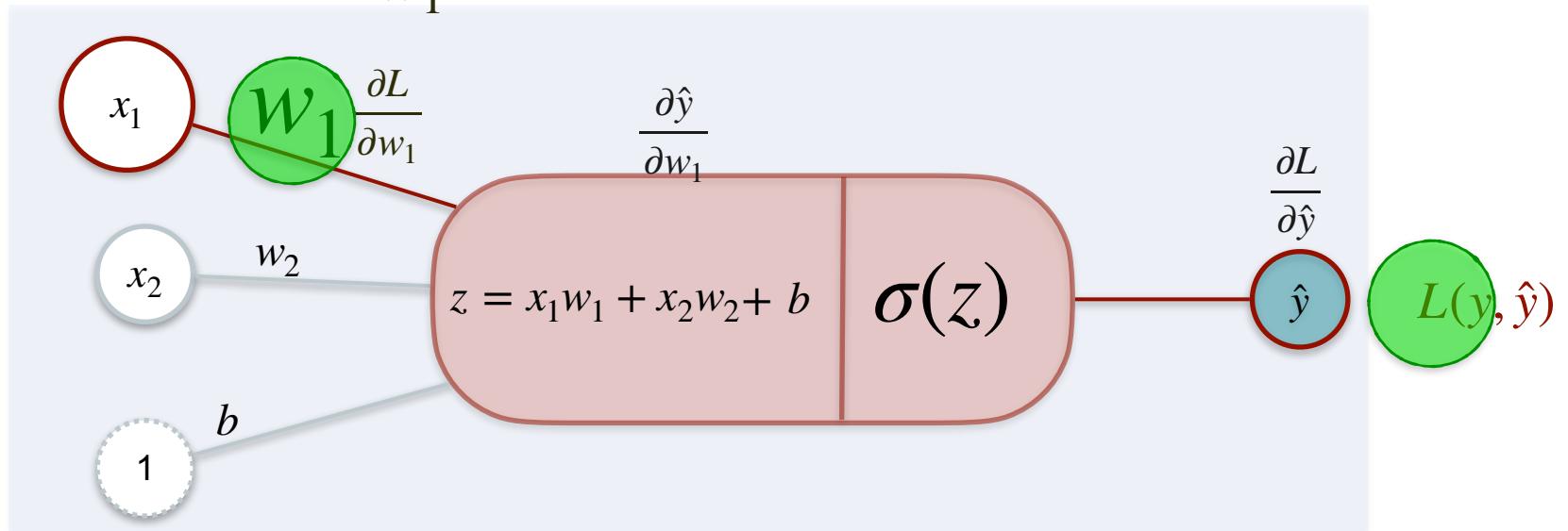
Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}}$$



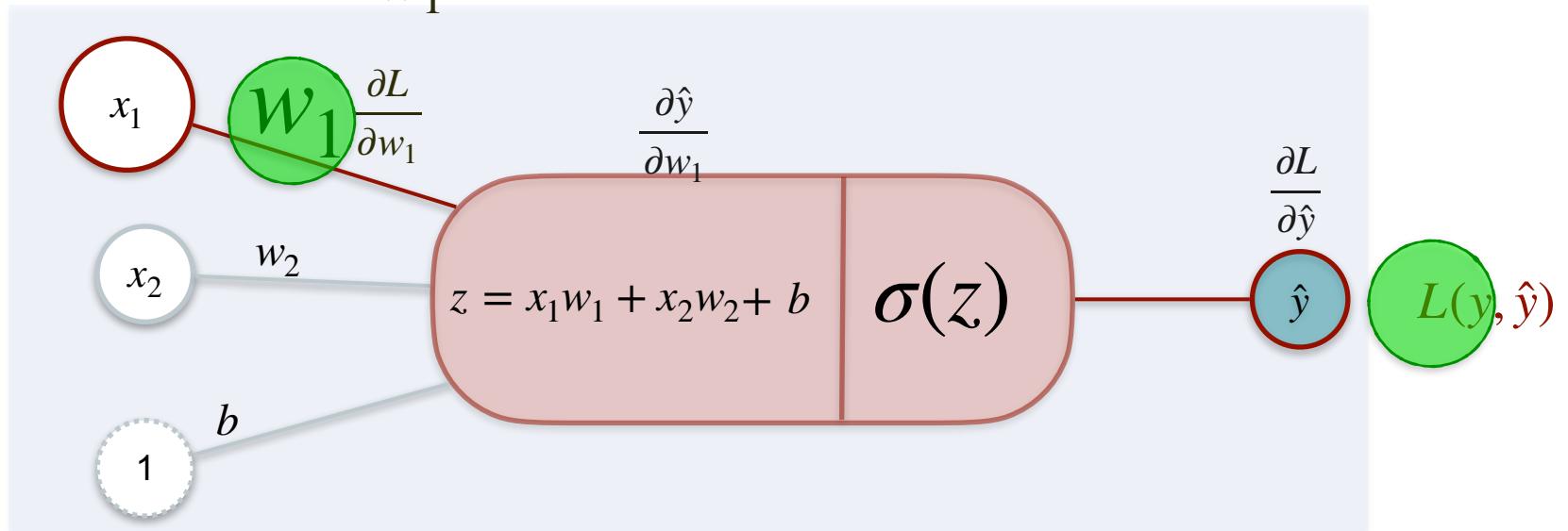
Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot$$

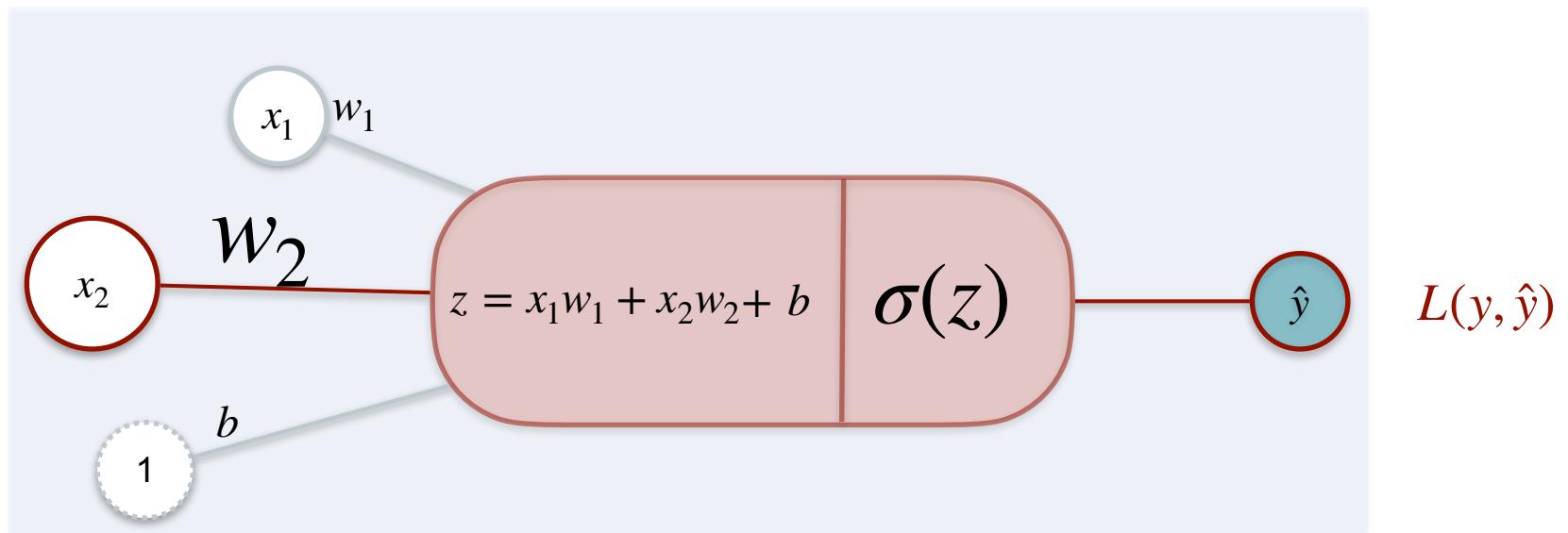


Classification With a Perceptron

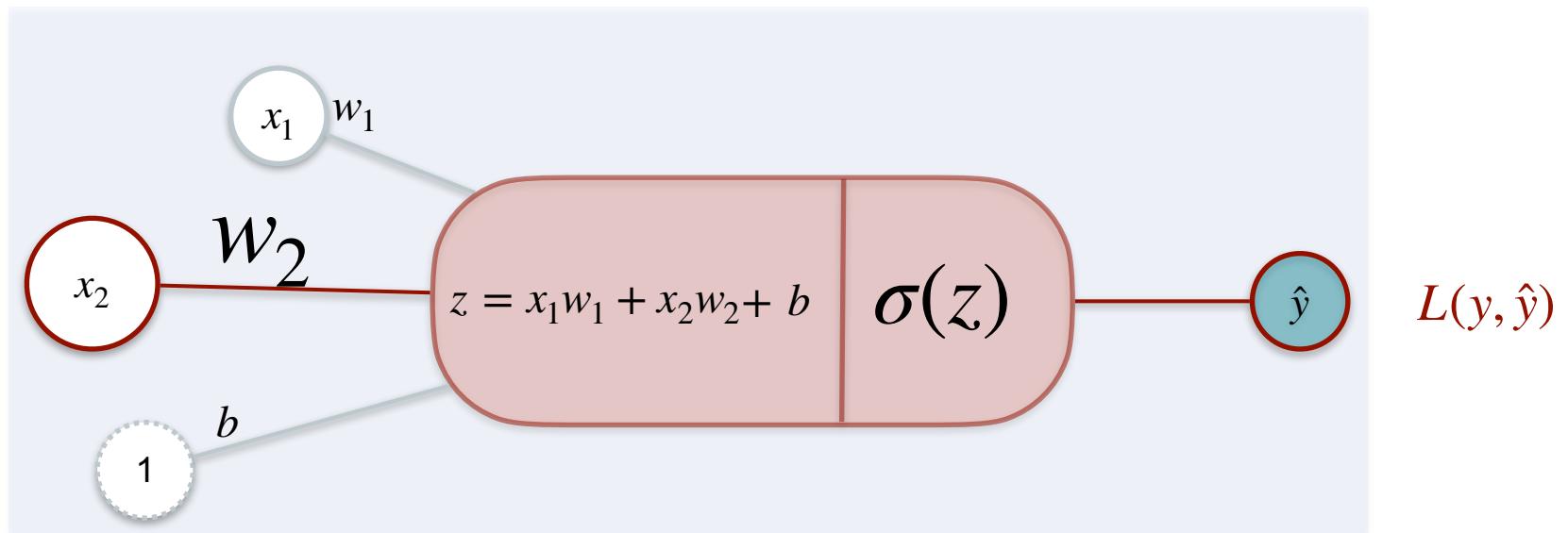
$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$



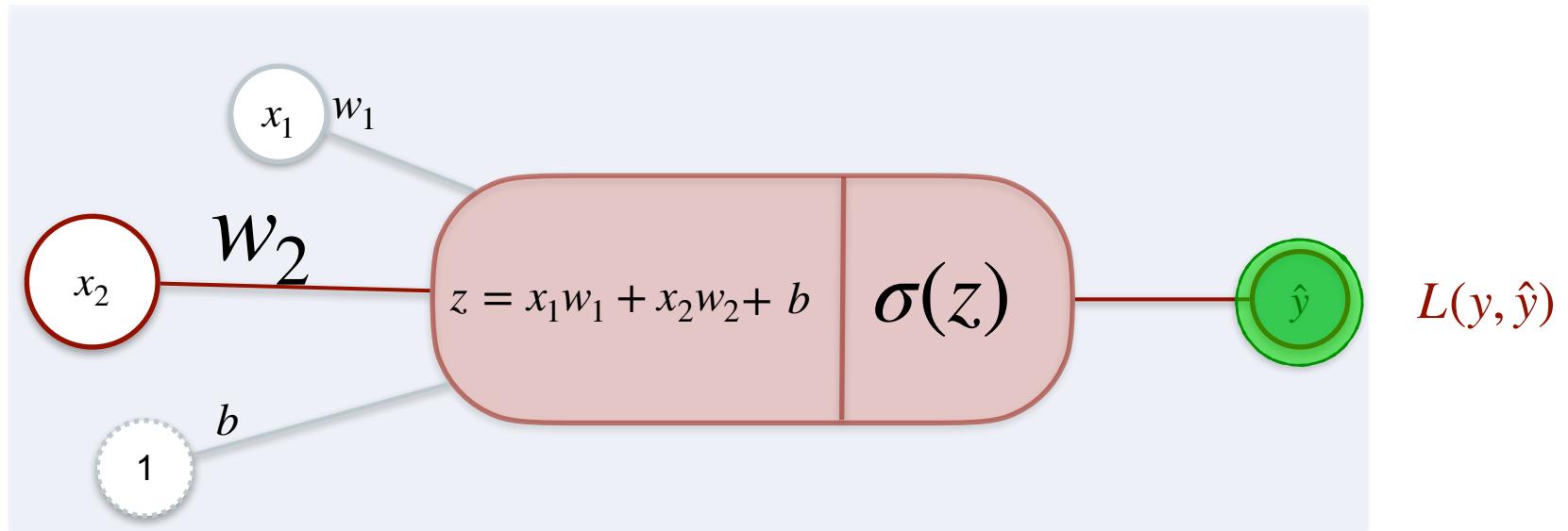
Classification With a Perceptron



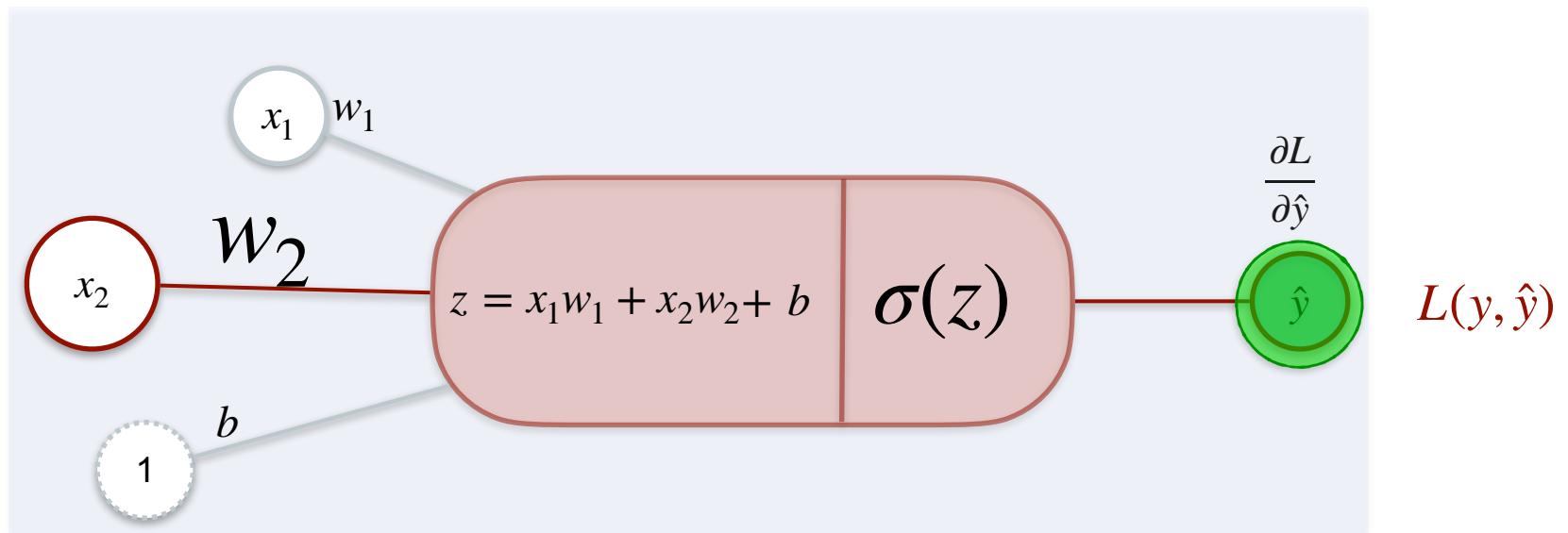
Classification With a Perceptron



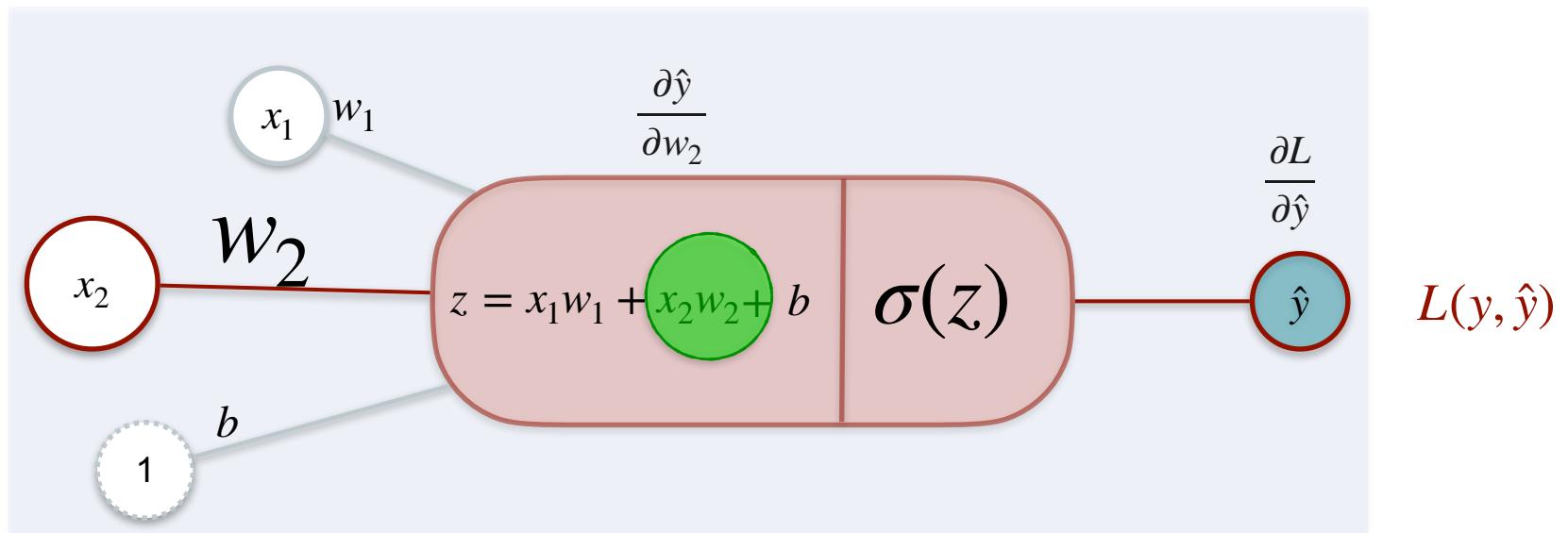
Classification With a Perceptron



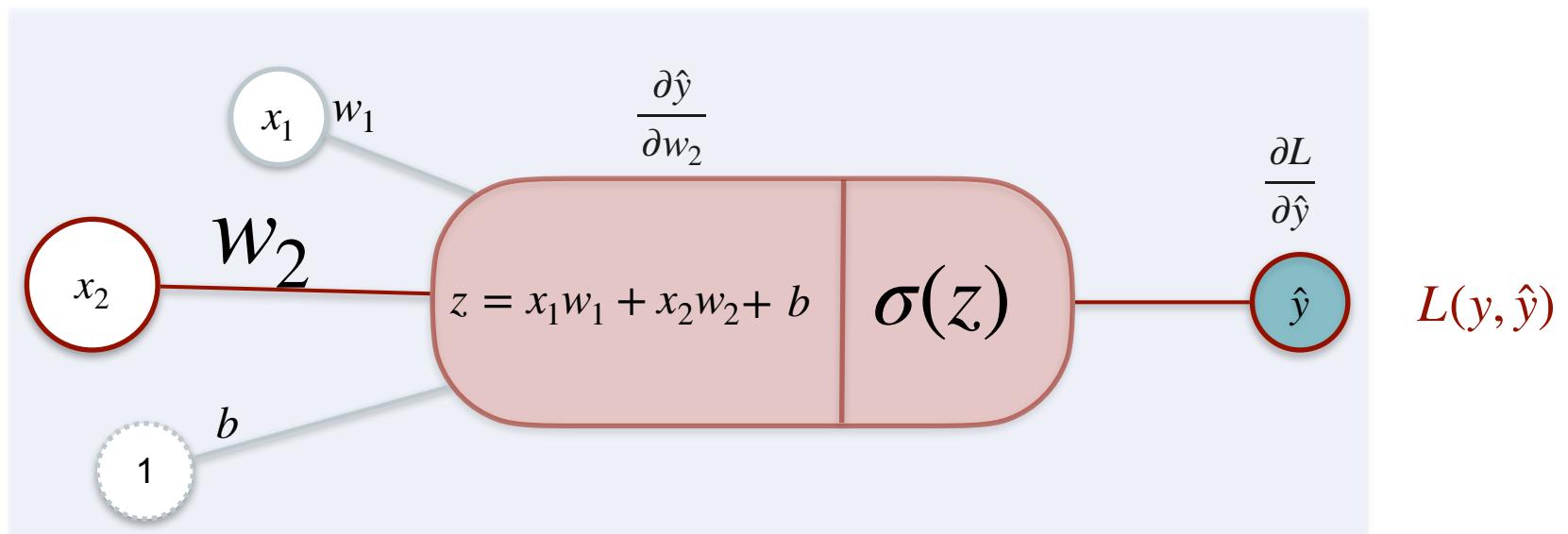
Classification With a Perceptron



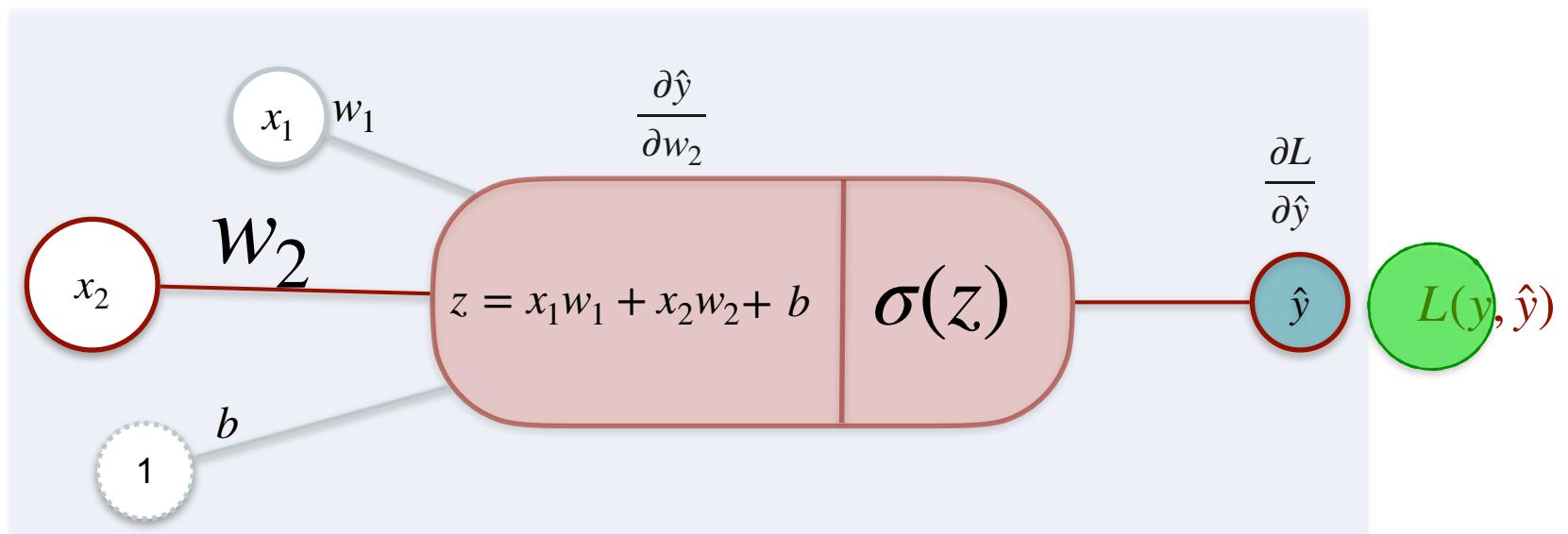
Classification With a Perceptron



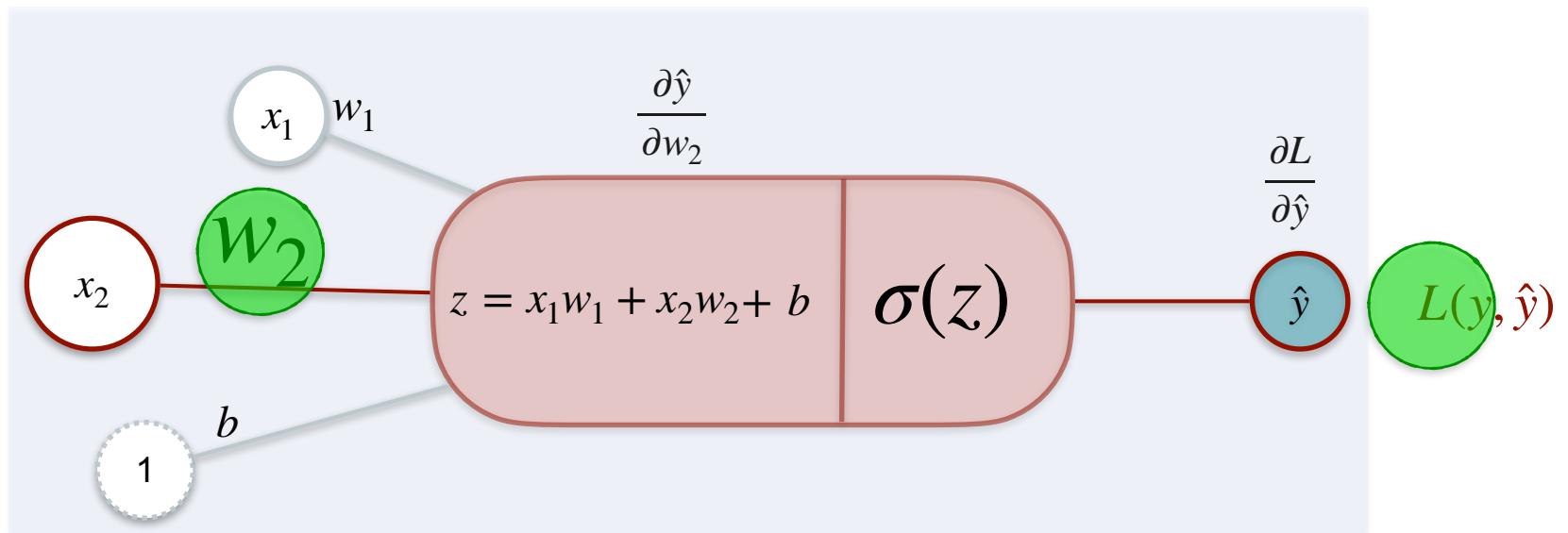
Classification With a Perceptron



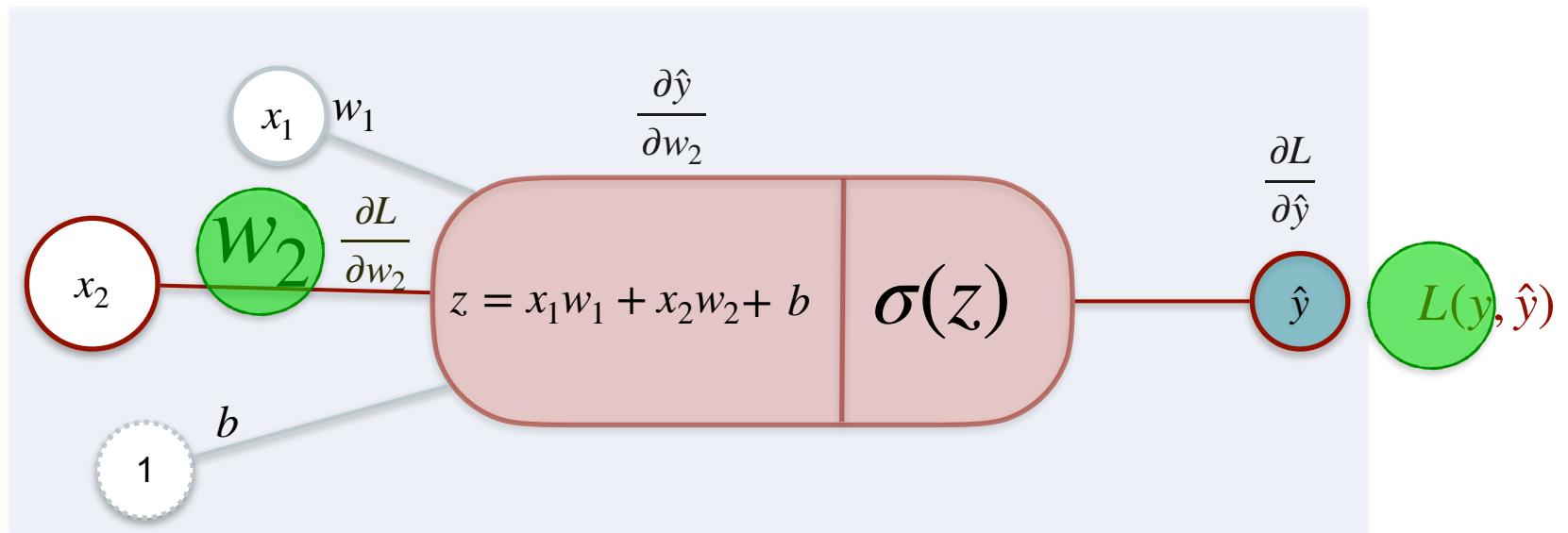
Classification With a Perceptron



Classification With a Perceptron

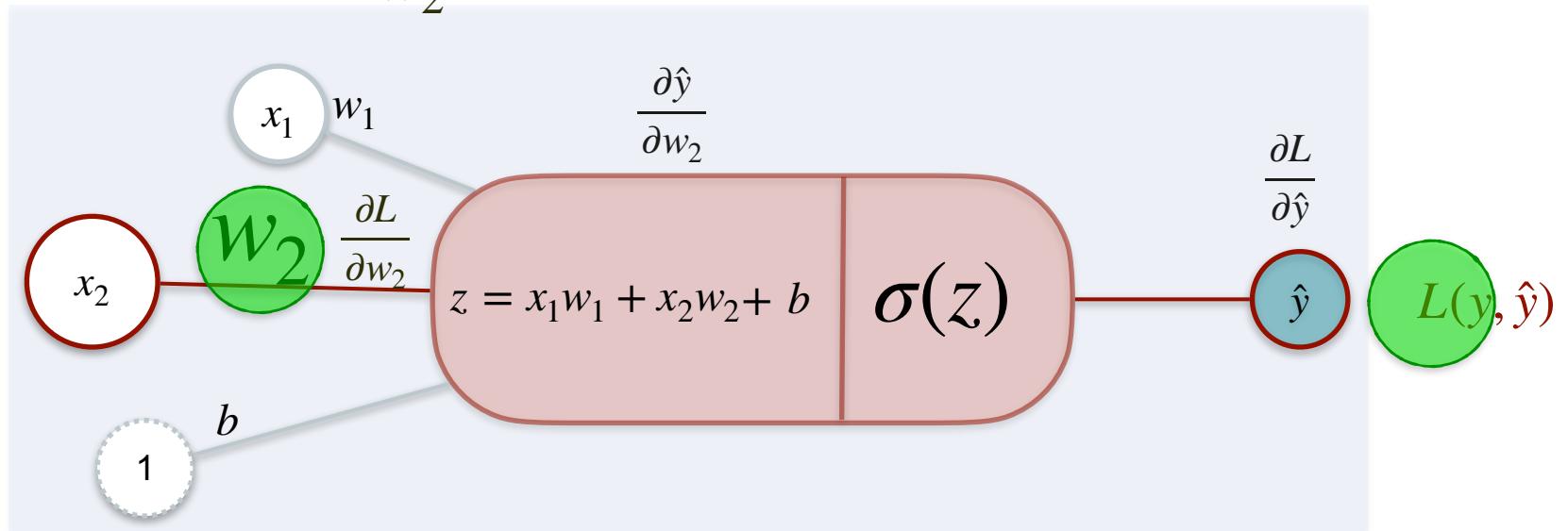


Classification With a Perceptron

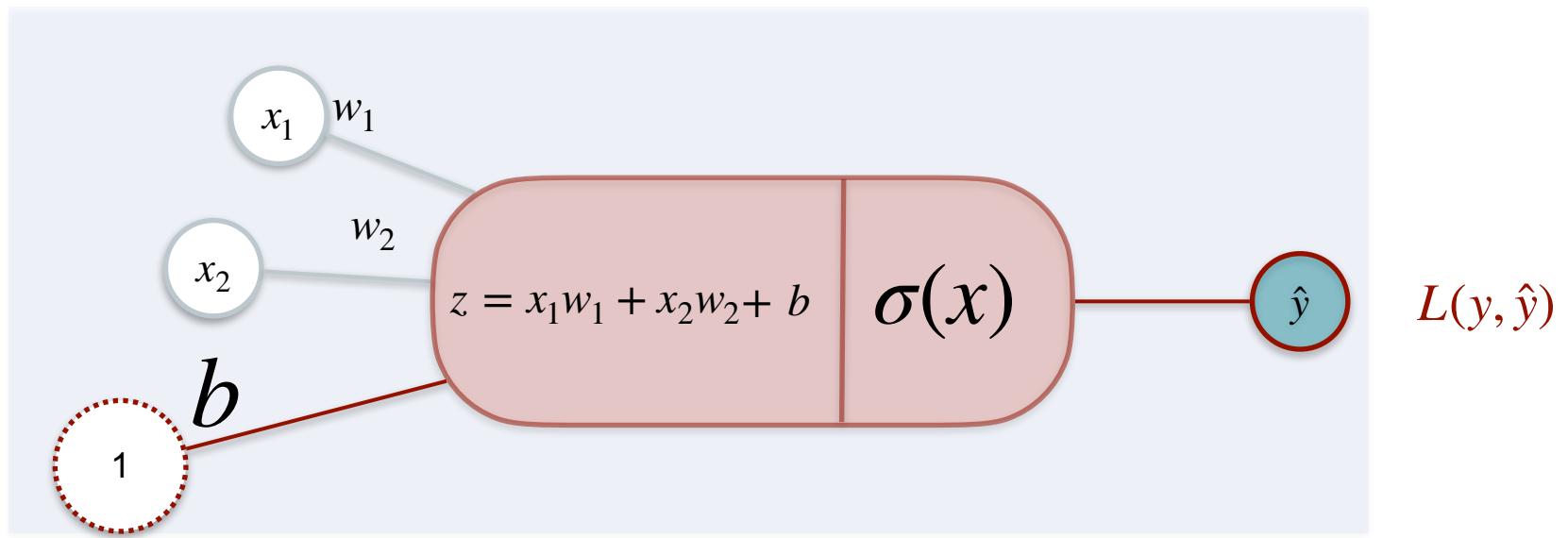


Classification With a Perceptron

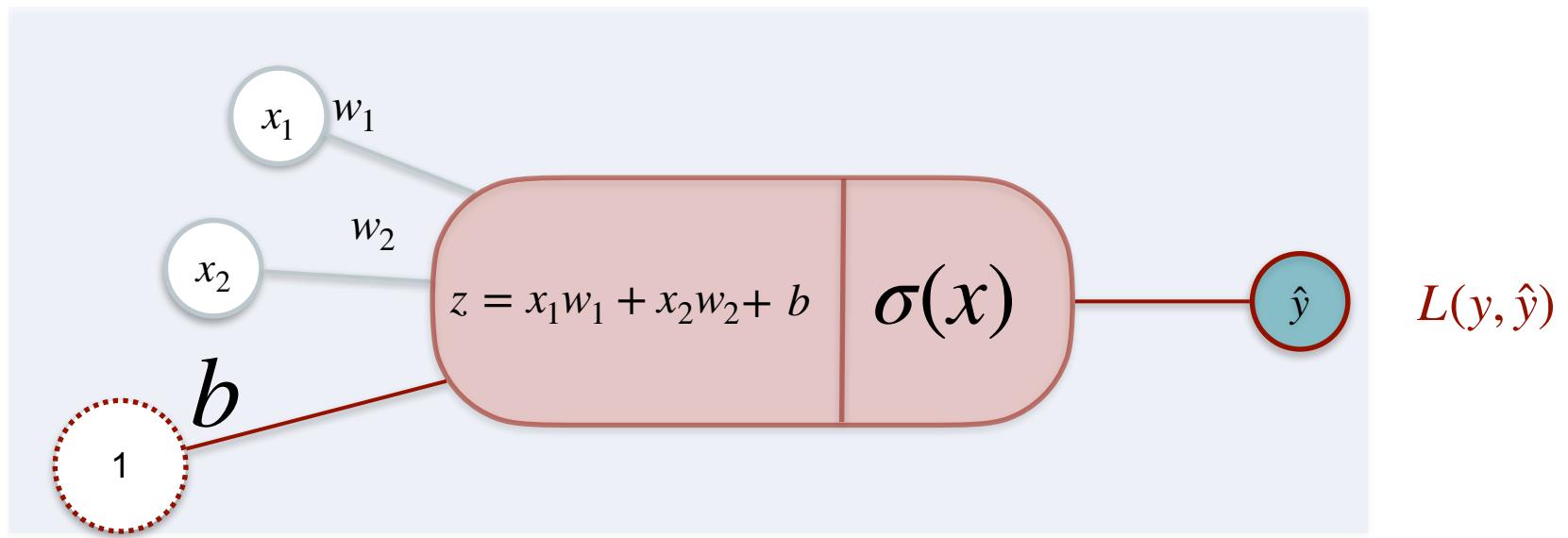
$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$



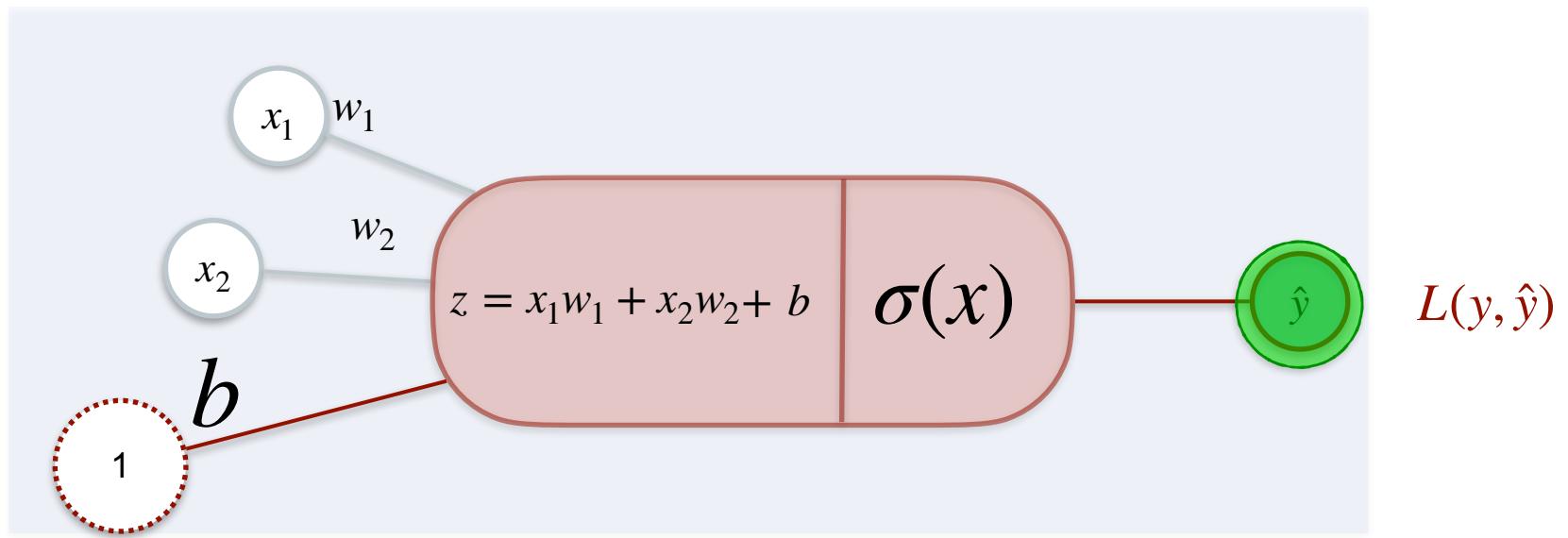
Classification With a Perceptron



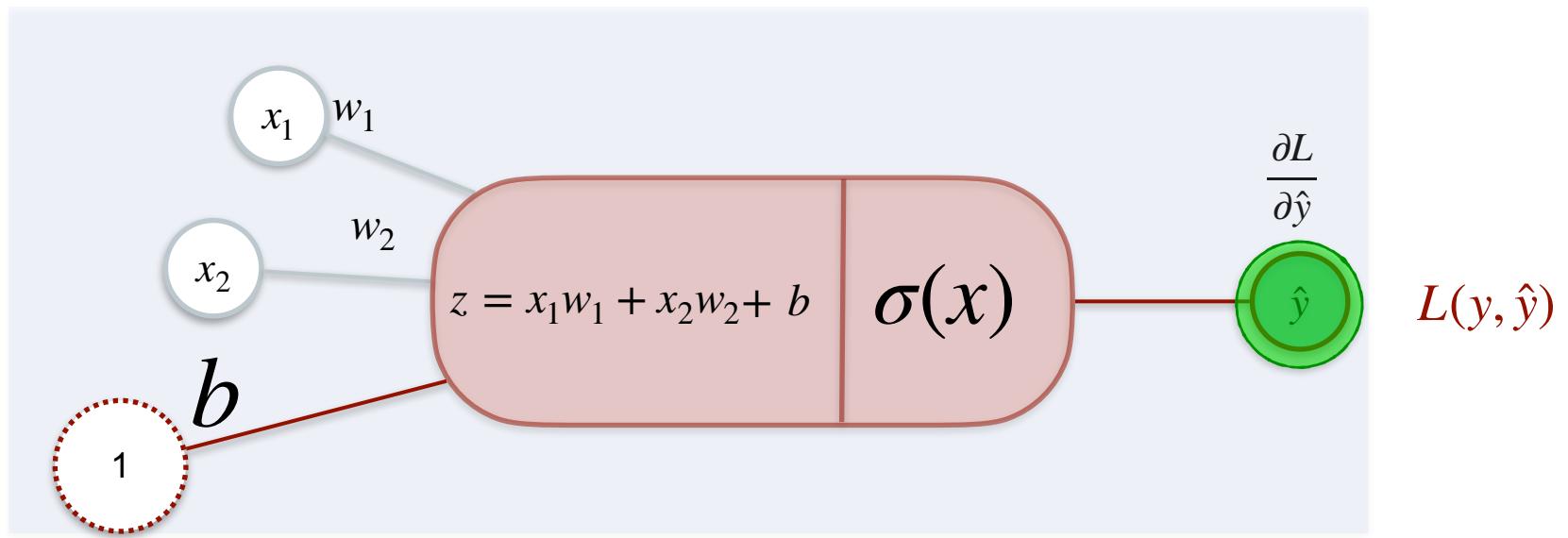
Classification With a Perceptron



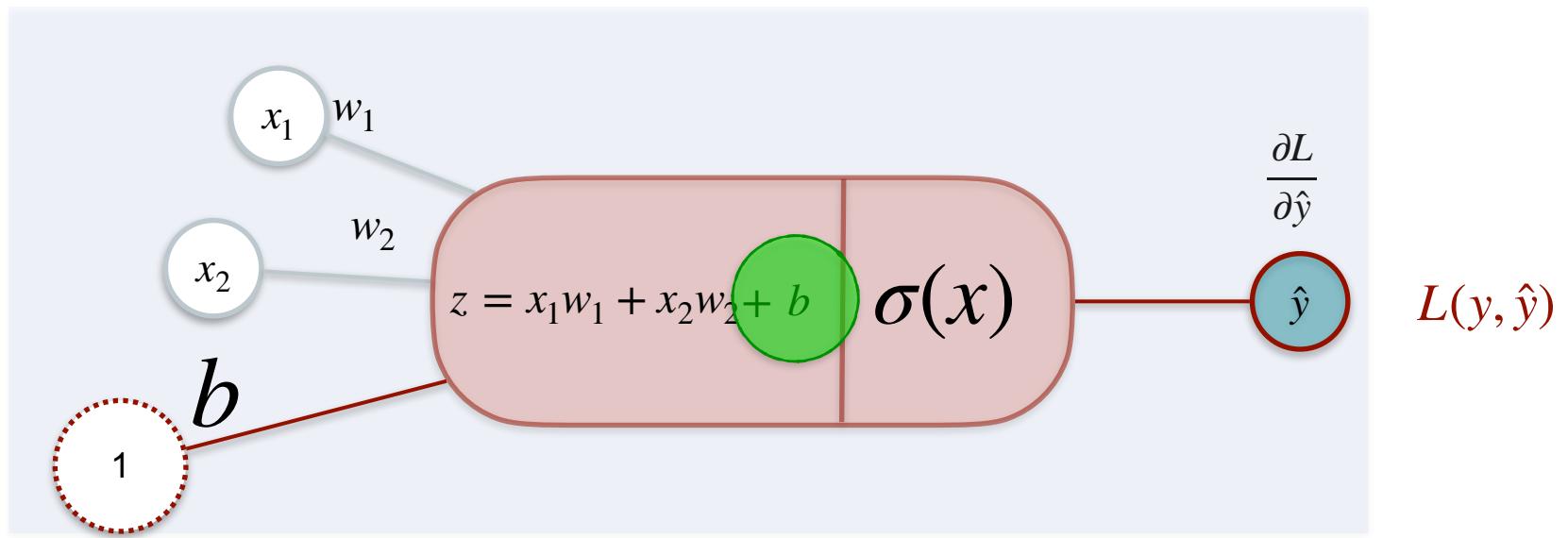
Classification With a Perceptron



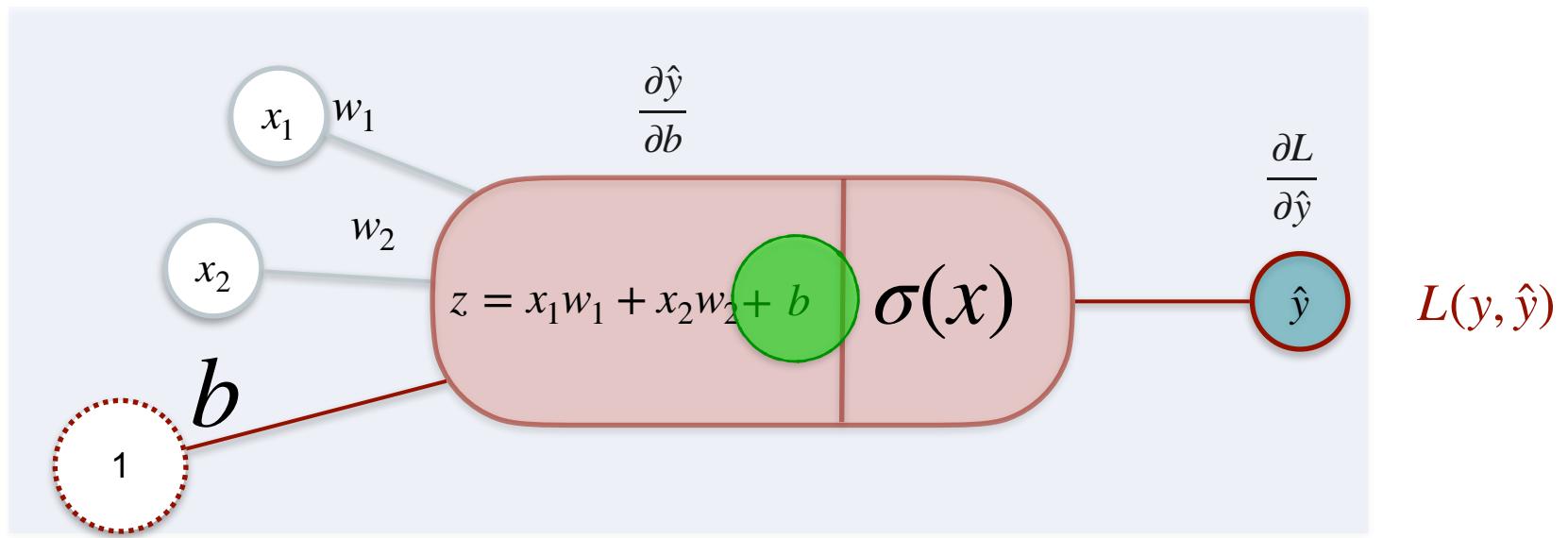
Classification With a Perceptron



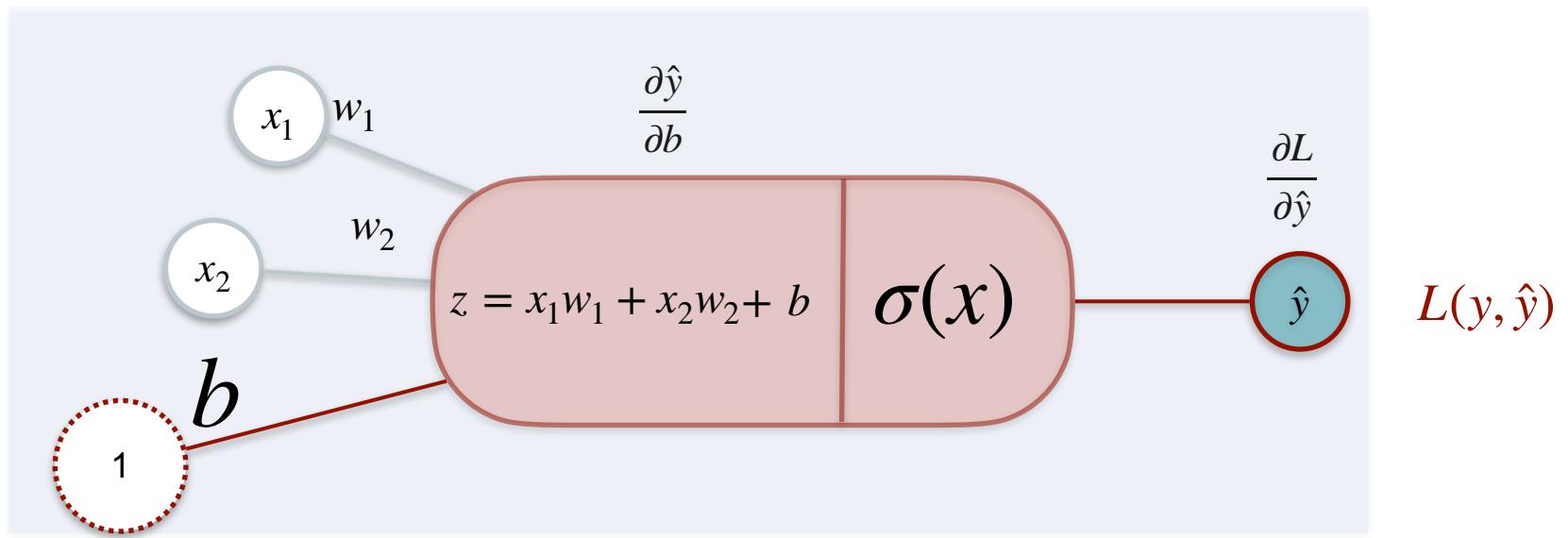
Classification With a Perceptron



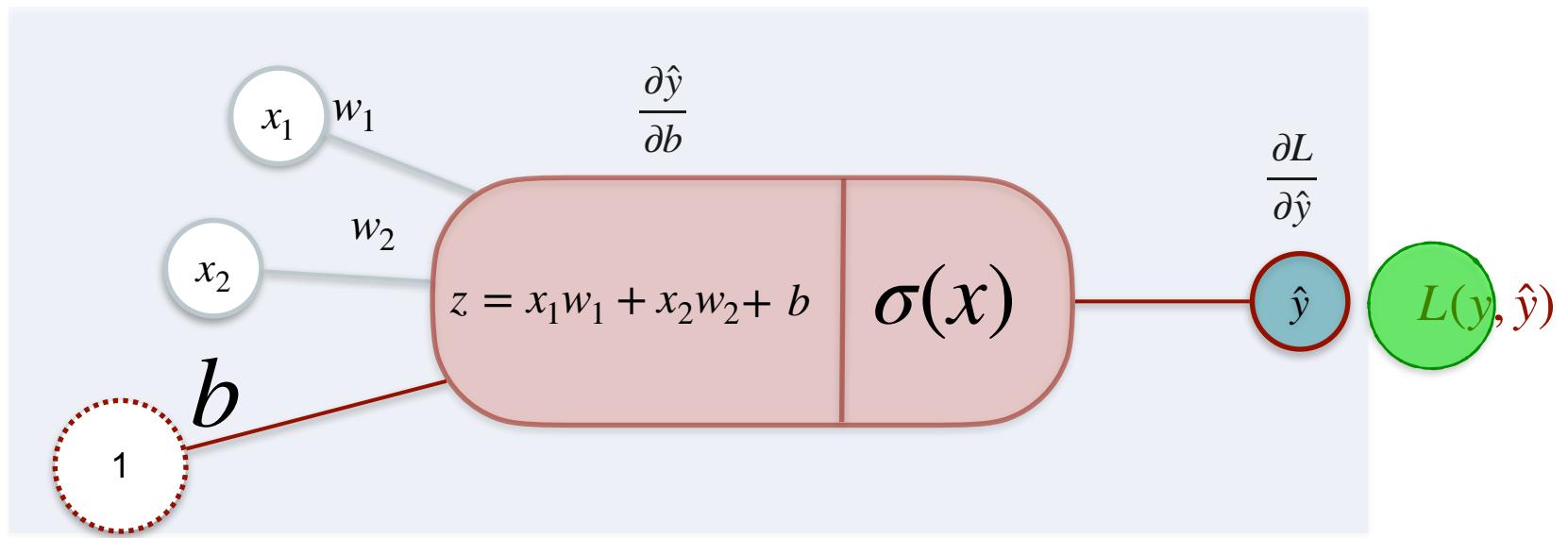
Classification With a Perceptron



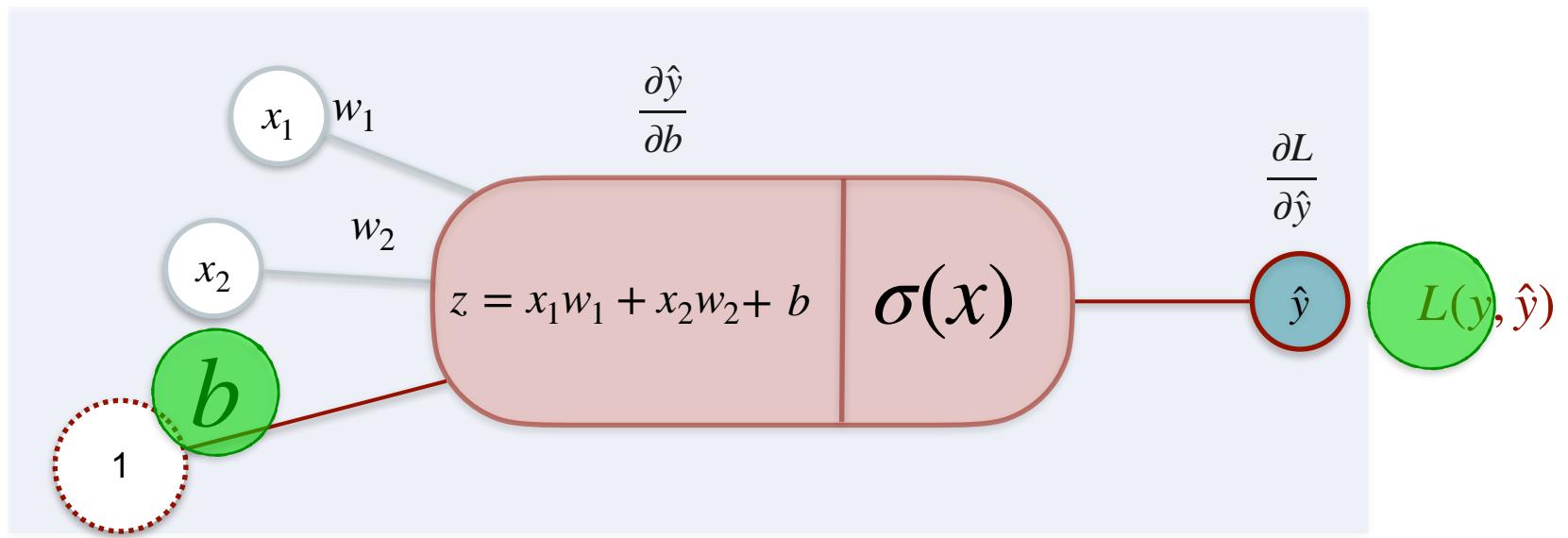
Classification With a Perceptron



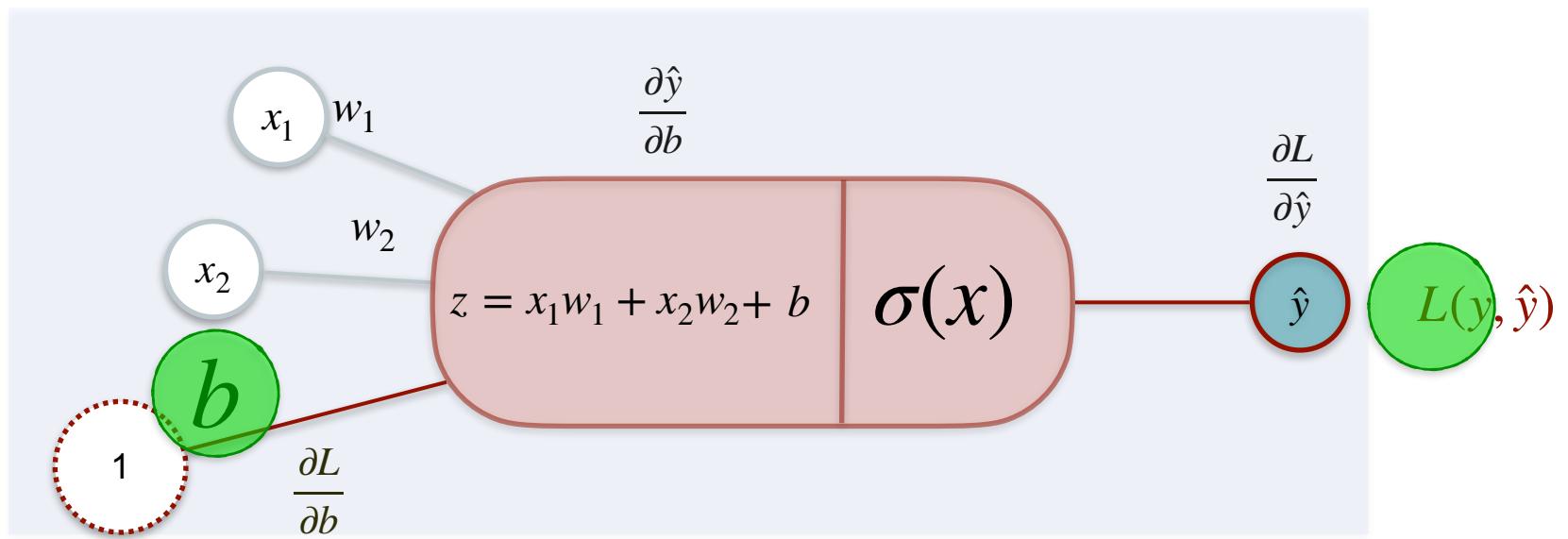
Classification With a Perceptron



Classification With a Perceptron

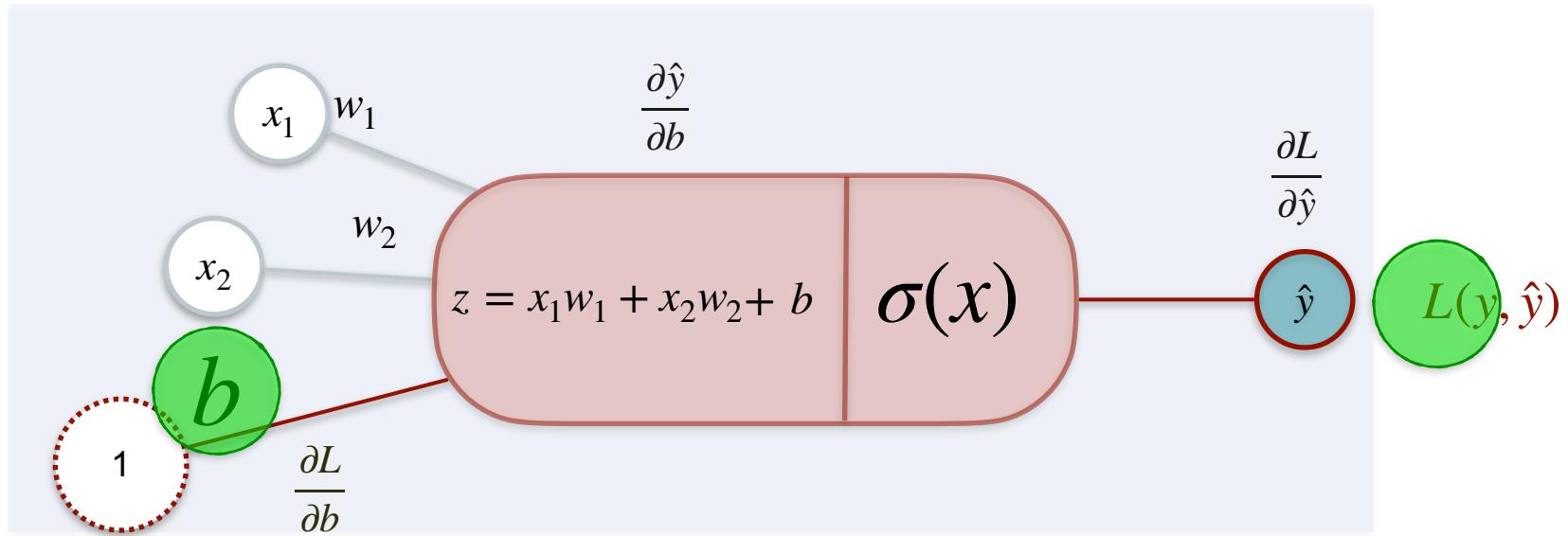


Classification With a Perceptron



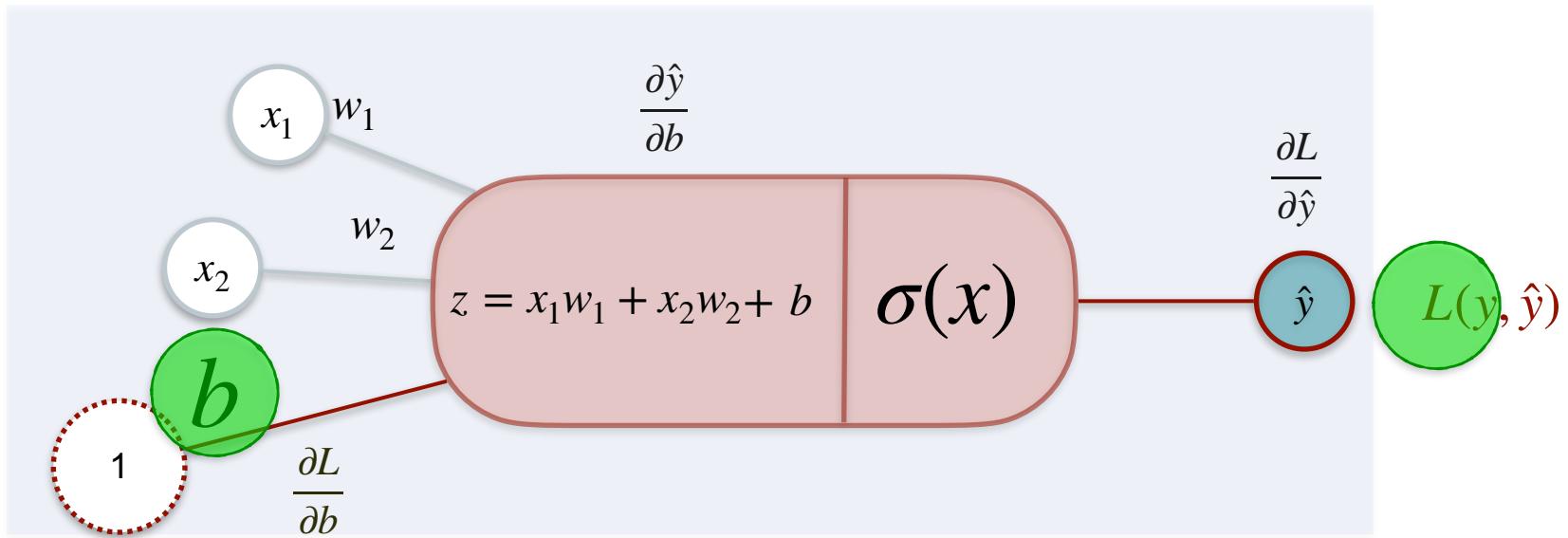
Classification With a Perceptron

$$\frac{\partial L}{\partial b} =$$



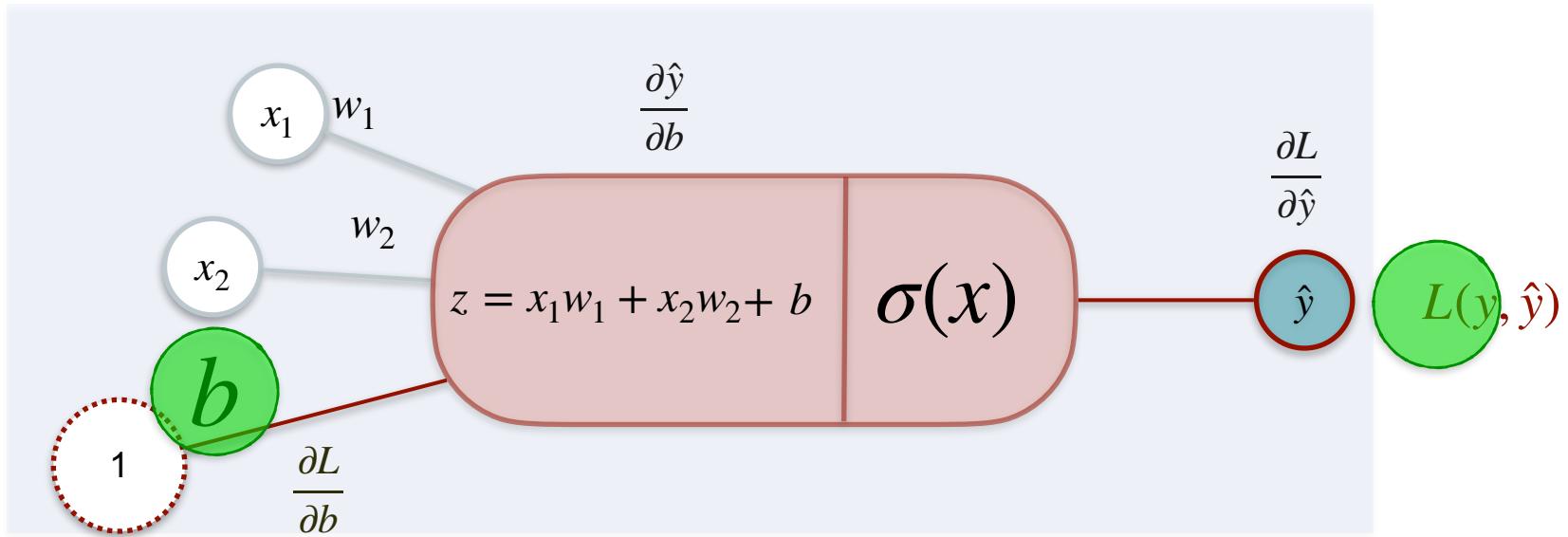
Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}}$$



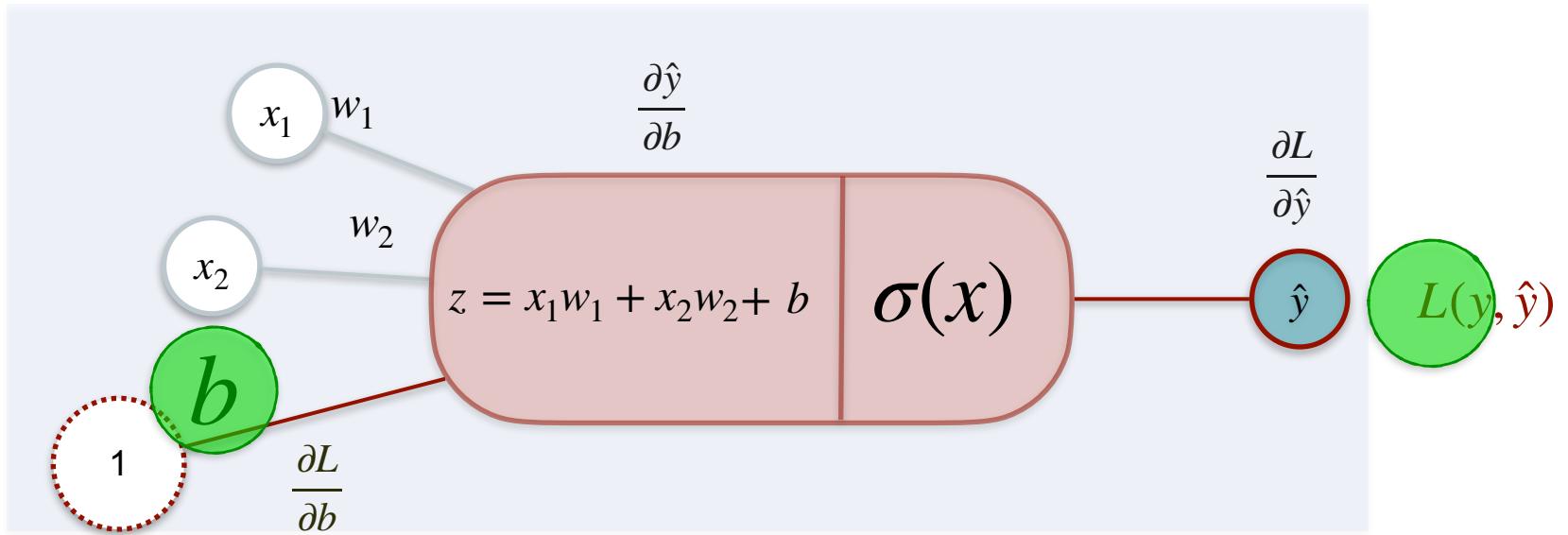
Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot$$



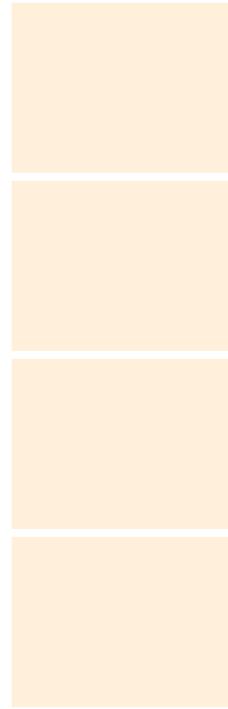
Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$



Classification With a Perceptron

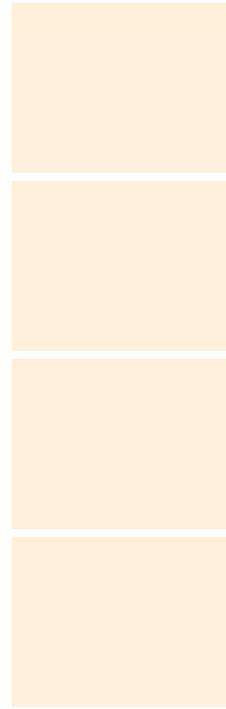
$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$



Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

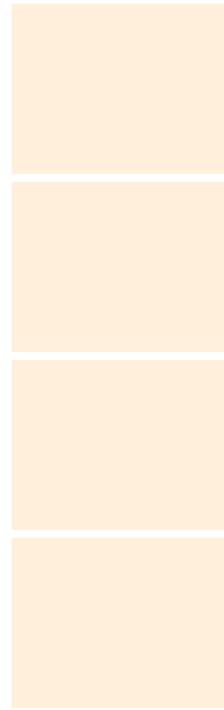


Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$



Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$

$$\frac{\partial L}{\partial \hat{y}} =$$

$$\frac{\partial \hat{y}}{\partial b} =$$

$$\frac{\partial \hat{y}}{\partial w_1} =$$

$$\frac{\partial \hat{y}}{\partial w_2} =$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$

$$\frac{\partial L}{\partial \hat{y}} =$$

$$\frac{\partial \hat{y}}{\partial b} =$$

?

$$\frac{\partial \hat{y}}{\partial w_1} =$$

$$\frac{\partial \hat{y}}{\partial w_2} =$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$

$$\frac{\partial L}{\partial \hat{y}} =$$

$$\frac{\partial \hat{y}}{\partial b} =$$

$$\frac{\partial \hat{y}}{\partial w_1} =$$

$$\frac{\partial \hat{y}}{\partial w_2} =$$

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$

$$\frac{\partial L}{\partial \hat{y}} =$$

$$\frac{\partial \hat{y}}{\partial b} =$$

$$\frac{\partial \hat{y}}{\partial w_1} =$$

$$\frac{\partial \hat{y}}{\partial w_2} =$$

$$\hat{y} = \sigma(w_1x_1 + w_2x_2 + b)$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}}$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}}$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{-y}{\hat{y}}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

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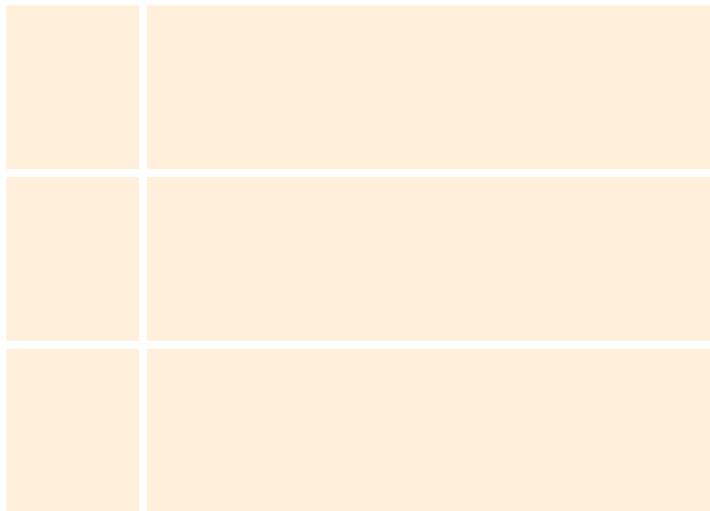
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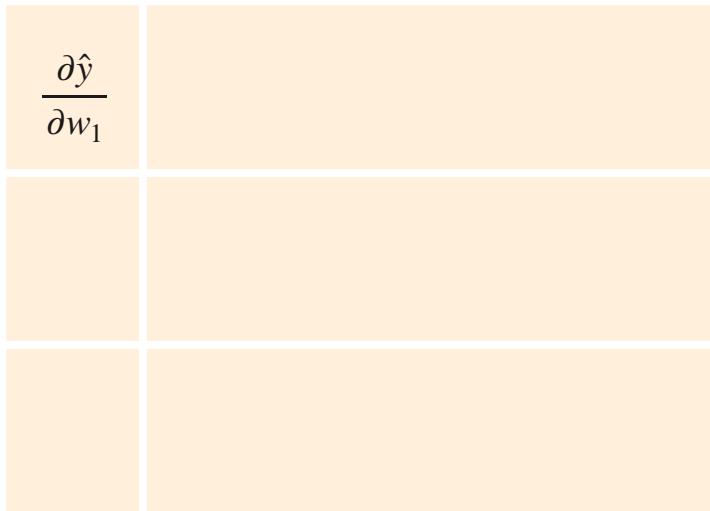
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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1 - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1 - \hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1 - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1 - \hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1 - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1 - \hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

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$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})} \hat{y}(1 - \hat{y})x_2$$

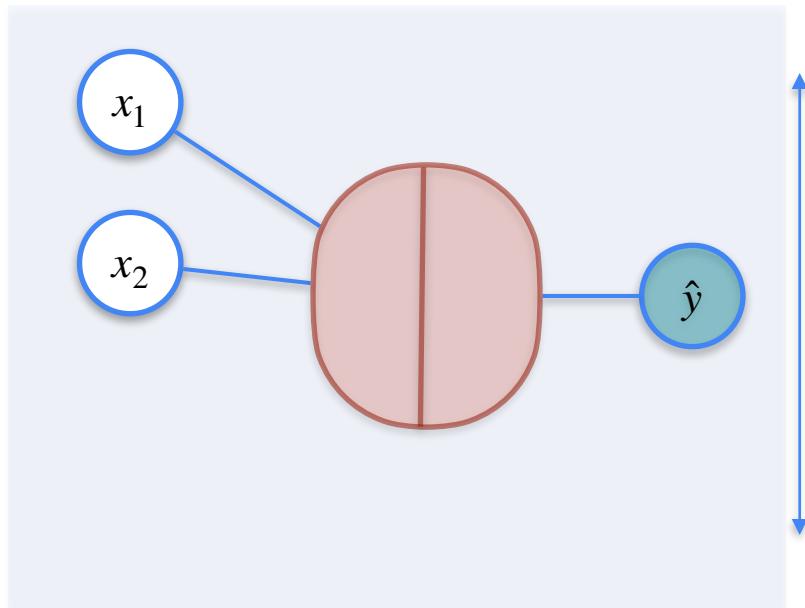
Classification With a Perceptron

$$\frac{\partial L}{\partial b} = -(y - \hat{y})$$

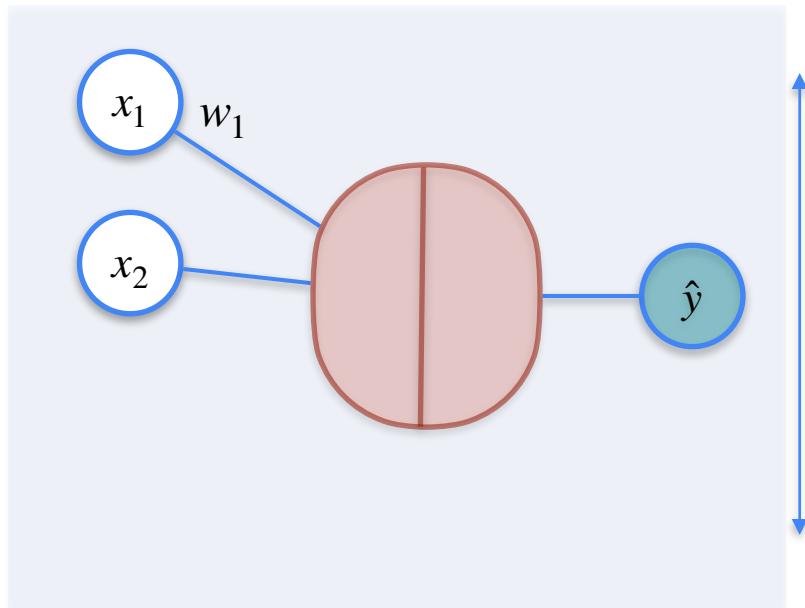
$$\frac{\partial L}{\partial w_1} = -(y - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = -(y - \hat{y})x_2$$

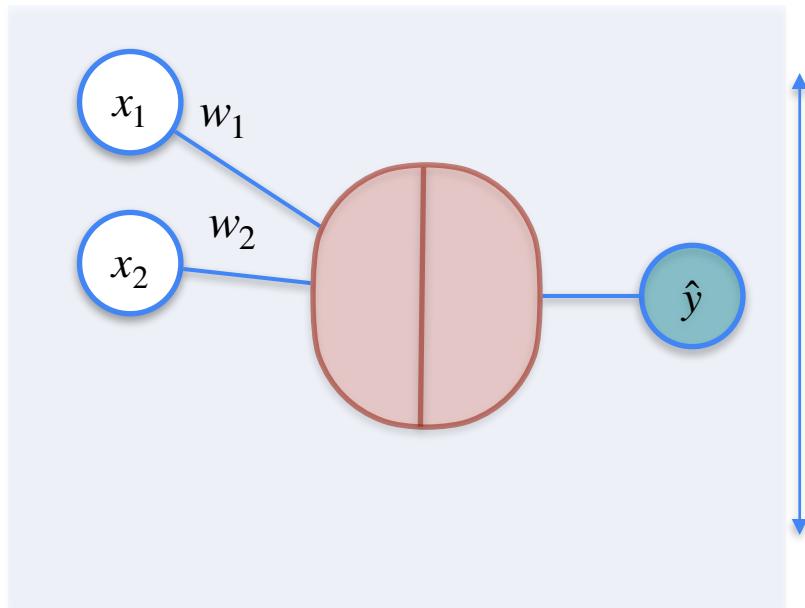
Classification With a Perceptron



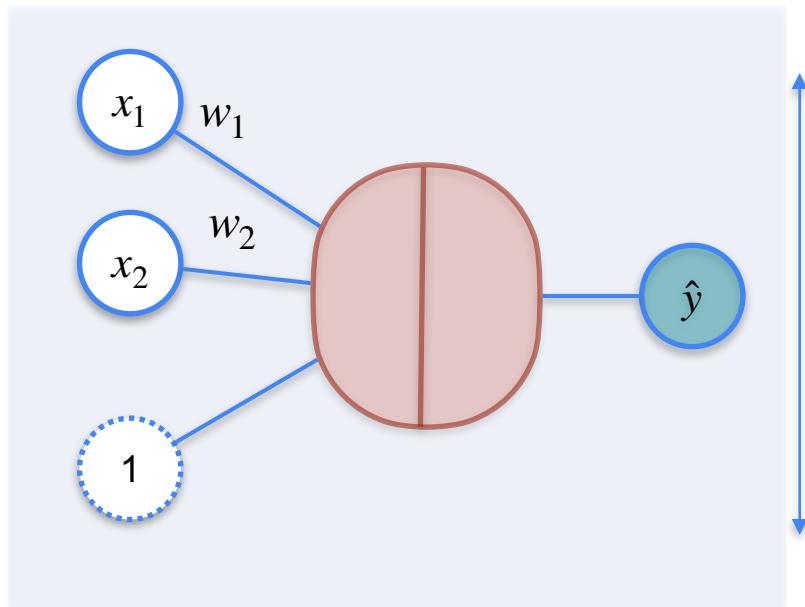
Classification With a Perceptron



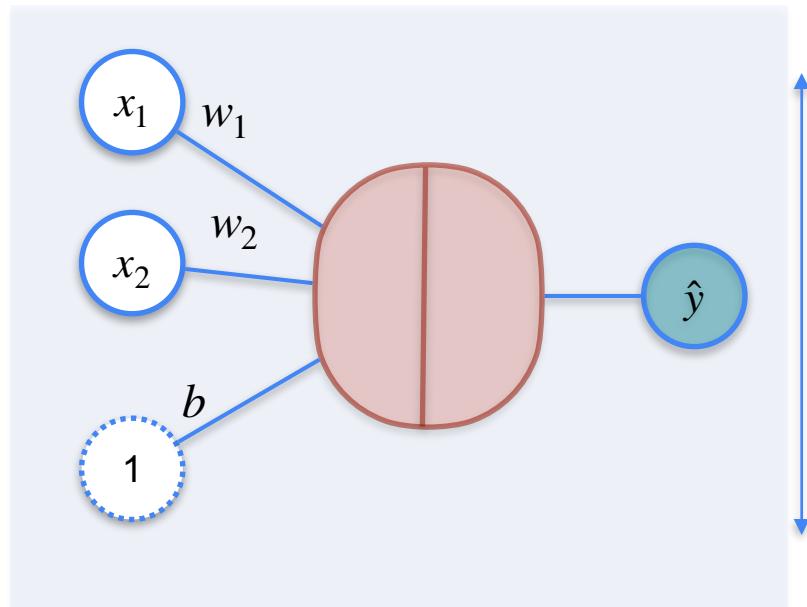
Classification With a Perceptron



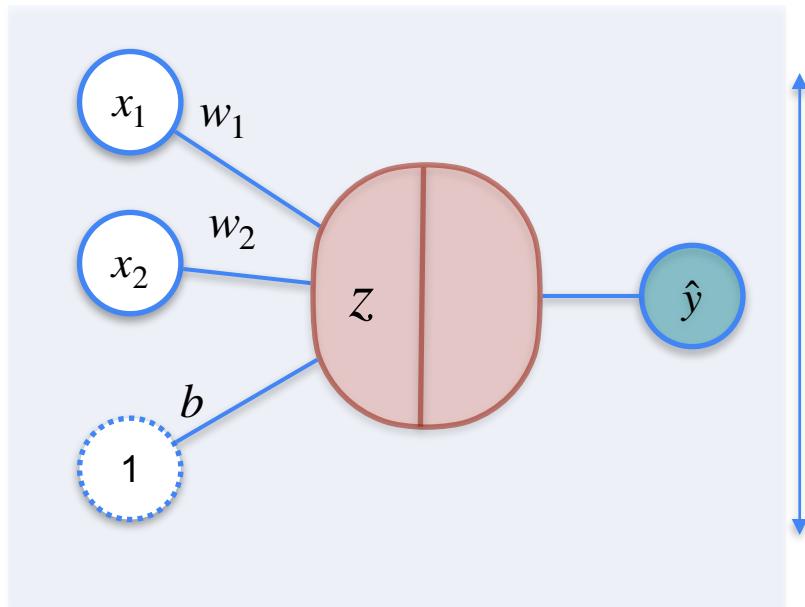
Classification With a Perceptron



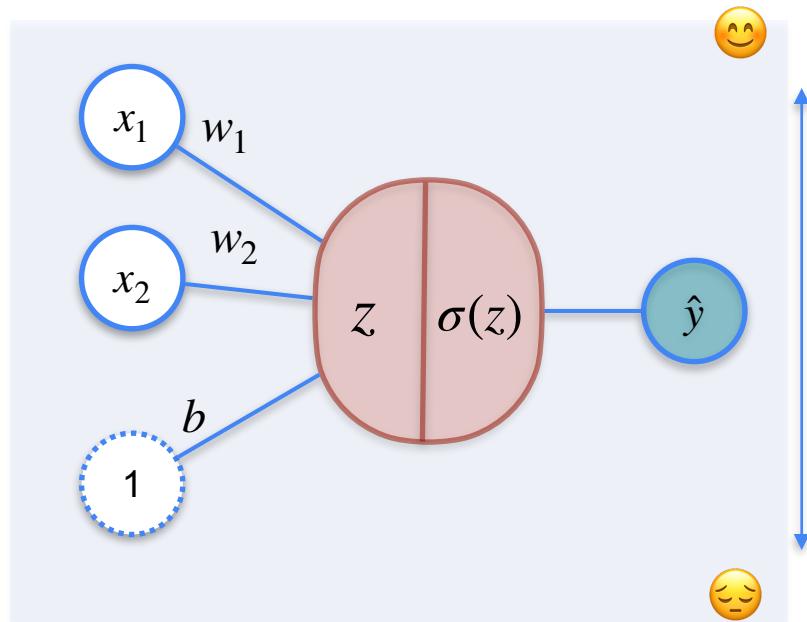
Classification With a Perceptron



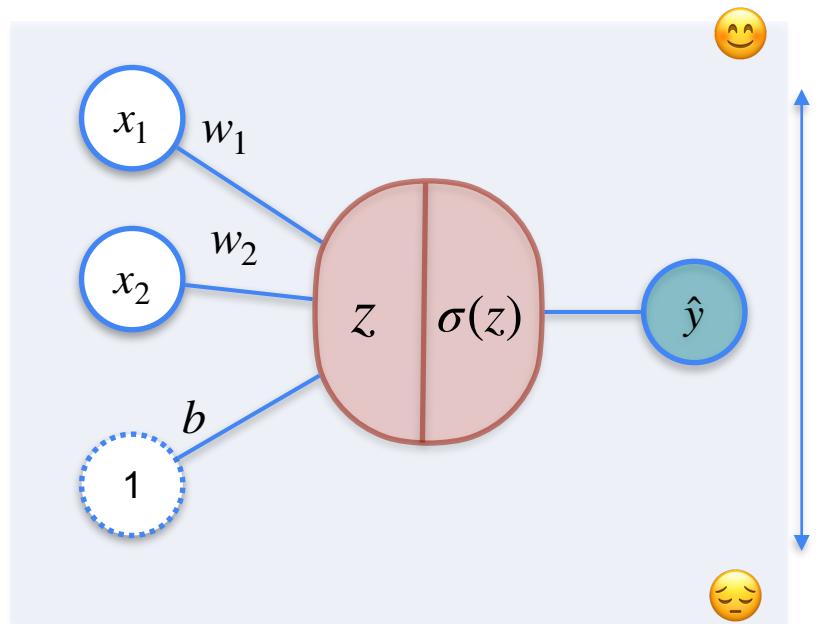
Classification With a Perceptron



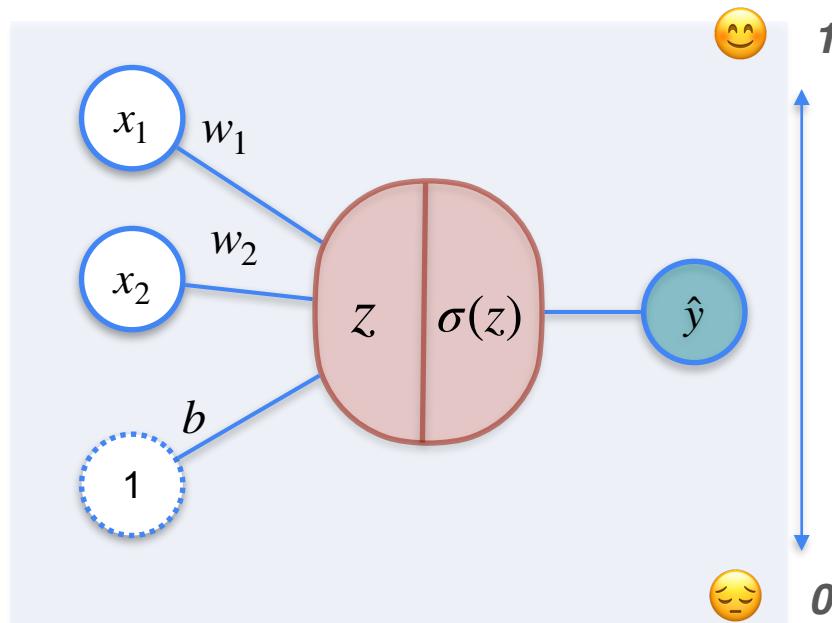
Classification With a Perceptron



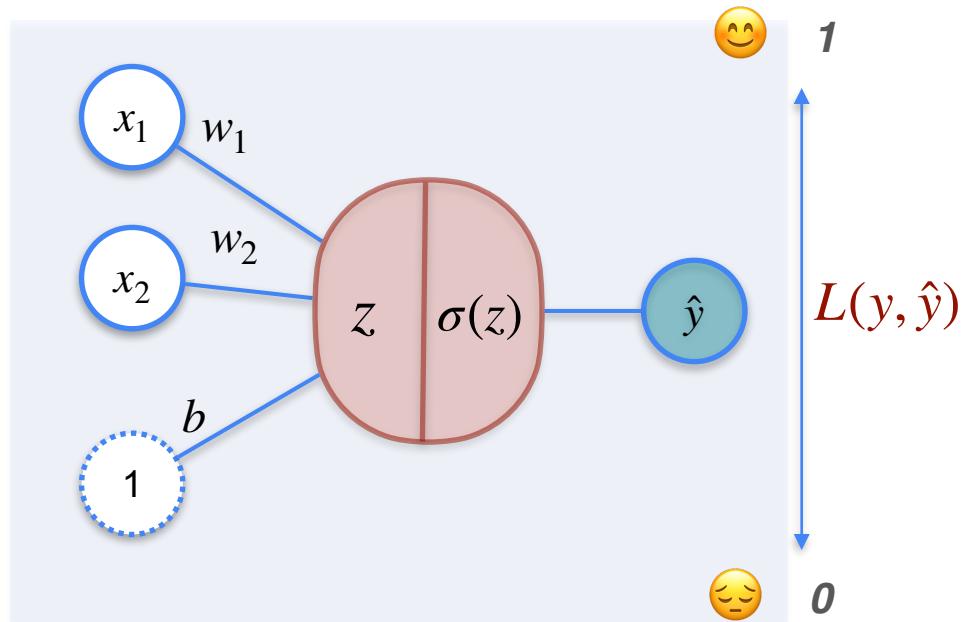
Classification With a Perceptron



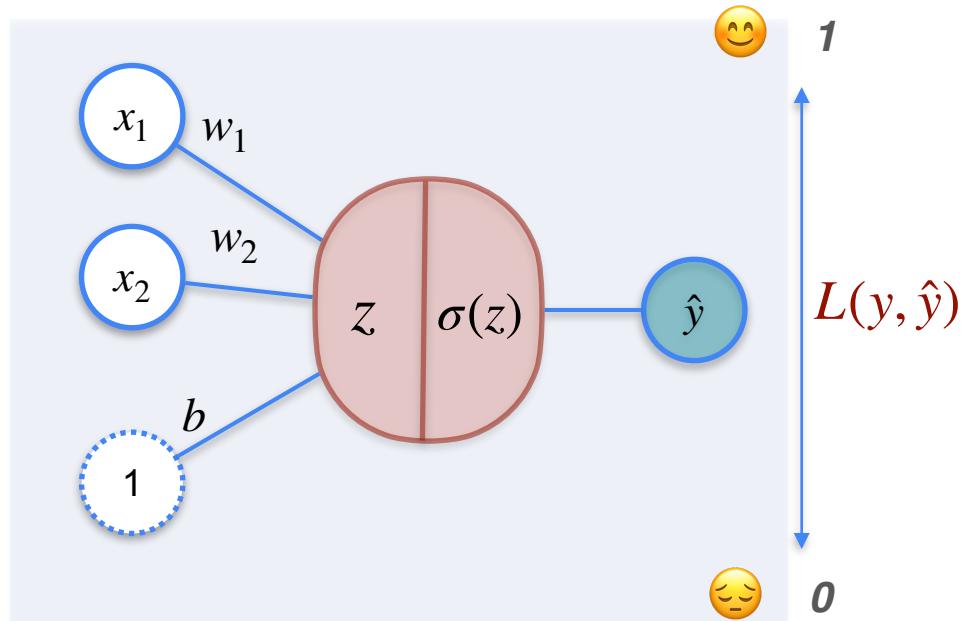
Classification With a Perceptron



Classification With a Perceptron

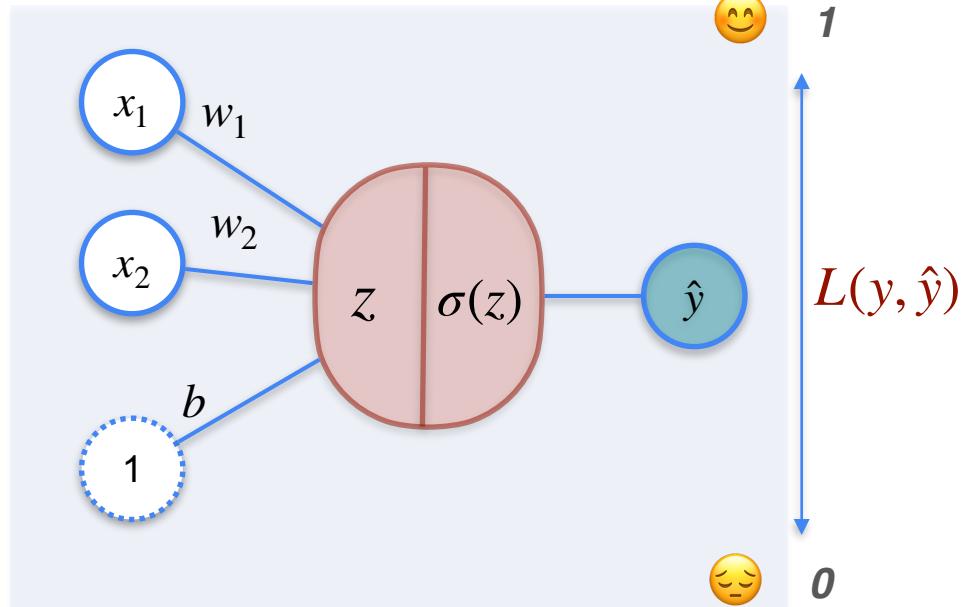


Classification With a Perceptron



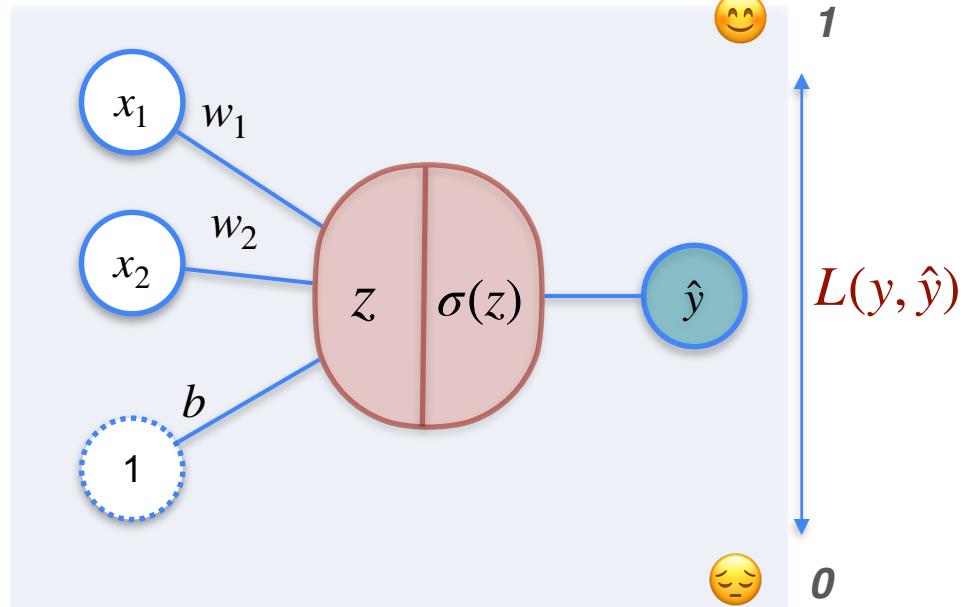
To find optimal values for:

Classification With a Perceptron



To find optimal values for:
 w_1 , w_2 , b

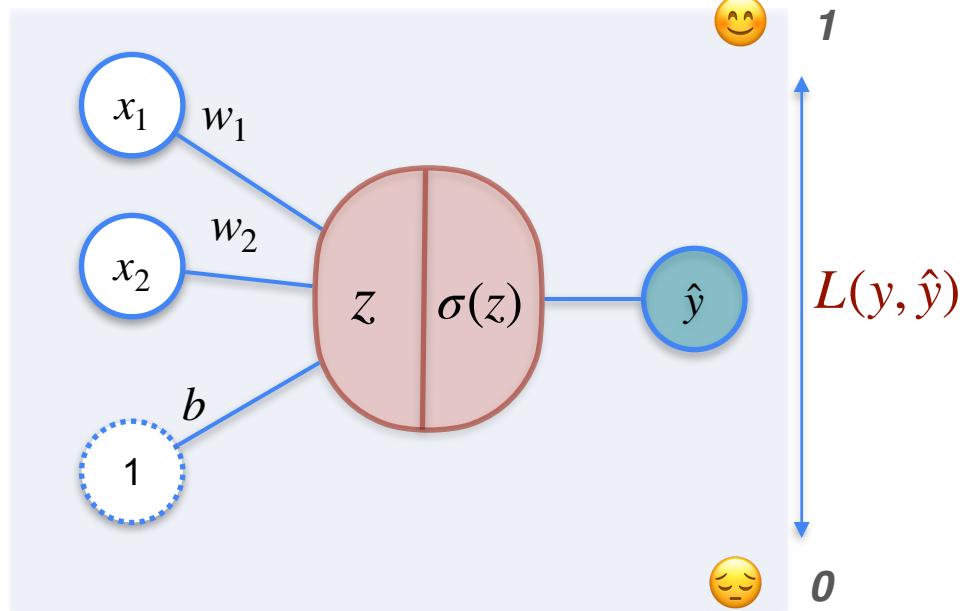
Classification With a Perceptron



To find optimal values for:
 w_1 , w_2 , b

You need gradient descent

Classification With a Perceptron

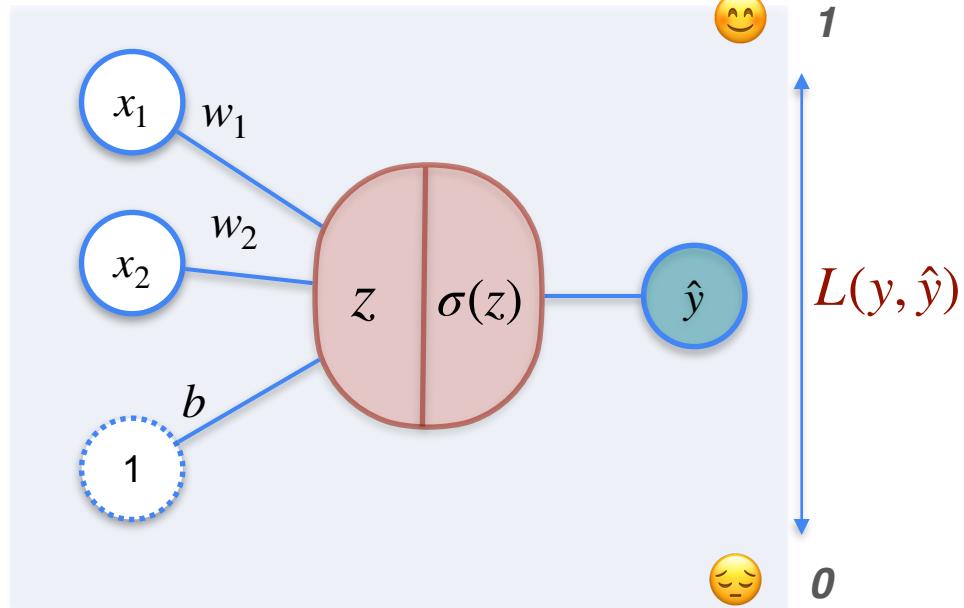


To find optimal values for:
 w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Classification With a Perceptron

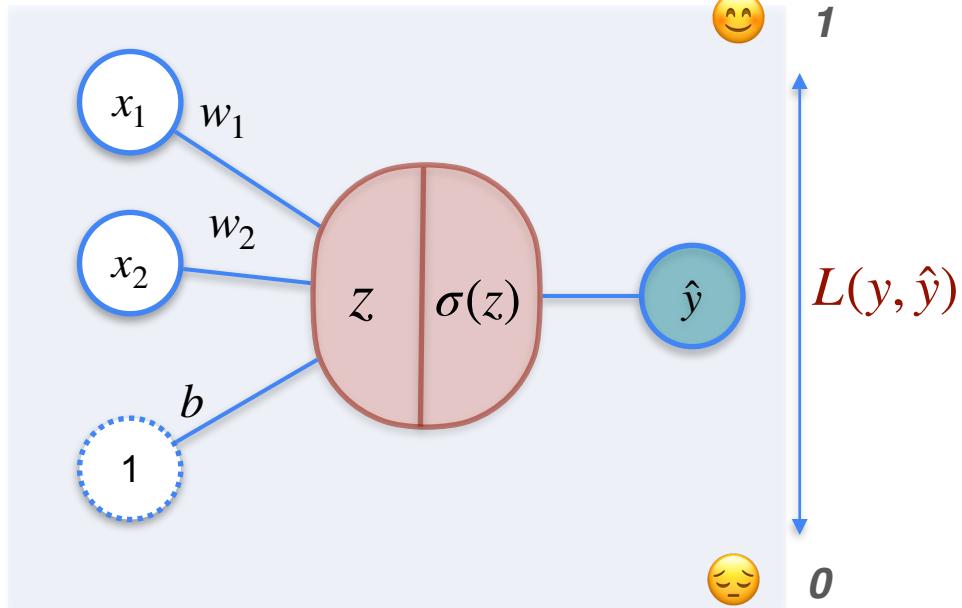


To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha$$

Classification With a Perceptron

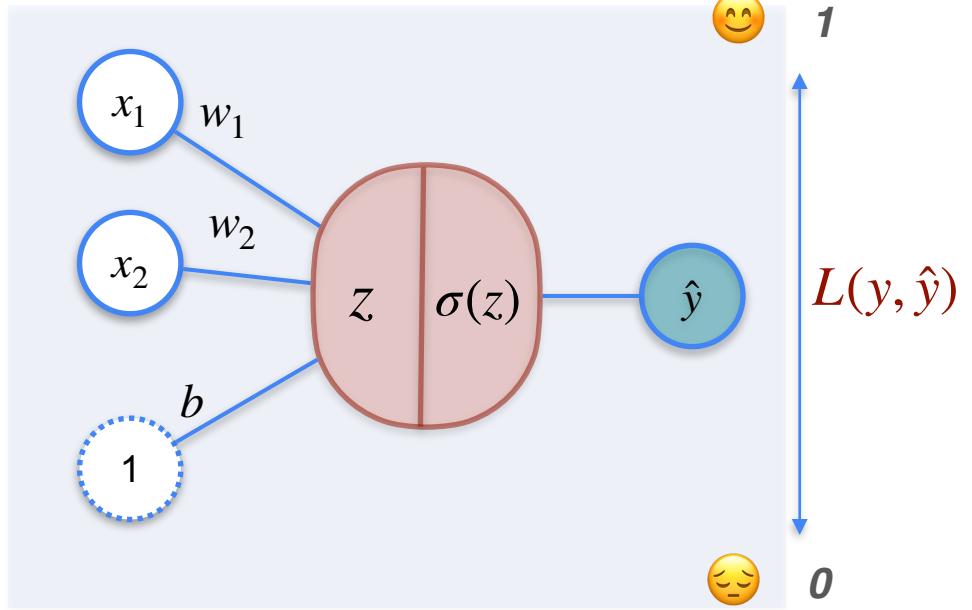


To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

Classification With a Perceptron



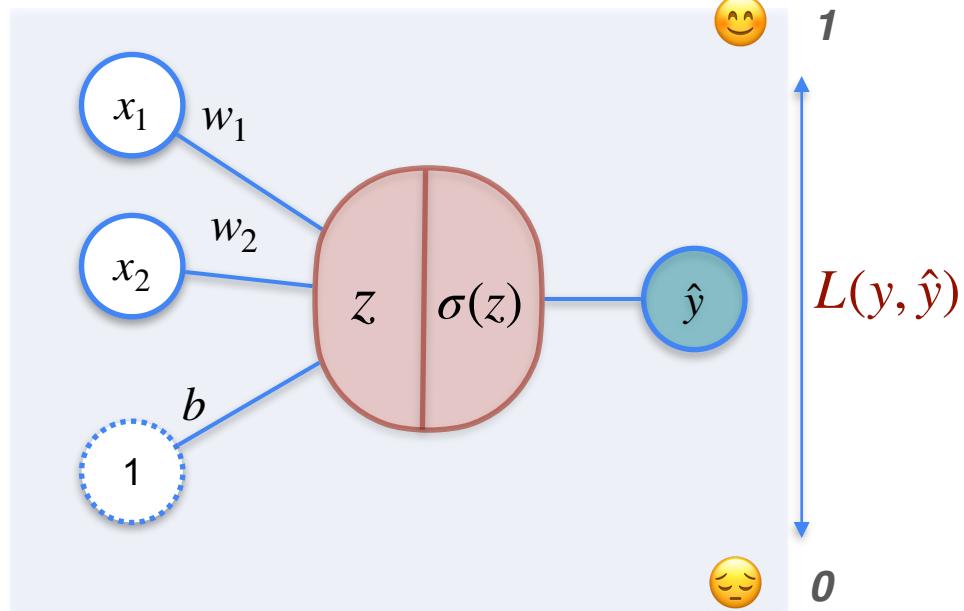
To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Classification With a Perceptron



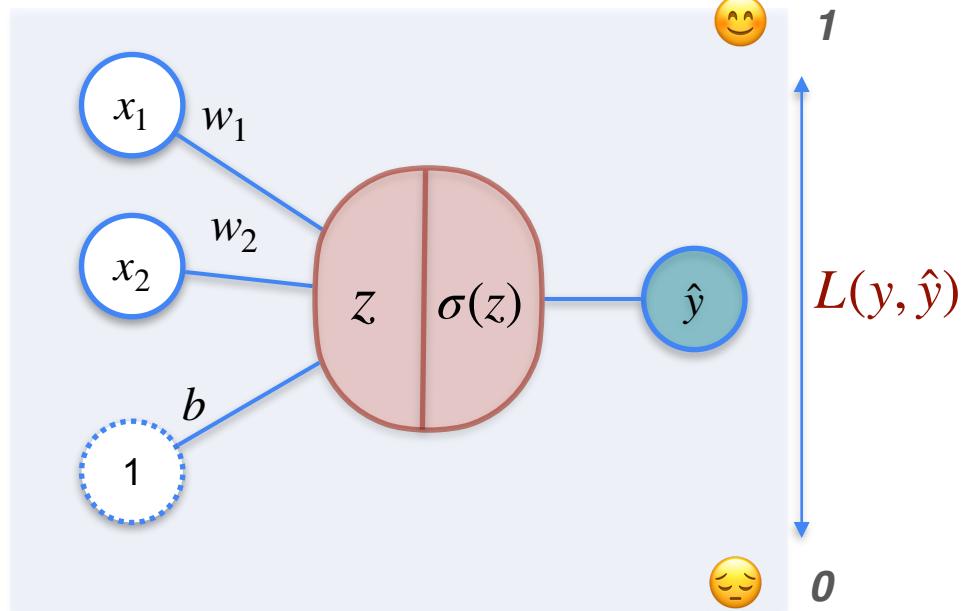
To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha$$

Classification With a Perceptron



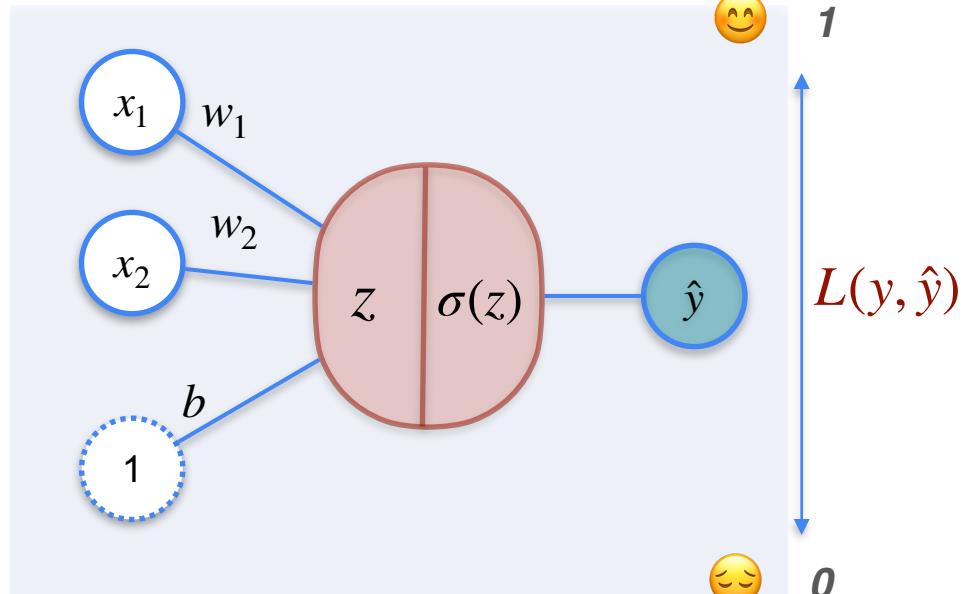
To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

Classification With a Perceptron



To find optimal values for:
 w_1, w_2, b

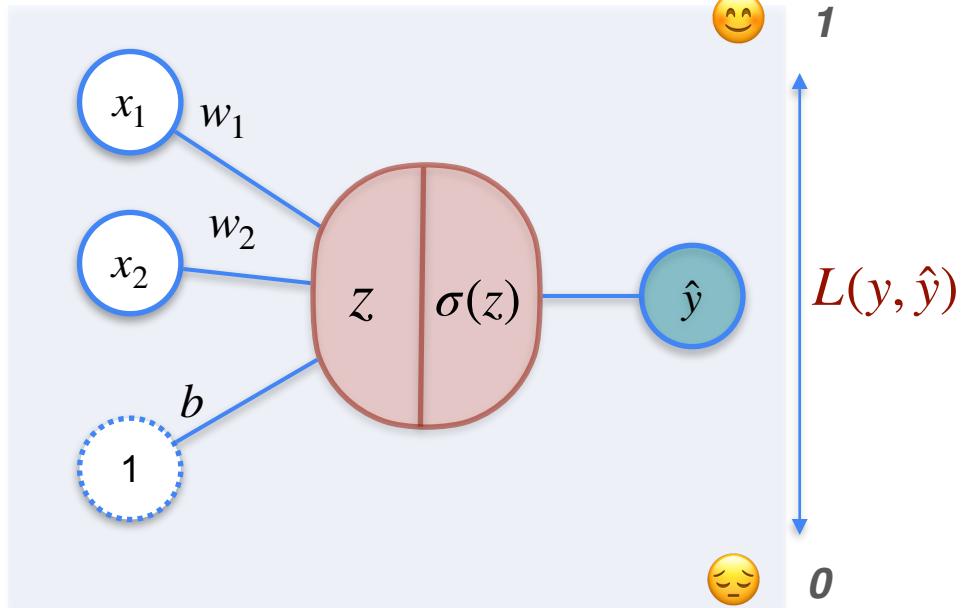
You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Classification With a Perceptron



To find optimal values for:
 w_1 , w_2 , b

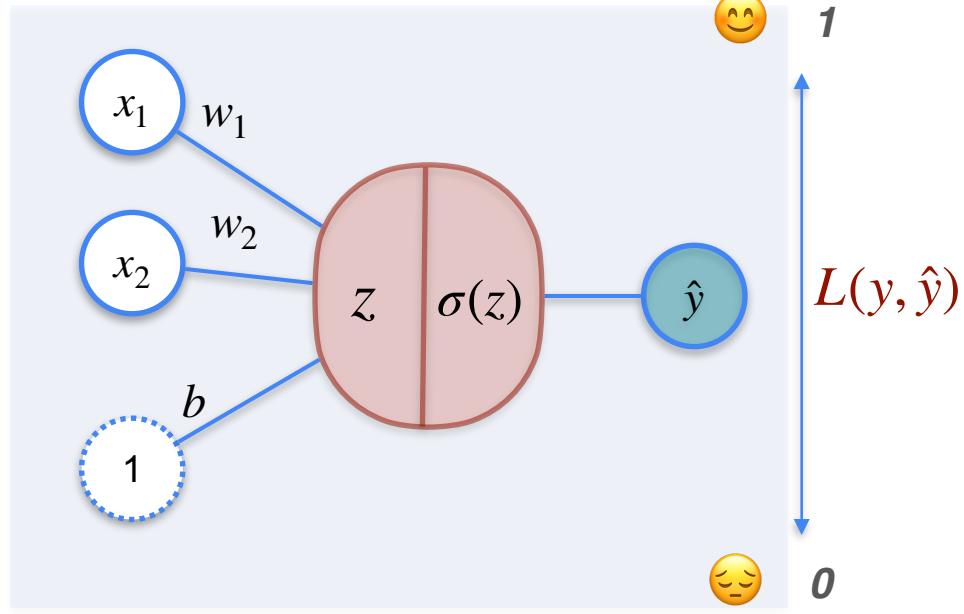
You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha$$

Classification With a Perceptron



To find optimal values for:
 w_1, w_2, b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha(-(y - \hat{y}))$$



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a Neural Network

Classification Problem Motivation

Classification Problem Motivation

<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	<i>Happy</i> 😊

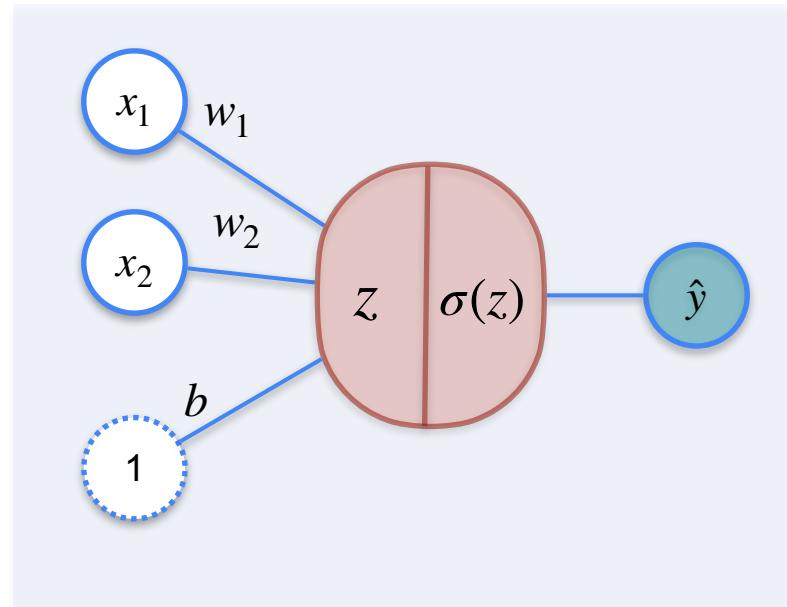
Beep beep! 0 2 *Sad* 😞

<i>Aack beep beep beep!</i>	1	3	<i>Sad</i> 😞
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Aack beep aack! 2 1 *Happy* 😊

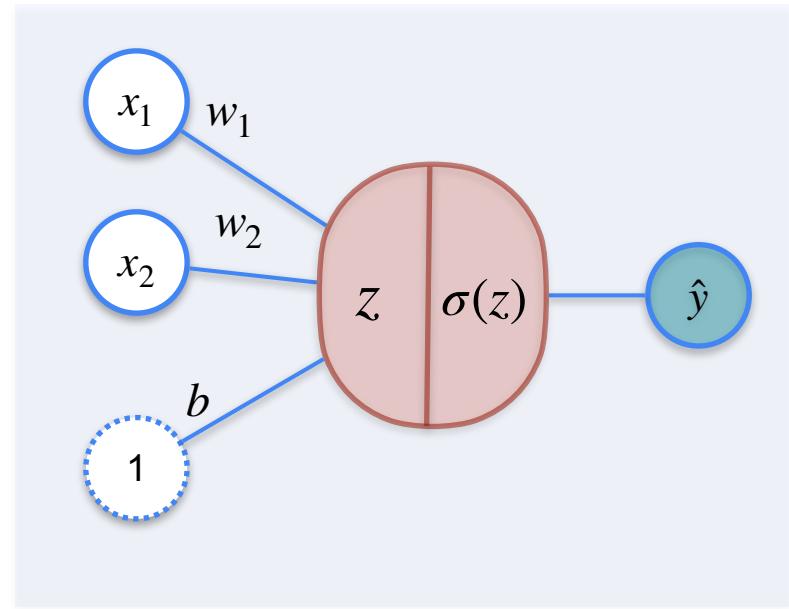
Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😞
Aack beep beep beep!	1	3	Sad 😞
Aack beep aack!	2	1	Happy 😊



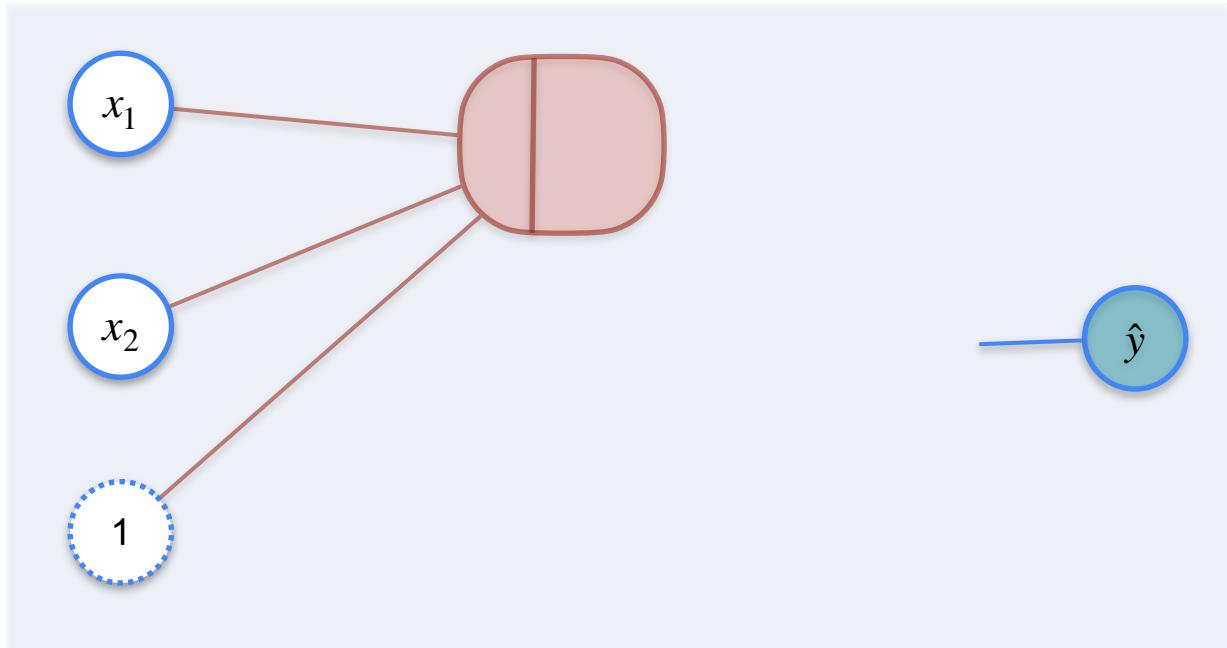
Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😞
Aack beep beep beep!	1	3	Sad 😞
Aack beep aack!	2	1	Happy 😊

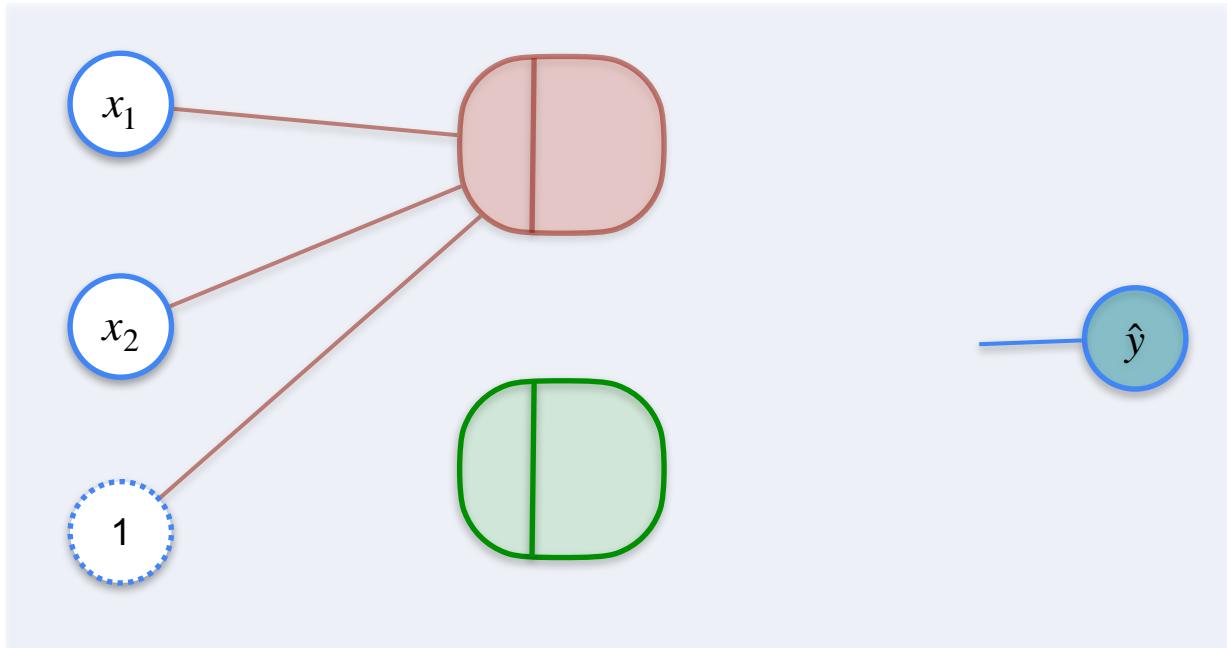


$$z = x_1 w_1 + x_2 w_2 + b$$

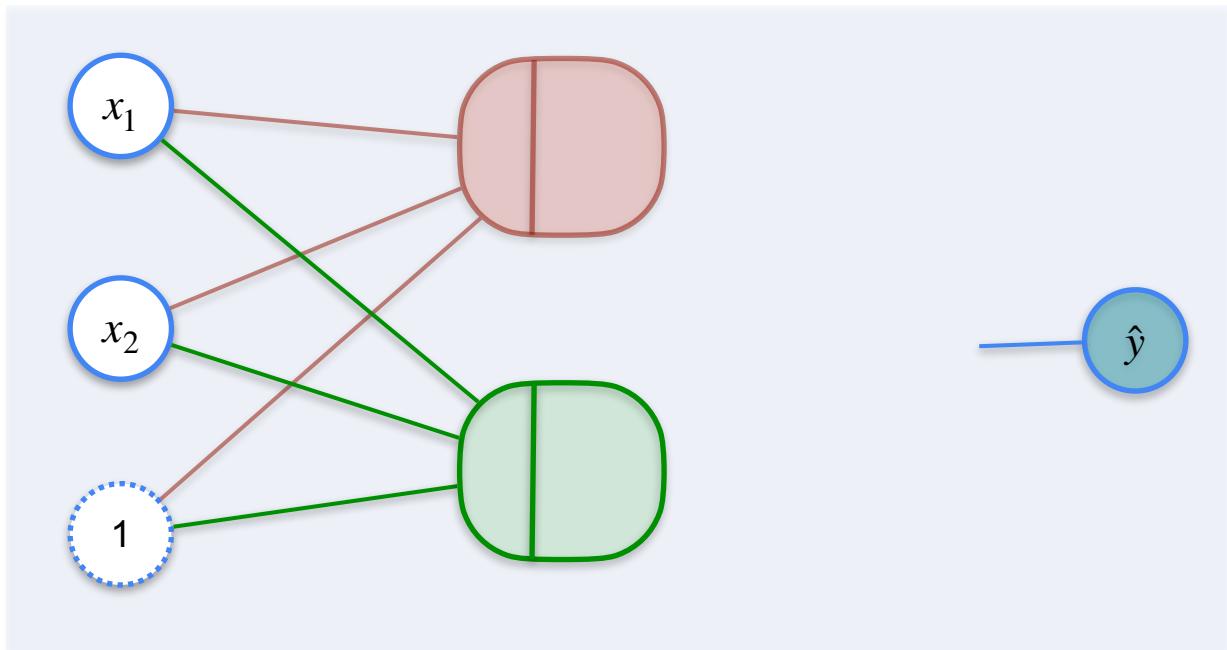
2,2,1 Neural Network



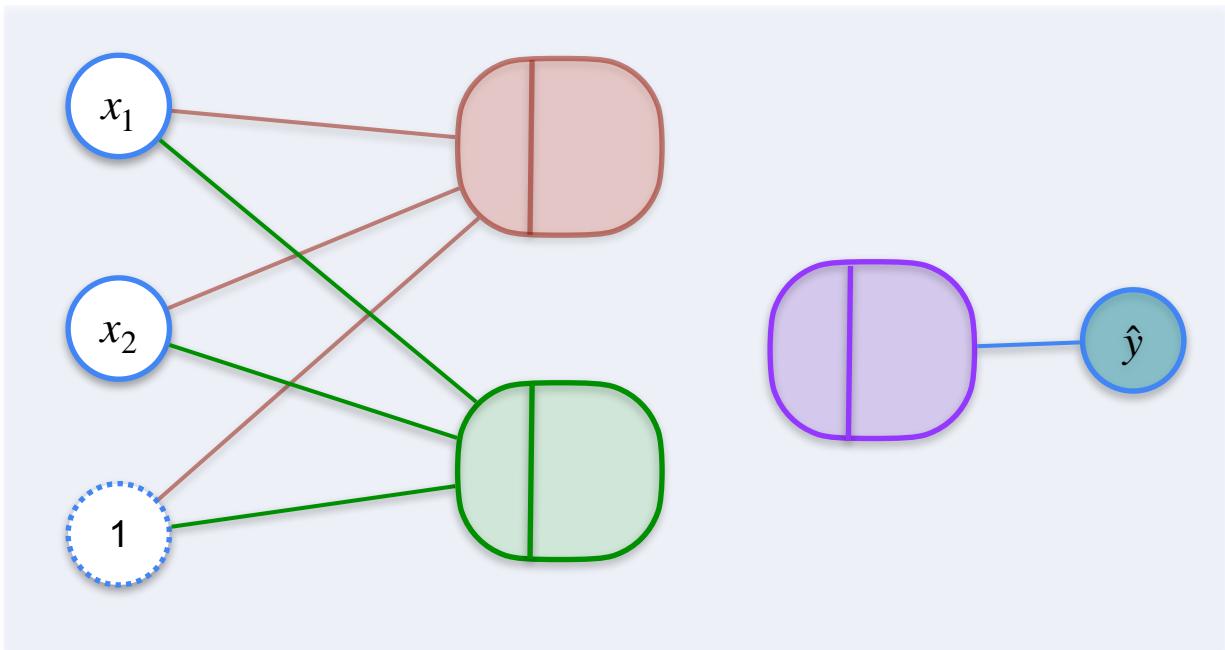
2,2,1 Neural Network



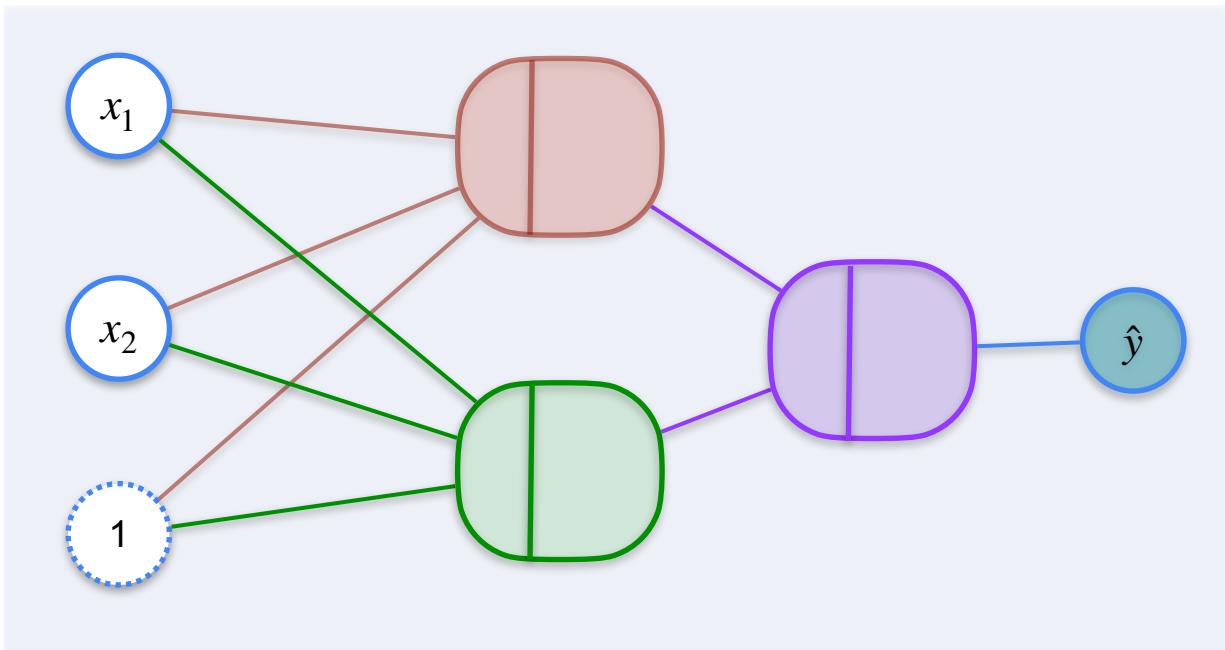
2,2,1 Neural Network



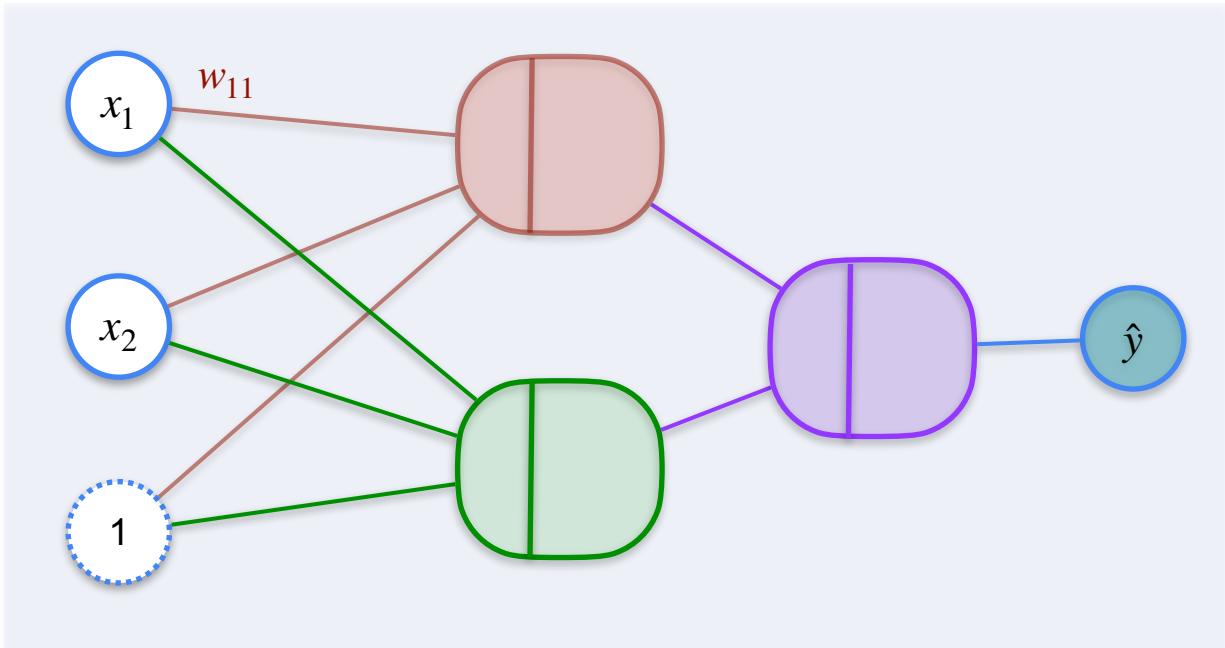
2,2,1 Neural Network



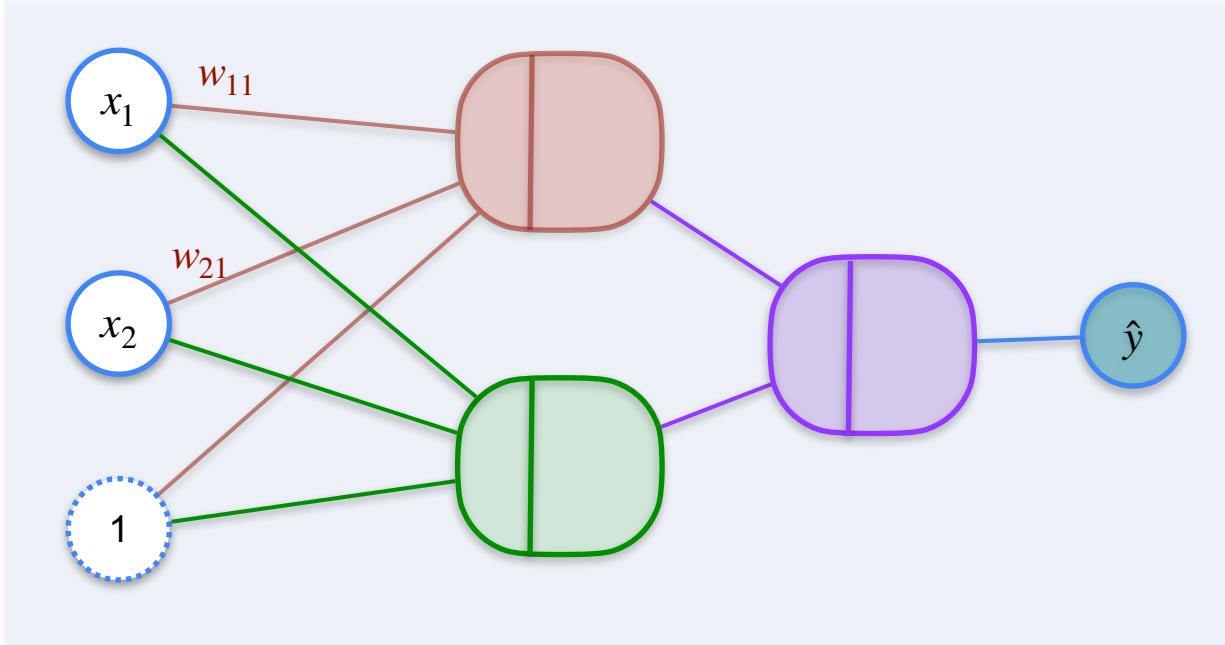
2,2,1 Neural Network



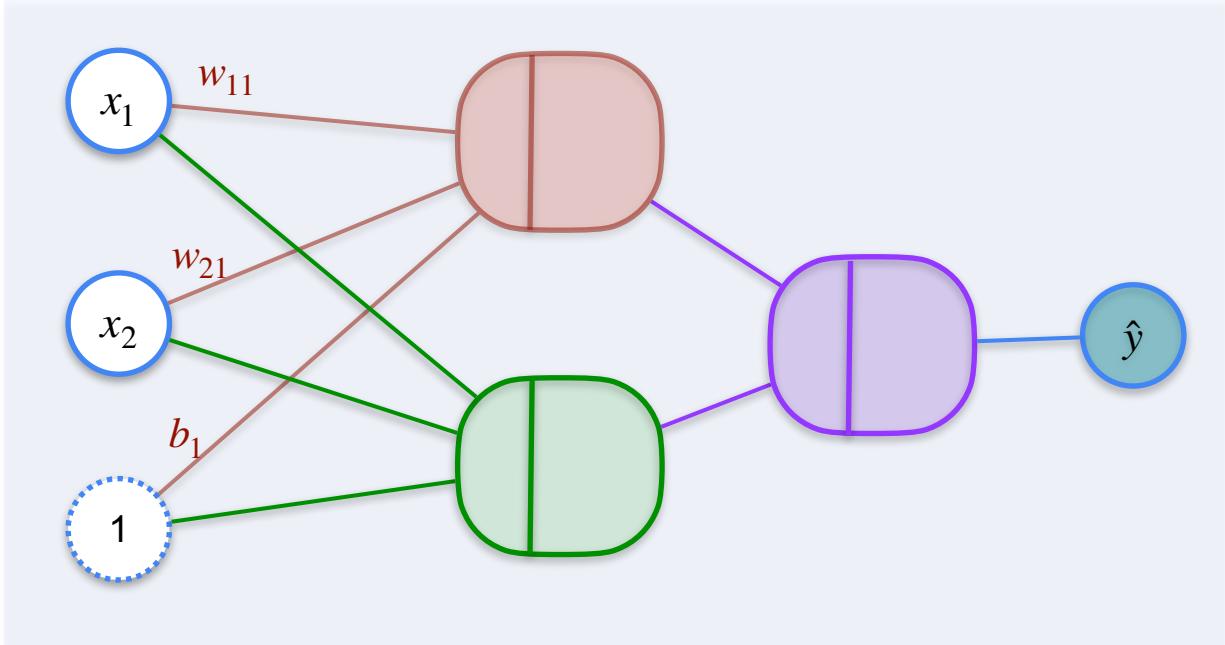
2,2,1 Neural Network



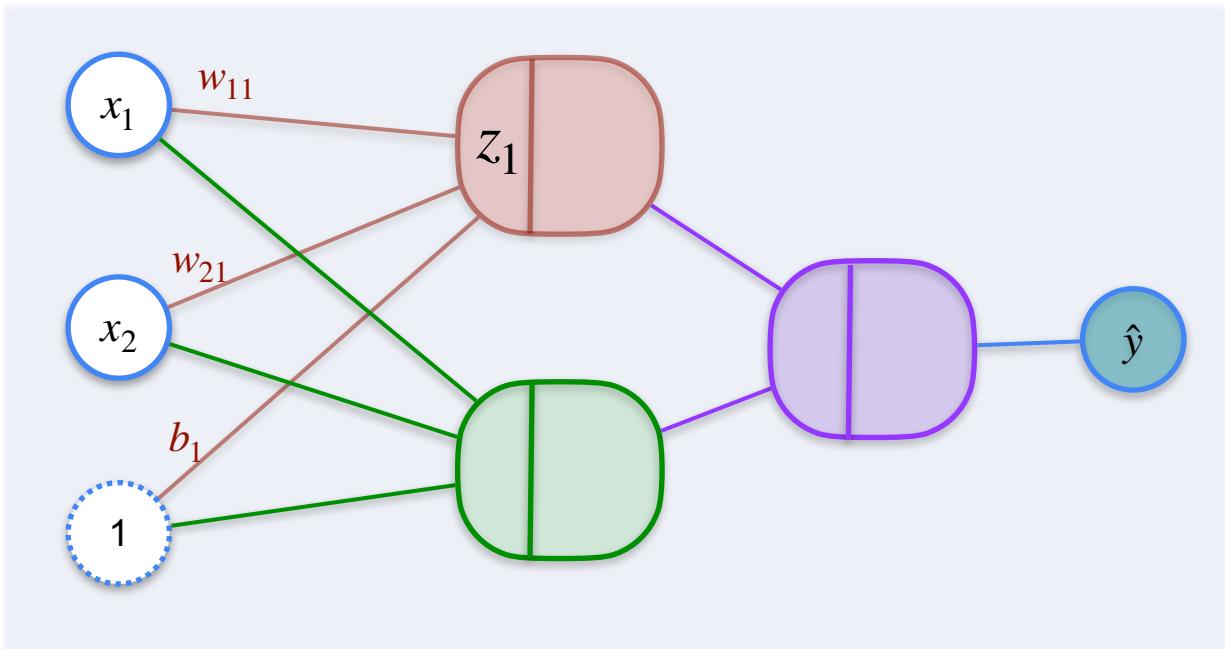
2,2,1 Neural Network



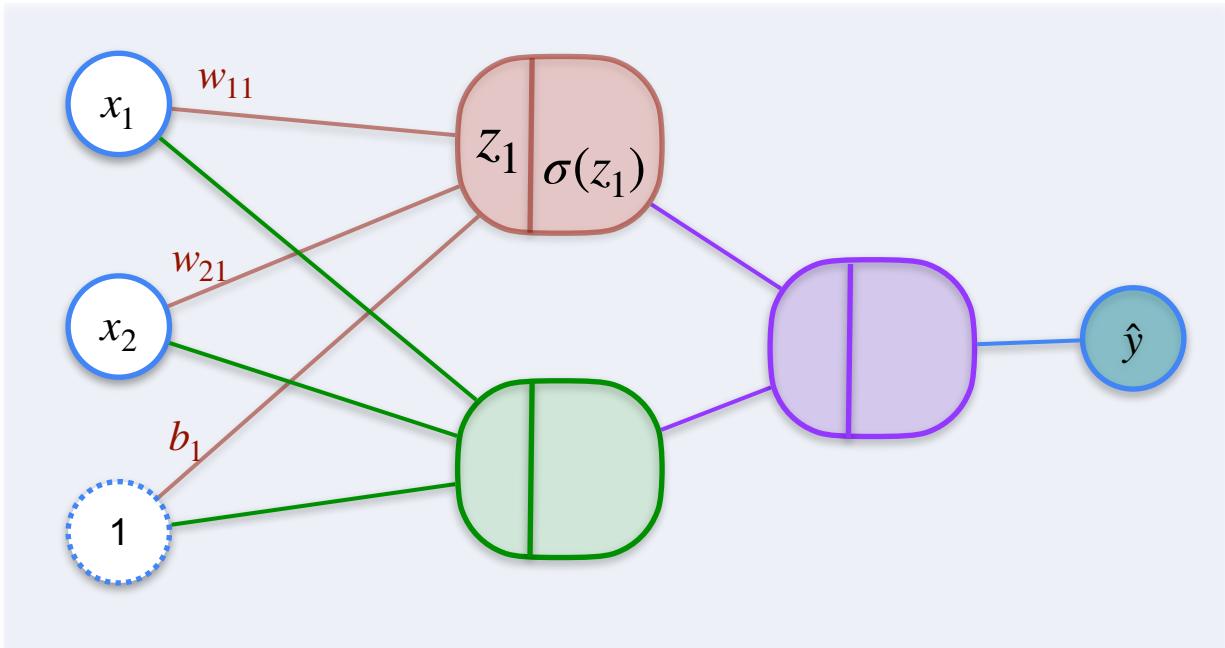
2,2,1 Neural Network



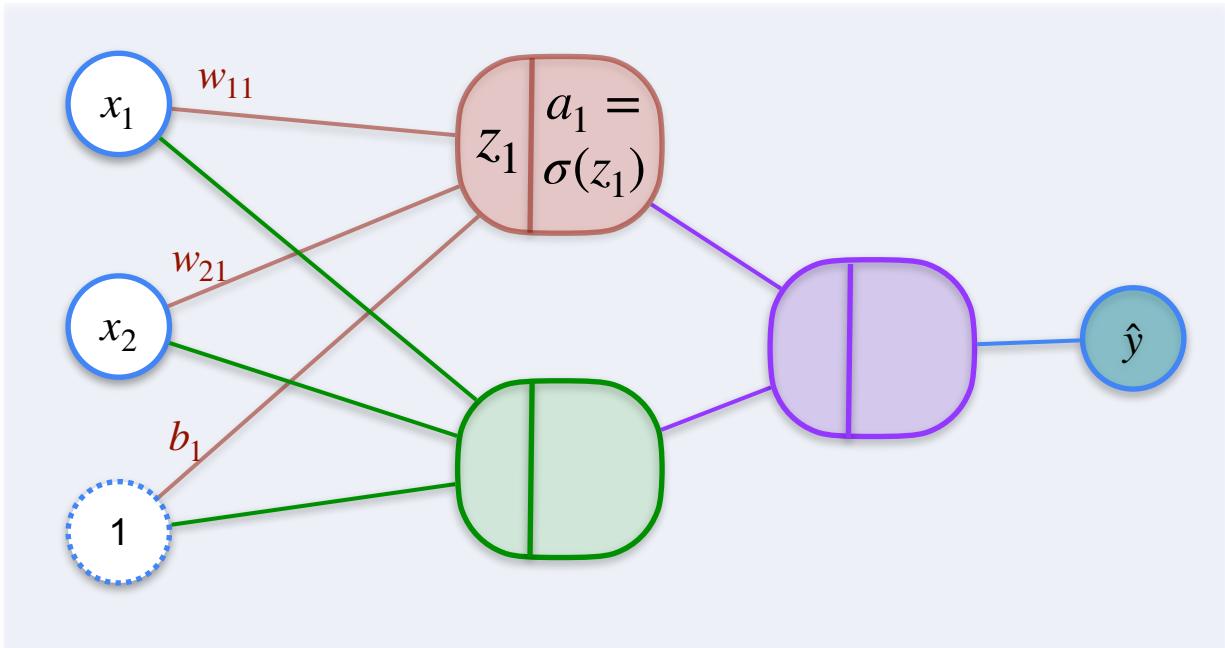
2,2,1 Neural Network



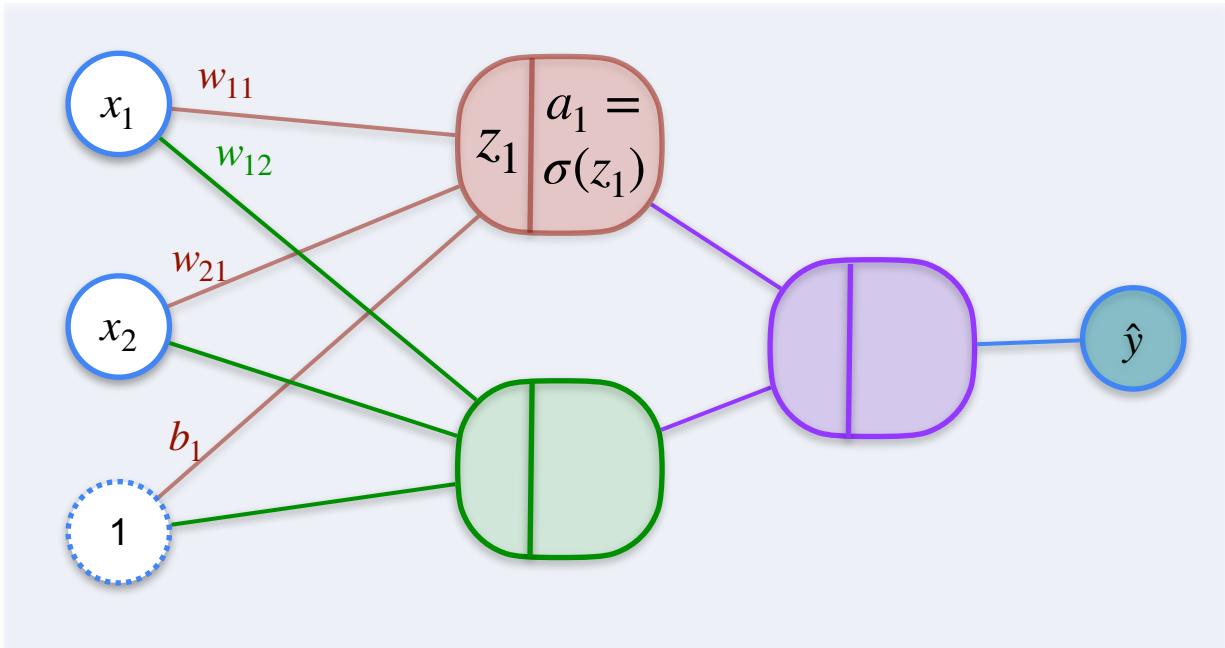
2,2,1 Neural Network



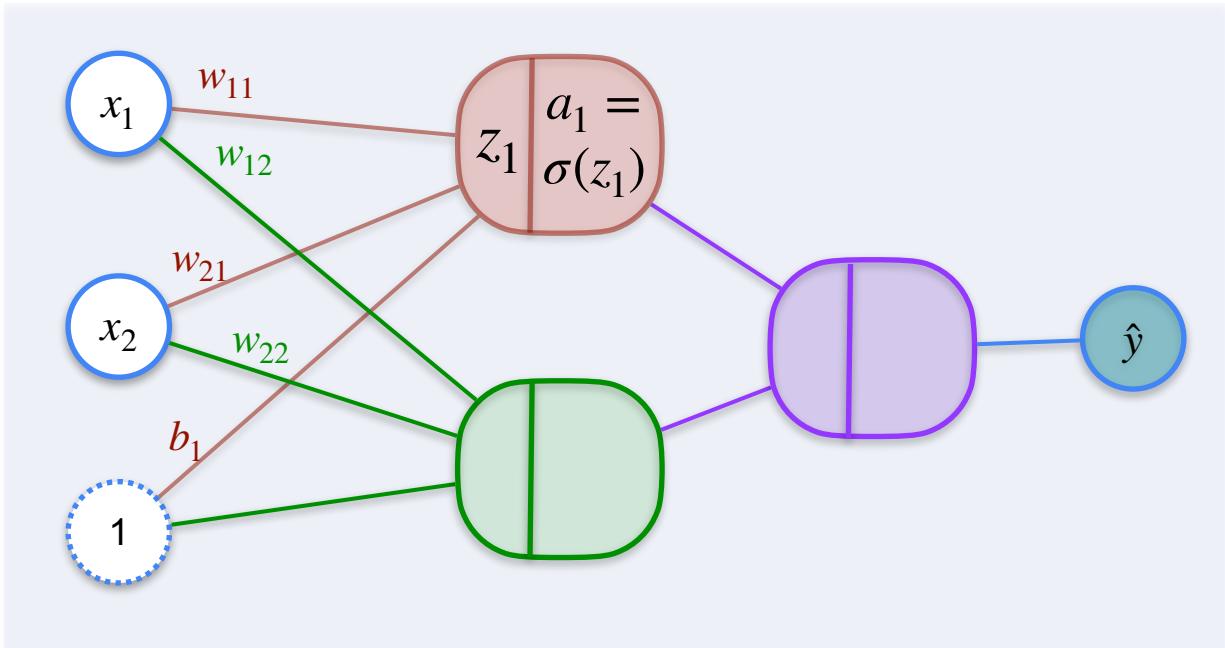
2,2,1 Neural Network



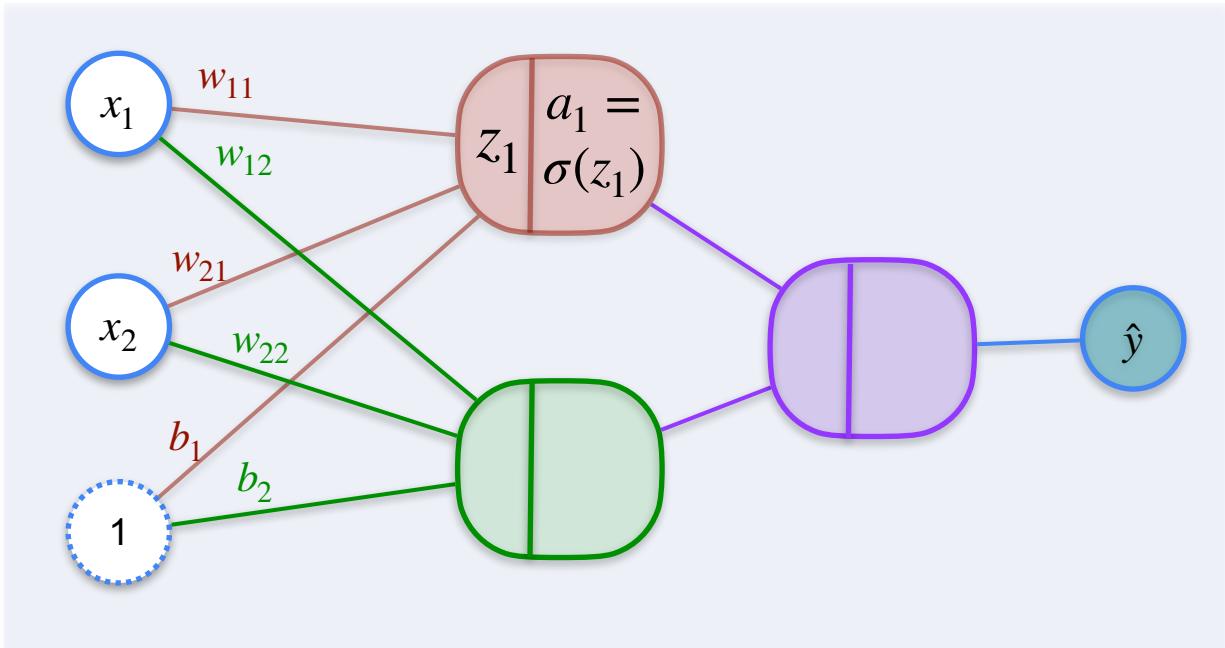
2,2,1 Neural Network



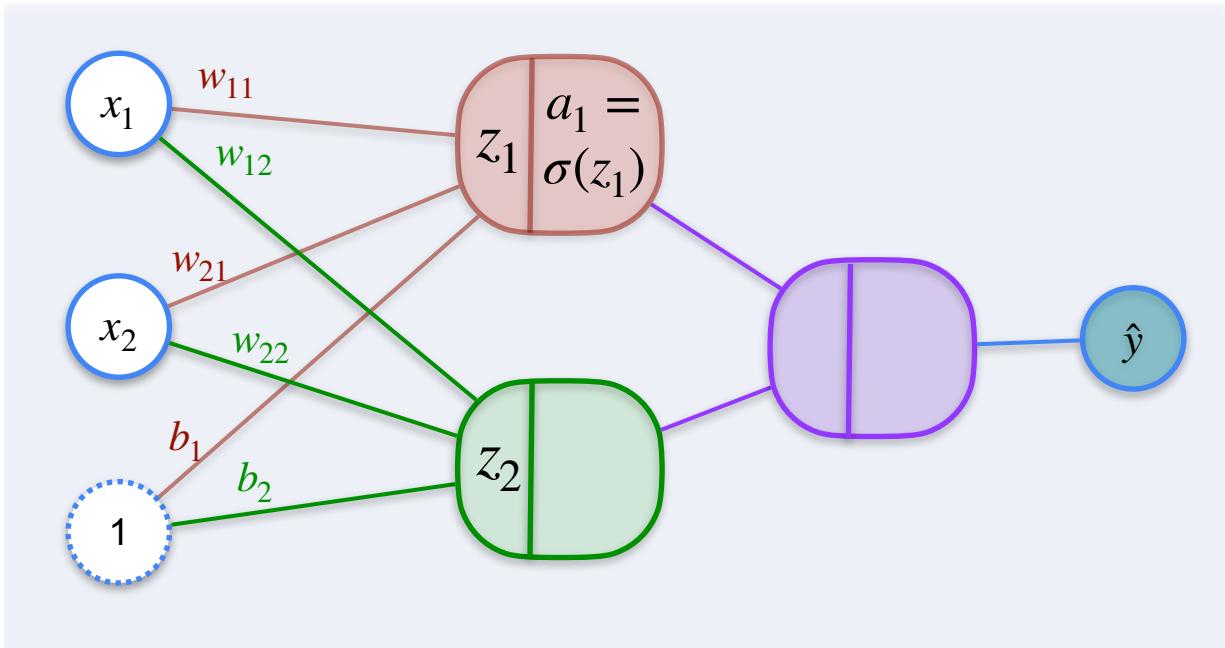
2,2,1 Neural Network



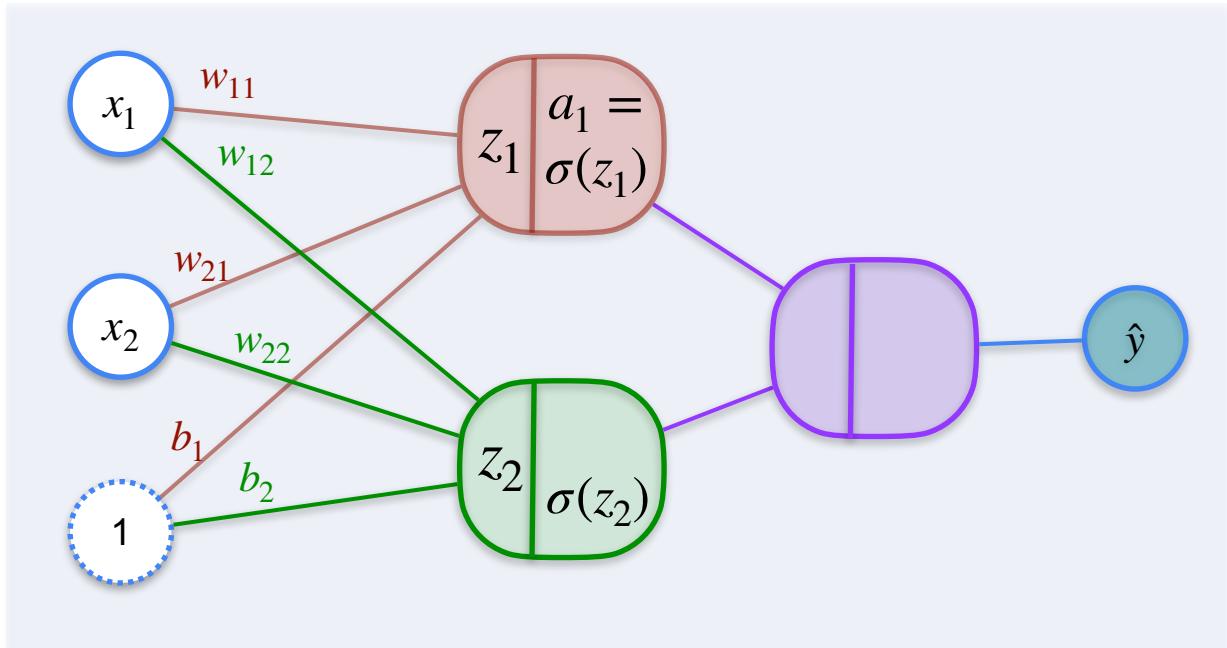
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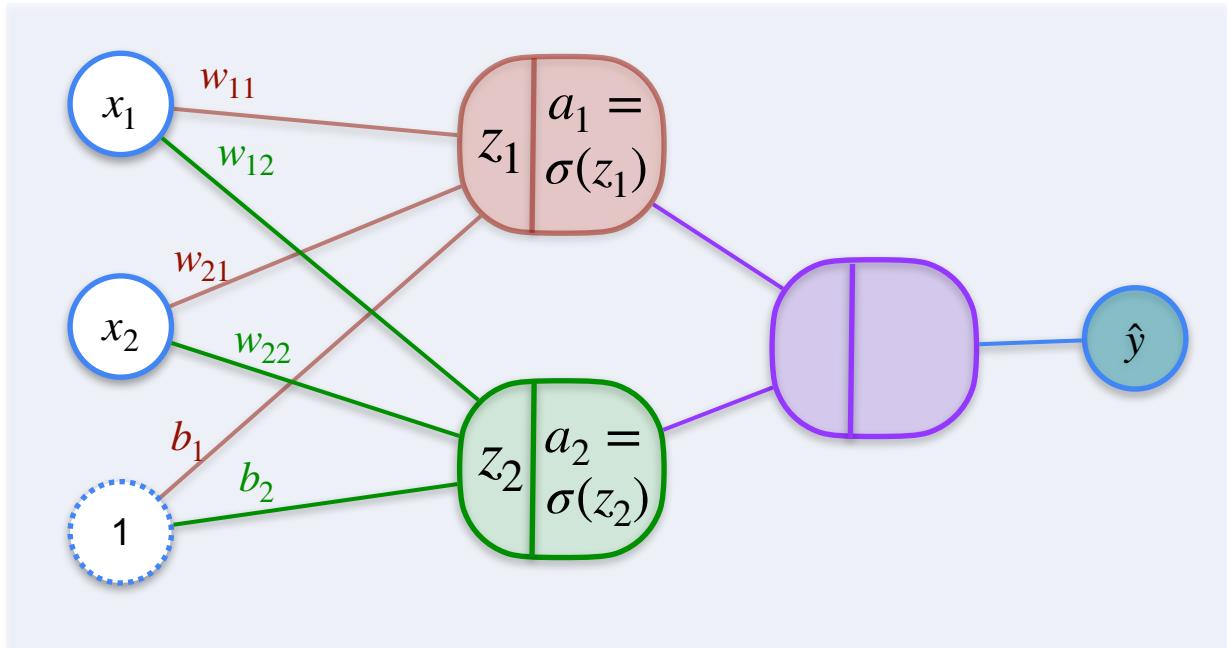
2,2,1 Neural Network



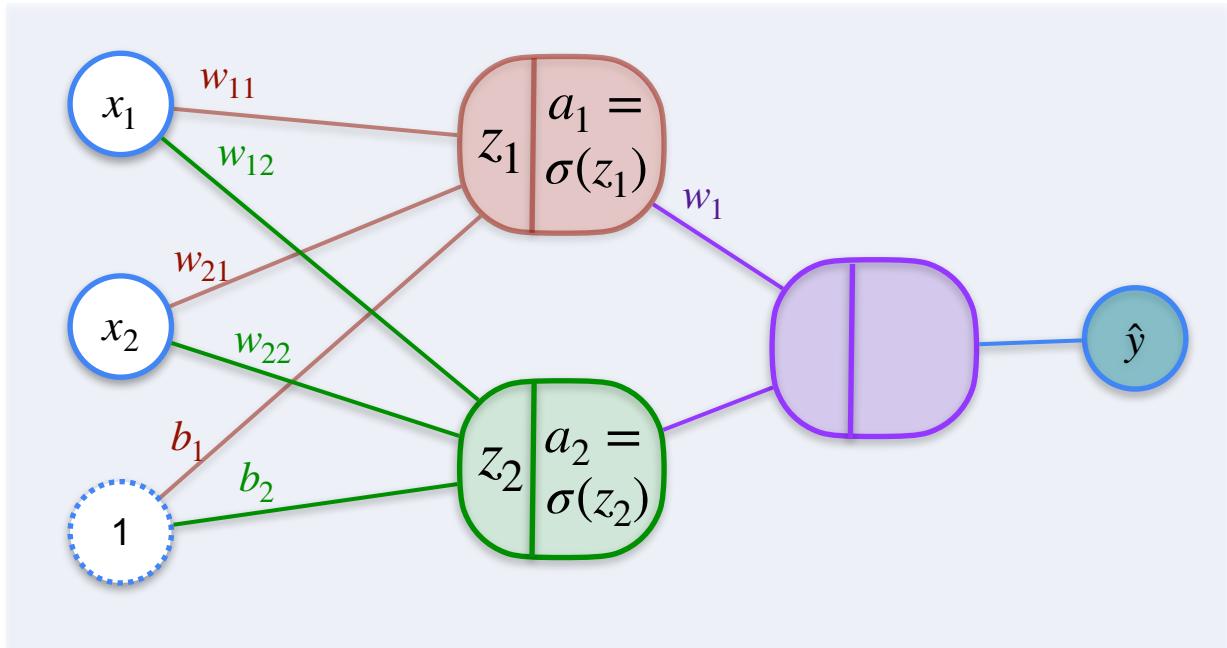
2,2,1 Neural Network



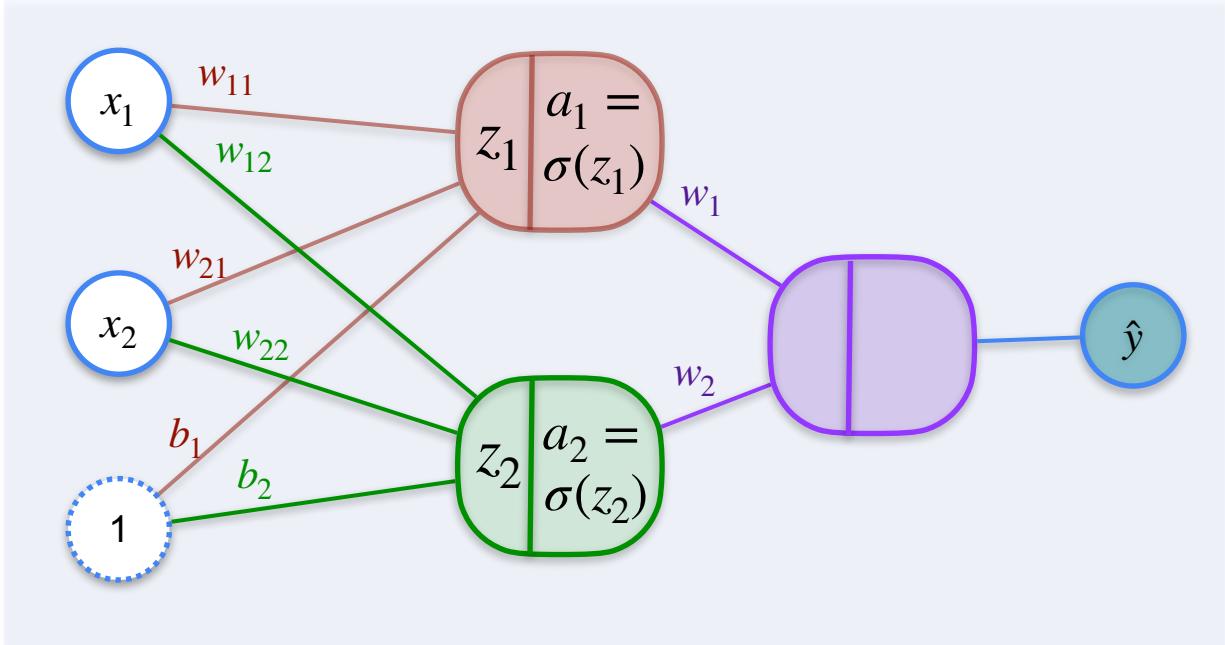
2,2,1 Neural Network



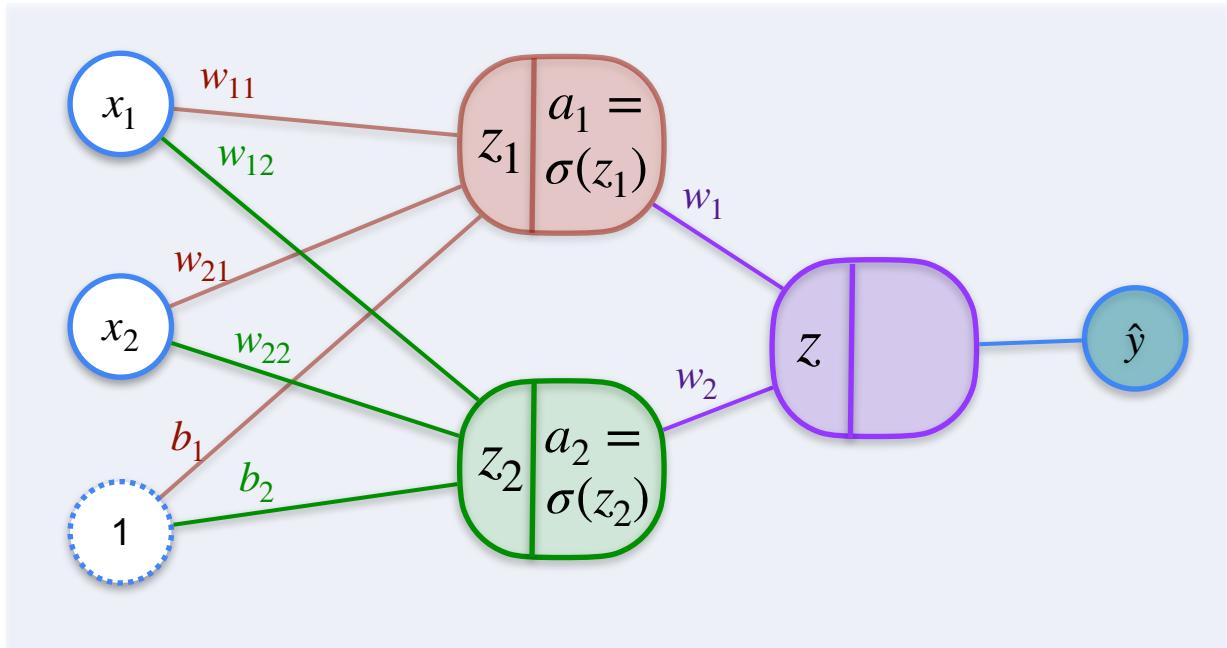
2,2,1 Neural Network



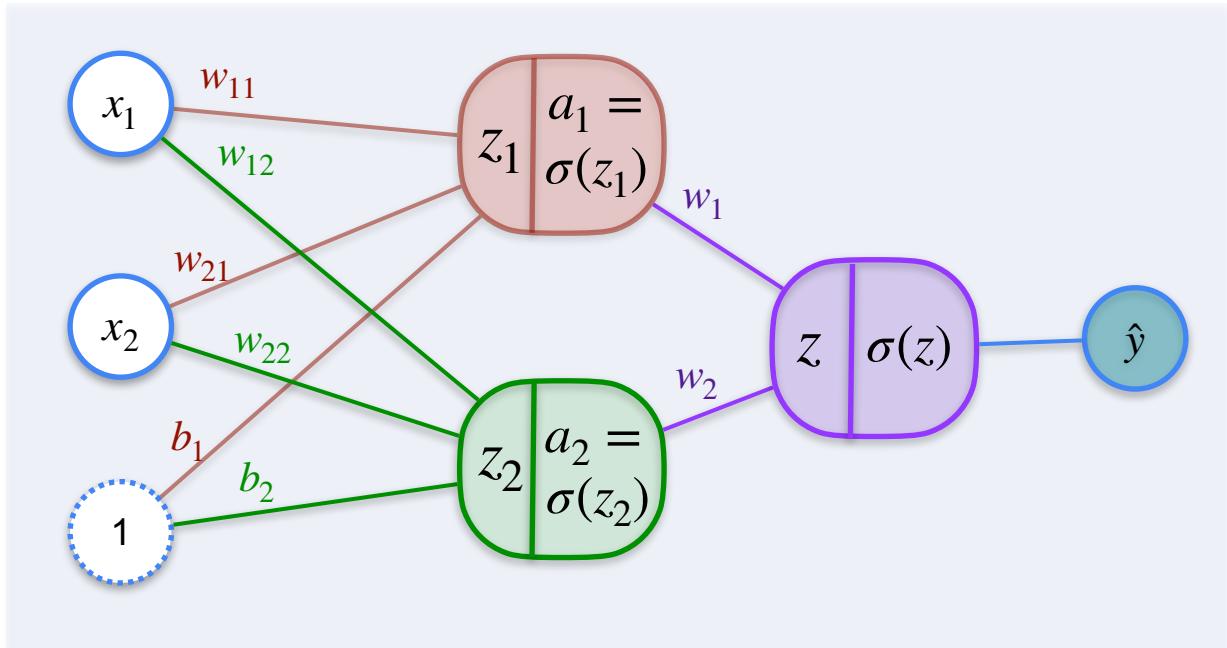
2,2,1 Neural Network



2,2,1 Neural Network



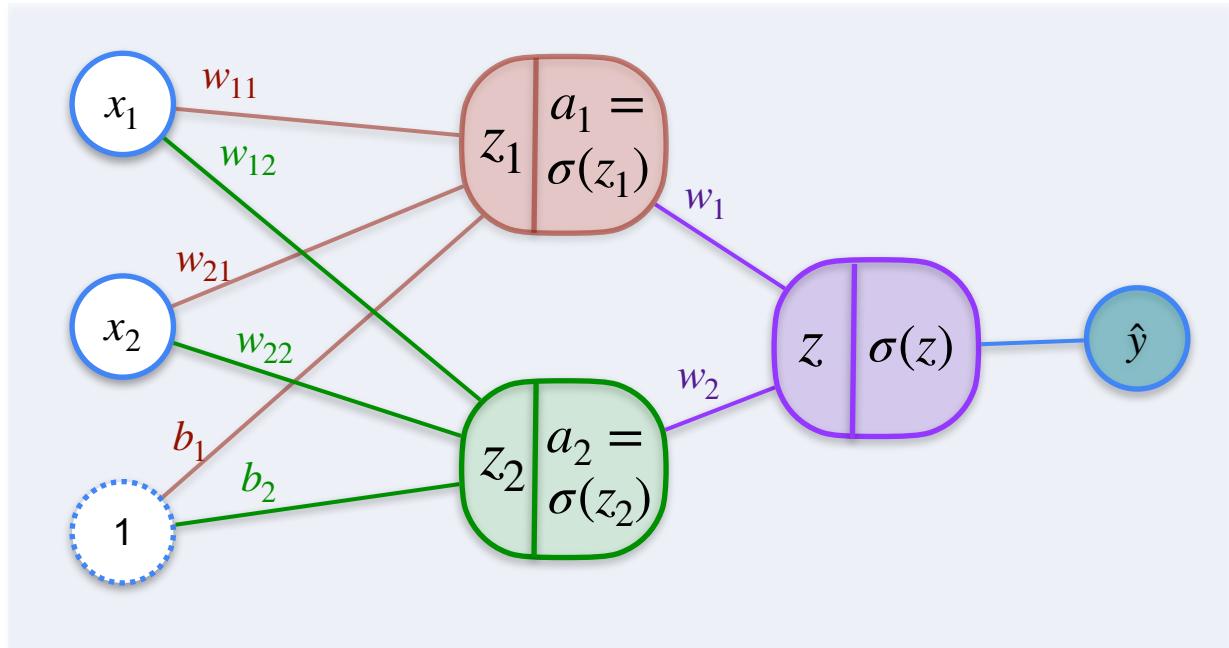
2,2,1 Neural Network



2,2,1 Neural Network

Neural network of depth 2

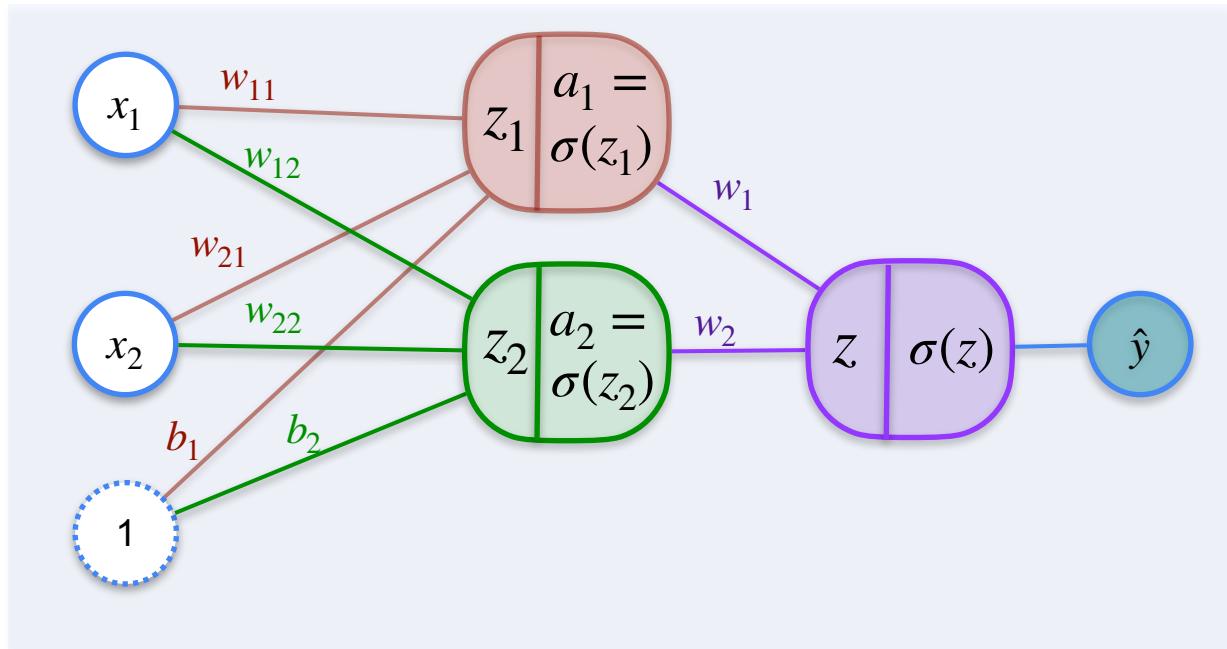
- one input layer
- one hidden layer
- one output layer



2,2,1 Neural Network

Neural network of depth 2

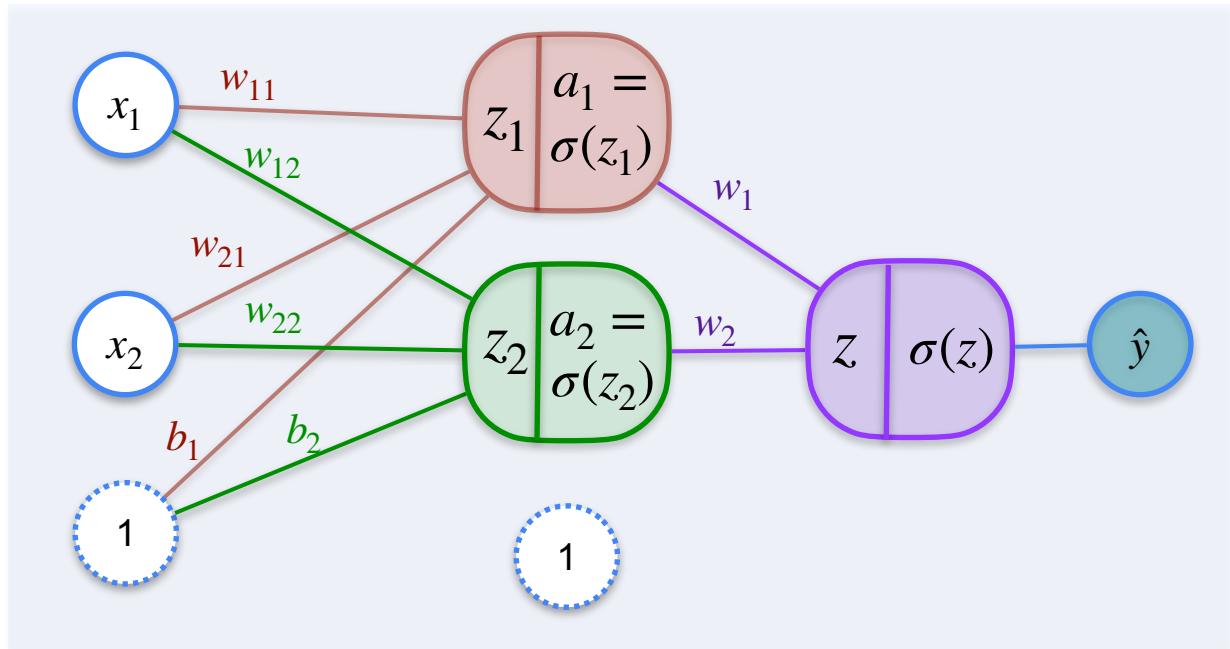
- one input layer
- one hidden layer
- one output layer



2,2,1 Neural Network

Neural network of depth 2

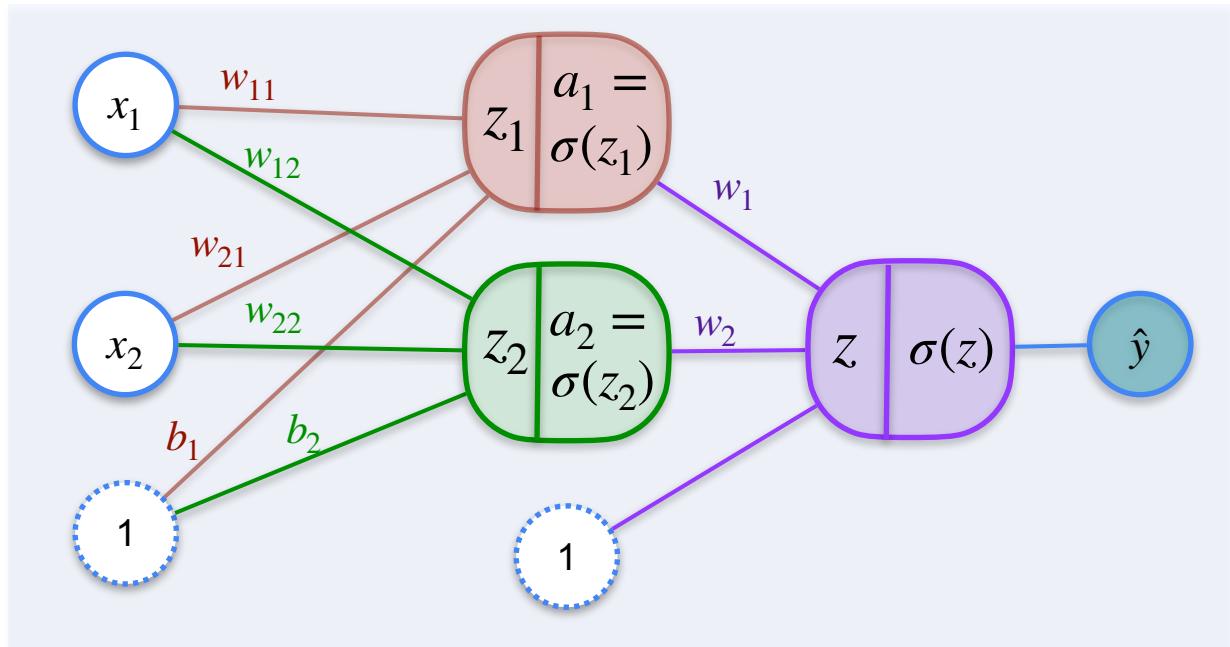
- one input layer
- one hidden layer
- one output layer



2,2,1 Neural Network

Neural network of depth 2

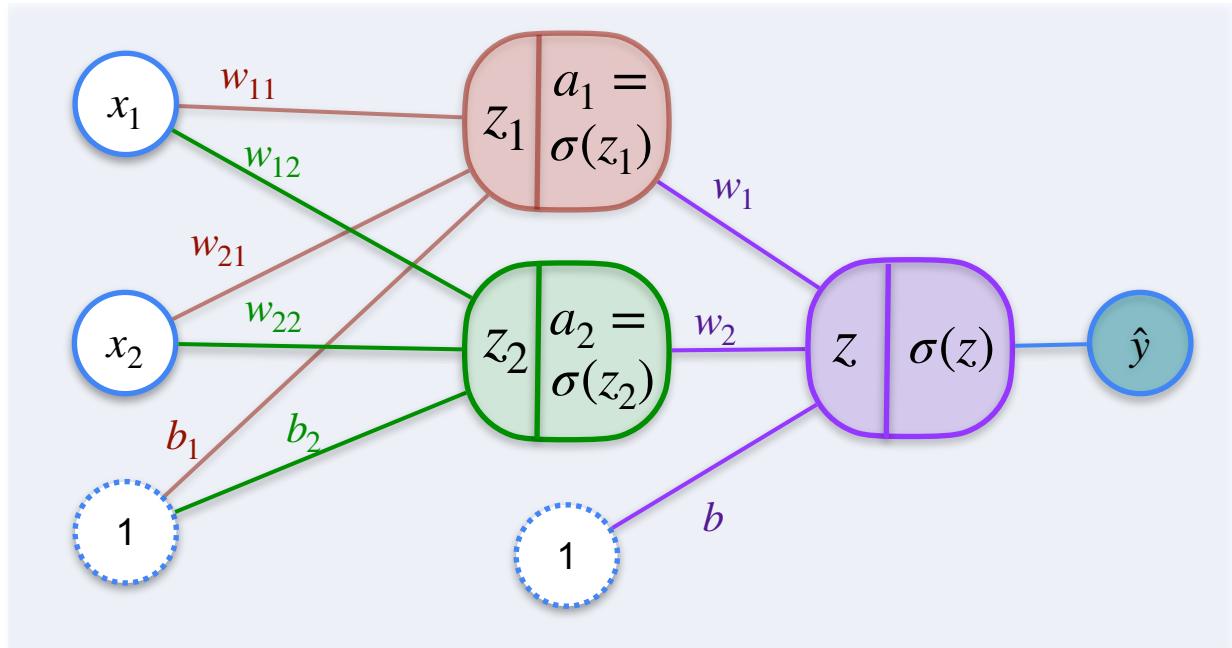
- one input layer
- one hidden layer
- one output layer



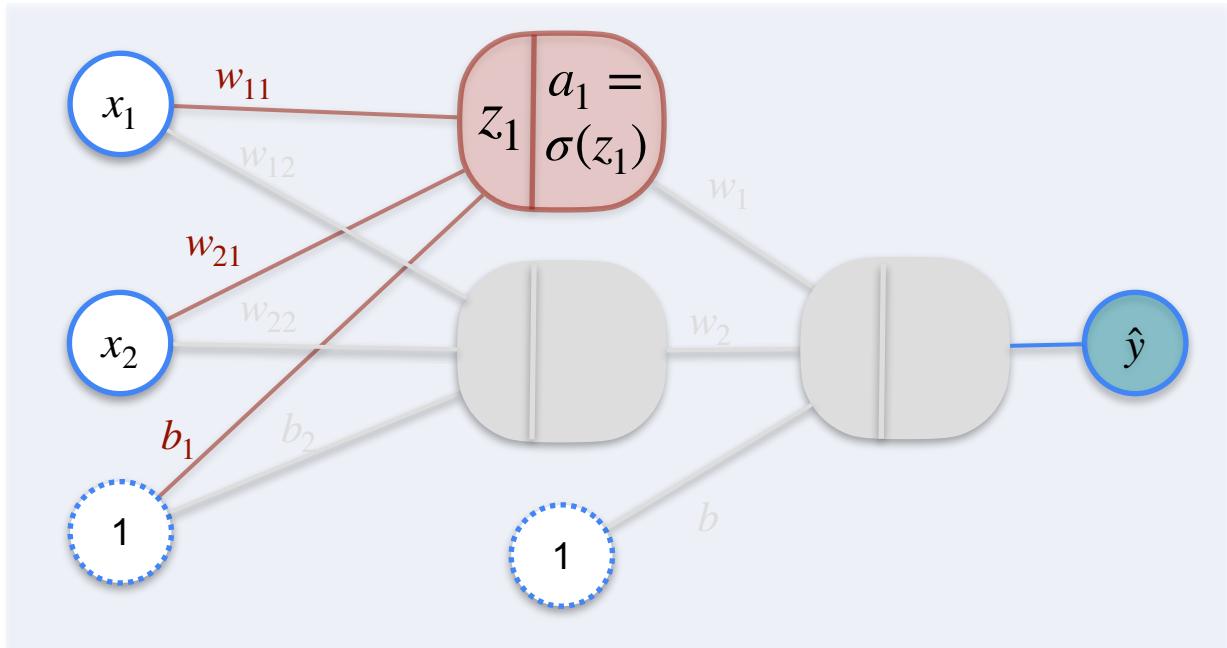
2,2,1 Neural Network

Neural network of depth 2

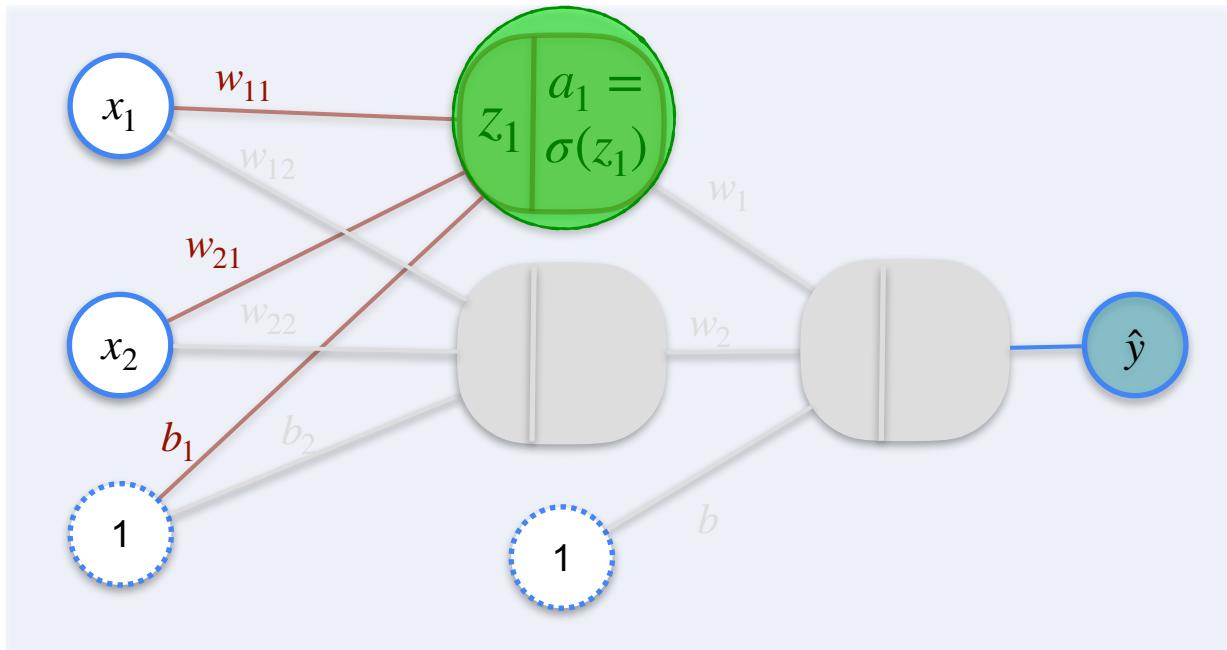
- one input layer
- one hidden layer
- one output layer



2,2,1 Neural Network

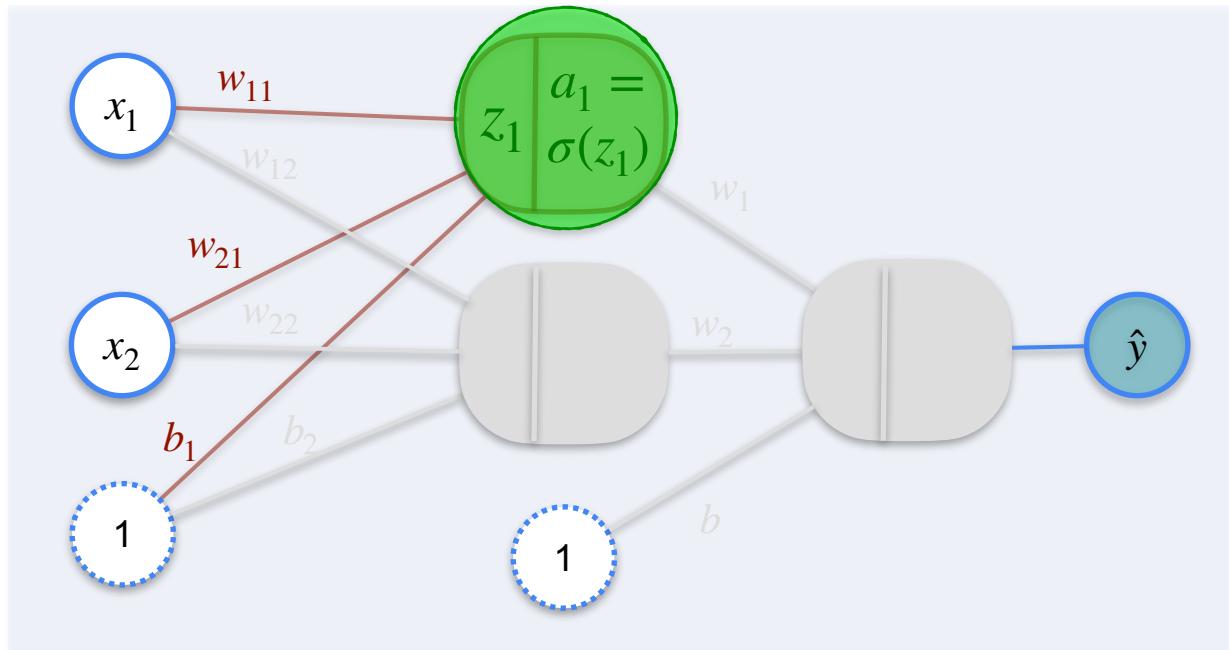


2,2,1 Neural Network



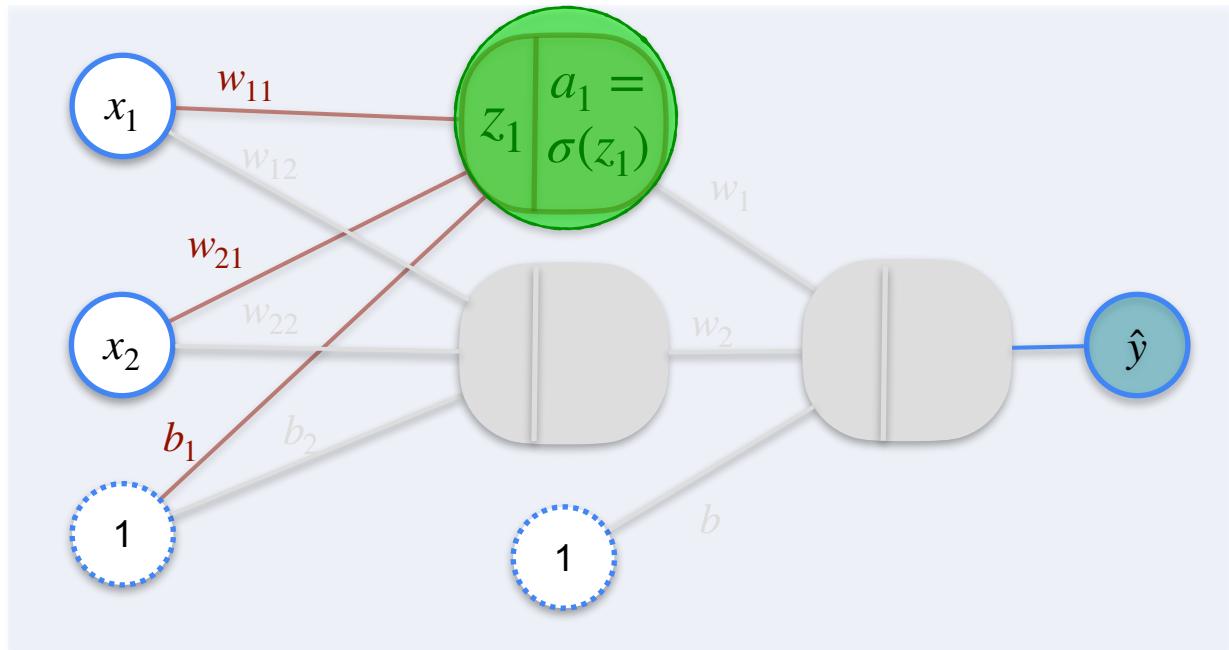
2,2,1 Neural Network

a_1



2,2,1 Neural Network

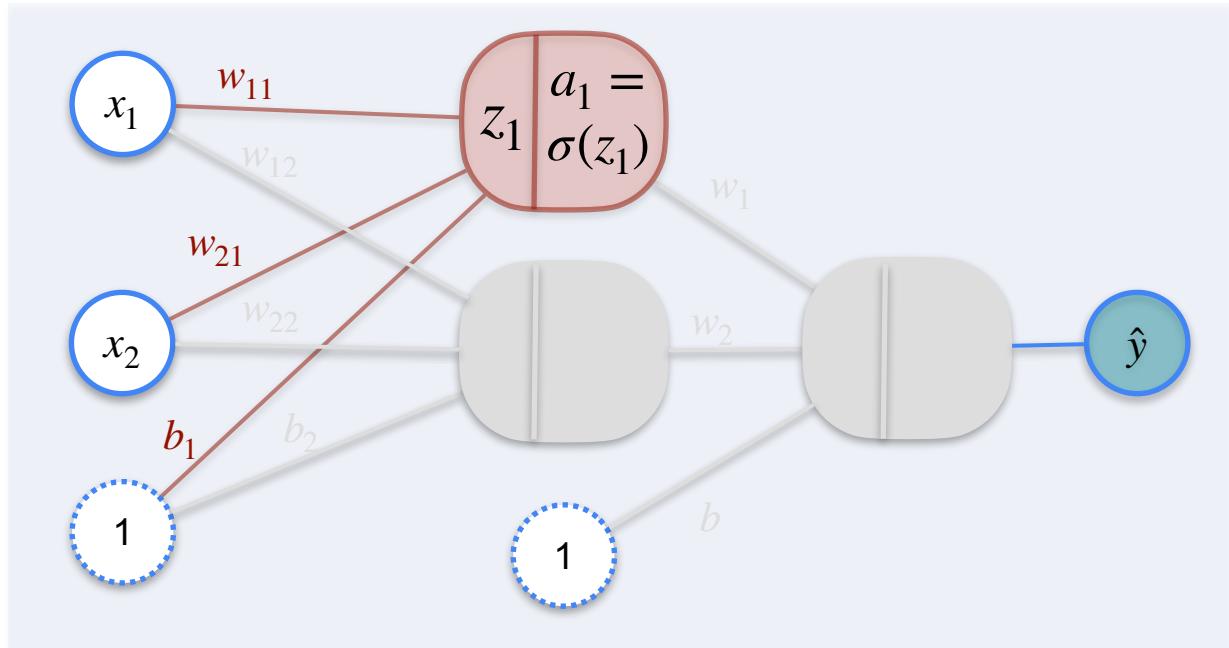
$$a_1 = \sigma(z_1)$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

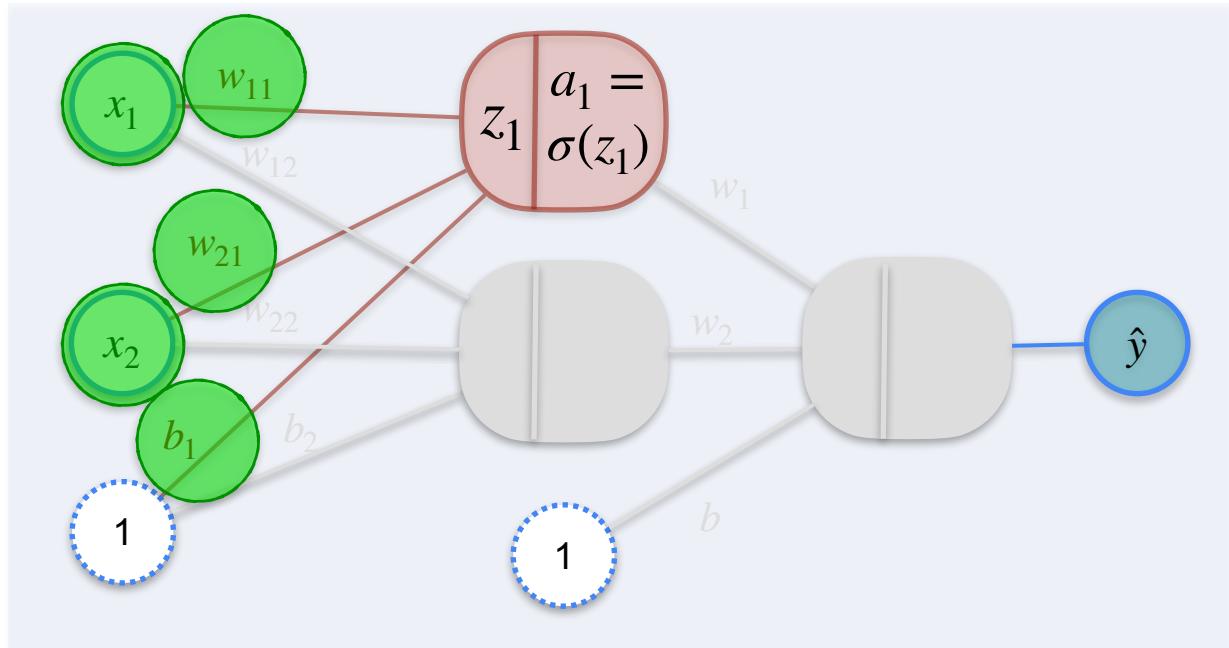
$$z_1$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

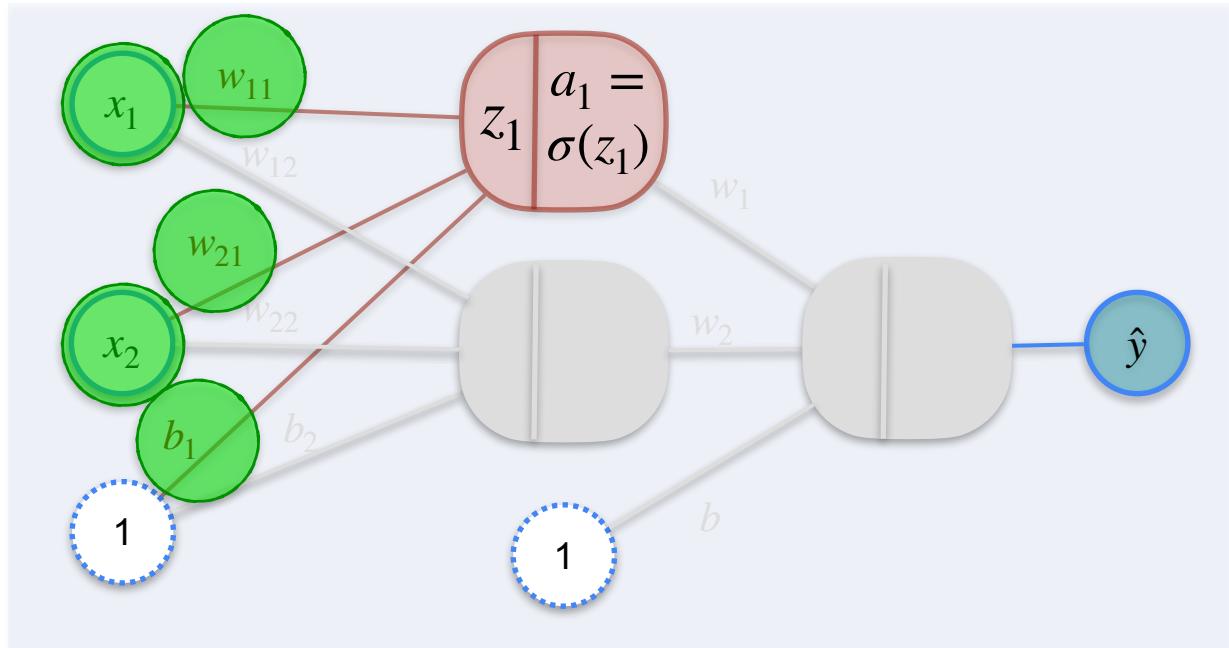
$$z_1$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

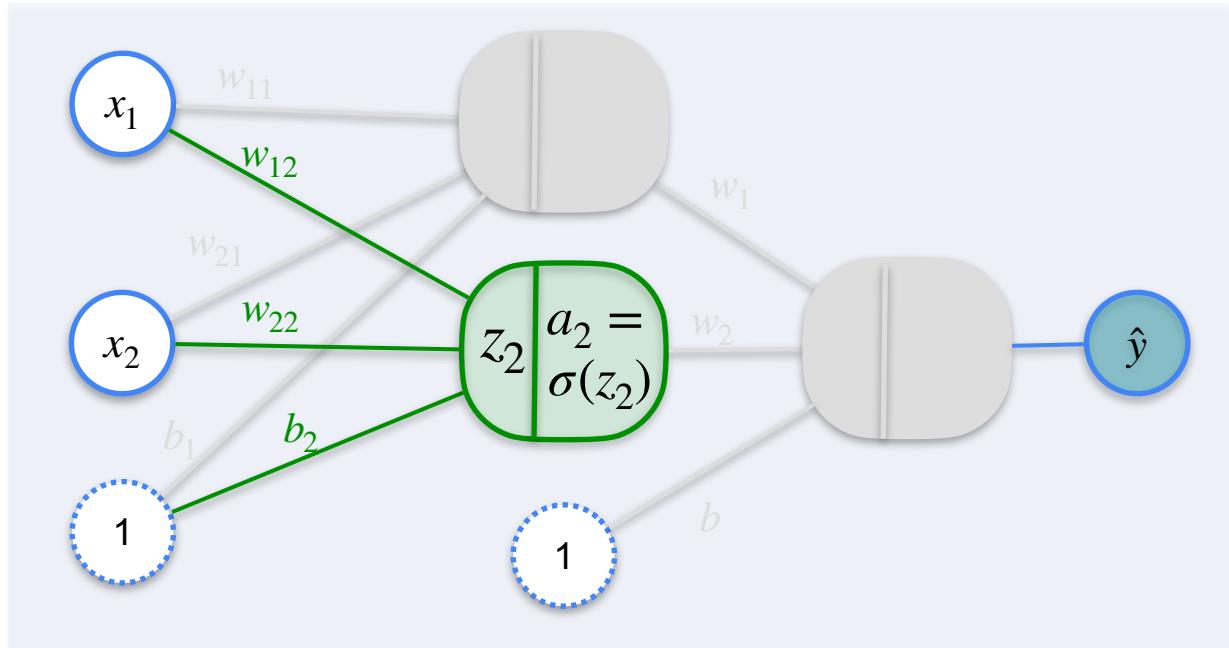
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

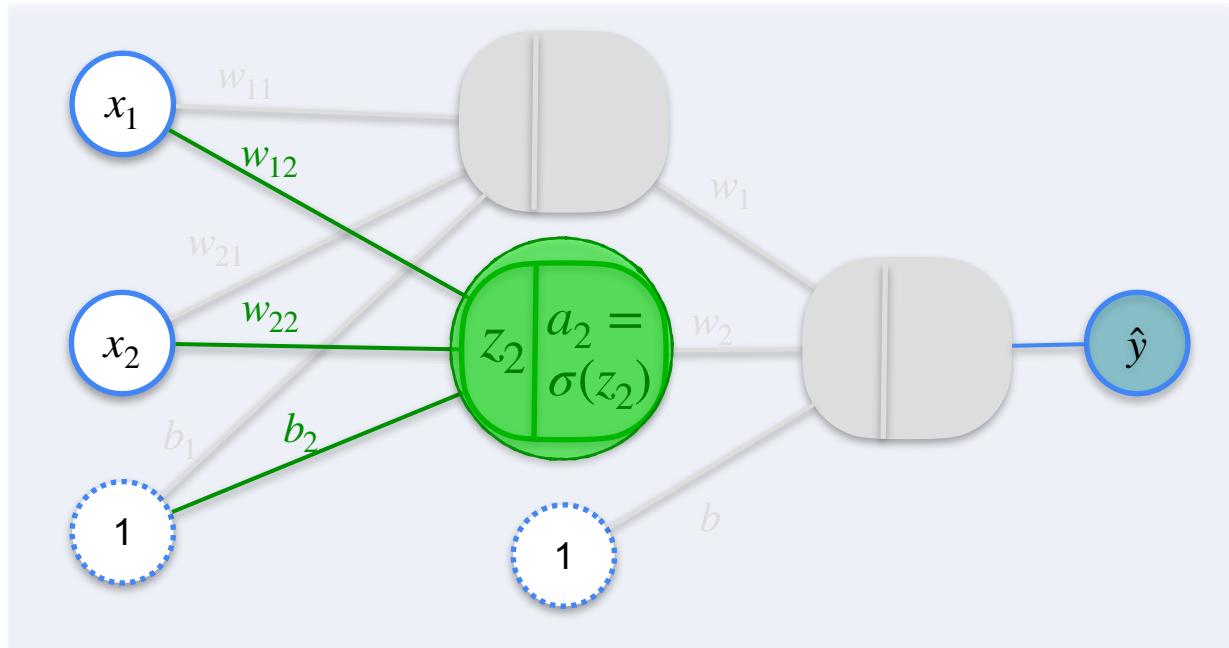
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

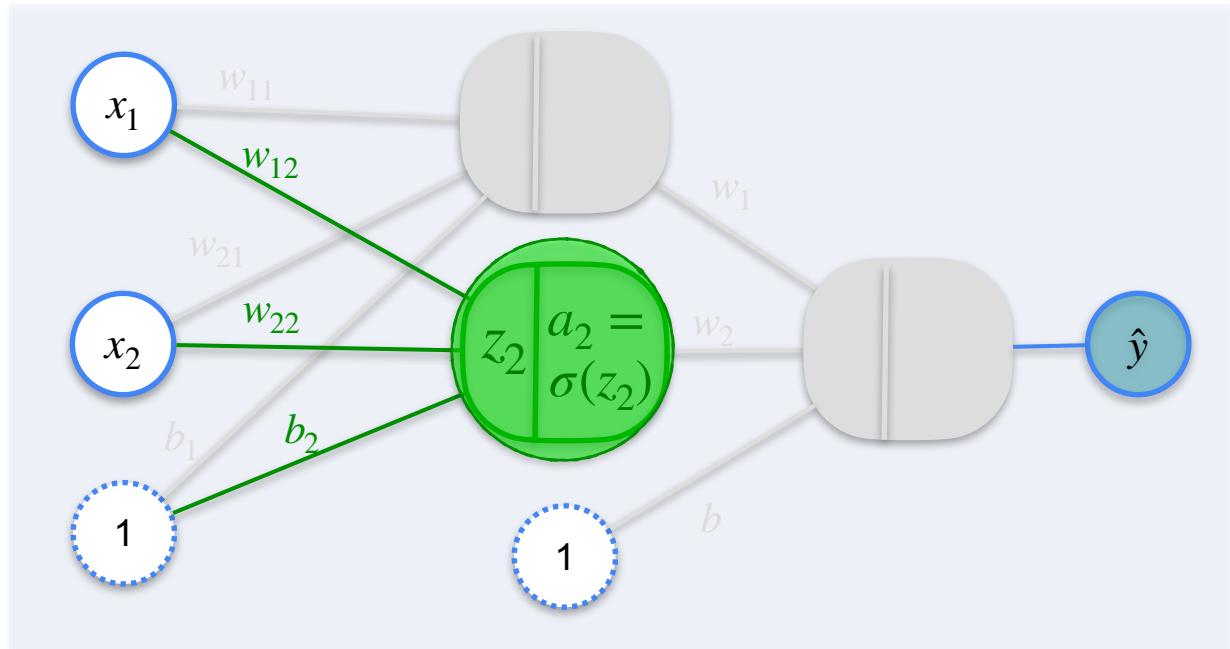


2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2$$

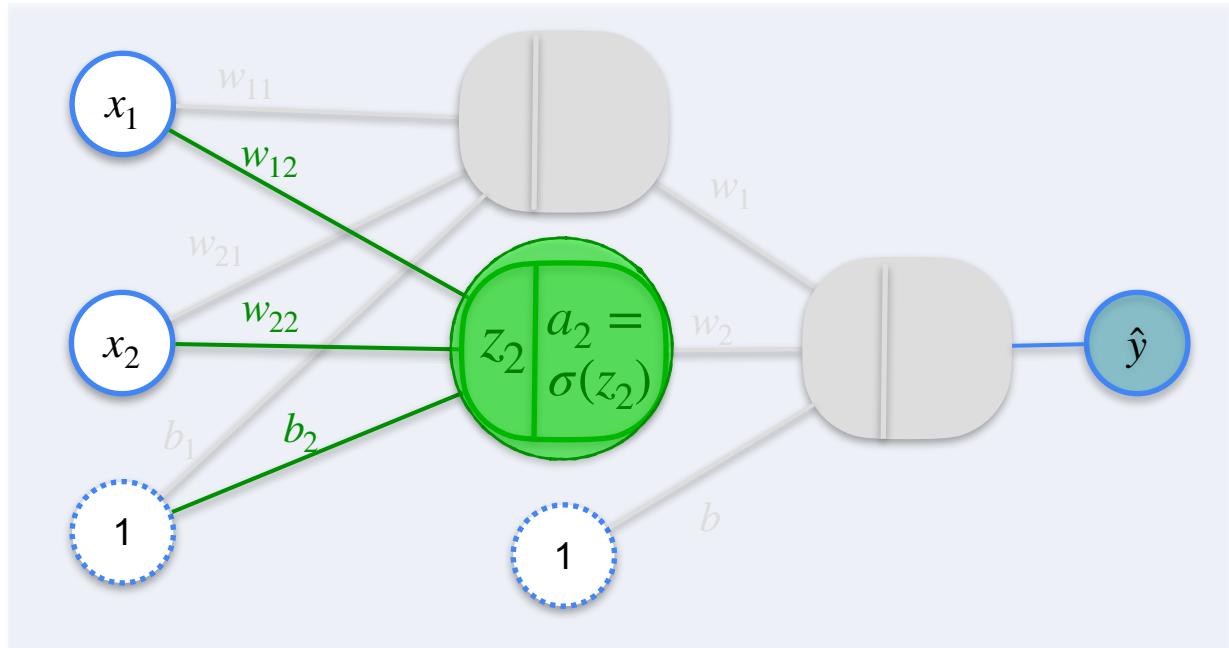


2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$



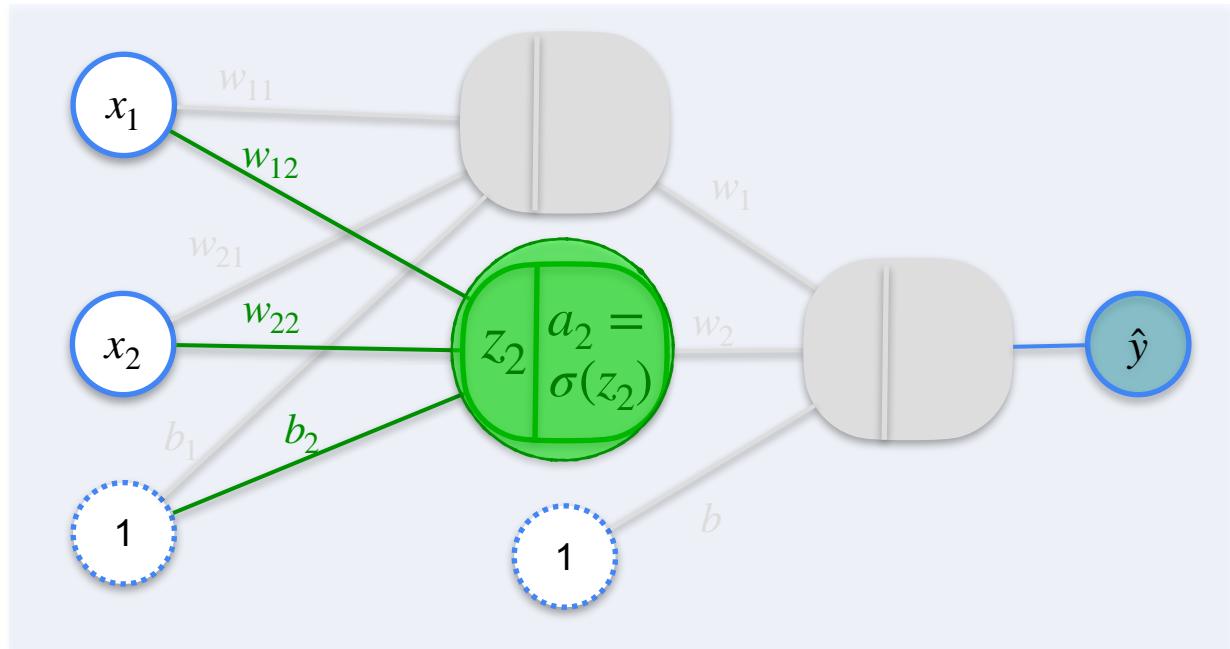
2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$



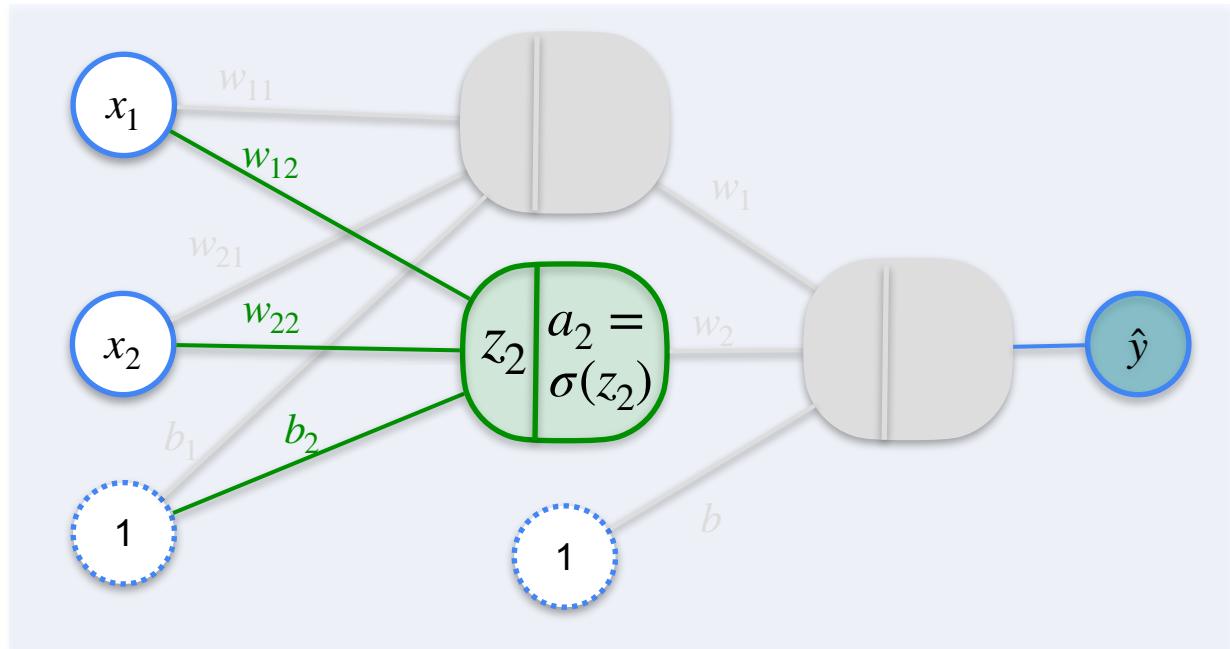
2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$



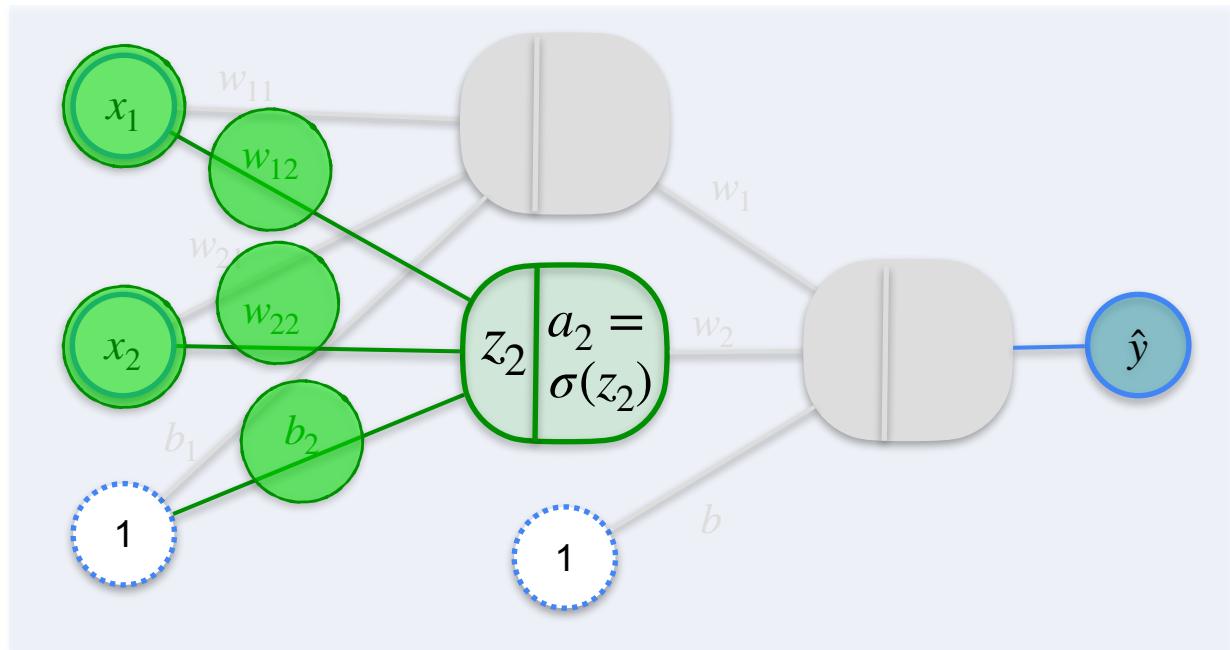
2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$



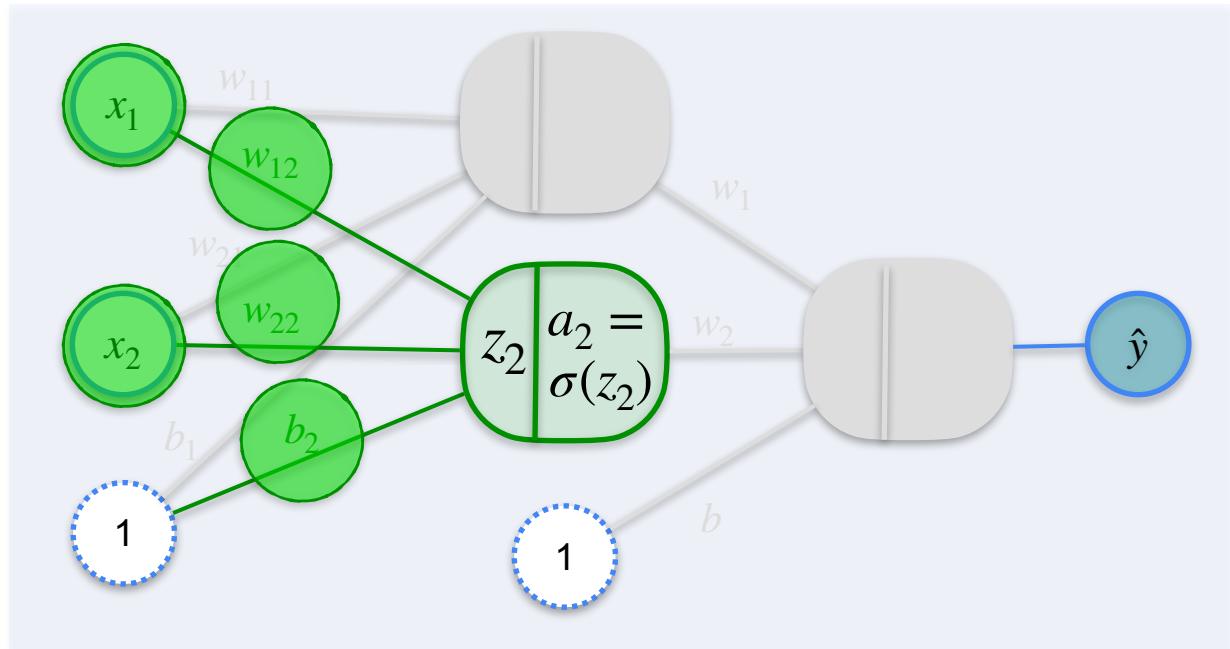
2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$



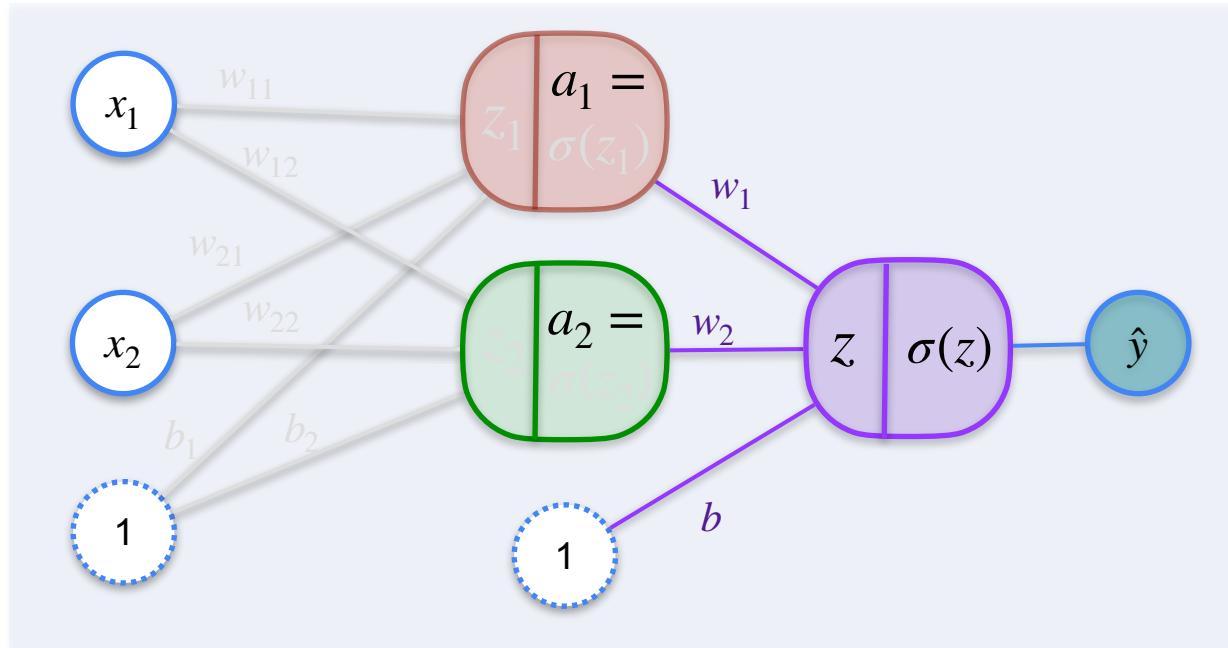
2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$



2,2,1 Neural Network

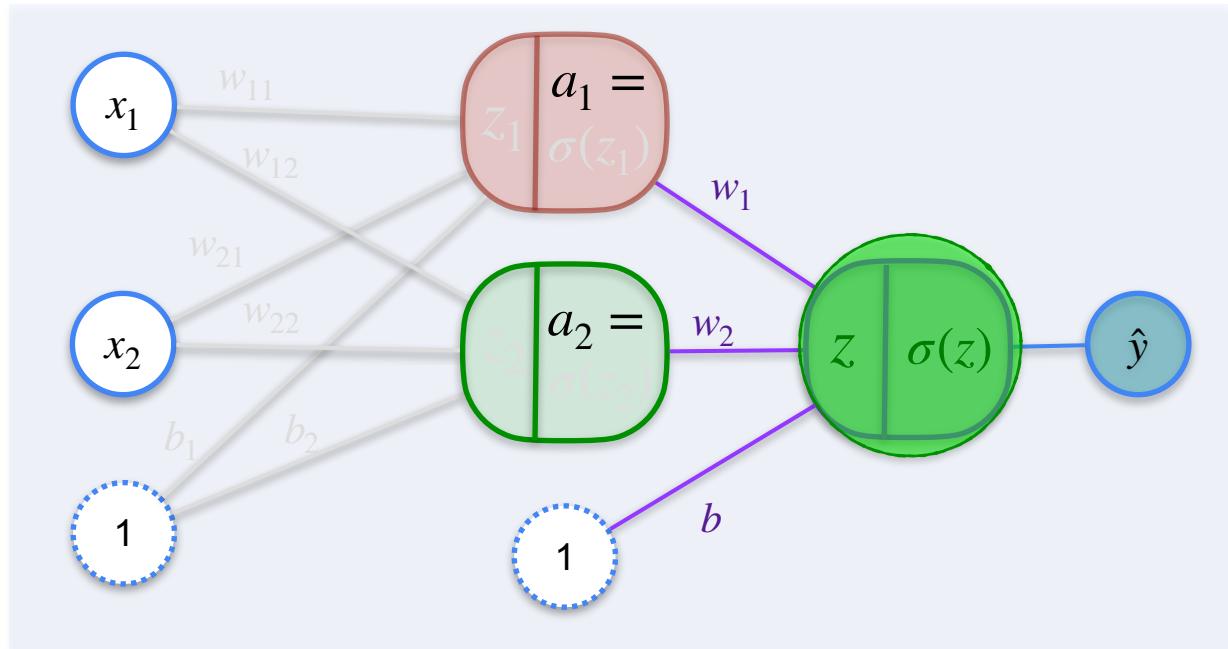
$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$



2,2,1 Neural Network

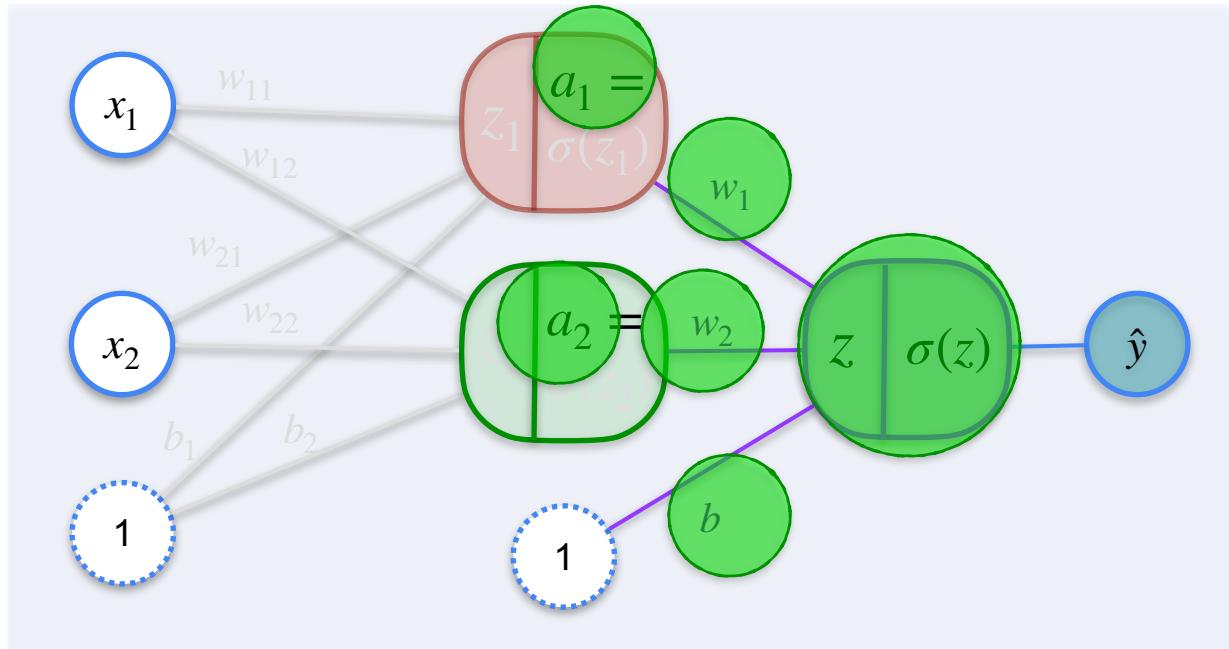
$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

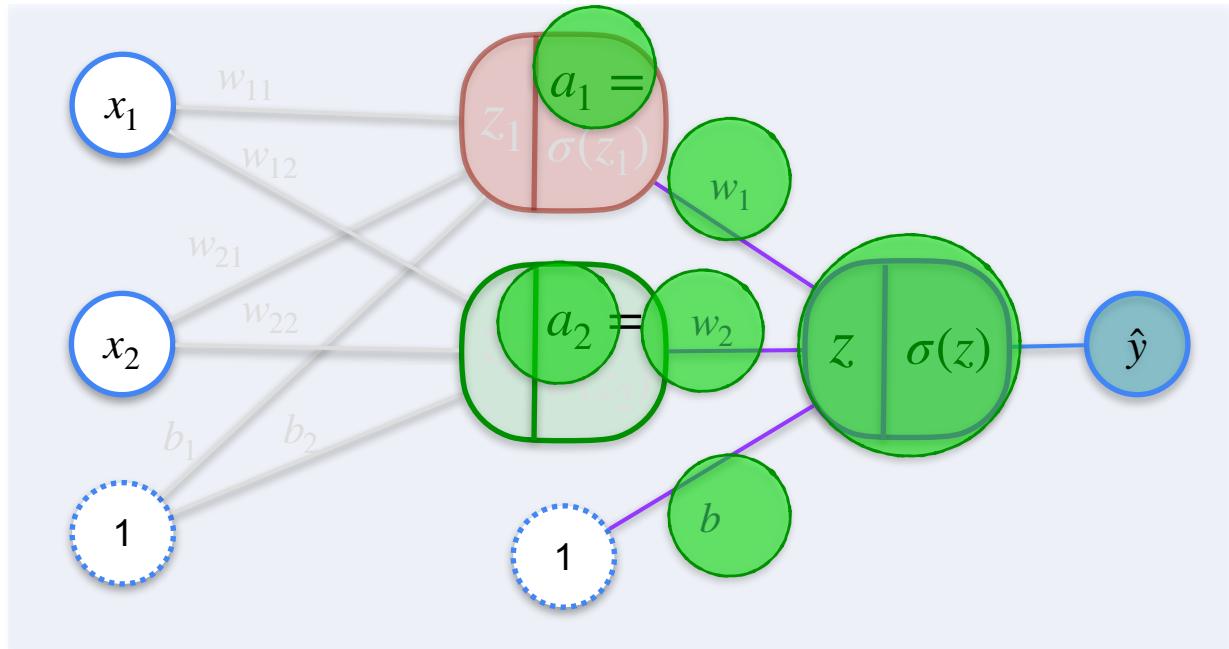
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

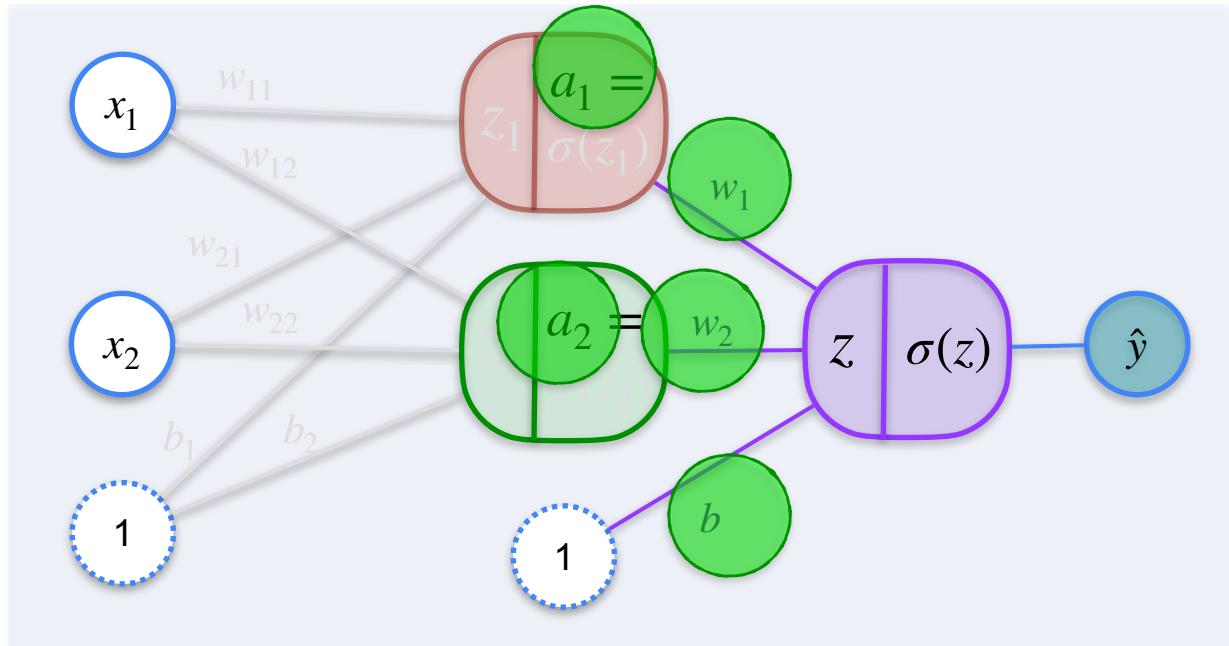
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

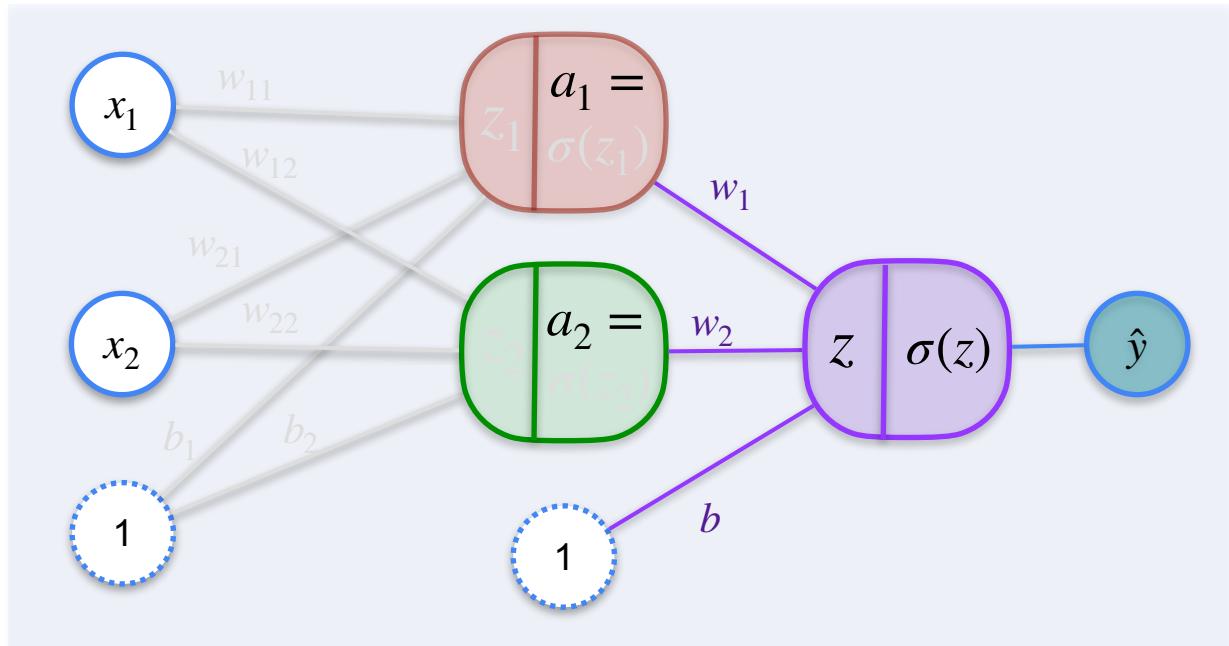
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

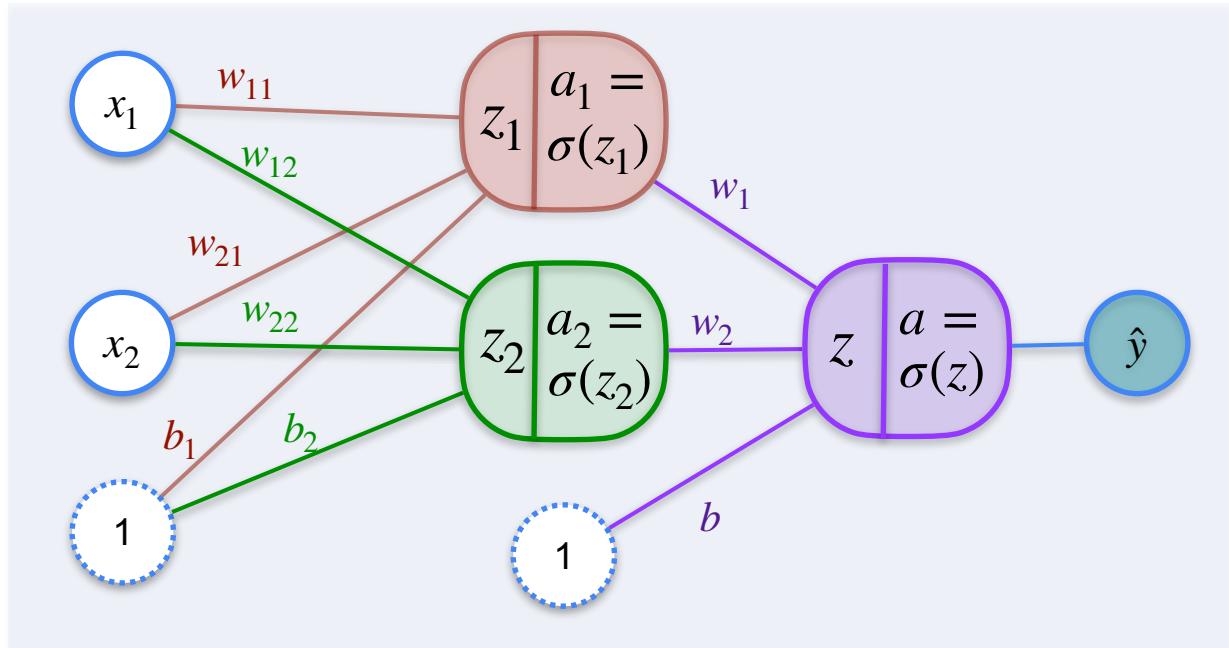
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$



2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

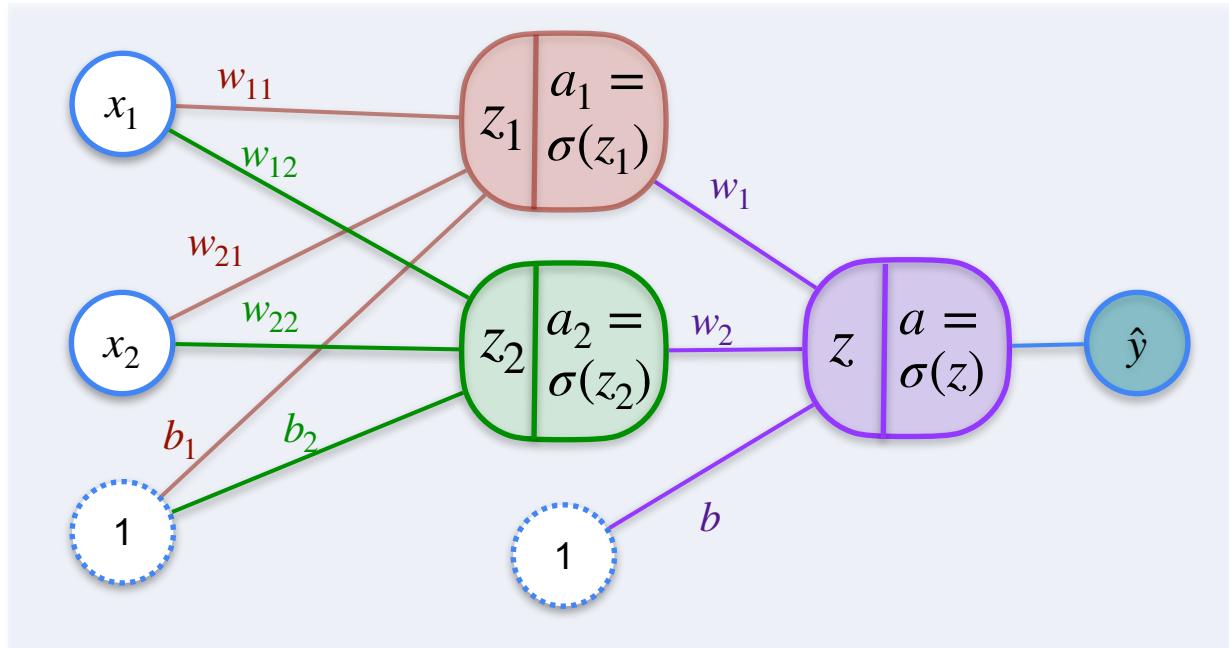
$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1 - y) \log(1 - \hat{y})$$



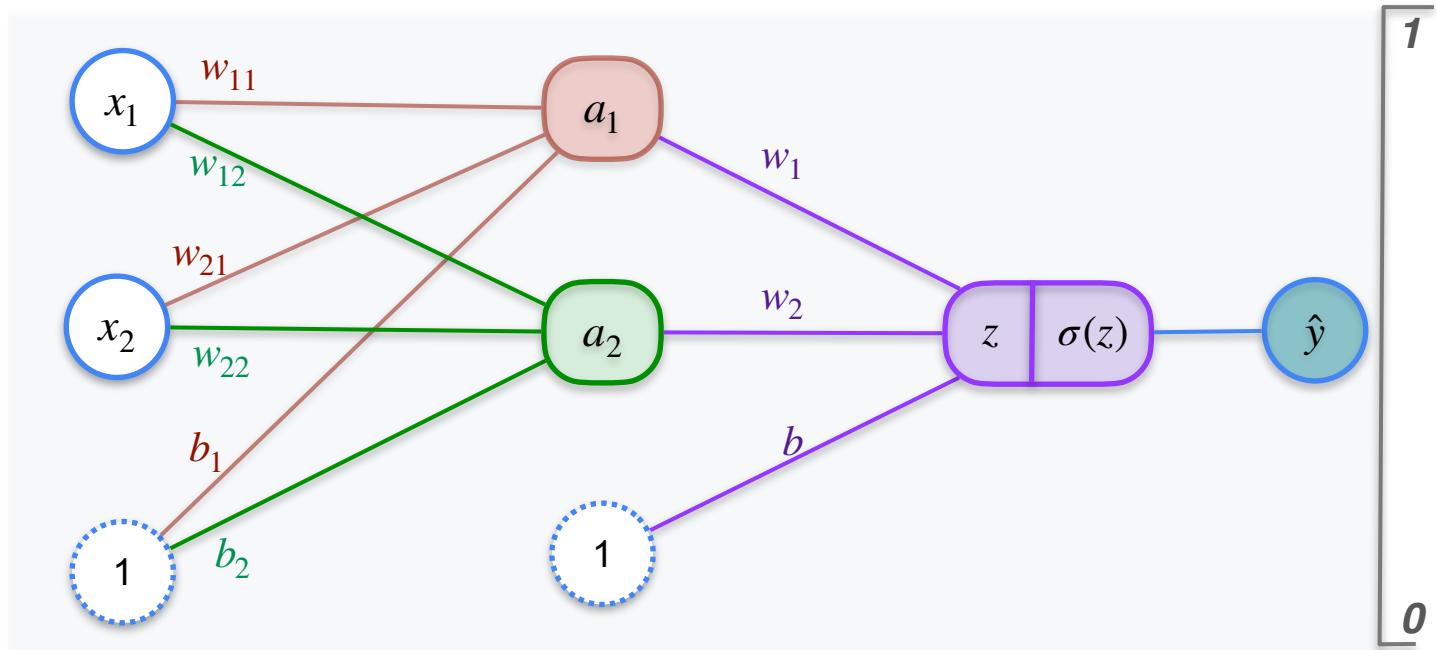


DeepLearning.AI

Optimization in Neural Networks and Newton's Method

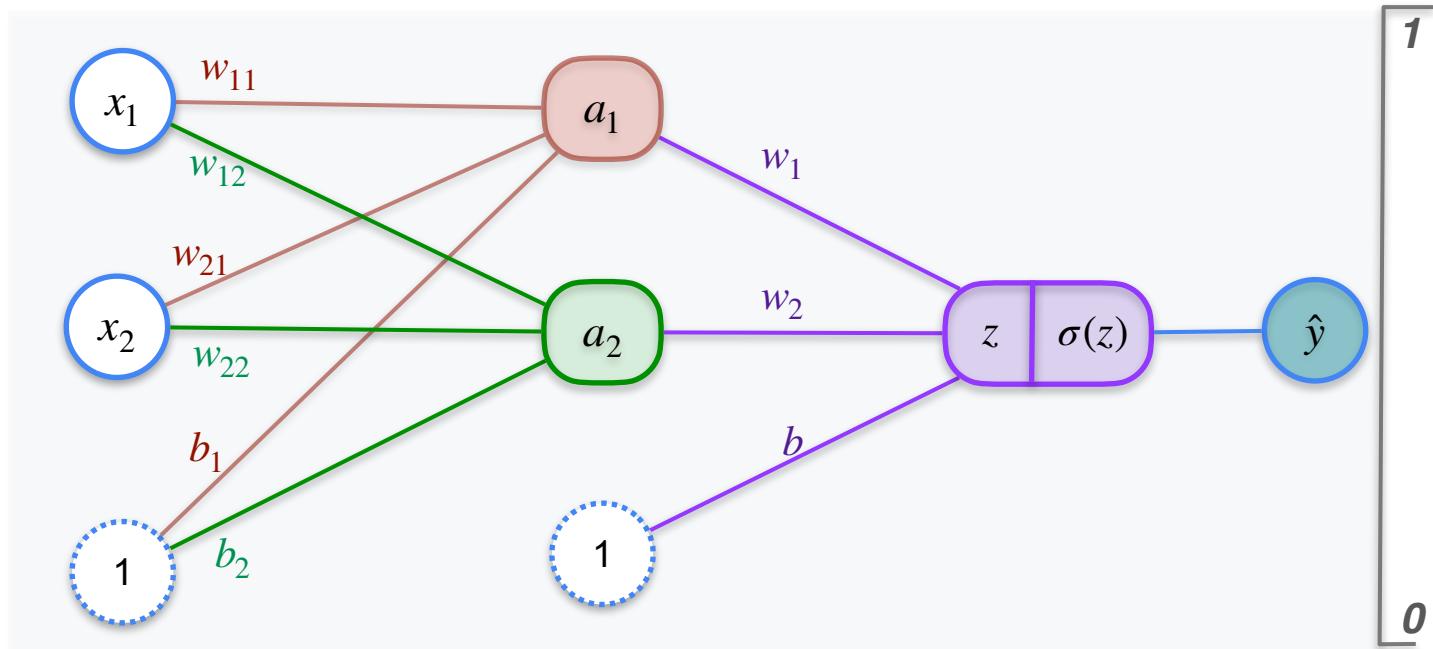
**Classification with a
Neural Network:
Minimizing log-loss**

2,2,1 Neural Network



2,2,1 Neural Network

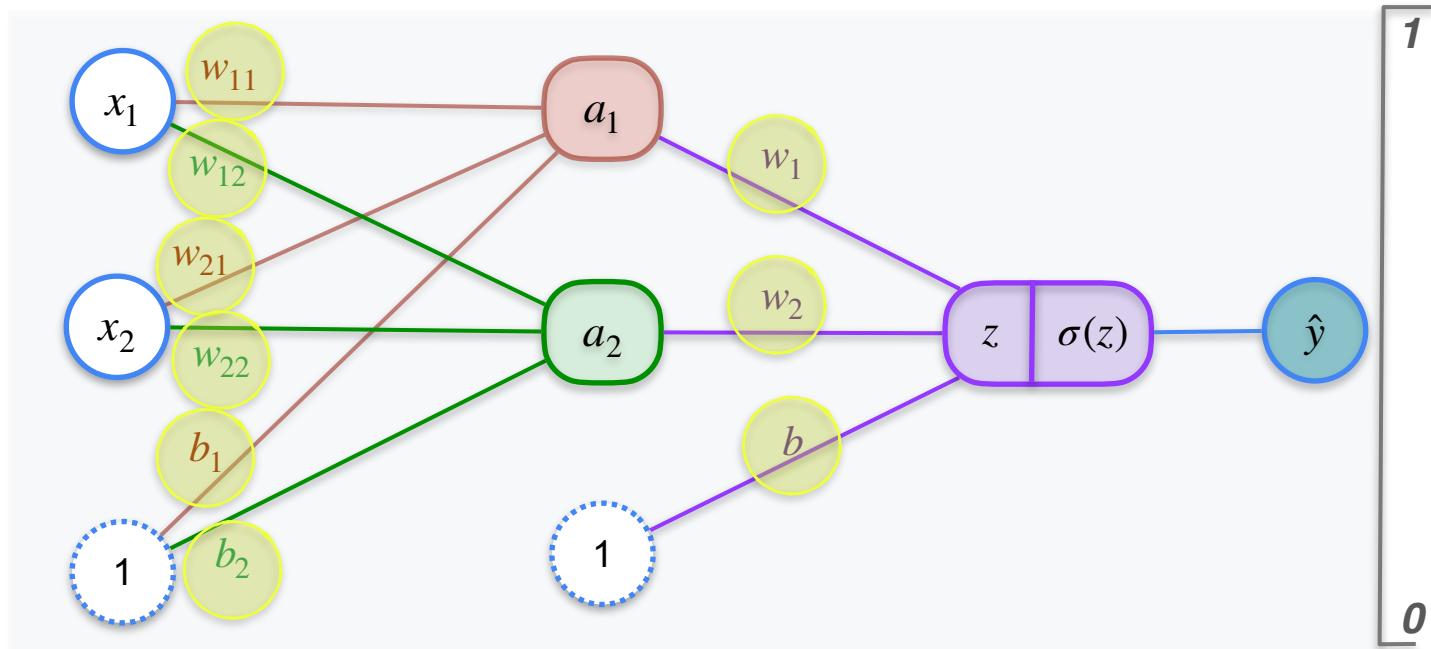
Goal



2,2,1 Neural Network

Goal

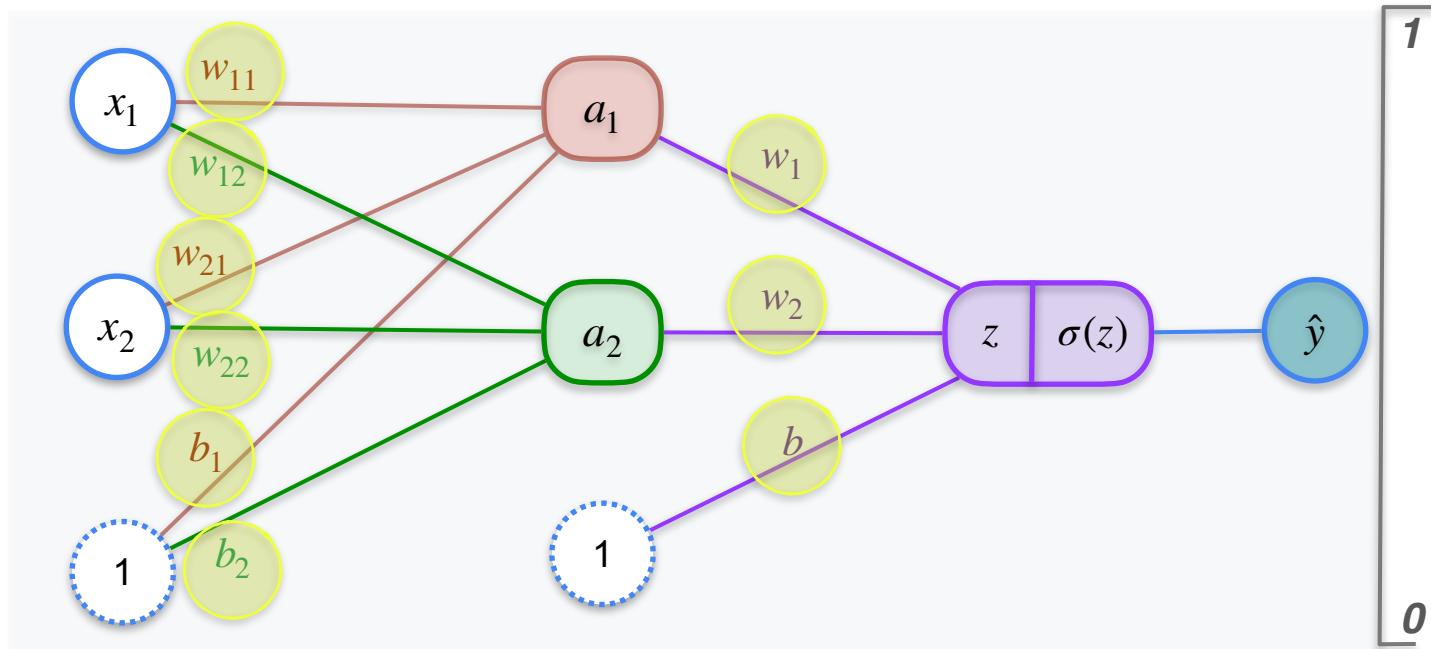
Adjust each of the highlighted weights and biases



2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases

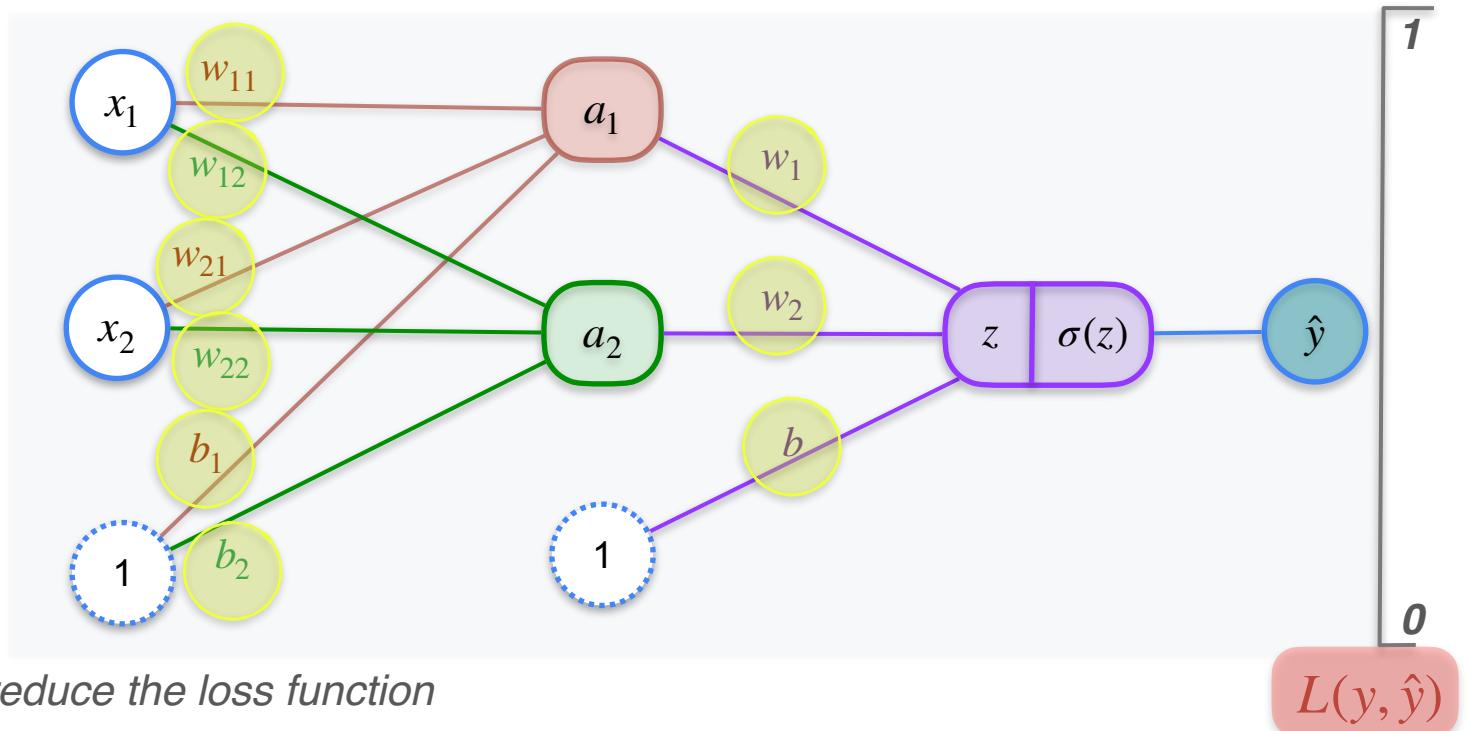


To reduce the loss function

2,2,1 Neural Network

Goal

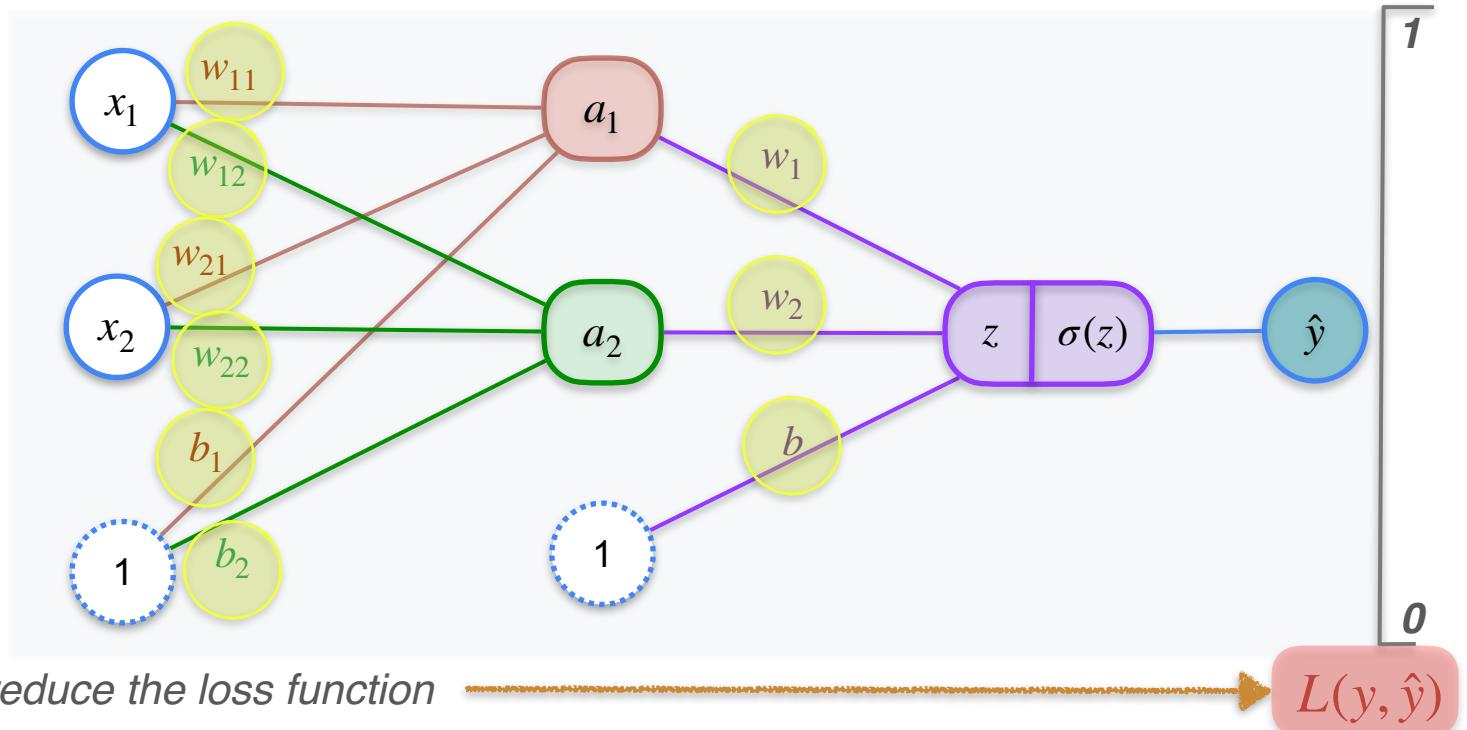
Adjust each of the highlighted weights and biases



2,2,1 Neural Network

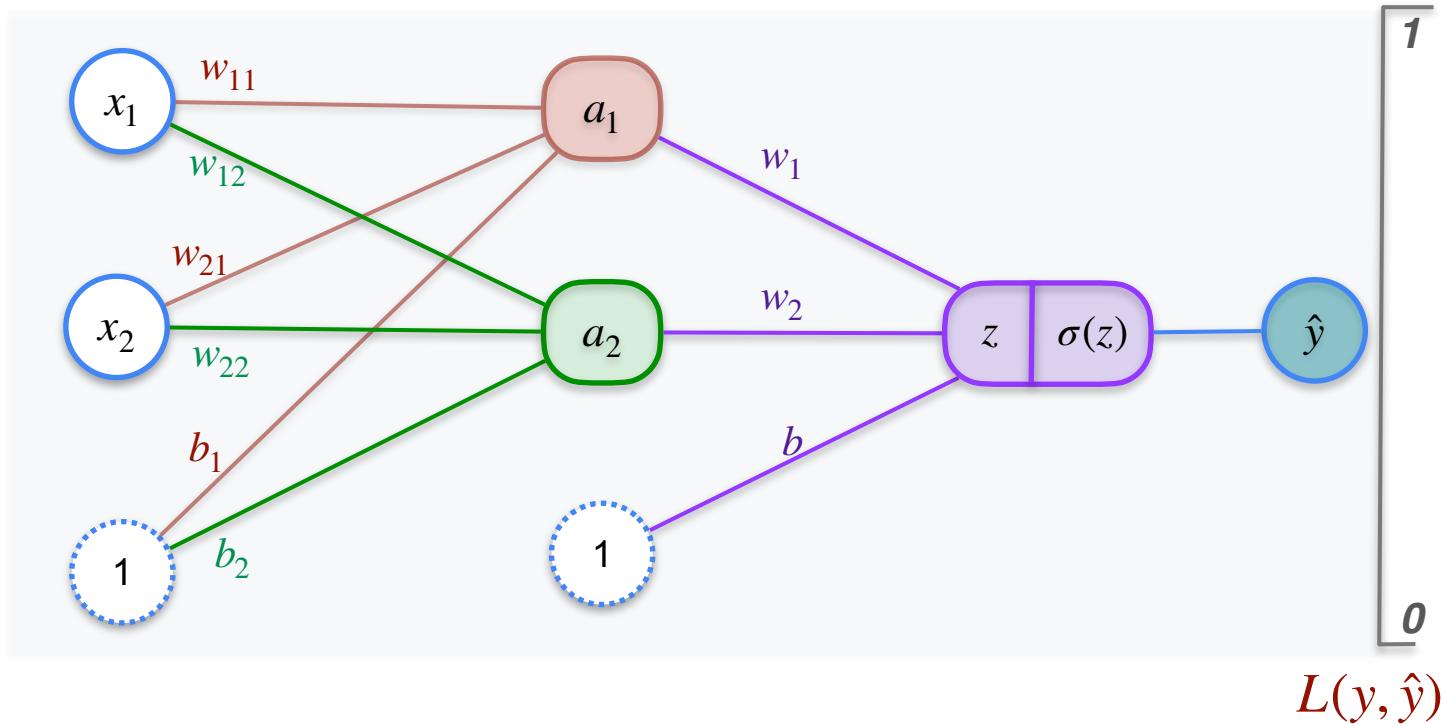
Goal

Adjust each of the highlighted weights and biases

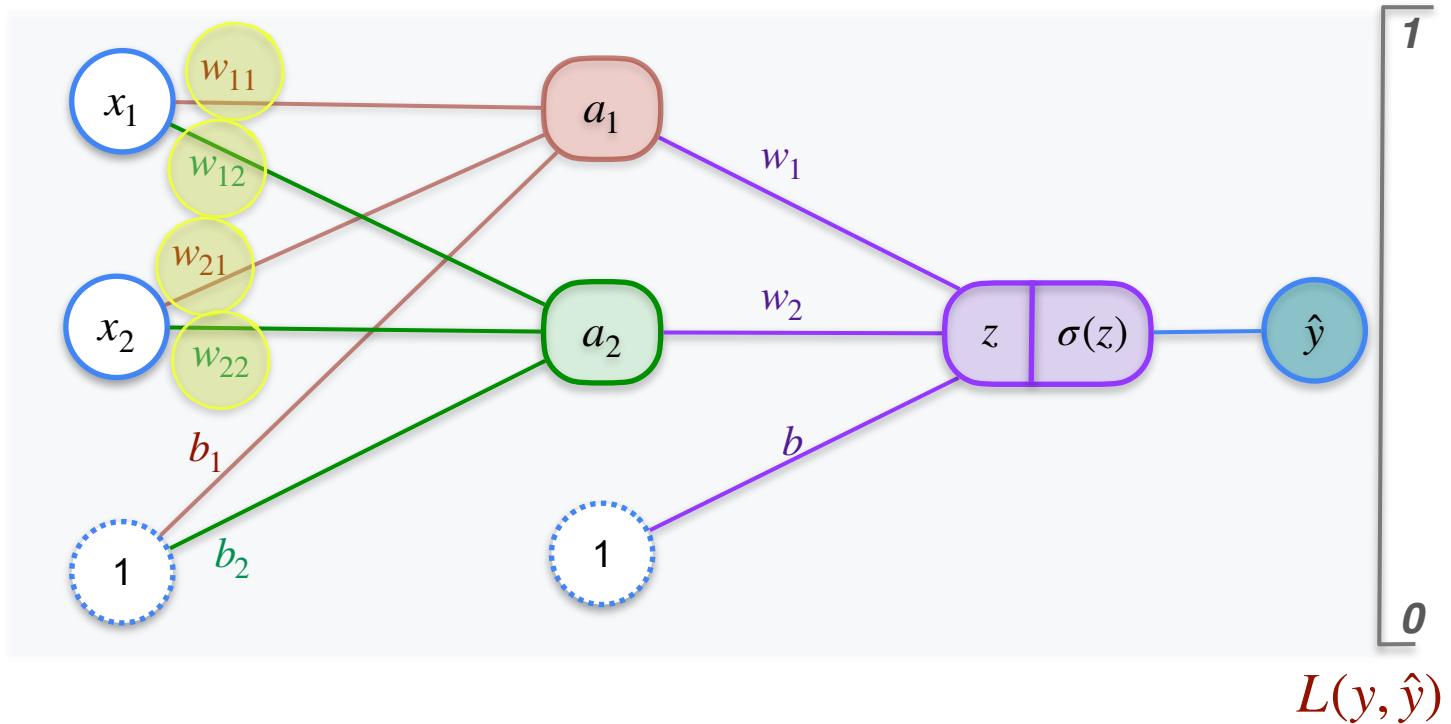


To reduce the loss function

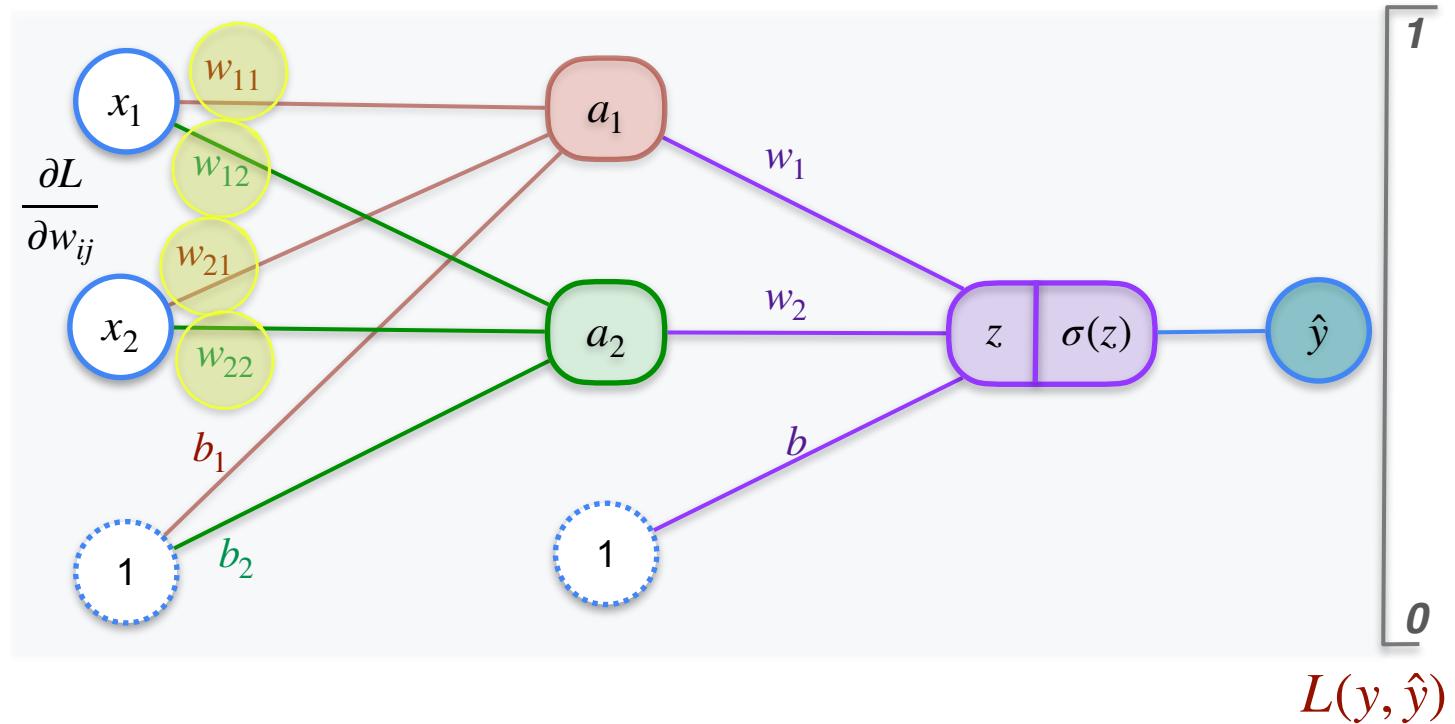
2,2,1 Neural Network



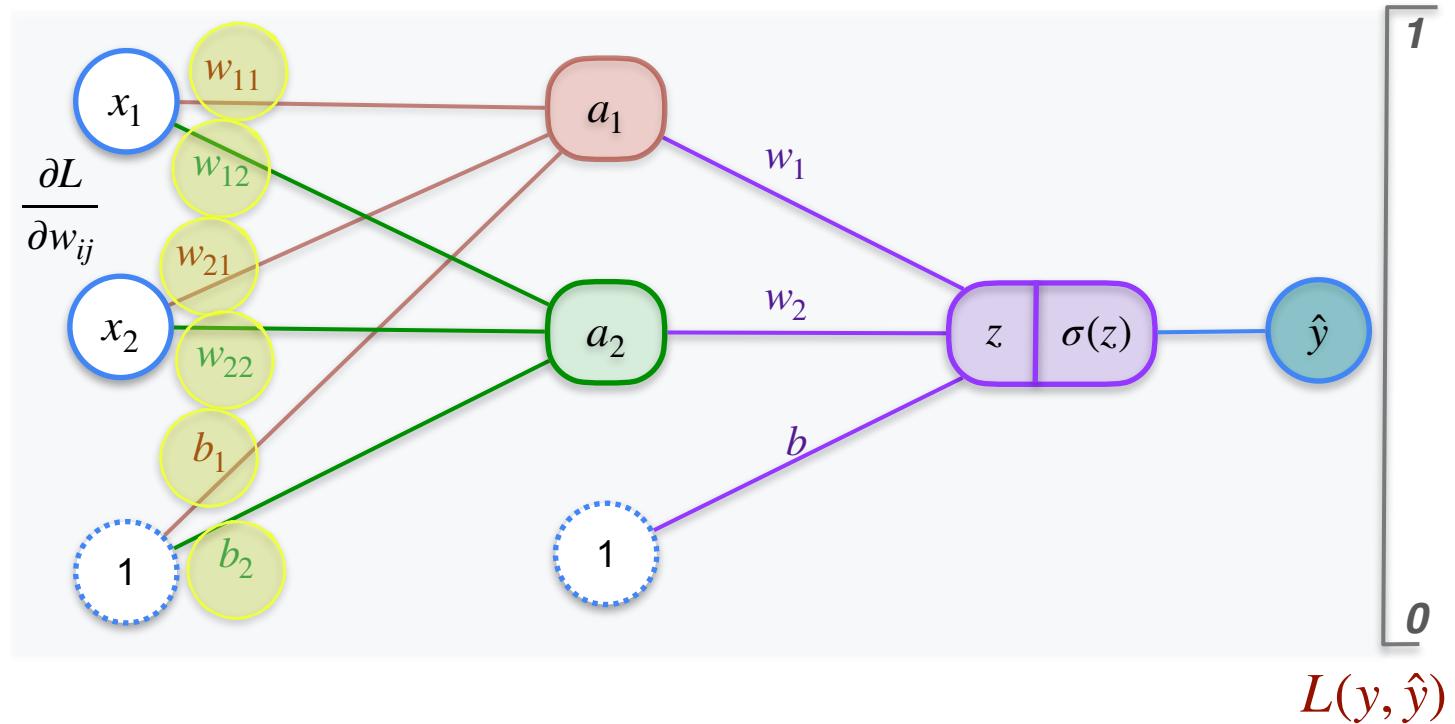
2,2,1 Neural Network



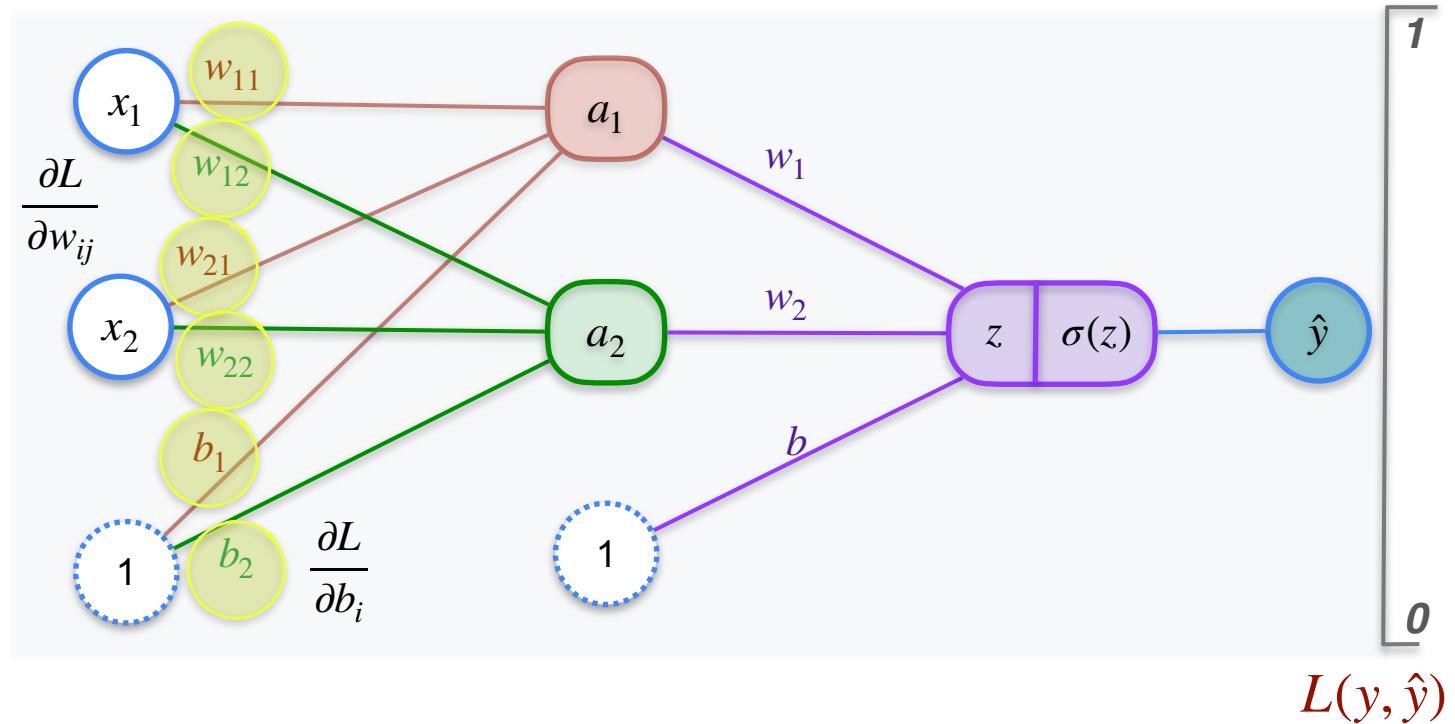
2,2,1 Neural Network



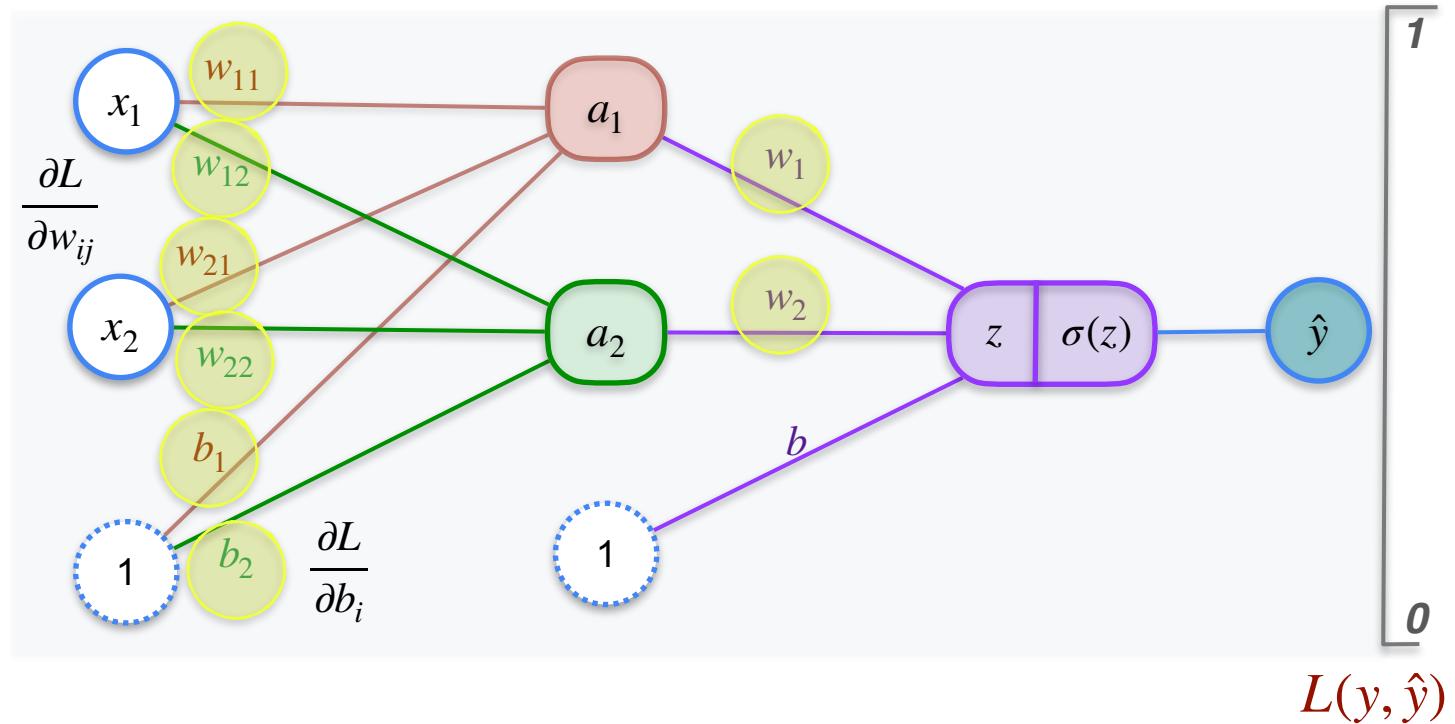
2,2,1 Neural Network



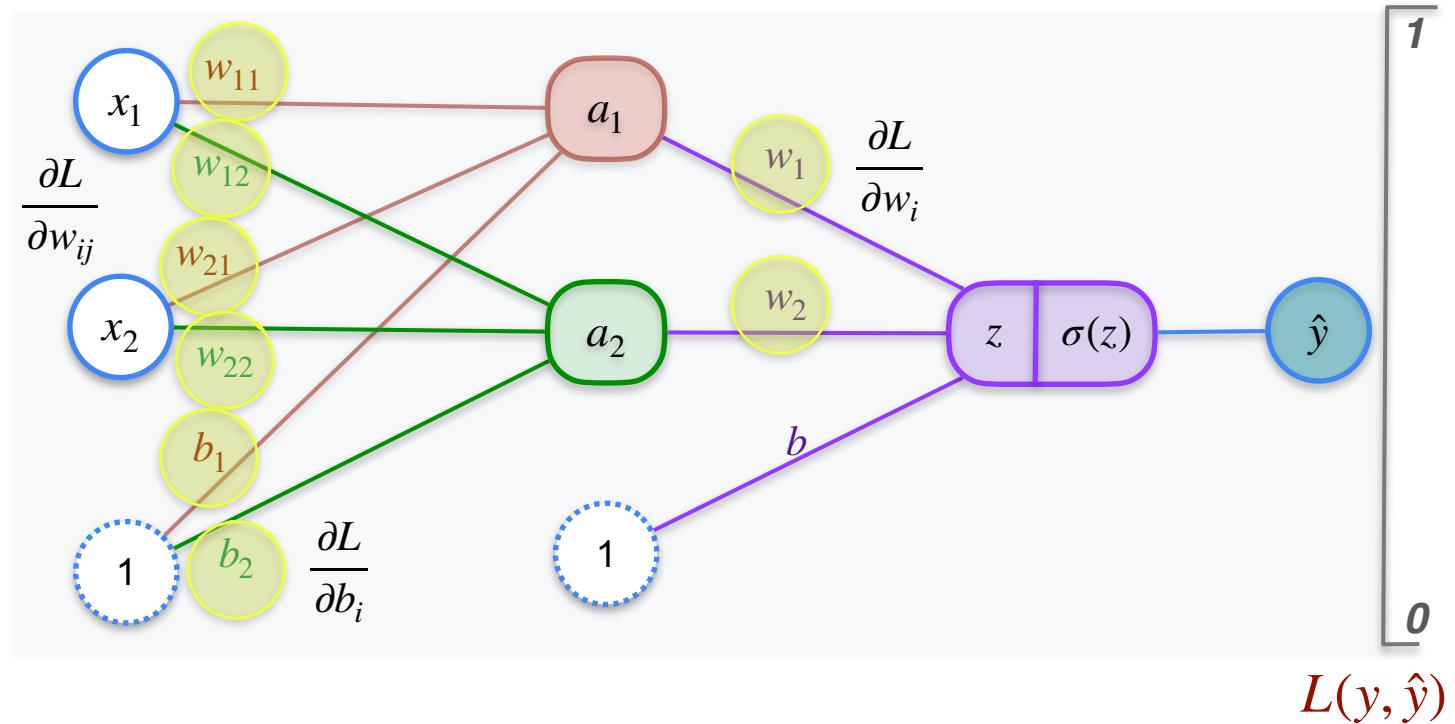
2,2,1 Neural Network



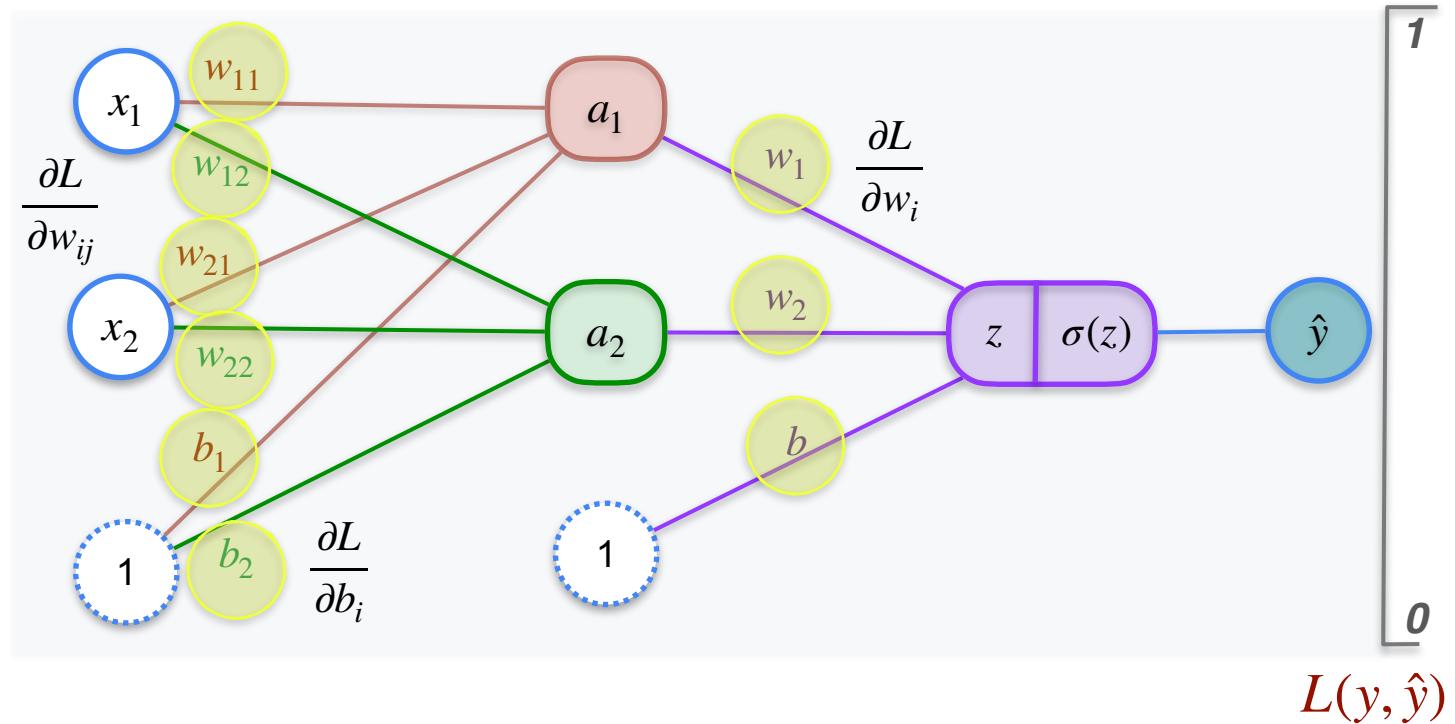
2,2,1 Neural Network



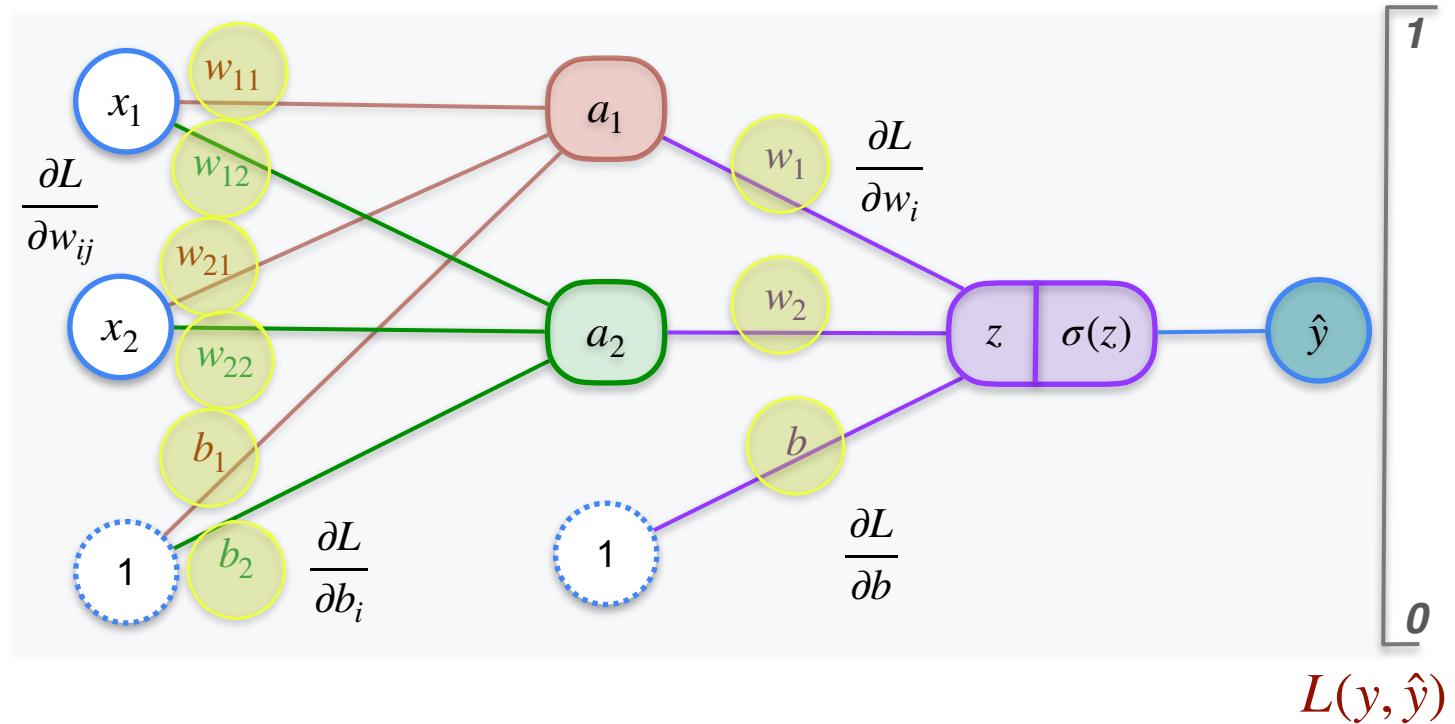
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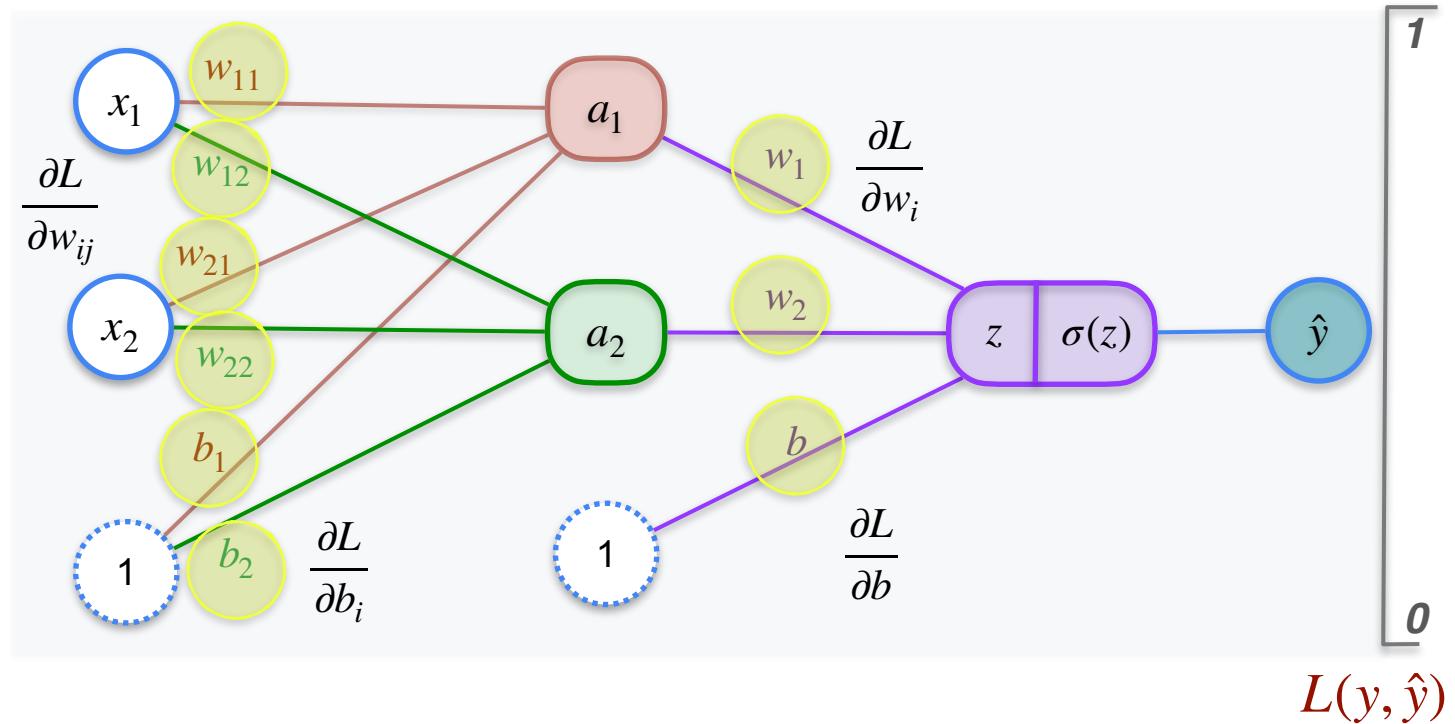
2,2,1 Neural Network



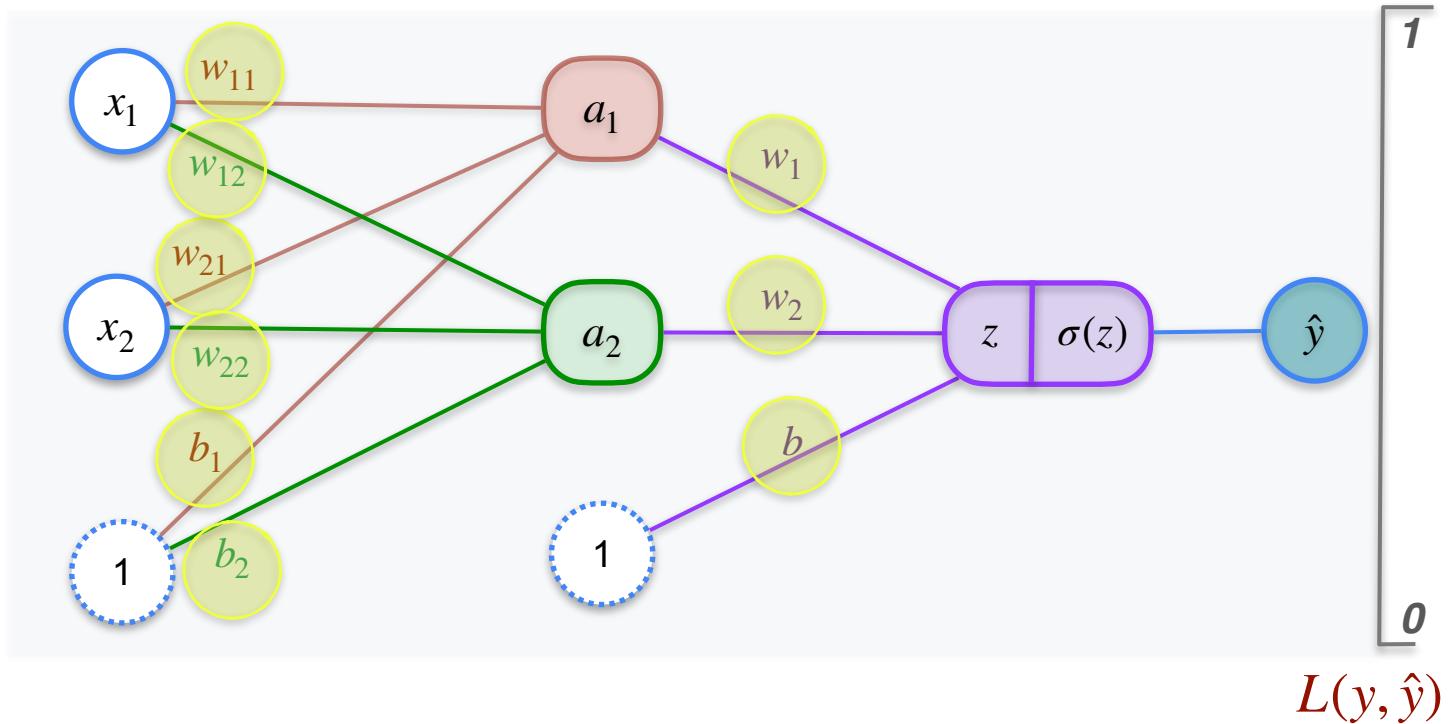
2,2,1 Neural Network



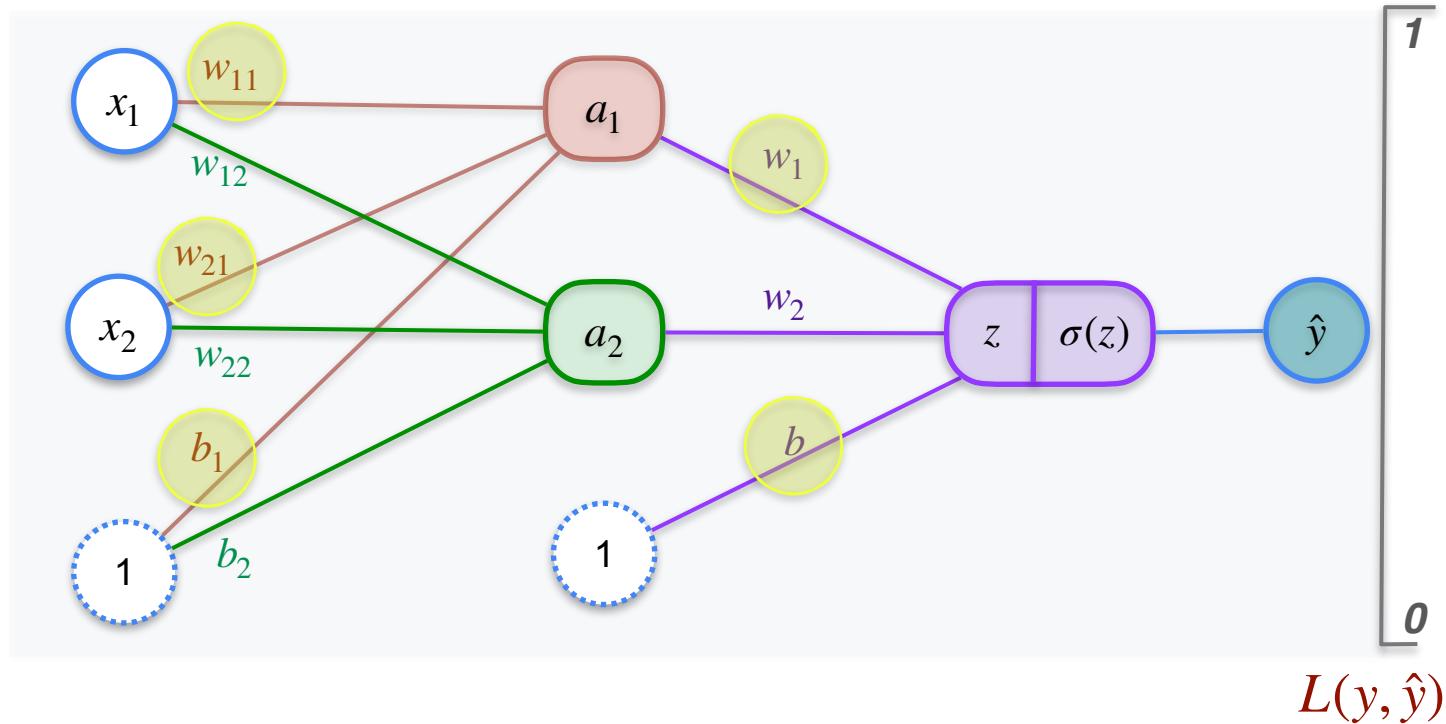
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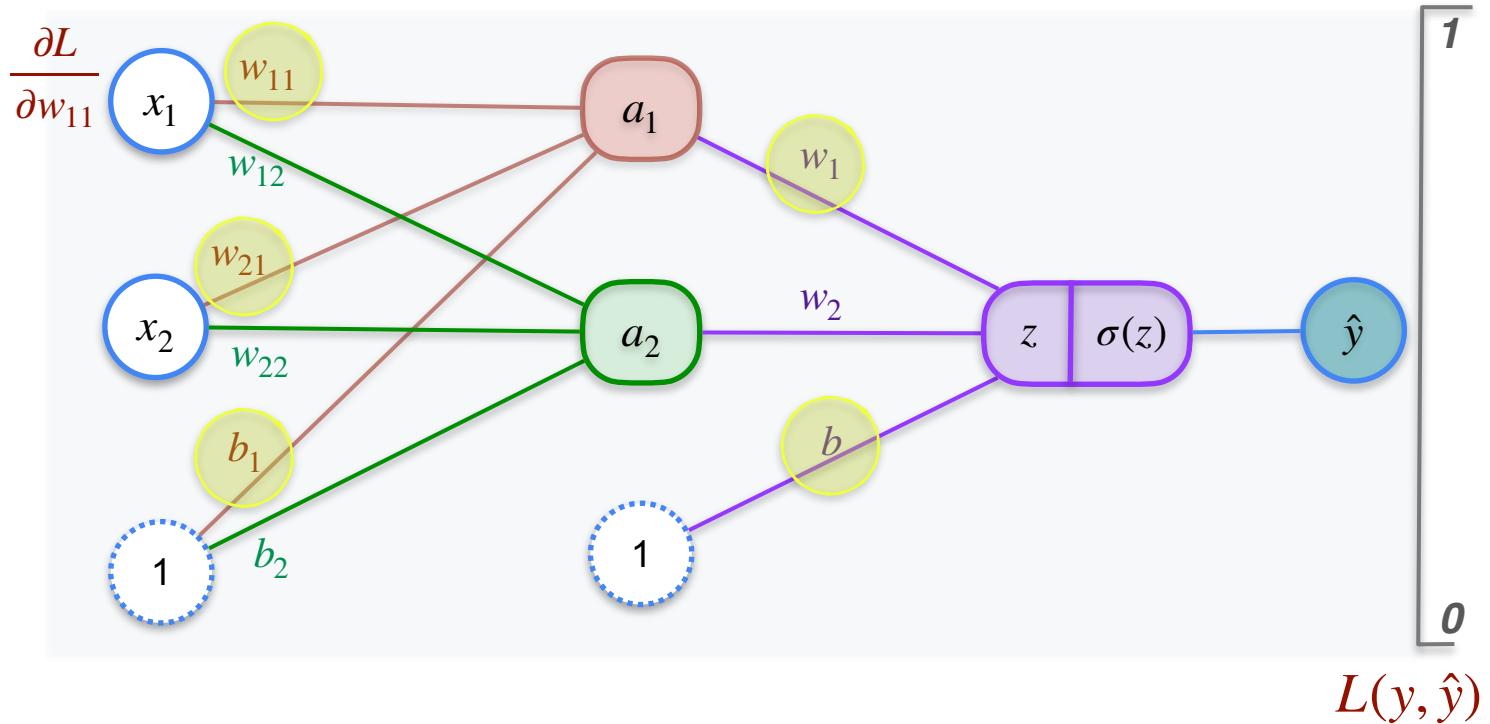
2,2,1 Neural Network



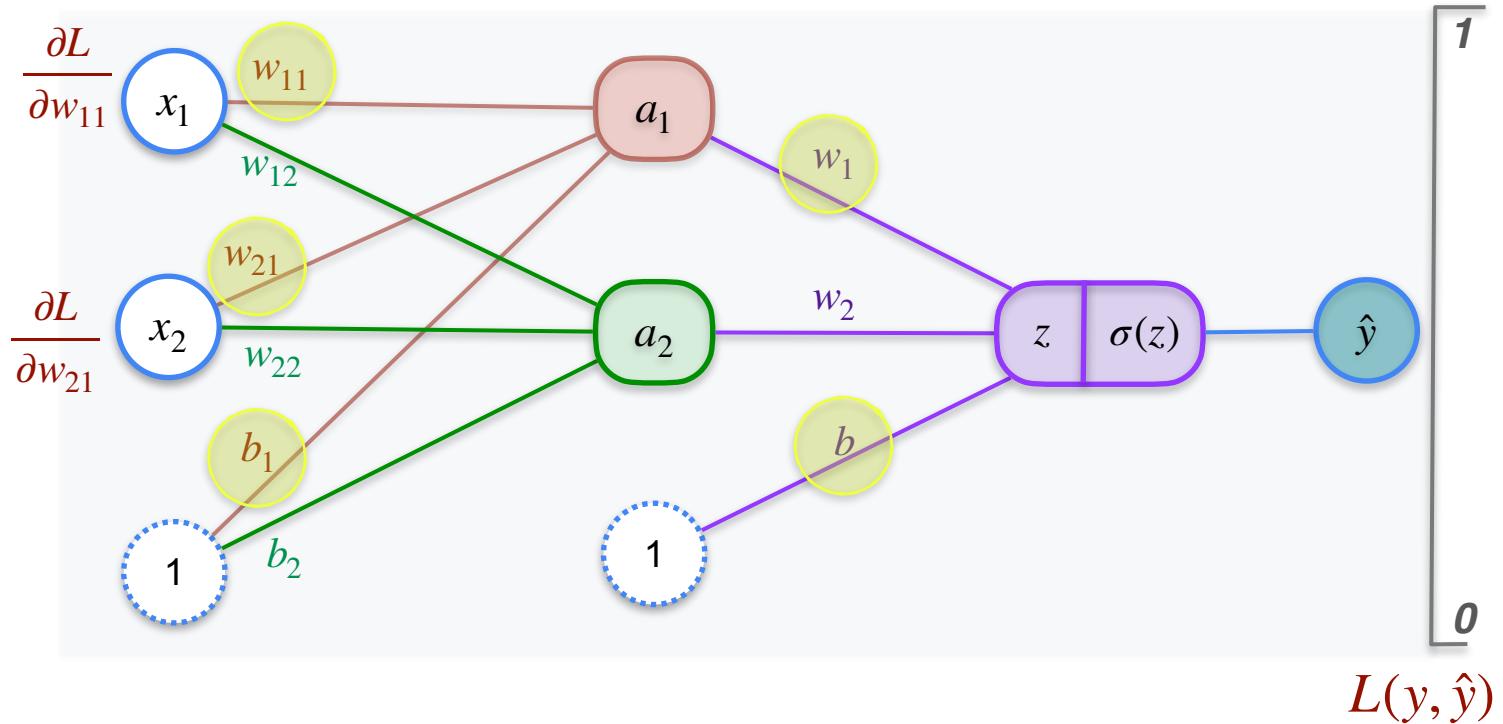
2,2,1 Neural Network



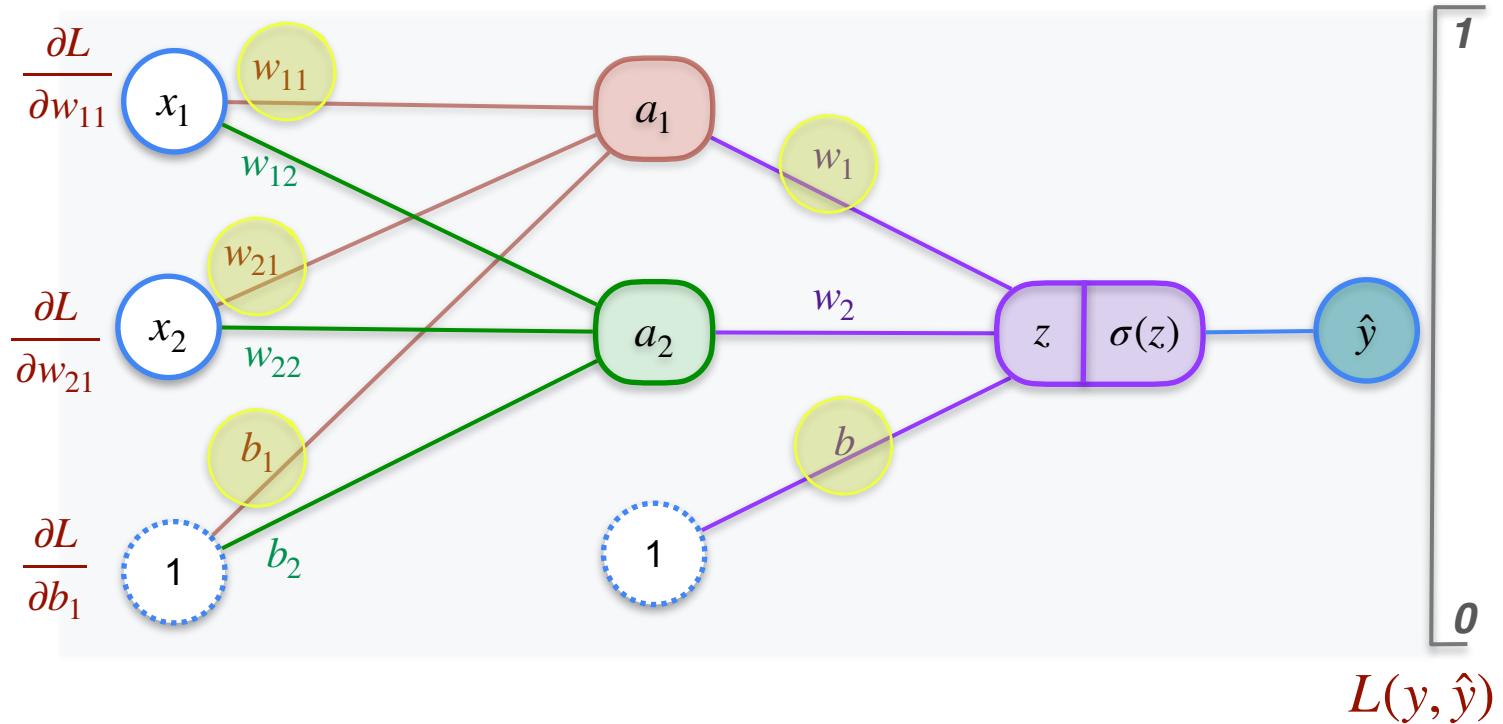
2,2,1 Neural Network



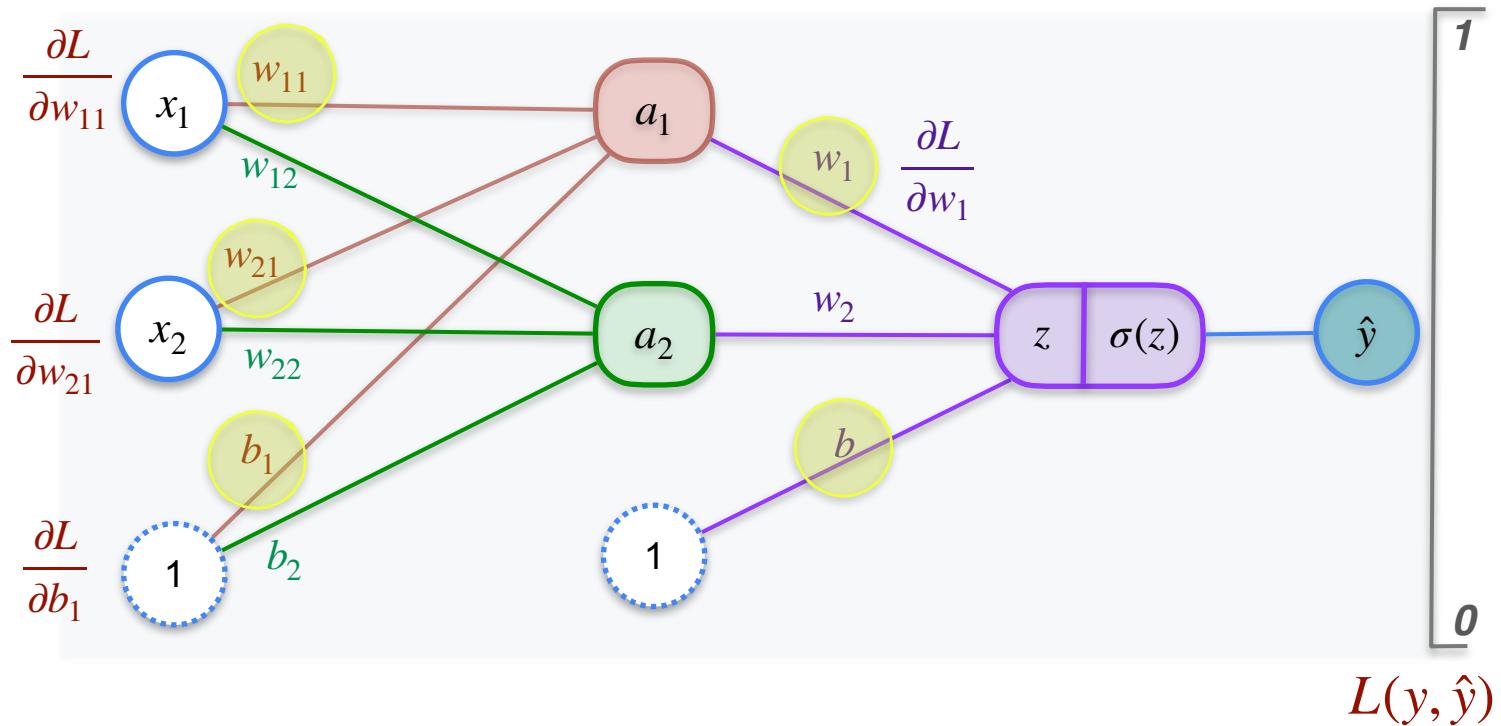
2,2,1 Neural Network



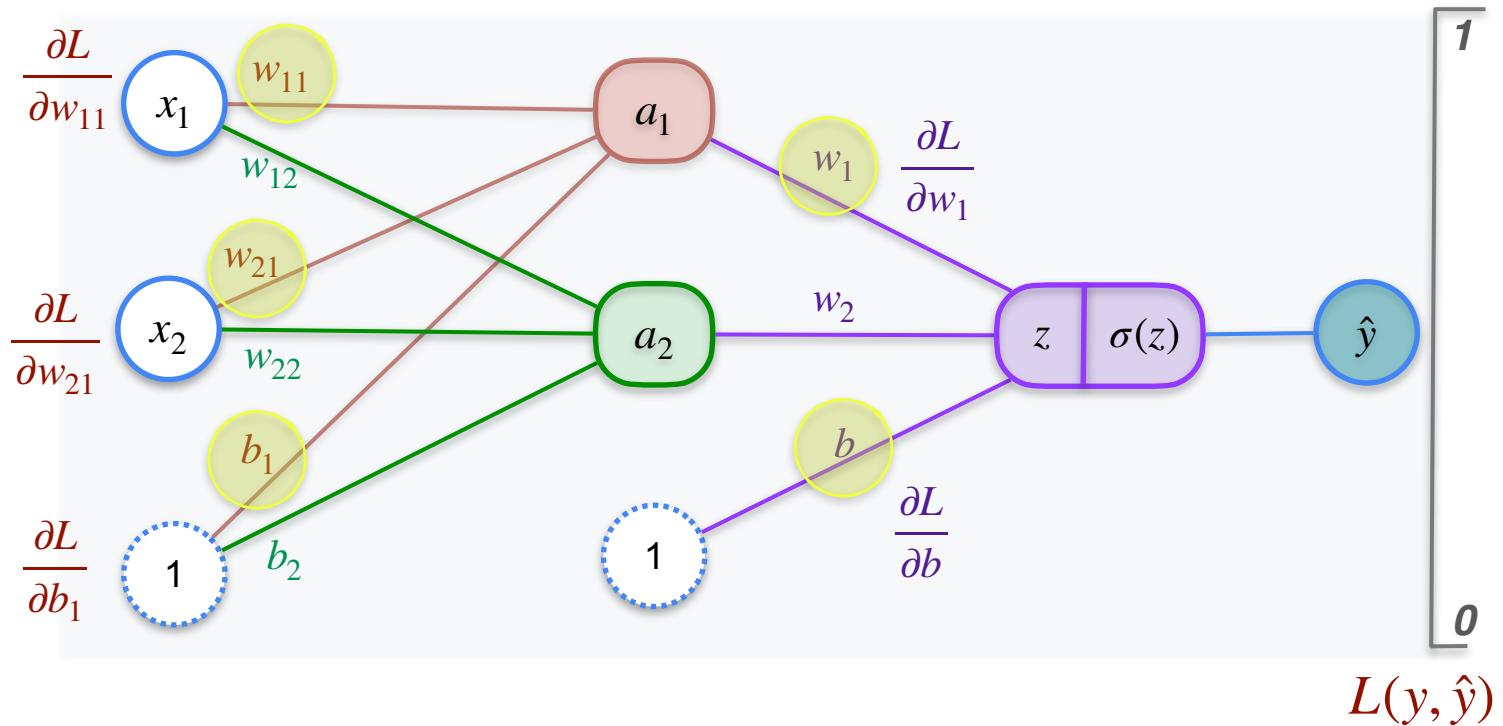
2,2,1 Neural Network



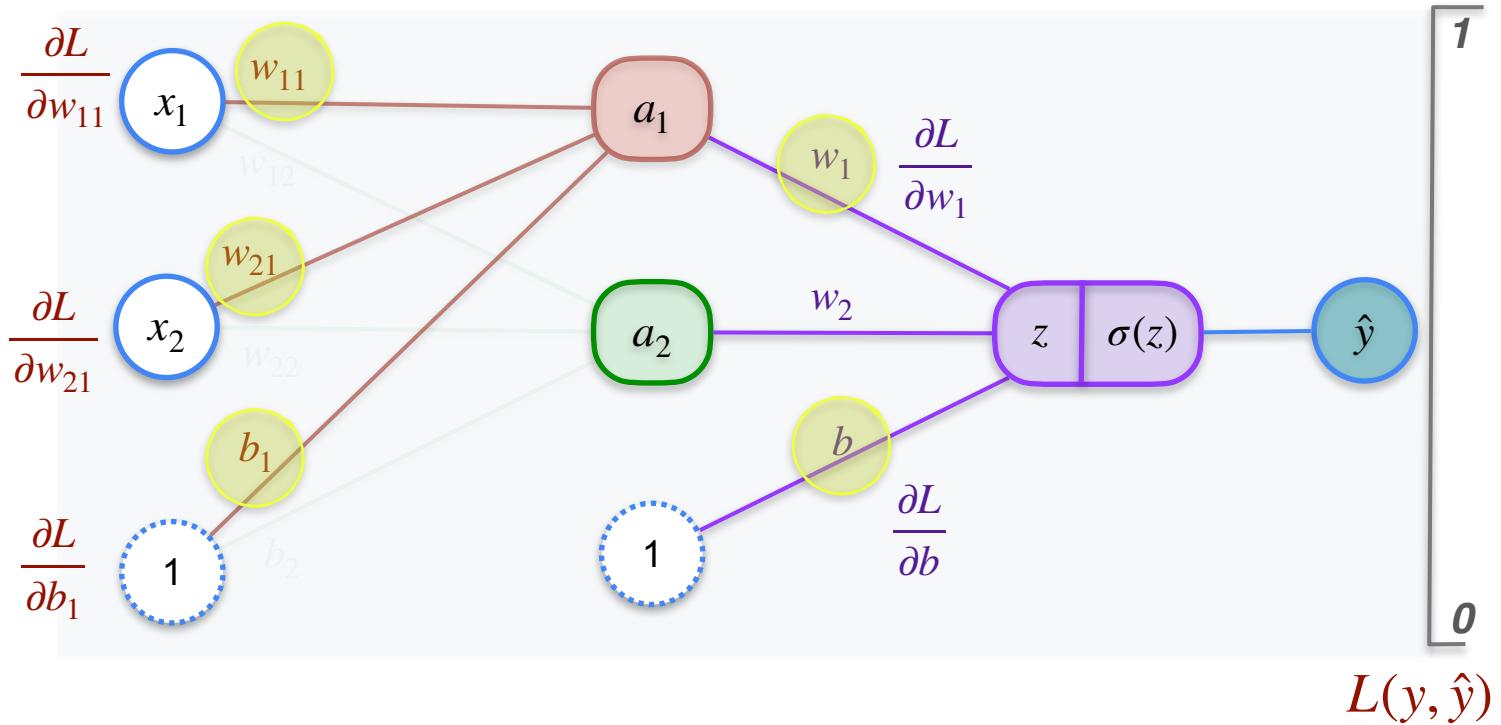
2,2,1 Neural Network



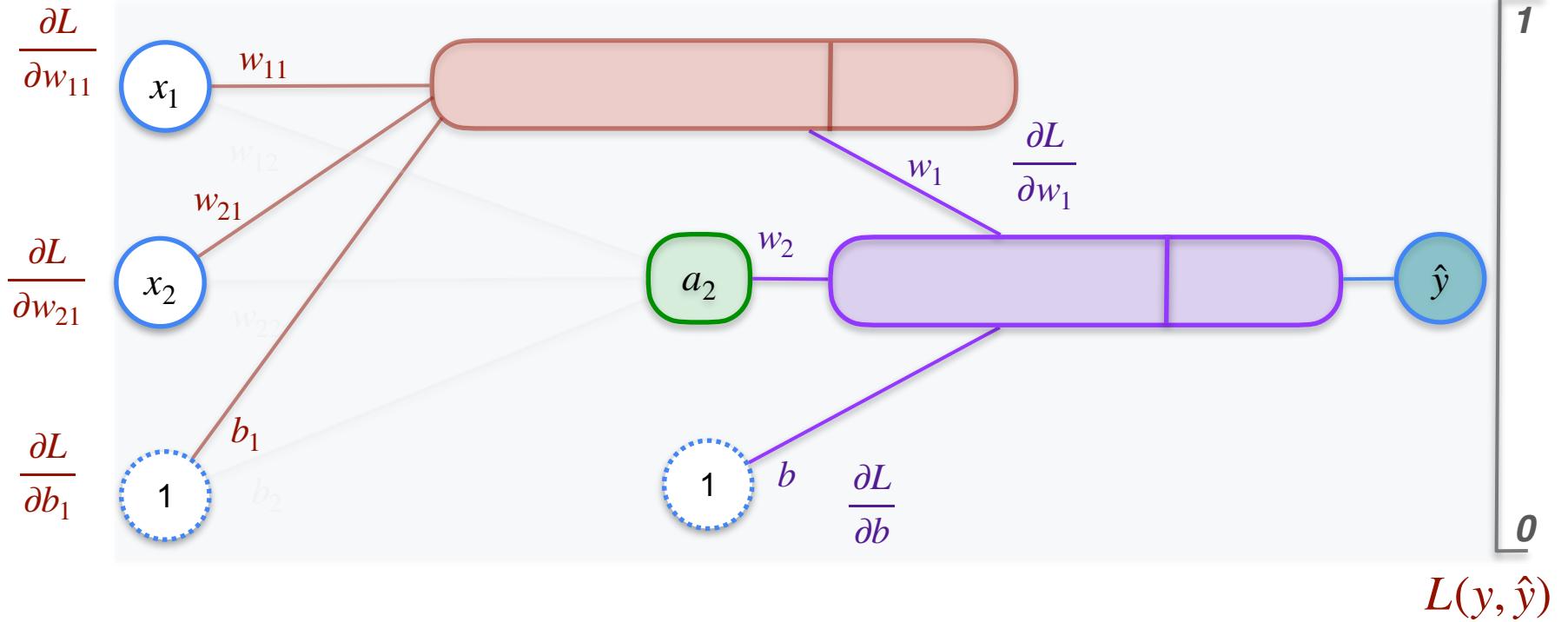
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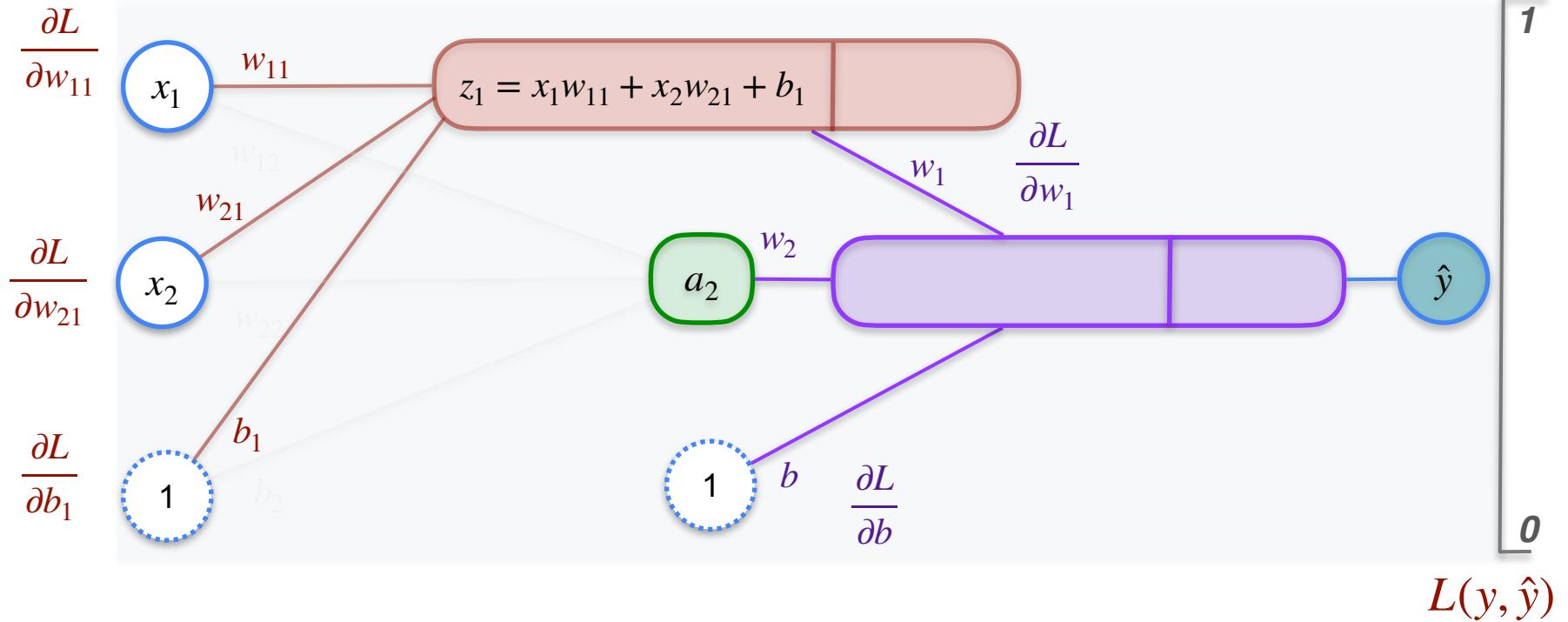
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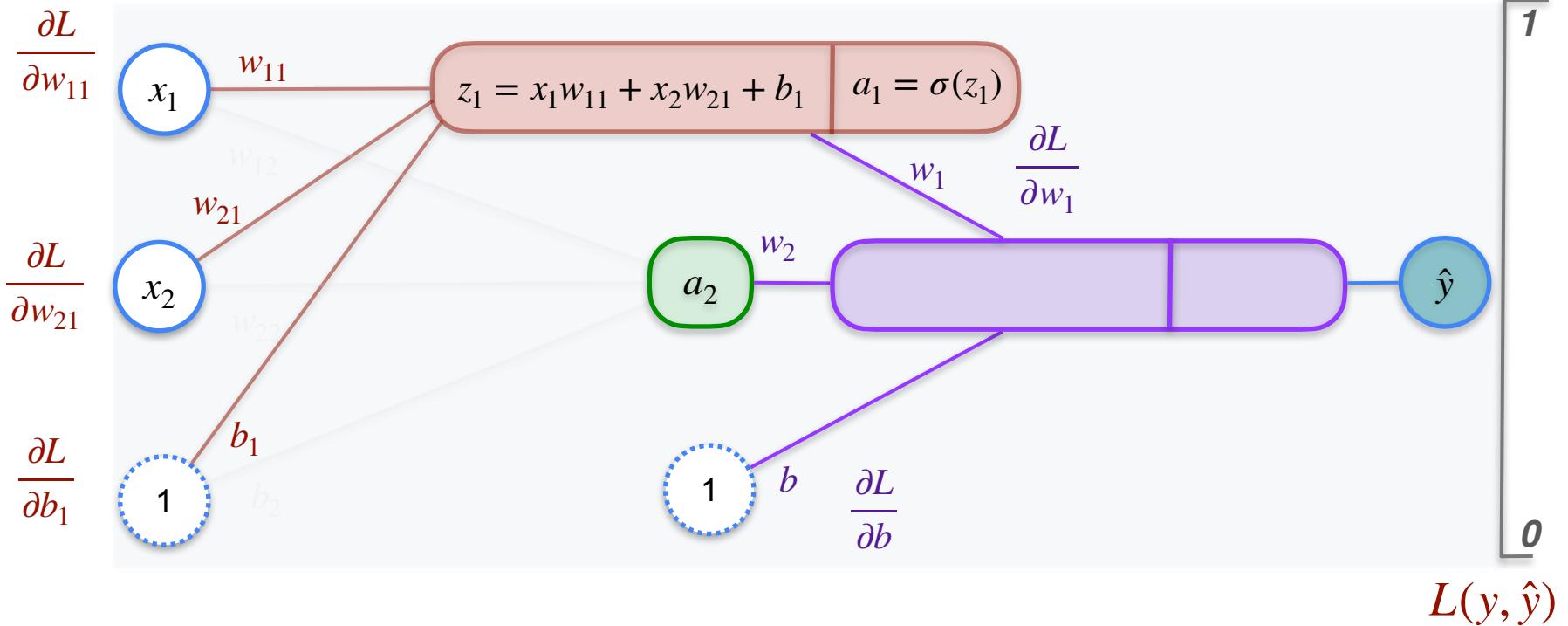
2,2,1 Neural Network



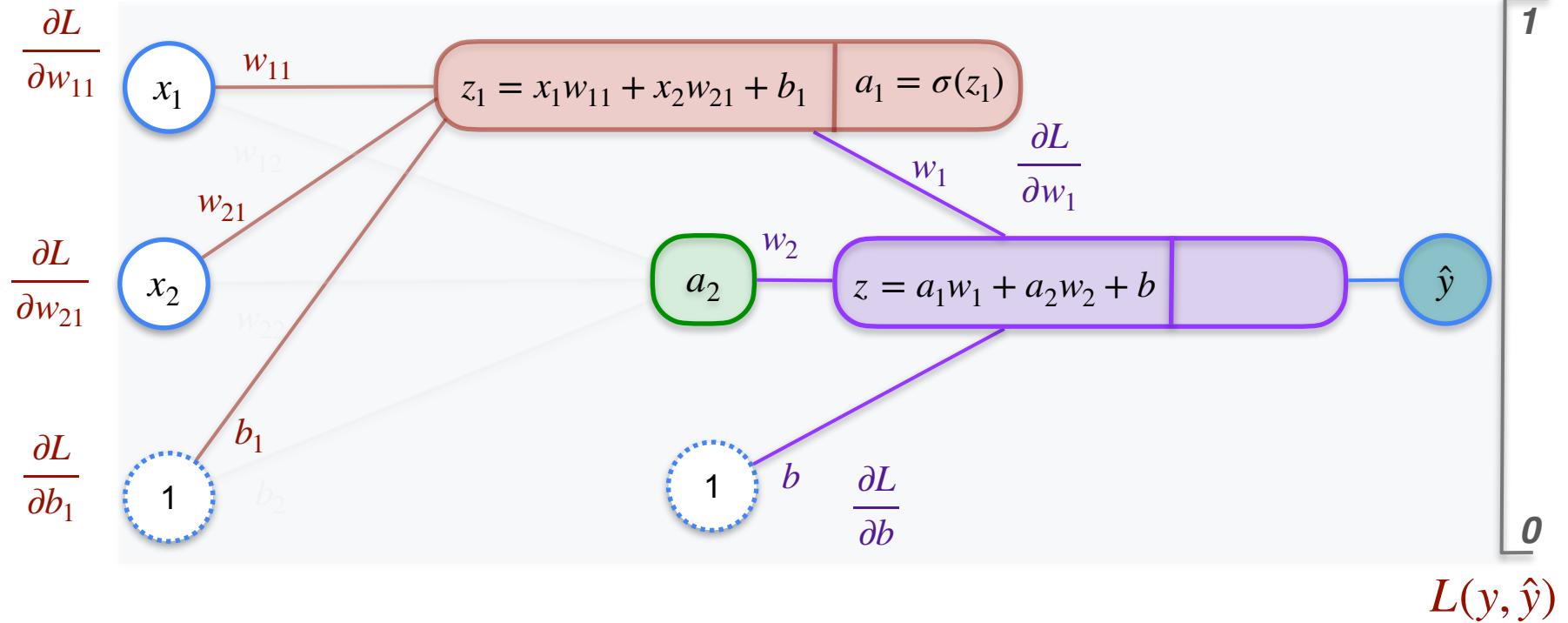
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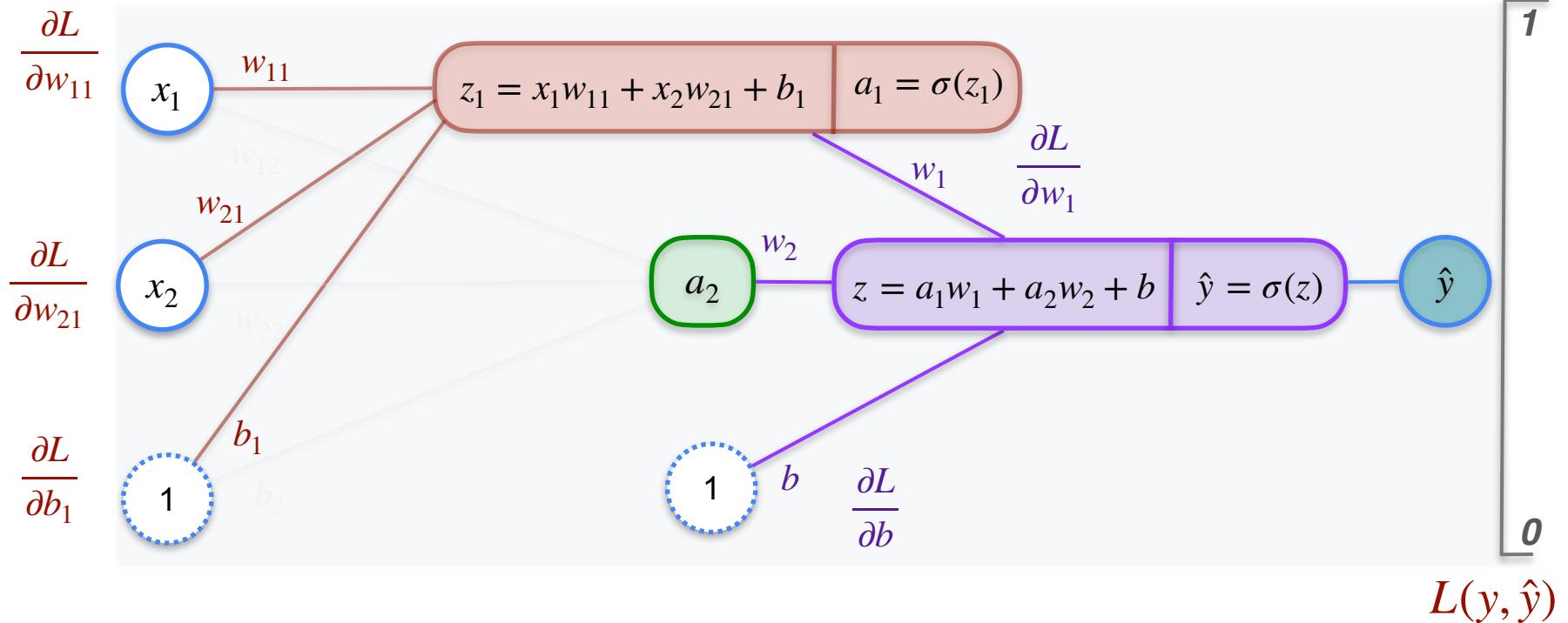
2,2,1 Neural Network



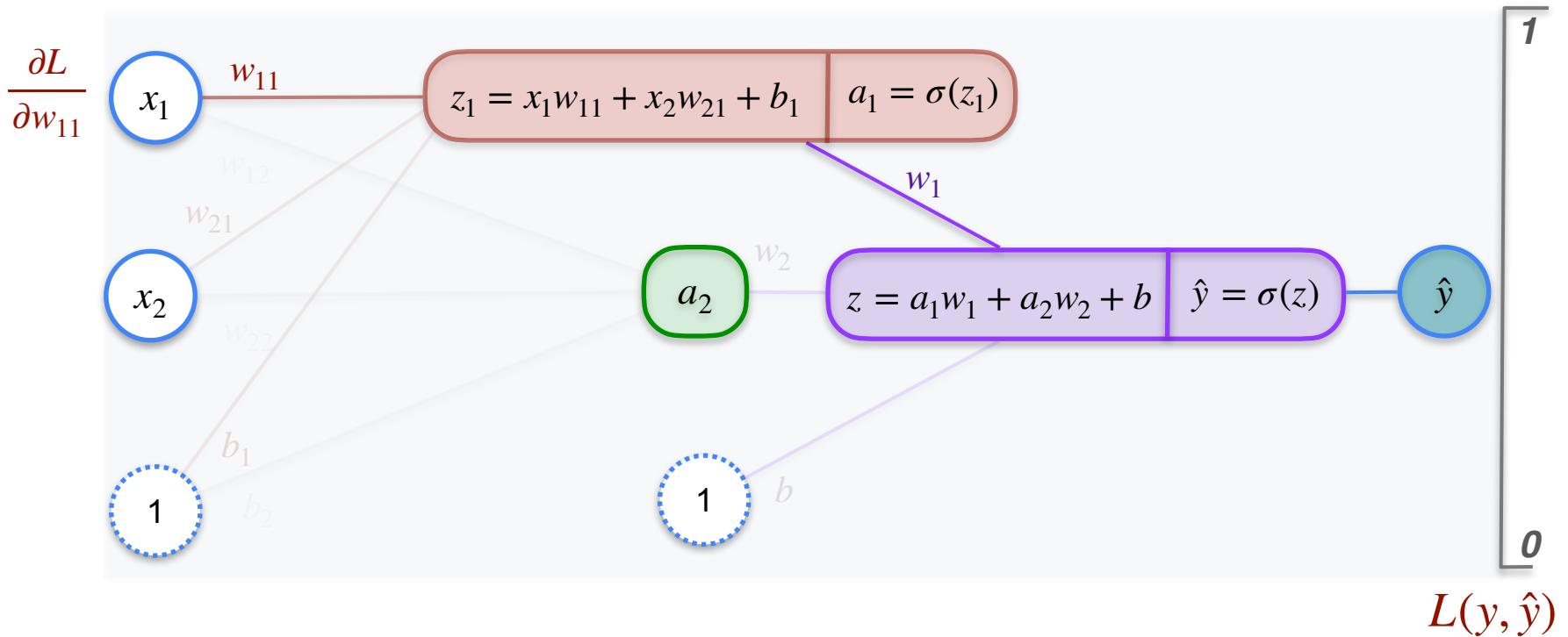
2,2,1 Neural Network



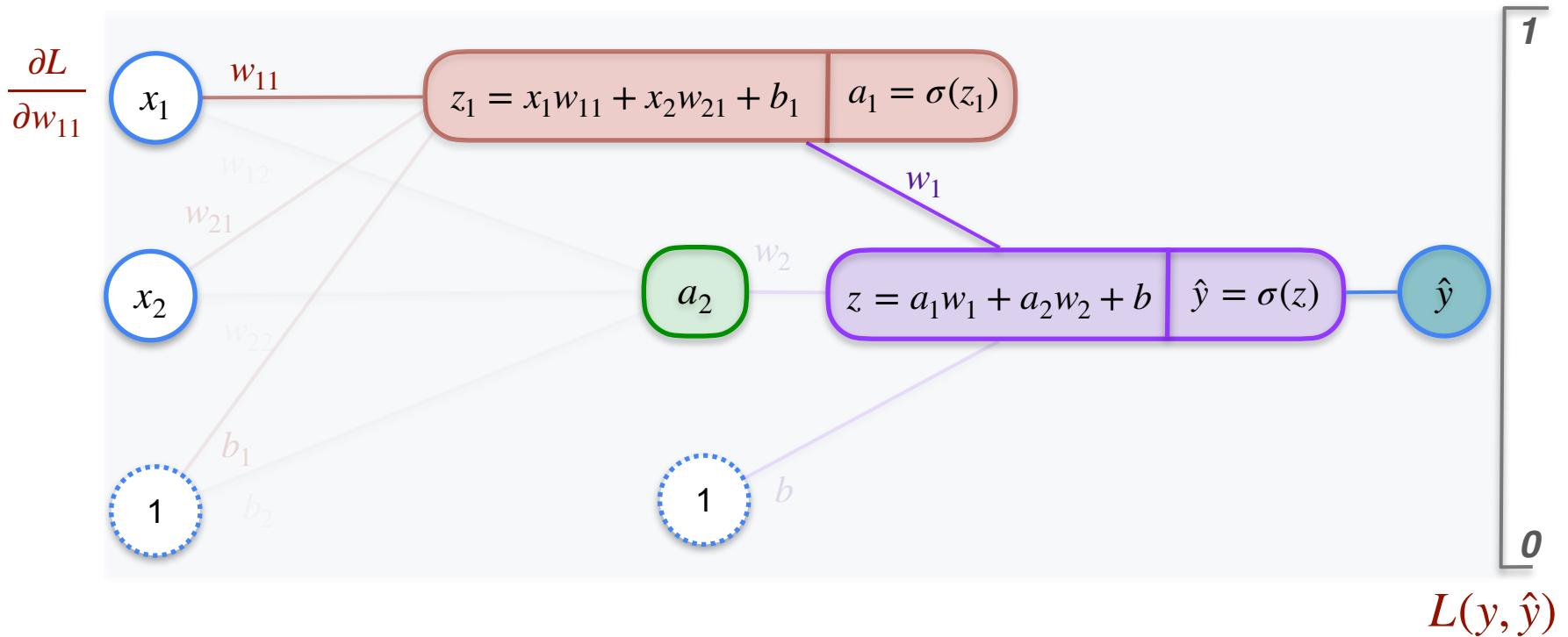
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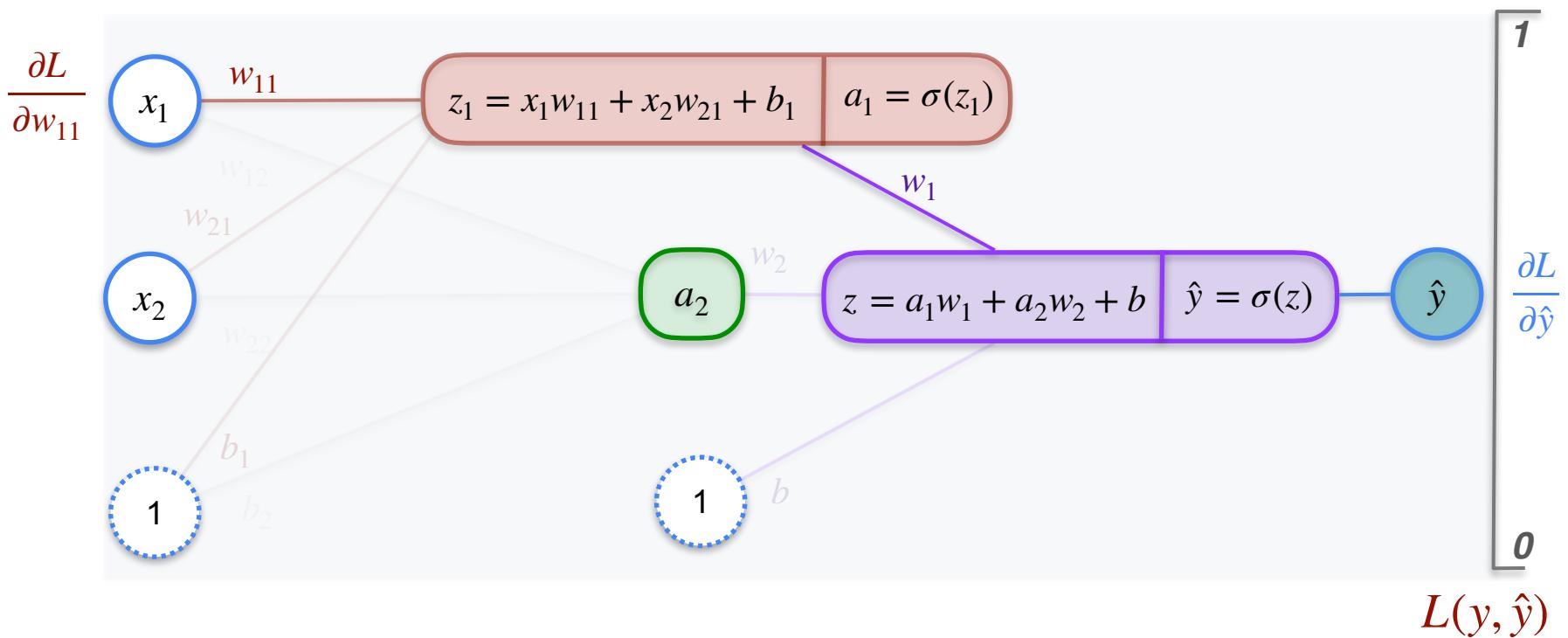
2,2,1 Neural Network



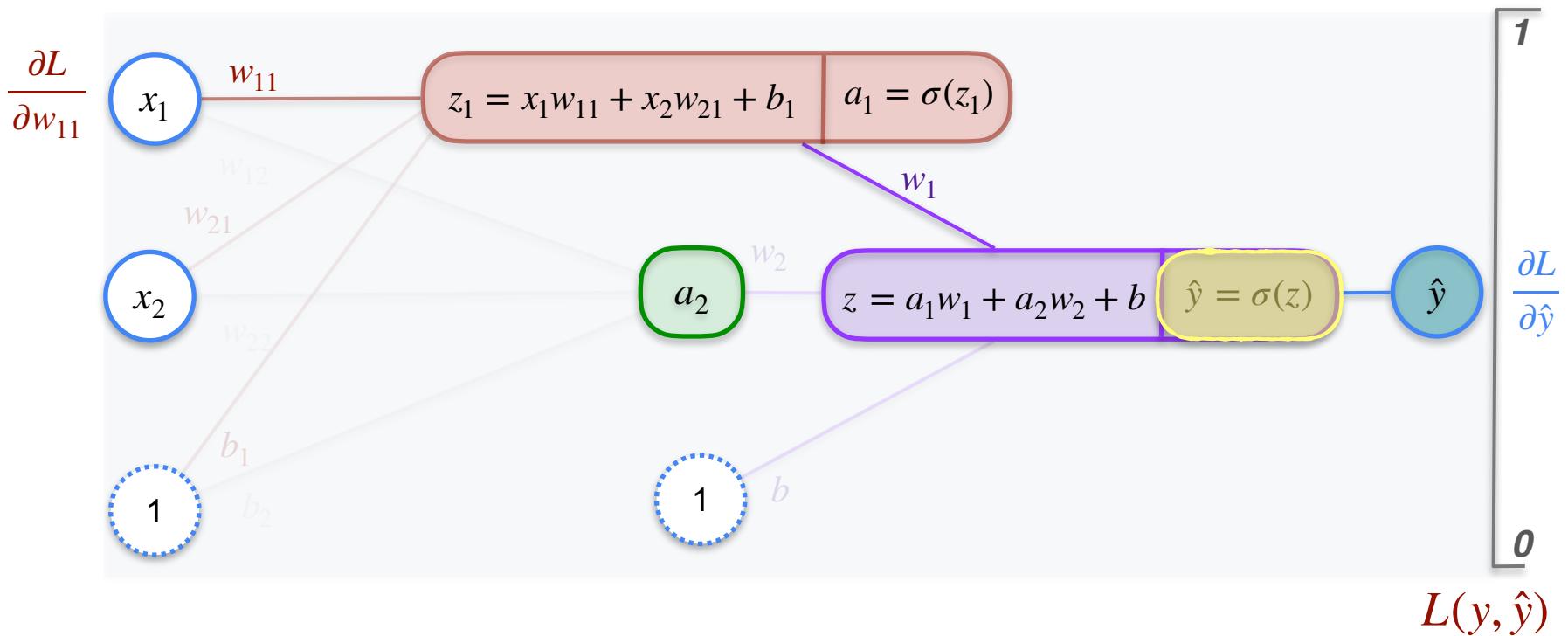
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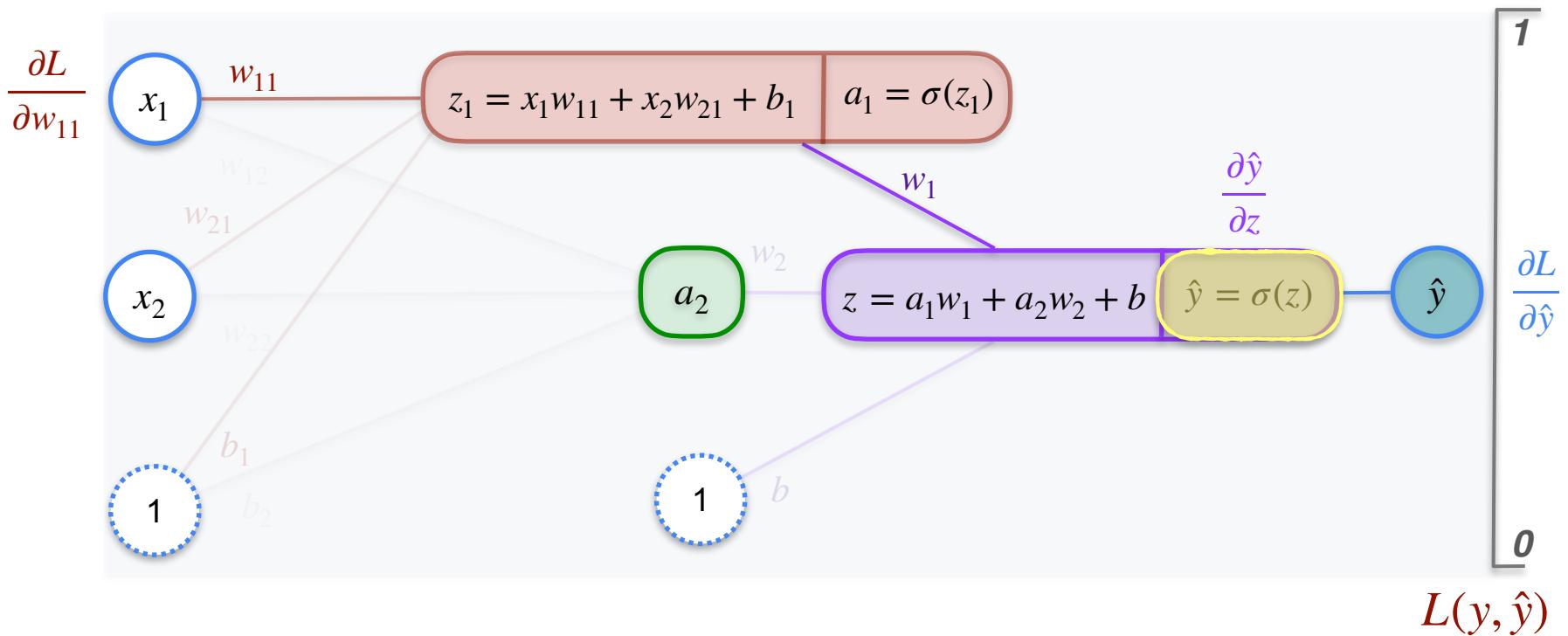
2,2,1 Neural Network



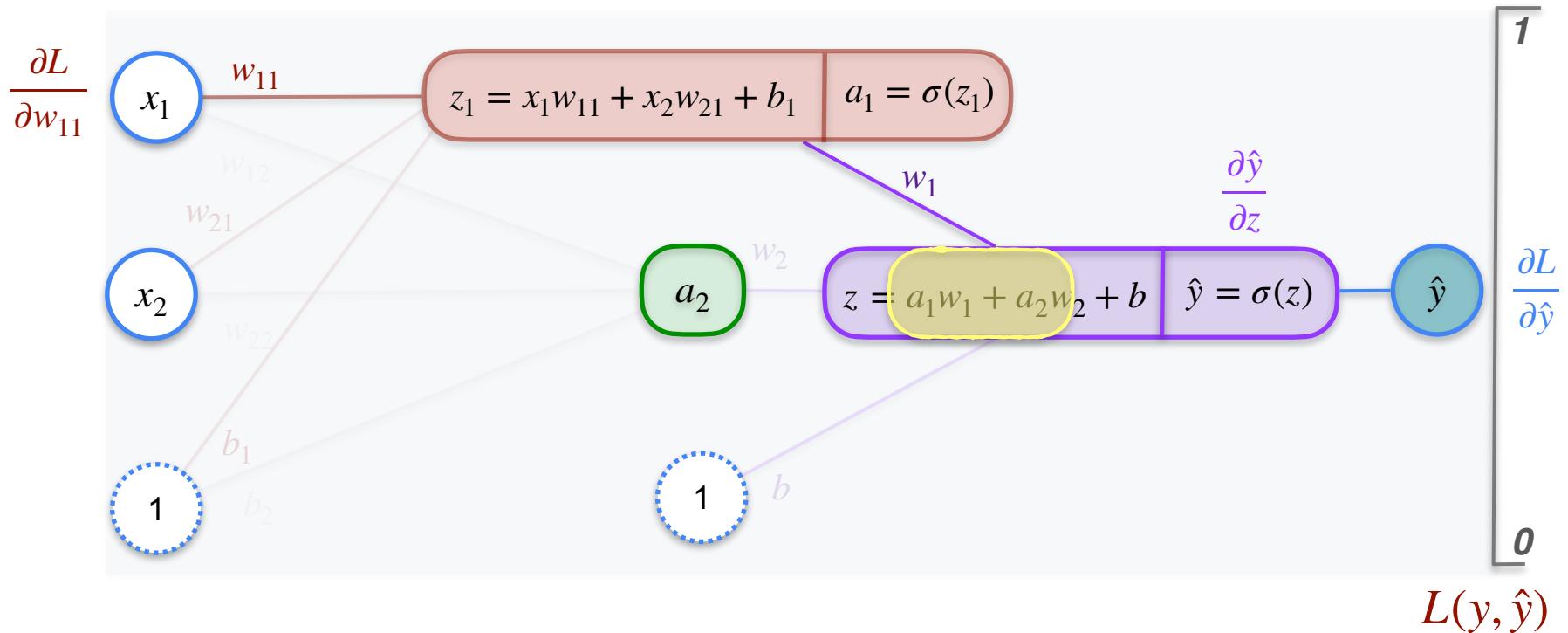
2,2,1 Neural Network



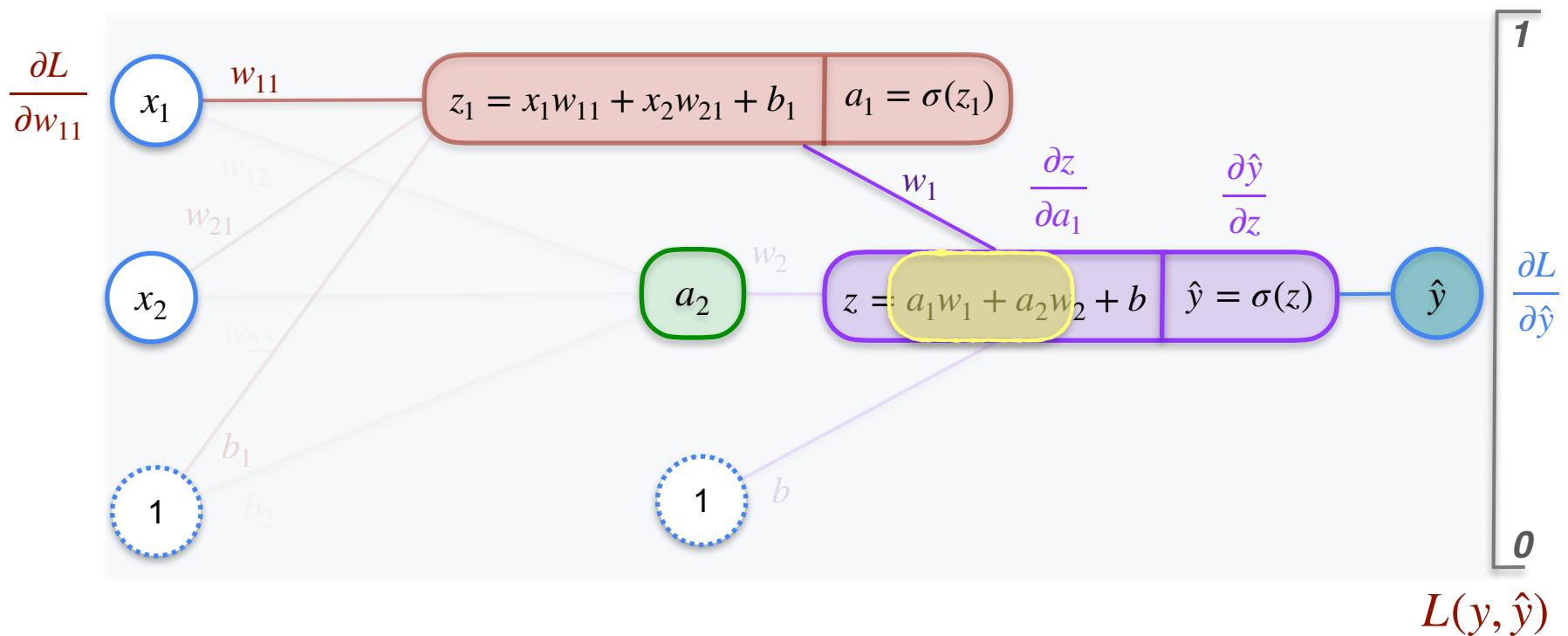
2,2,1 Neural Network



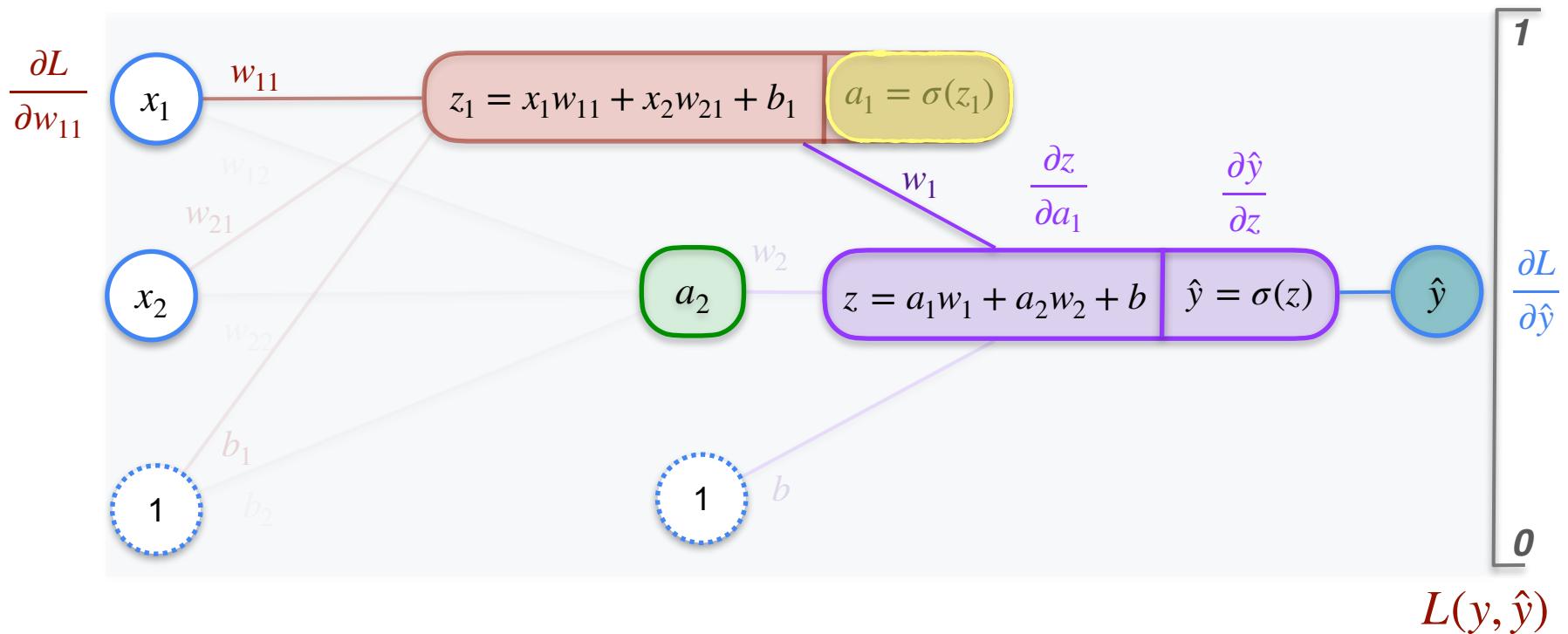
2,2,1 Neural Network



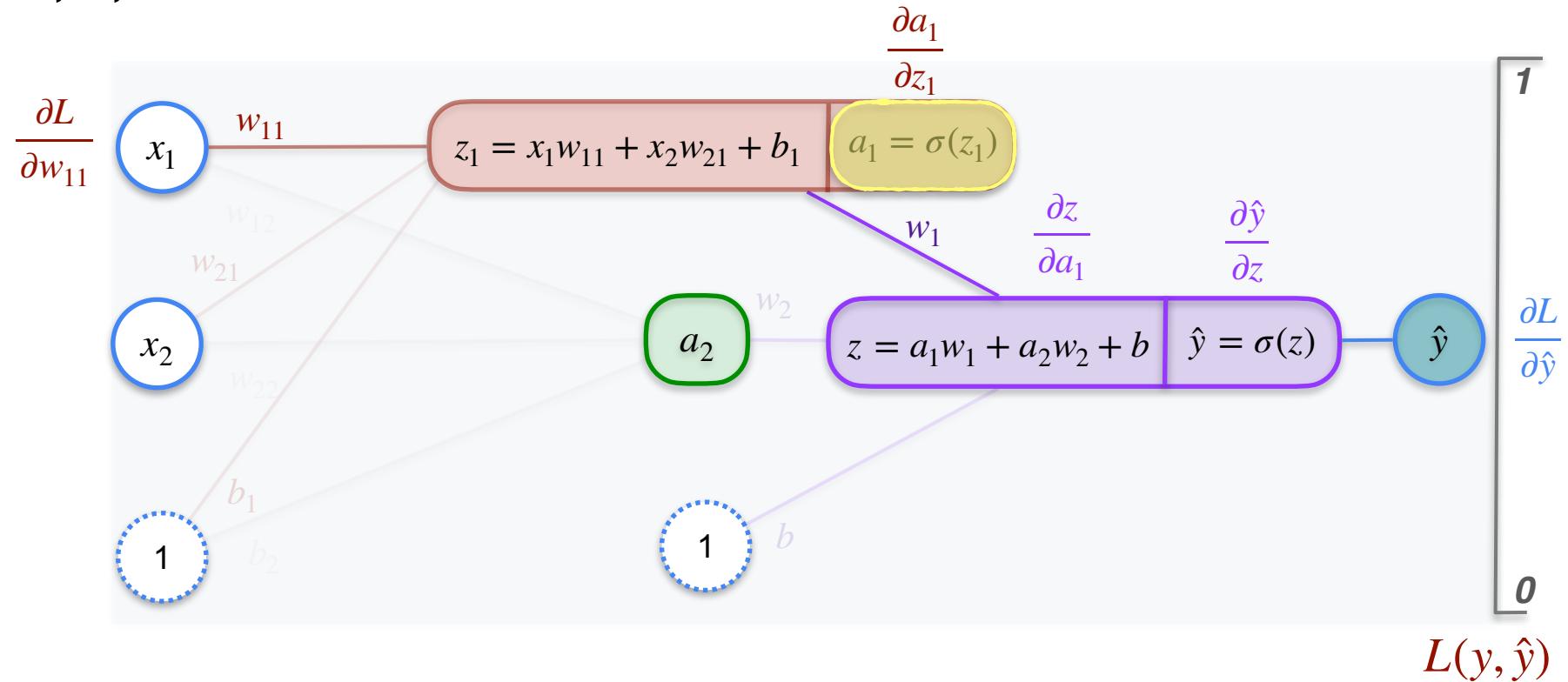
2,2,1 Neural Network



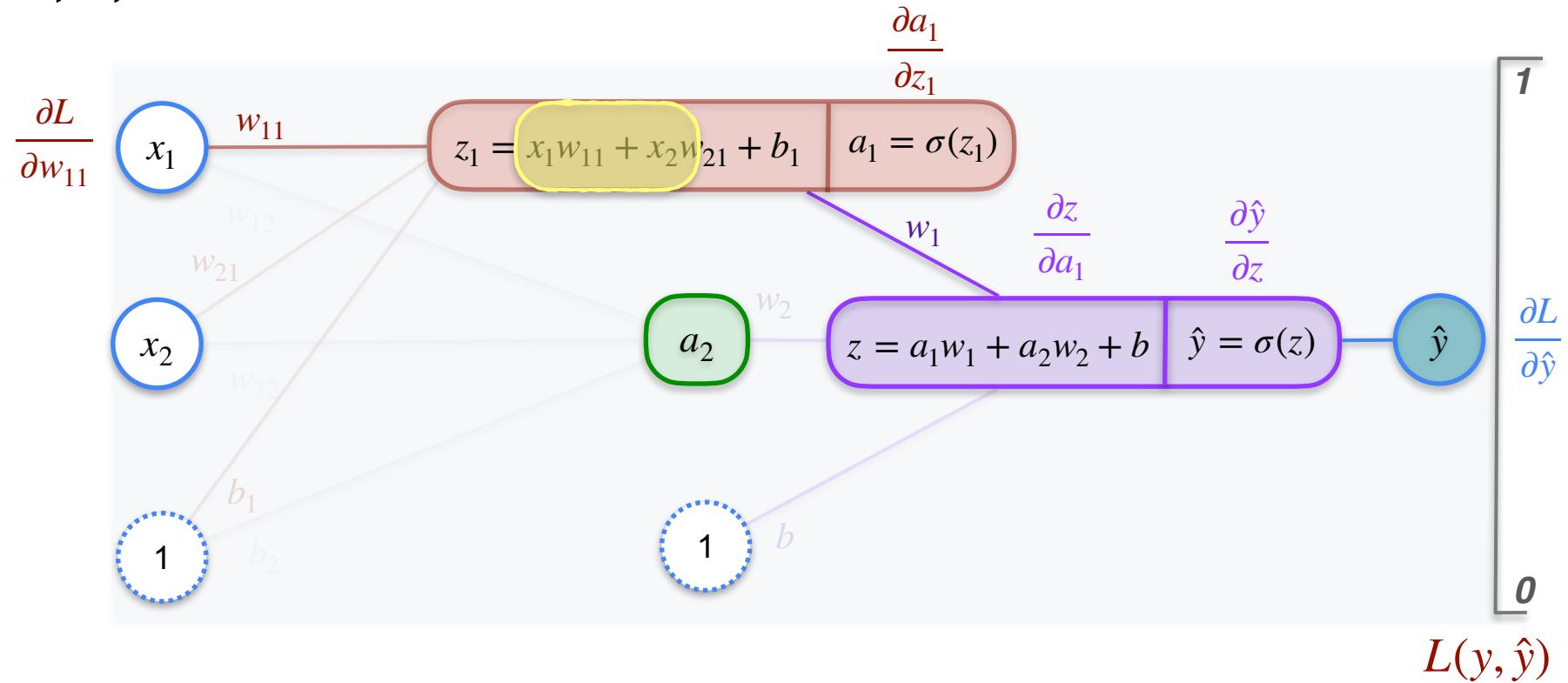
2,2,1 Neural Network



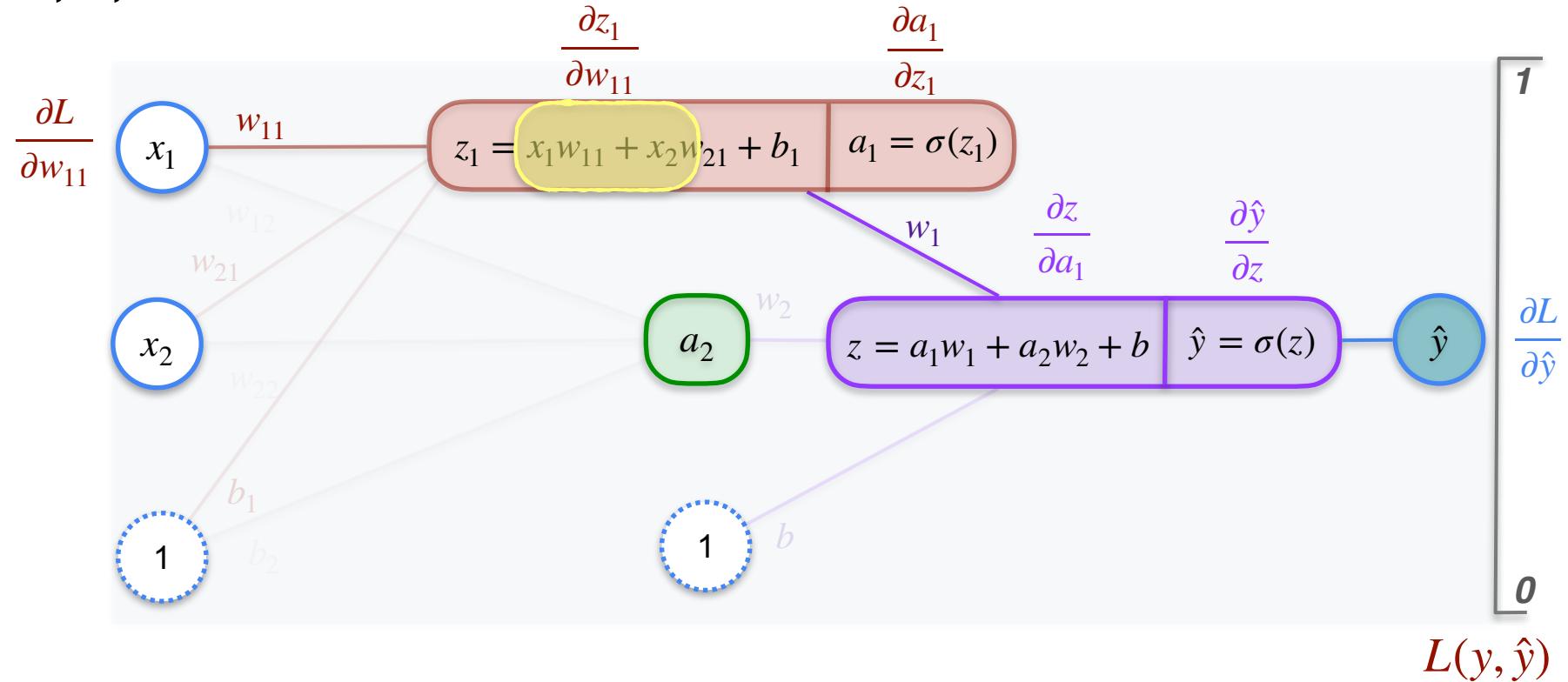
2,2,1 Neural Network



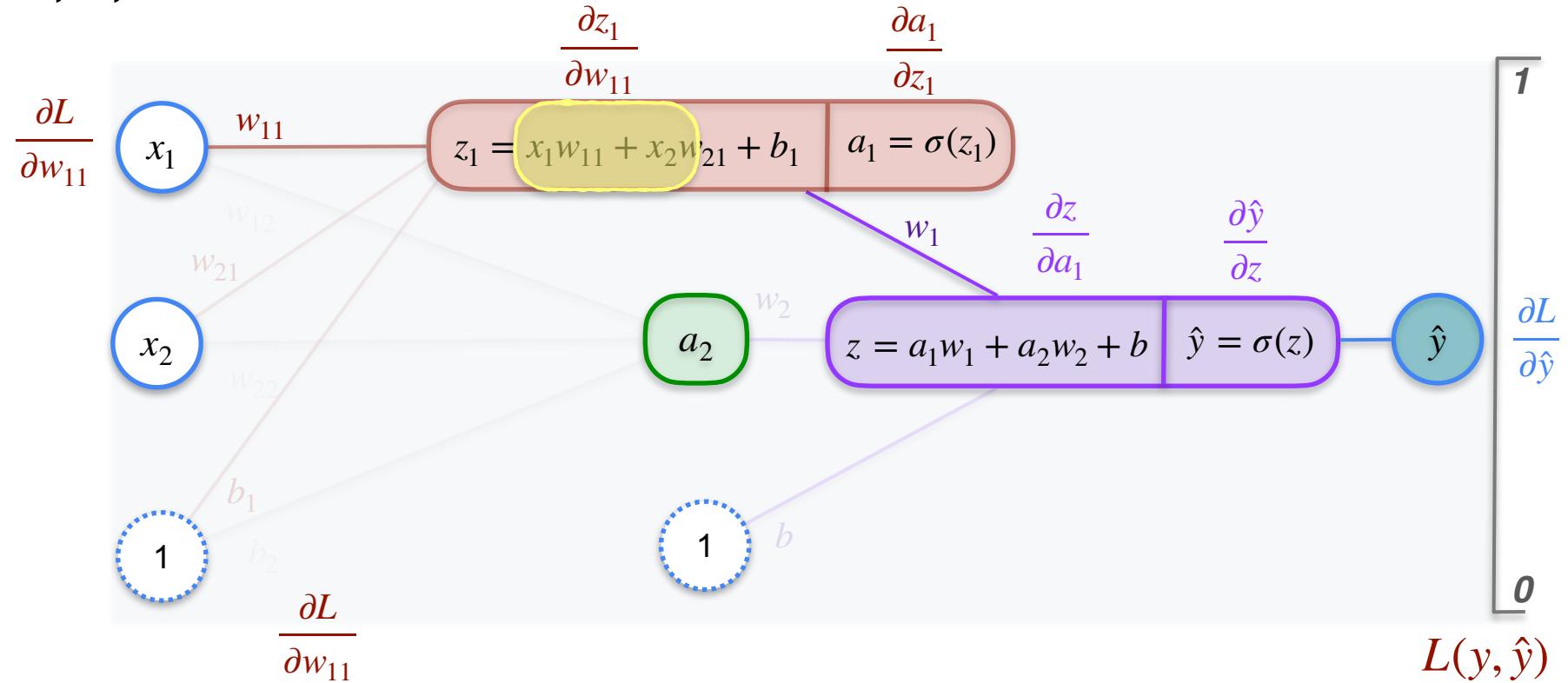
2,2,1 Neural Network



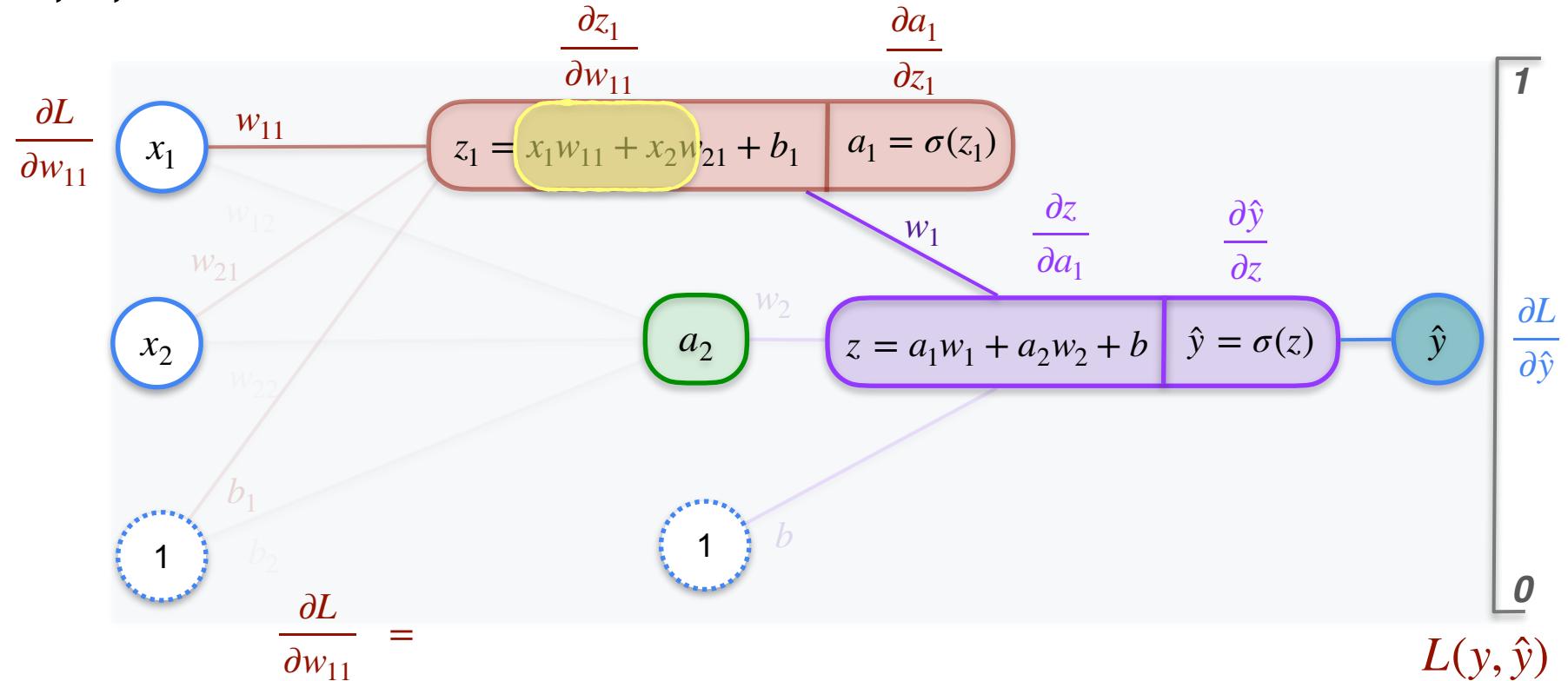
2,2,1 Neural Network



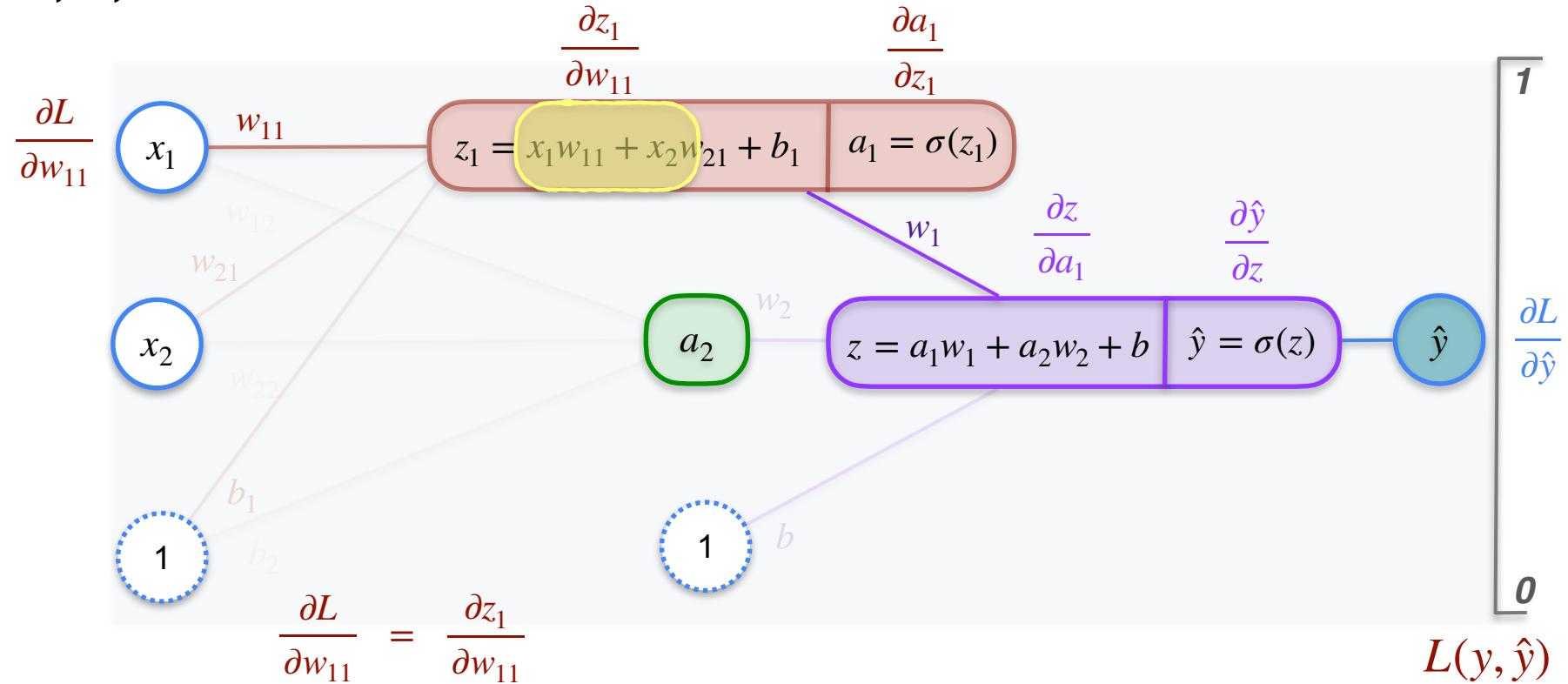
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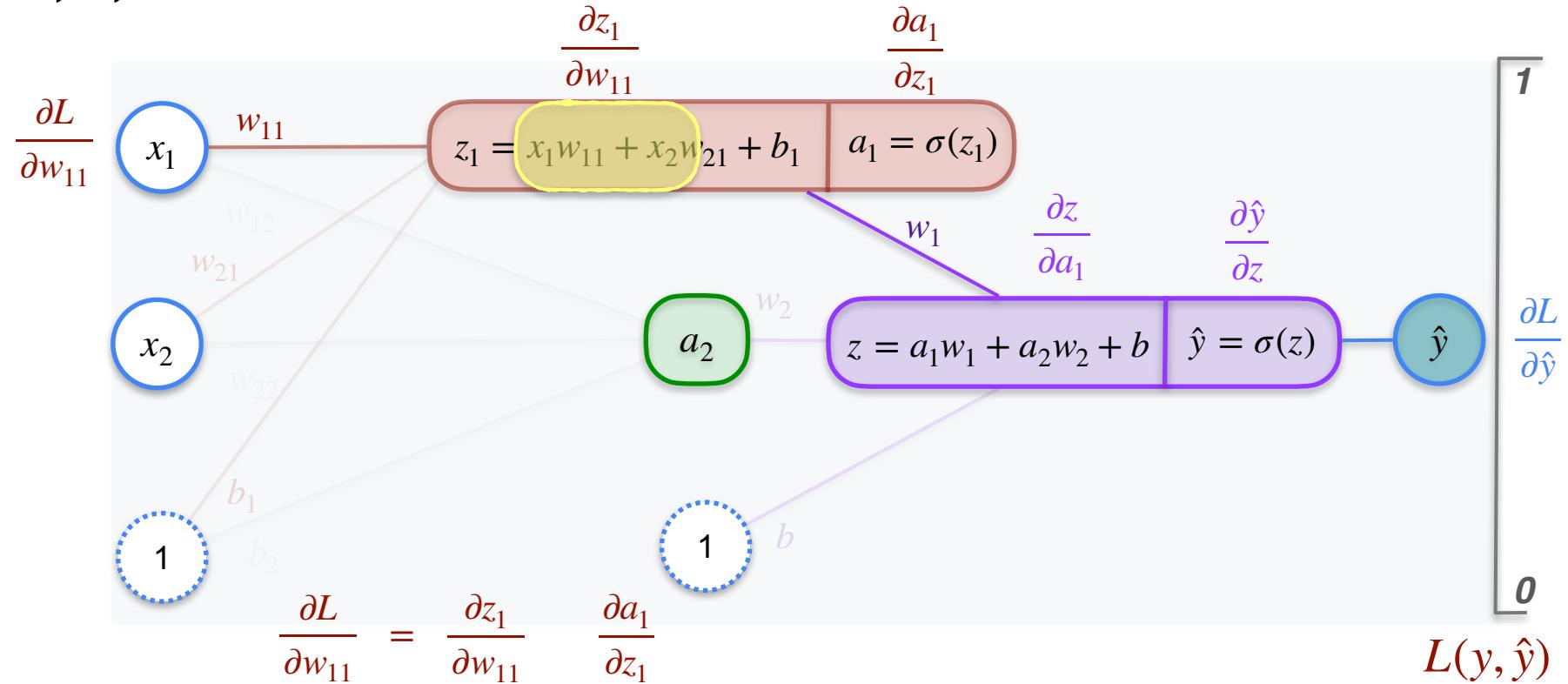
2,2,1 Neural Network



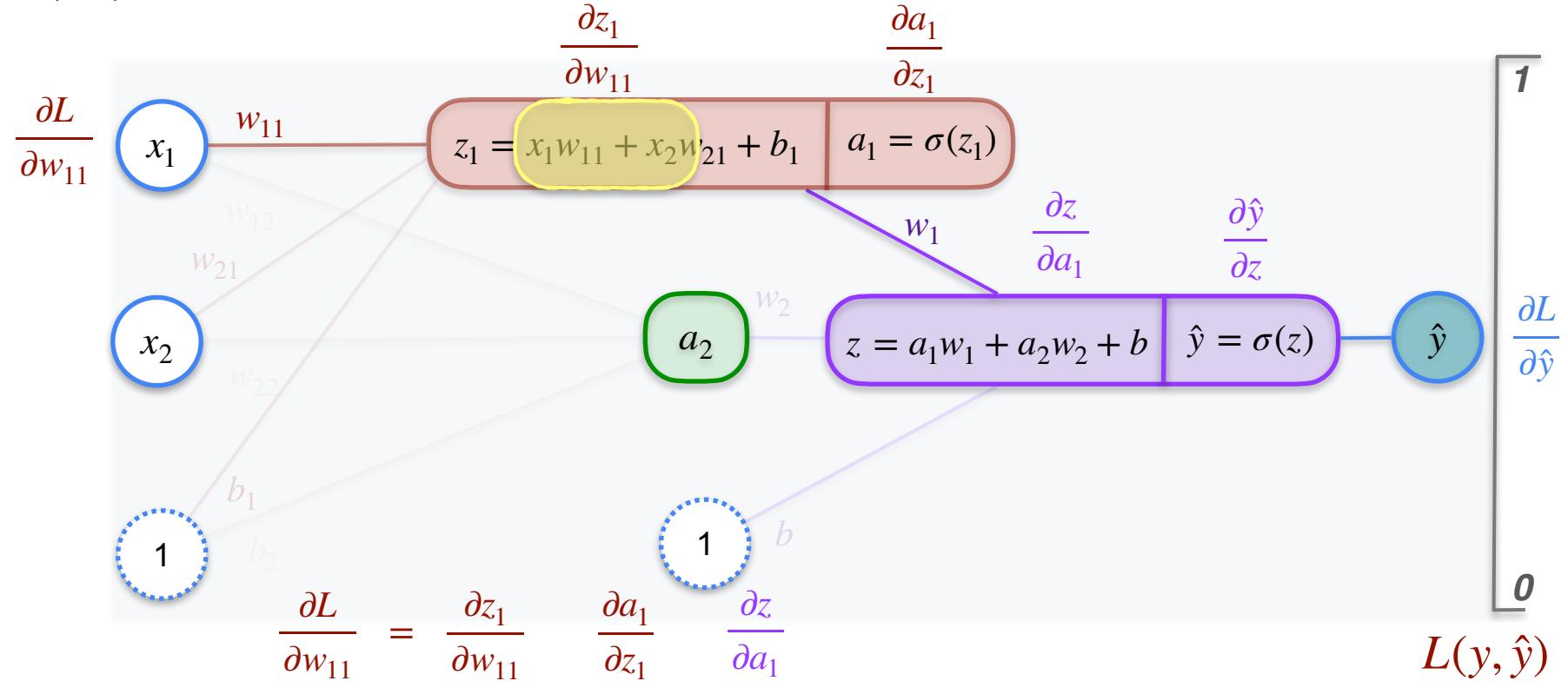
2,2,1 Neural Network



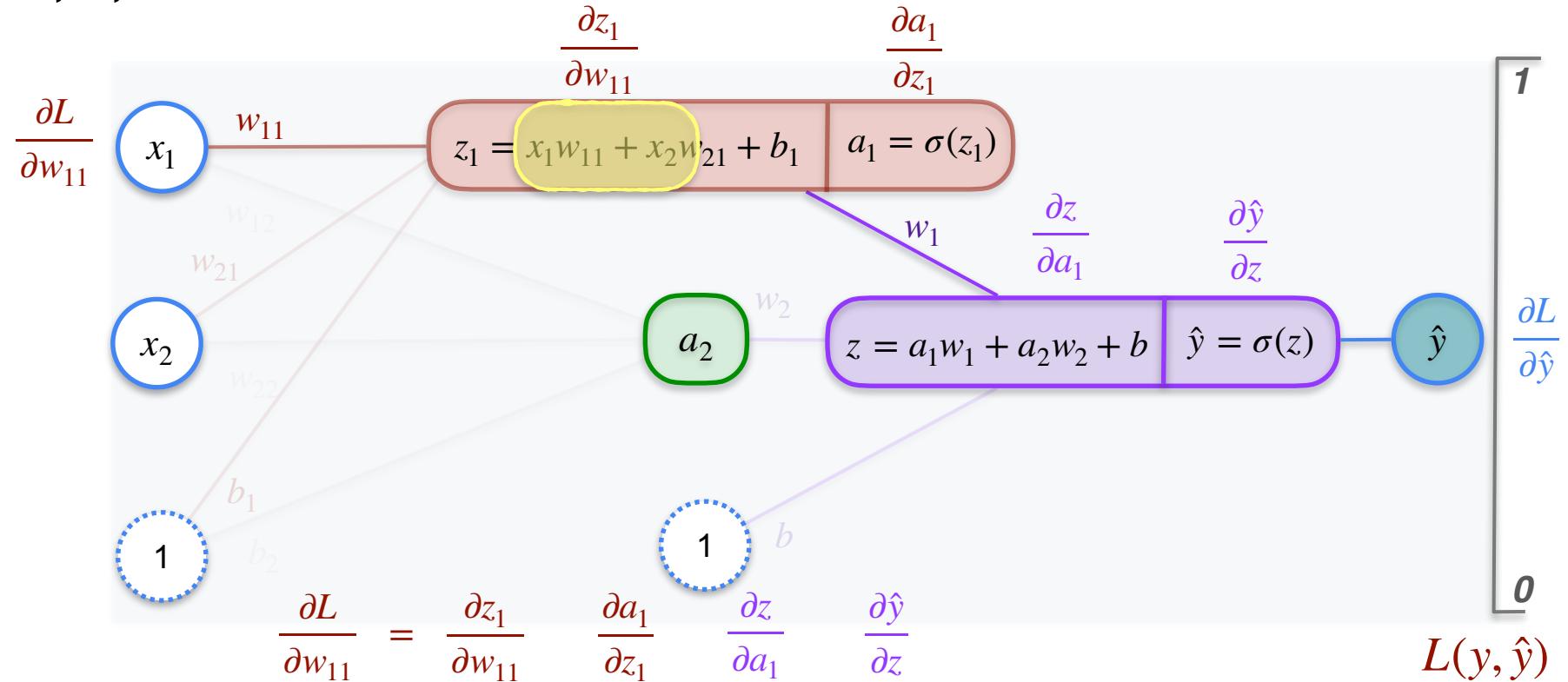
2,2,1 Neural Network



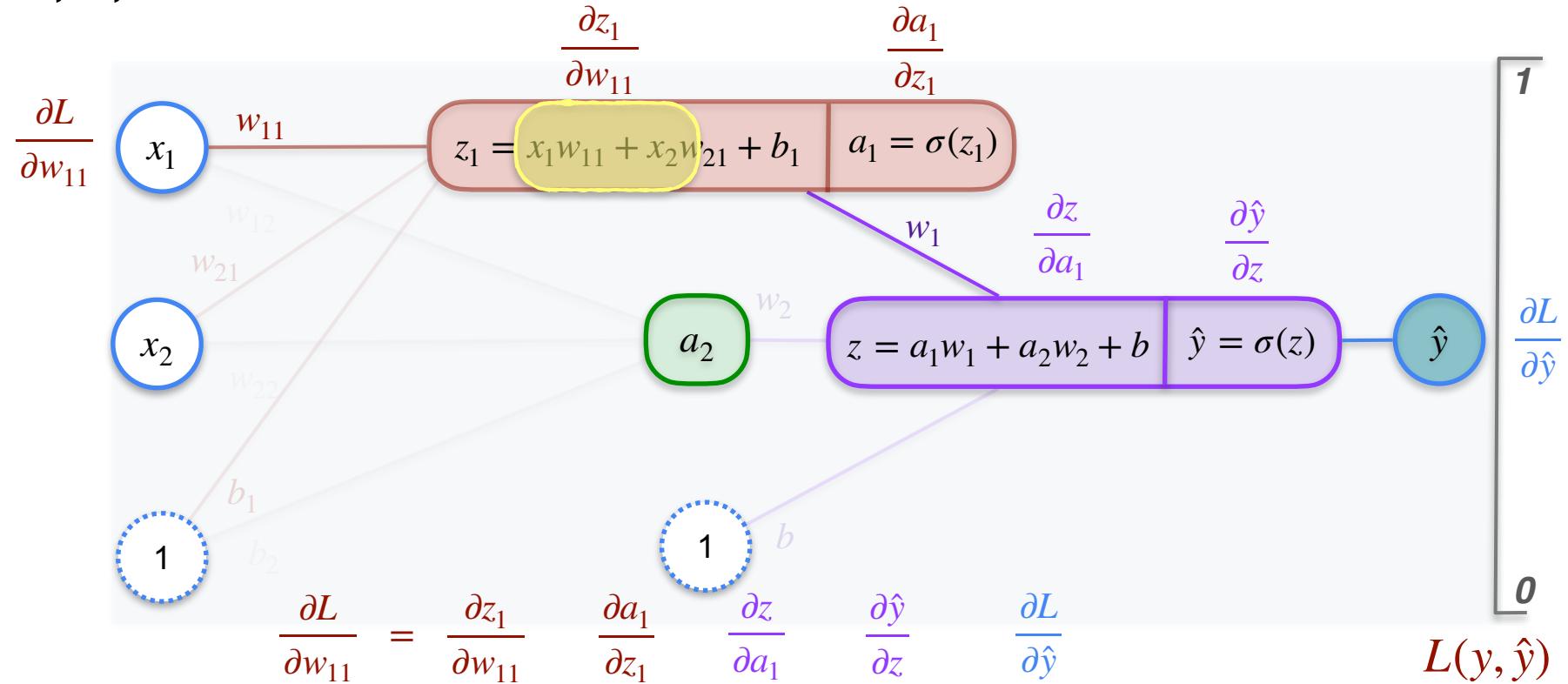
2,2,1 Neural Network



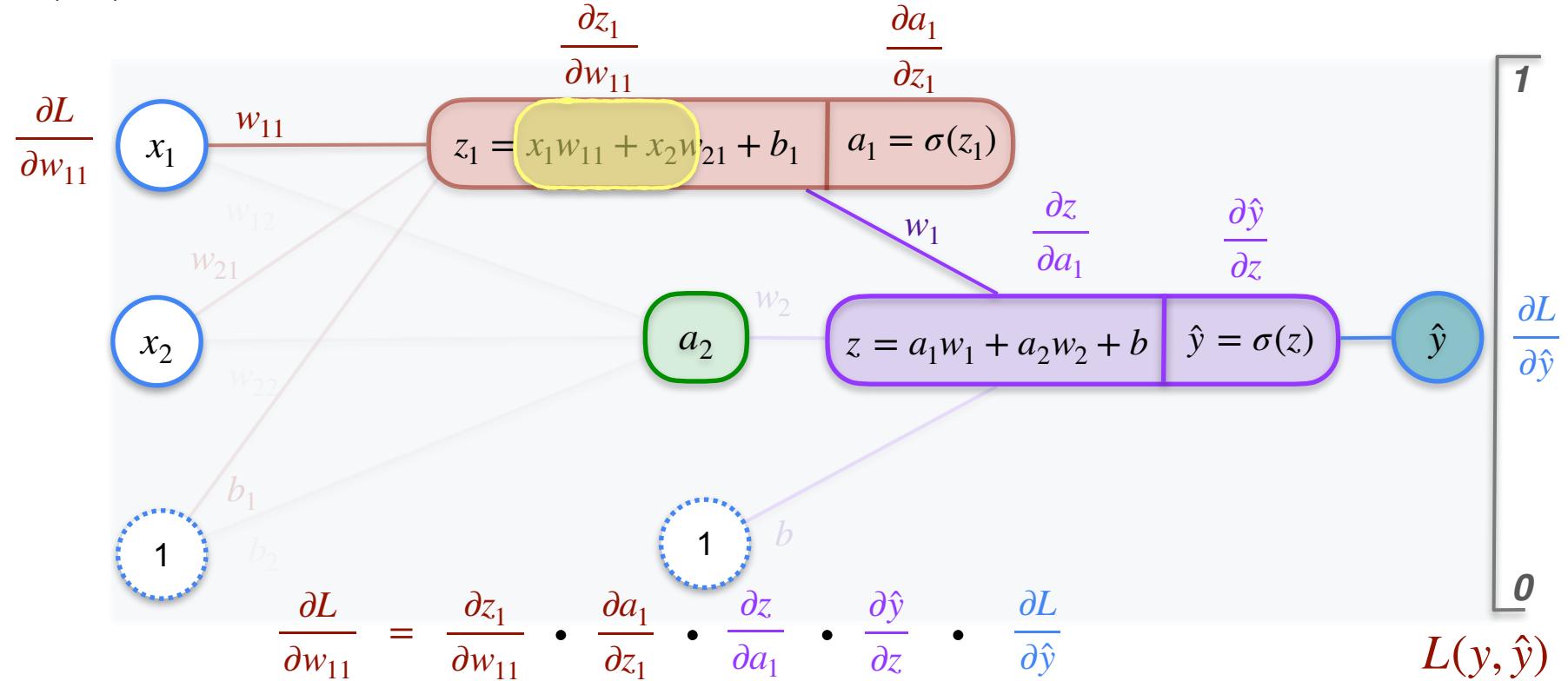
2,2,1 Neural Network



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$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \bullet \frac{\partial a_1}{\partial z_1} \bullet \frac{\partial z}{\partial a_1} \bullet \frac{\partial \hat{y}}{\partial z} \bullet \frac{\partial L}{\partial \hat{y}}$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}) \quad \frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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Perform gradient descent with

*to find optimal
value of w_{11} that
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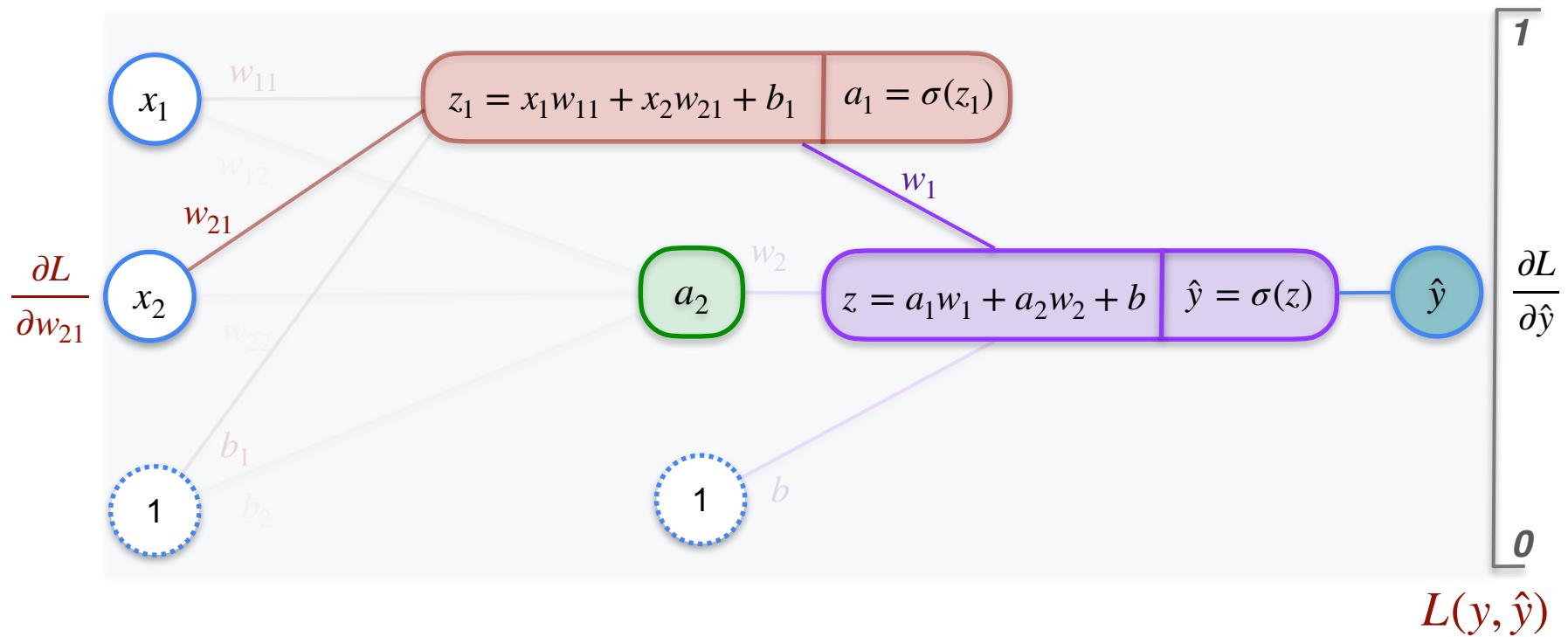
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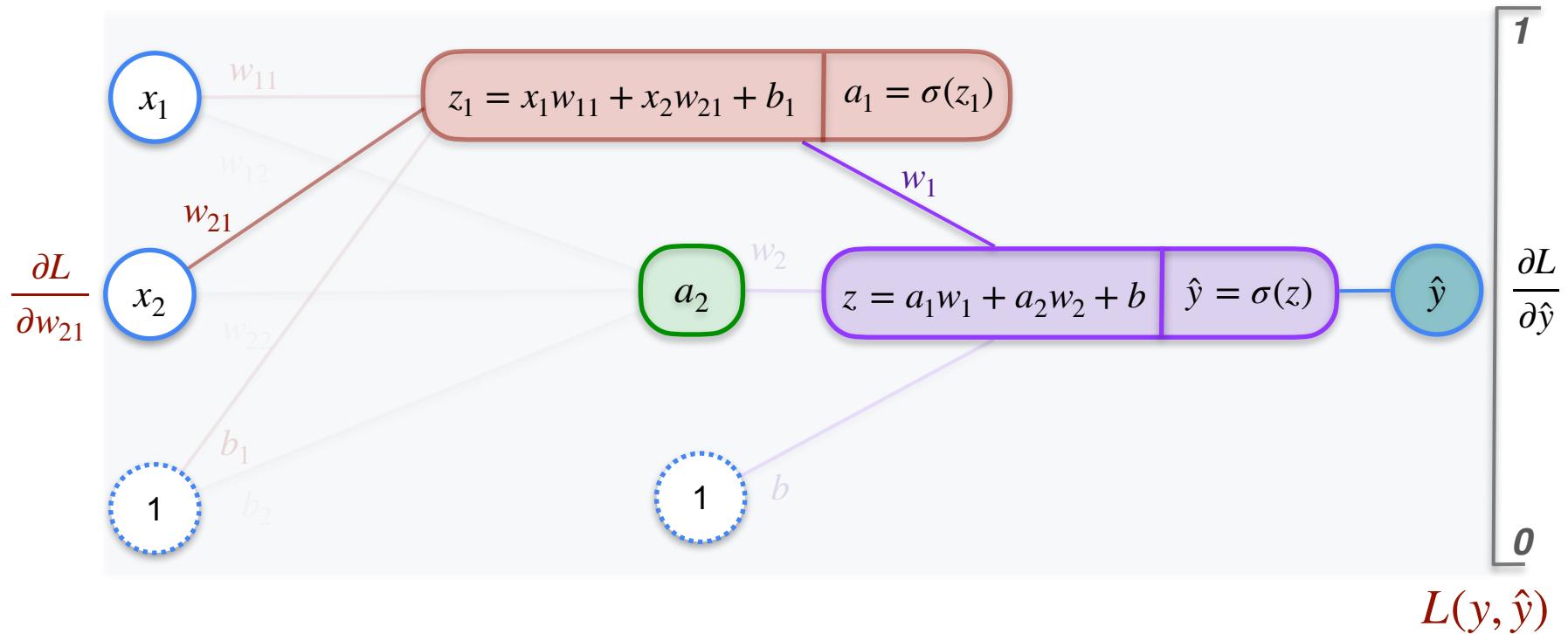
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to find optimal value of w_{11} that gives the least error

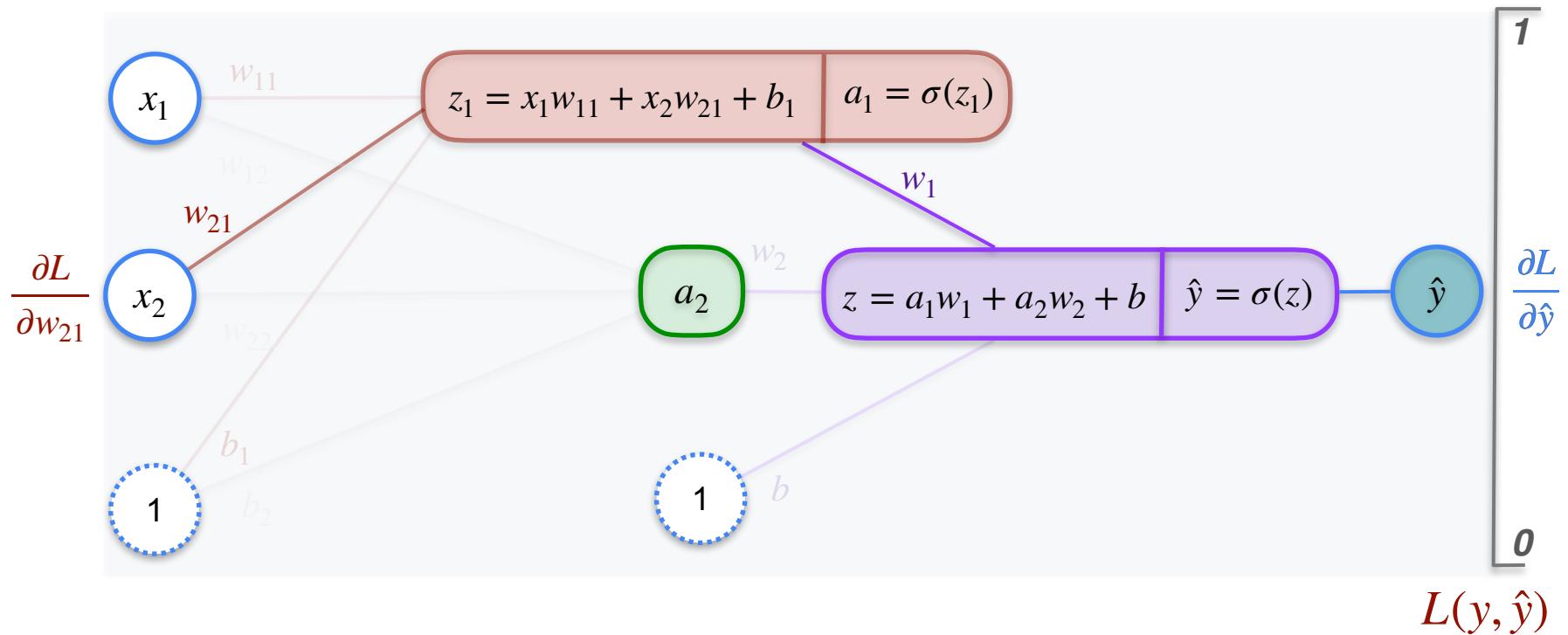
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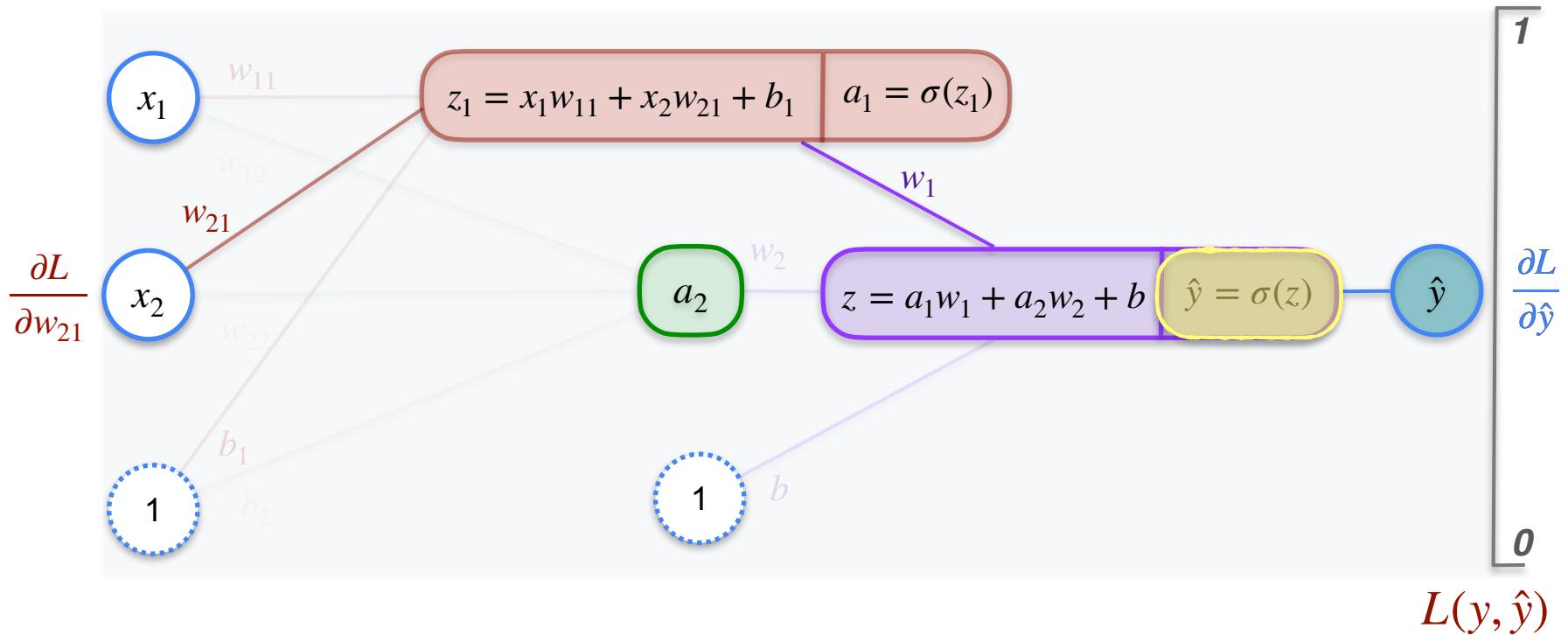
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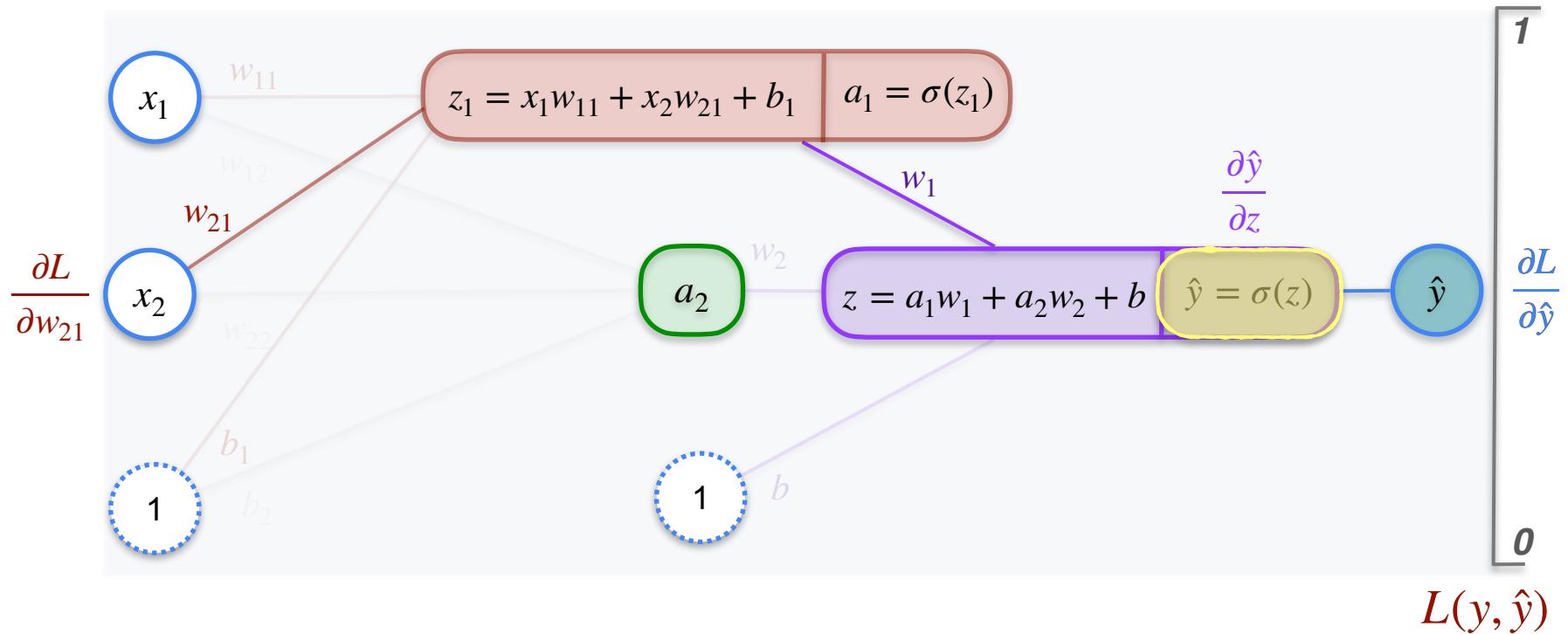
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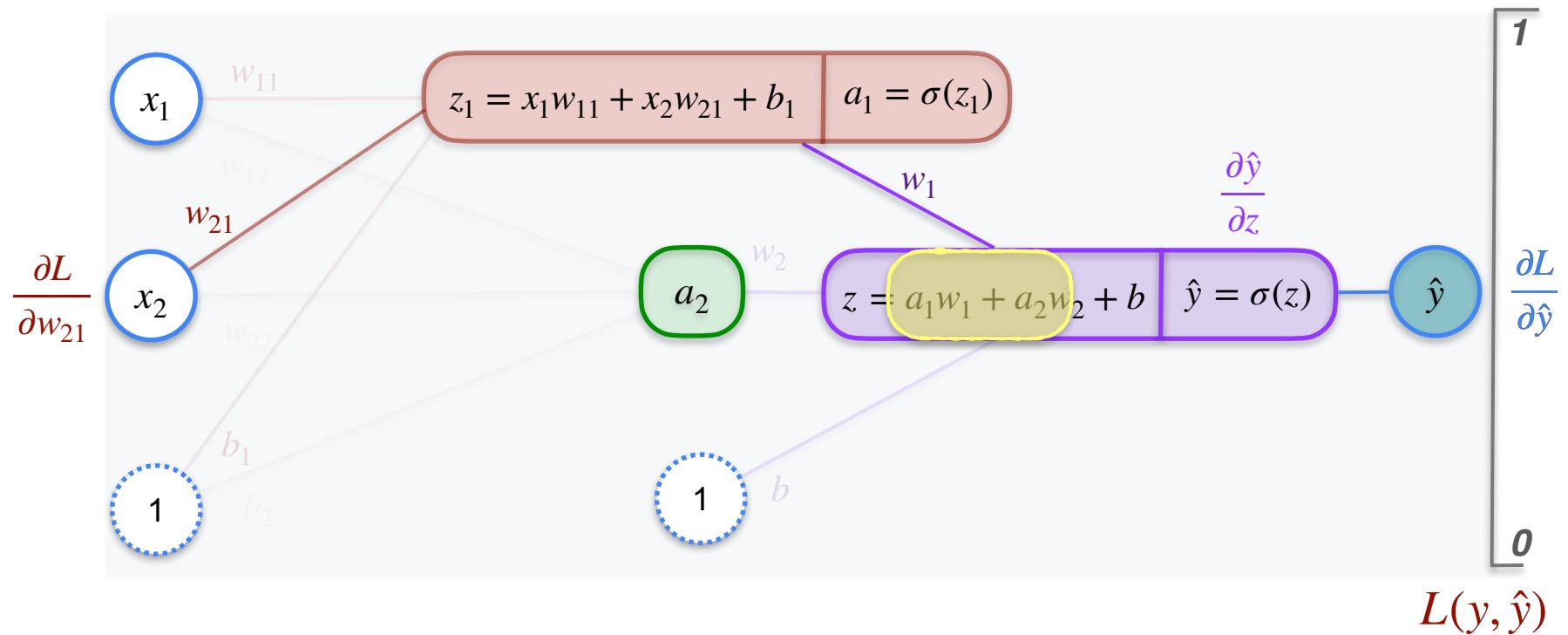
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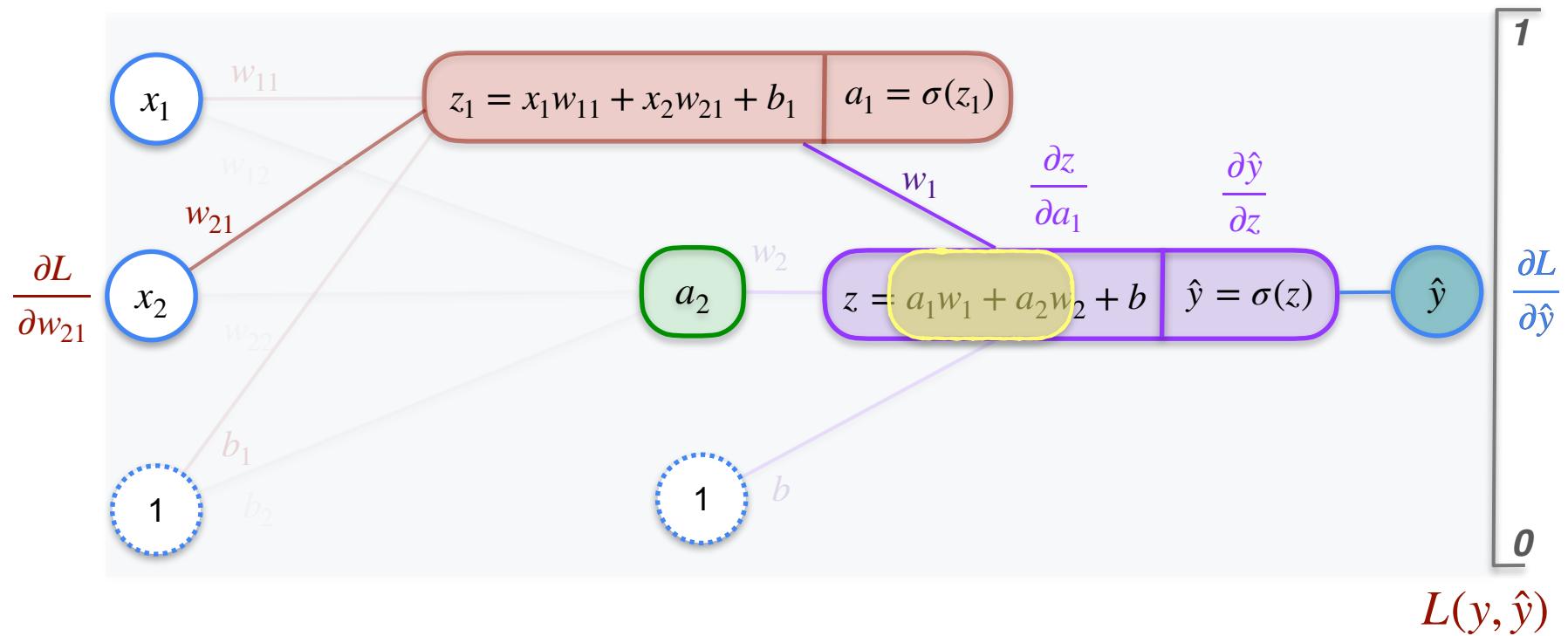
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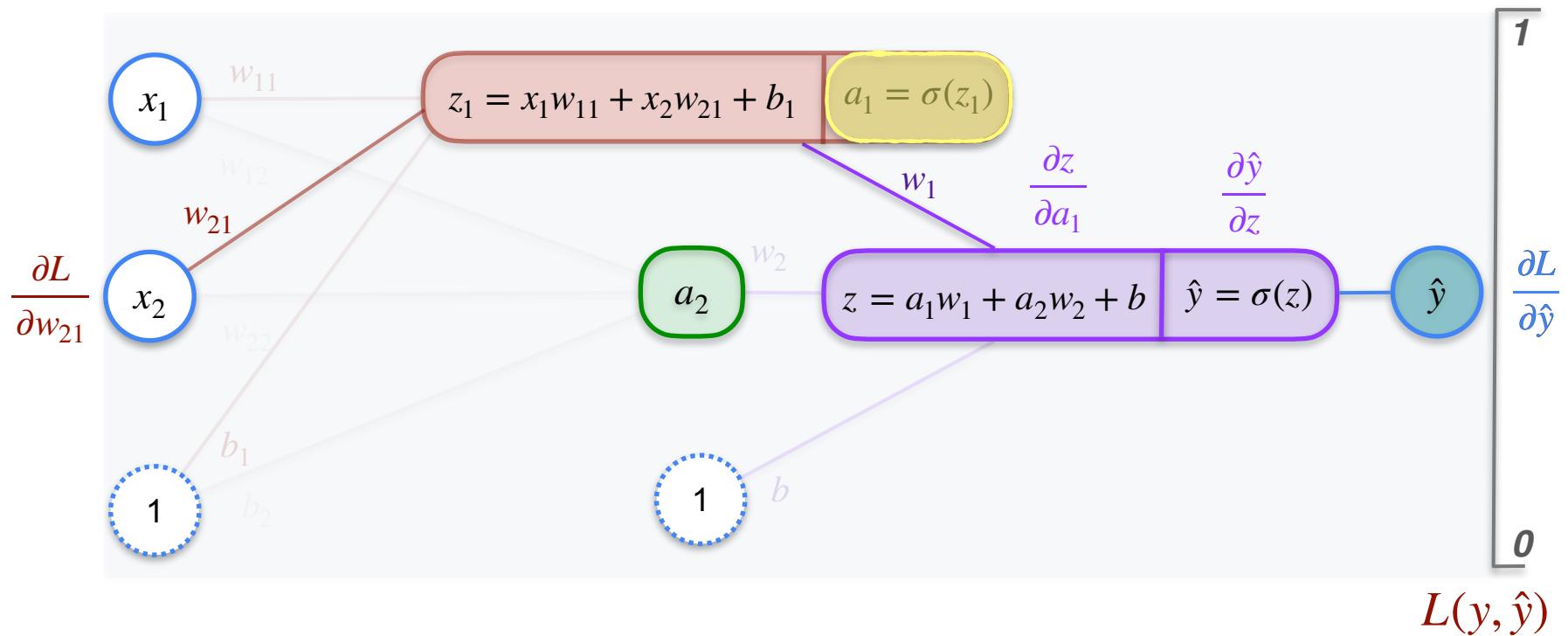
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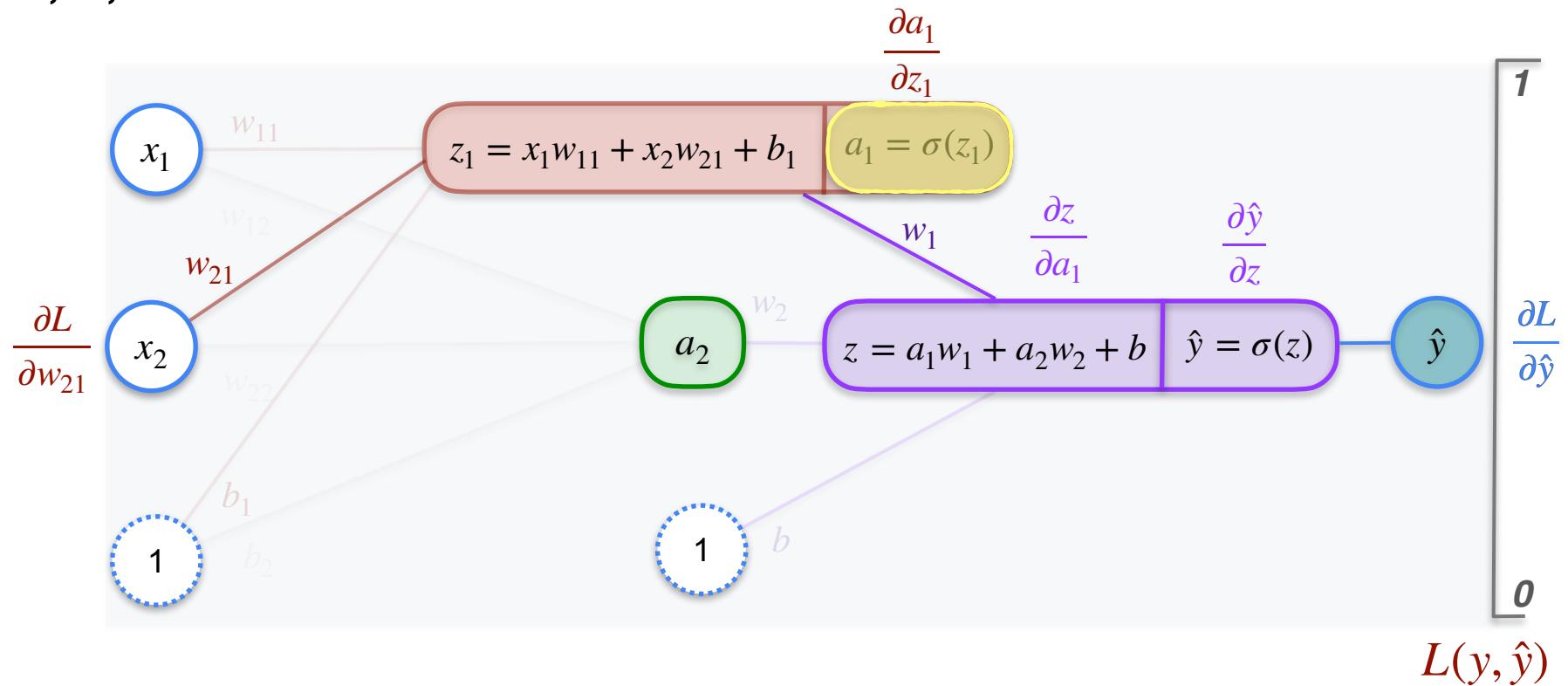
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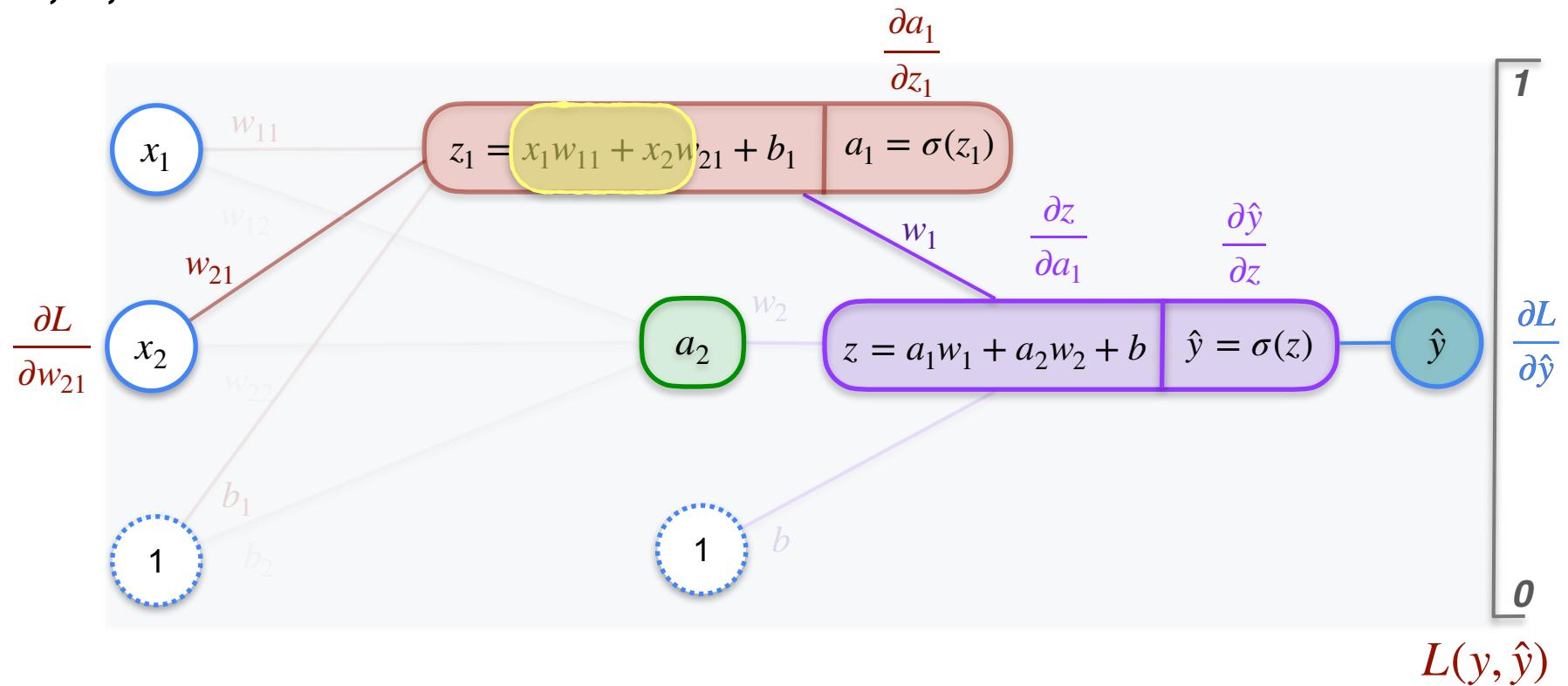
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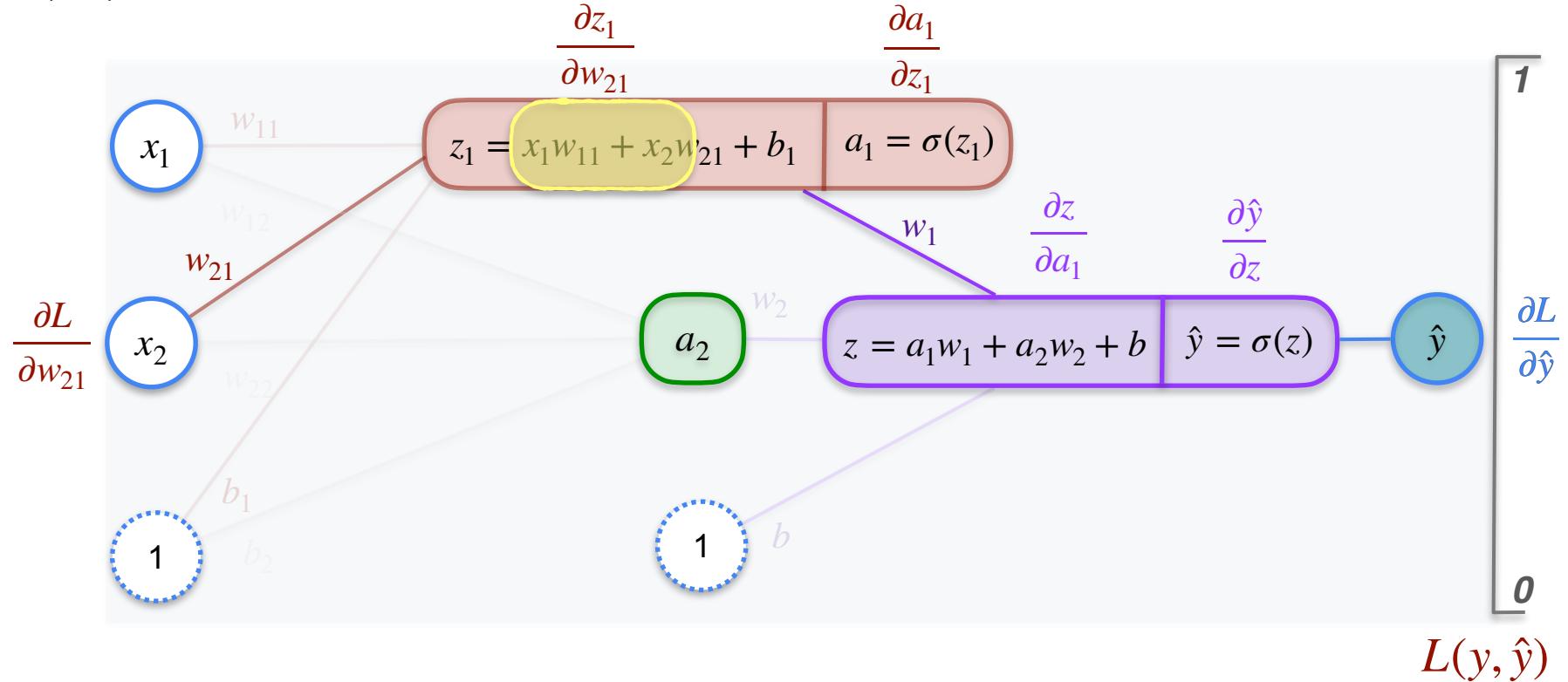
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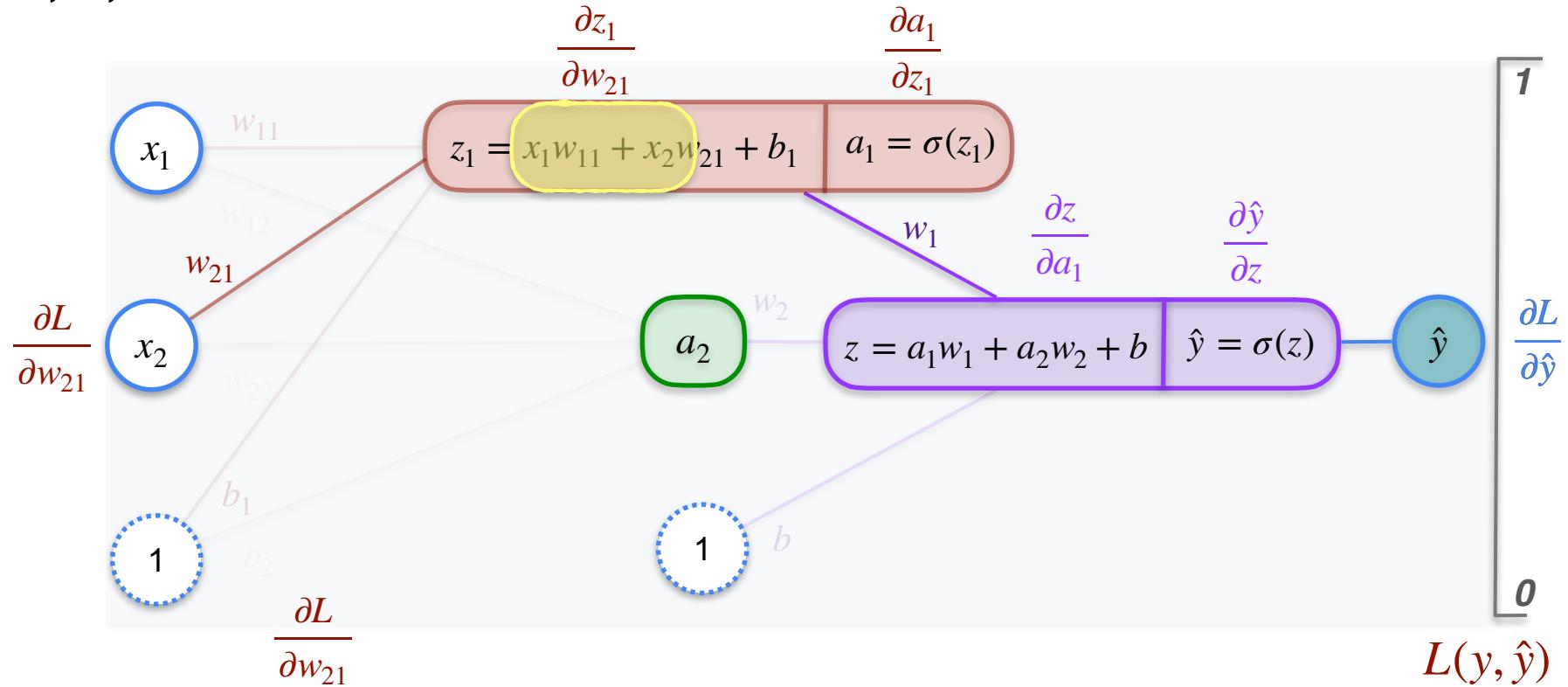
2,2,1 Neural Network



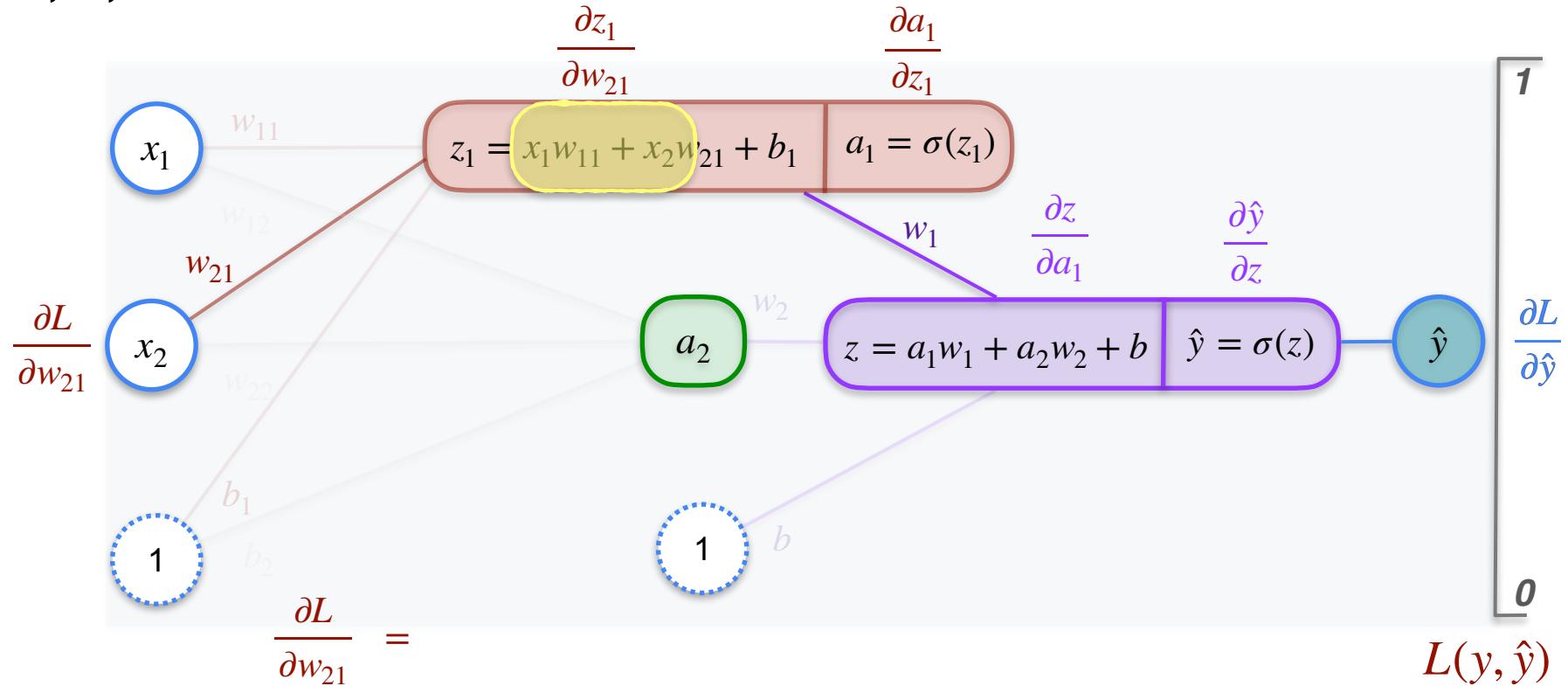
2,2,1 Neural Network



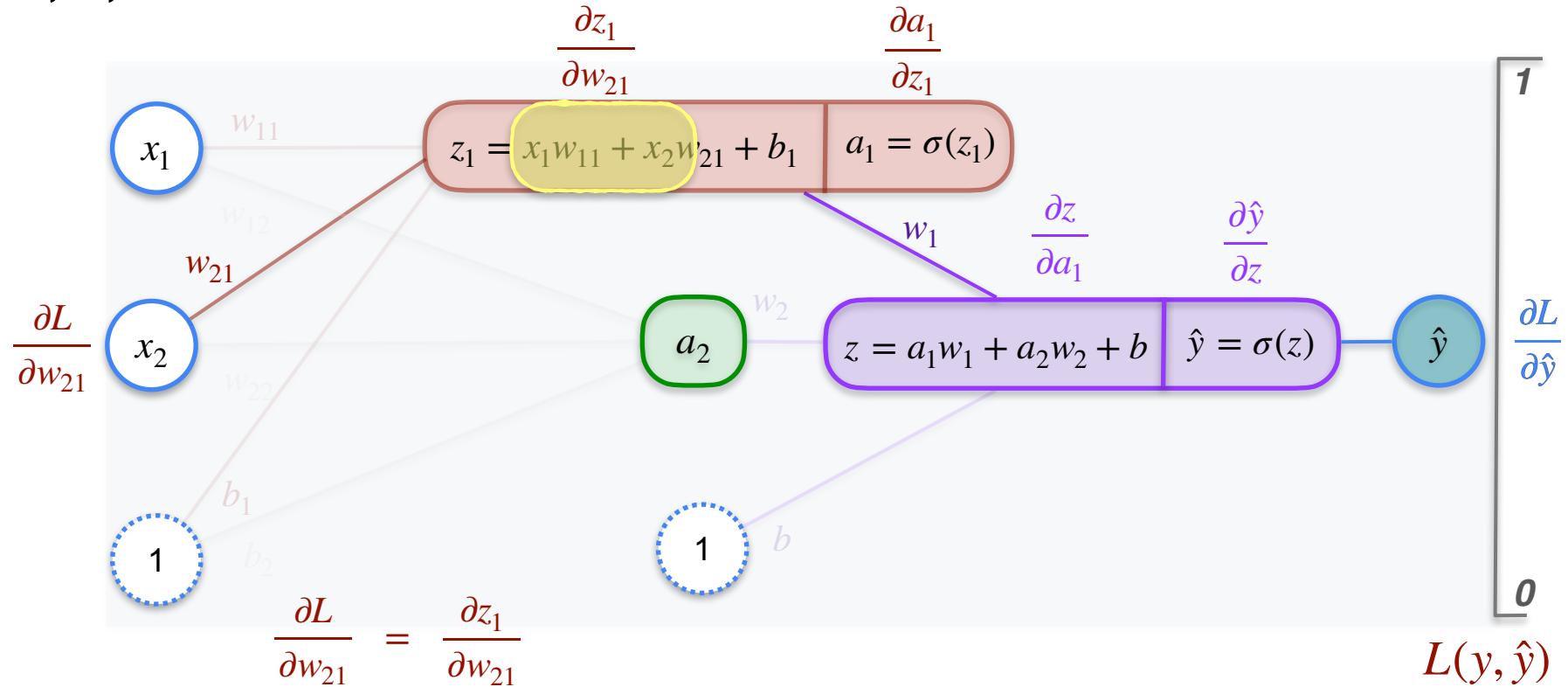
2,2,1 Neural Network



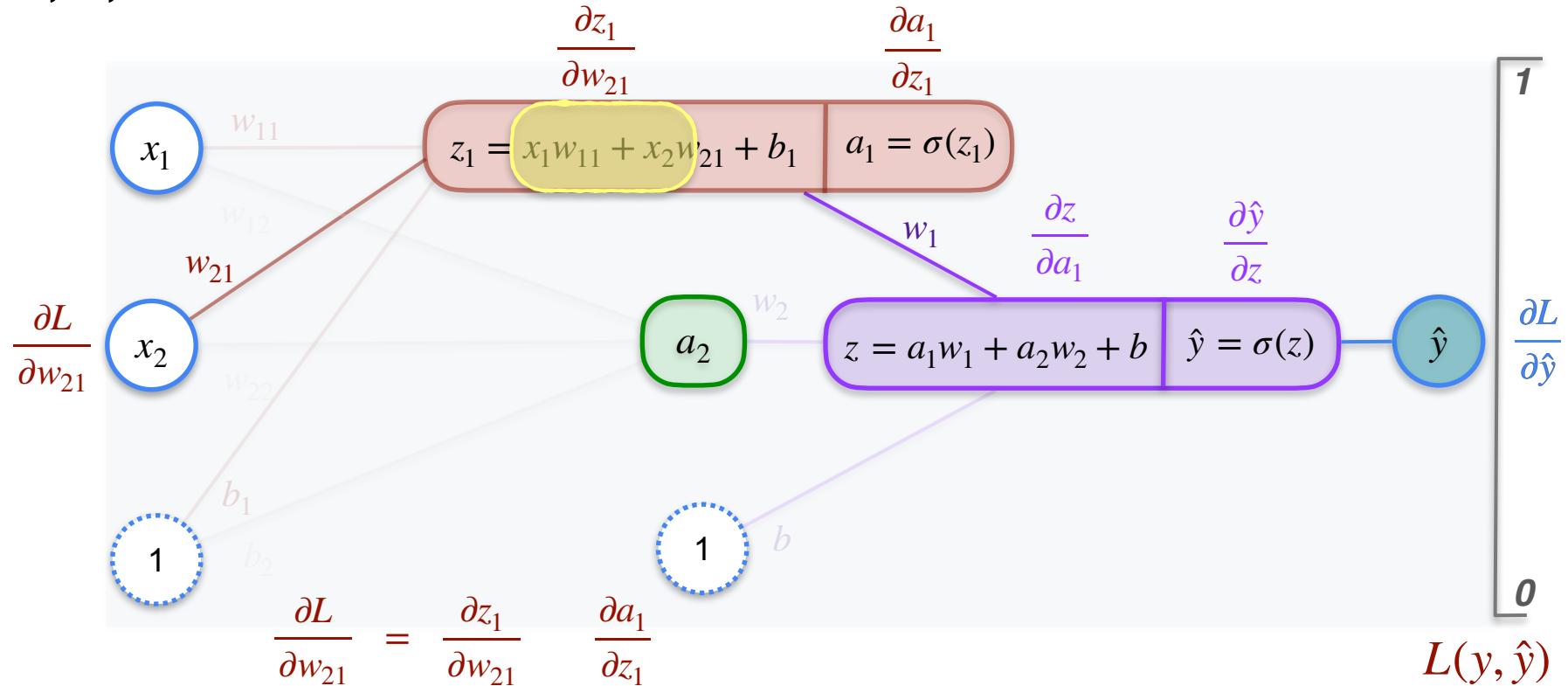
2,2,1 Neural Network



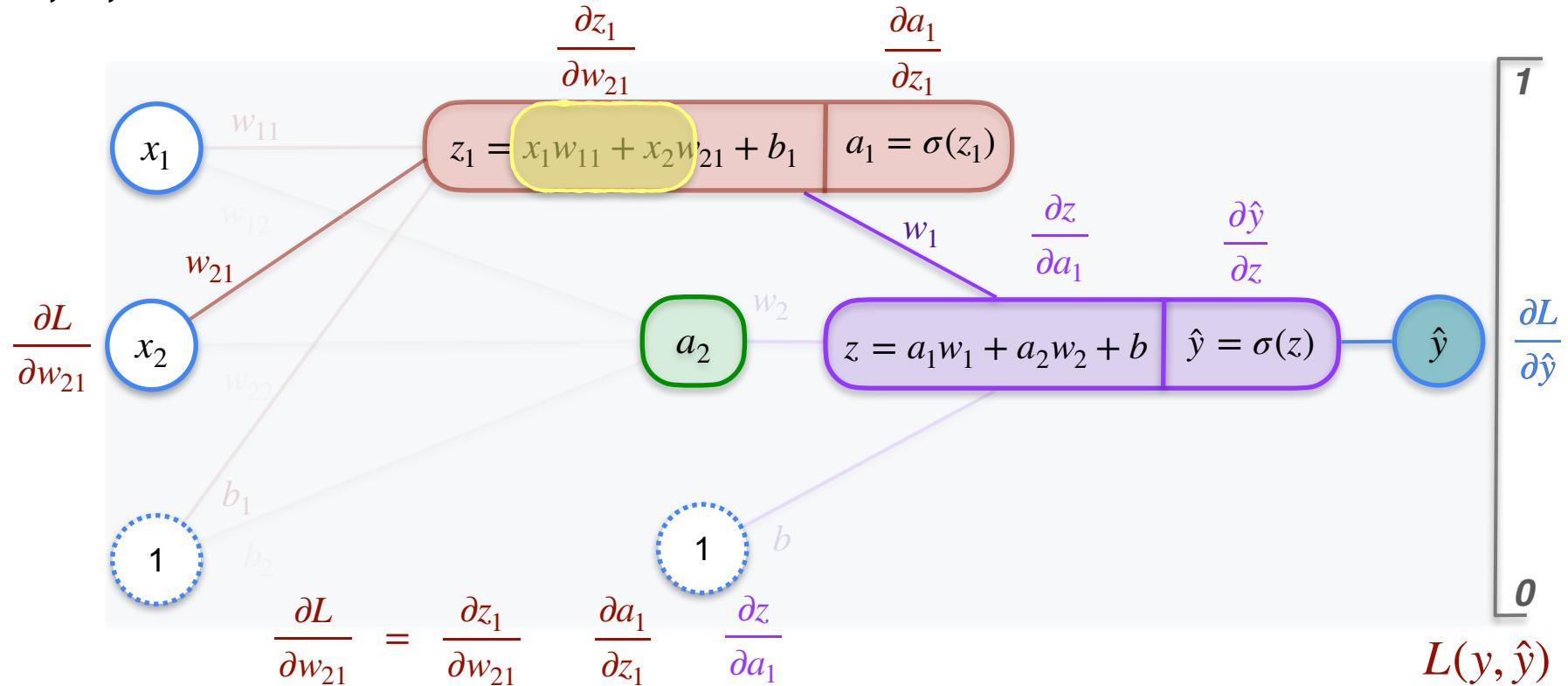
2,2,1 Neural Network



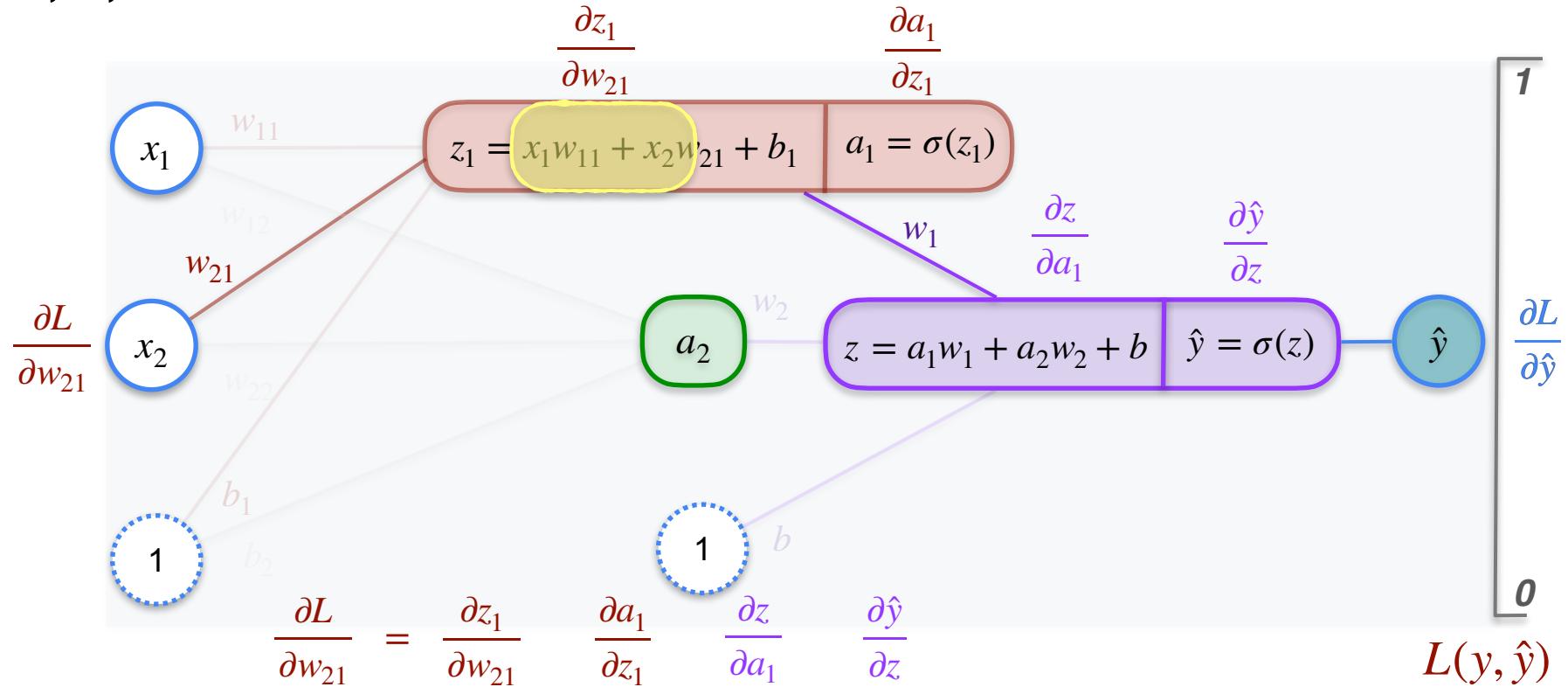
2,2,1 Neural Network



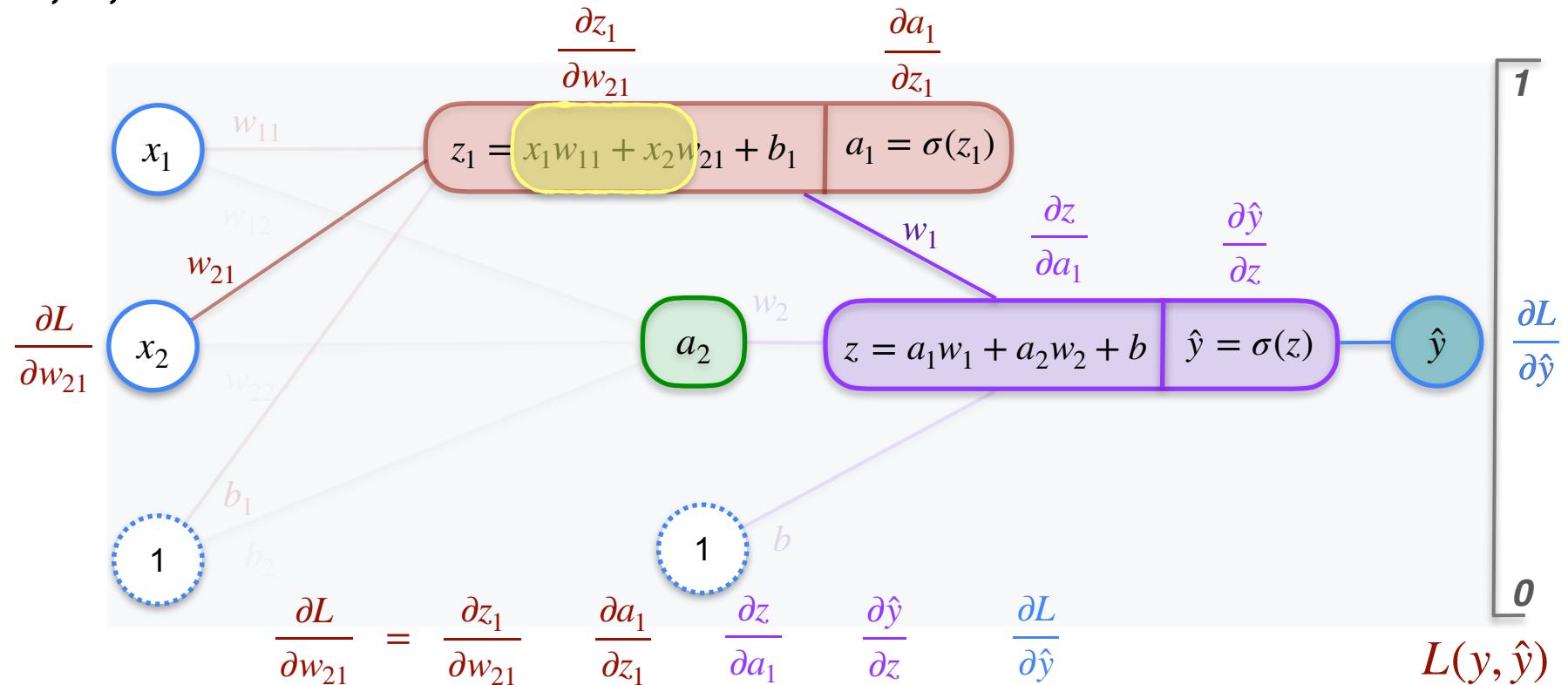
2,2,1 Neural Network



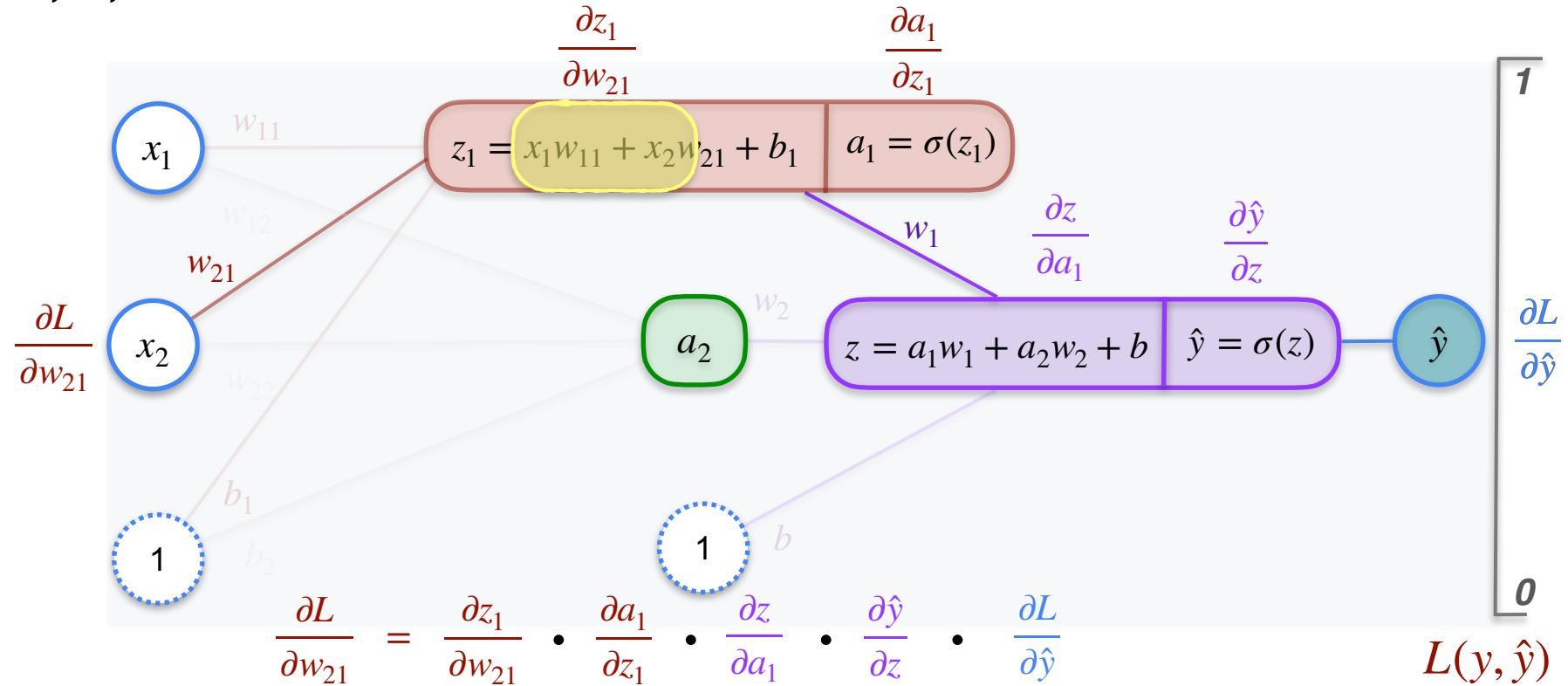
2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial z_1}{\partial w_{21}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}) \quad \frac{\partial L}{\partial w_{21}} = \frac{\partial z_1}{\partial w_{21}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}}$$

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2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}} = \frac{\partial z_1}{\partial w_{21}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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$$\frac{\partial L}{\partial w_{21}} =$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}} = x_2$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}} = x_2 - a_1(1 - a_1)$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}} = x_2 - a_1(1-a_1) w_1$$

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$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

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$$\frac{\partial L}{\partial w_{21}} = x_2 - a_1(1-a_1) w_1 - \hat{y}(1-\hat{y})$$

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$$\frac{\partial L}{\partial w_{21}} = x_2 \quad a_1(1-a_1) \quad w_1 \quad \hat{y}(1-\hat{y})$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_{21}} = x_2 \quad a_1(1-a_1) \quad w_1 \quad \hat{y}(1-\hat{y}) \quad \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

2,2,1 Neural Network

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2,2,1 Neural Network

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2,2,1 Neural Network

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Perform gradient descent with

*to find optimal
value of w_{21} that
gives the least error*

2,2,1 Neural Network

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Perform gradient descent with

$$w_{21} \rightarrow w_{21} - \alpha \frac{\partial L}{\partial w_{21}}$$

to find optimal value of w_{21} that gives the least error

2,2,1 Neural Network

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Perform gradient descent with

$$w_{21} \rightarrow w_{21} - \alpha$$

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2,2,1 Neural Network

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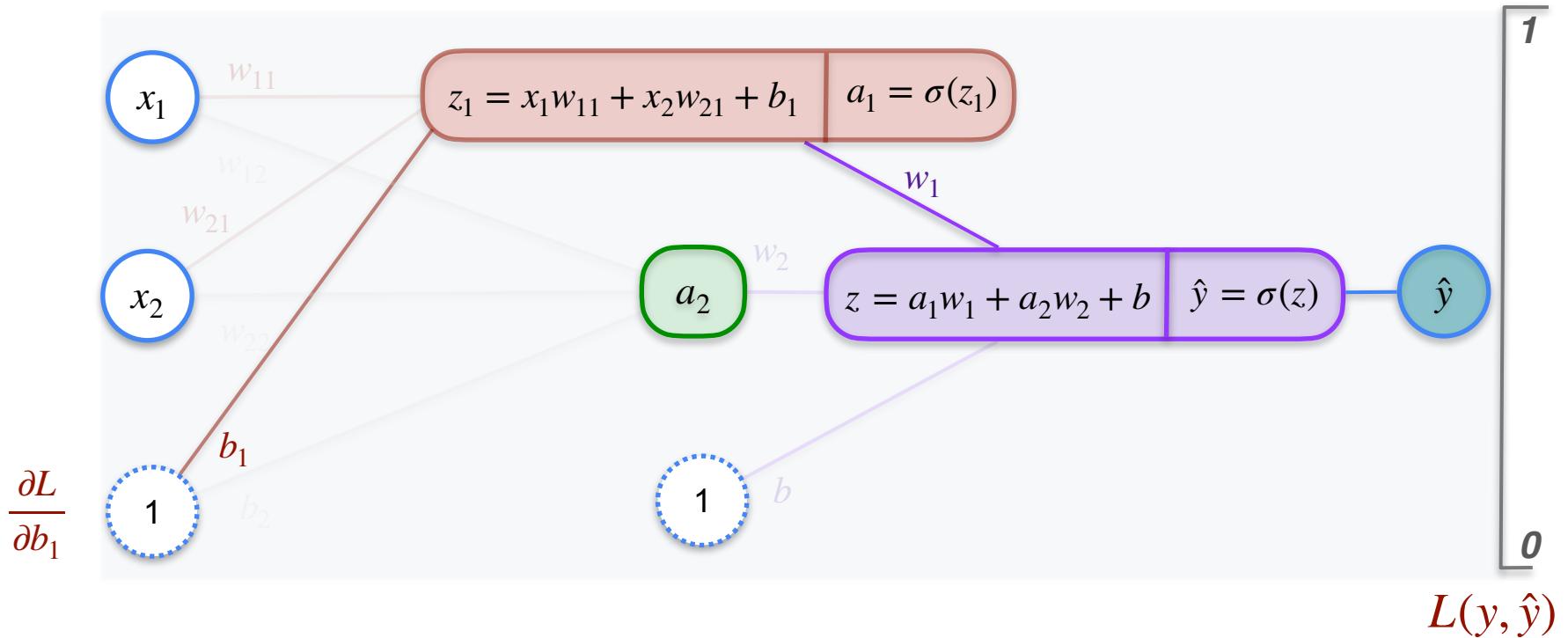
$$\begin{aligned}\frac{\partial L}{\partial w_{21}} &= \frac{\partial z_1}{\partial w_{21}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}} \\ \frac{\partial L}{\partial w_{21}} &= x_2 \cdot a_1(1-a_1) \cdot w_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}} \\ &= -x_2 w_1 a_1 (1-a_1) (y - \hat{y})\end{aligned}$$

Perform gradient descent with

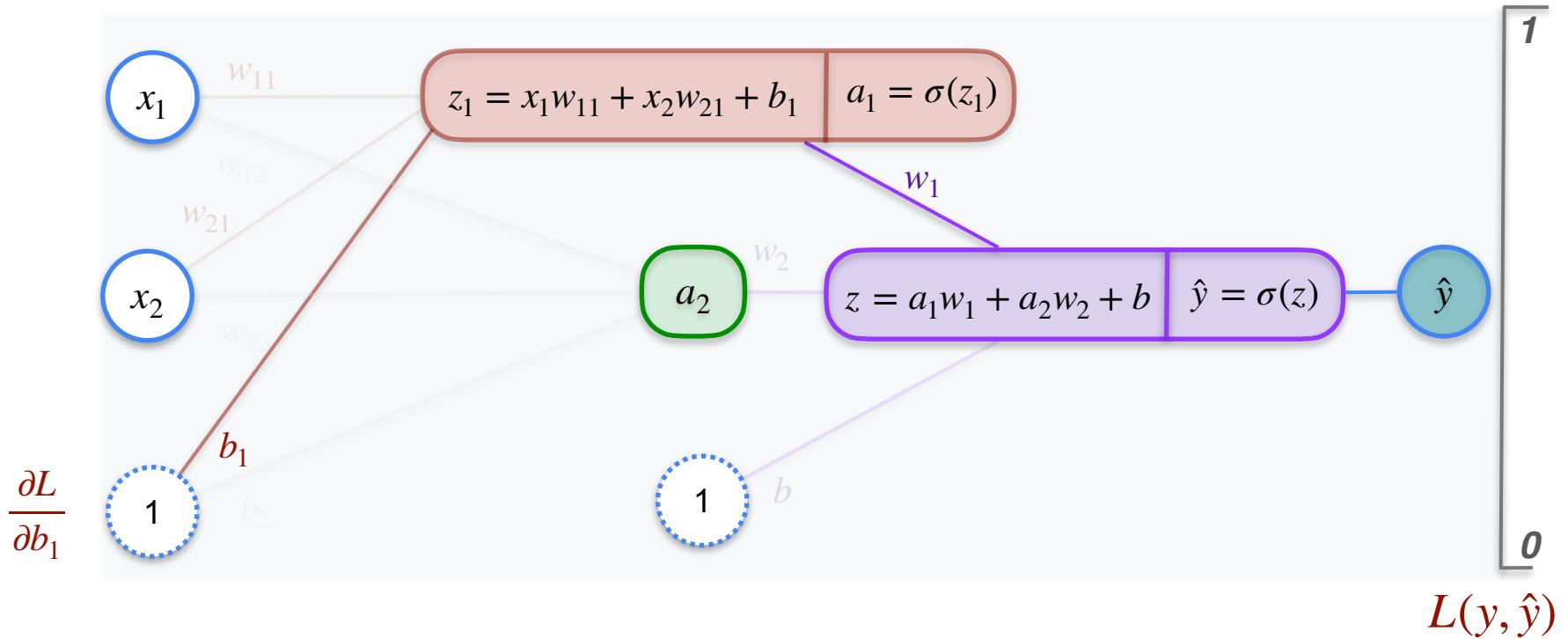
$$w_{21} \rightarrow w_{21} - \alpha \cdot x_2 w_1 a_1 (1-a_1) (y - \hat{y})$$

to find optimal value of w_{21} that gives the least error

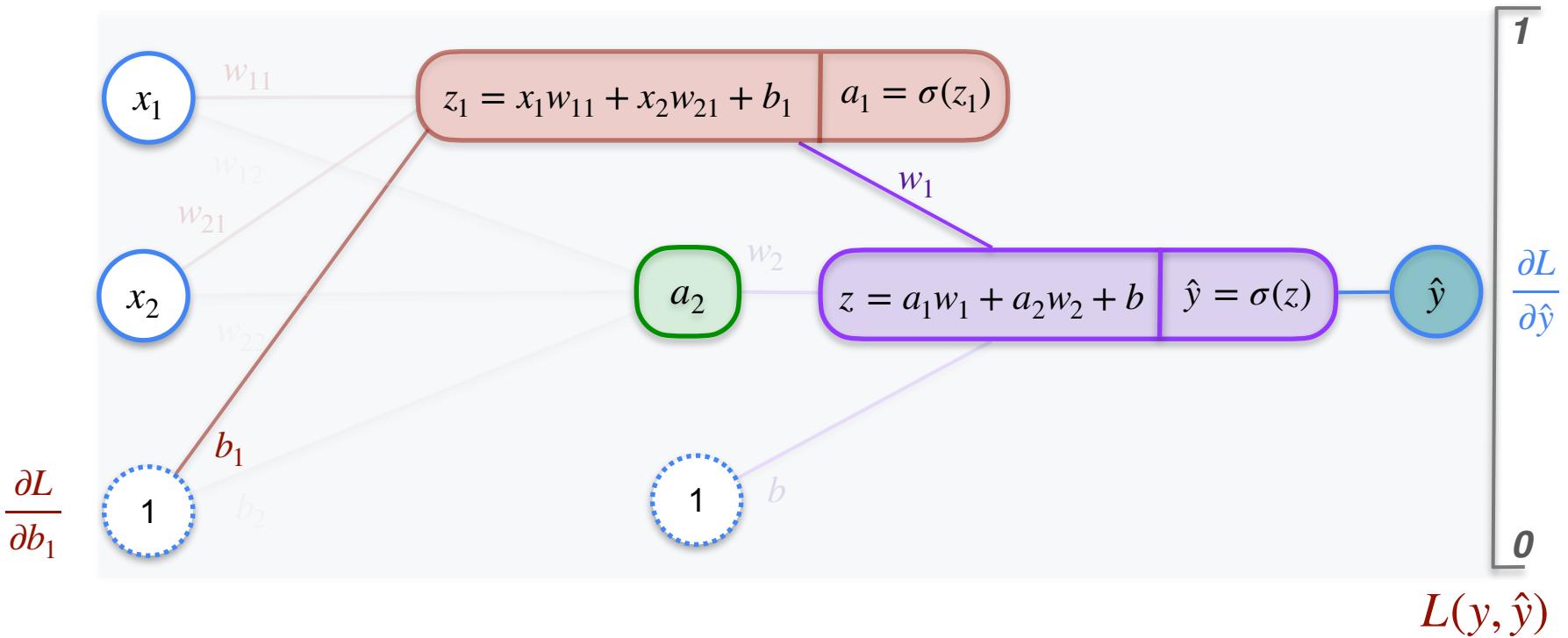
2,2,1 Neural Network



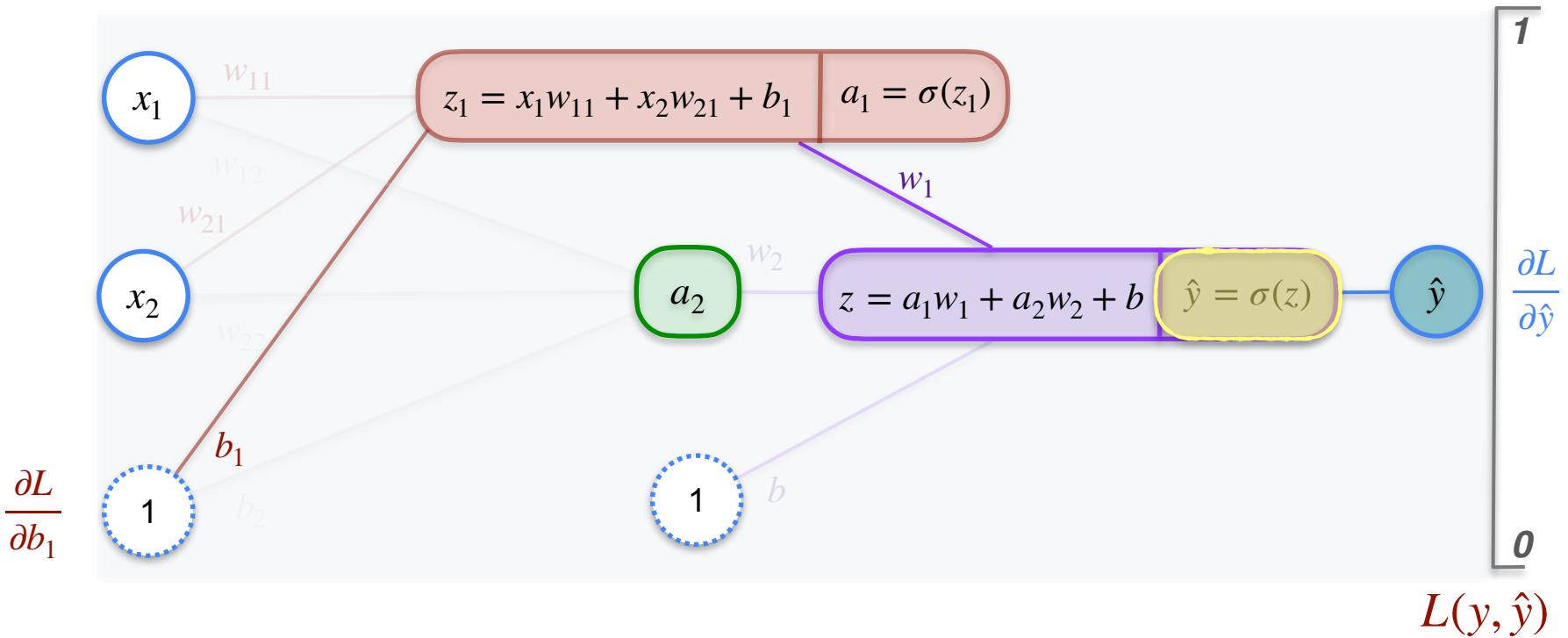
2,2,1 Neural Network



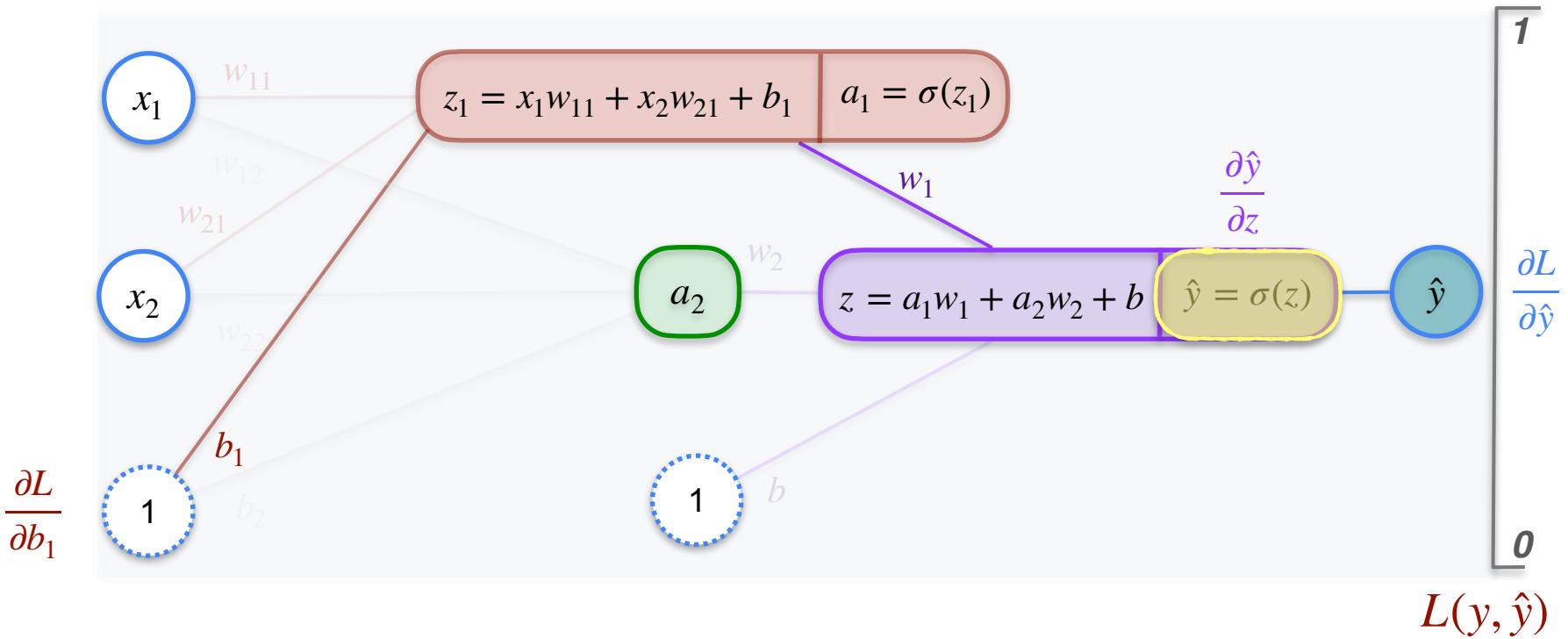
2,2,1 Neural Network



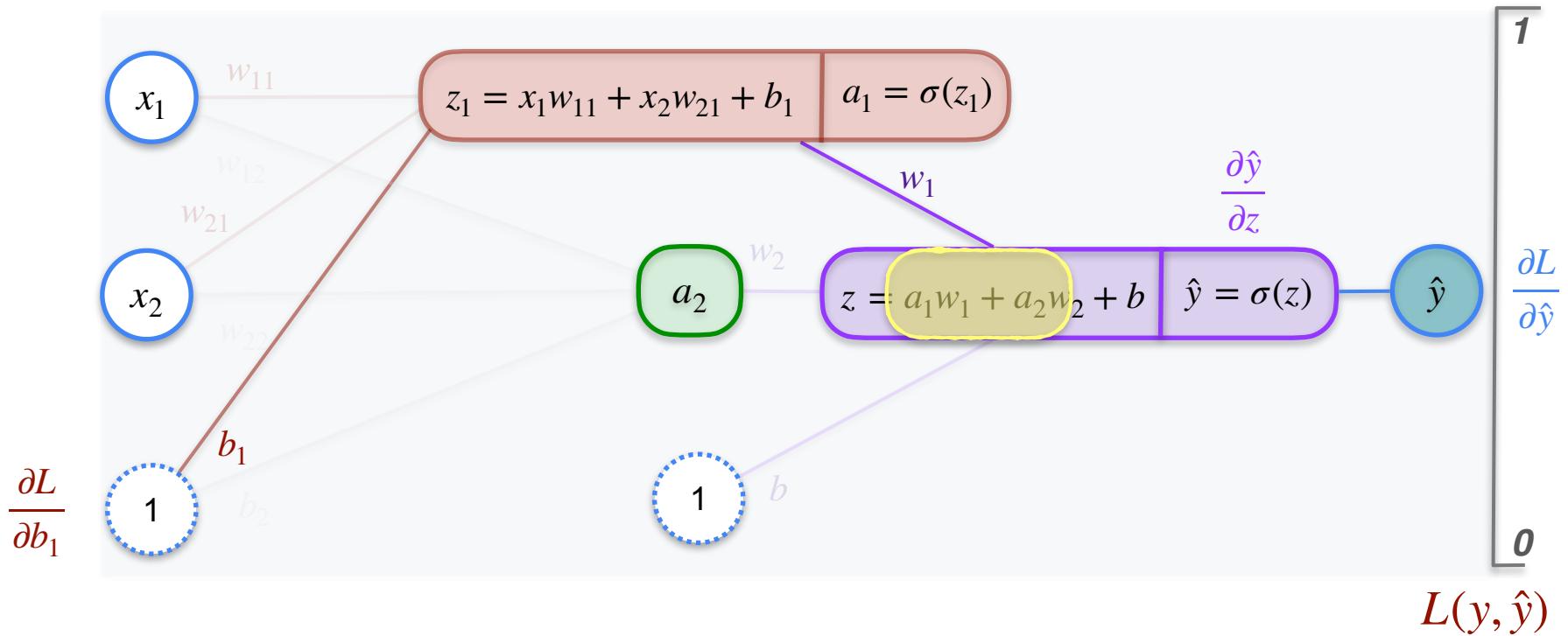
2,2,1 Neural Network



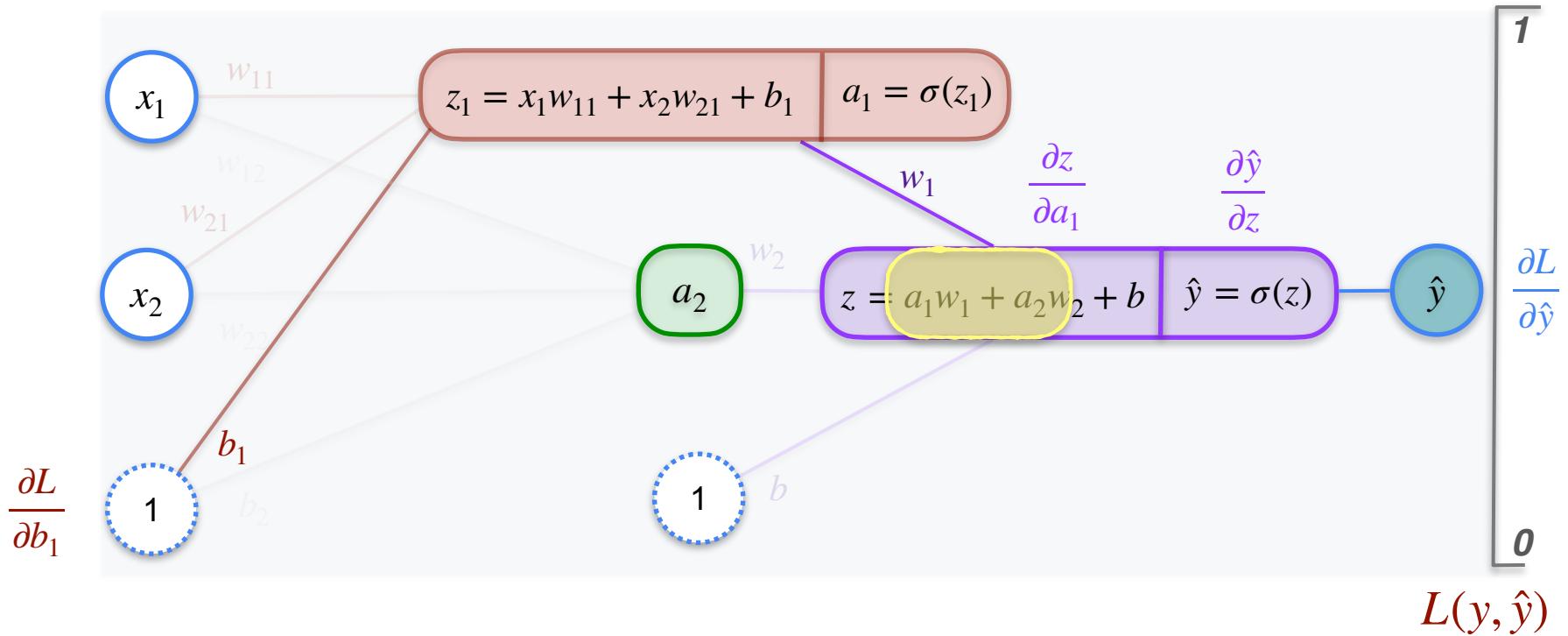
2,2,1 Neural Network



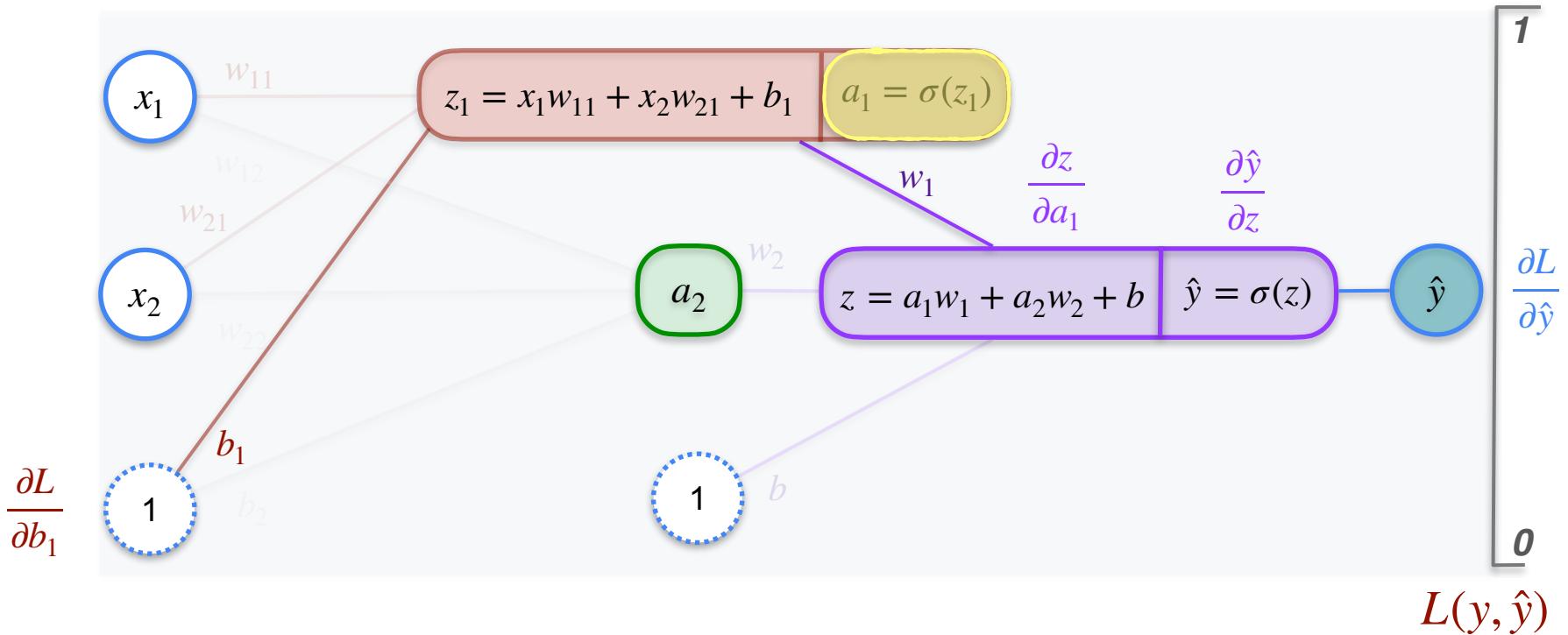
2,2,1 Neural Network



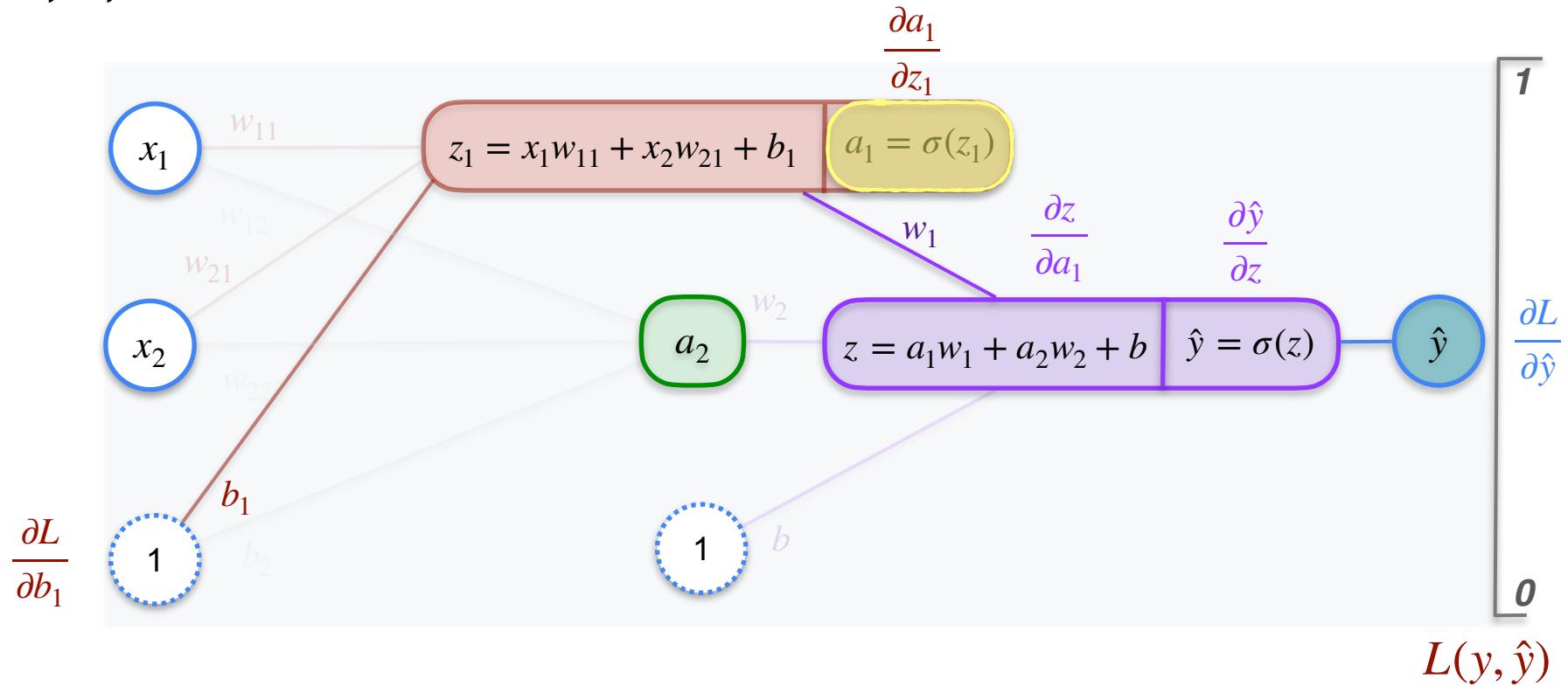
2,2,1 Neural Network



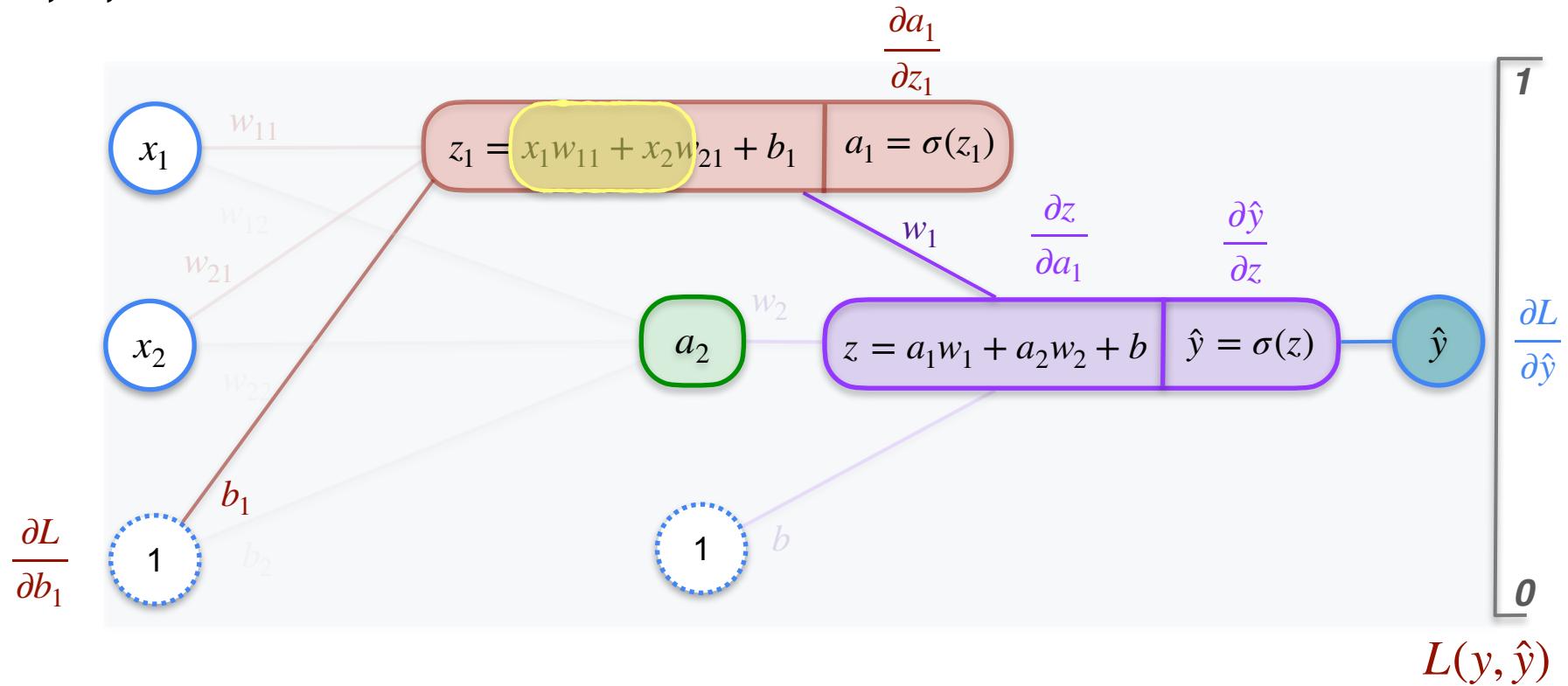
2,2,1 Neural Network



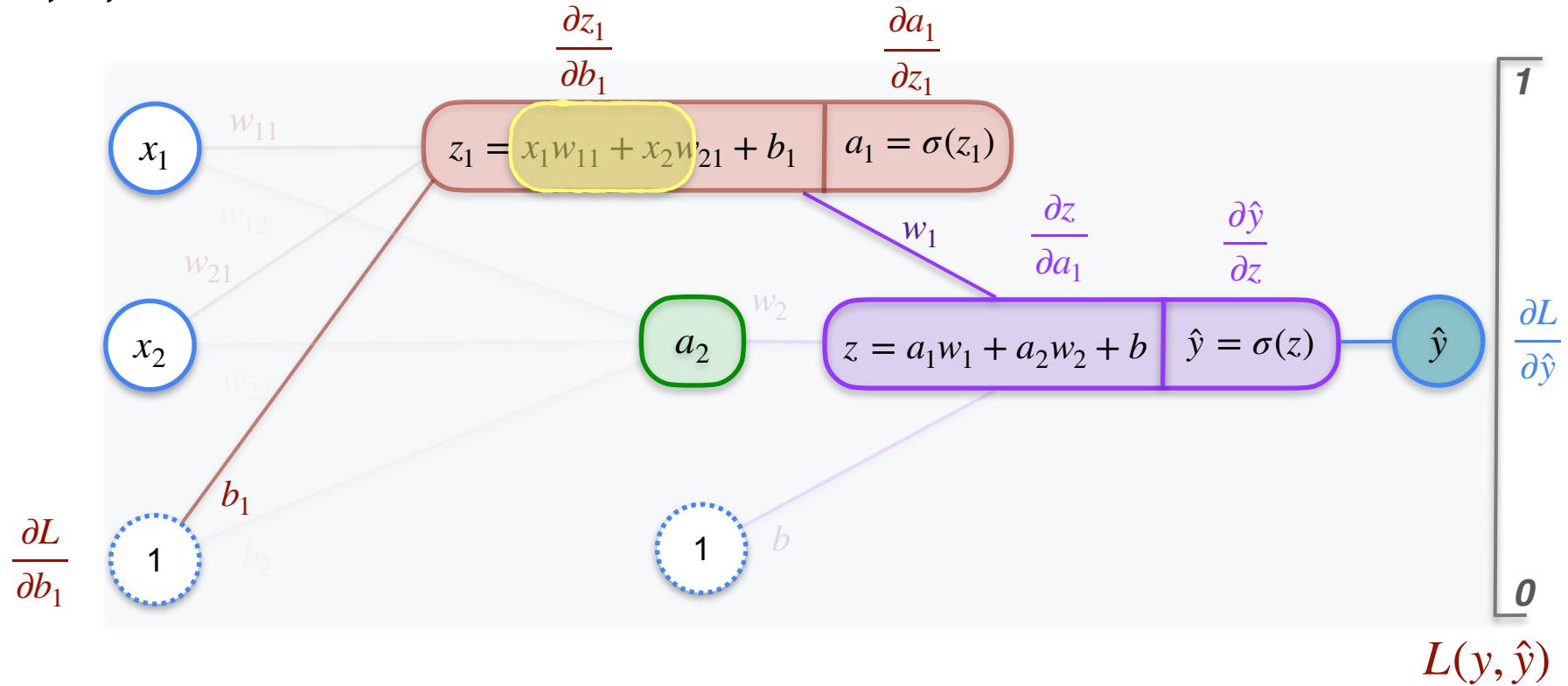
2,2,1 Neural Network



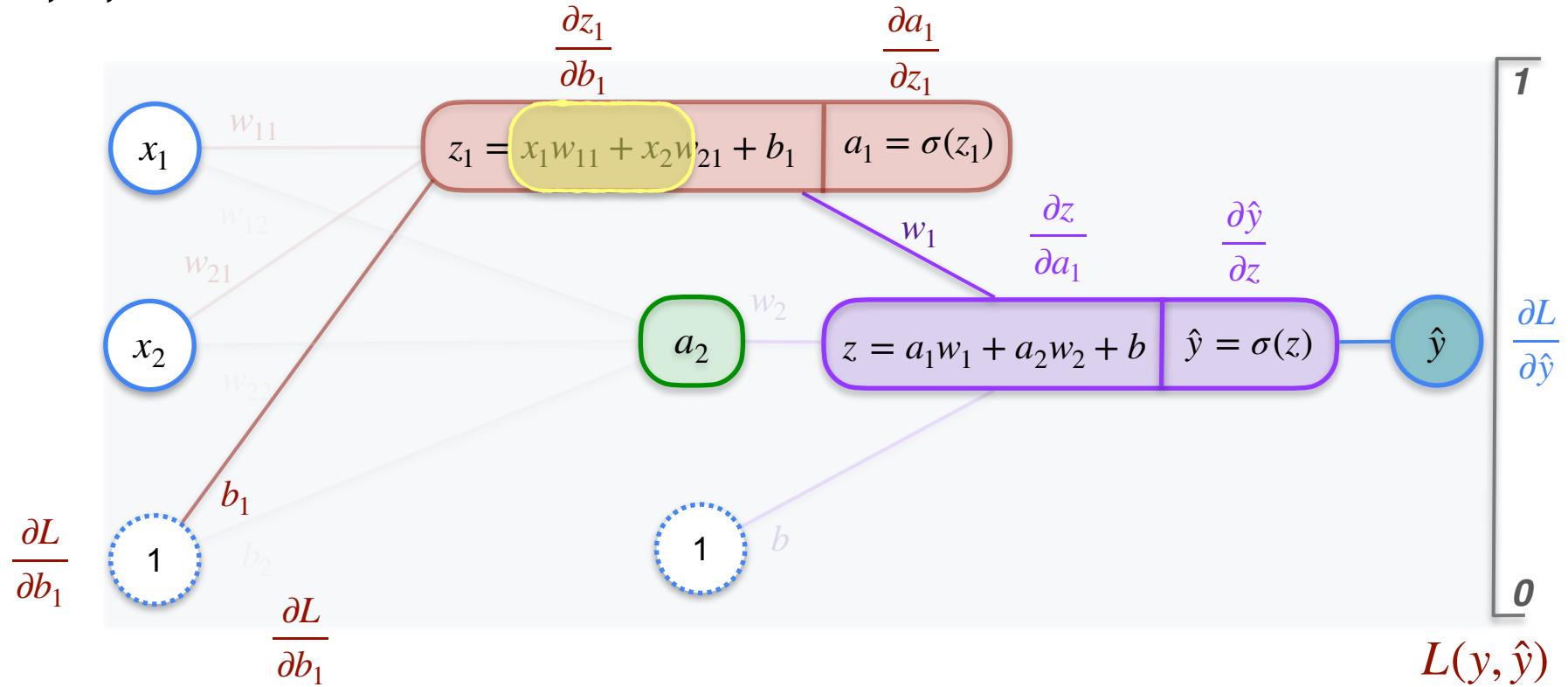
2,2,1 Neural Network



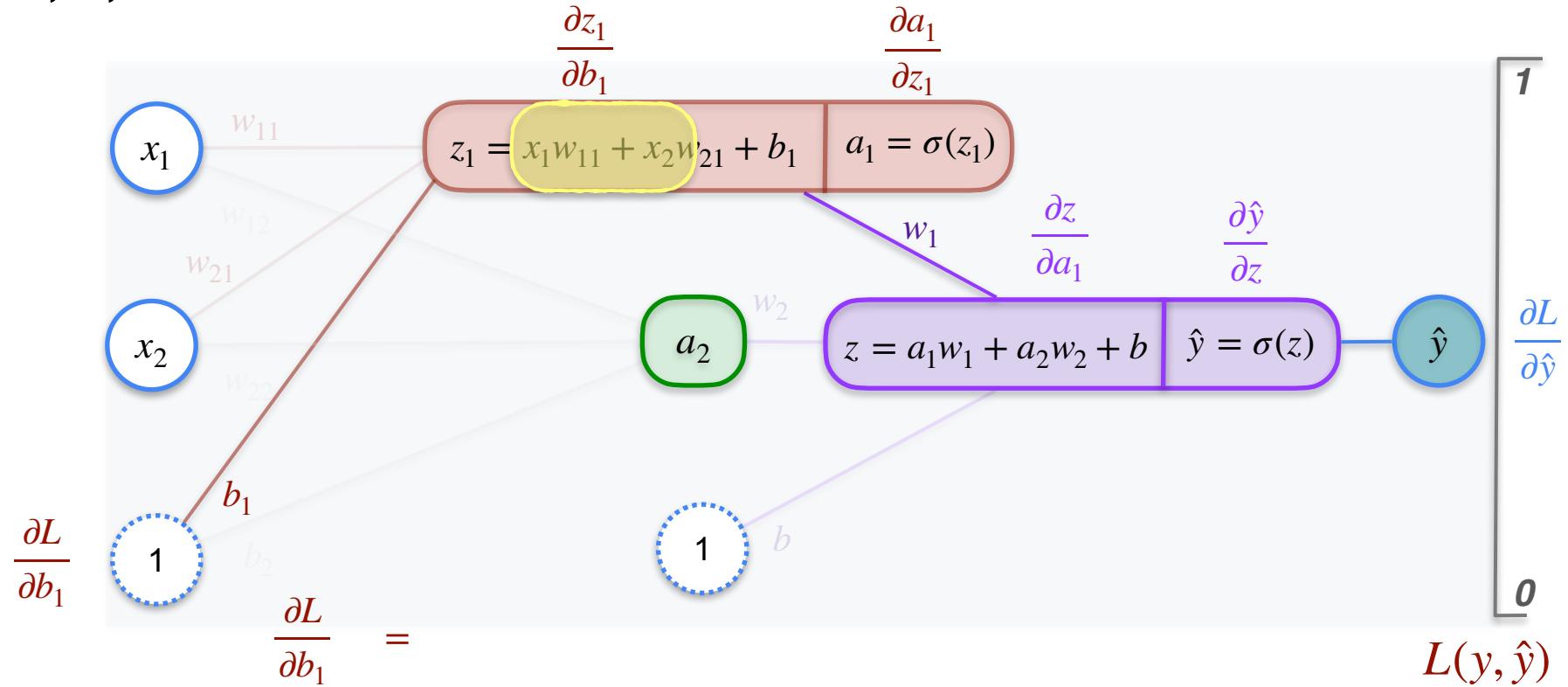
2,2,1 Neural Network



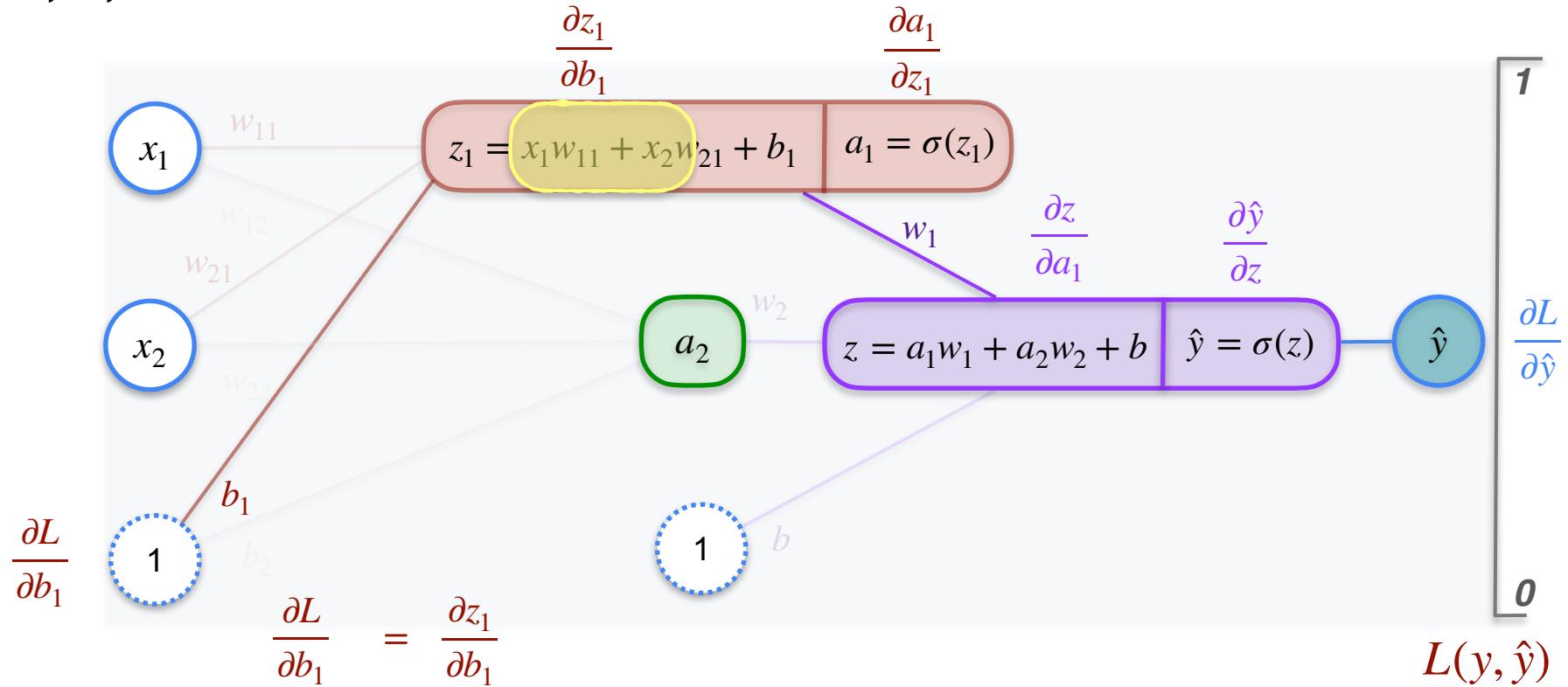
2,2,1 Neural Network



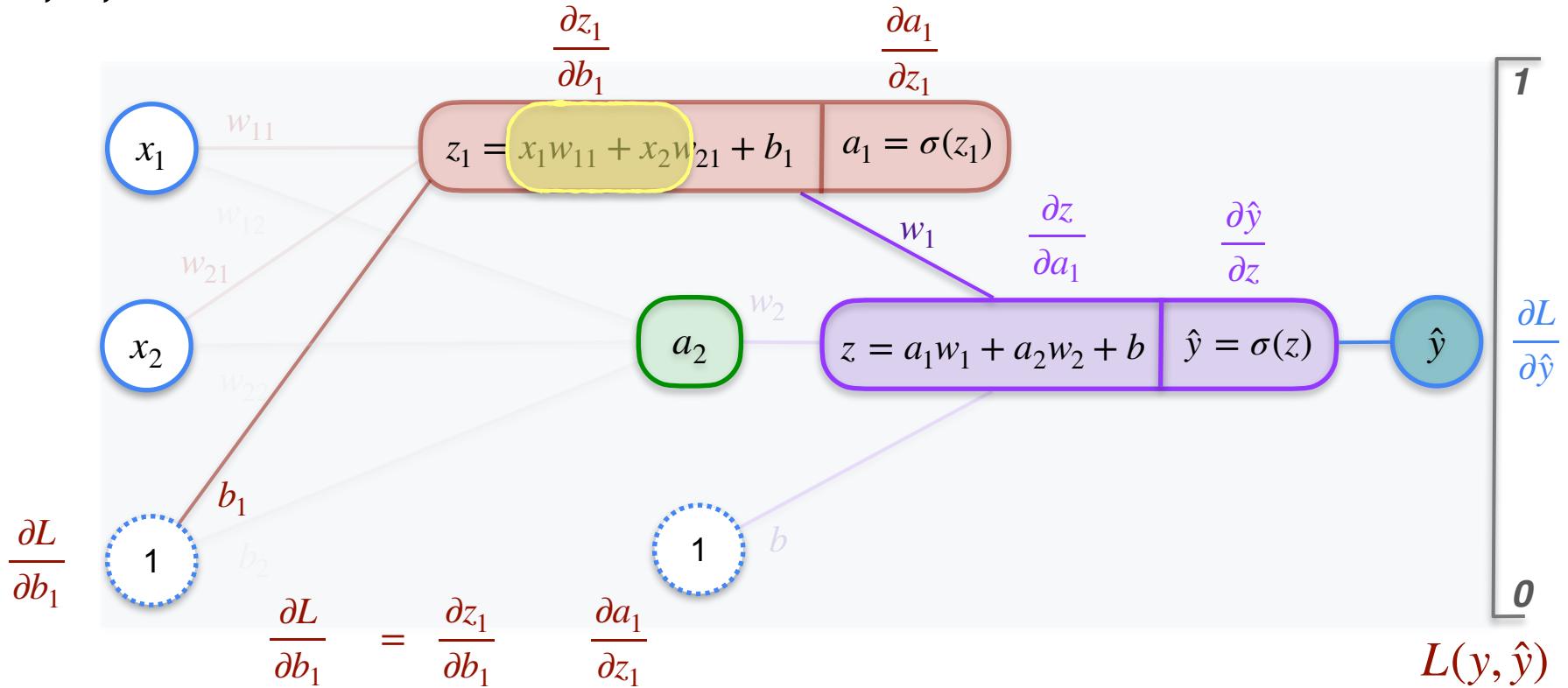
2,2,1 Neural Network



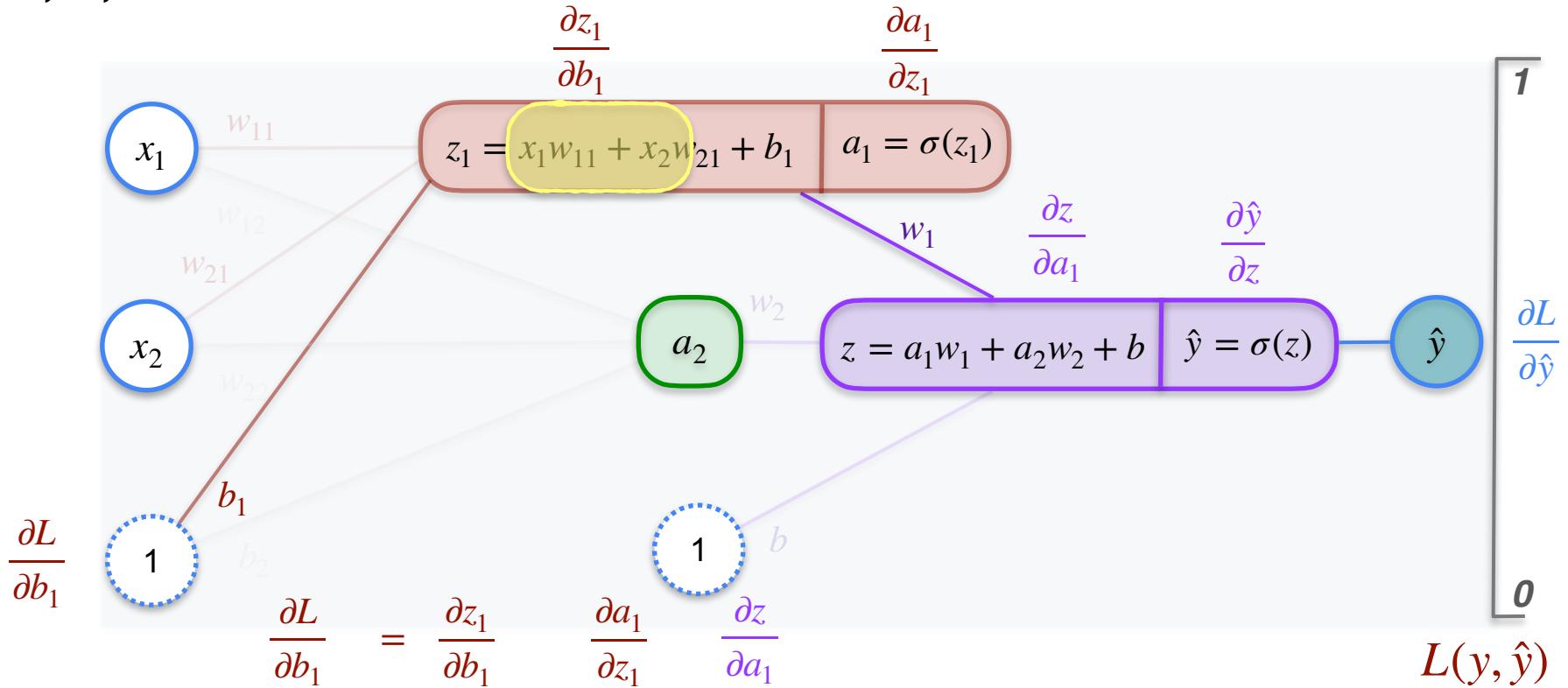
2,2,1 Neural Network



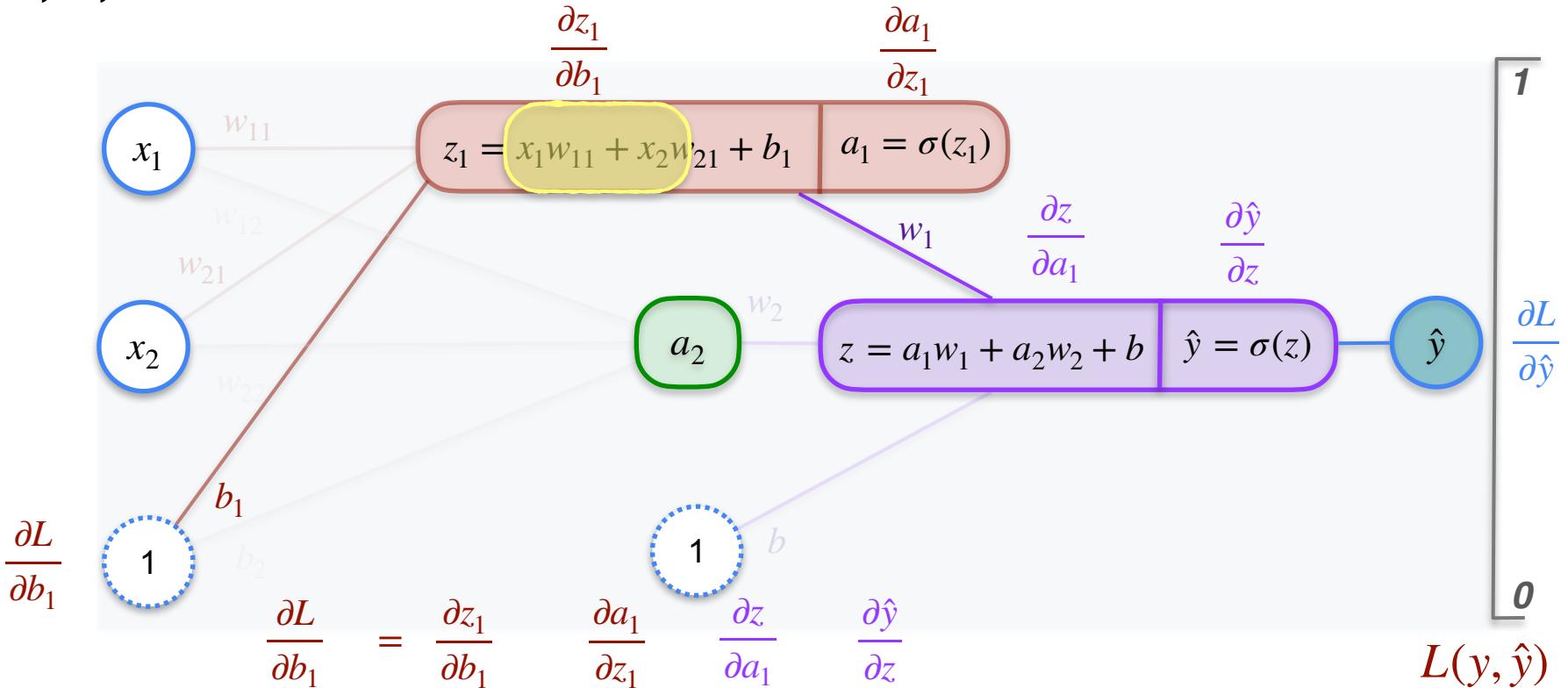
2,2,1 Neural Network



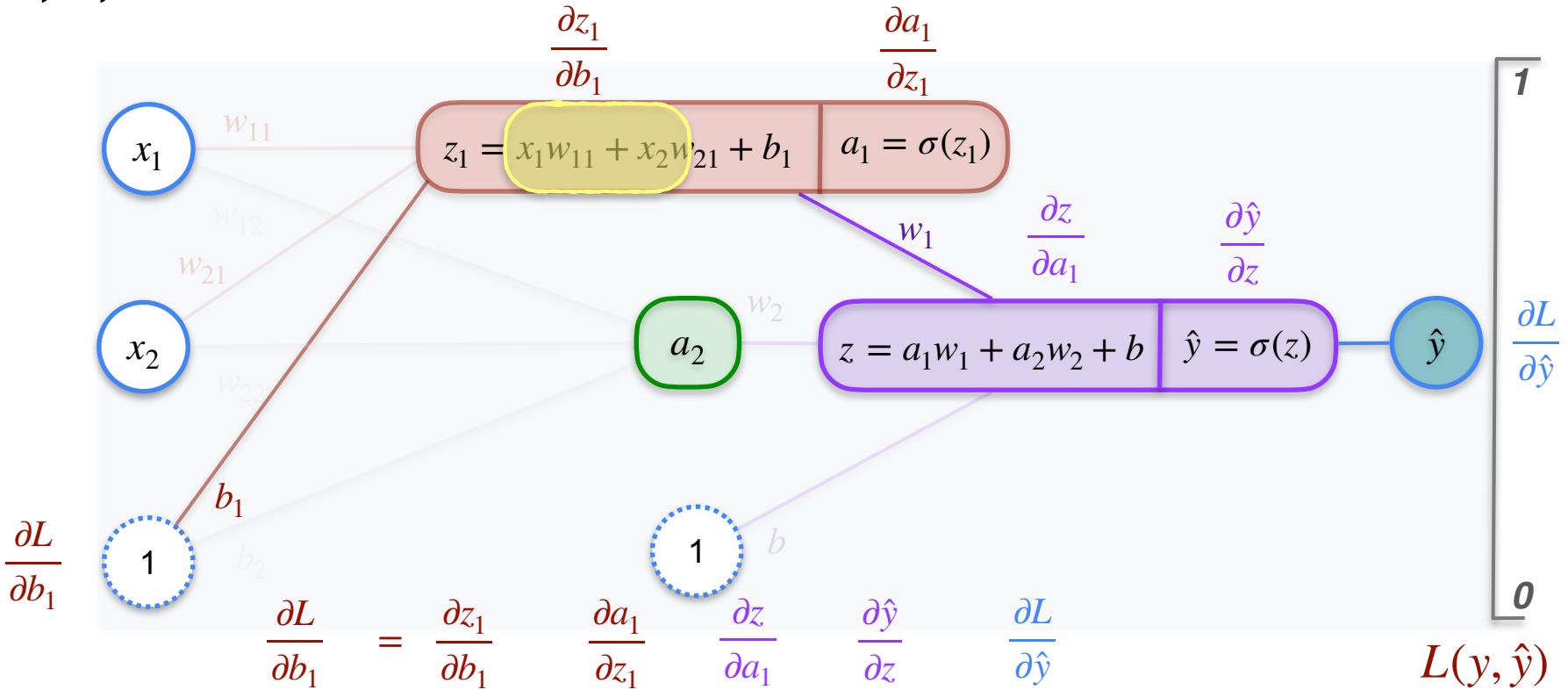
2,2,1 Neural Network



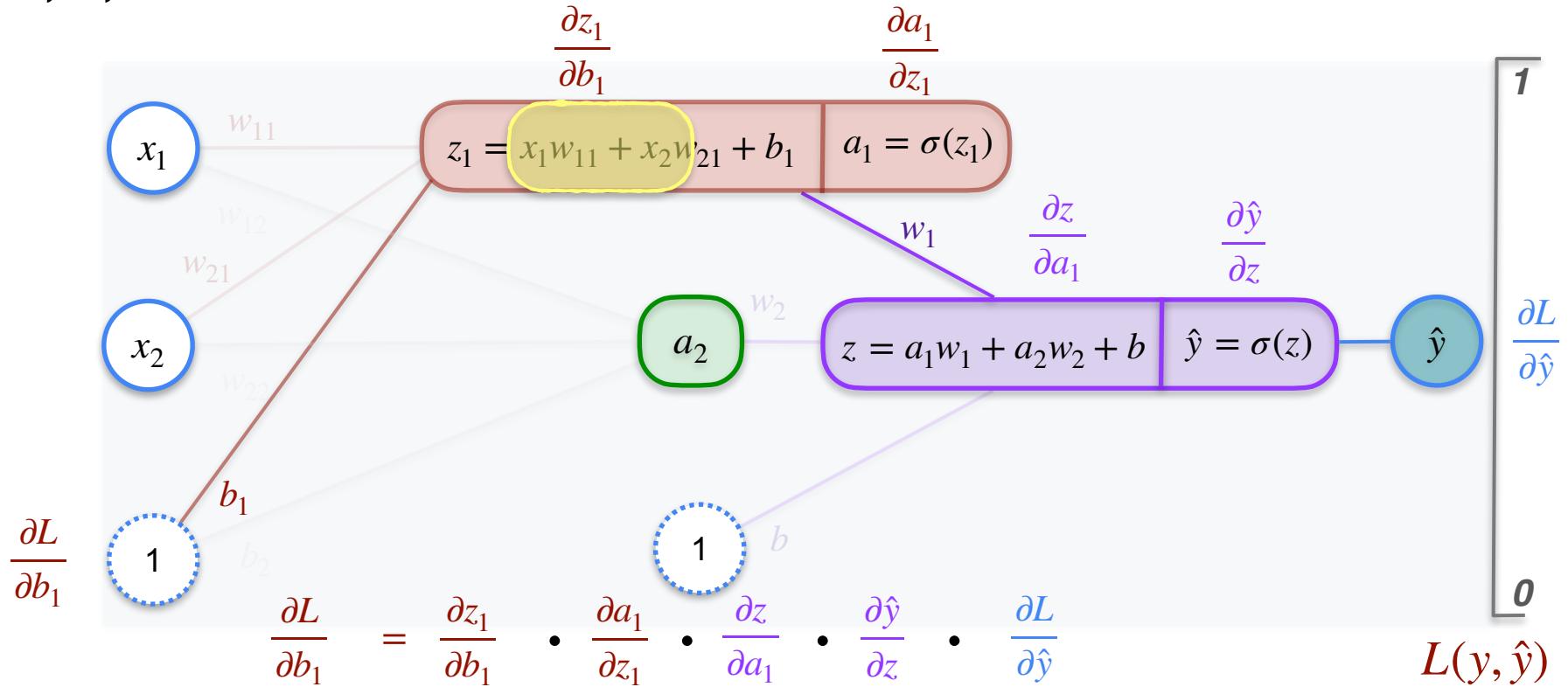
2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network

$$\frac{\partial L}{\partial b_1} = \frac{\partial z_1}{\partial b_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}) \quad \frac{\partial L}{\partial b_1} = \frac{\partial z_1}{\partial b_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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$$\frac{\partial L}{\partial b_1} = 1 - a_1(1 - a_1)$$

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Perform gradient descent with

*to find optimal
value of b_1 that
gives the least error*

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Perform gradient descent with

$$b_1 \rightarrow b_1 - \alpha \frac{\partial L}{\partial b_1}$$

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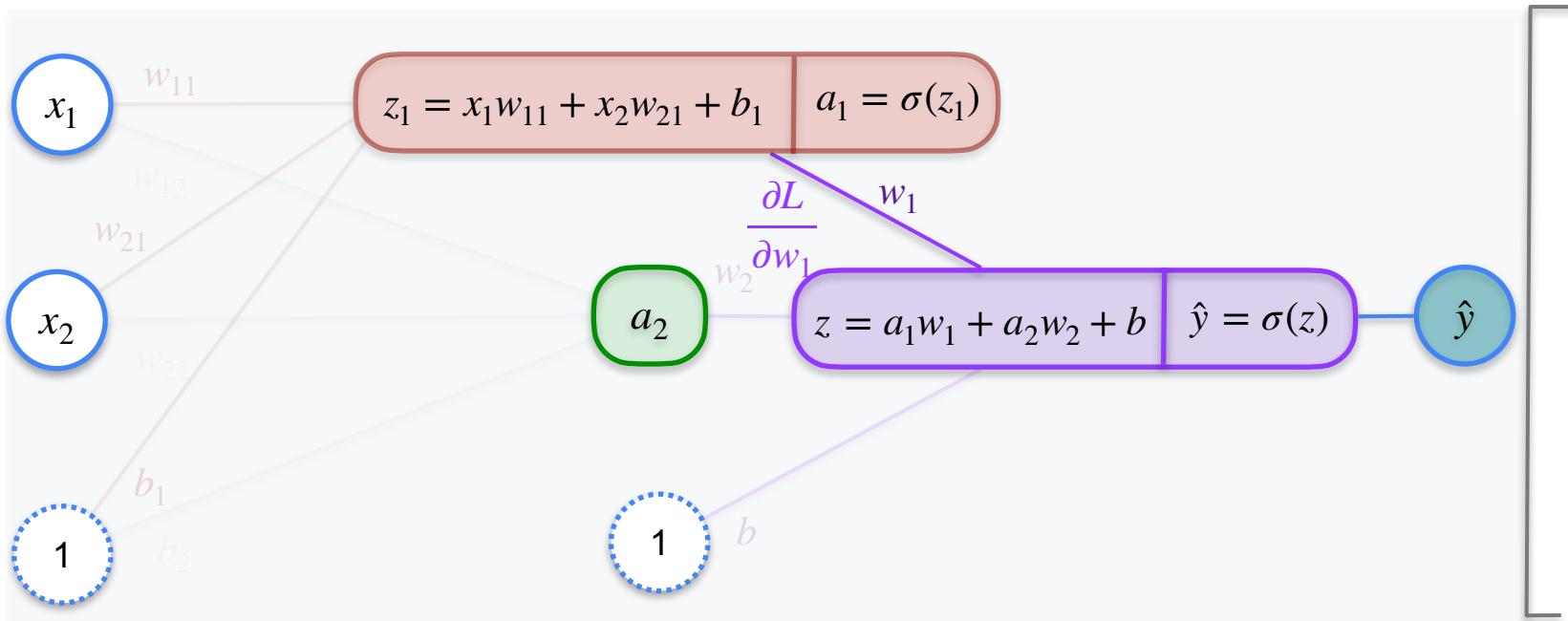
$$\begin{aligned}\frac{\partial L}{\partial b_1} &= \frac{\partial z_1}{\partial b_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}} \\ \frac{\partial L}{\partial b_1} &= 1 \cdot a_1(1-a_1) \cdot w_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}} \\ &= -w_1 a_1 (1-a_1) (y - \hat{y})\end{aligned}$$

Perform gradient descent with

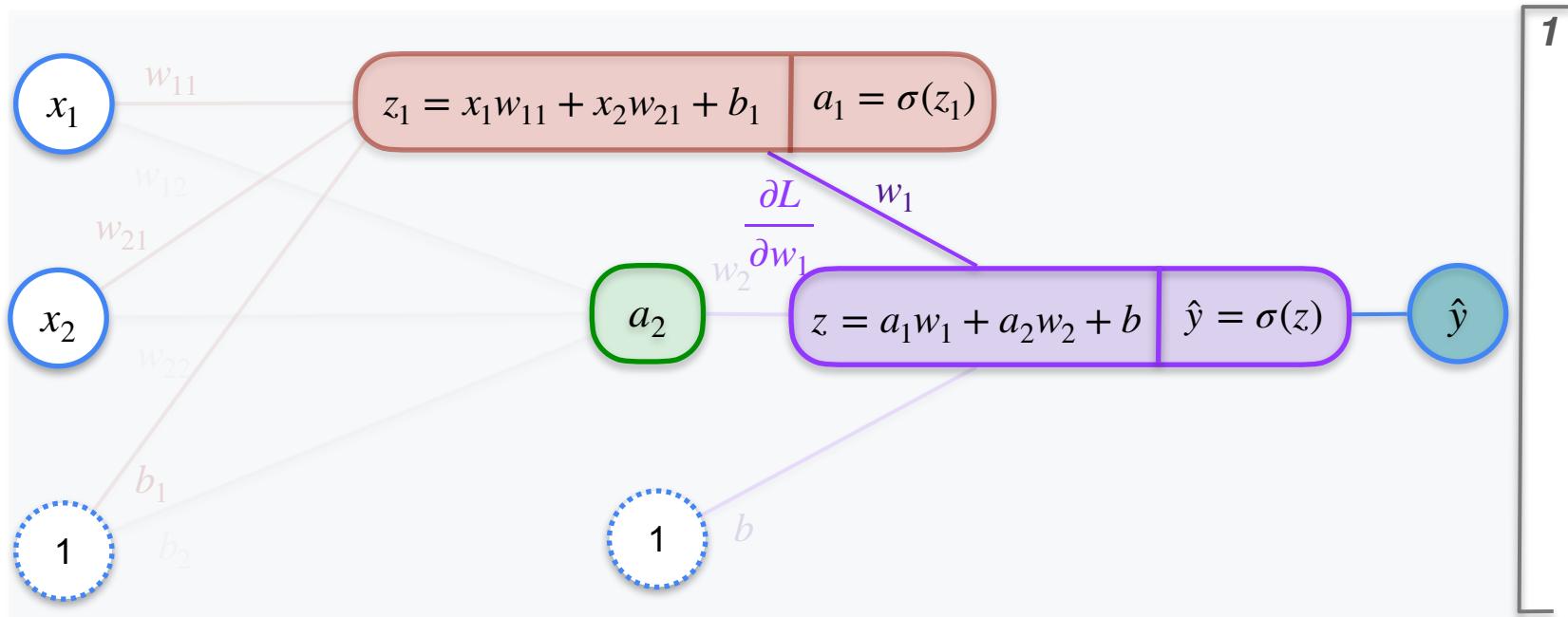
$$b_1 \rightarrow b_1 - \alpha (-w_1 a_1 (1-a_1) (y - \hat{y}))$$

to find optimal value of b_1 that gives the least error

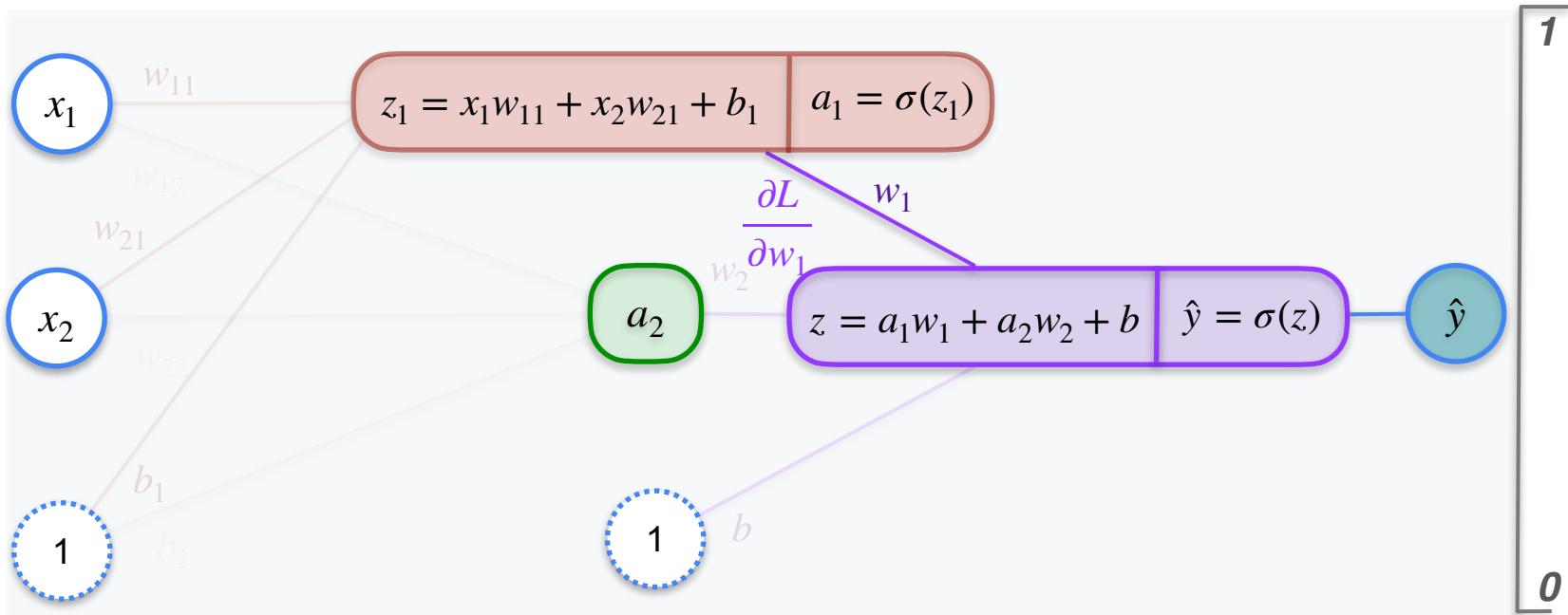
2,2,1 Neural Network



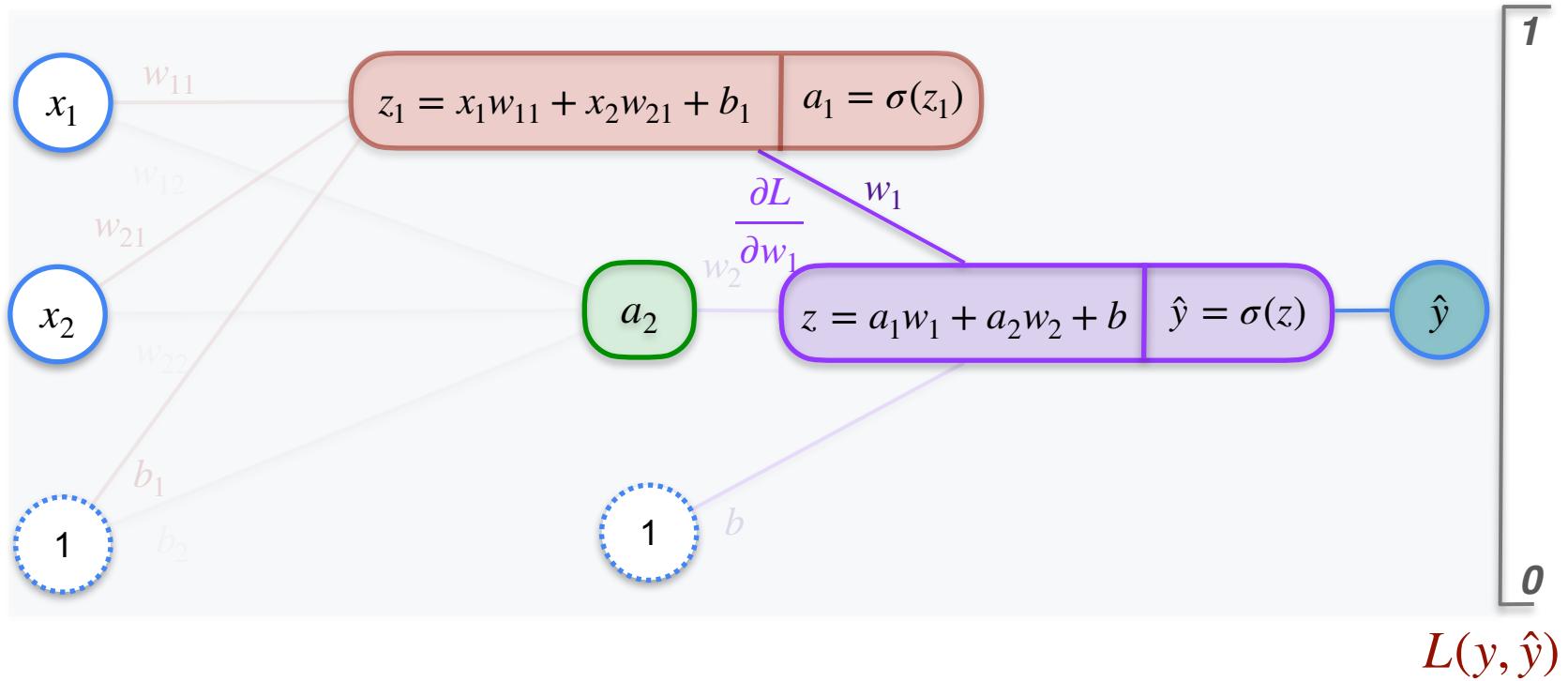
2,2,1 Neural Network



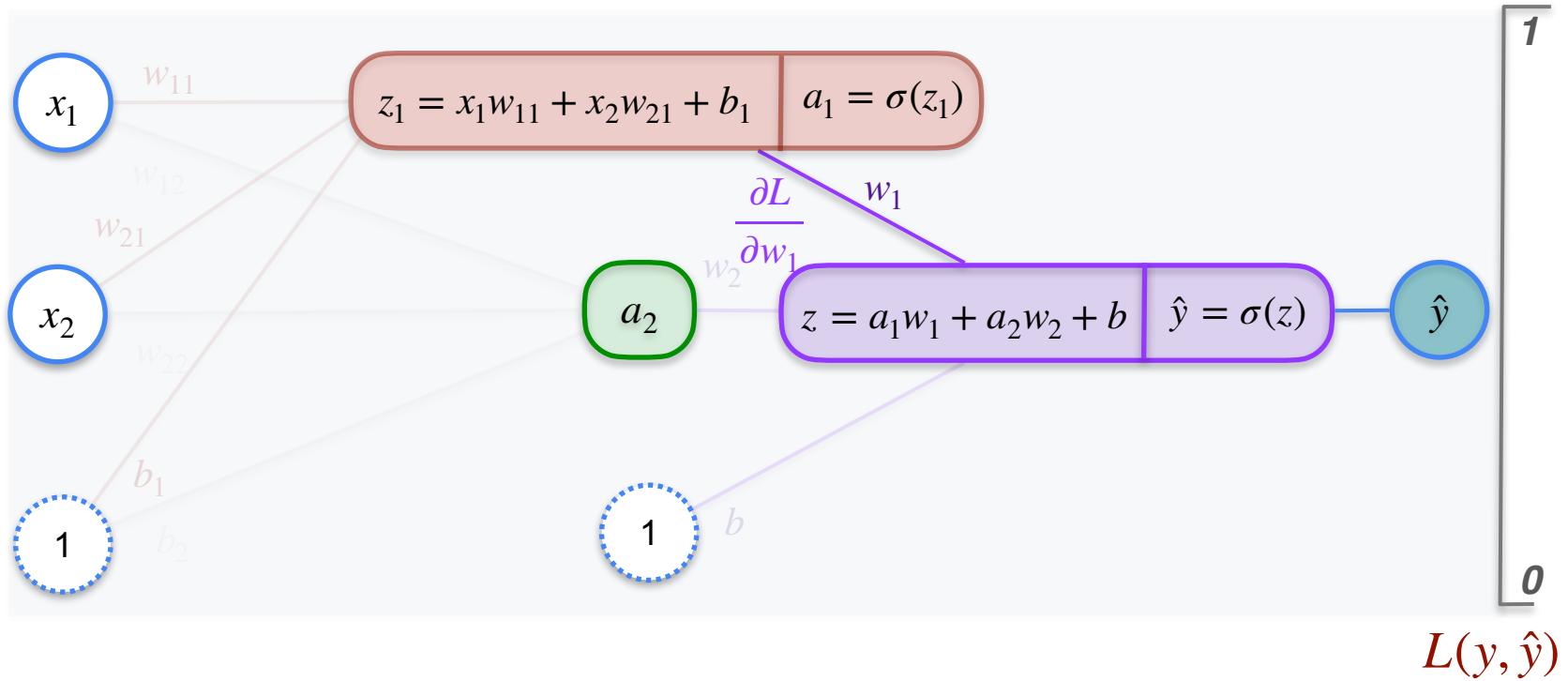
2,2,1 Neural Network



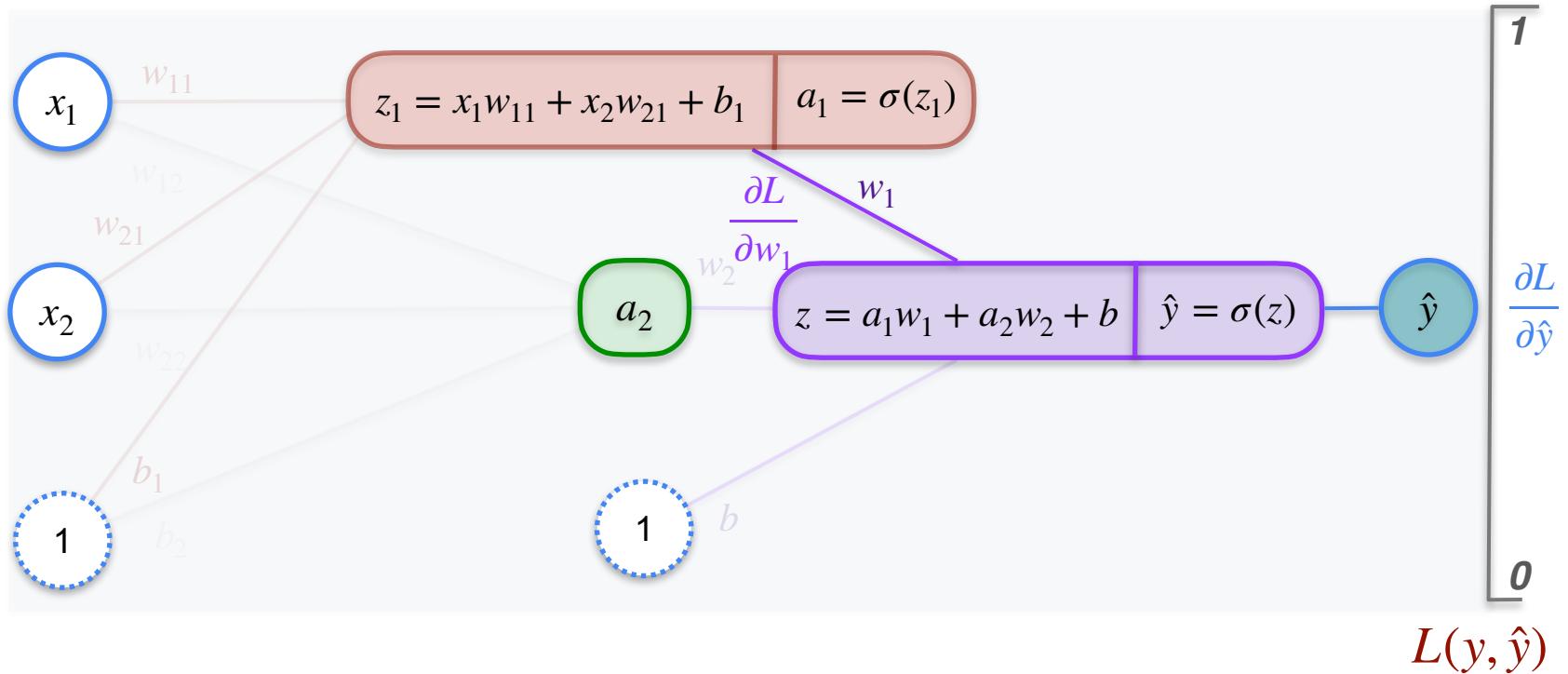
2,2,1 Neural Network



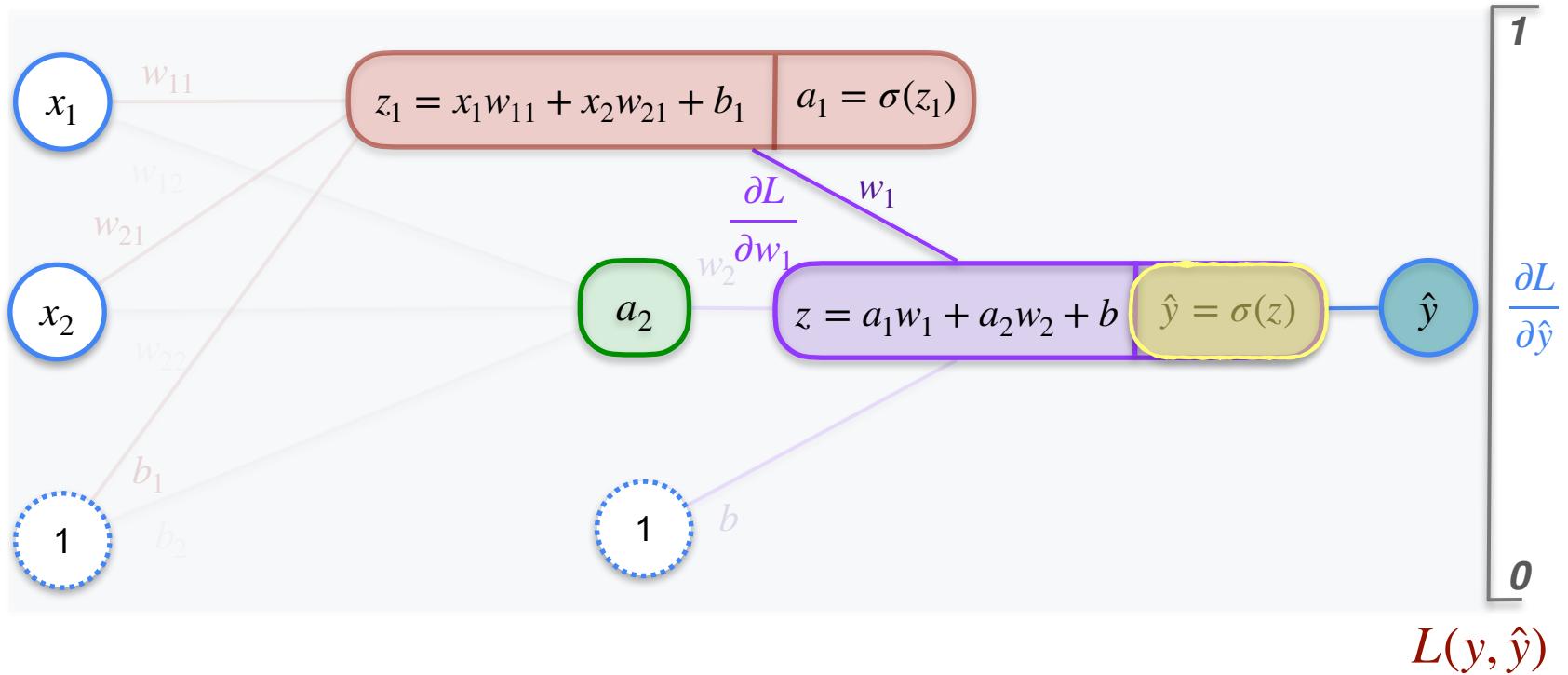
2,2,1 Neural Network



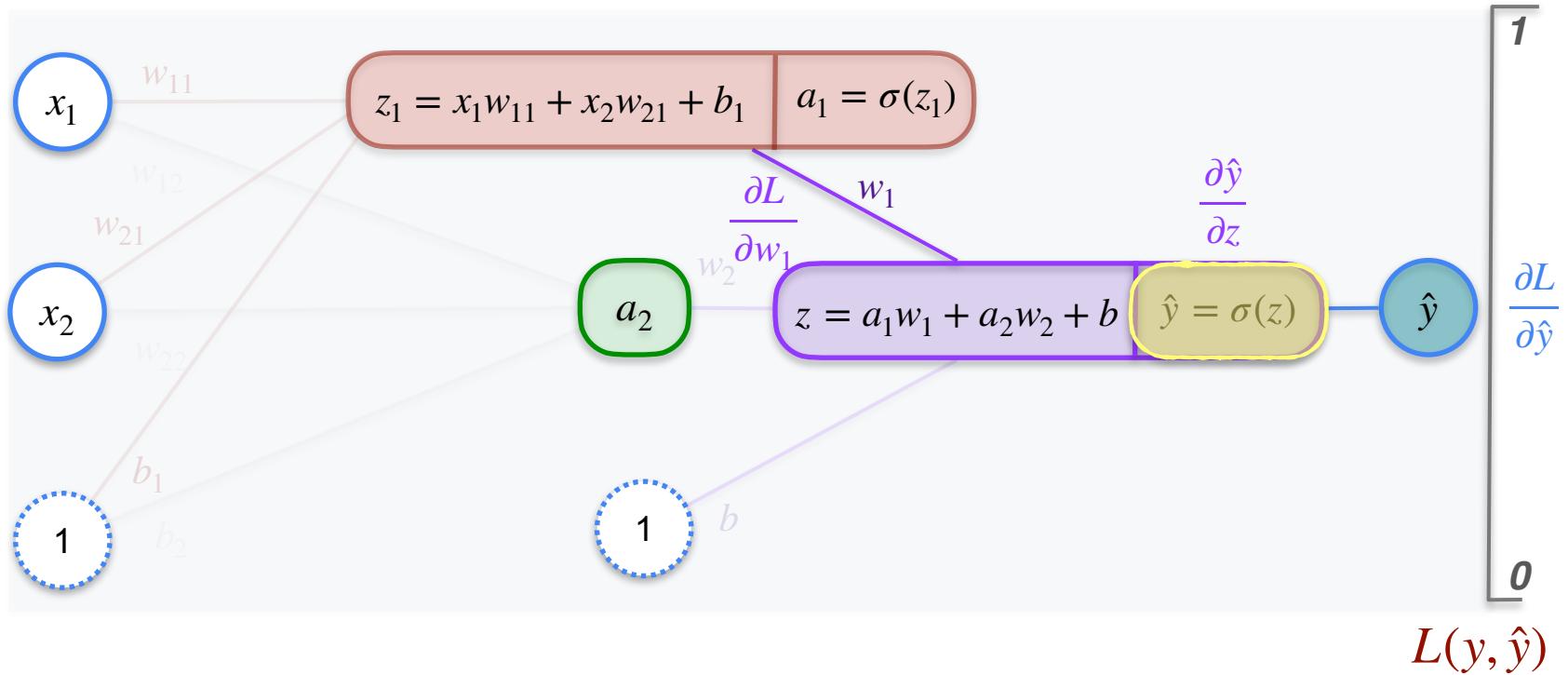
2,2,1 Neural Network



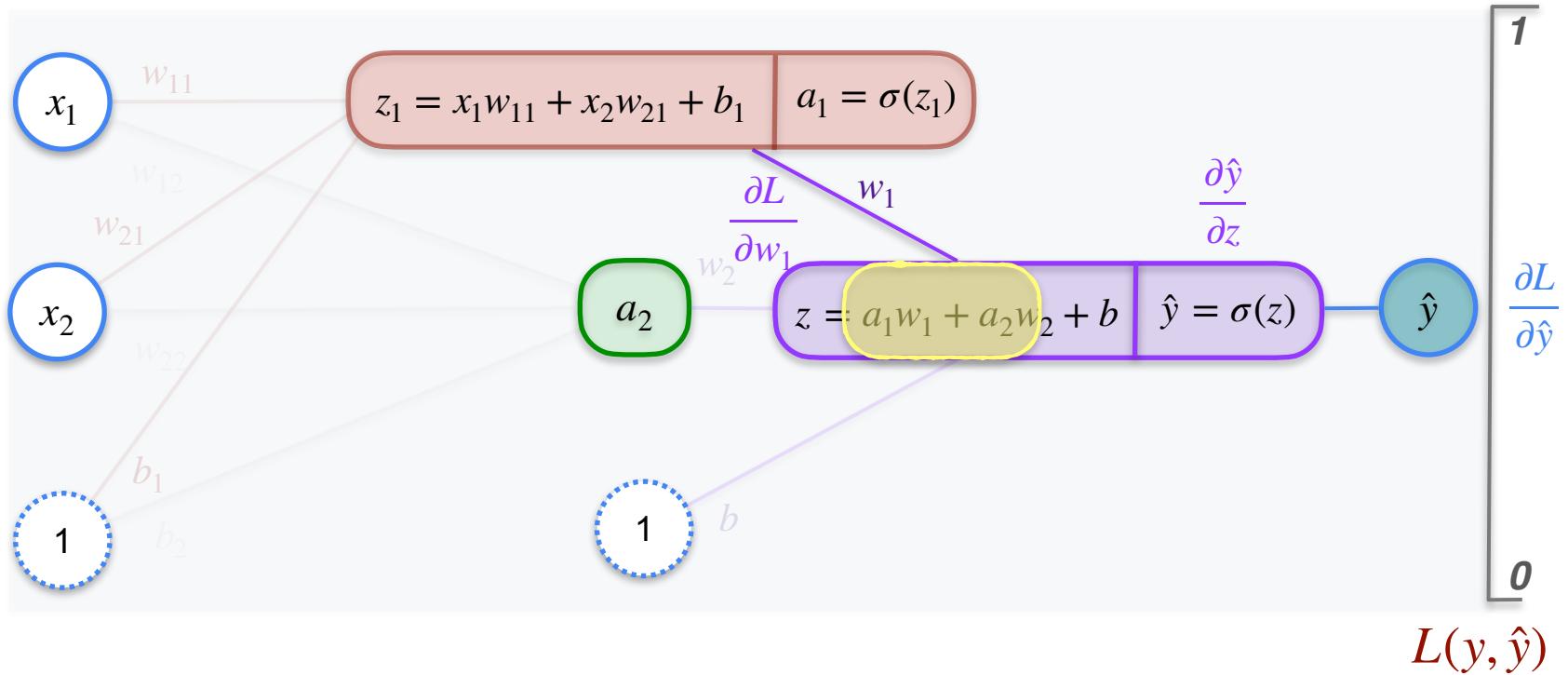
2,2,1 Neural Network



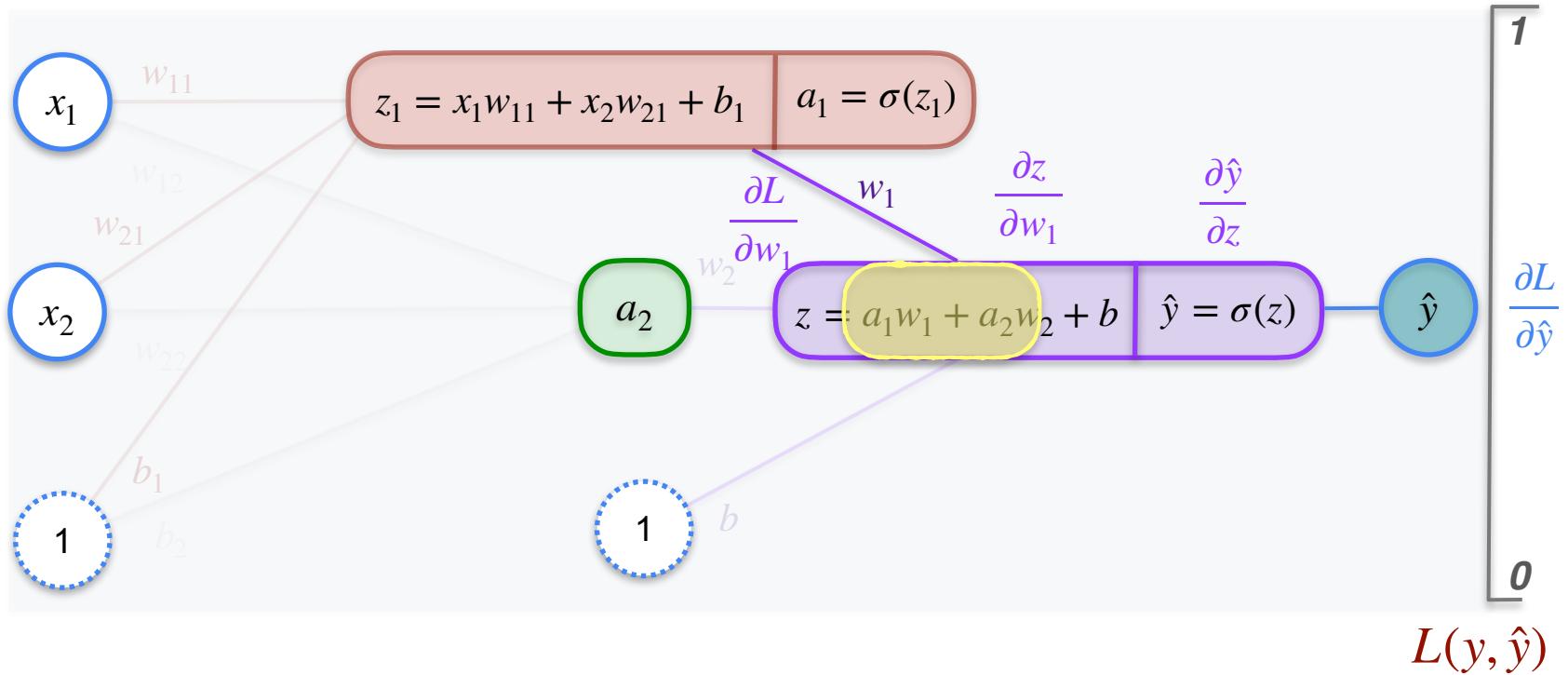
2,2,1 Neural Network



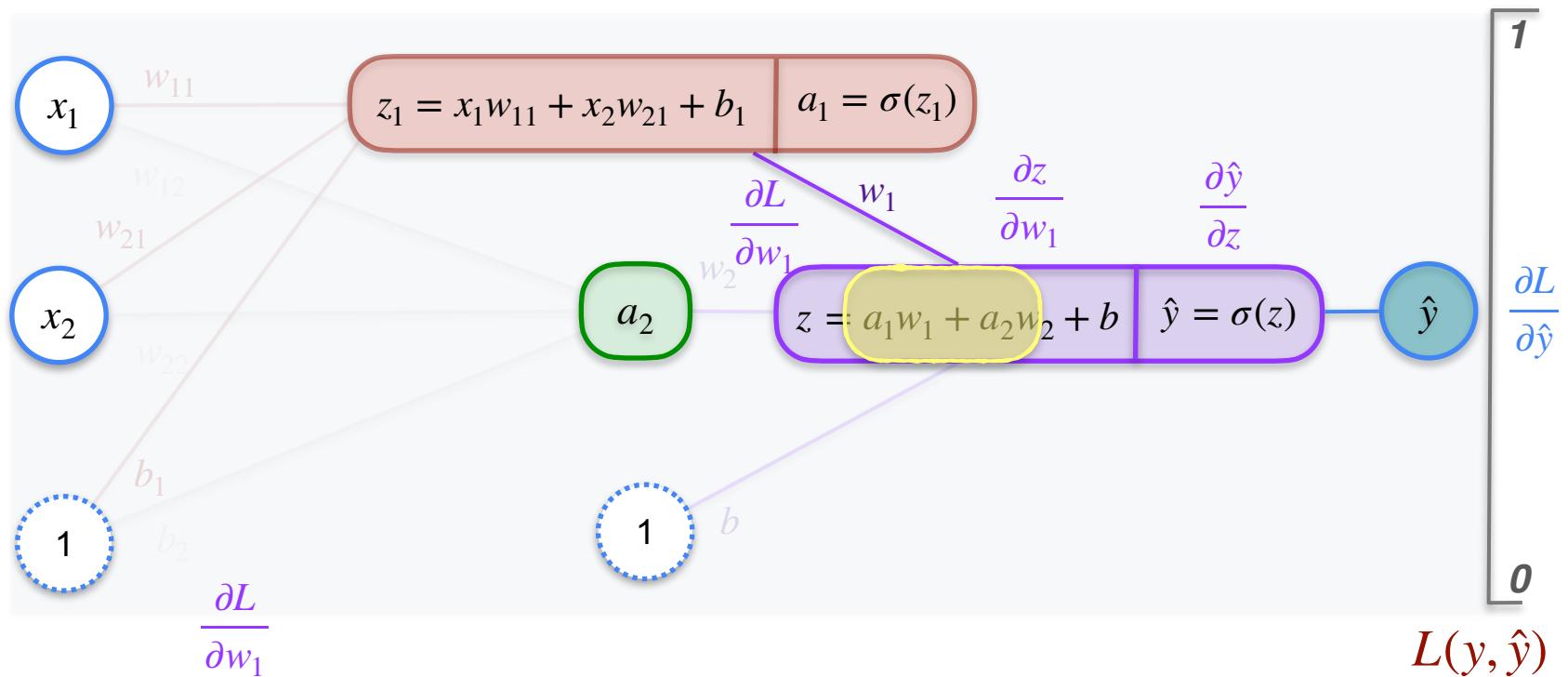
2,2,1 Neural Network



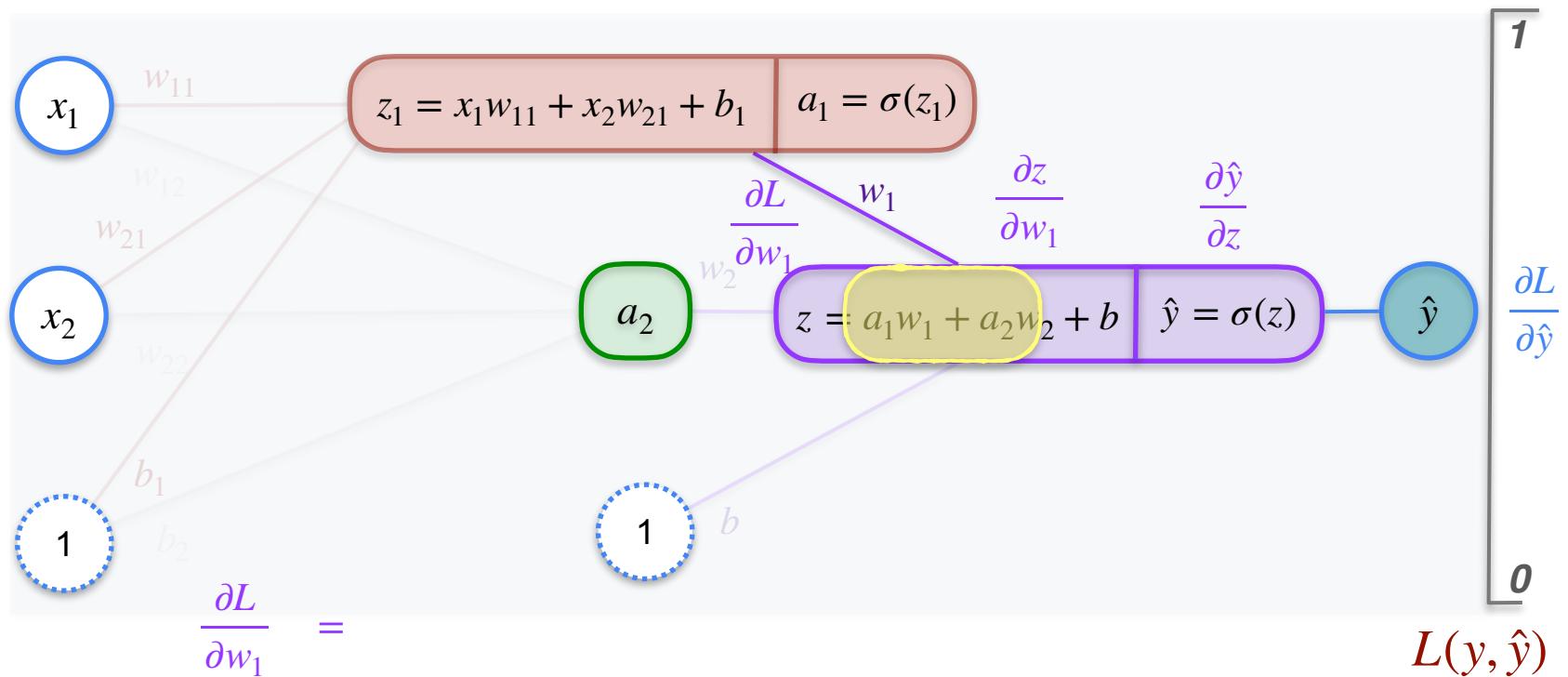
2,2,1 Neural Network



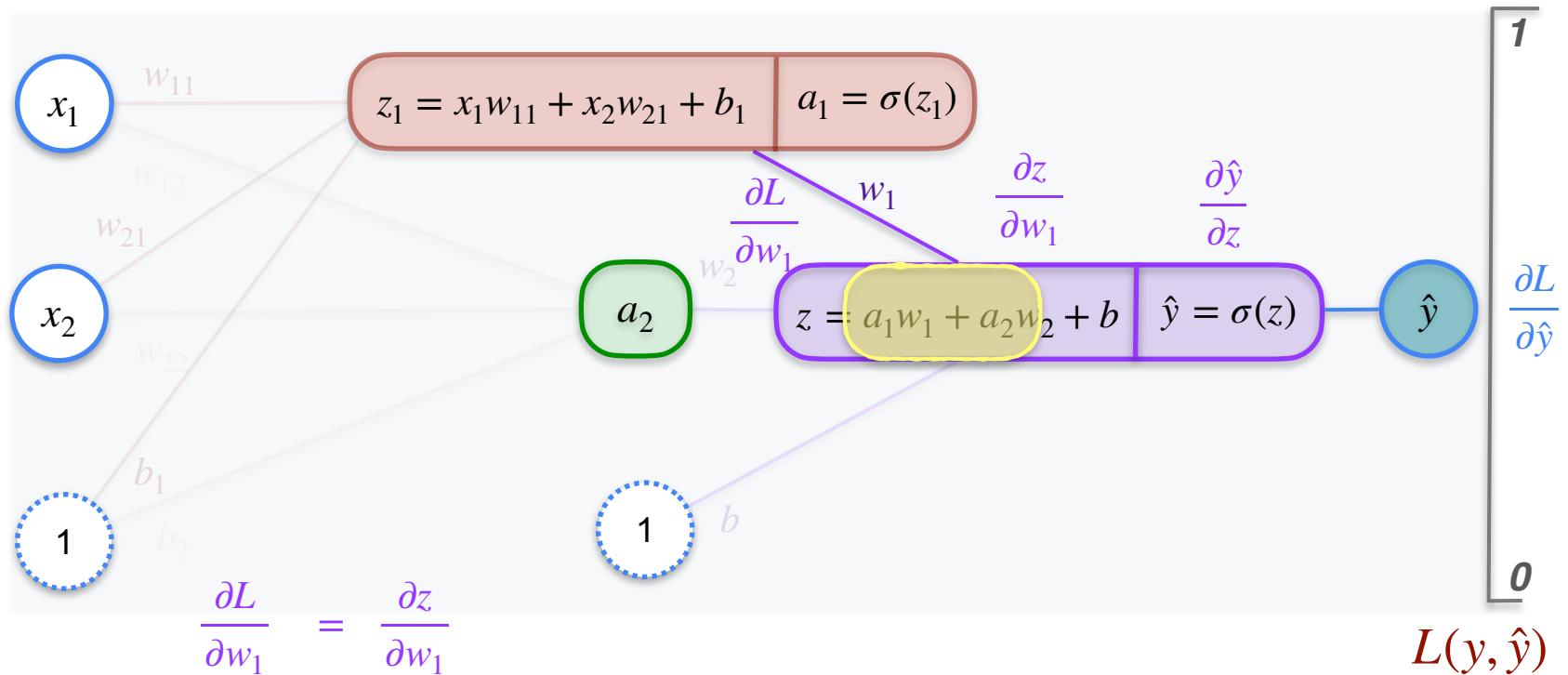
2,2,1 Neural Network



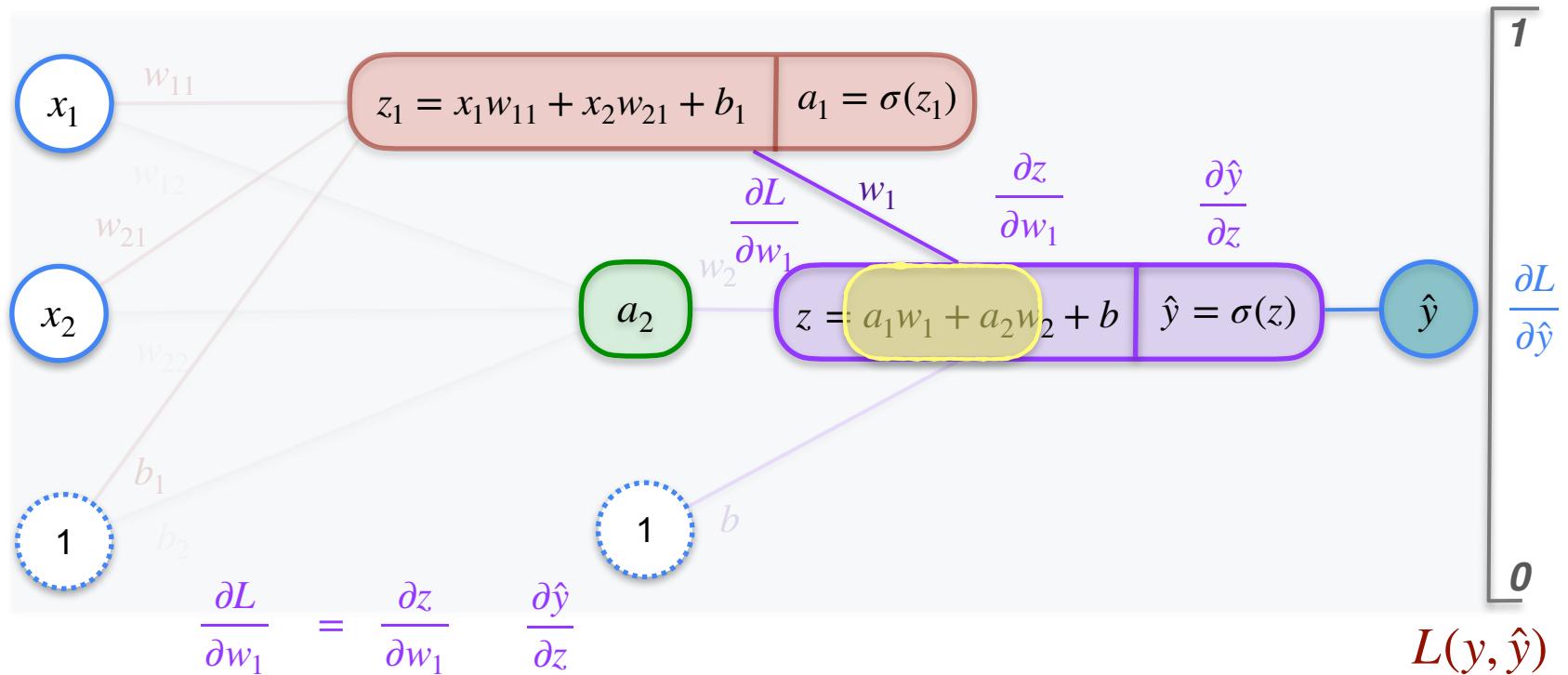
2,2,1 Neural Network



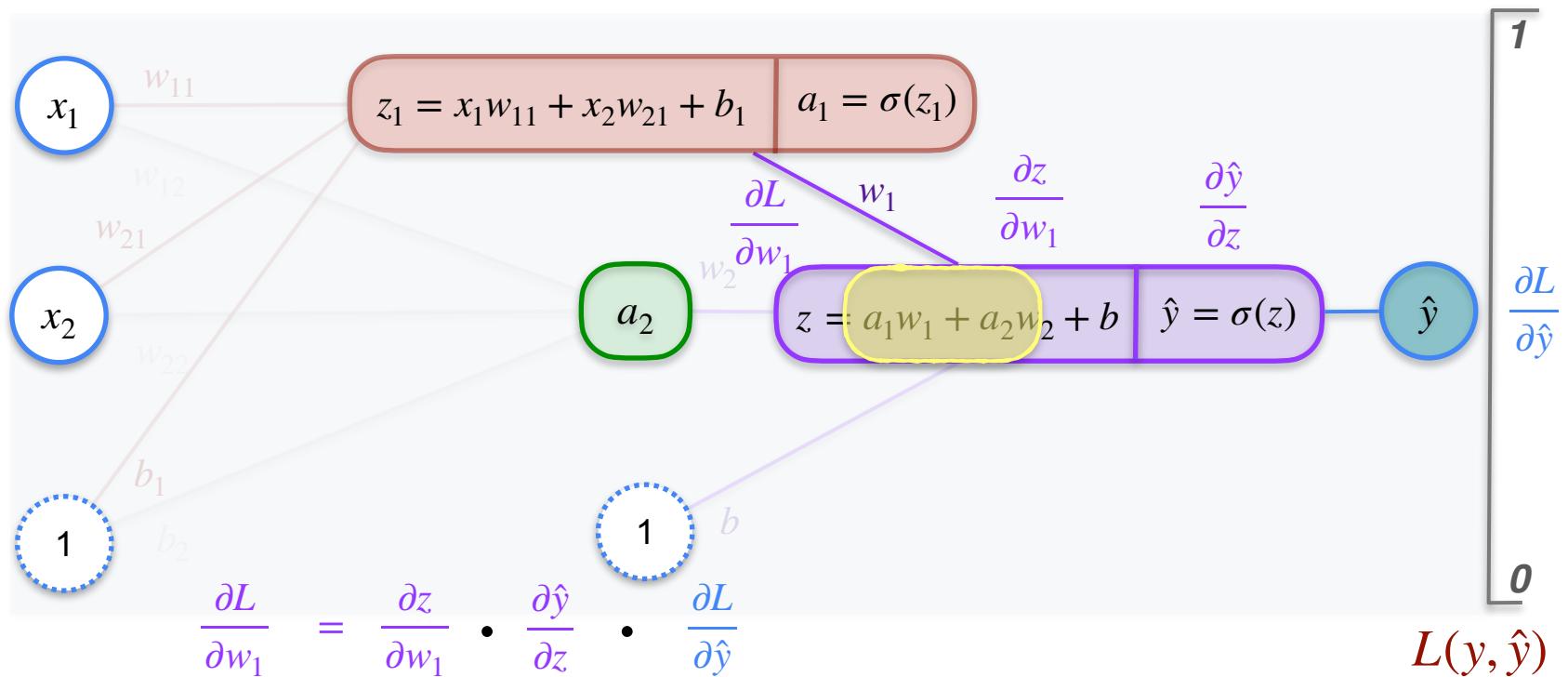
2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network

$$\frac{\partial L}{\partial w_1} = \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\hat{y} = \sigma(z)$$

$$z = a_1w_1 + a_2w_2 + b$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}) \quad \frac{\partial L}{\partial w_1} = \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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$$\frac{\partial L}{\partial w_1}$$

2,2,1 Neural Network

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$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$\frac{\partial L}{\partial w_1} =$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_1} = a_1$$

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$$\frac{\partial L}{\partial w_1} = a_1 \hat{y}(1-\hat{y})$$

2,2,1 Neural Network

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$$z = a_1 w_1 + a_2 w_2 + b$$

$$\frac{\partial L}{\partial w_1} = a_1 \hat{y}(1-\hat{y}) \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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2,2,1 Neural Network

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$$\frac{\partial L}{\partial w_1} = a_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}}$$

$$= -a_1(y - \hat{y})$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

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$$\begin{aligned}\frac{\partial L}{\partial w_1} &= \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}} \\ \frac{\partial L}{\partial w_1} &= a_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}} \\ &= -a_1(y - \hat{y})\end{aligned}$$

to find optimal value of w_1 that gives the least error

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

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Perform gradient descent with

*to find optimal
value of w_1 that
gives the least error*

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

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Perform gradient descent with

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

to find optimal value of w_1 that gives the least error

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

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Perform gradient descent with

$$w_1 \rightarrow w_1 - \alpha$$

to find optimal value of w_1 that gives the least error

2,2,1 Neural Network

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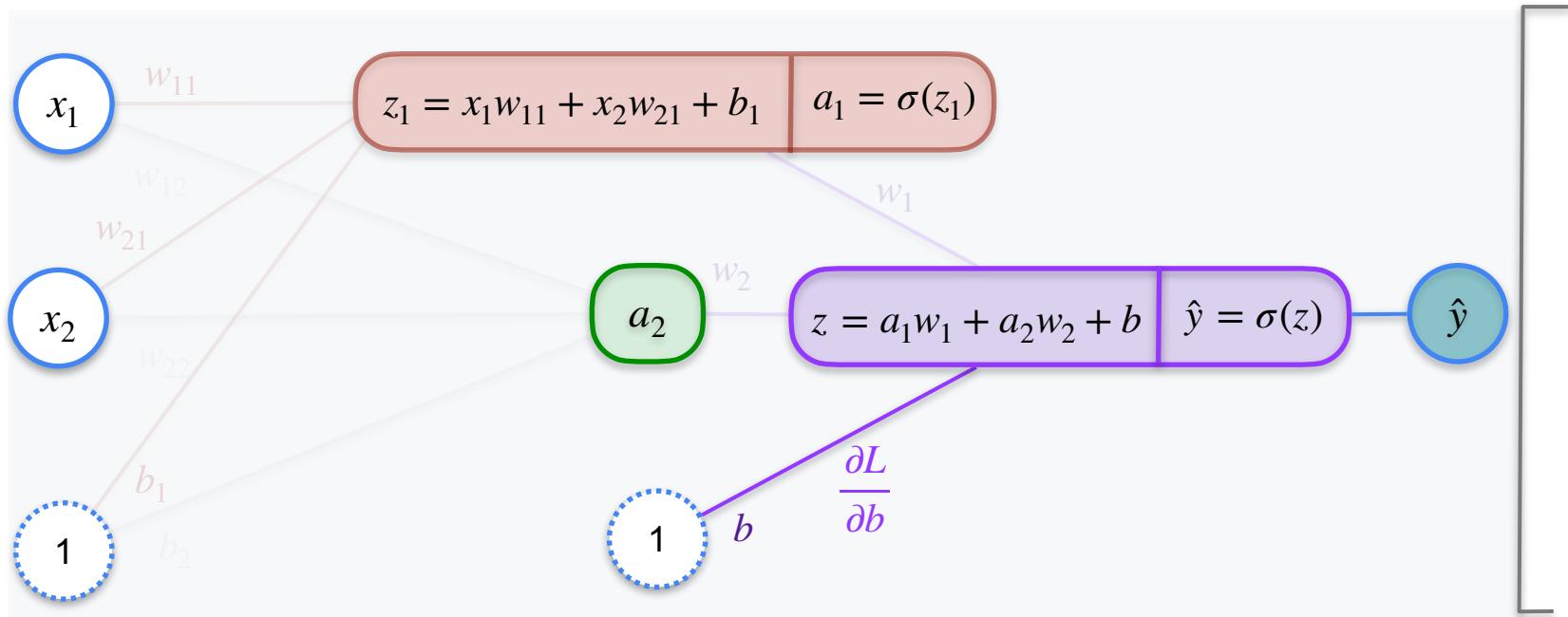
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Perform gradient descent with

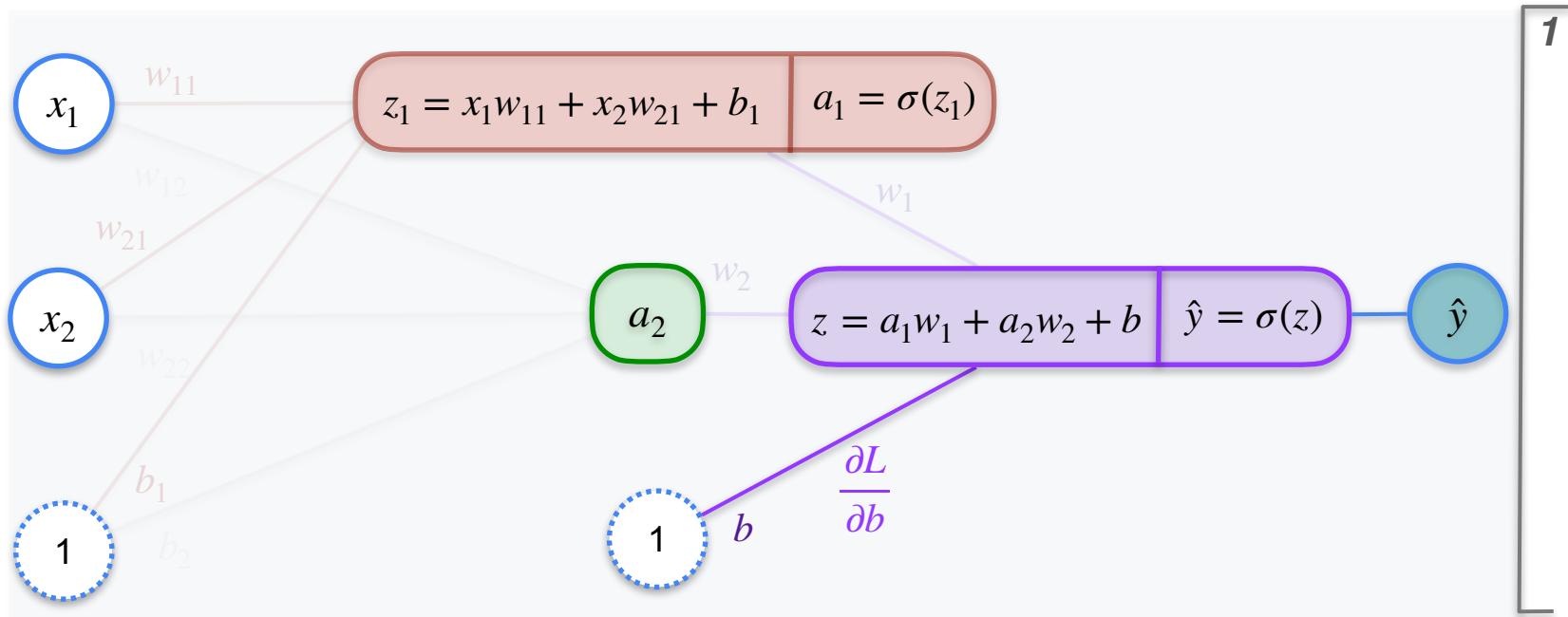
$$w_1 \rightarrow w_1 - \alpha(-a_1(y - \hat{y}))$$

to find optimal value of w_1 that gives the least error

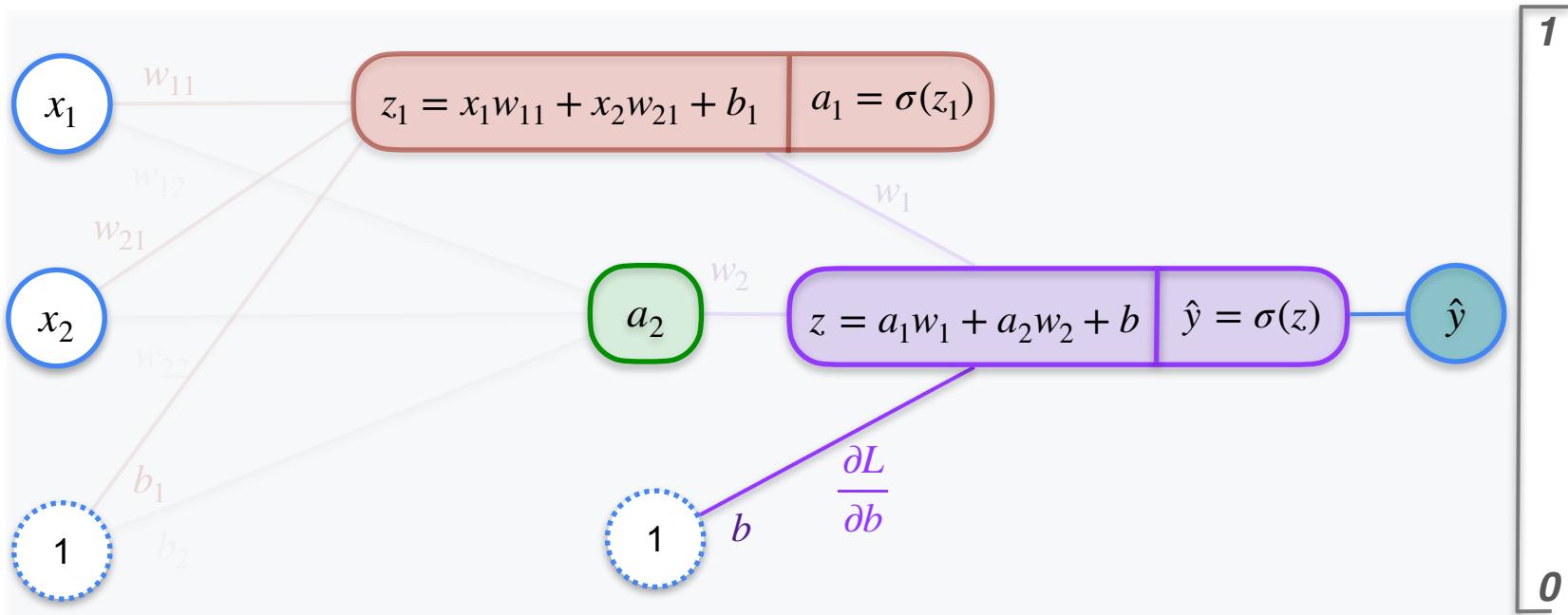
2,2,1 Neural Network



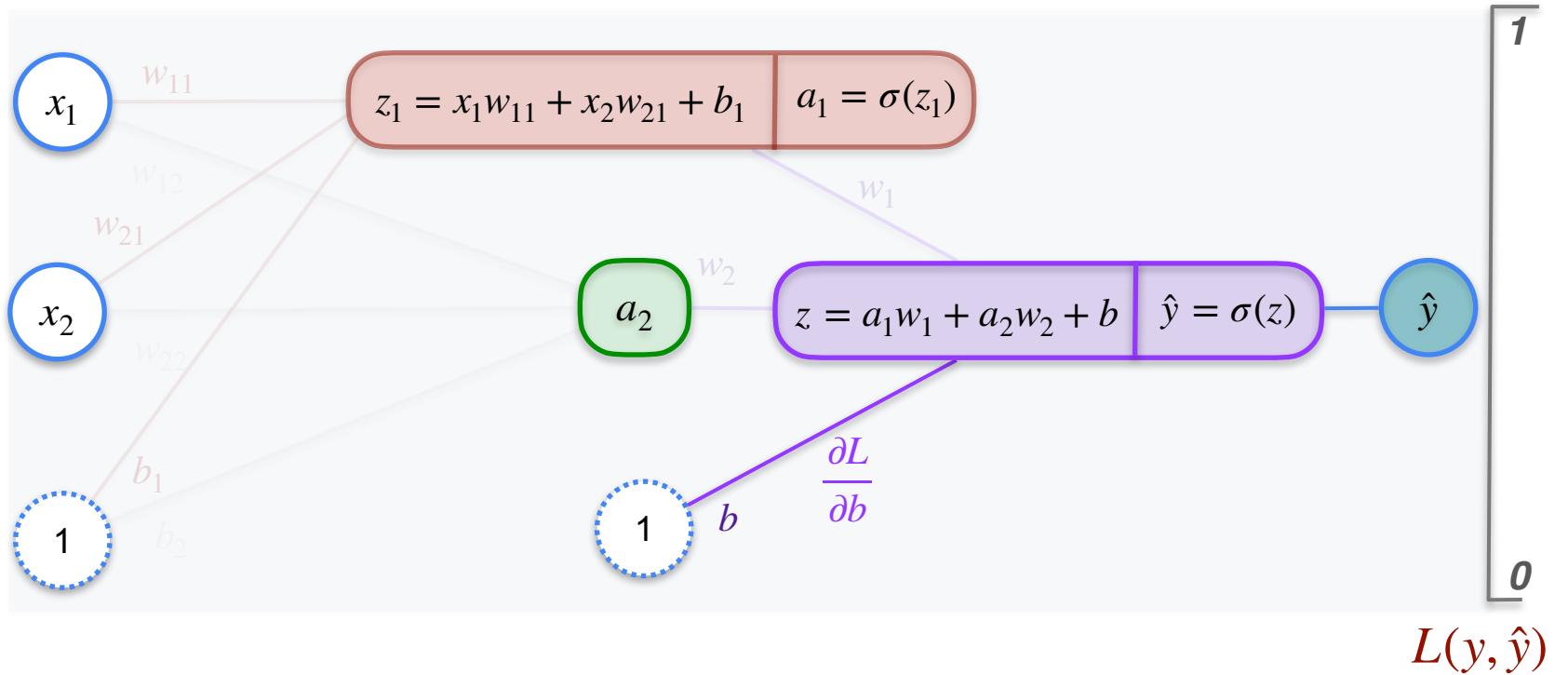
2,2,1 Neural Network



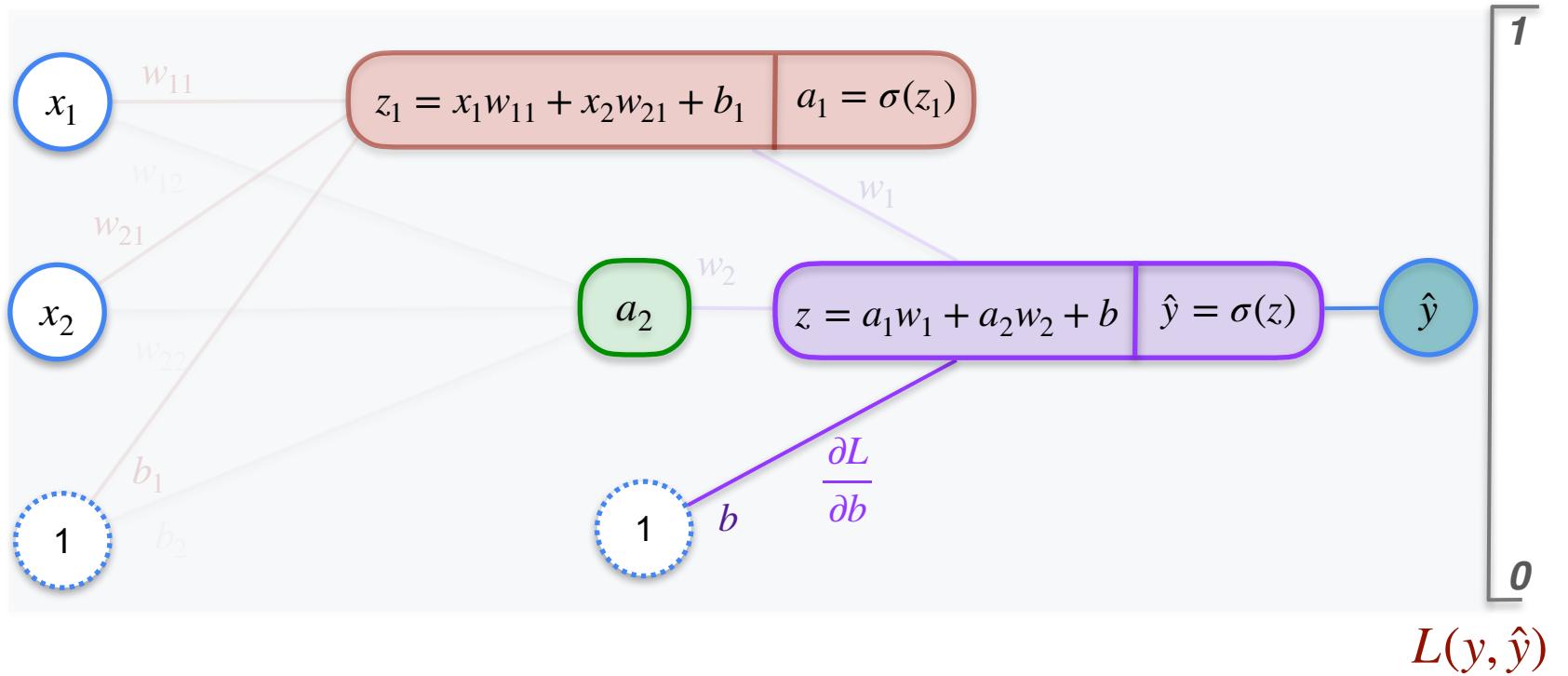
2,2,1 Neural Network



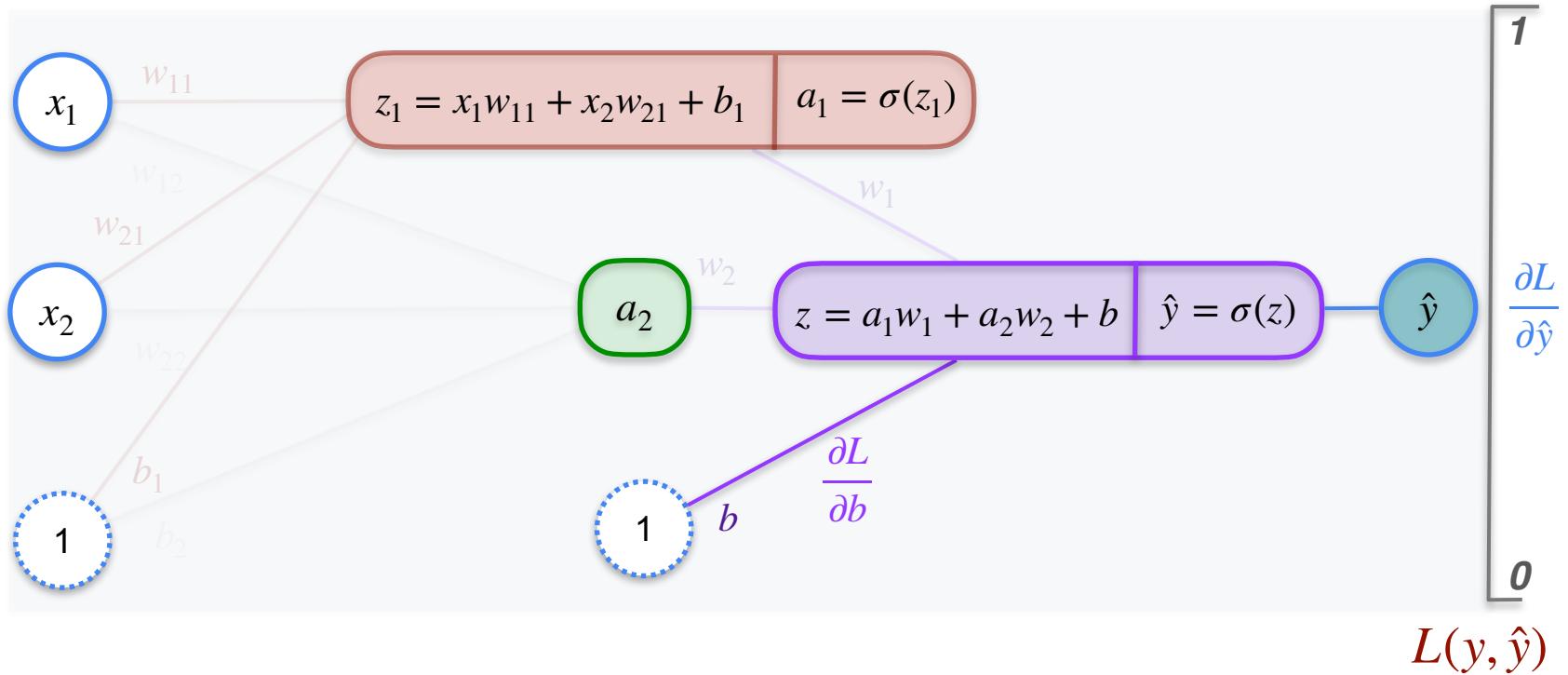
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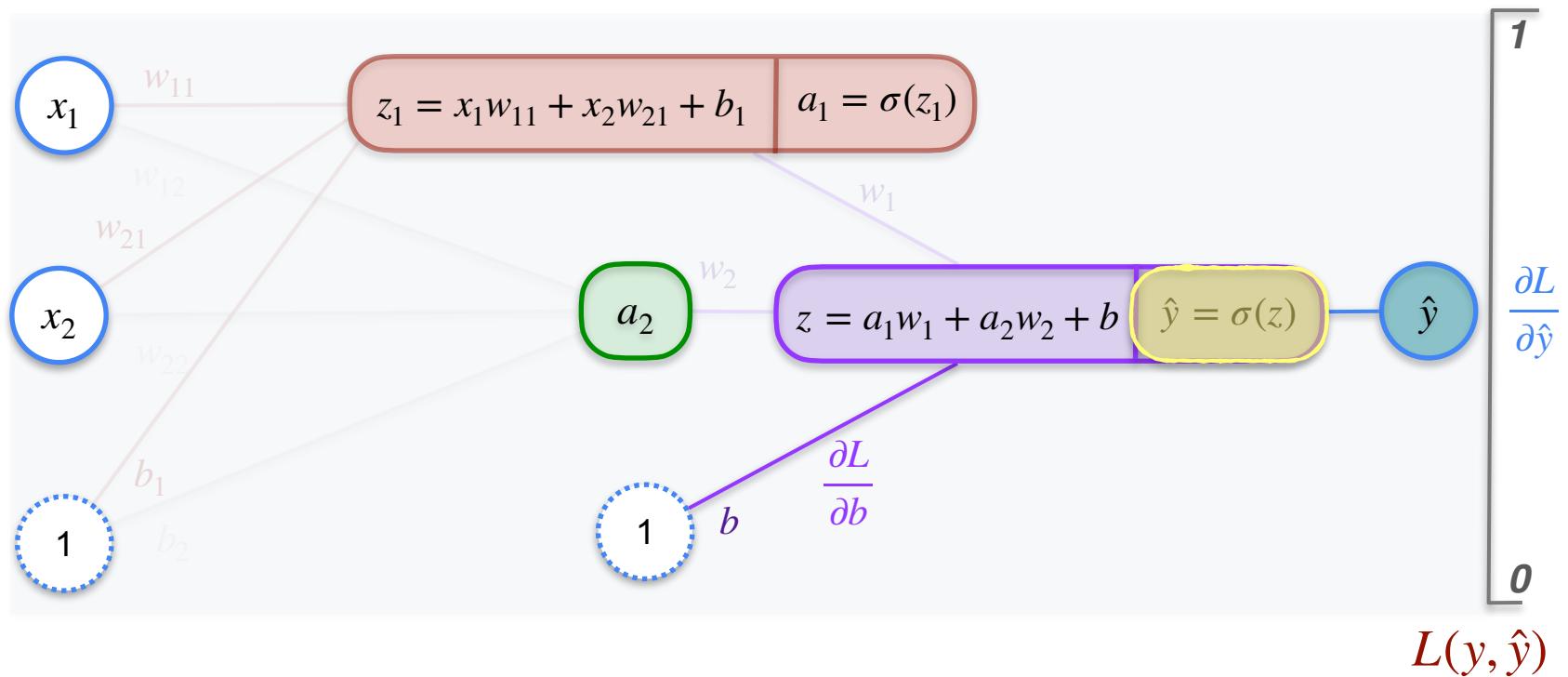
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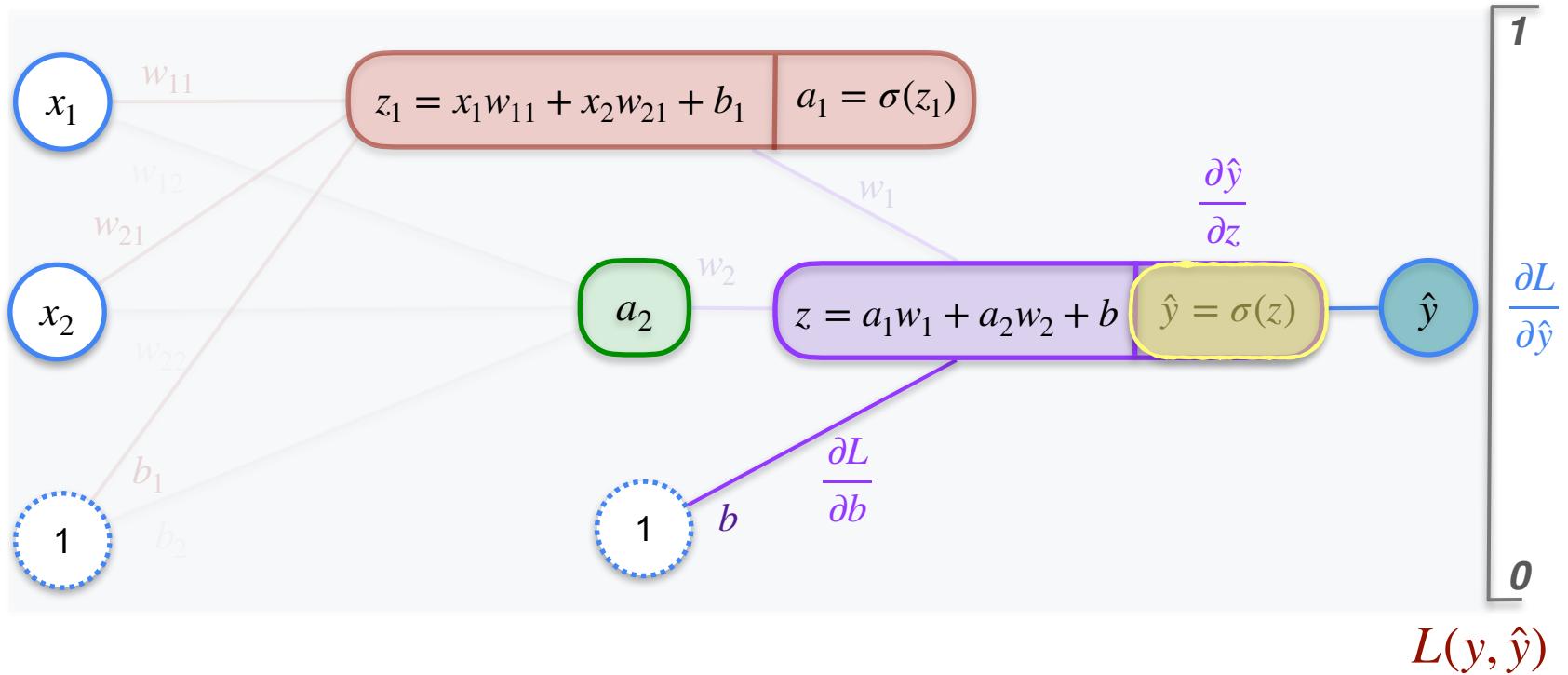
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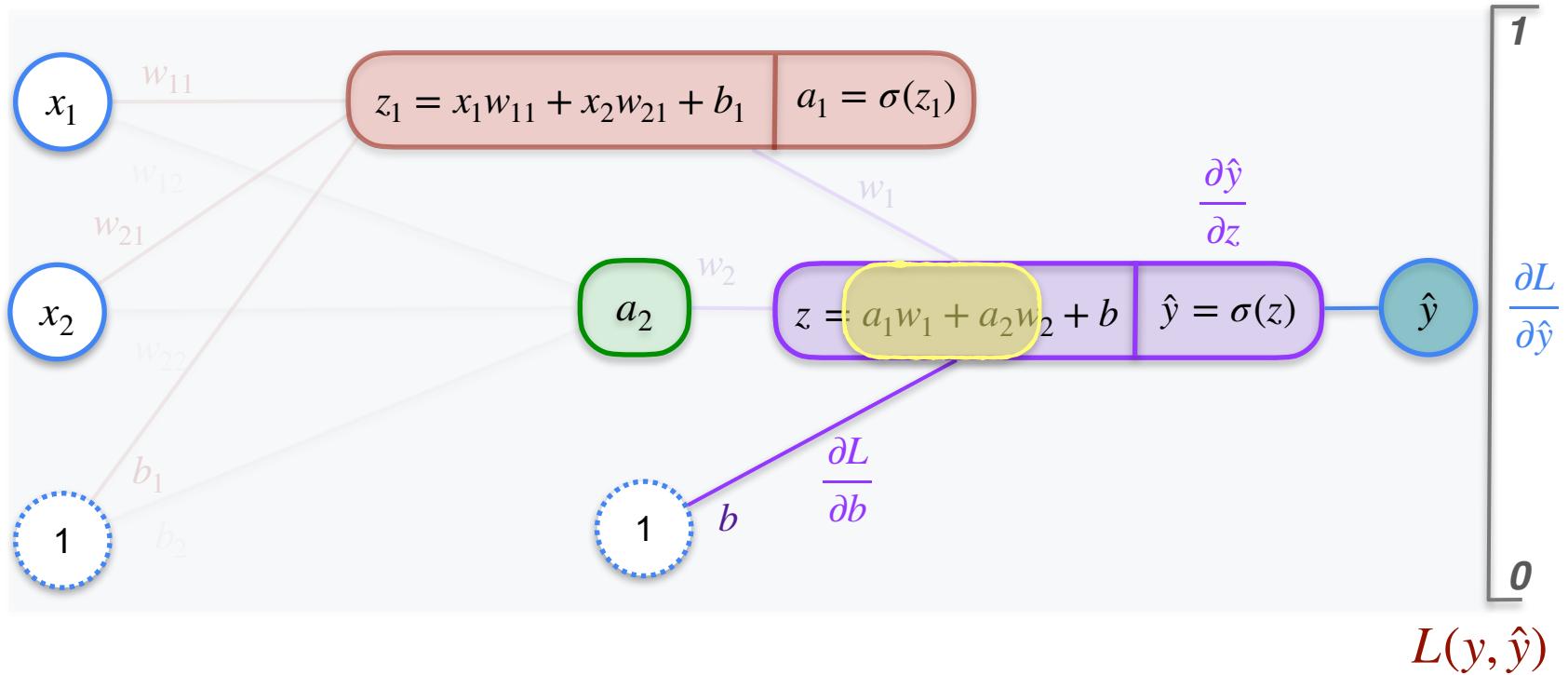
2,2,1 Neural Network



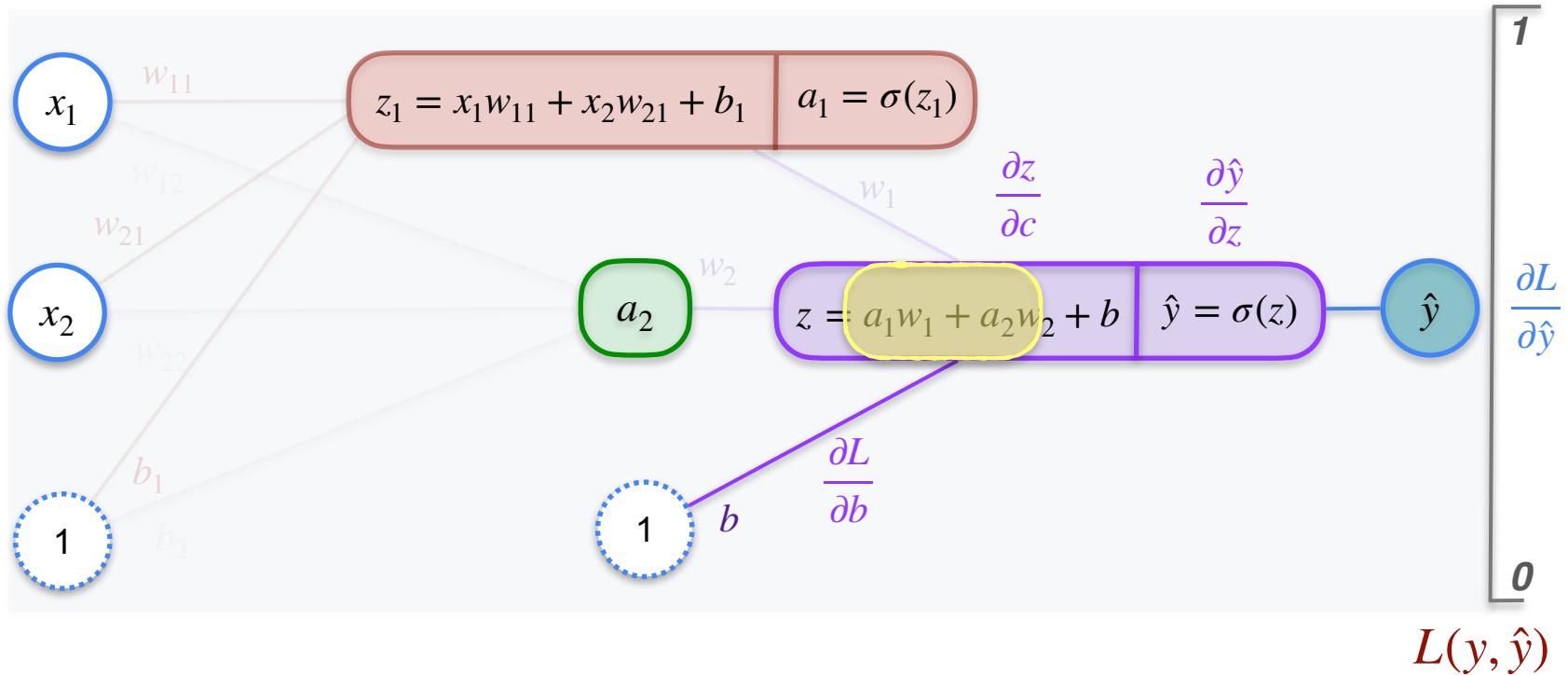
2,2,1 Neural Network



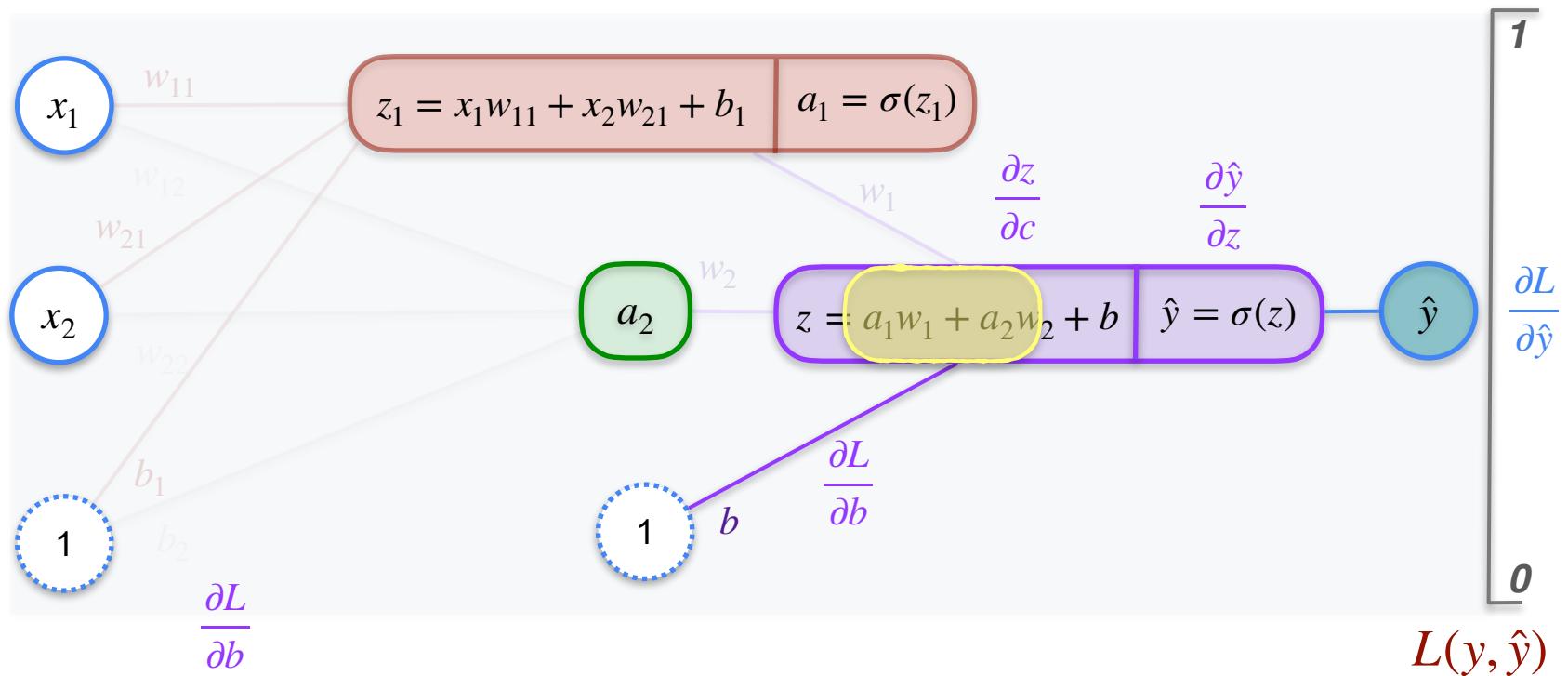
2,2,1 Neural Network



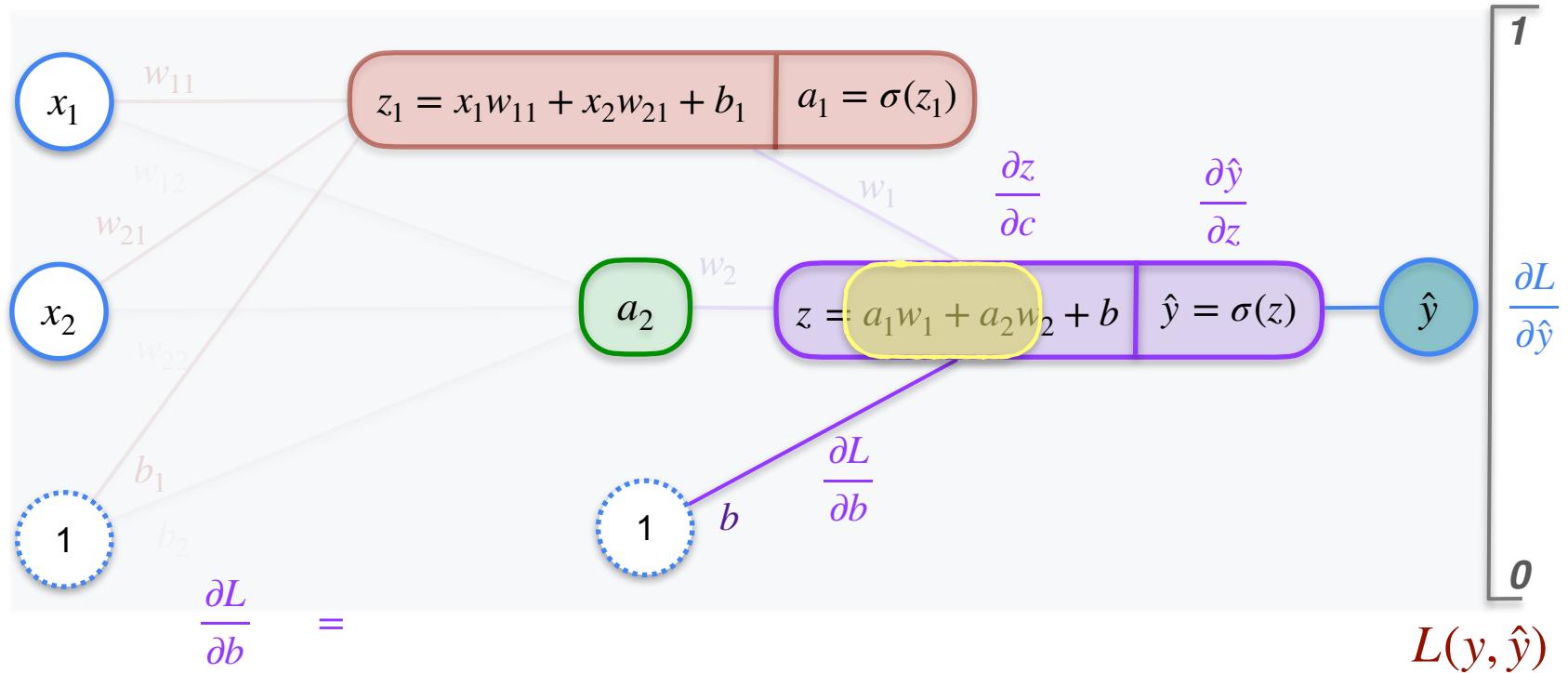
2,2,1 Neural Network



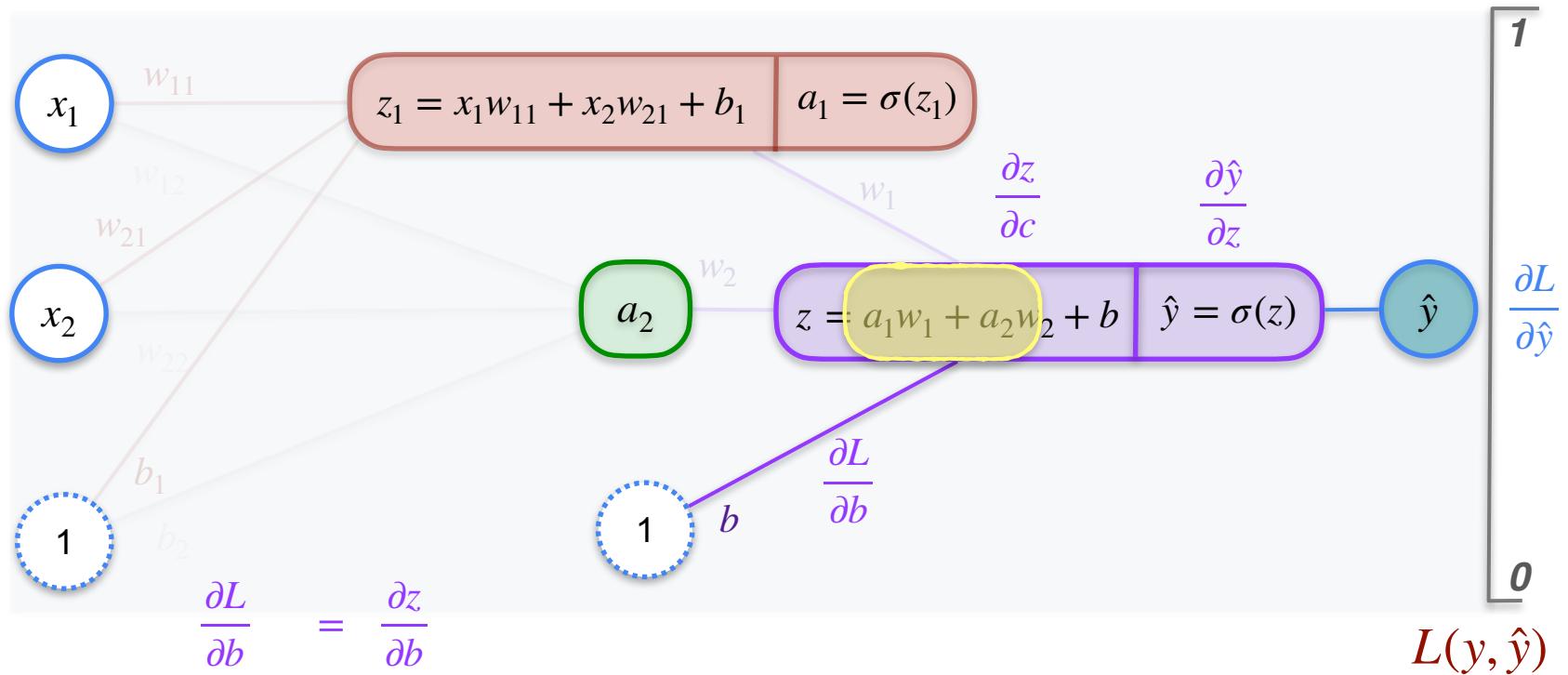
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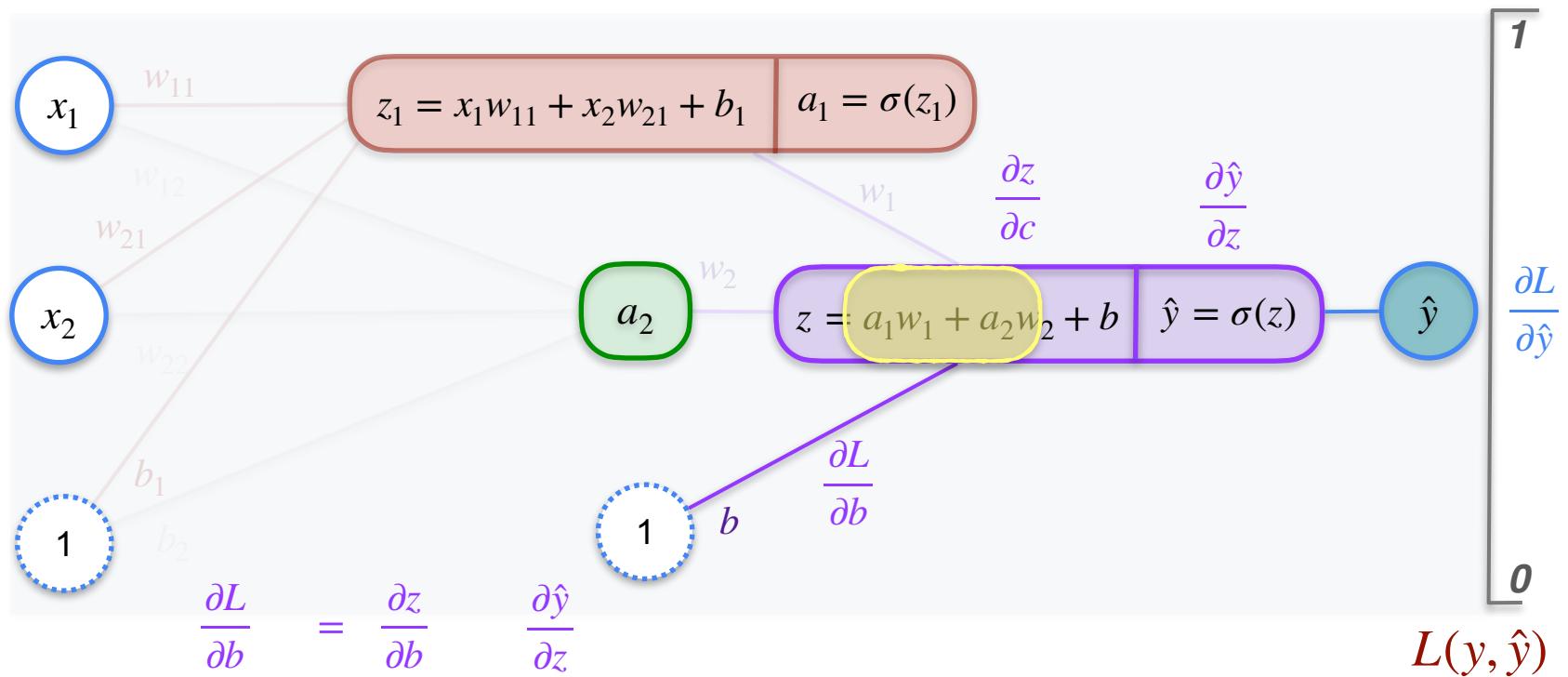
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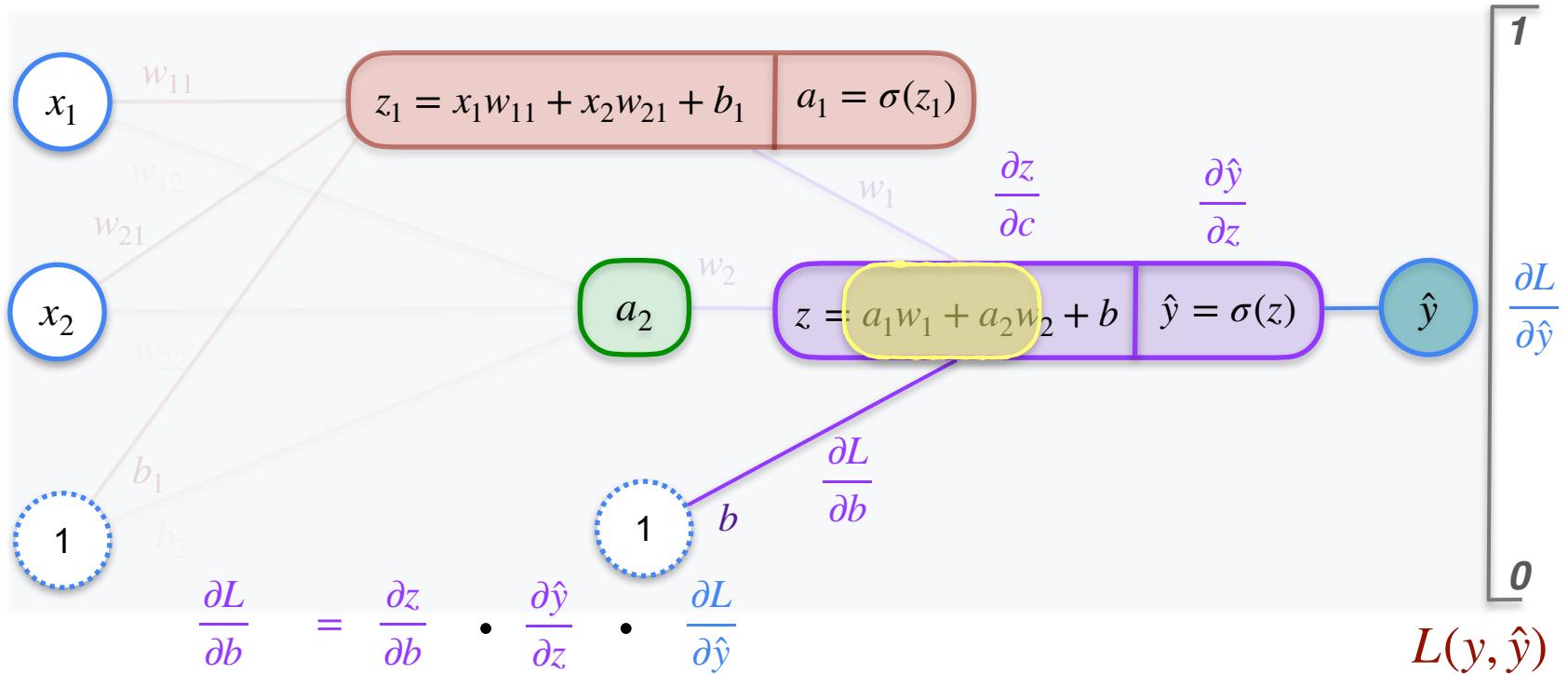
2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network



2,2,1 Neural Network

$$\frac{\partial L}{\partial b} = \frac{\partial z}{\partial b} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\hat{y} = \sigma(z)$$

$$z = a_1w_1 + a_2w_2 + b$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}) \quad \frac{\partial L}{\partial b} = \frac{\partial z}{\partial b} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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$$\frac{\partial L}{\partial b}$$

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$$\frac{\partial L}{\partial b} =$$

2,2,1 Neural Network

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$$\frac{\partial L}{\partial b} = 1$$

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$$\frac{\partial L}{\partial b} = 1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1 - y) \log(1 - \hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$\frac{\partial L}{\partial b} = \frac{\partial z}{\partial b} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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$$\frac{\partial L}{\partial b} = 1 - \hat{y}(1 - \hat{y})$$

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$$\frac{\partial L}{\partial b} = 1 - \hat{y}(1 - \hat{y}) \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

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$$= -(y - \hat{y})$$

to find optimal value of b that gives the least error

2,2,1 Neural Network

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$$= -(y - \hat{y})$$

Perform gradient descent with

*to find optimal
value of b that gives
the least error*

2,2,1 Neural Network

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$$= -(y - \hat{y})$$

Perform gradient descent with

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

to find optimal value of b that gives the least error

2,2,1 Neural Network

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Perform gradient descent with

$$b \rightarrow b - \alpha$$

to find optimal value of b that gives the least error

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$$= -(y - \hat{y})$$

Perform gradient descent with

$$b \rightarrow b - \alpha(-(y - \hat{y}))$$

to find optimal value of b that gives the least error



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Gradient Descent and Backpropagation

Back Propagation Introduction

Back Propagation Introduction



Back Propagation Introduction



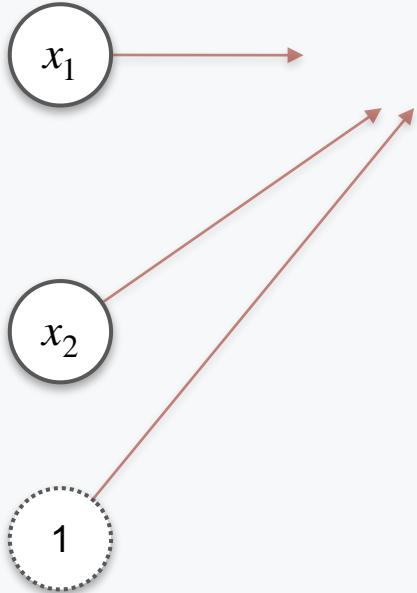
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x_1

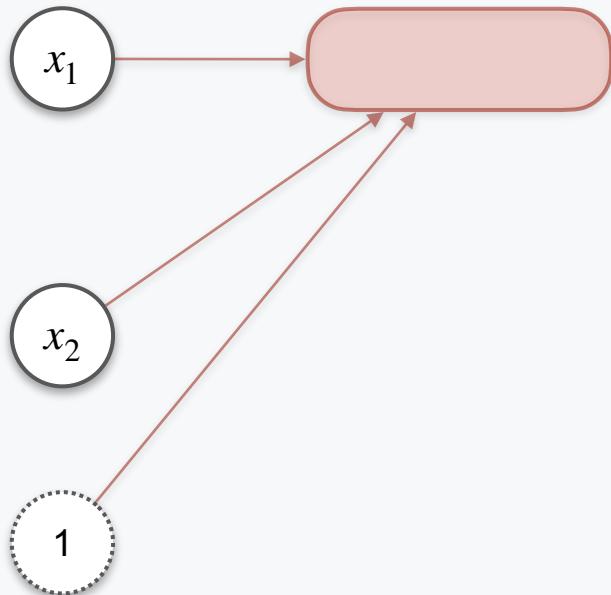
x_2

1

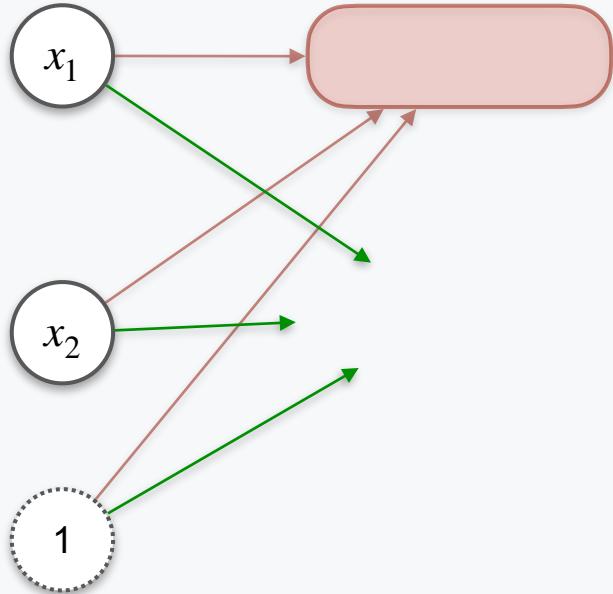
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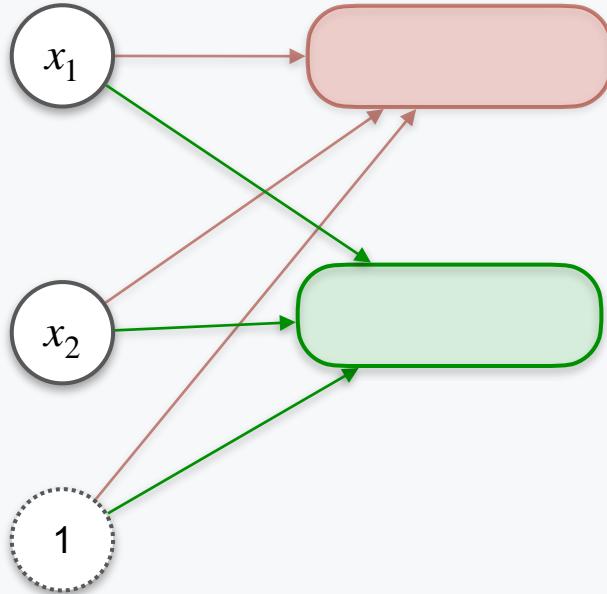
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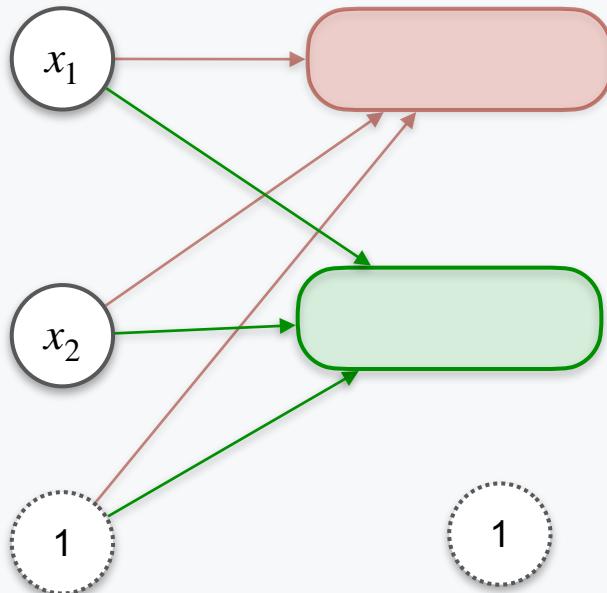
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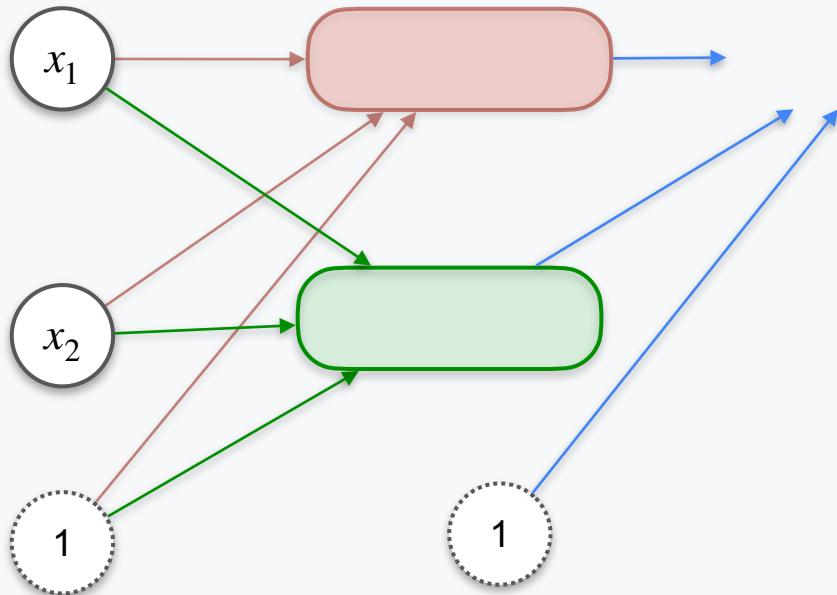
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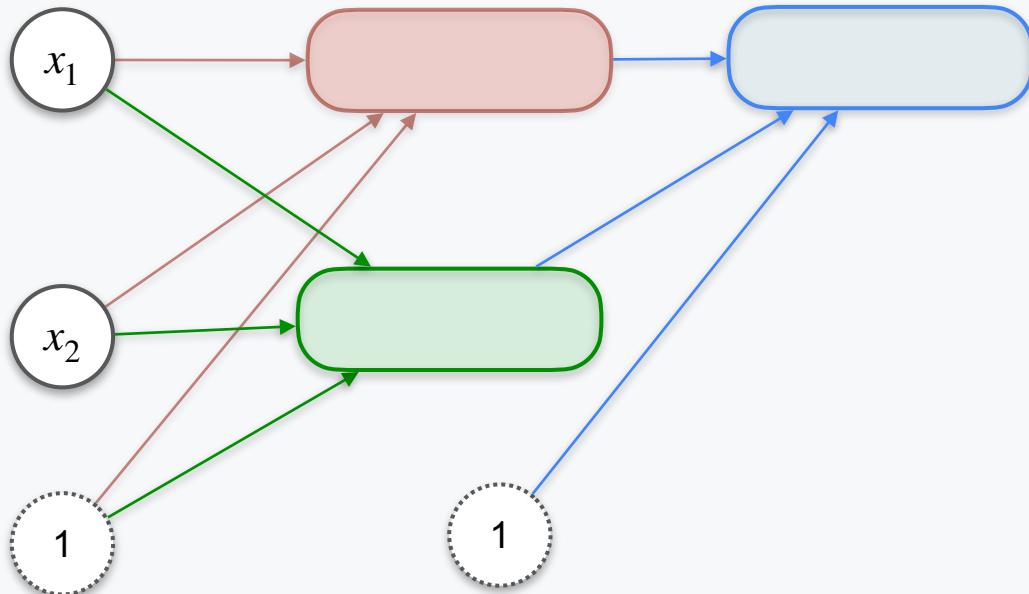
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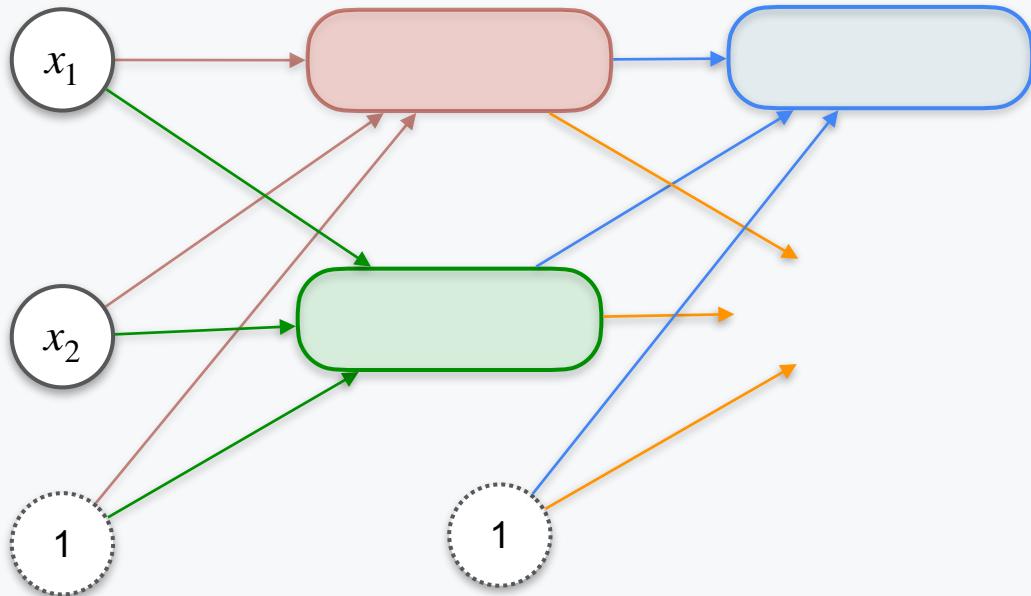
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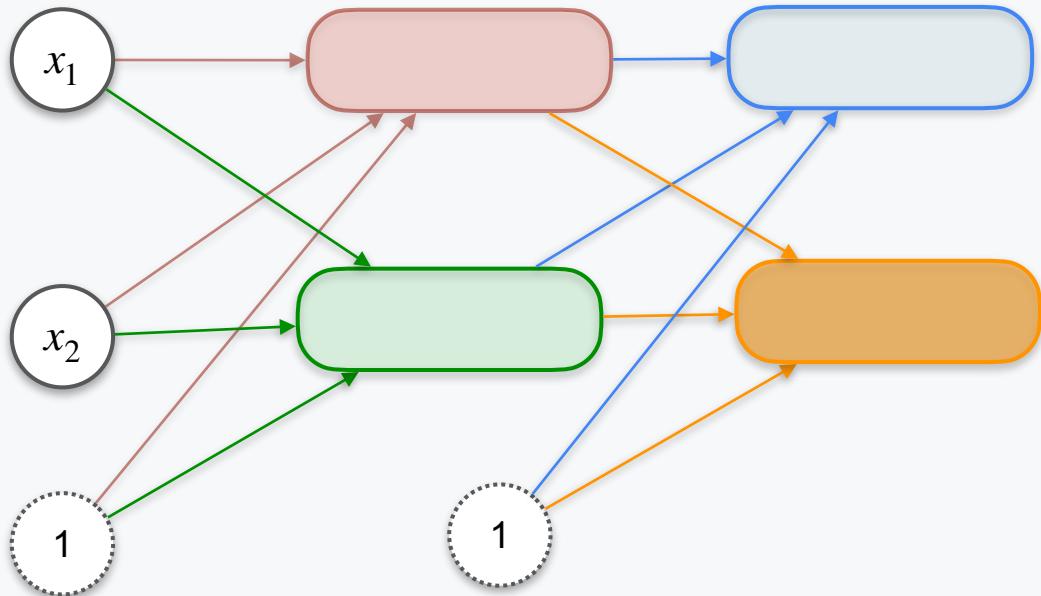
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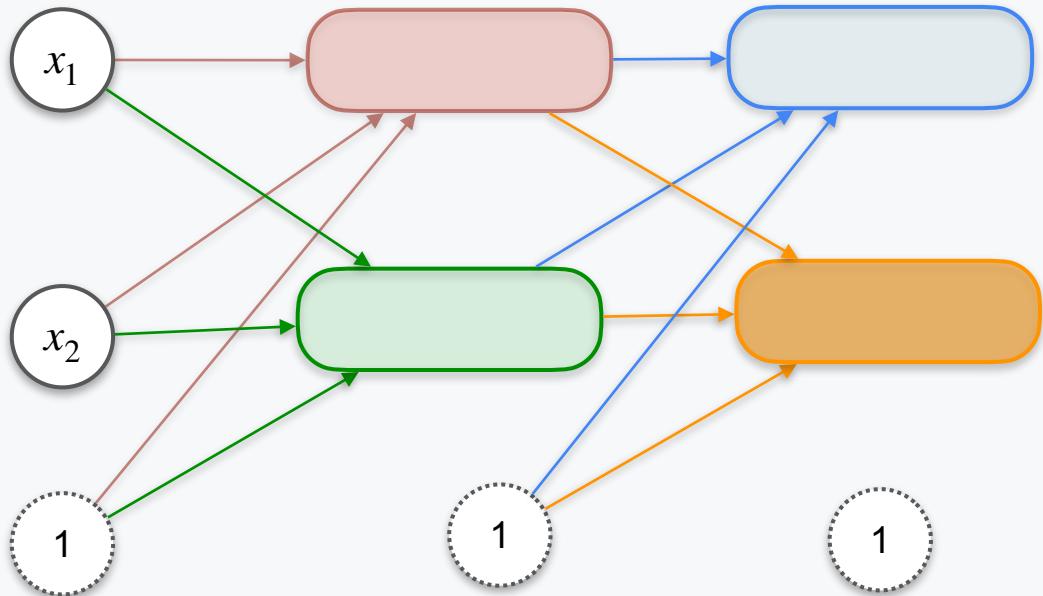
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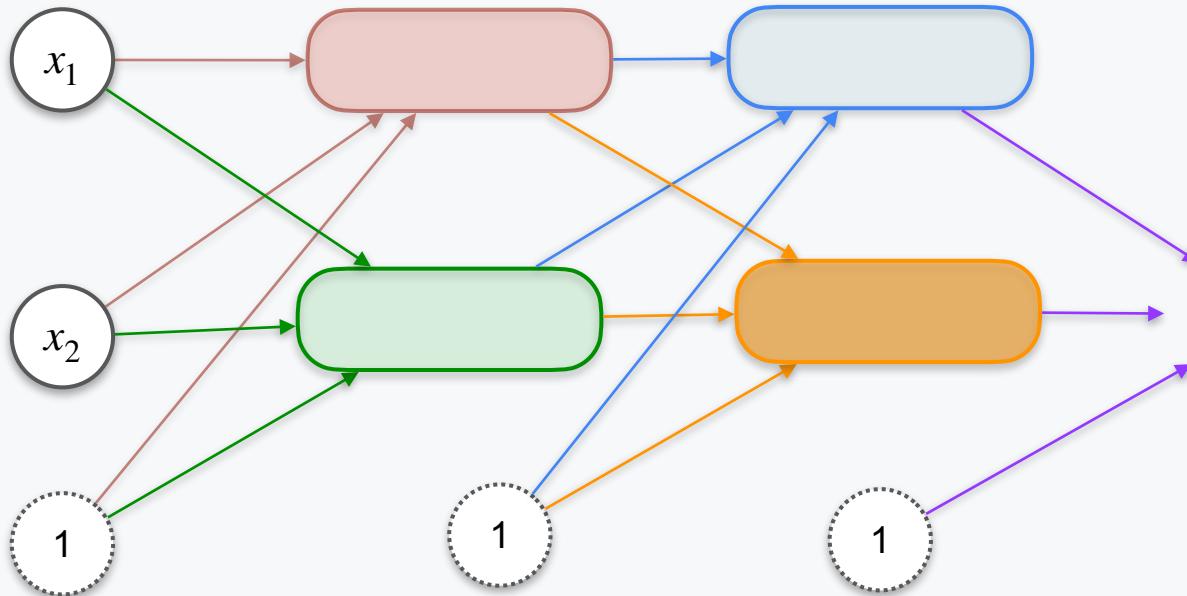
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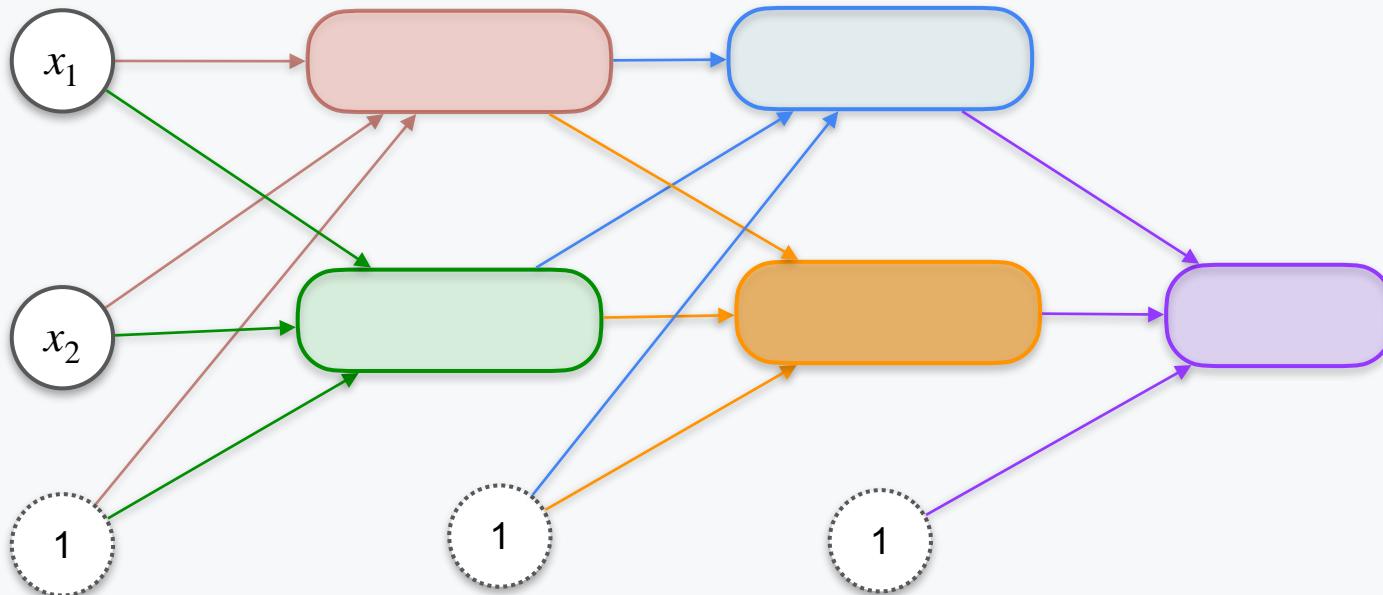
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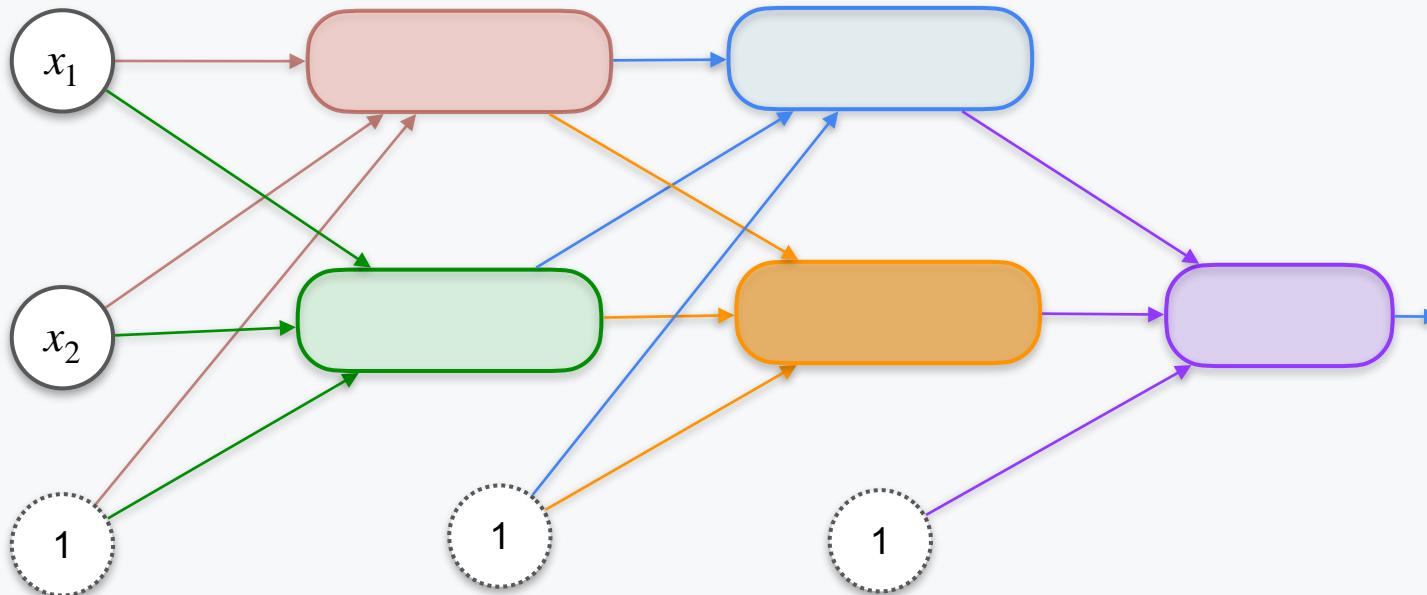
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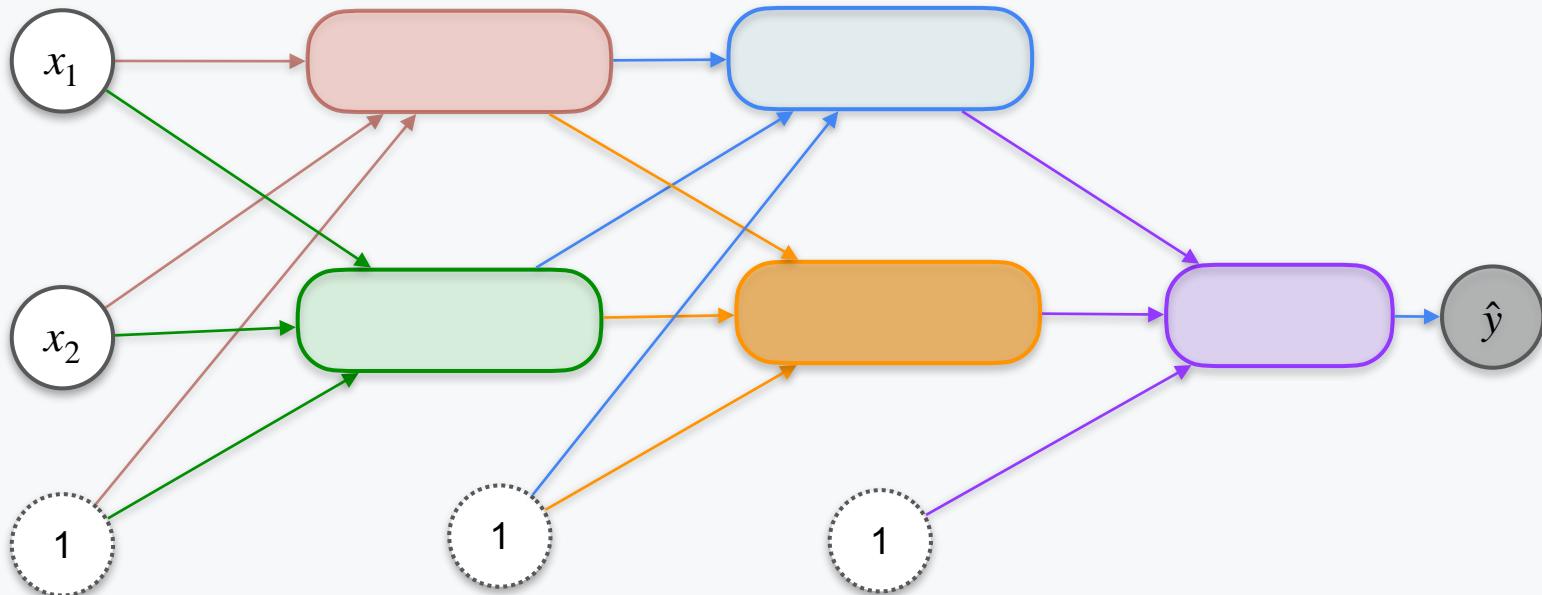
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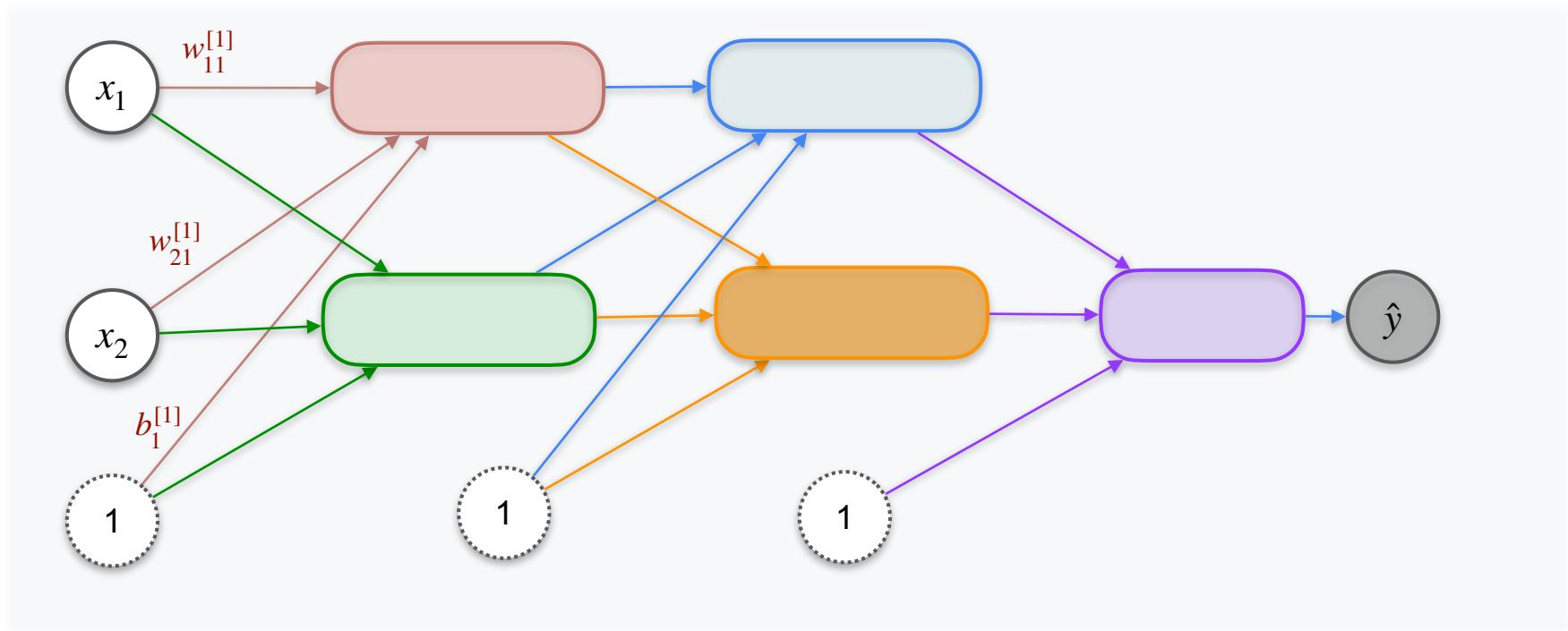
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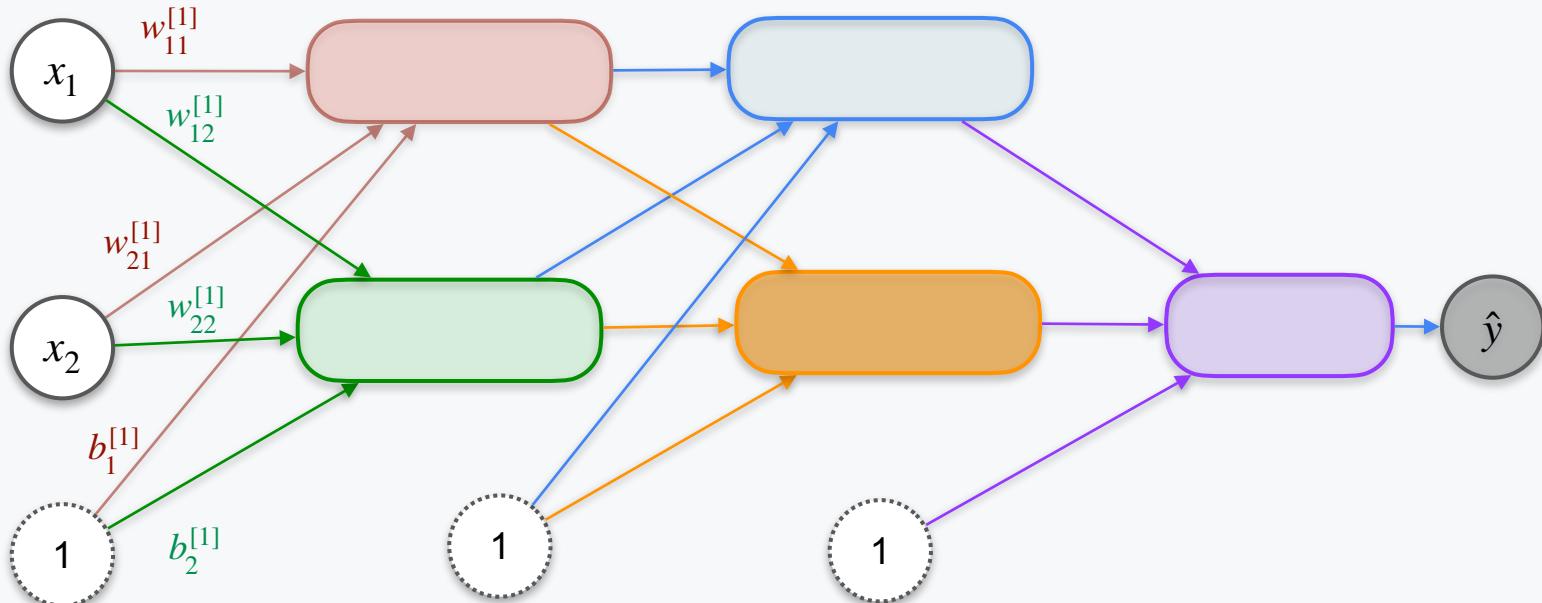
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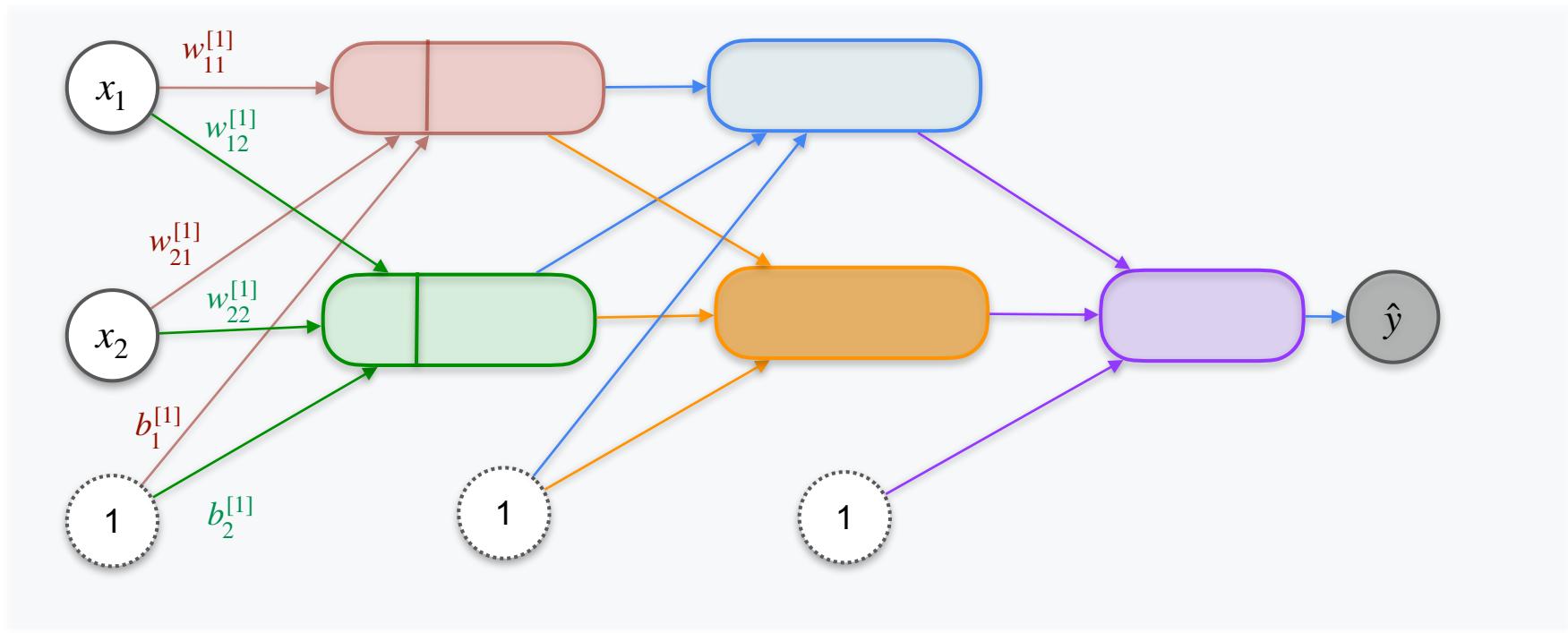
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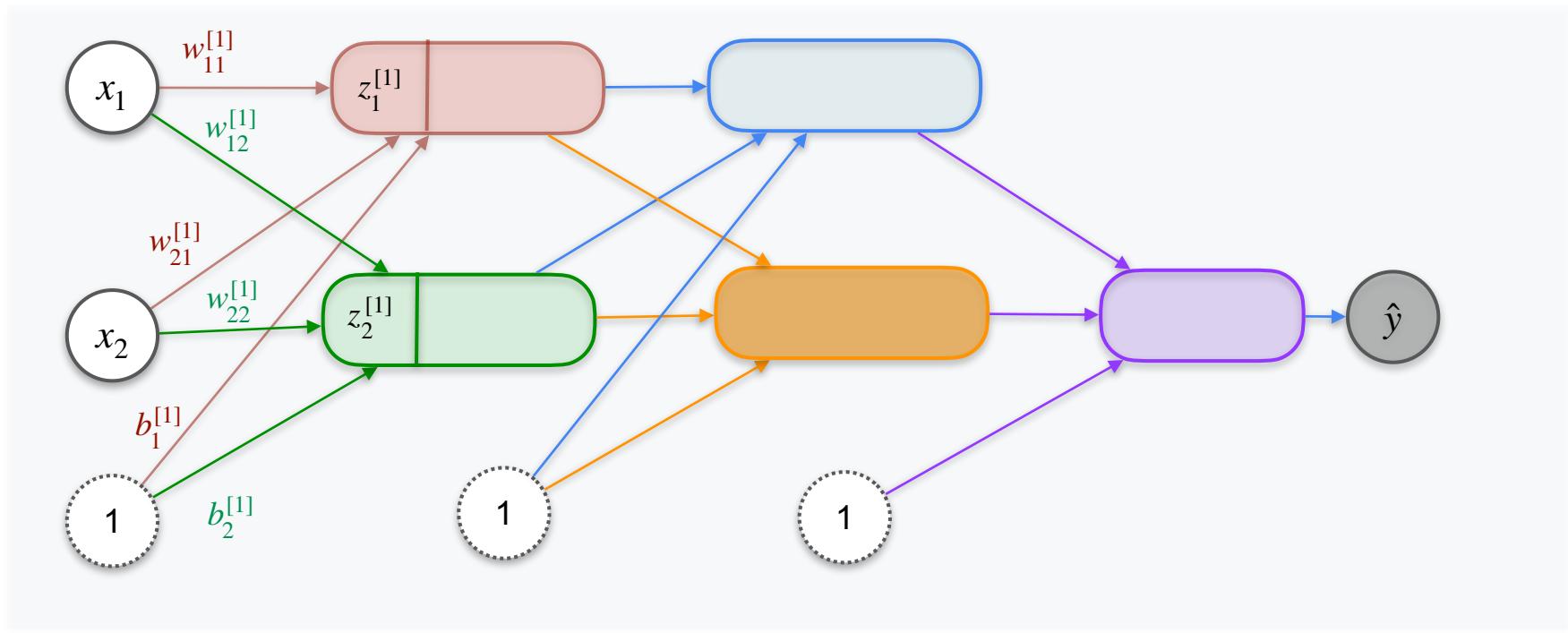
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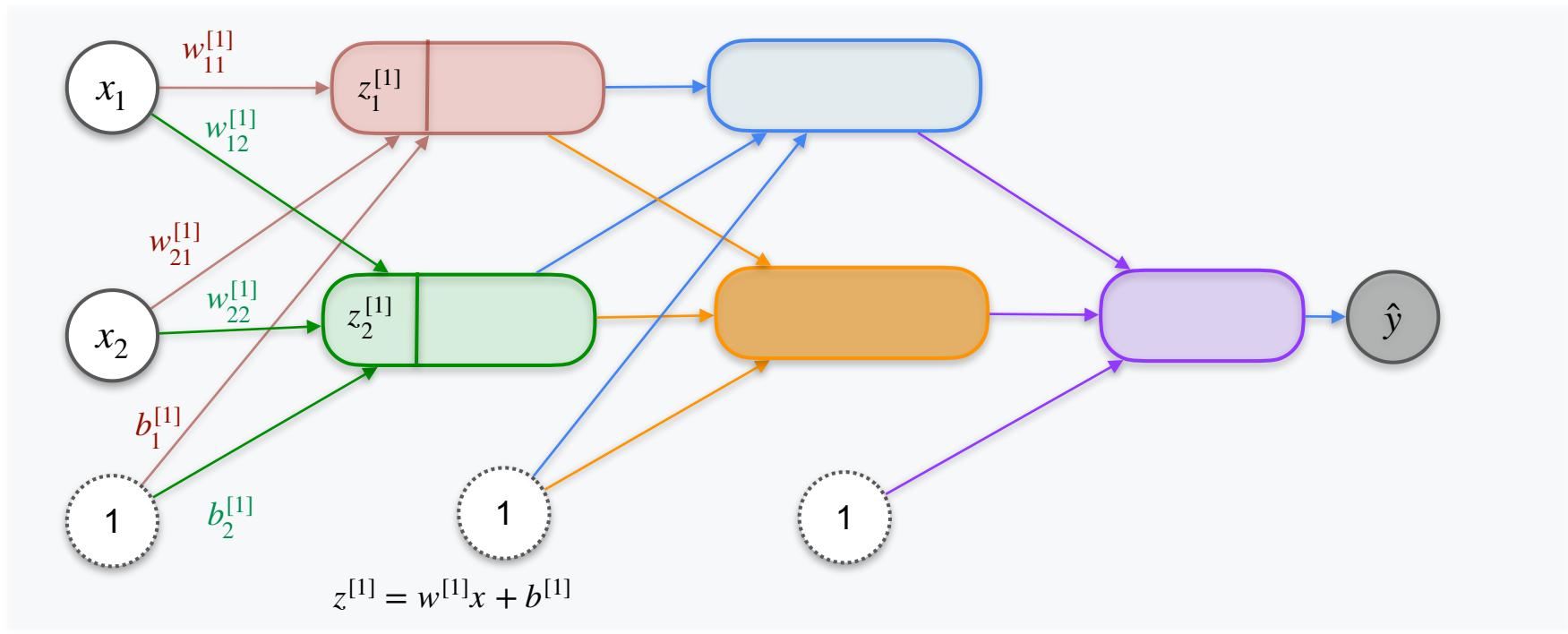
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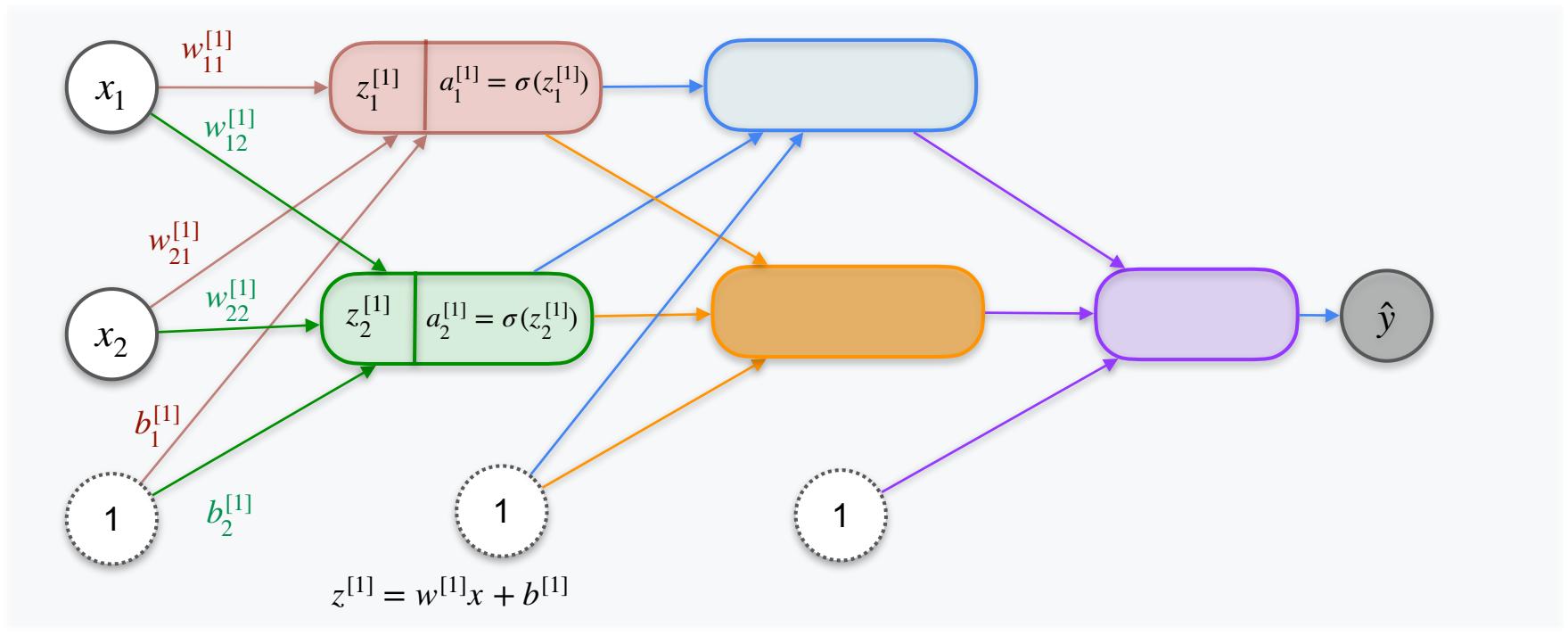
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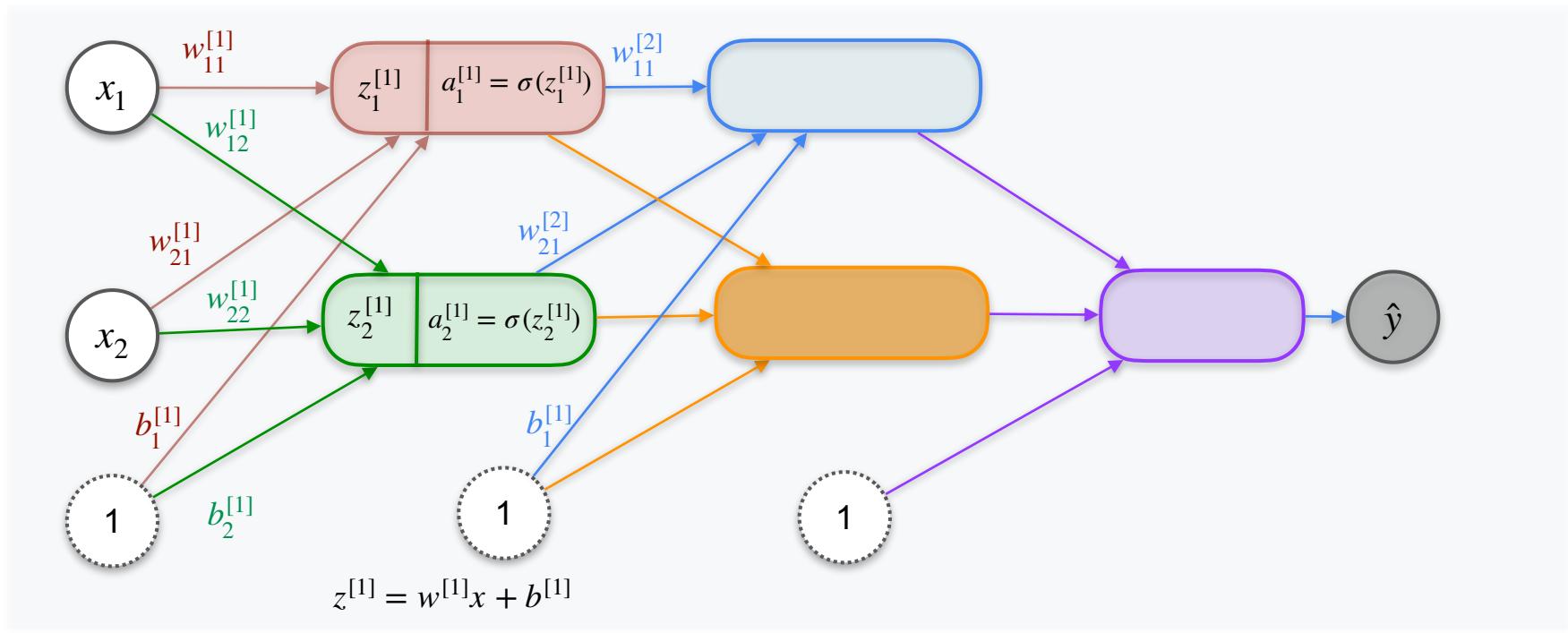
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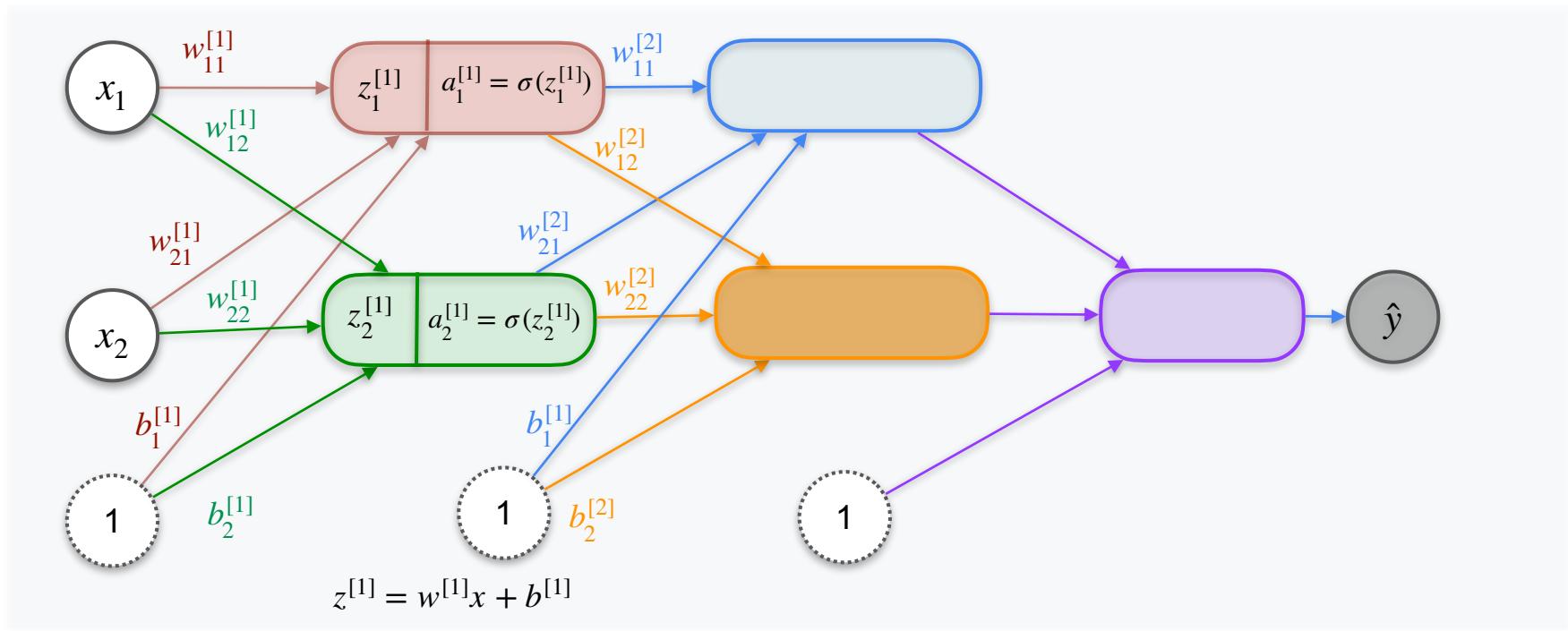
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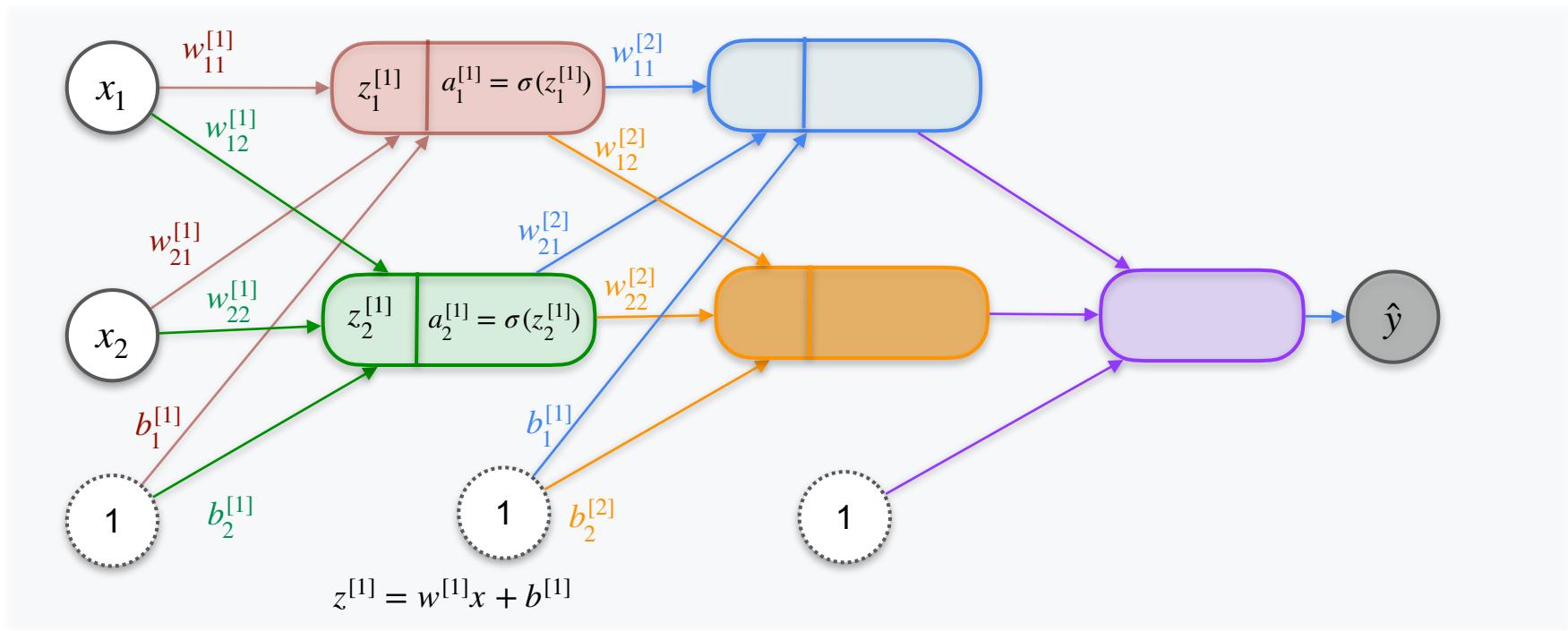
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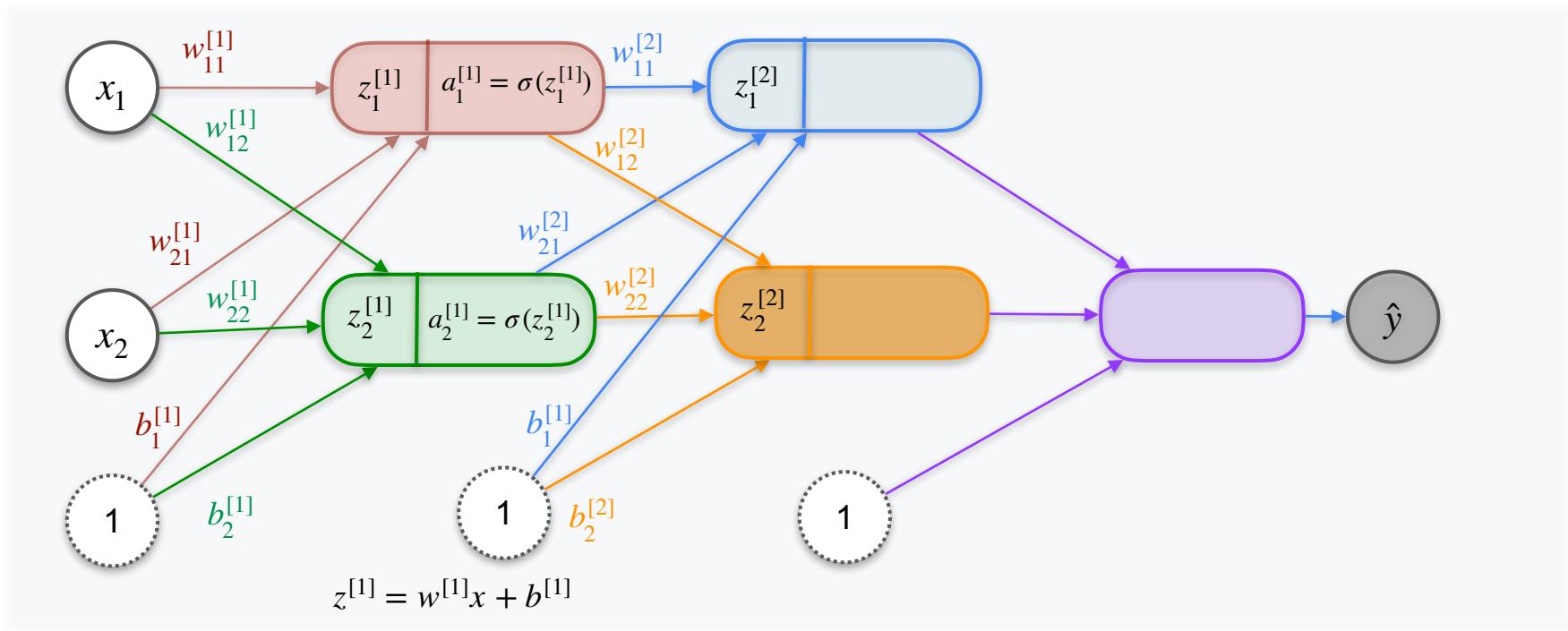
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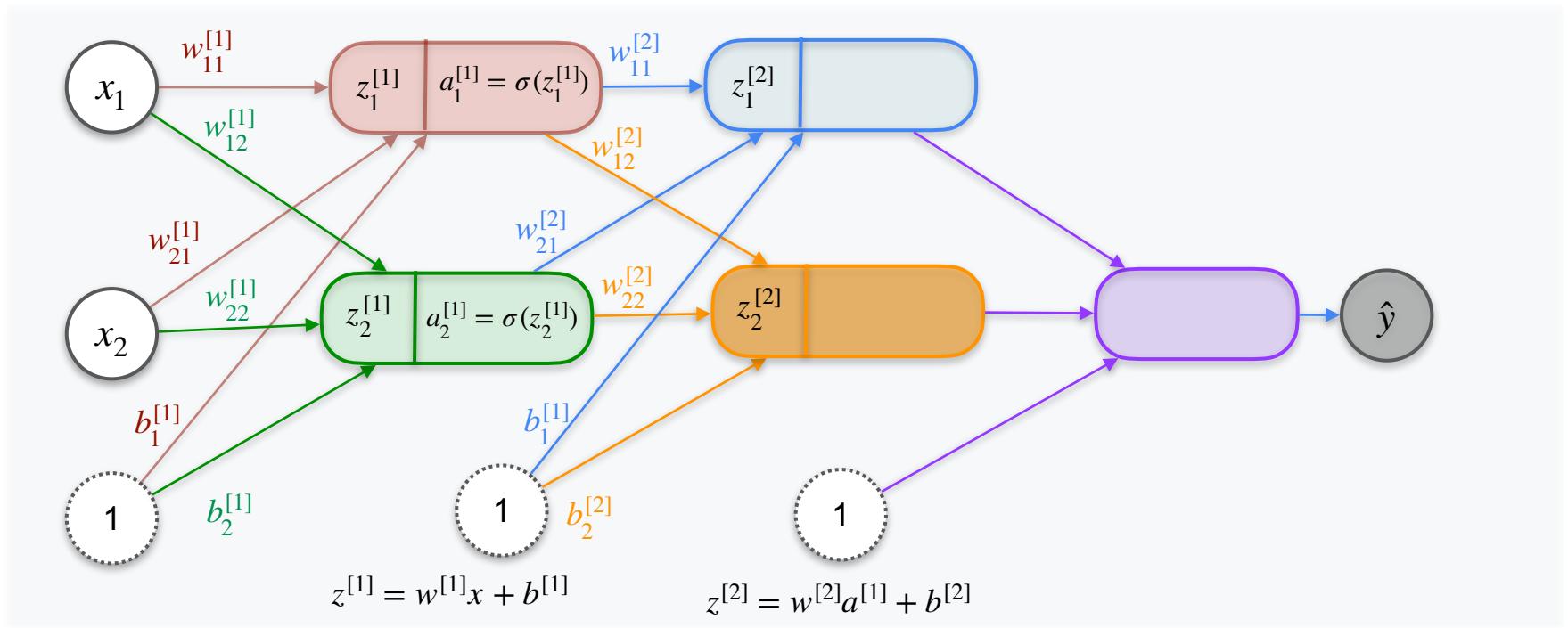
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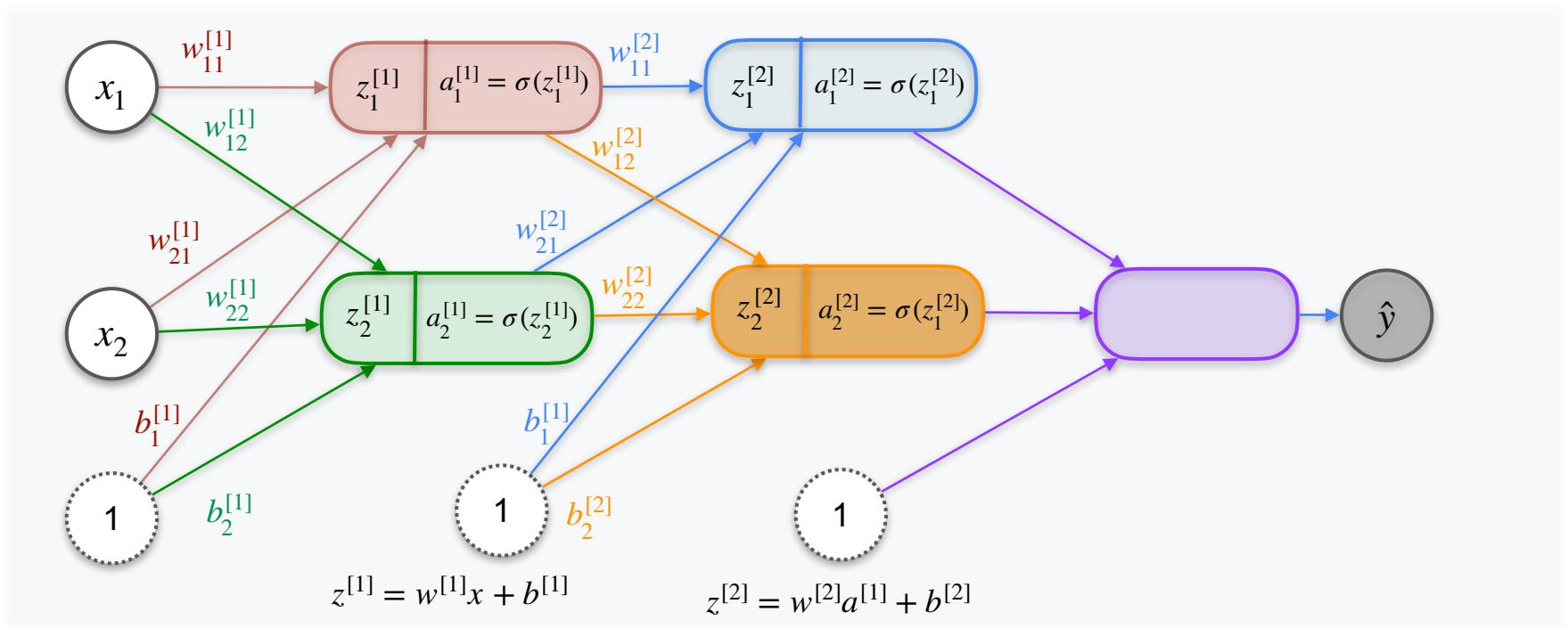
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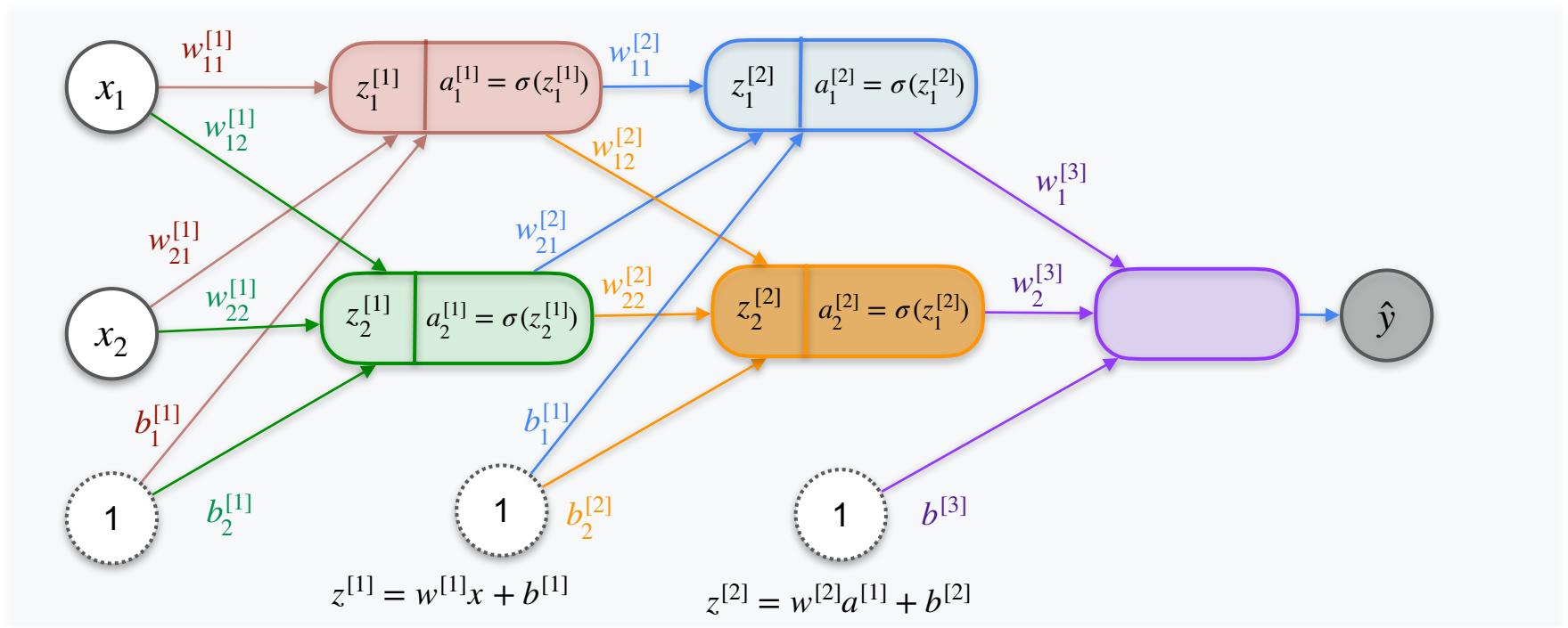
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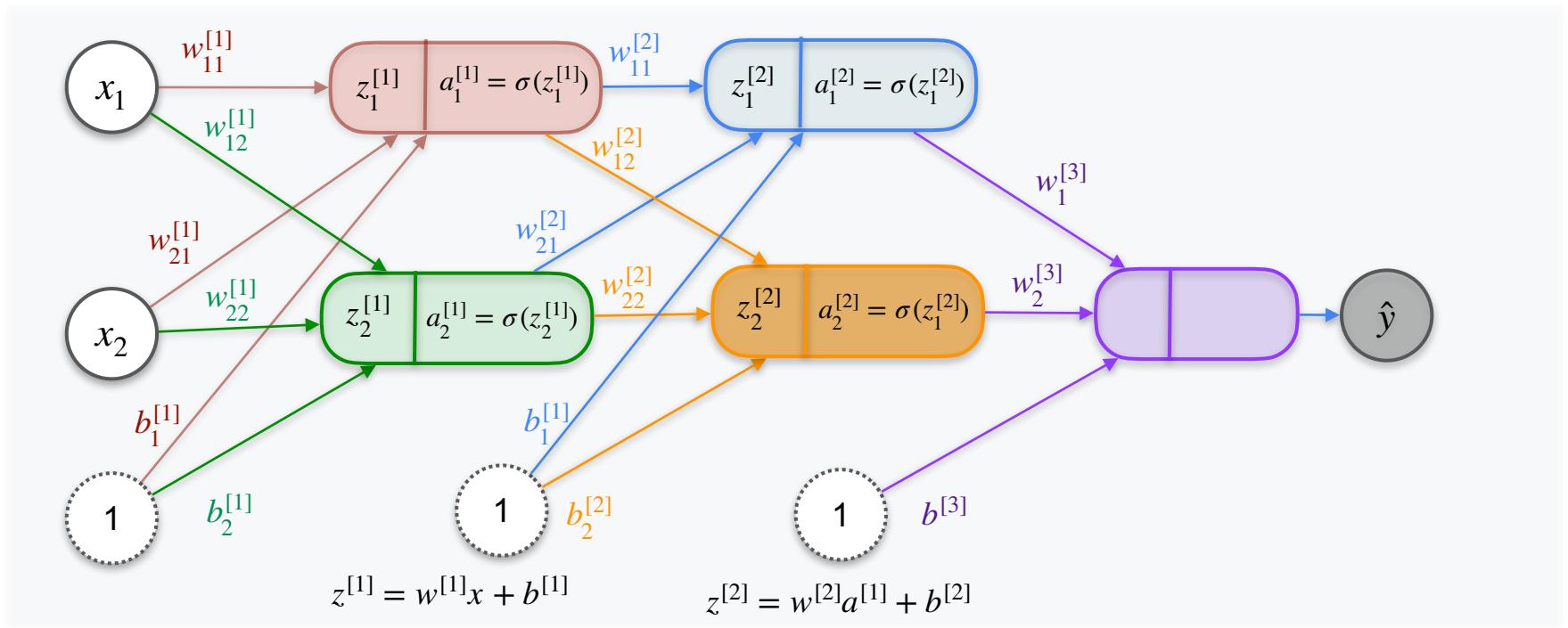
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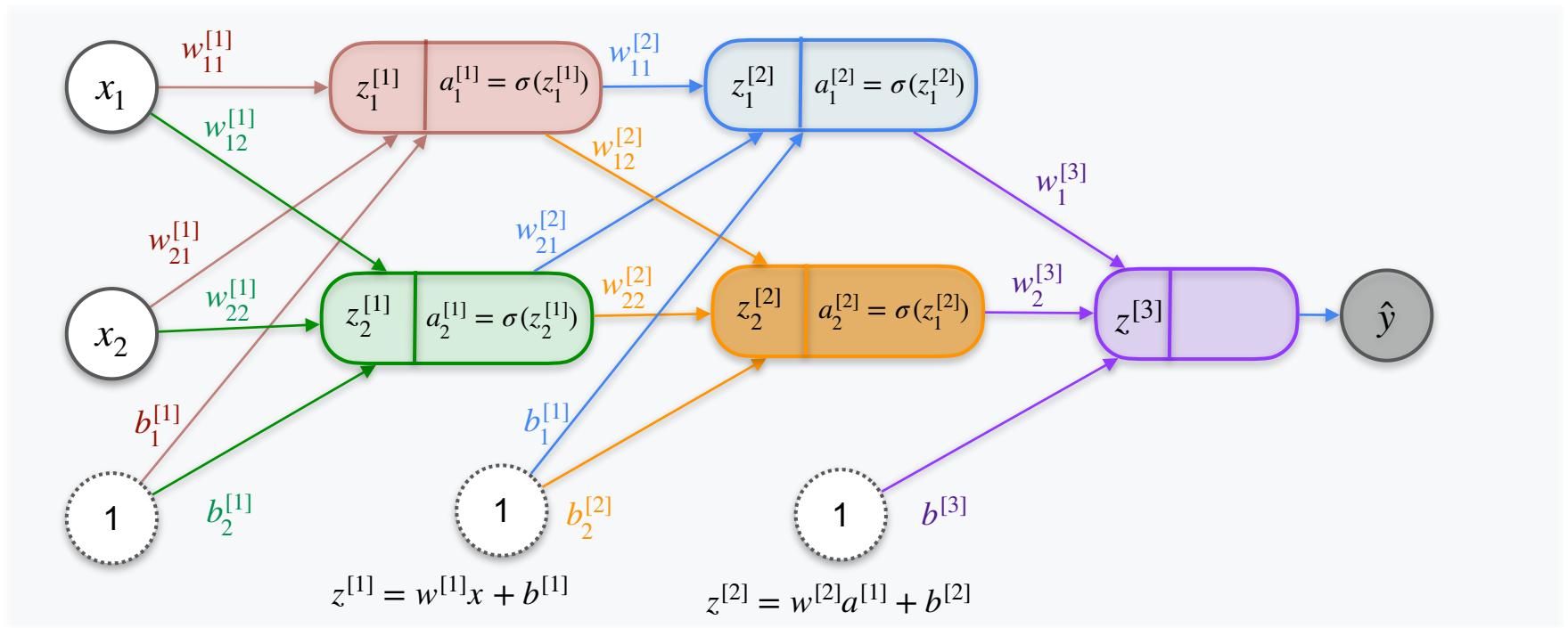
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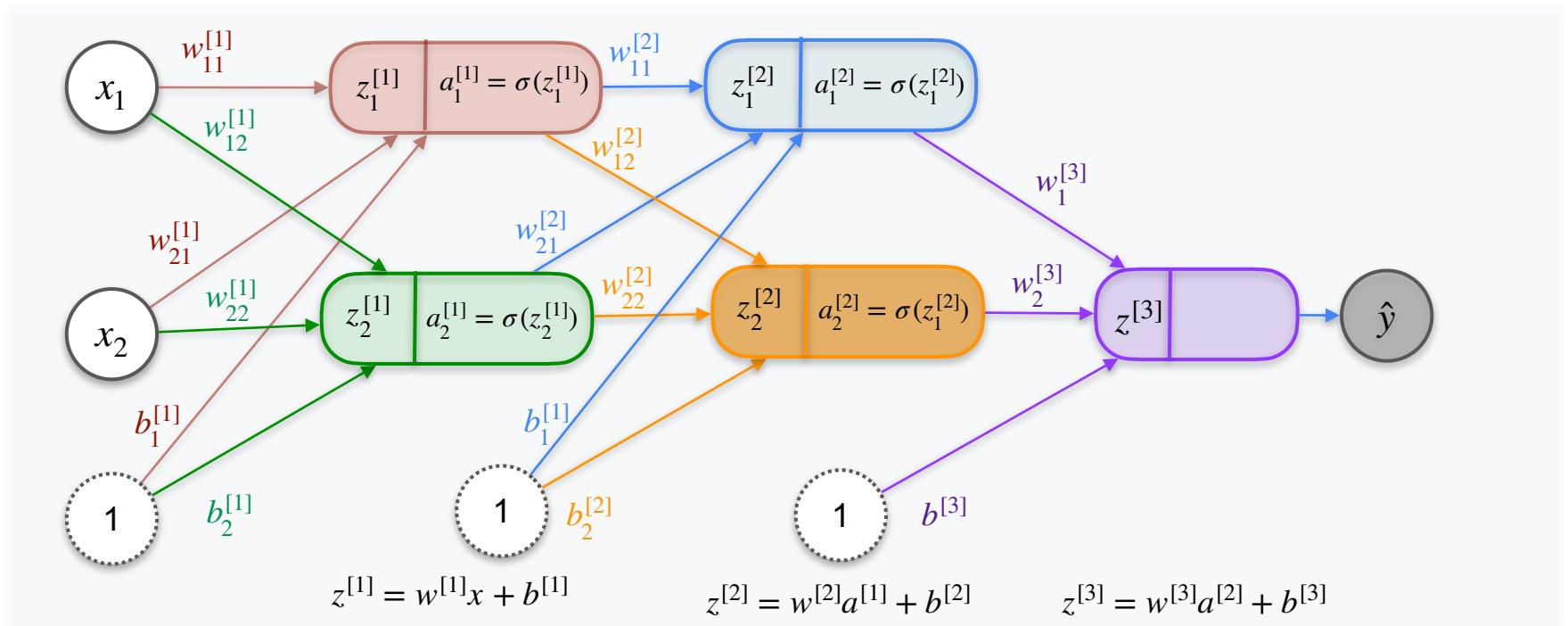
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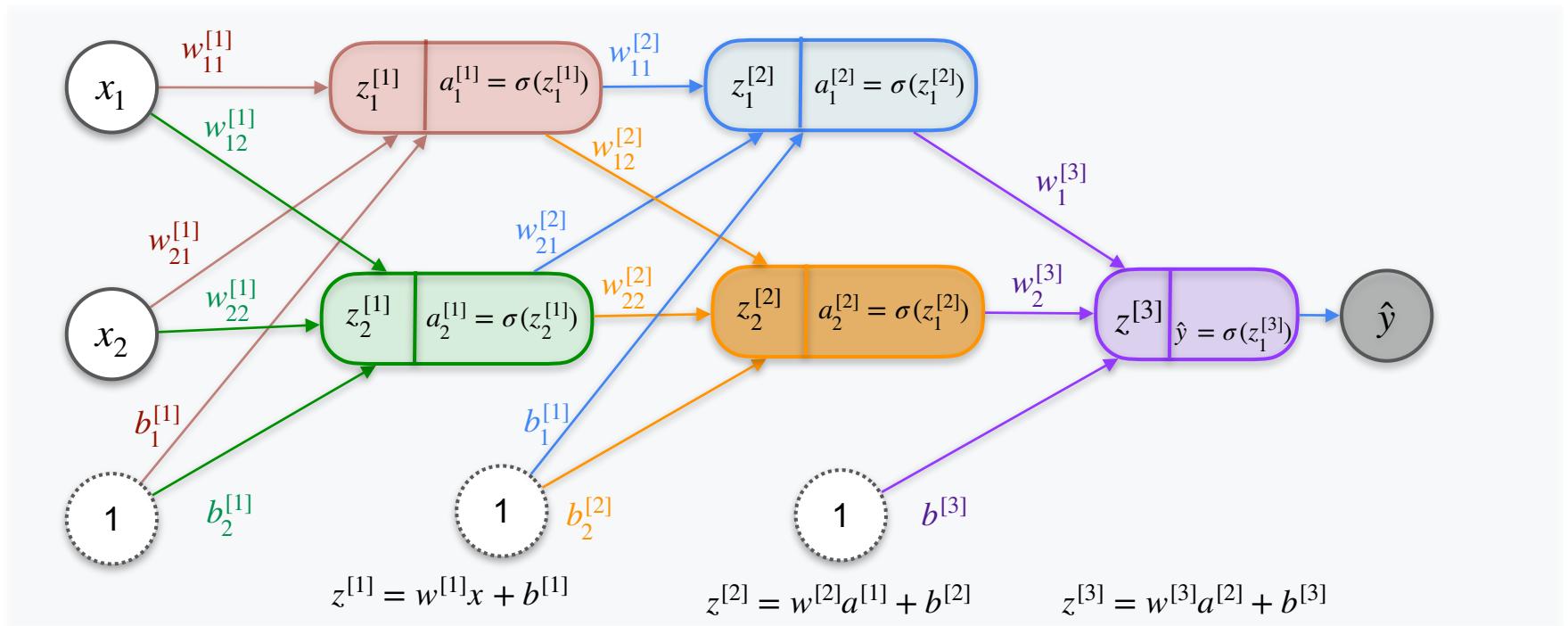
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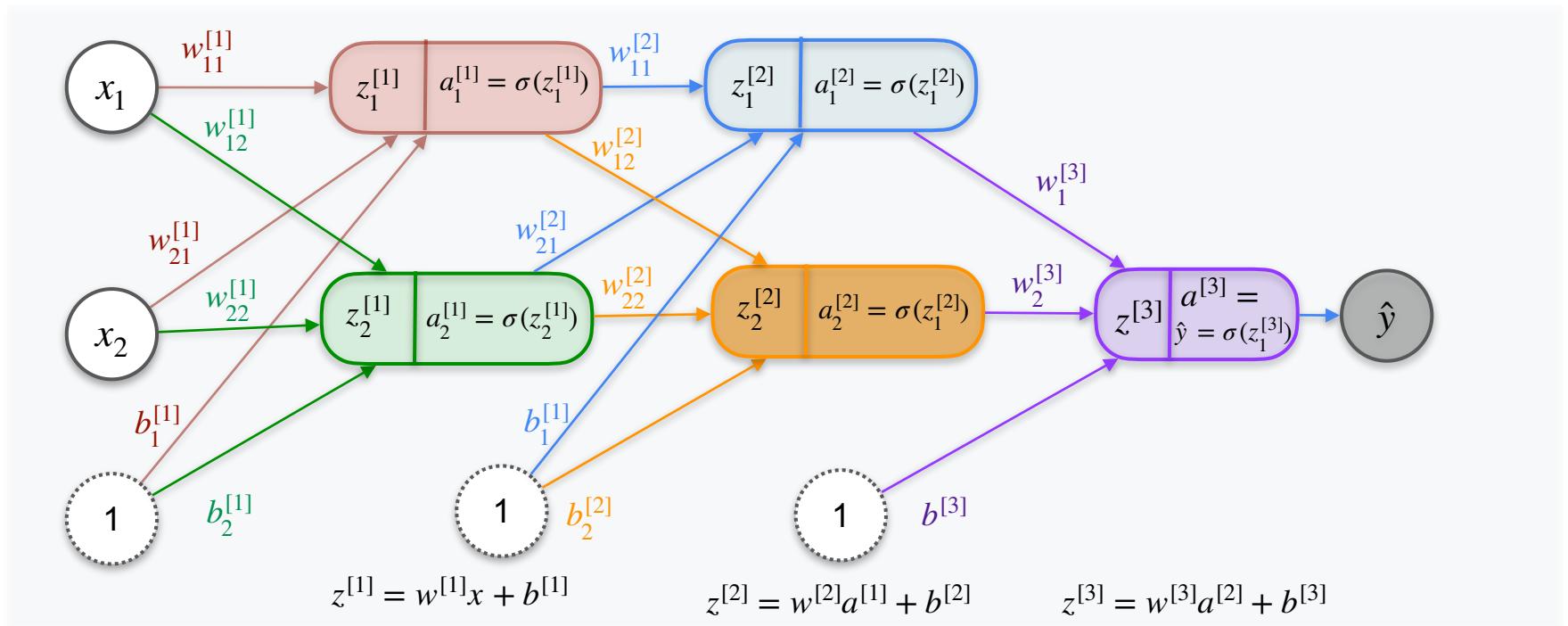
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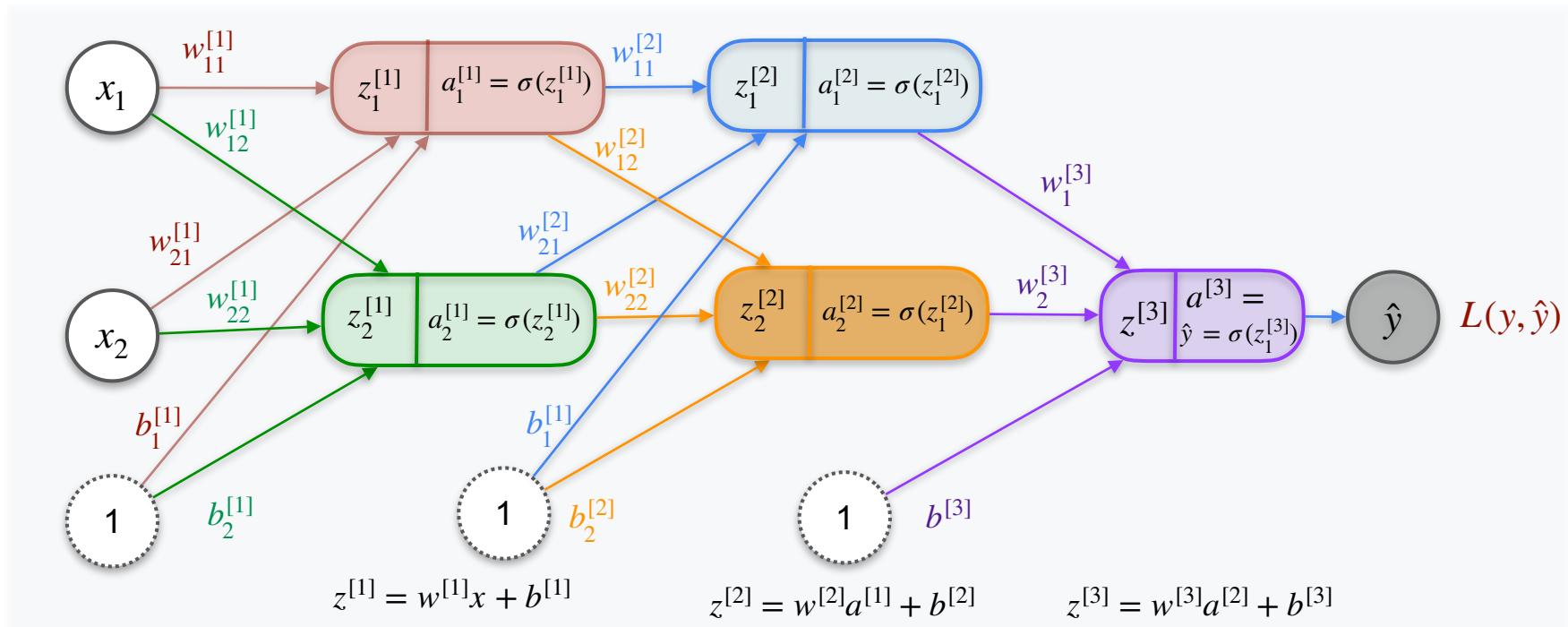
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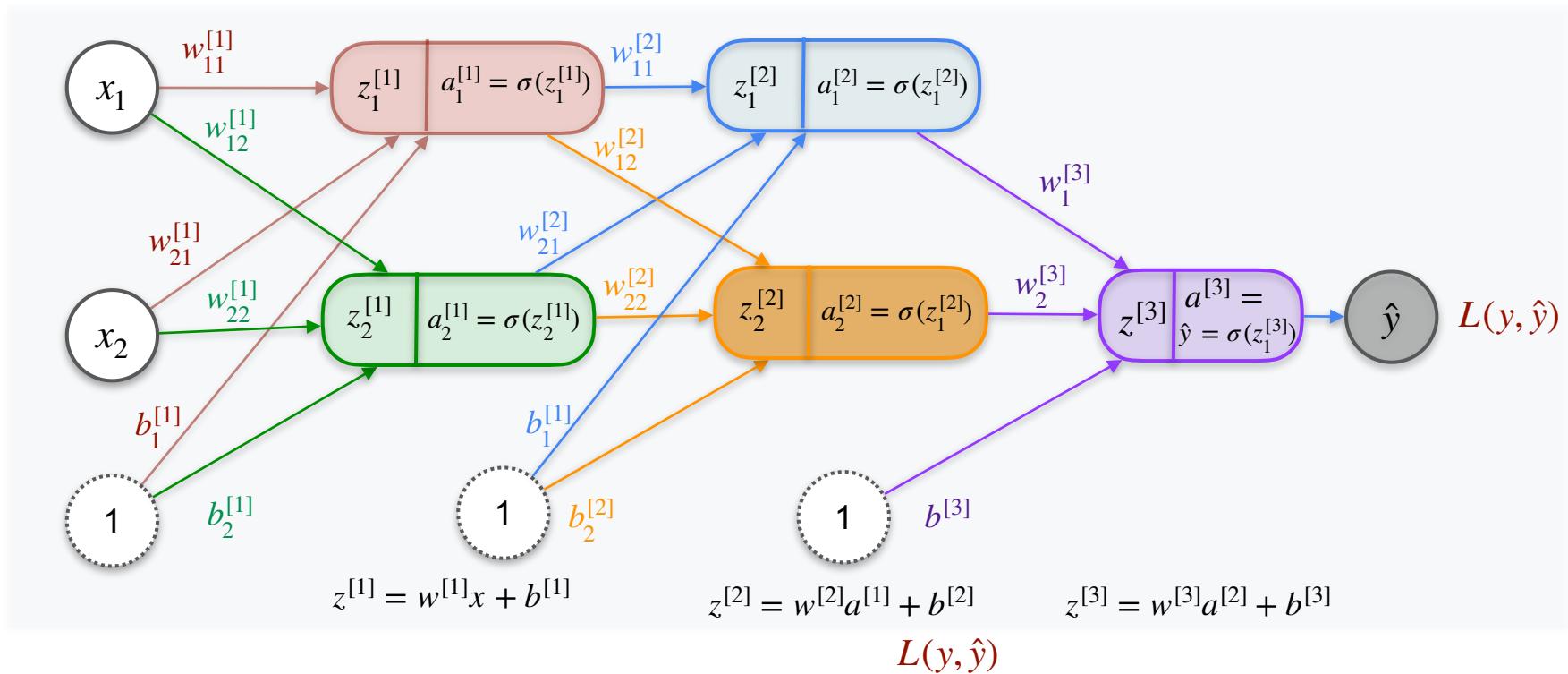
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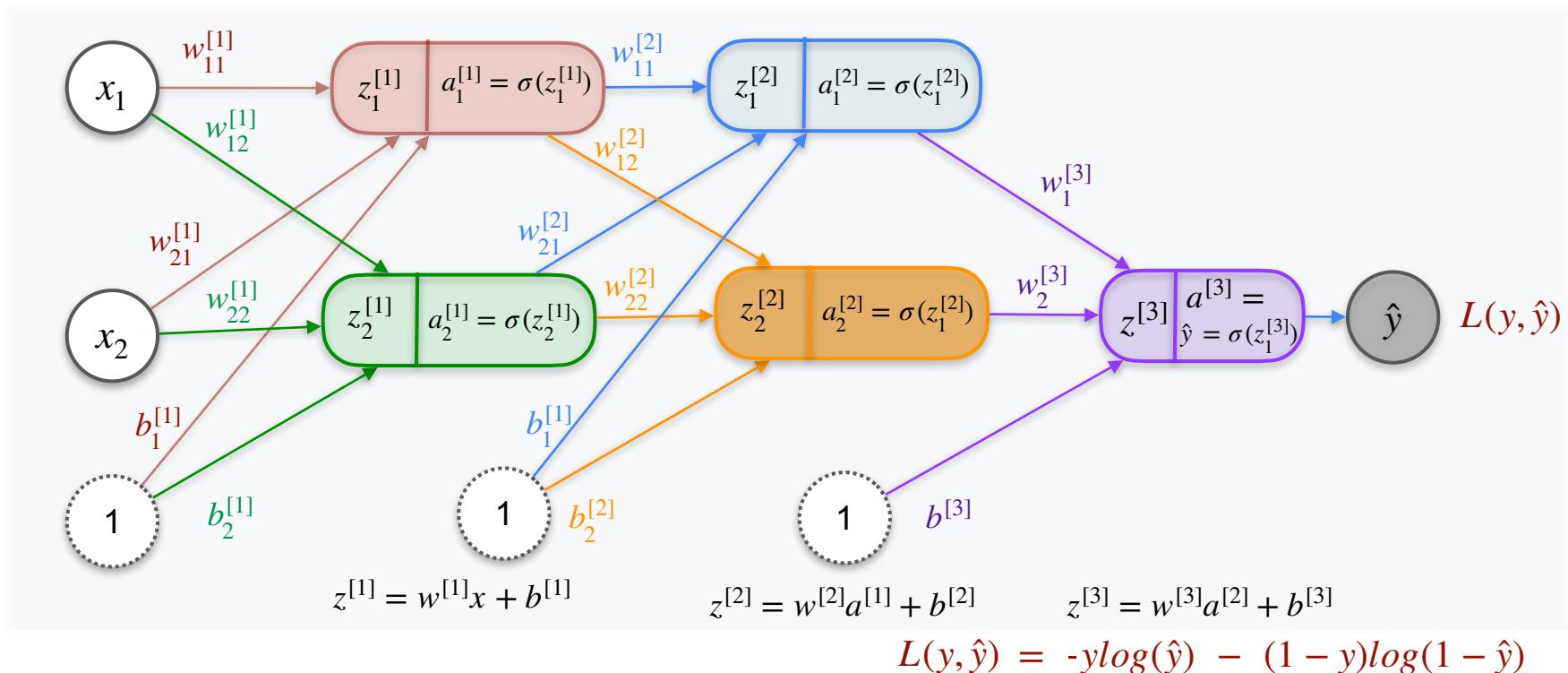
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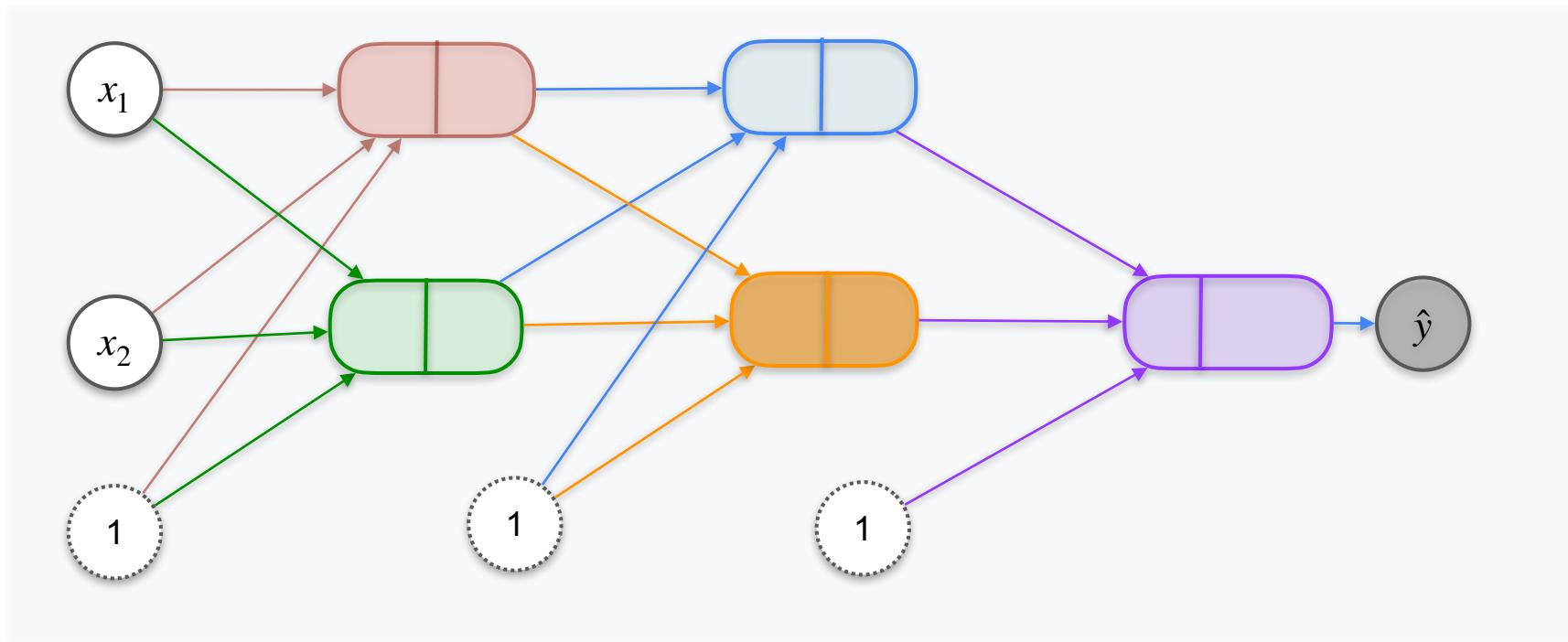
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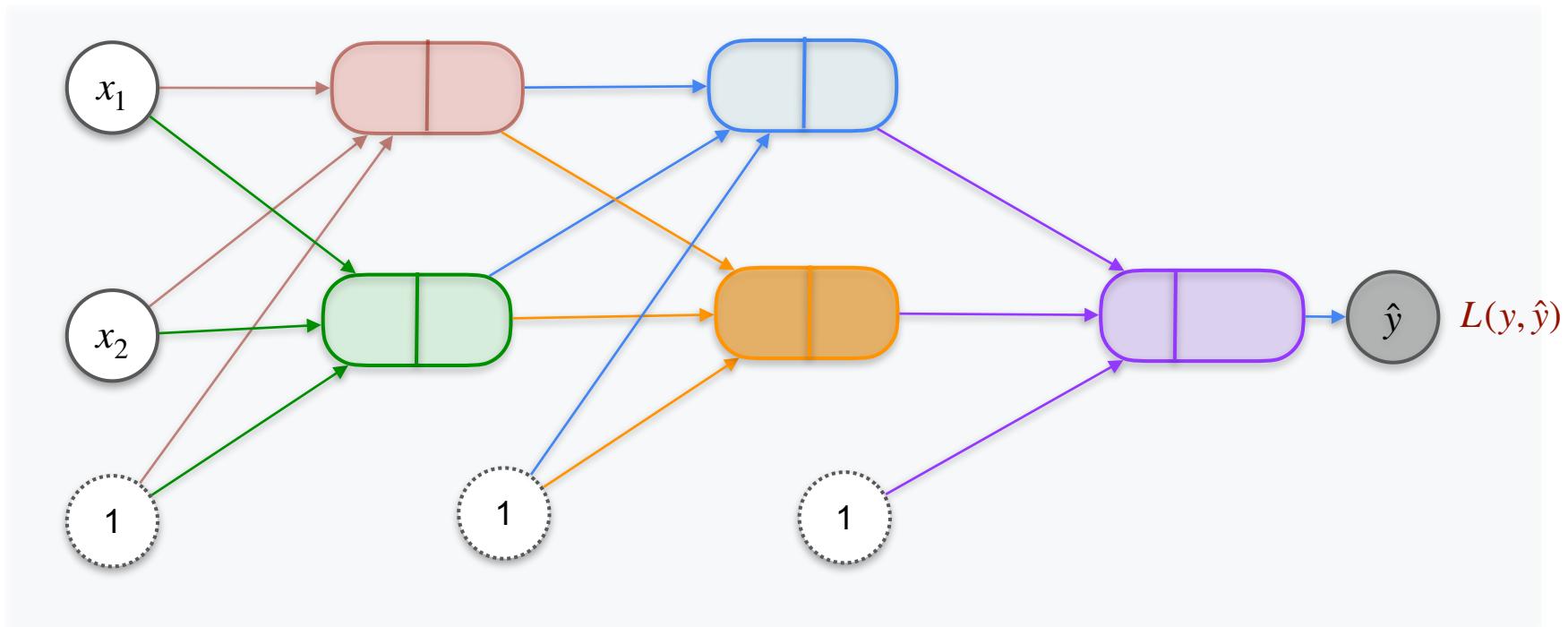
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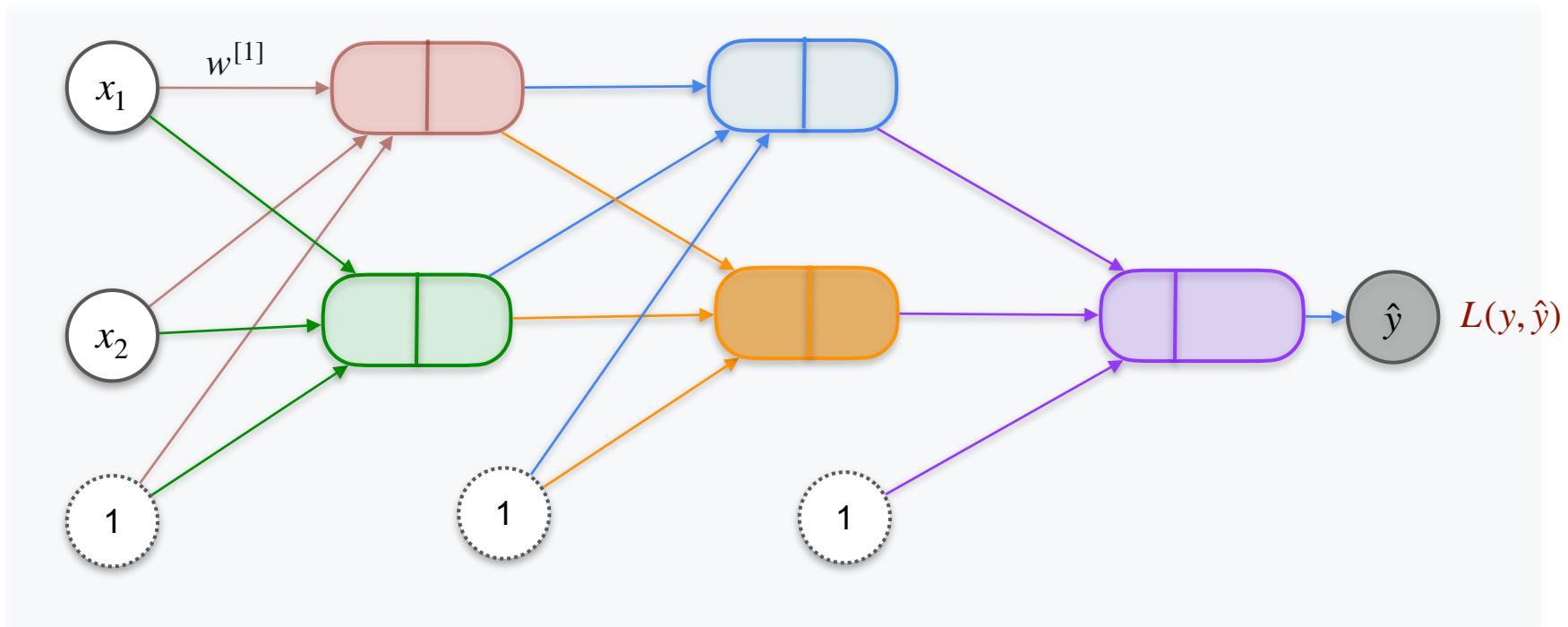
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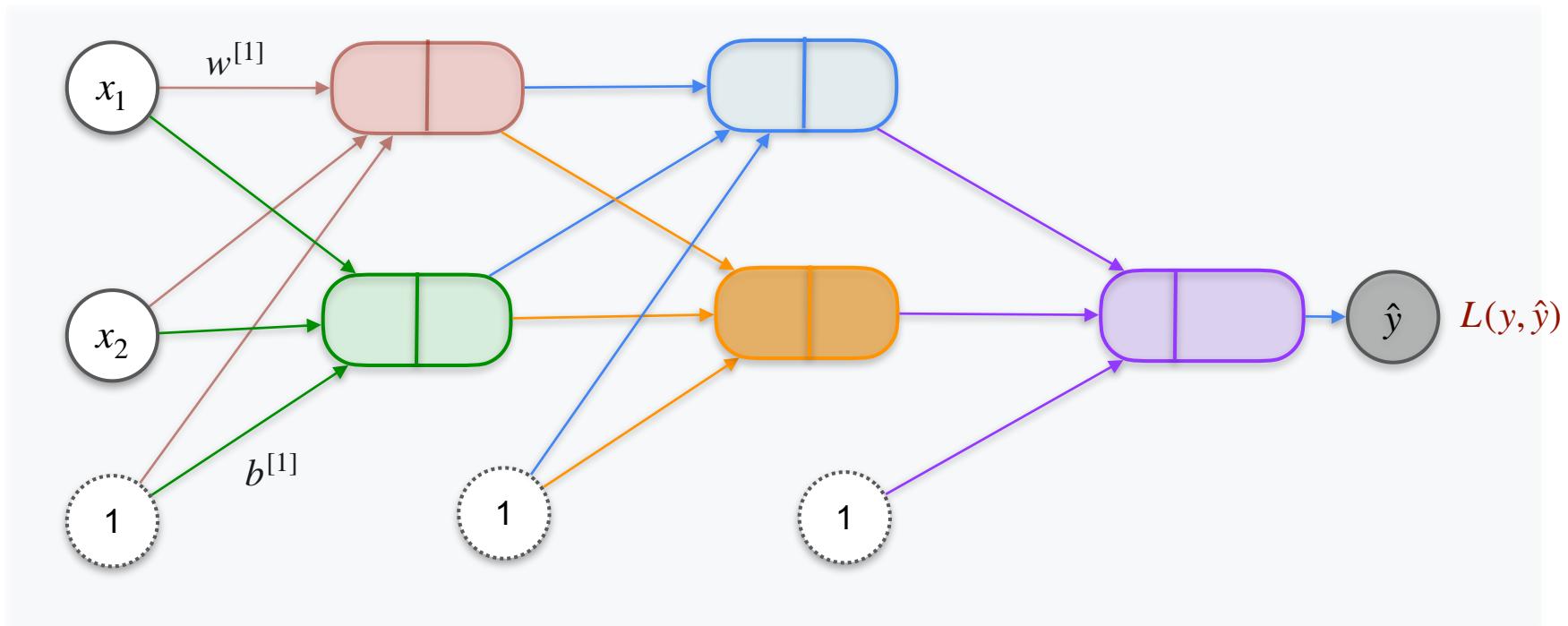
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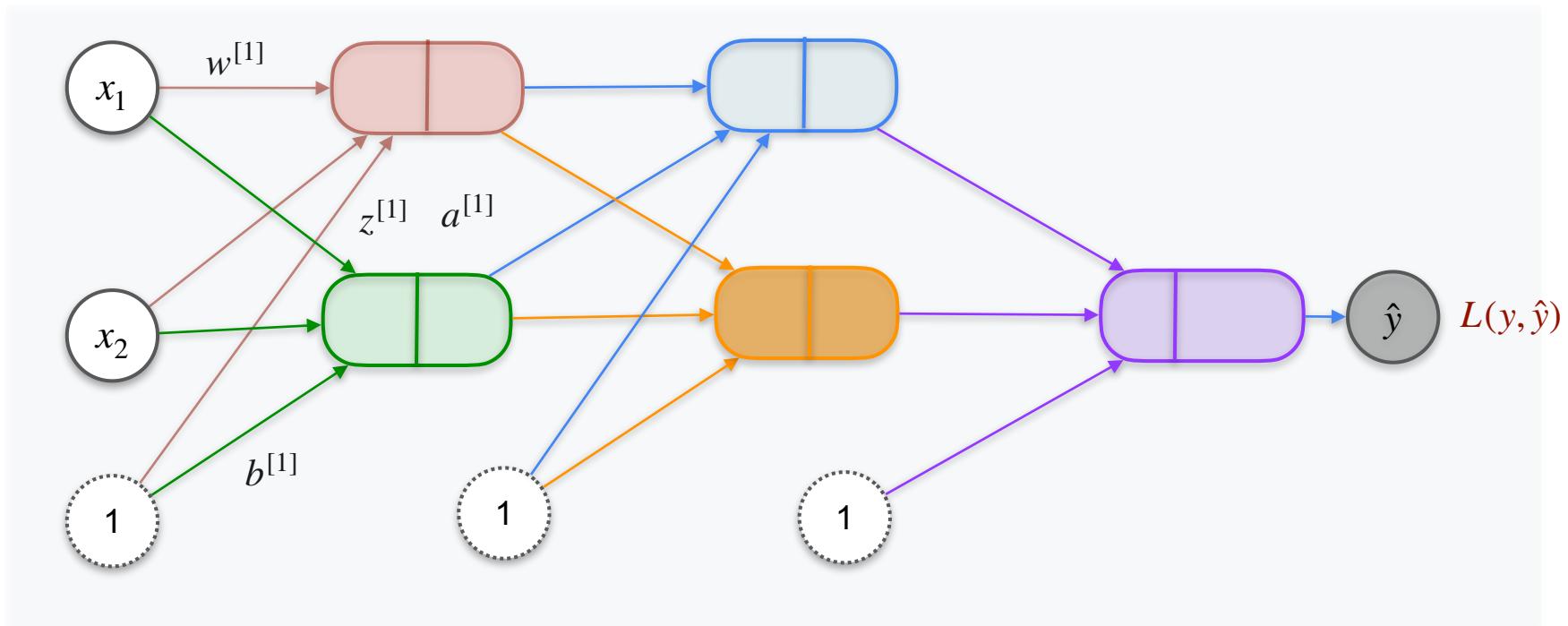
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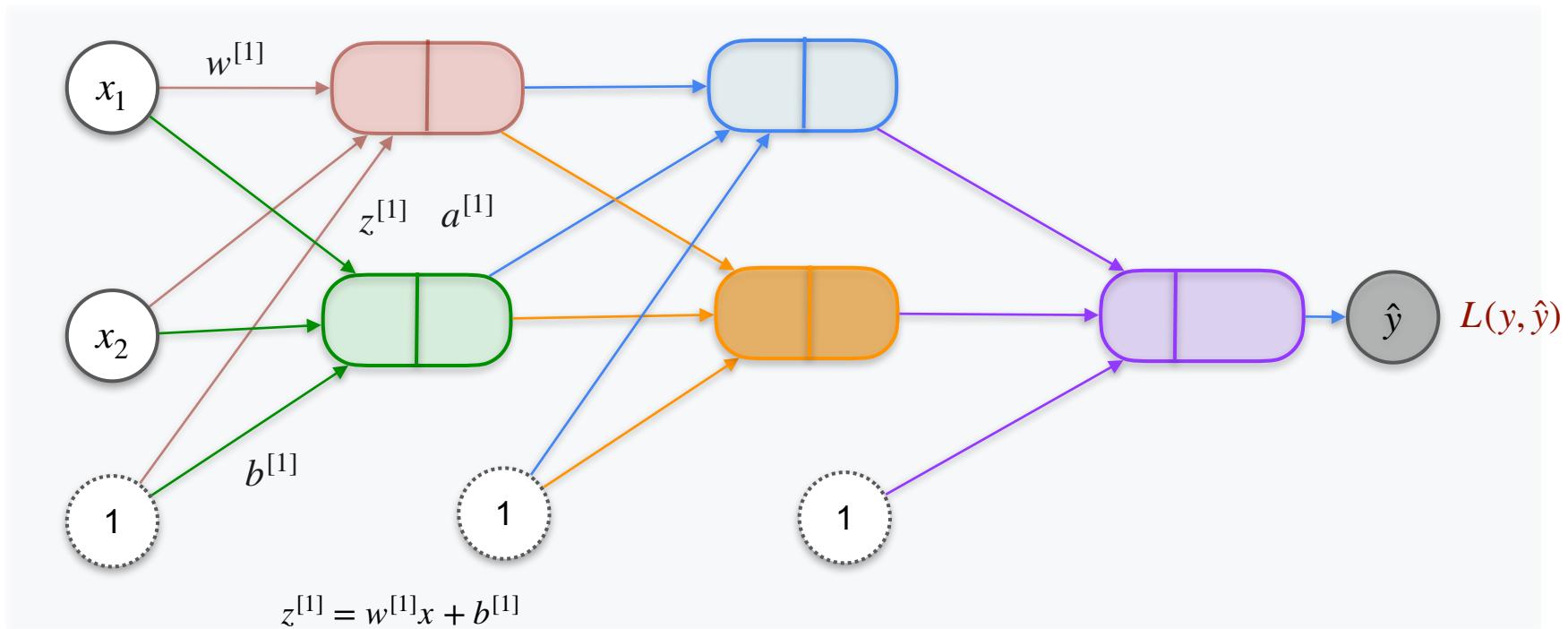
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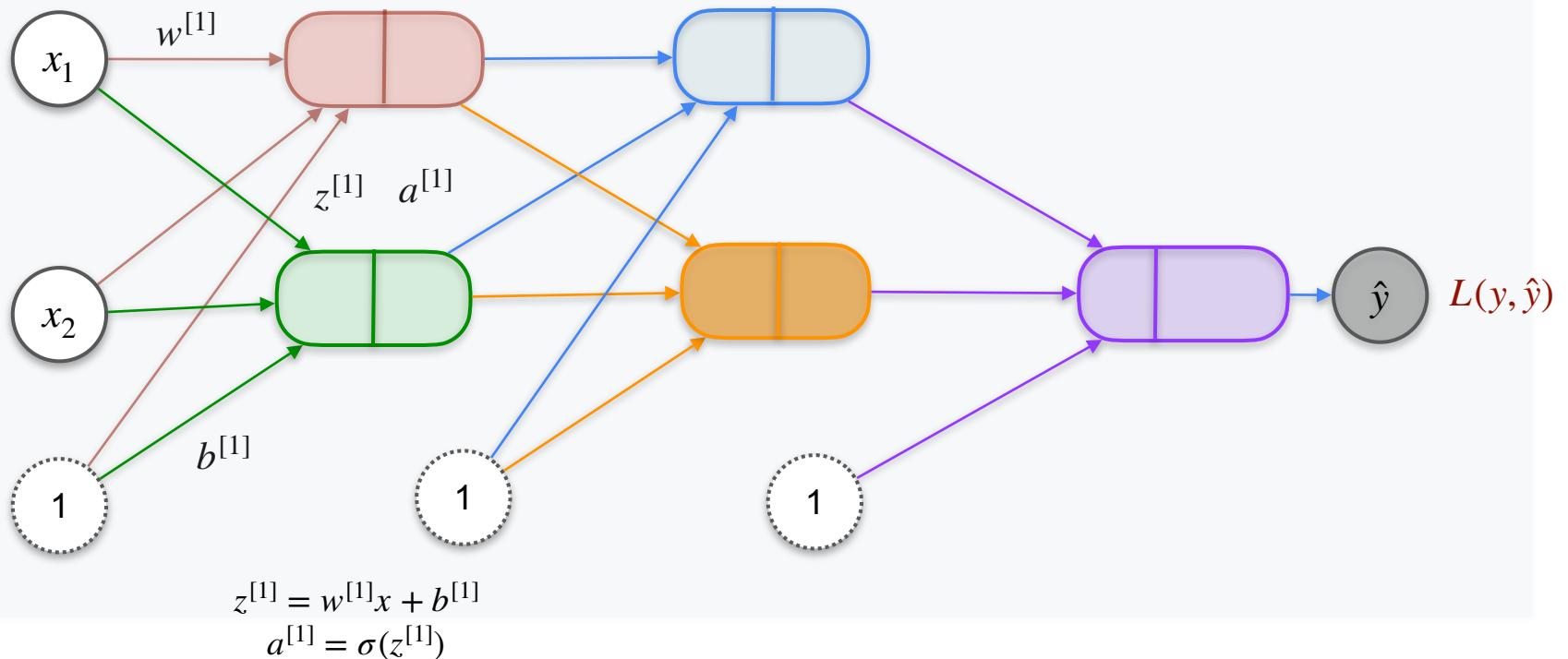
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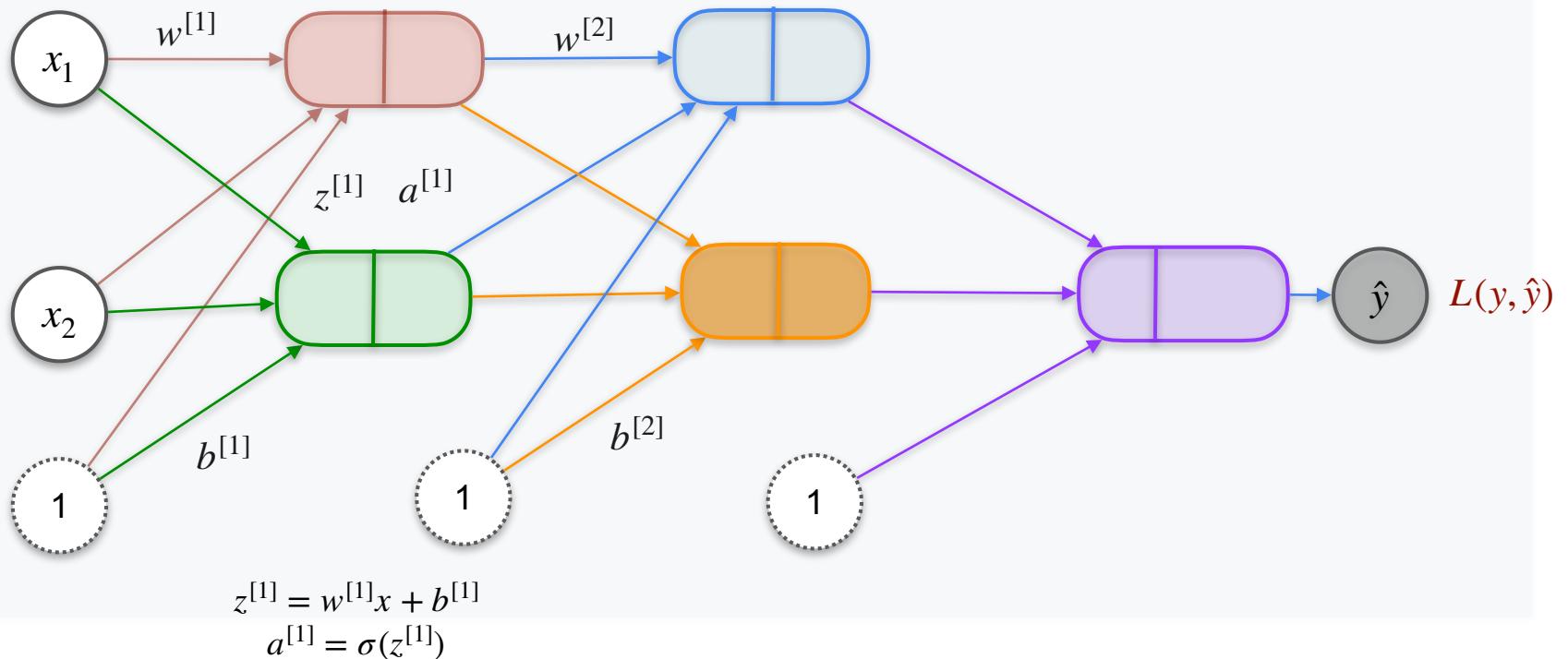
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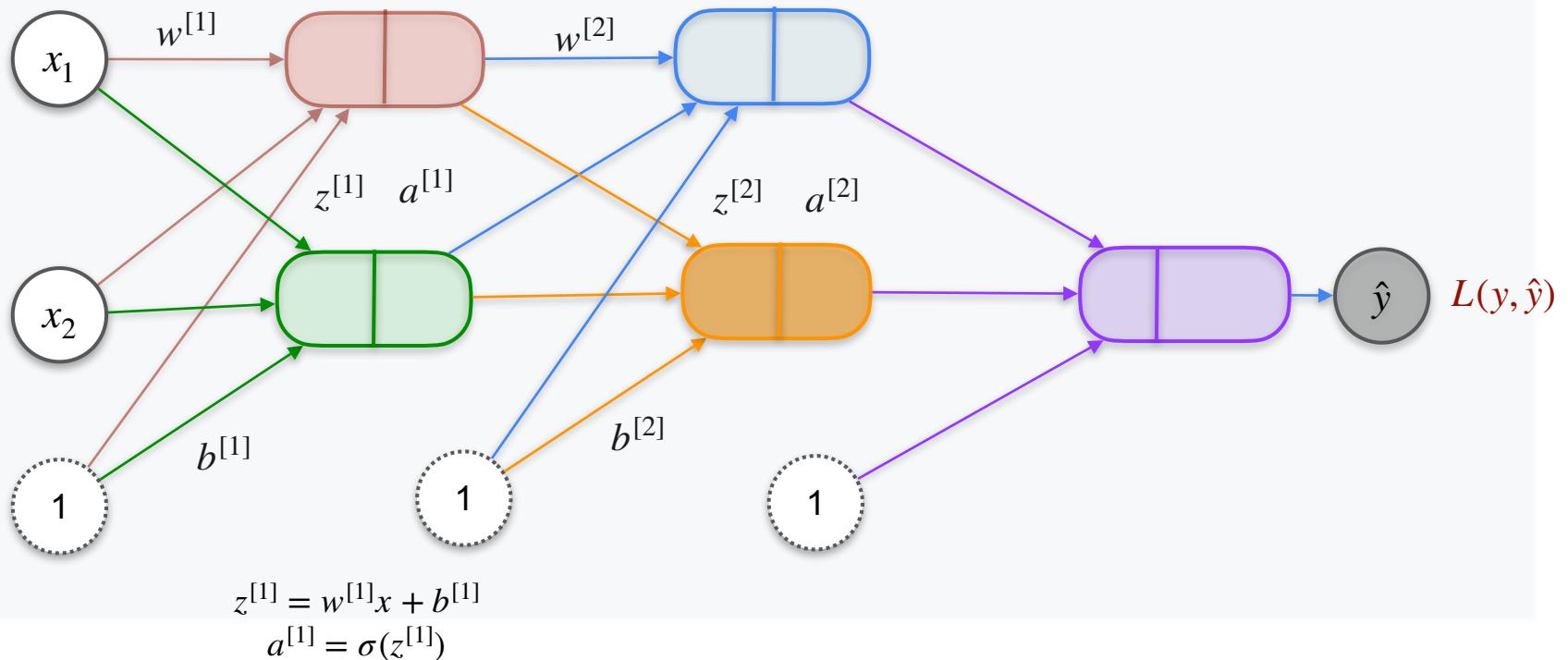
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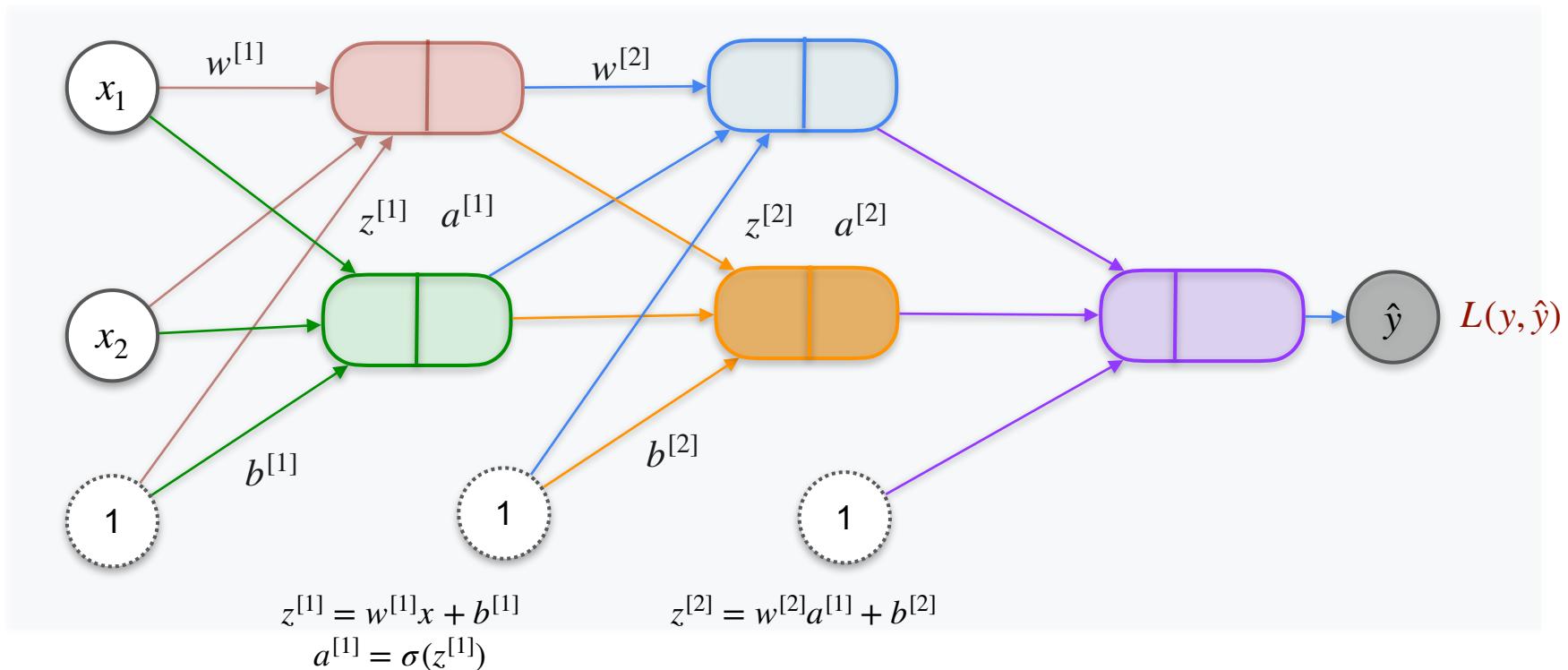
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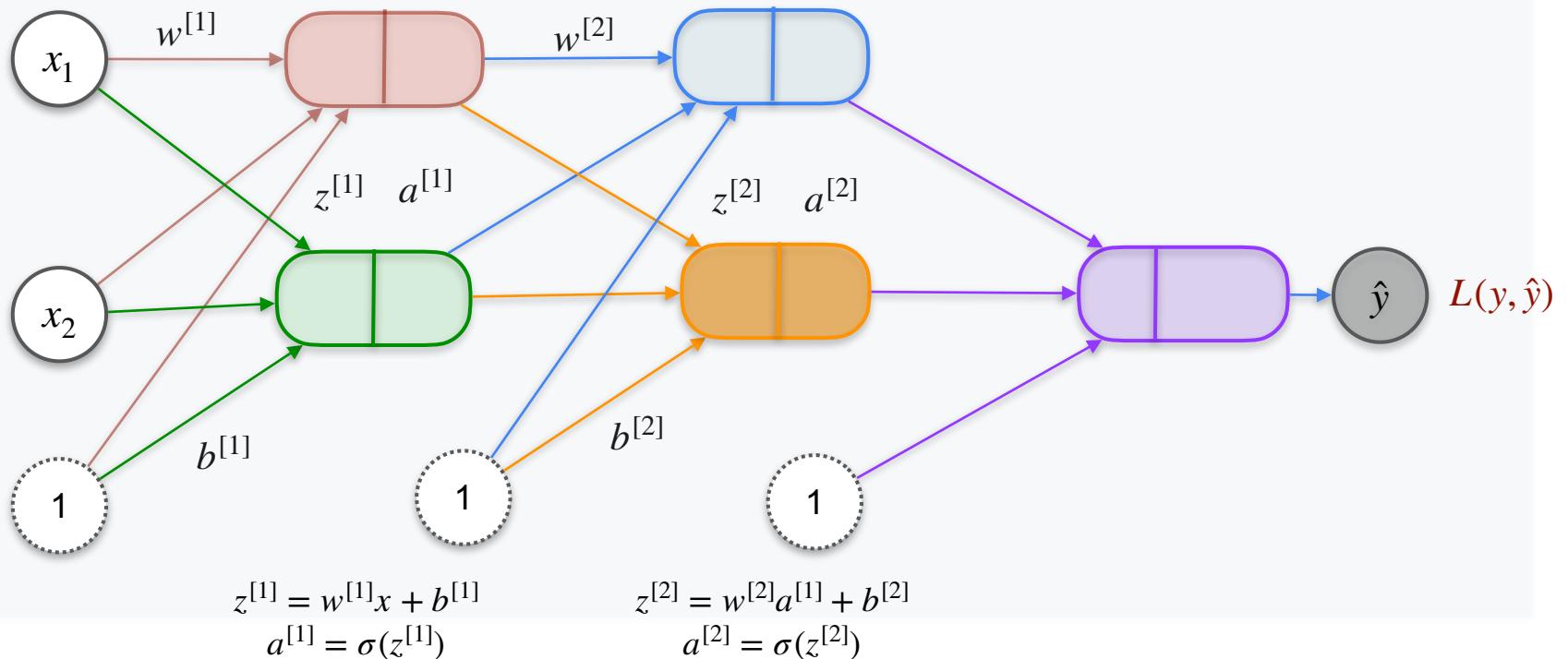
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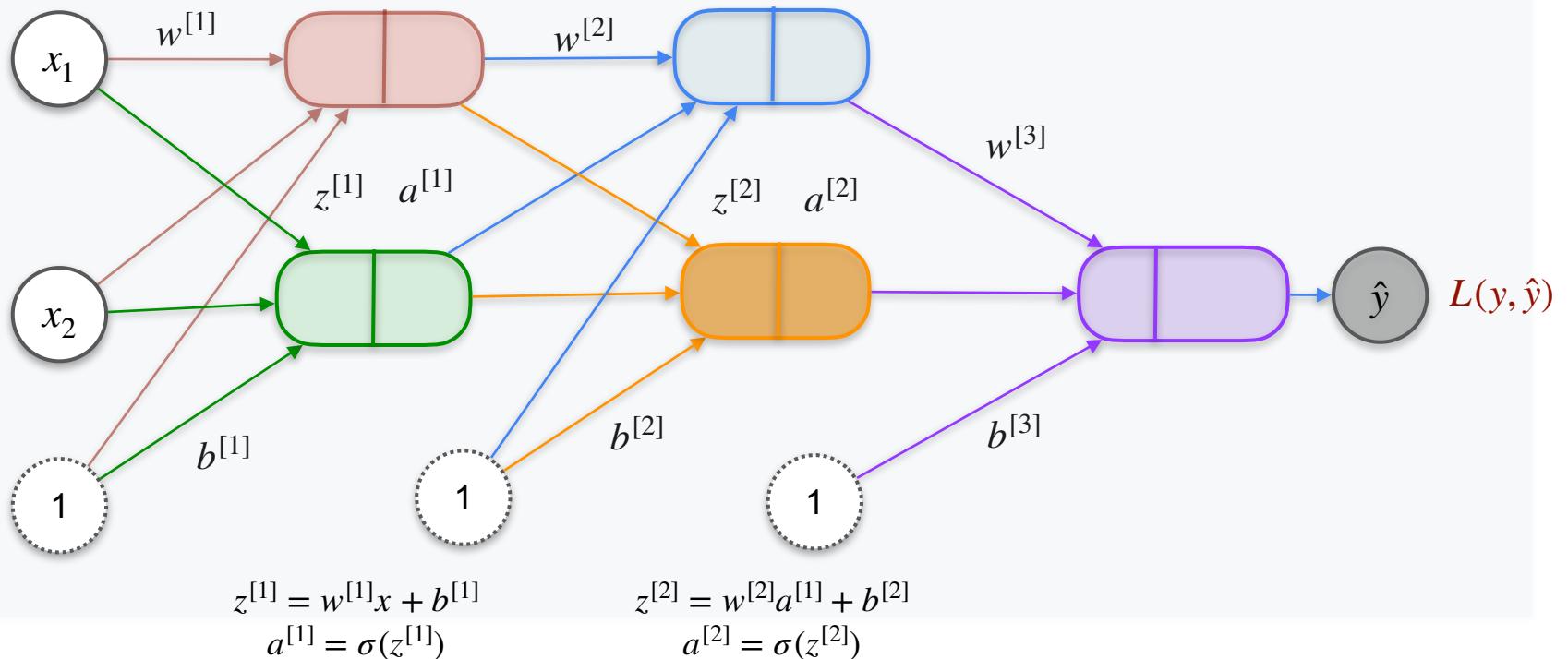
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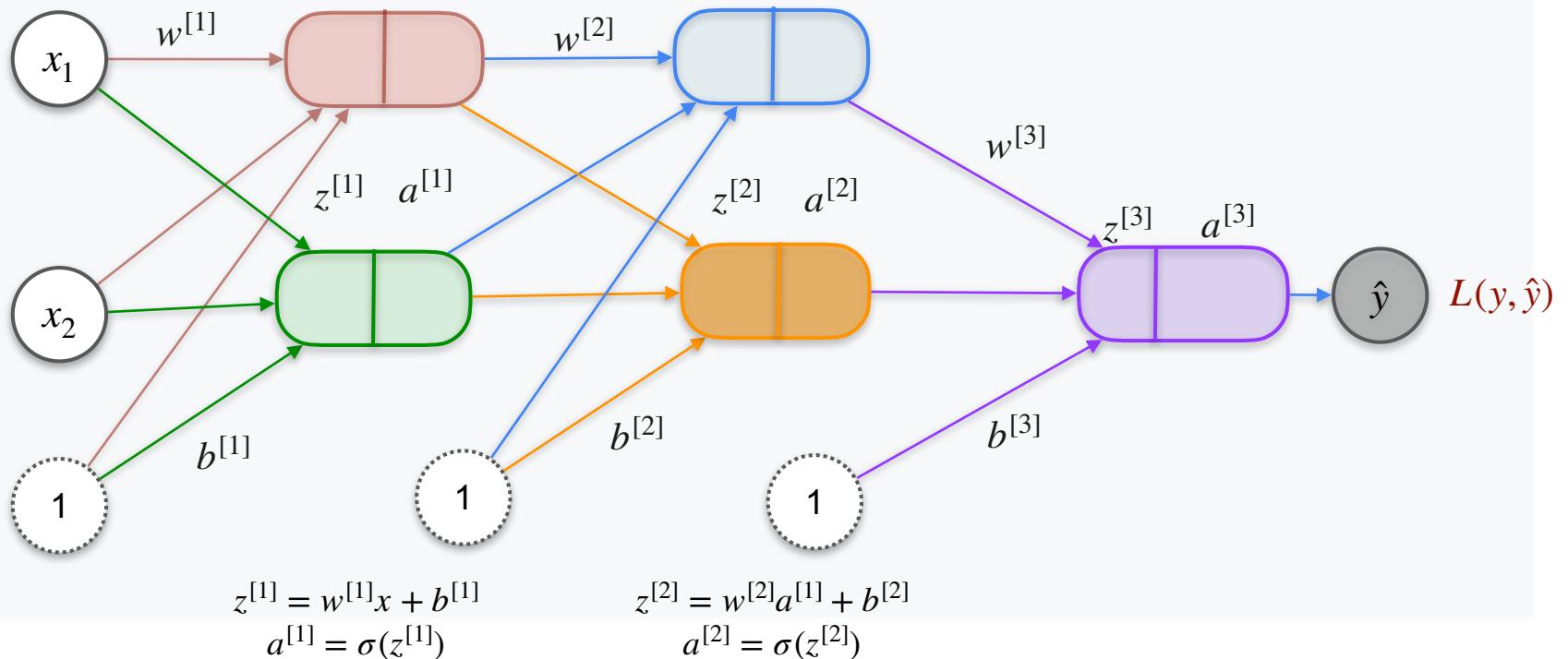
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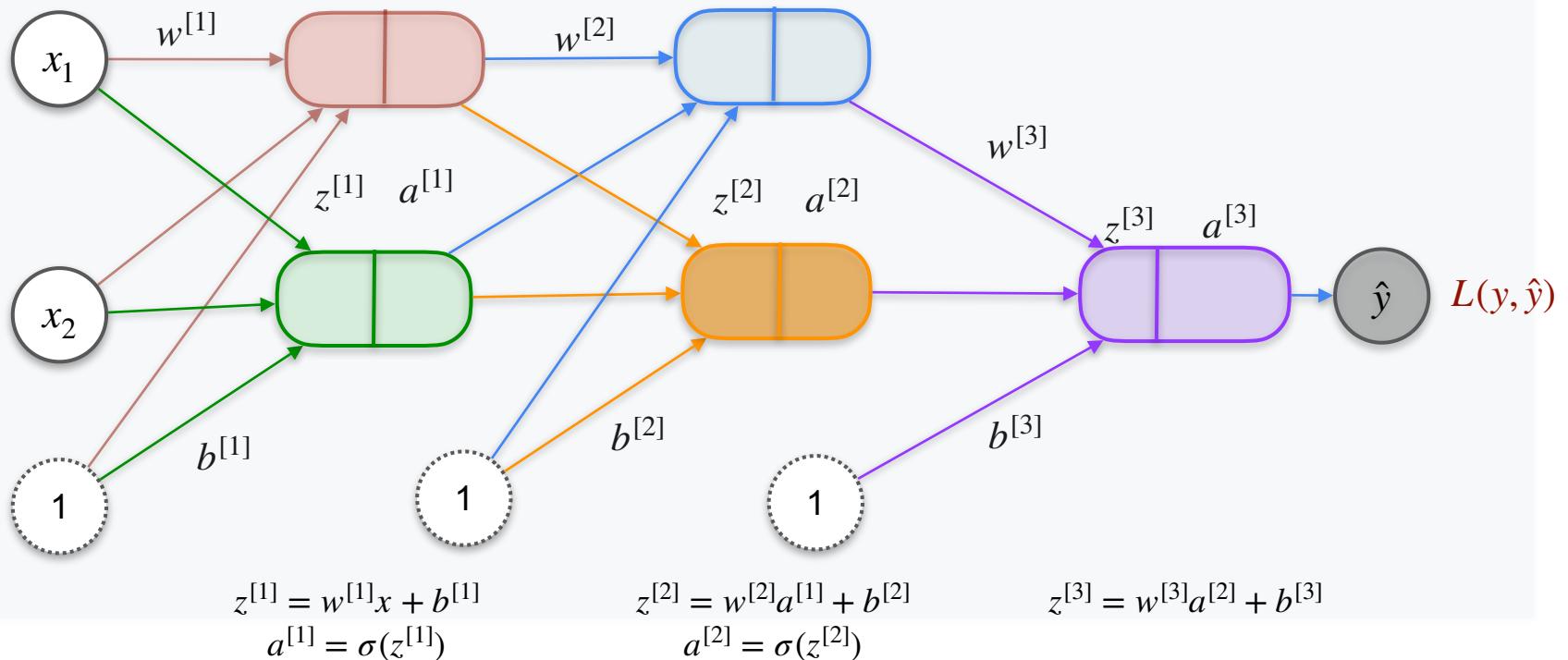
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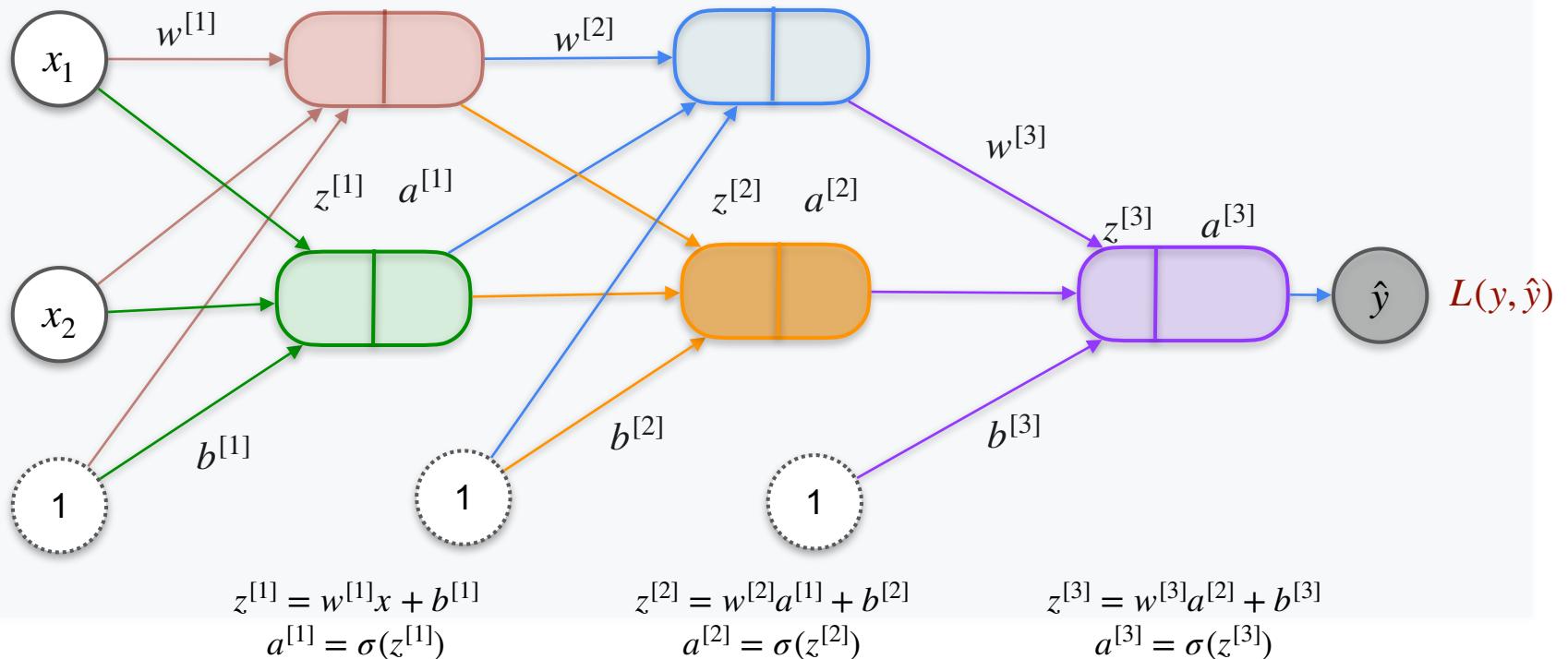
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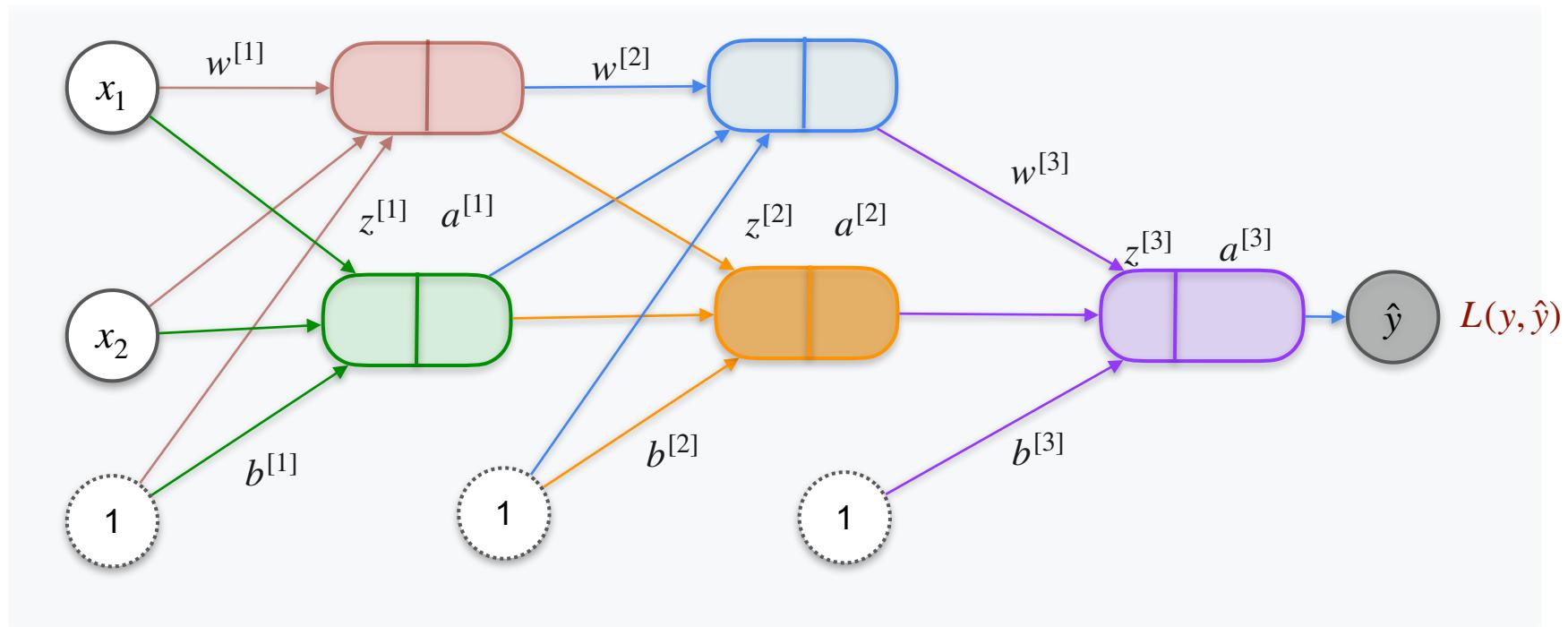
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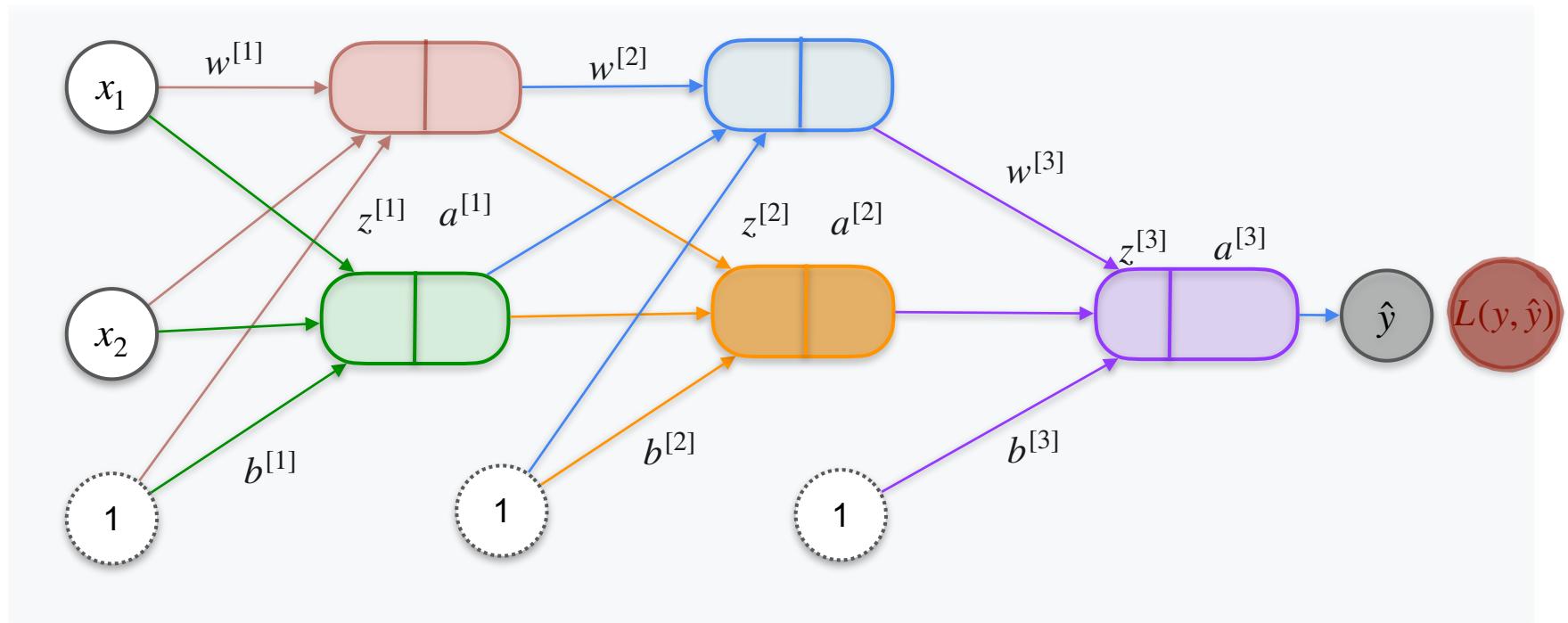
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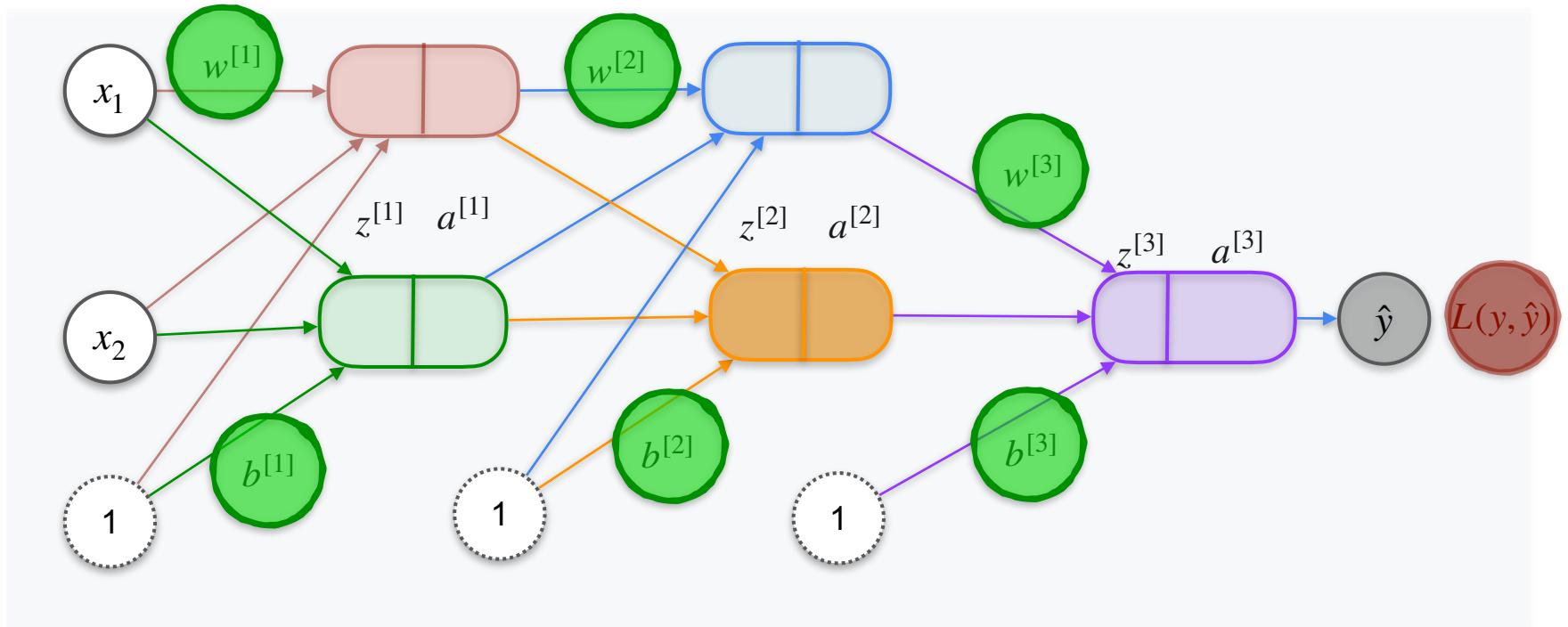
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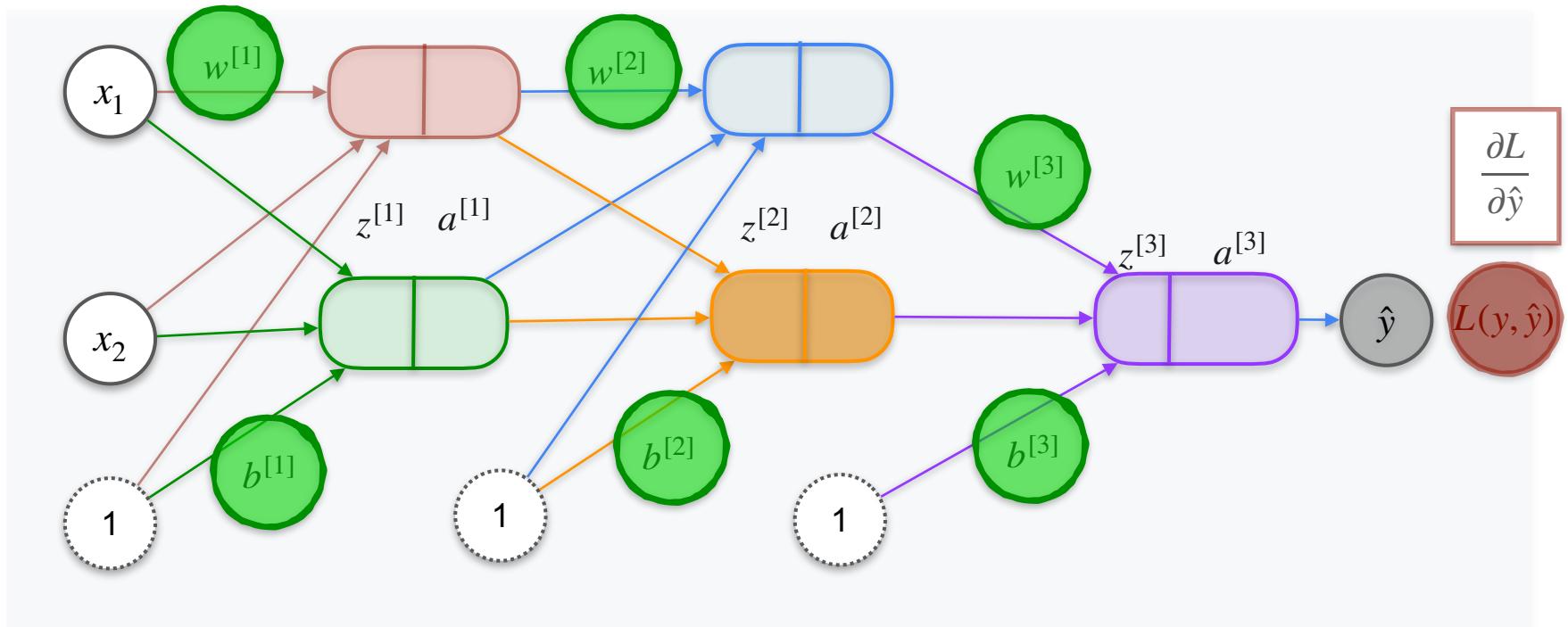
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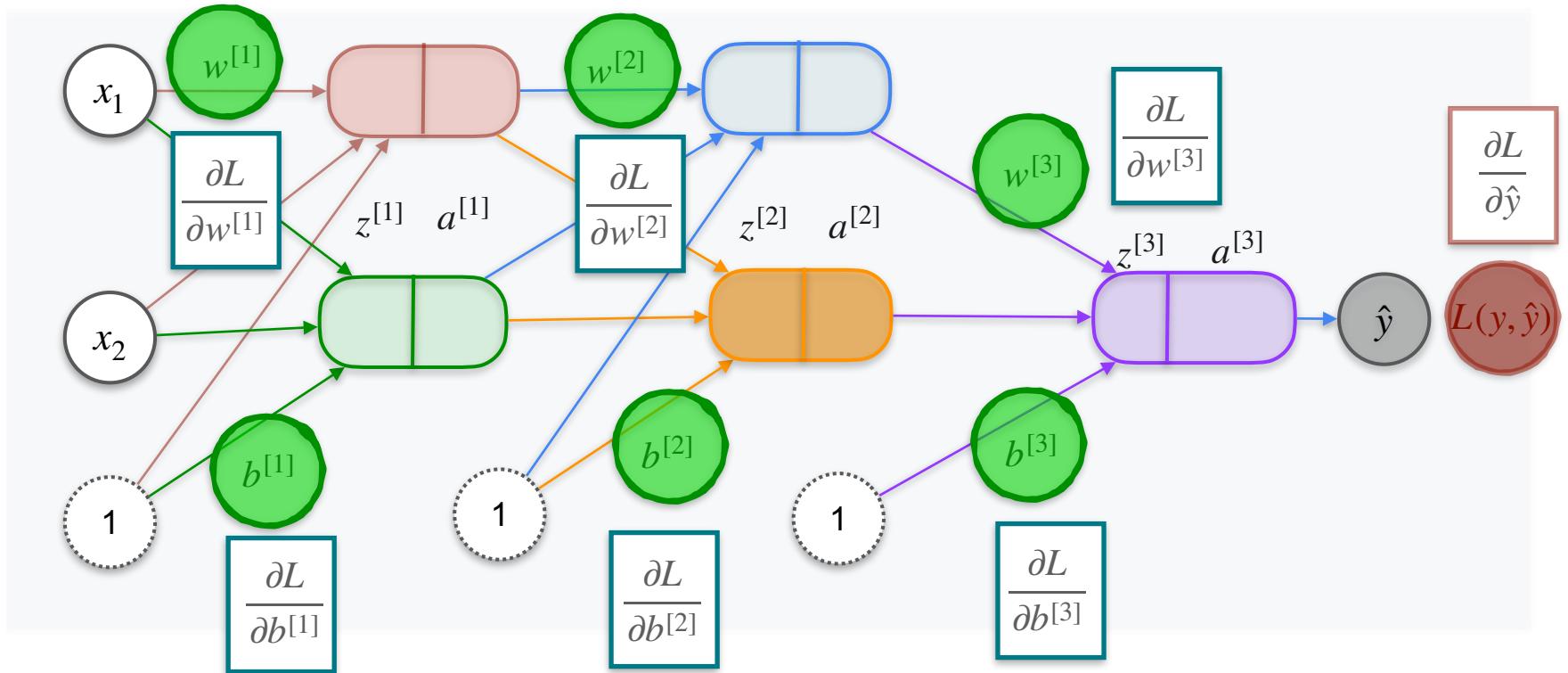
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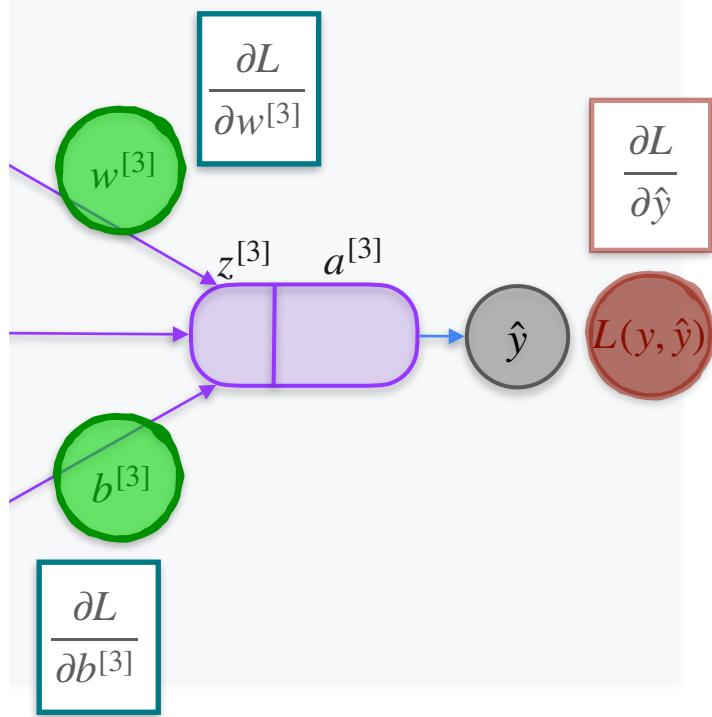
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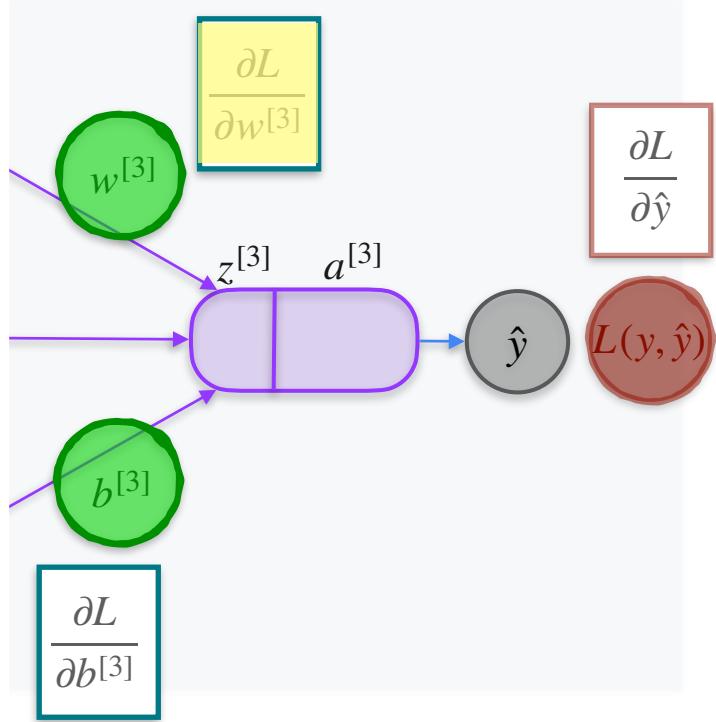
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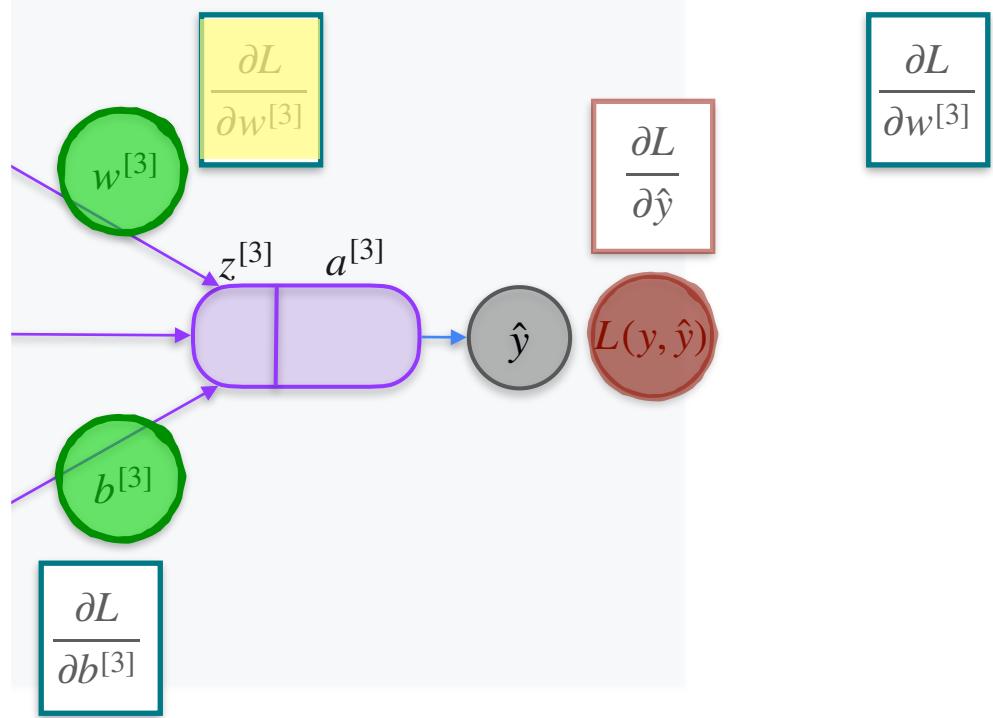
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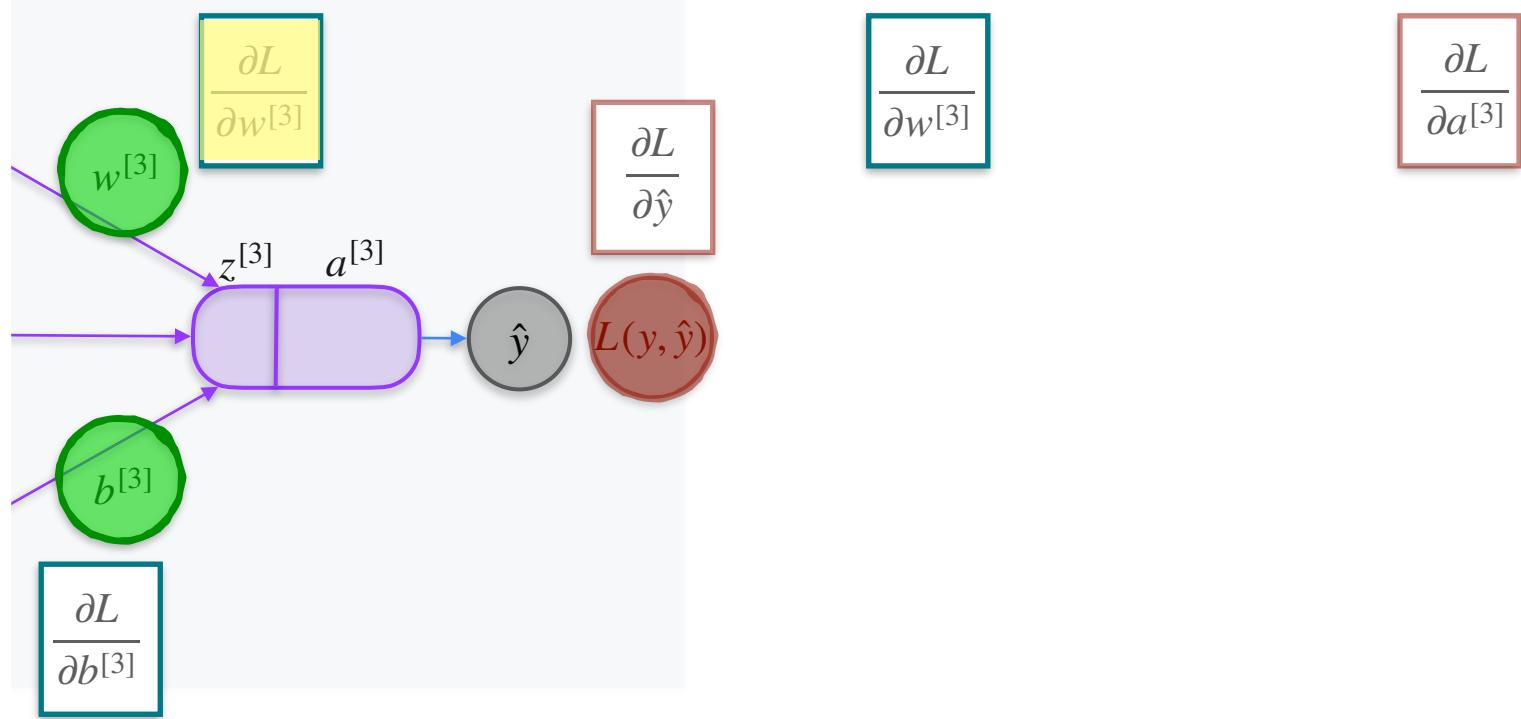
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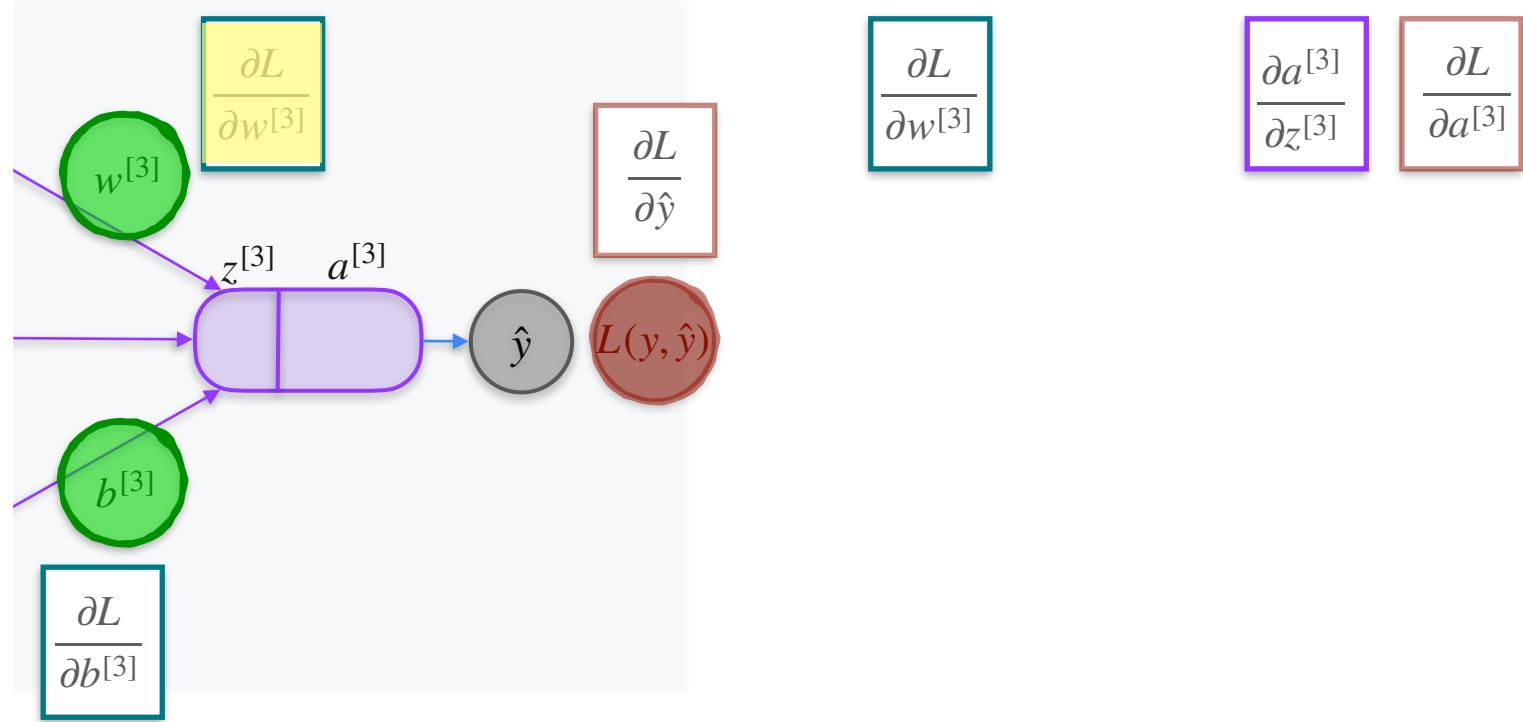
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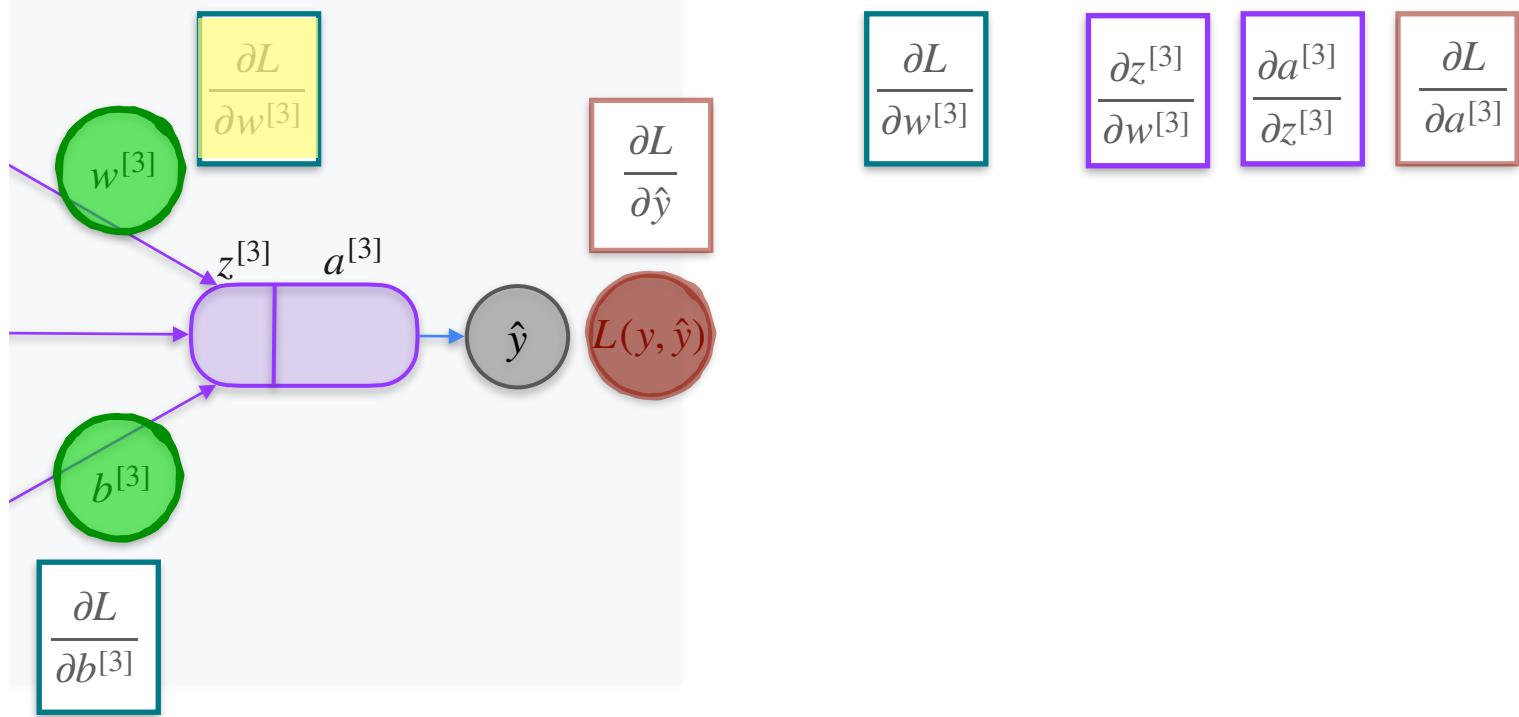
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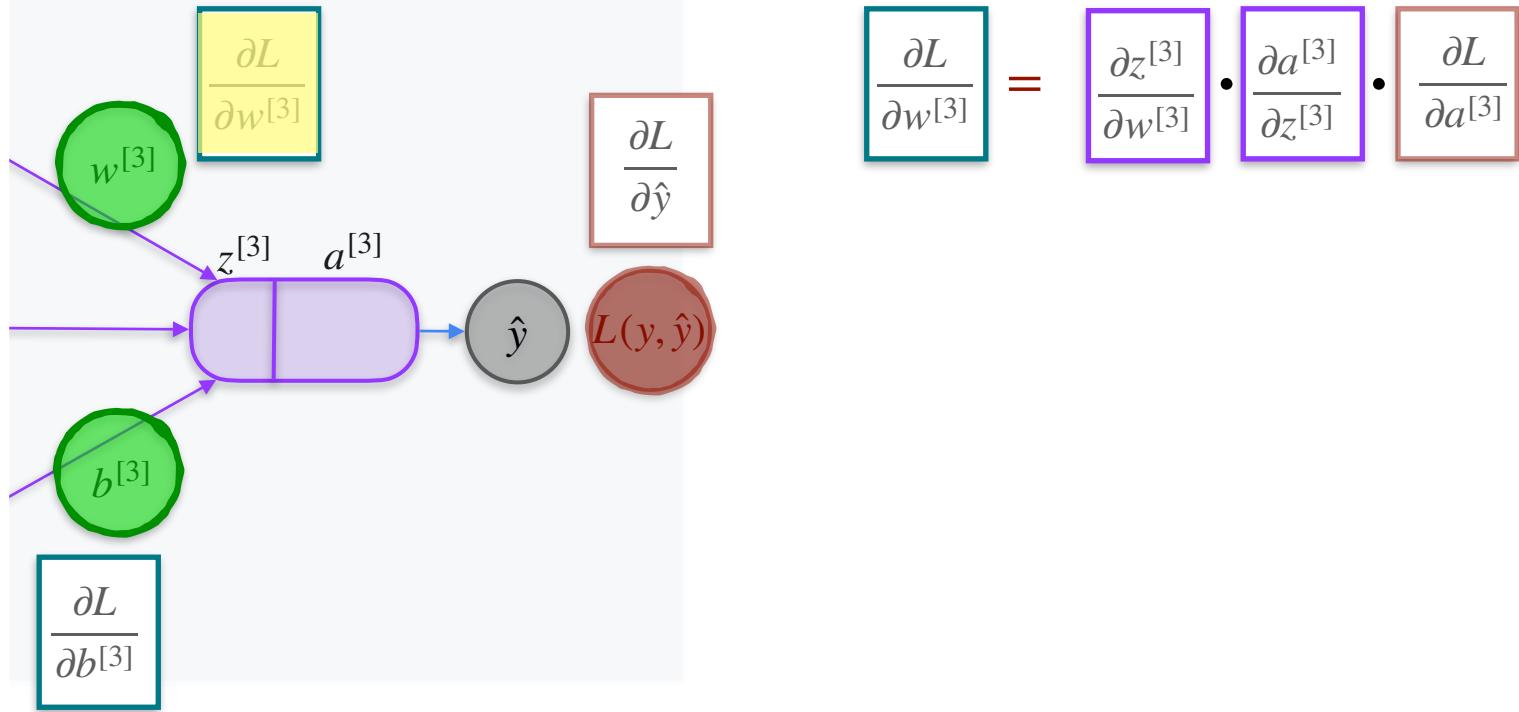
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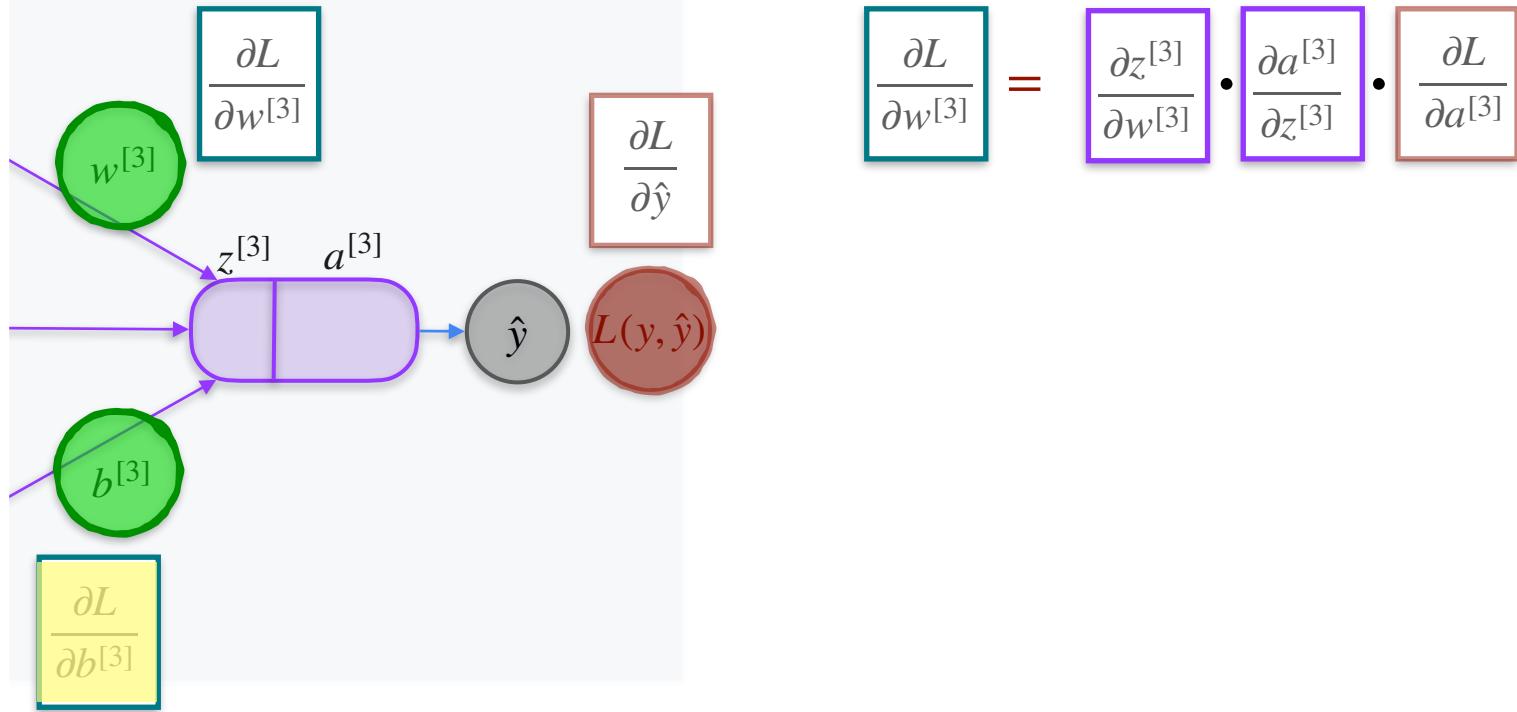
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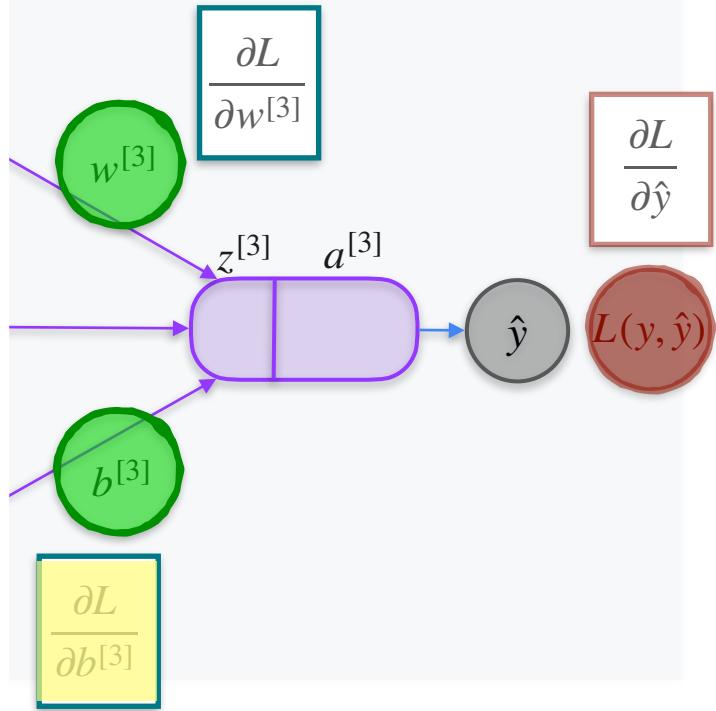
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Back Propagation Introduction

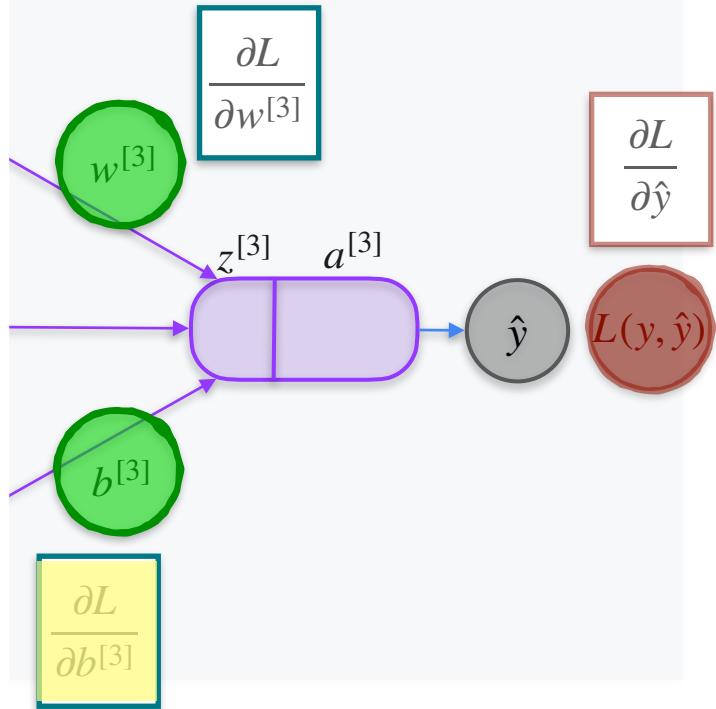


Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

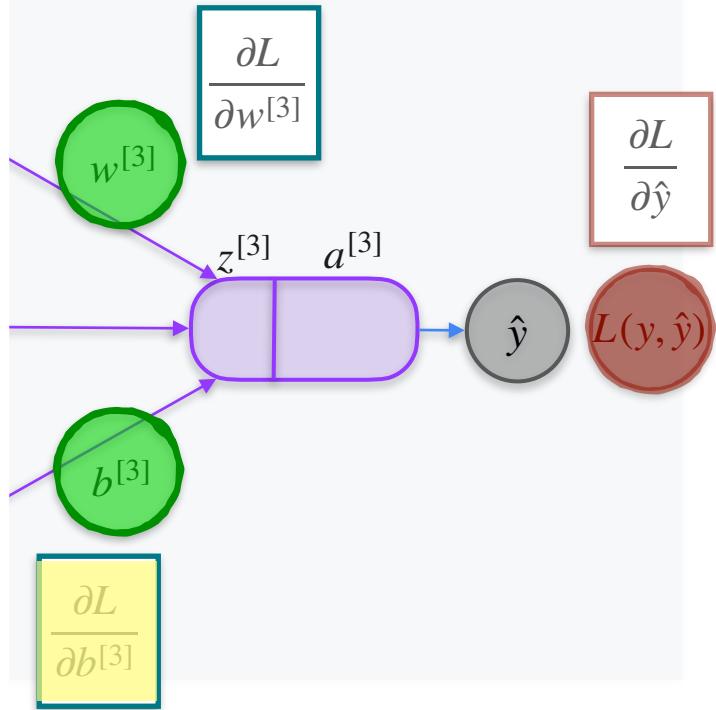
Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\begin{array}{c} \frac{\partial L}{\partial b^{[3]}} \\ \frac{\partial z^{[3]}}{\partial b^{[3]}} \\ \frac{\partial a^{[3]}}{\partial z^{[3]}} \\ \frac{\partial L}{\partial a^{[3]}} \end{array}$$

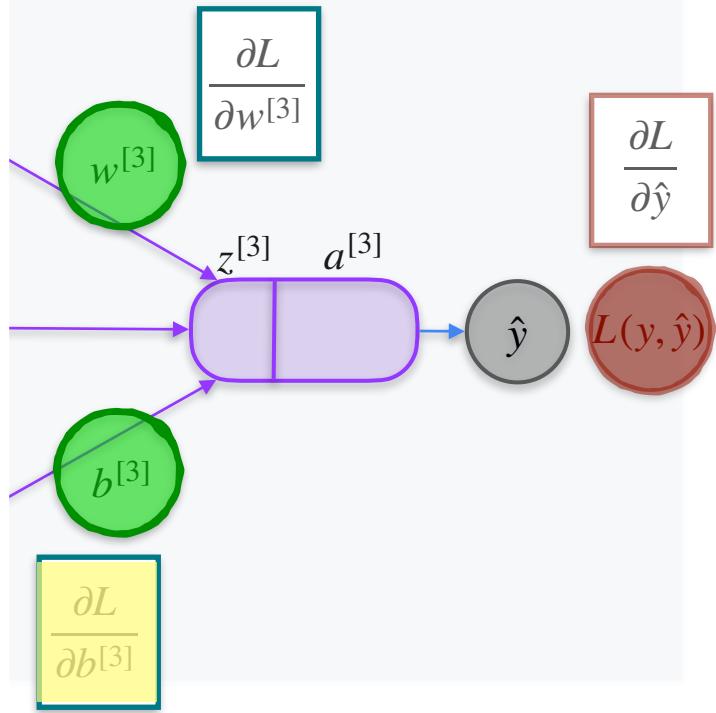
Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

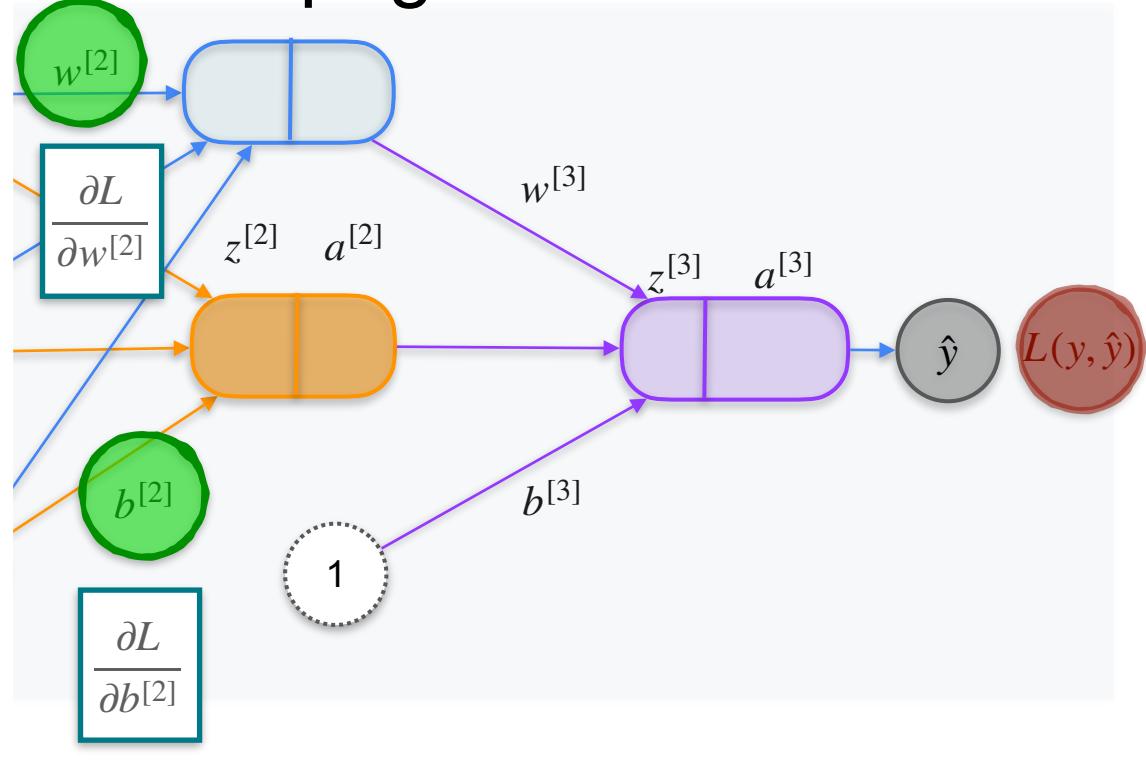


$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

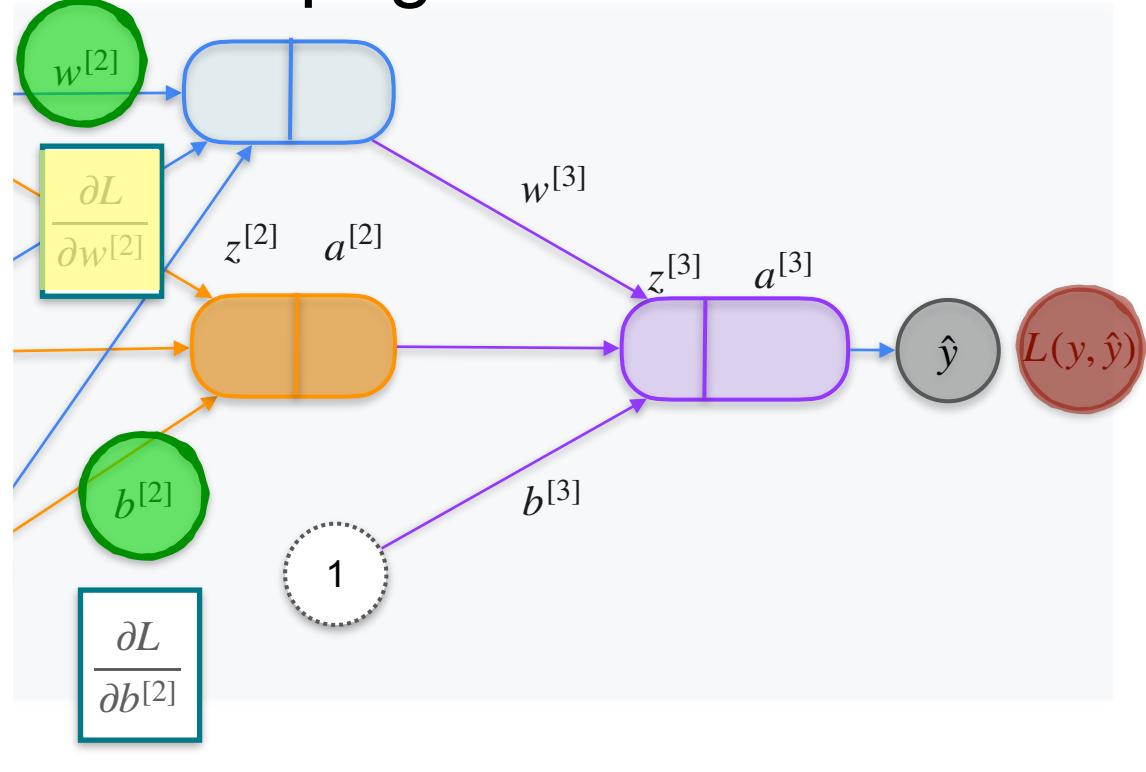
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction



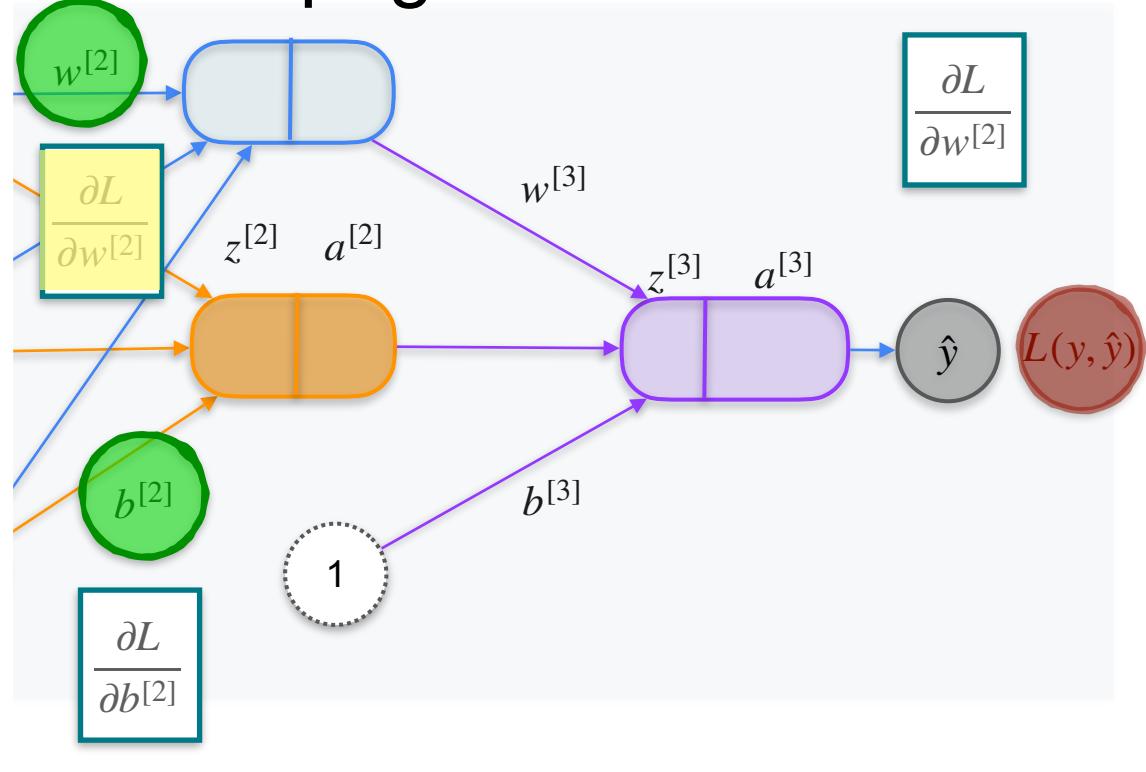
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction



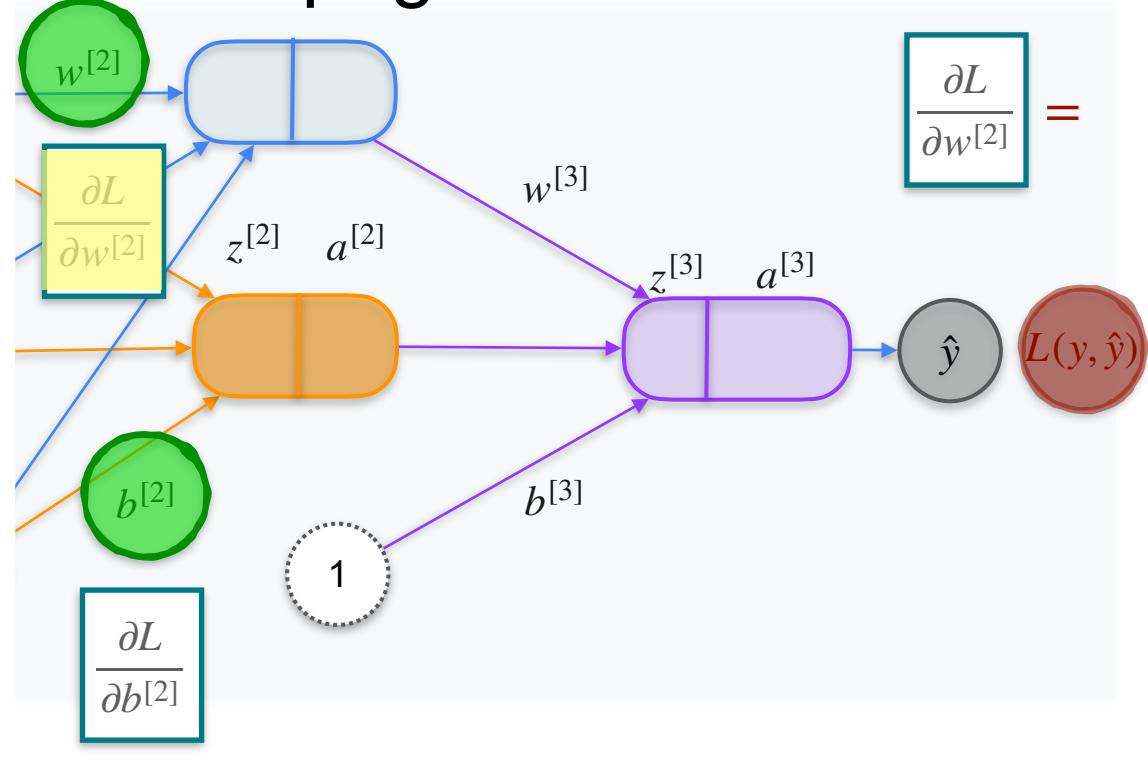
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

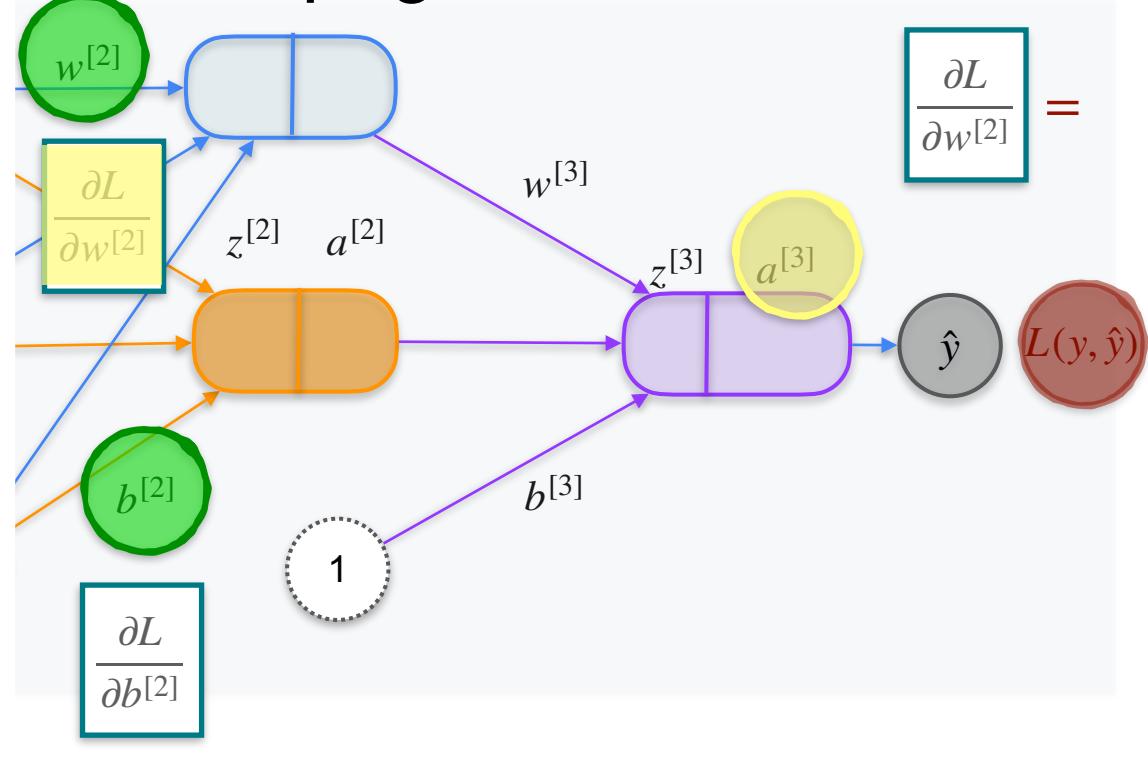


$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

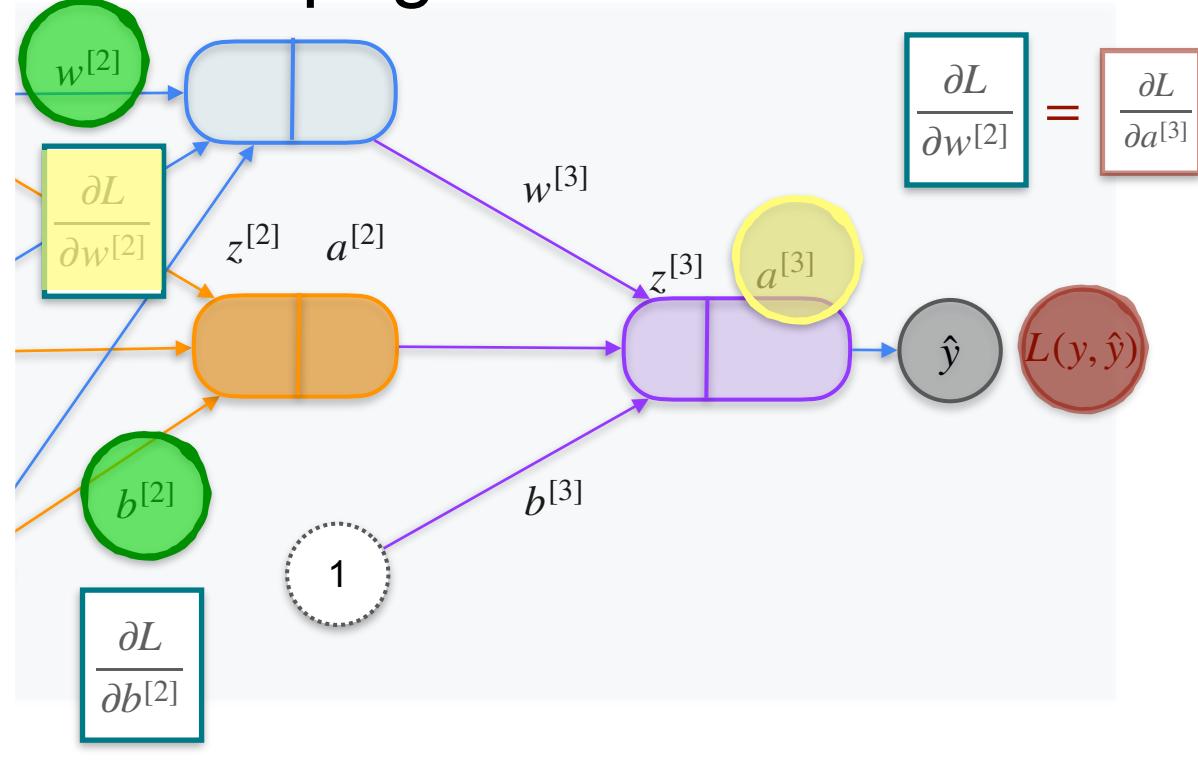


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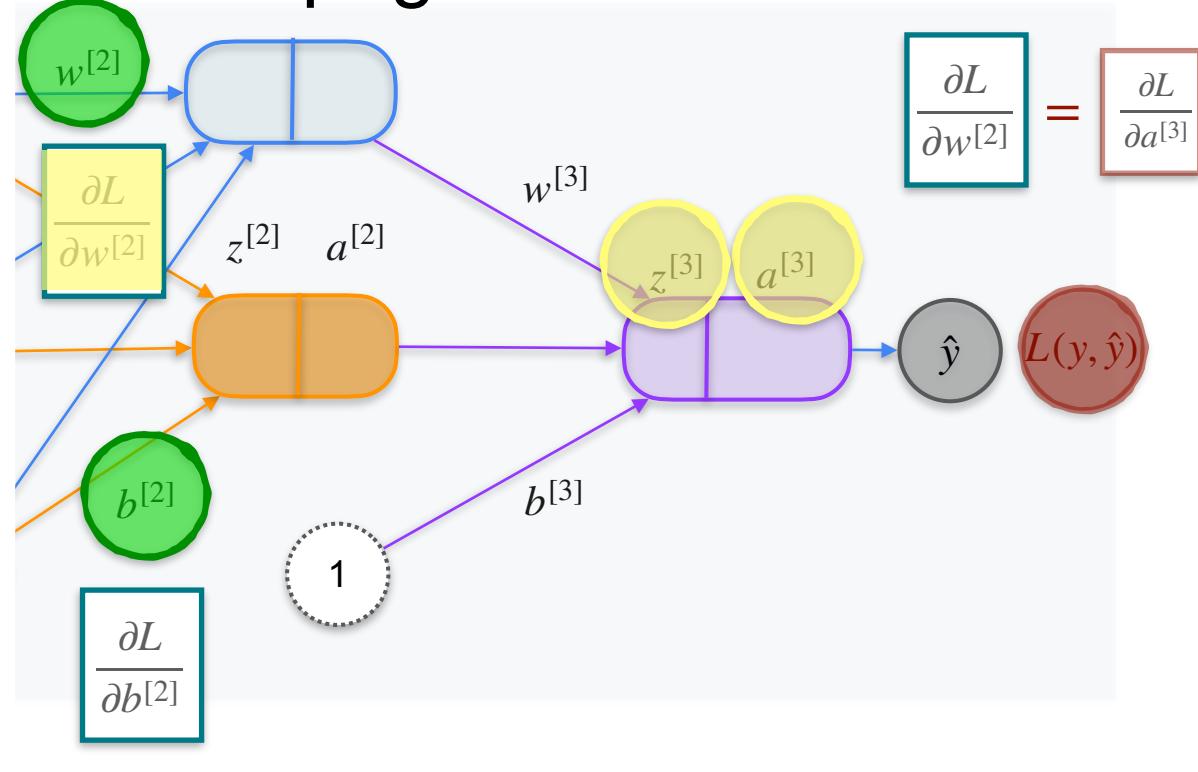
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction



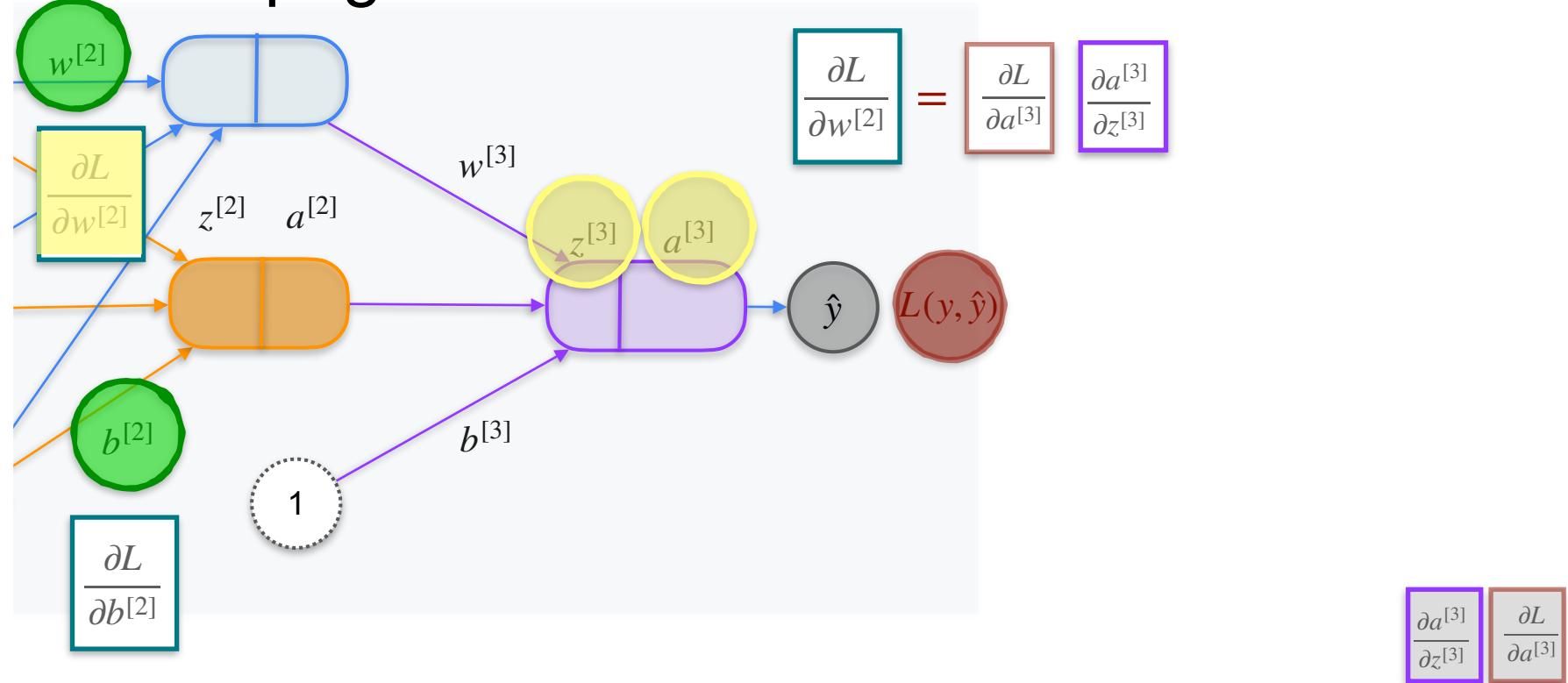
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

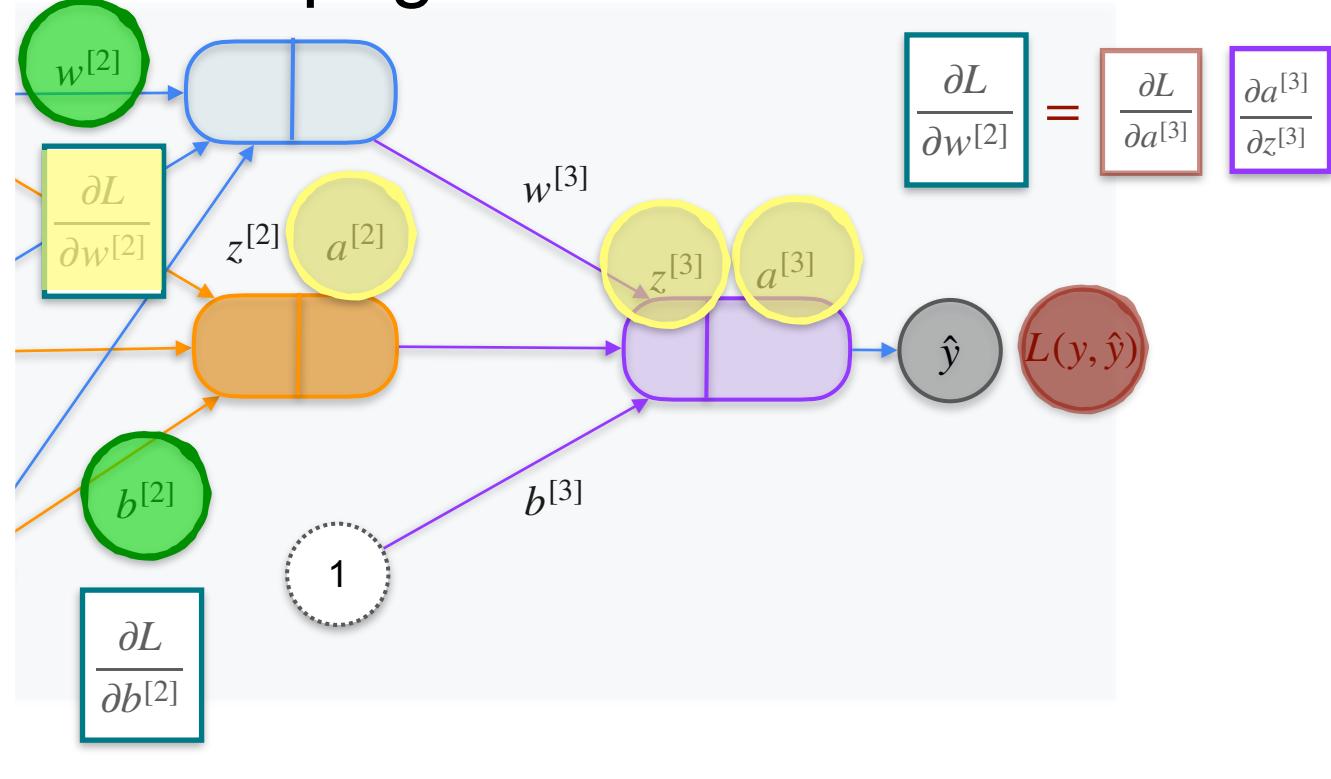


$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

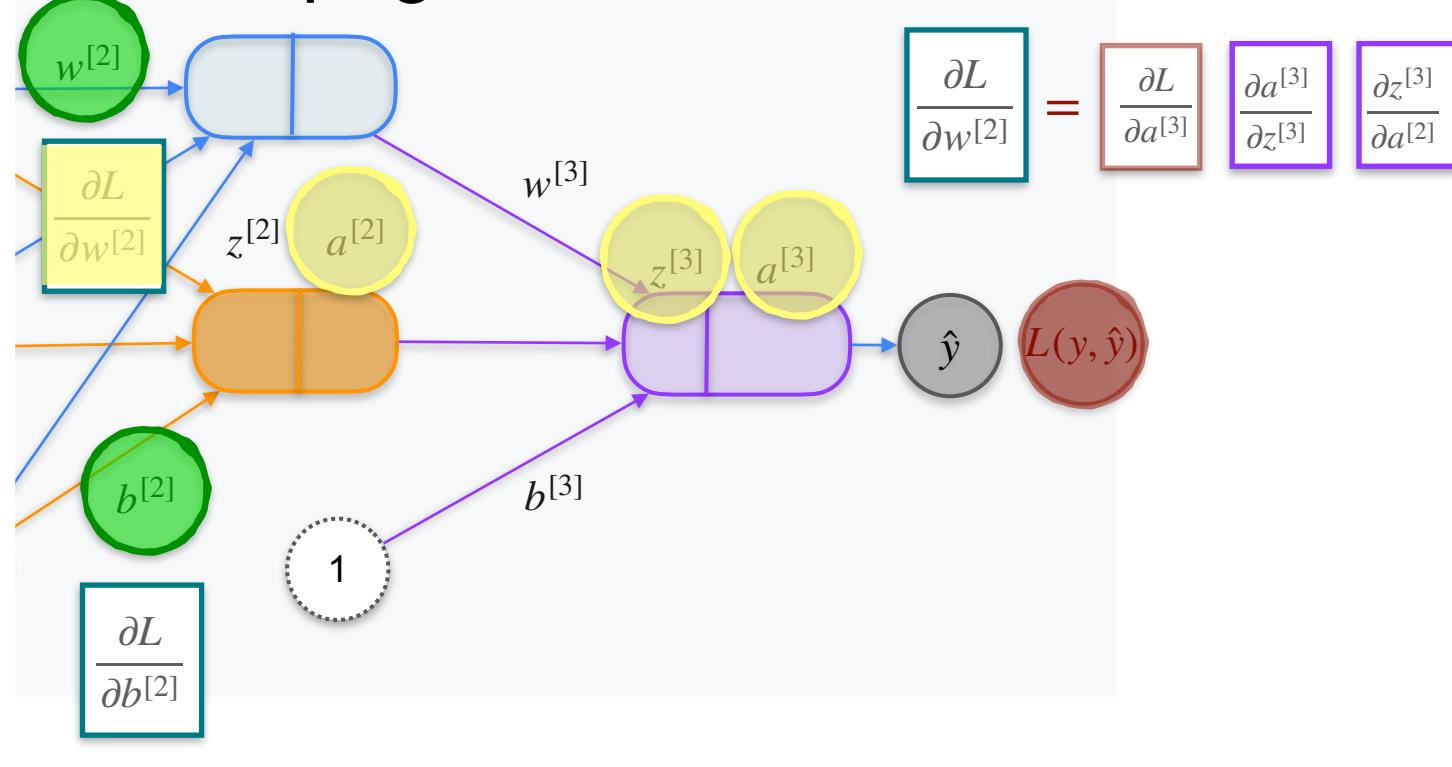
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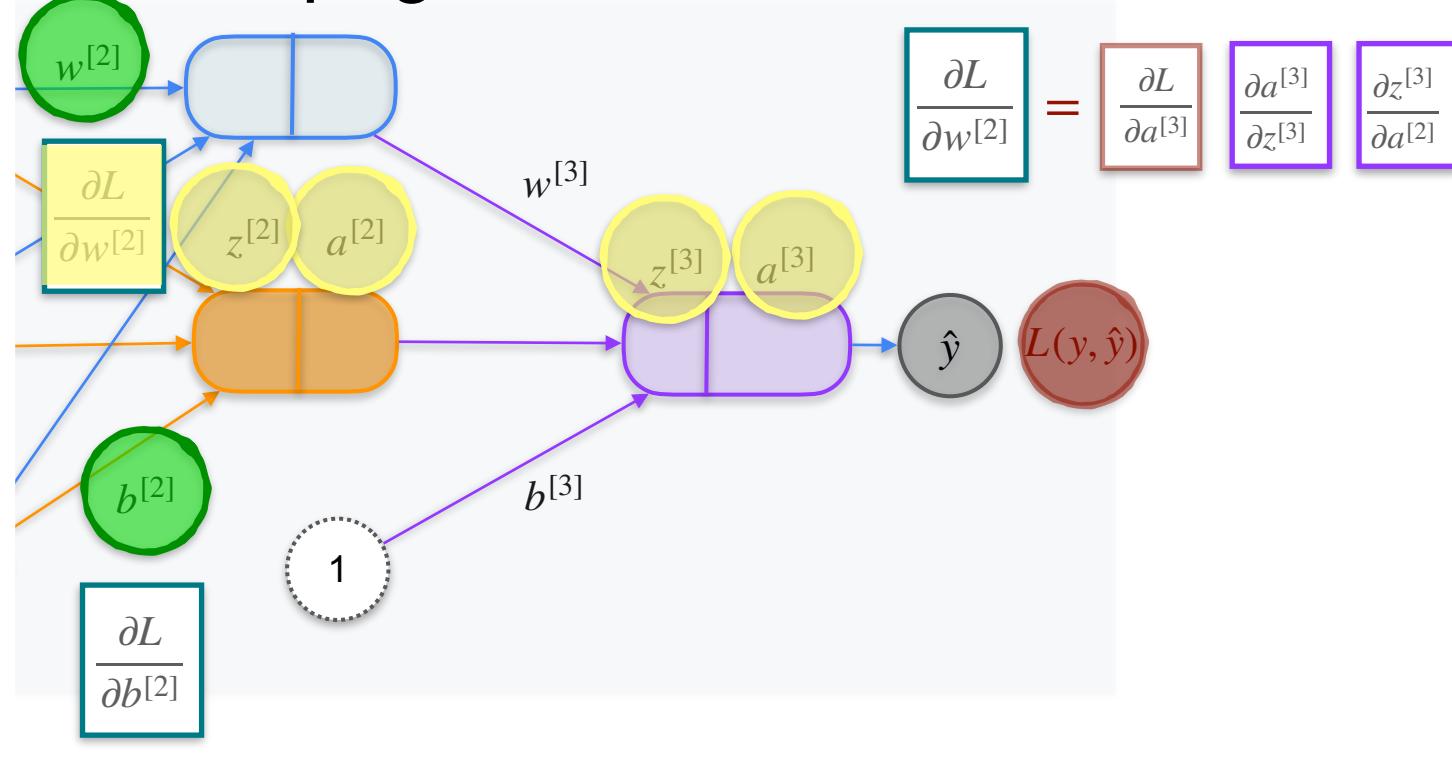
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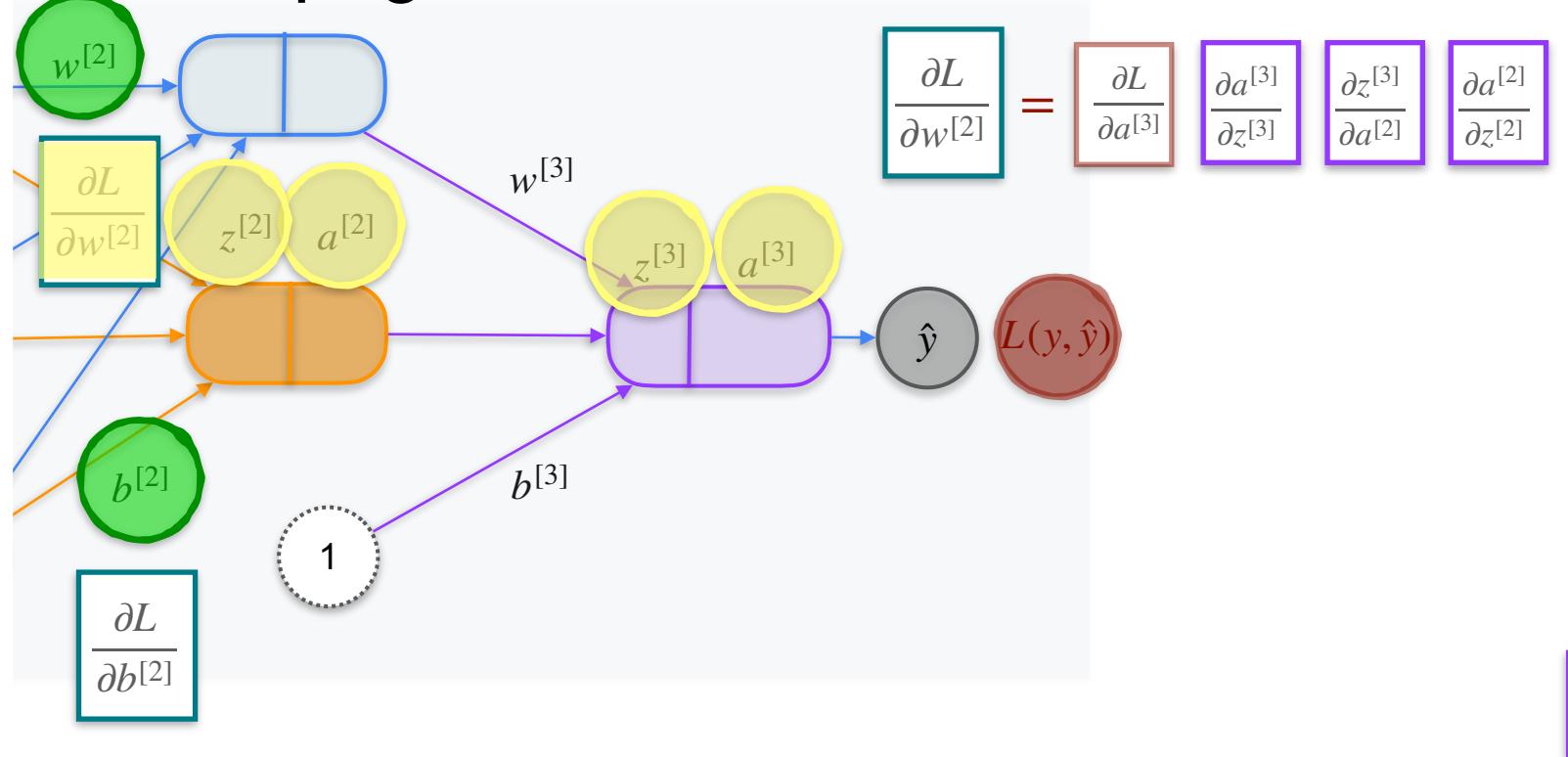


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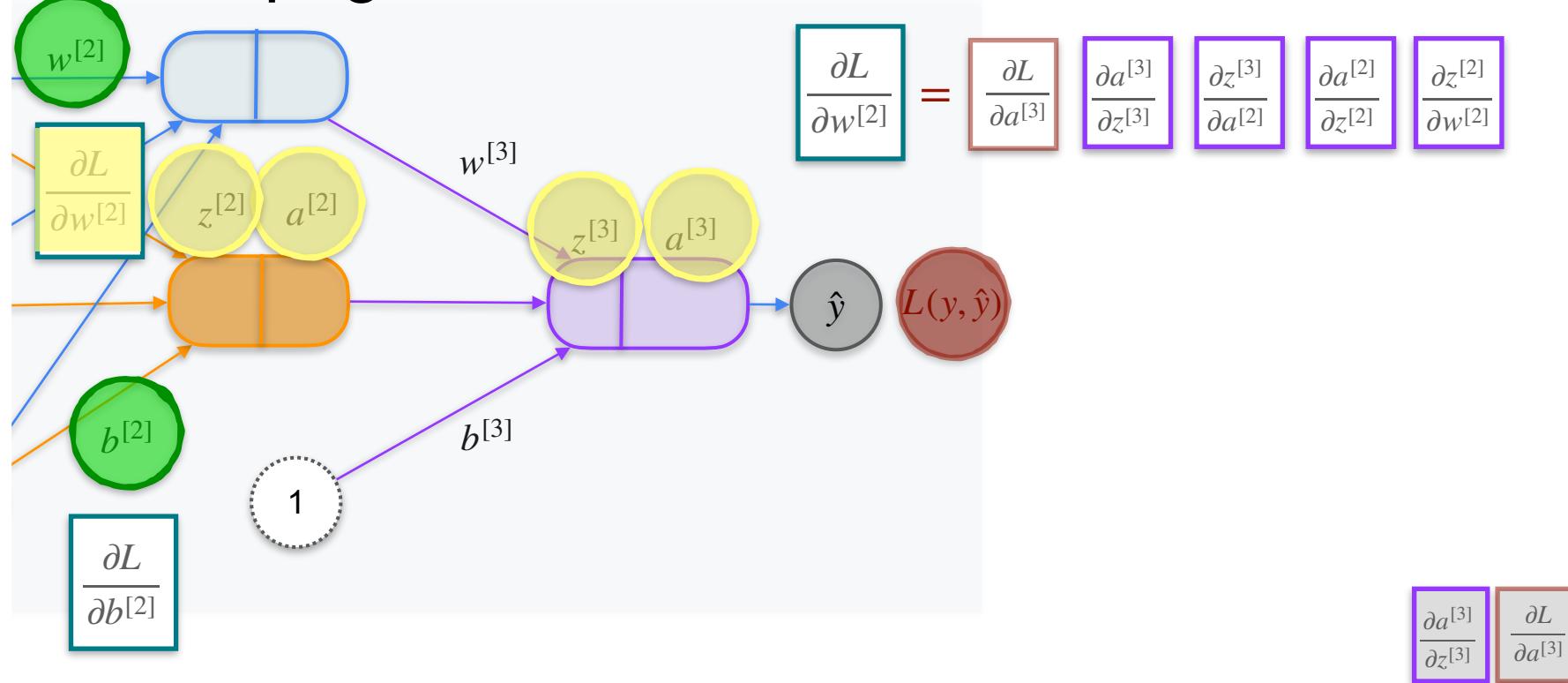


$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

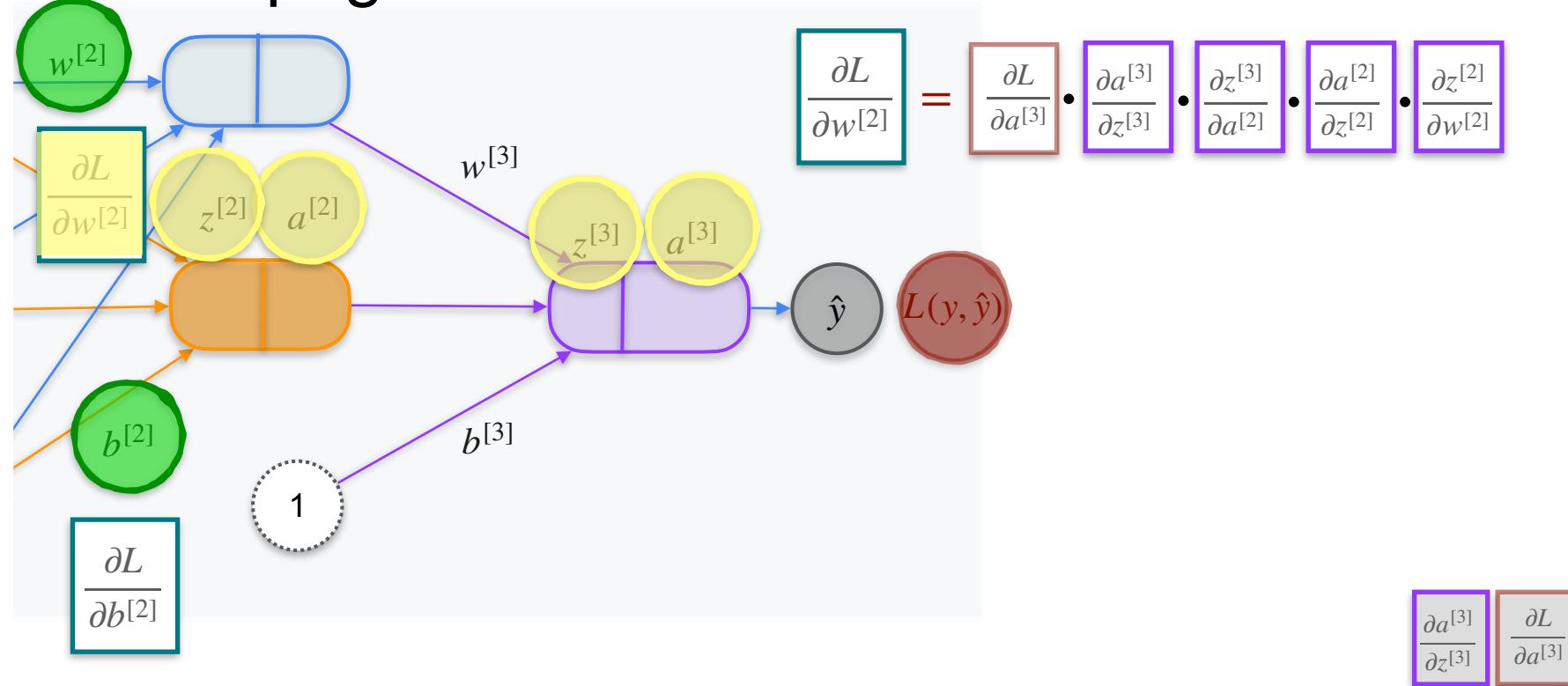
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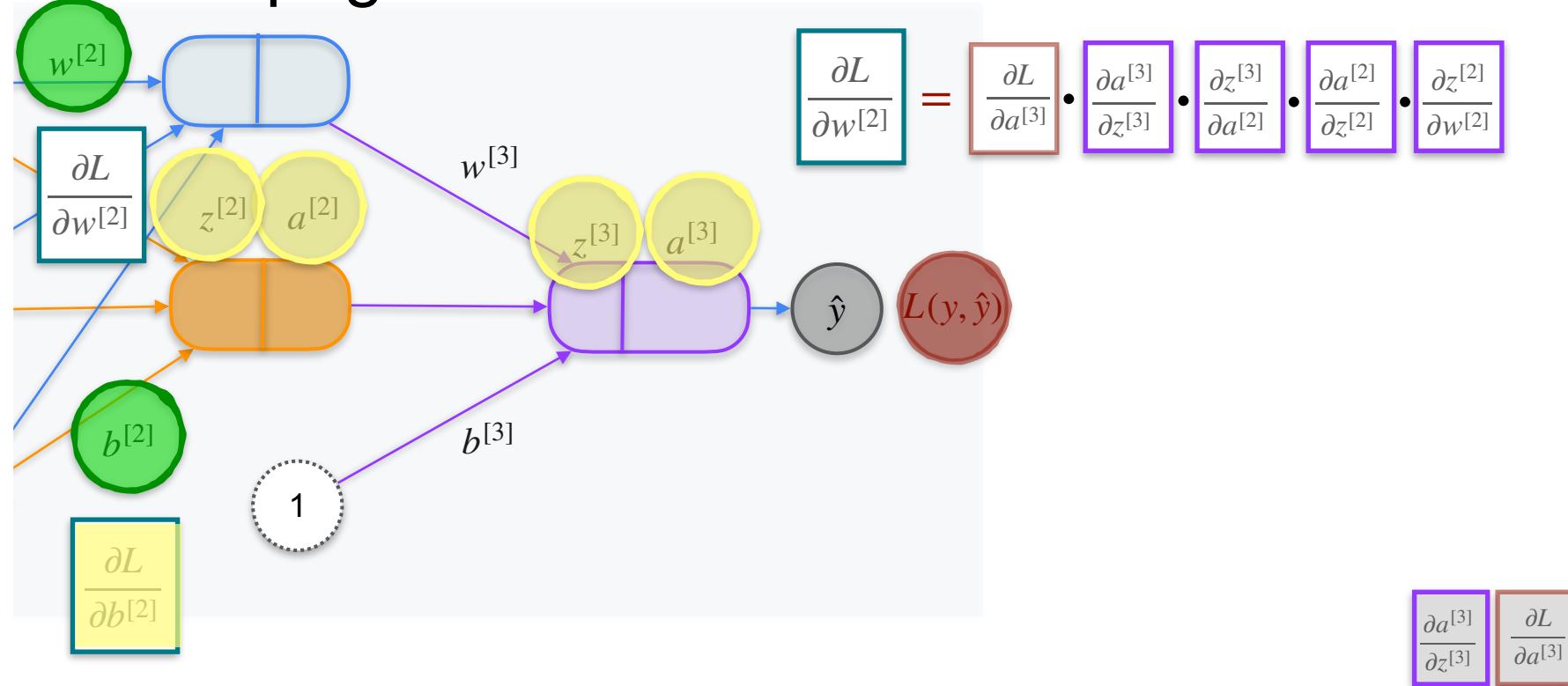
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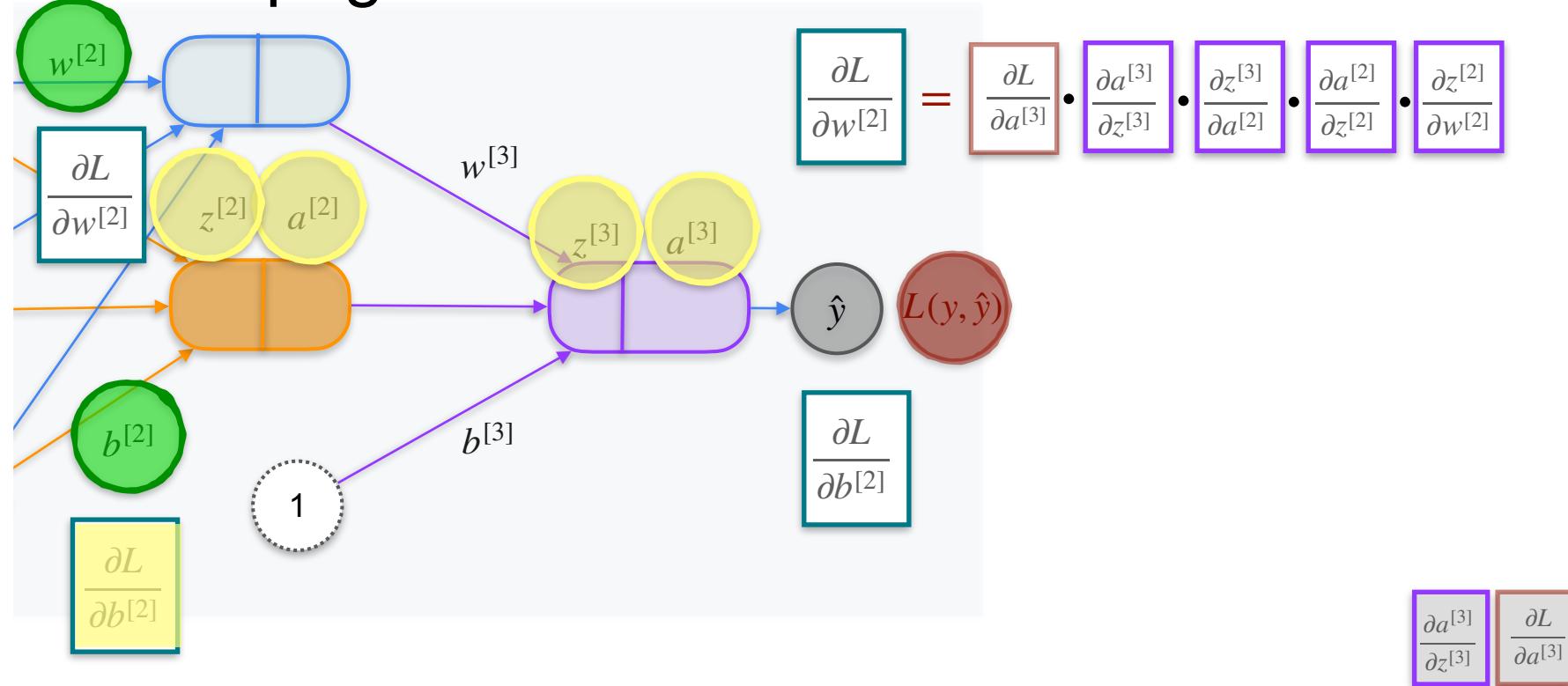
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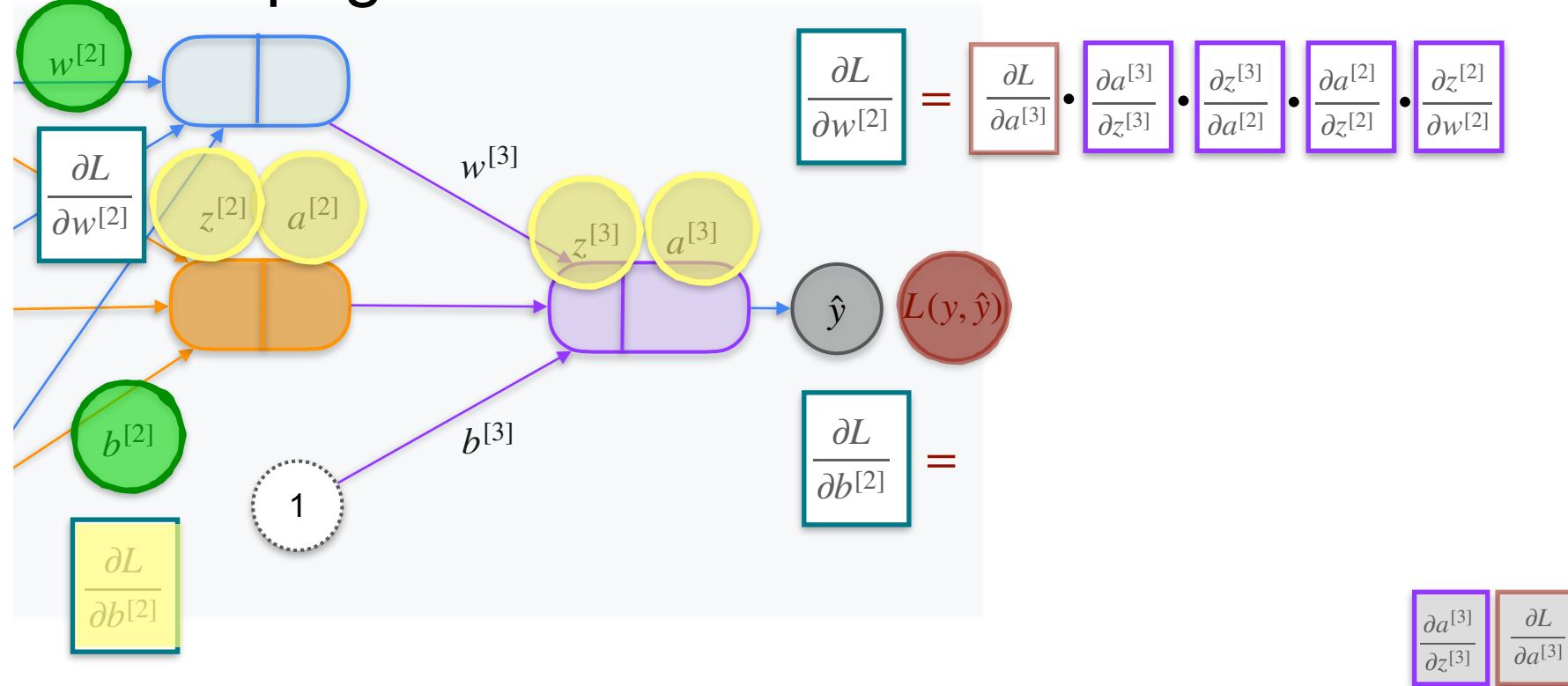
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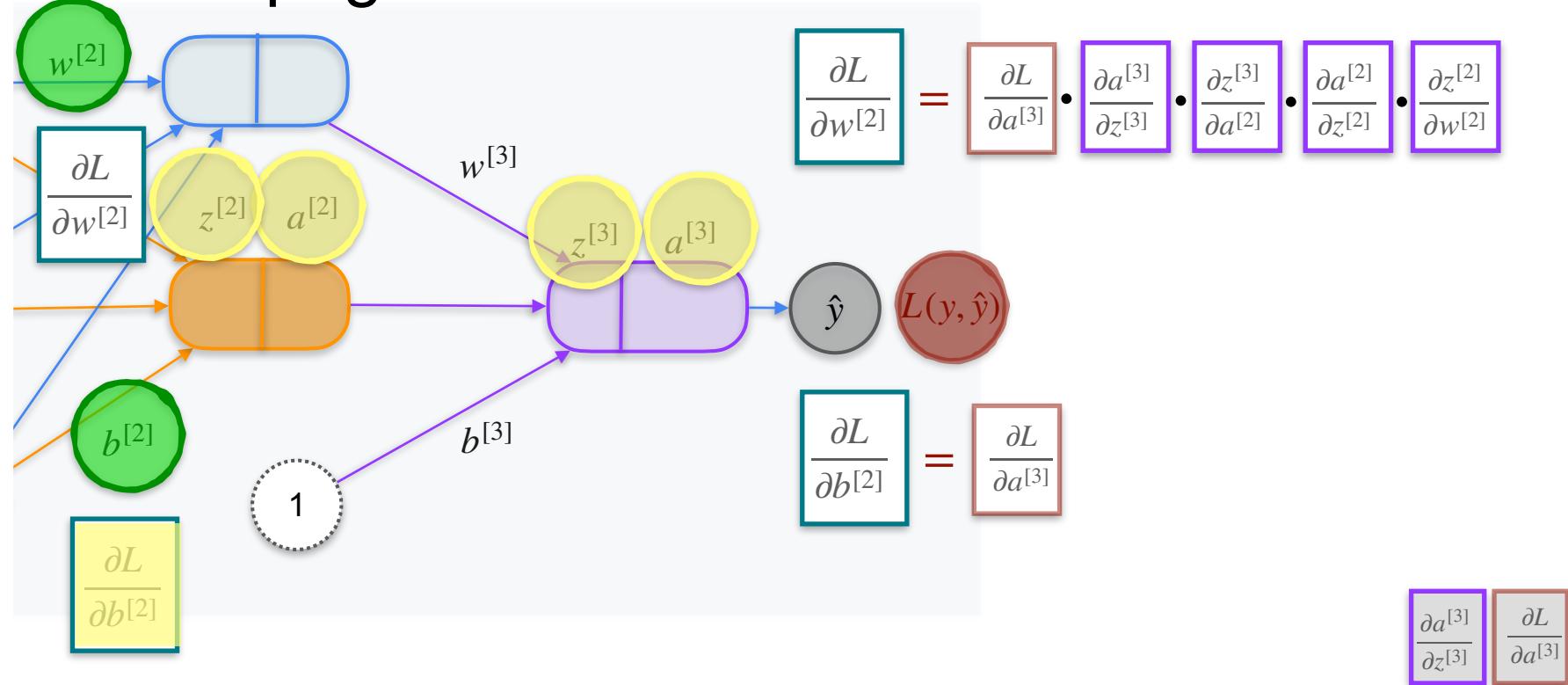
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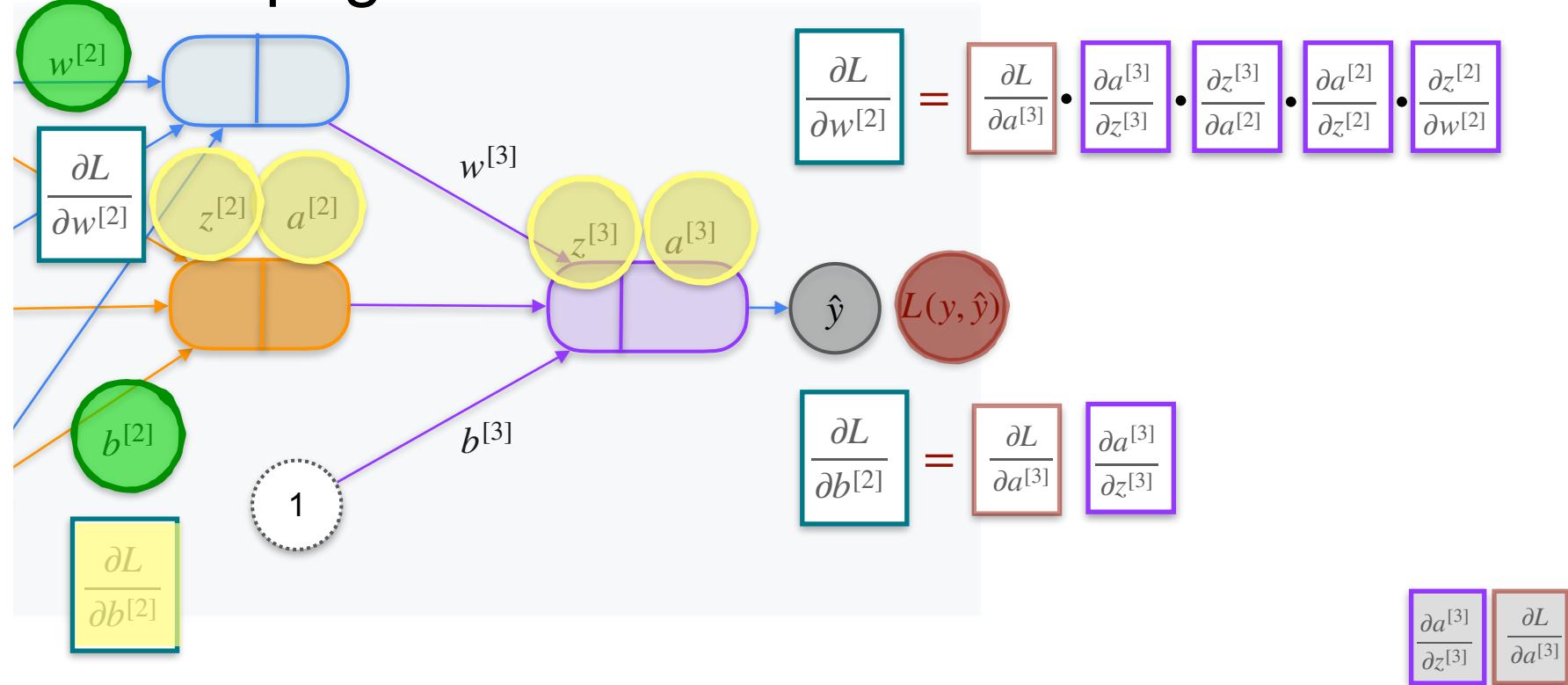
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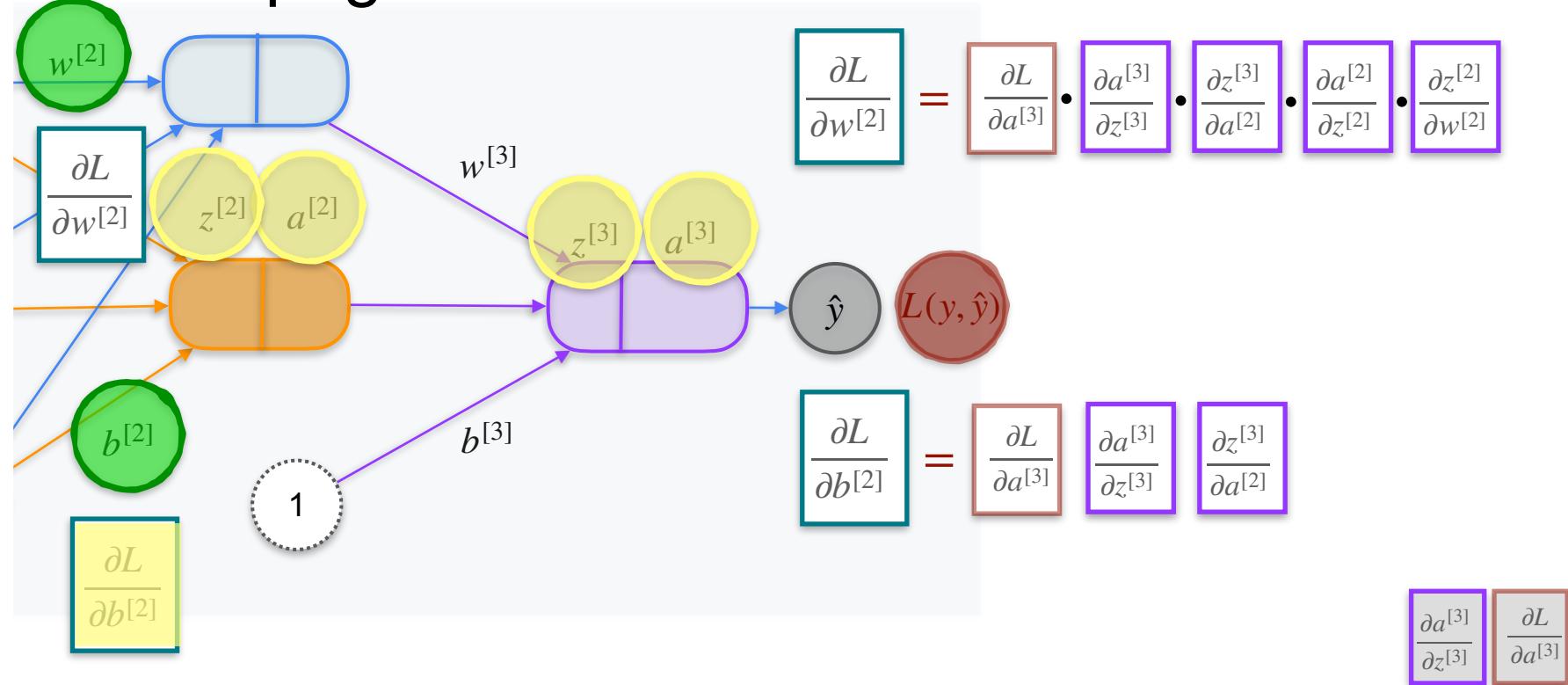
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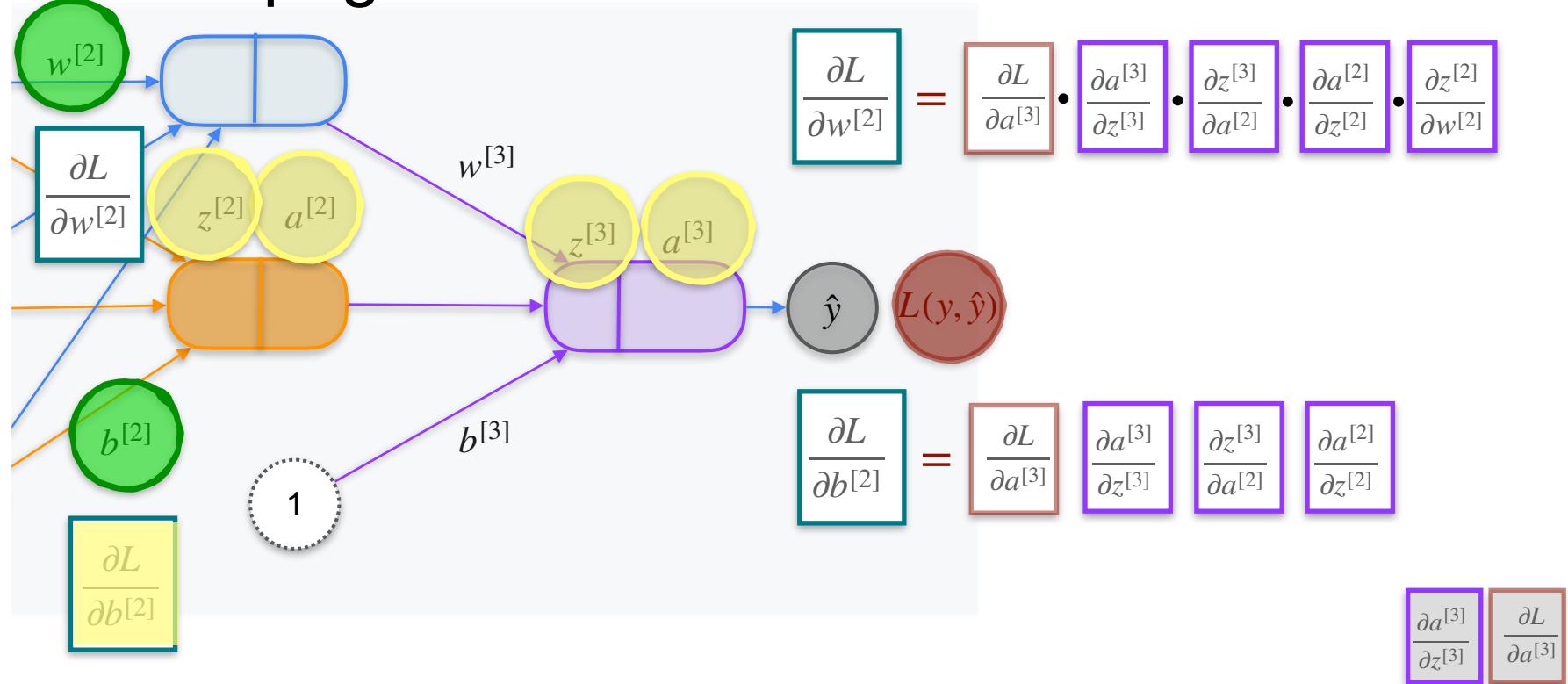
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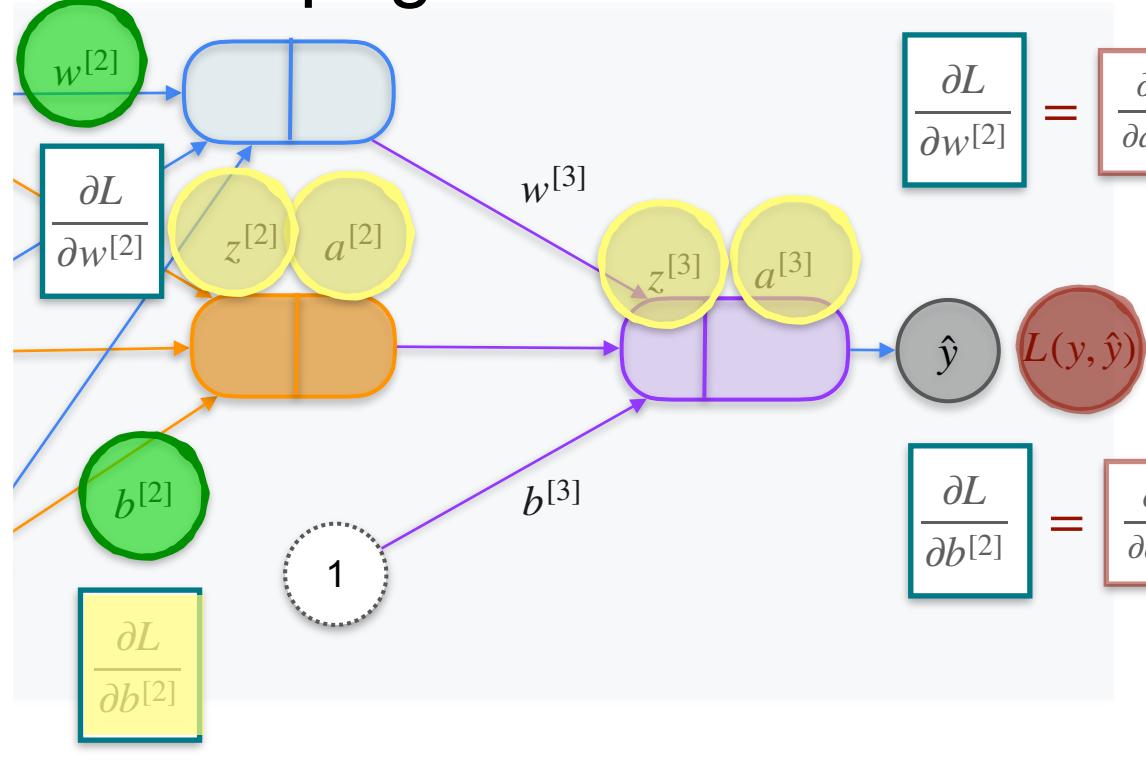
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Back Propagation Introduction

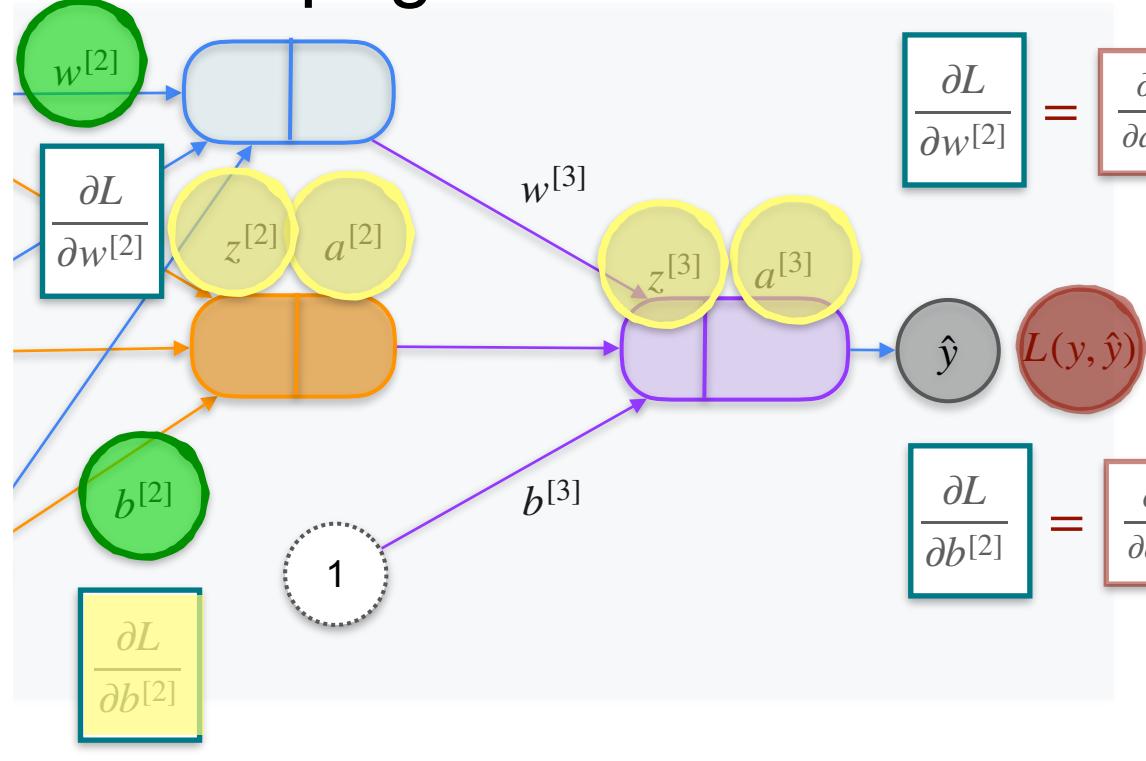


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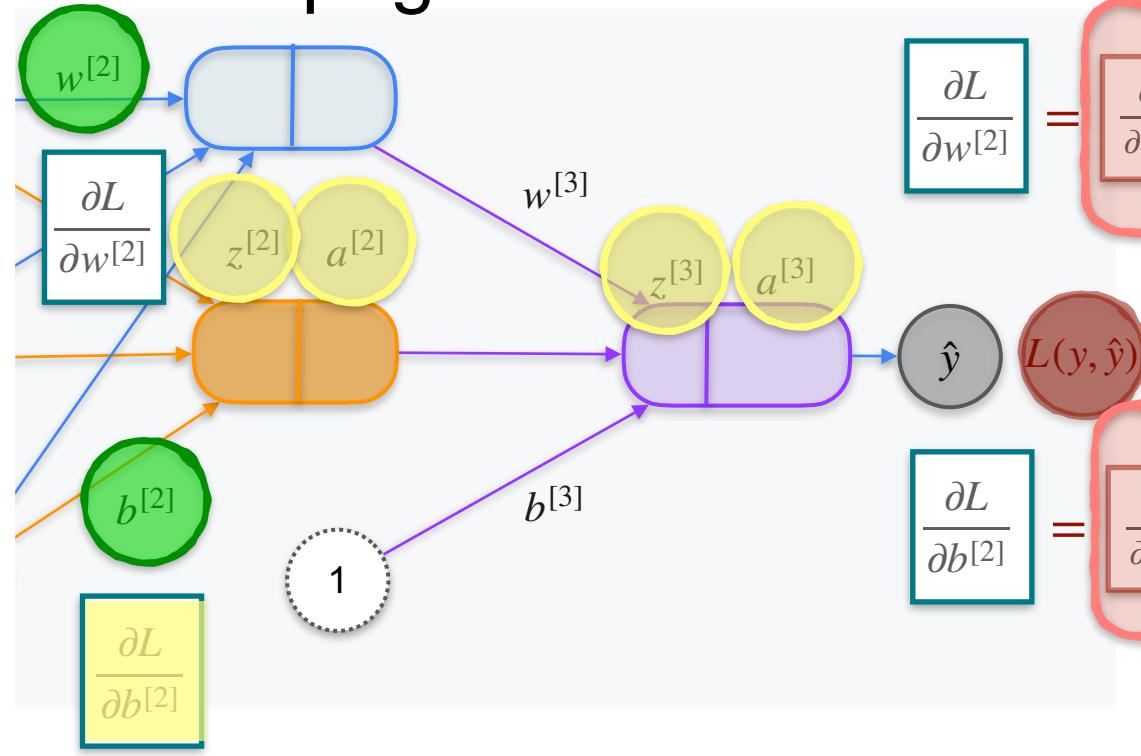
$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction



$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

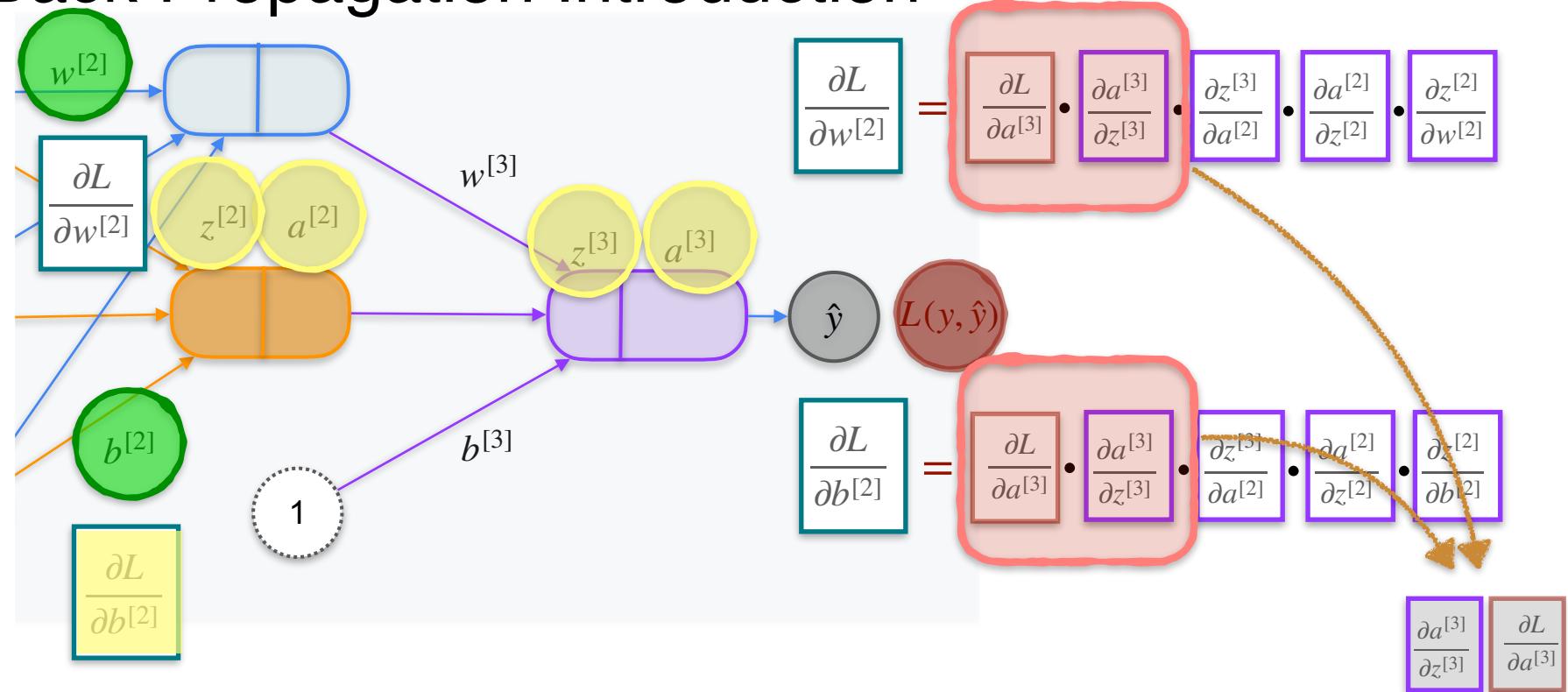


$$\frac{\partial L}{\partial w^{[2]}} = \boxed{\frac{\partial L}{\partial a^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial z^{[3]}}{\partial a^{[2]}}} \cdot \boxed{\frac{\partial a^{[2]}}{\partial z^{[2]}}} \cdot \boxed{\frac{\partial z^{[2]}}{\partial w^{[2]}}}$$

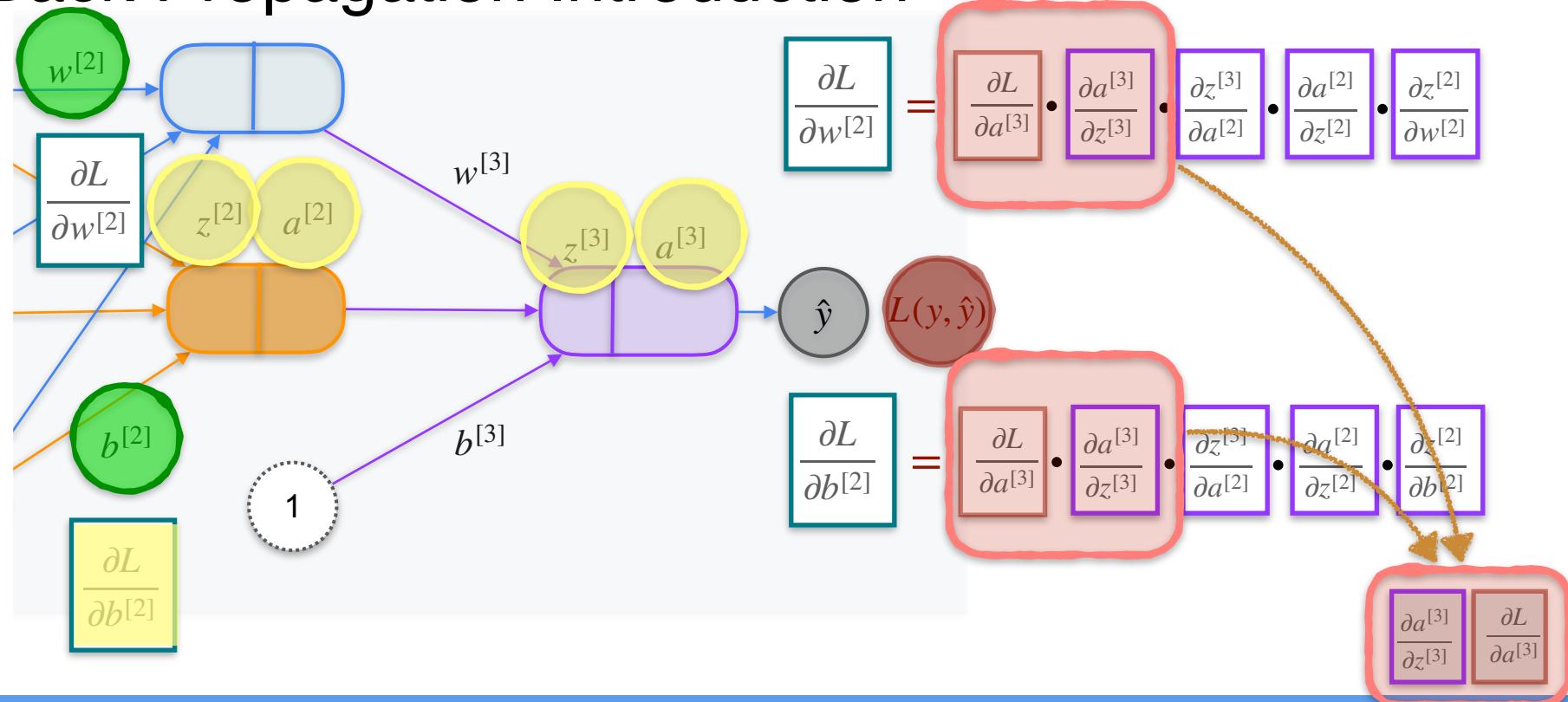
$$\frac{\partial L}{\partial b^{[2]}} = \boxed{\frac{\partial L}{\partial a^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial z^{[3]}}{\partial a^{[2]}}} \cdot \boxed{\frac{\partial a^{[2]}}{\partial z^{[2]}}} \cdot \boxed{\frac{\partial z^{[2]}}{\partial b^{[2]}}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

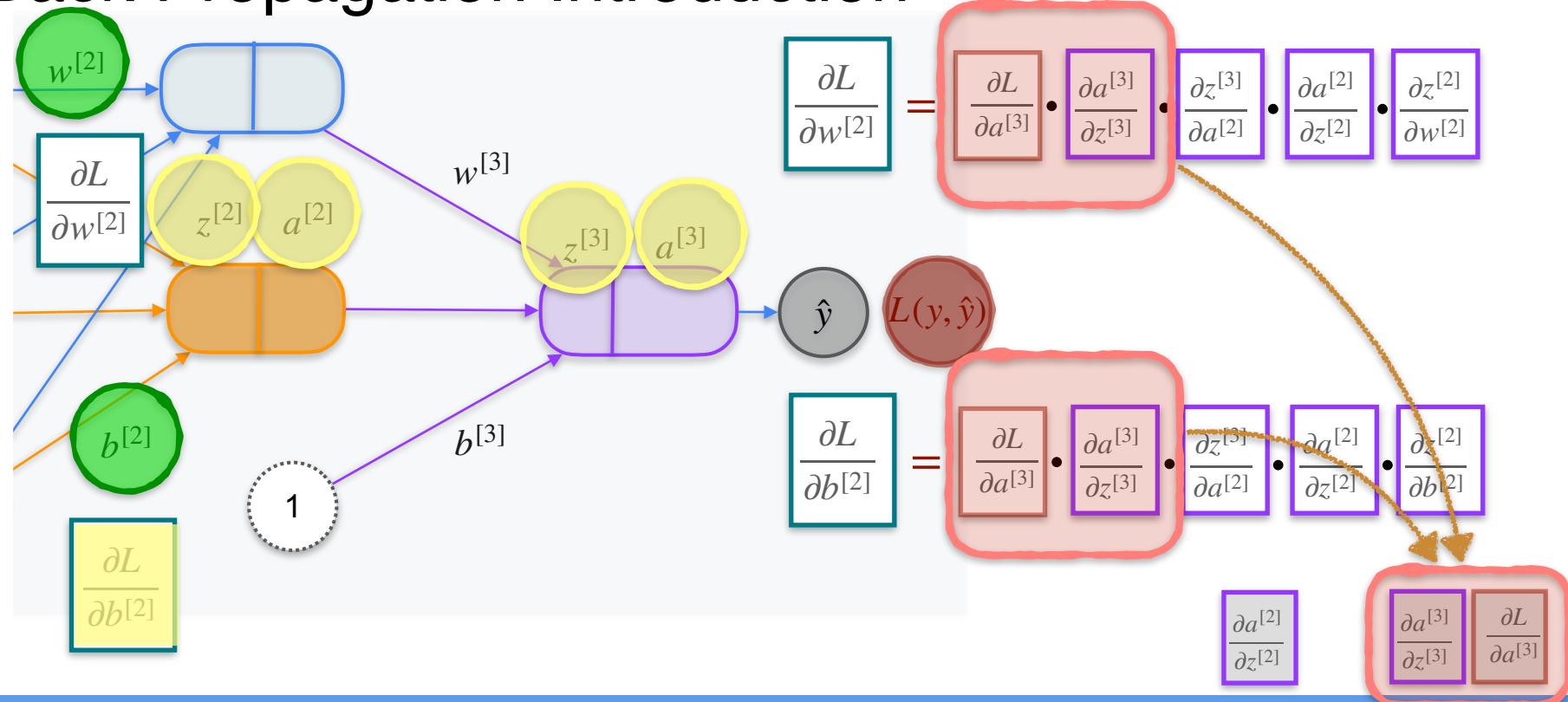
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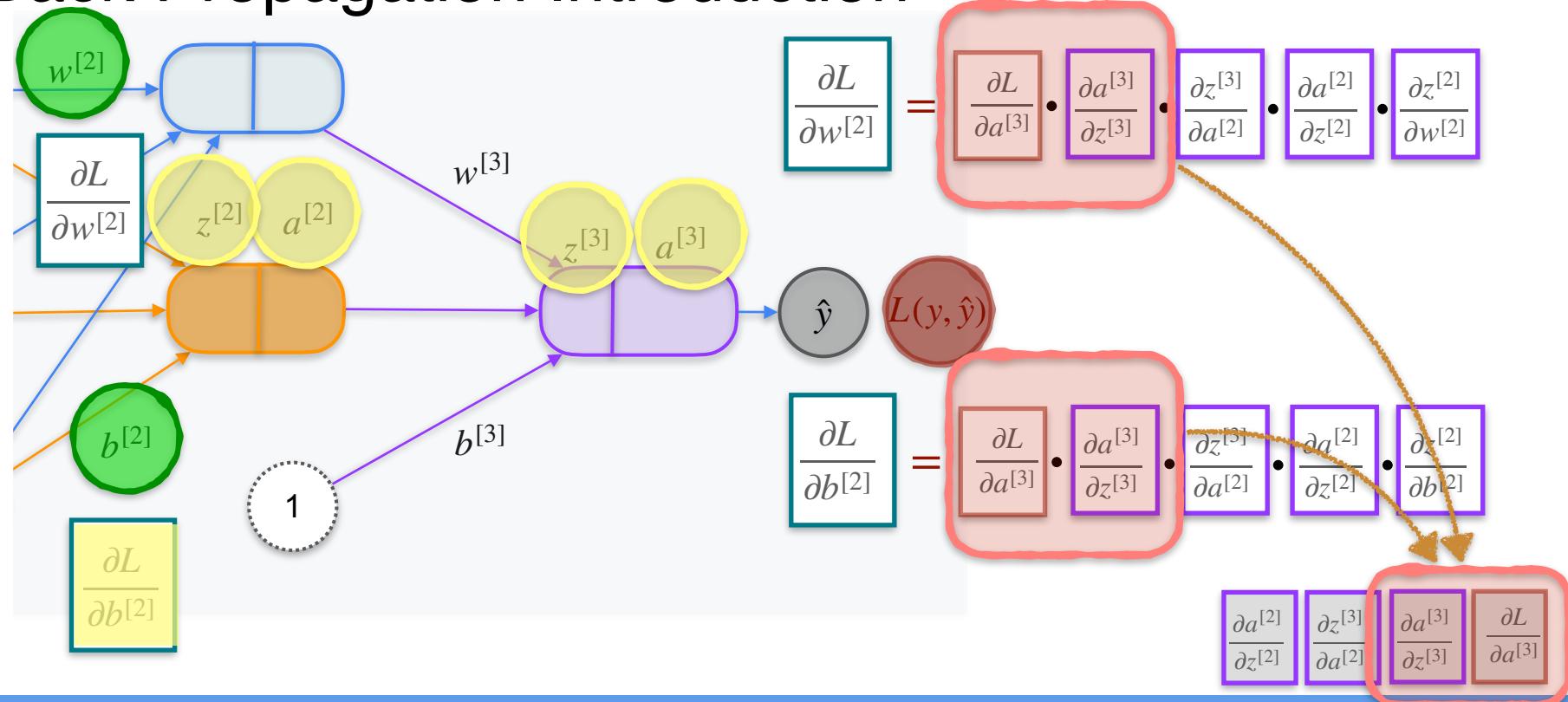
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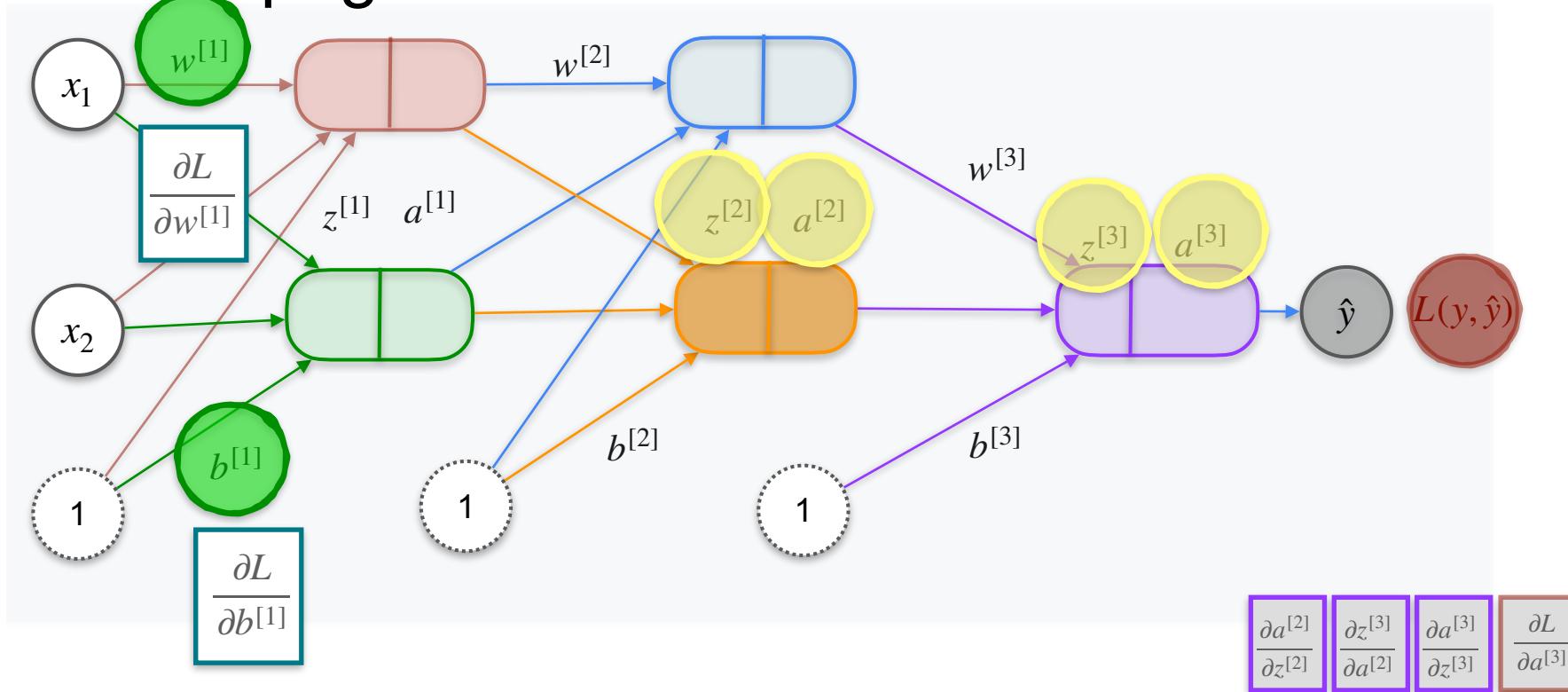
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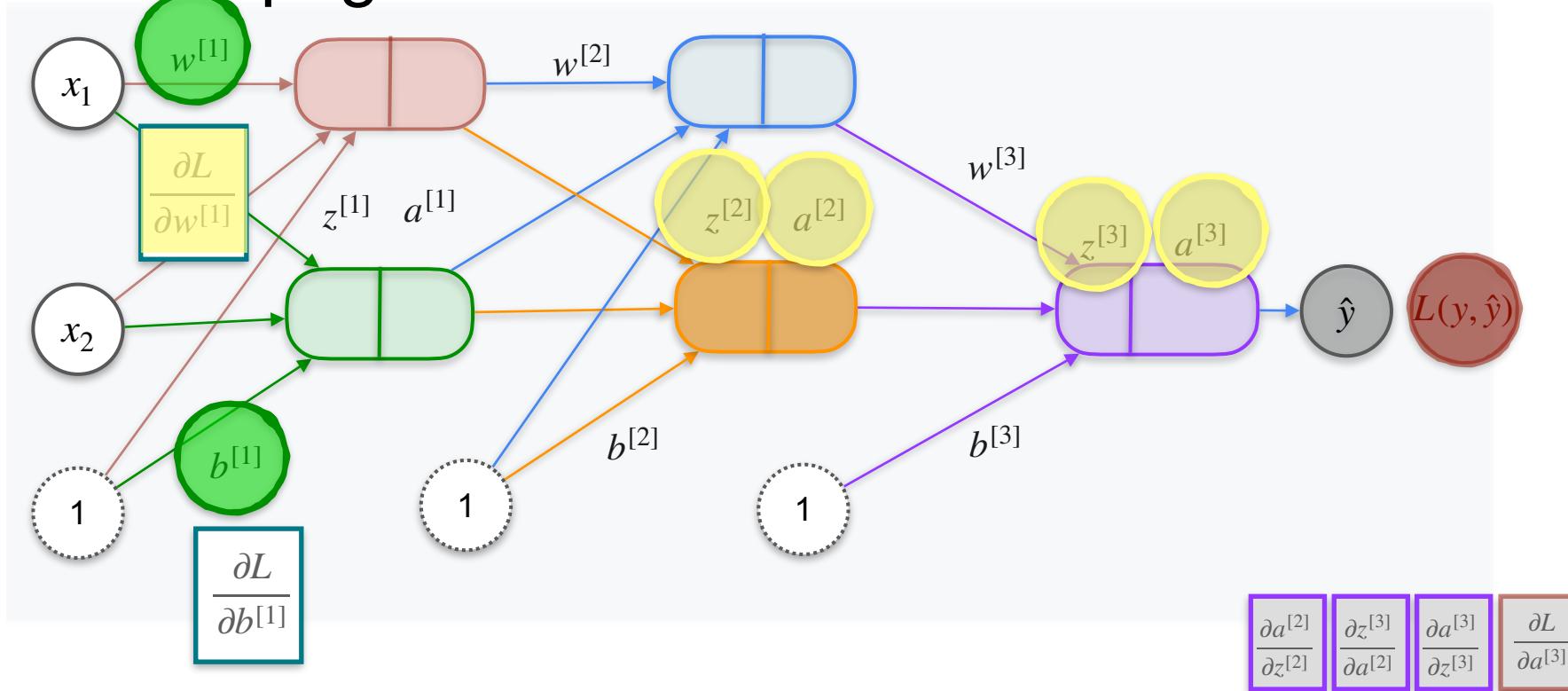
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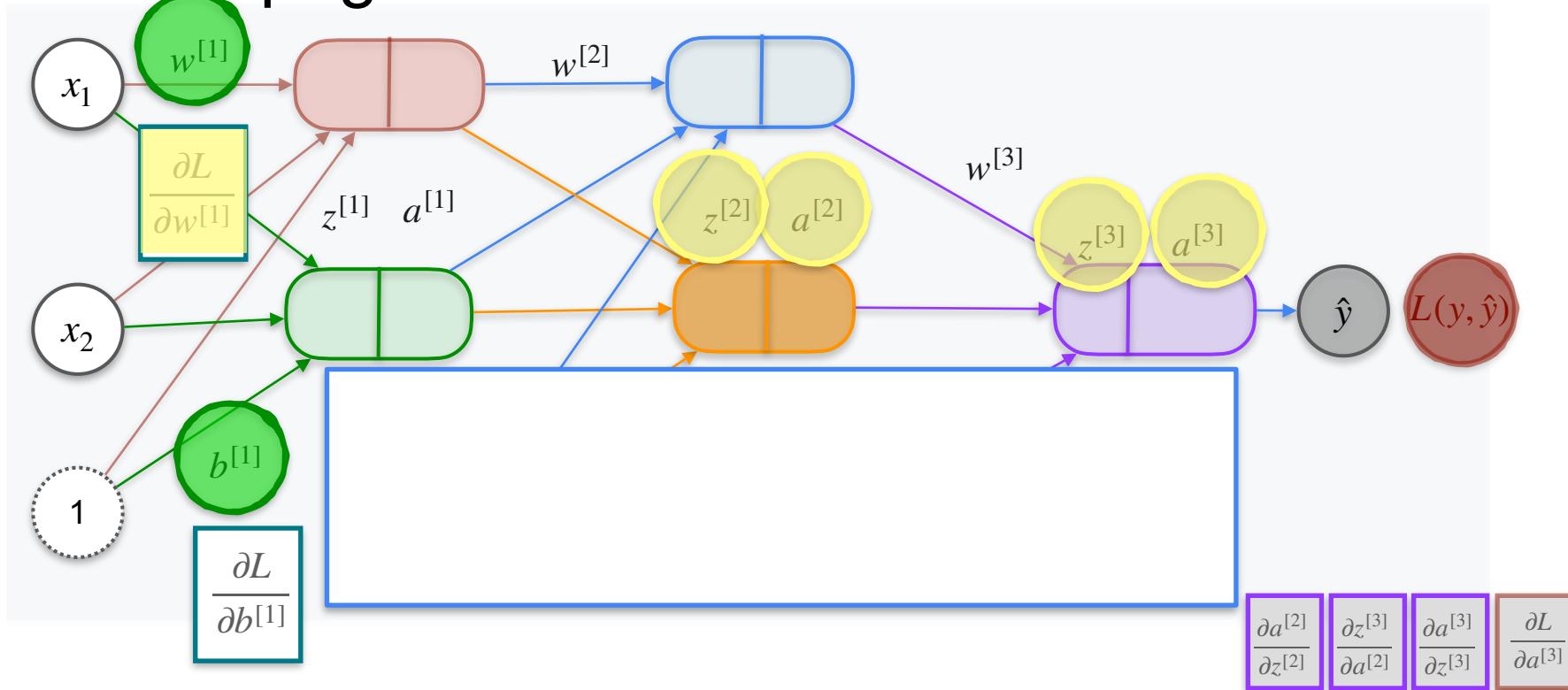
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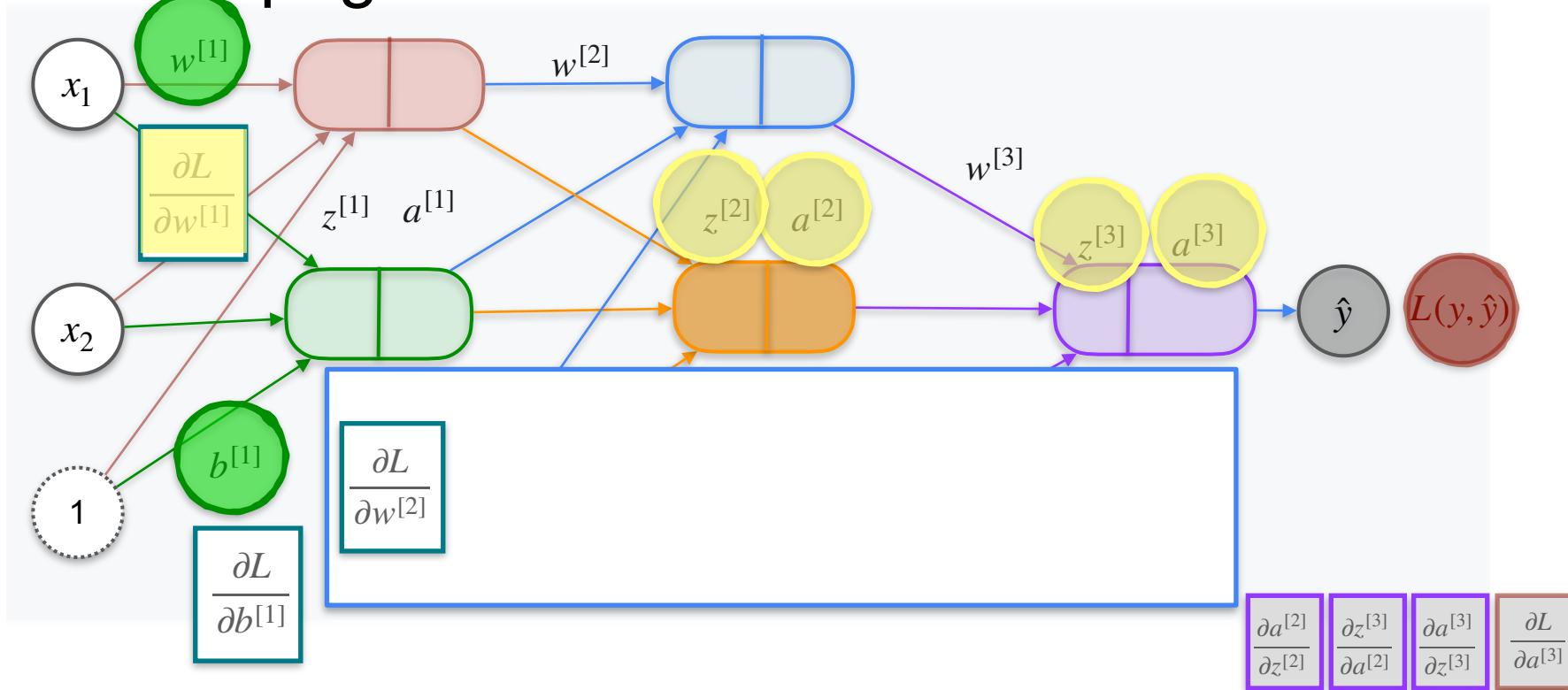
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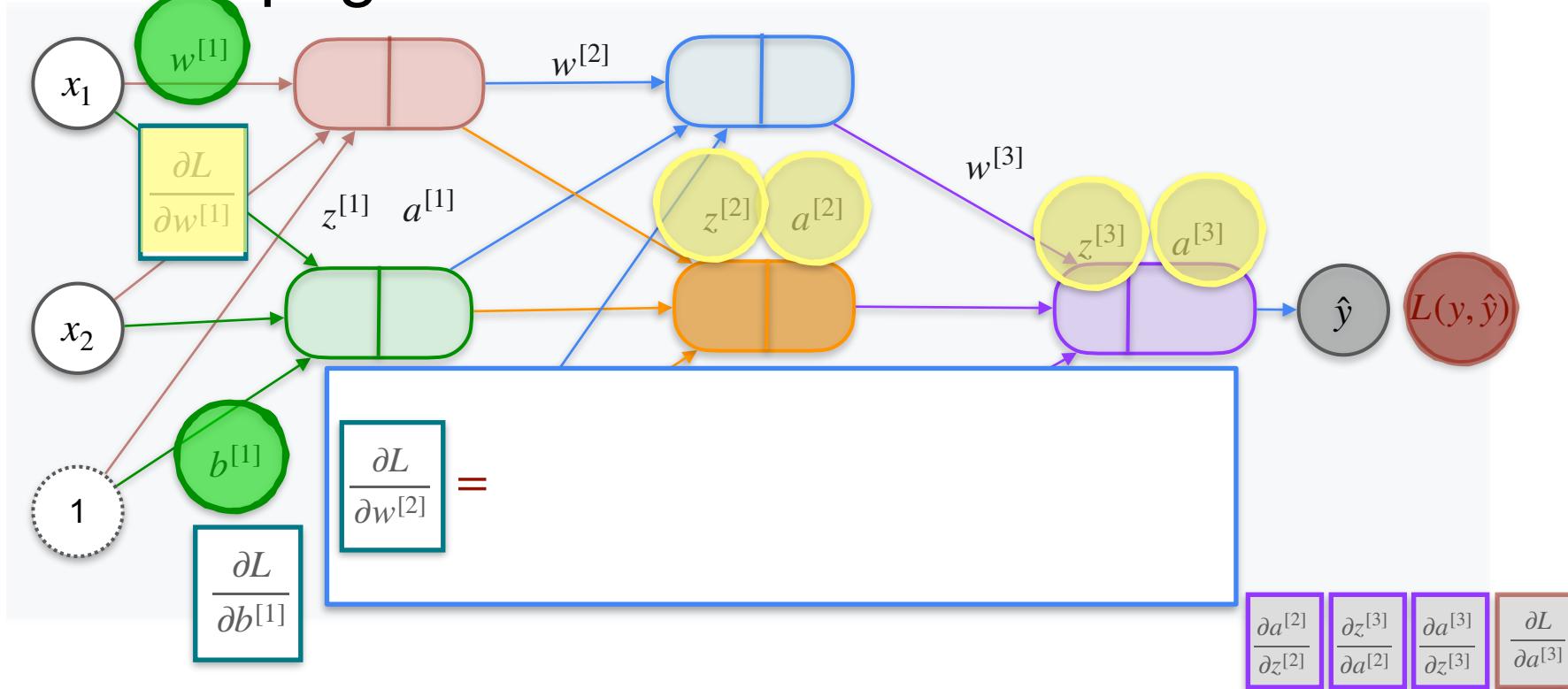
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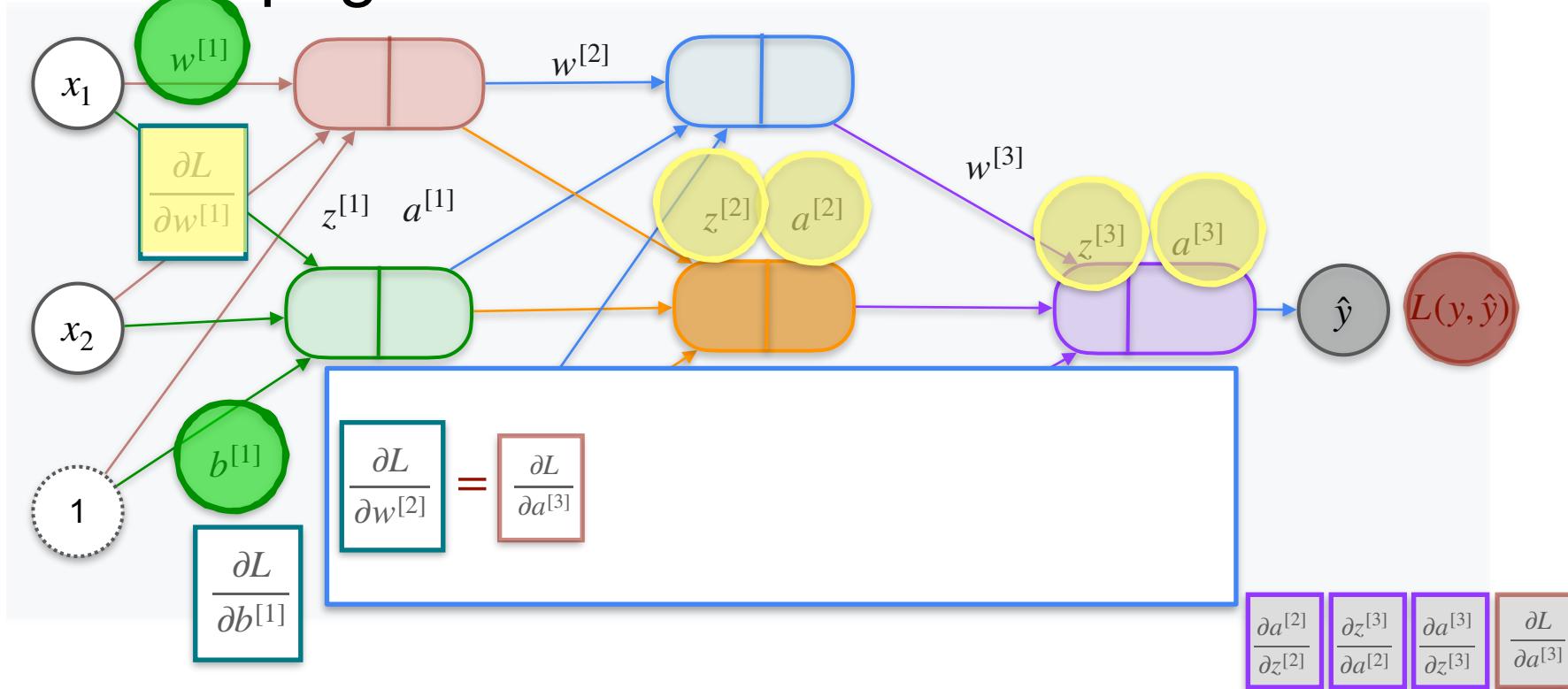
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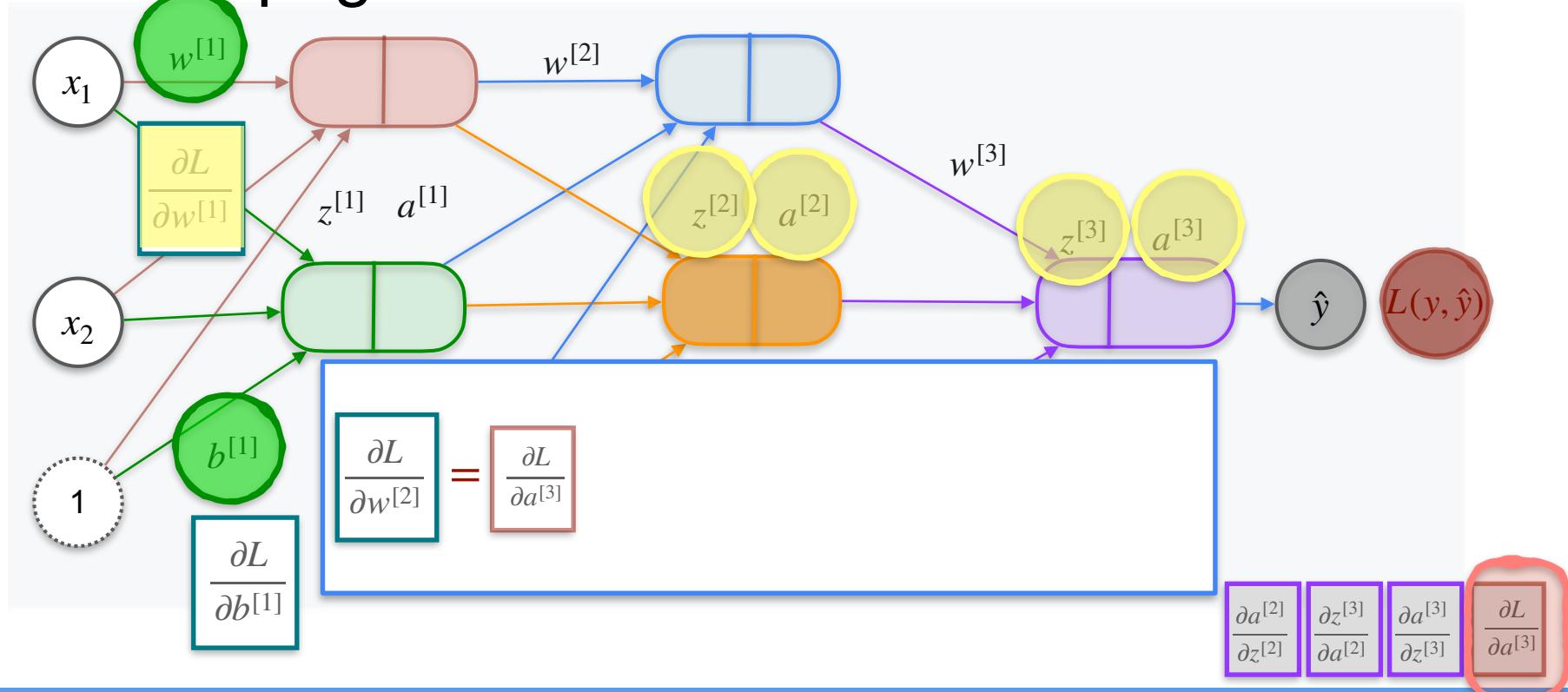
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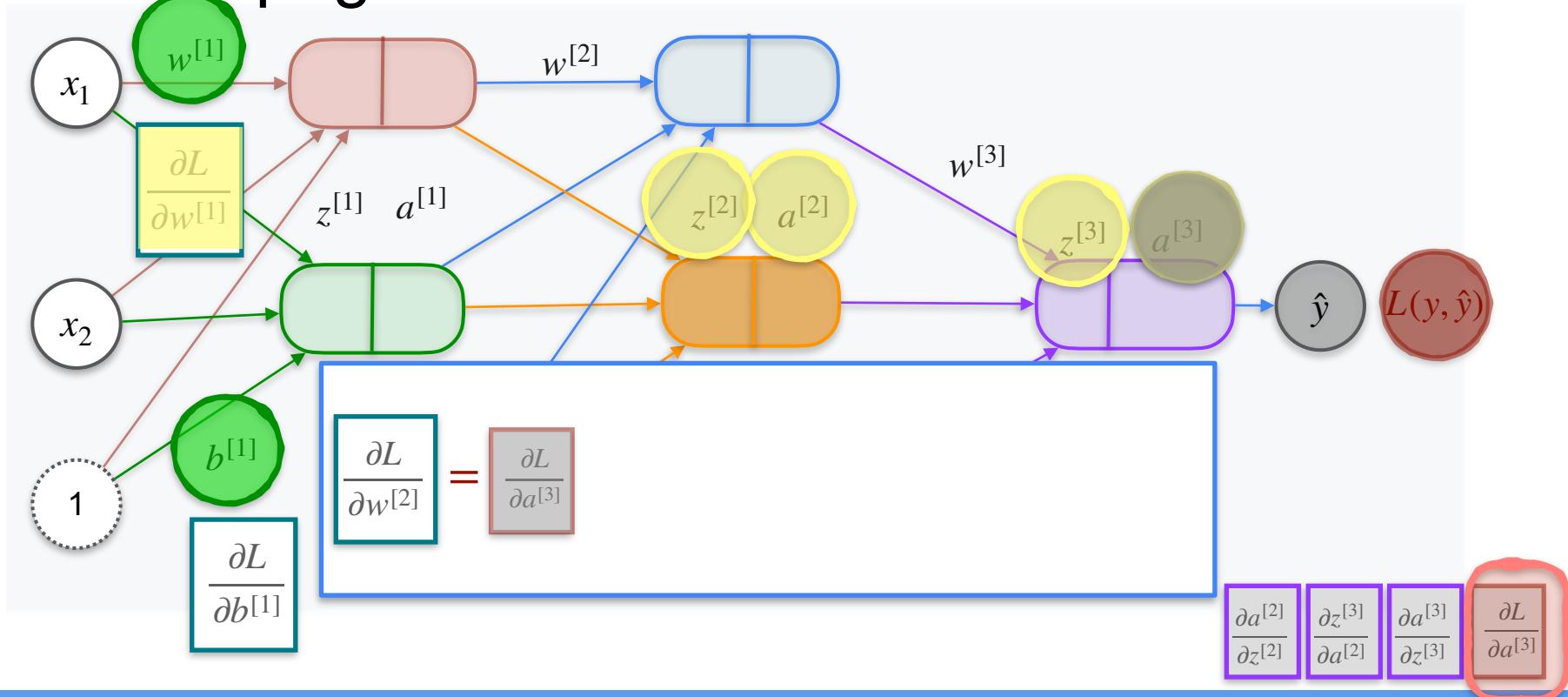
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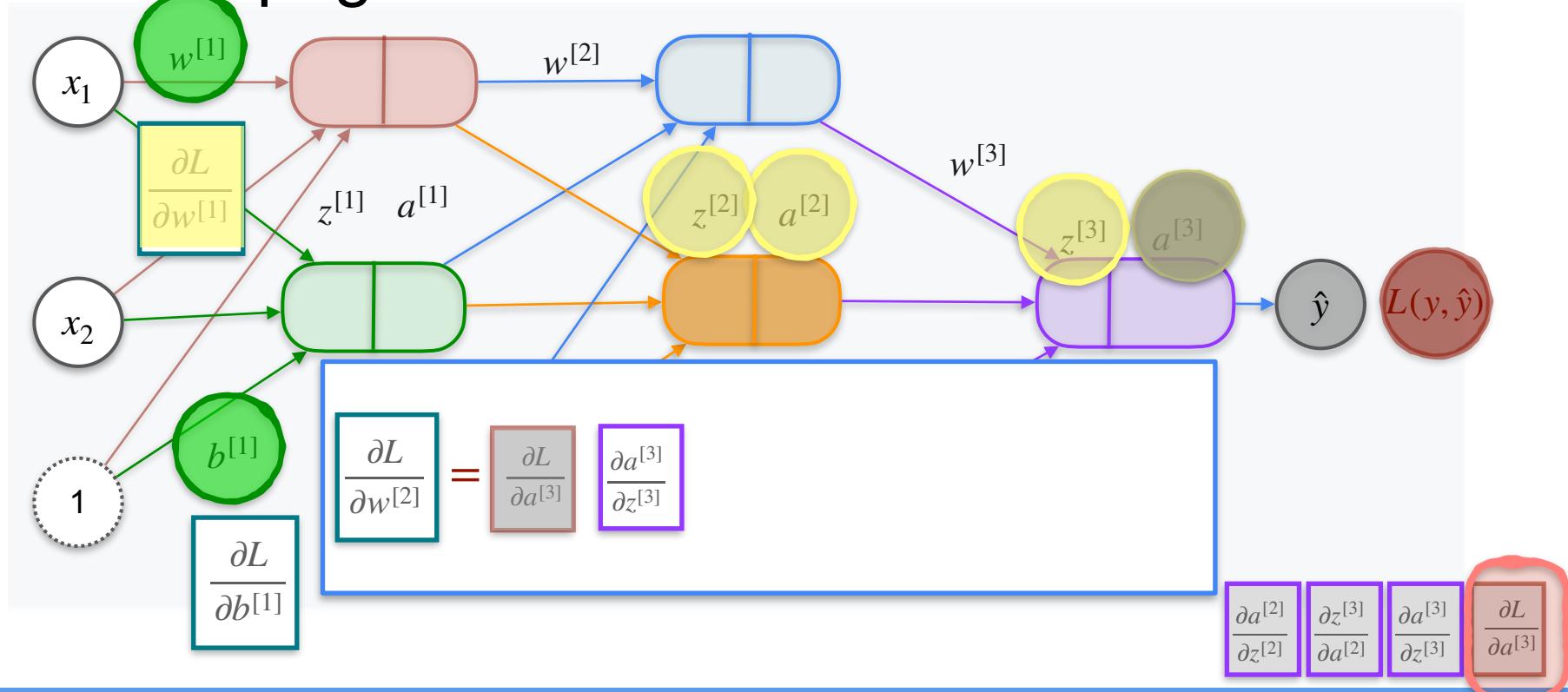
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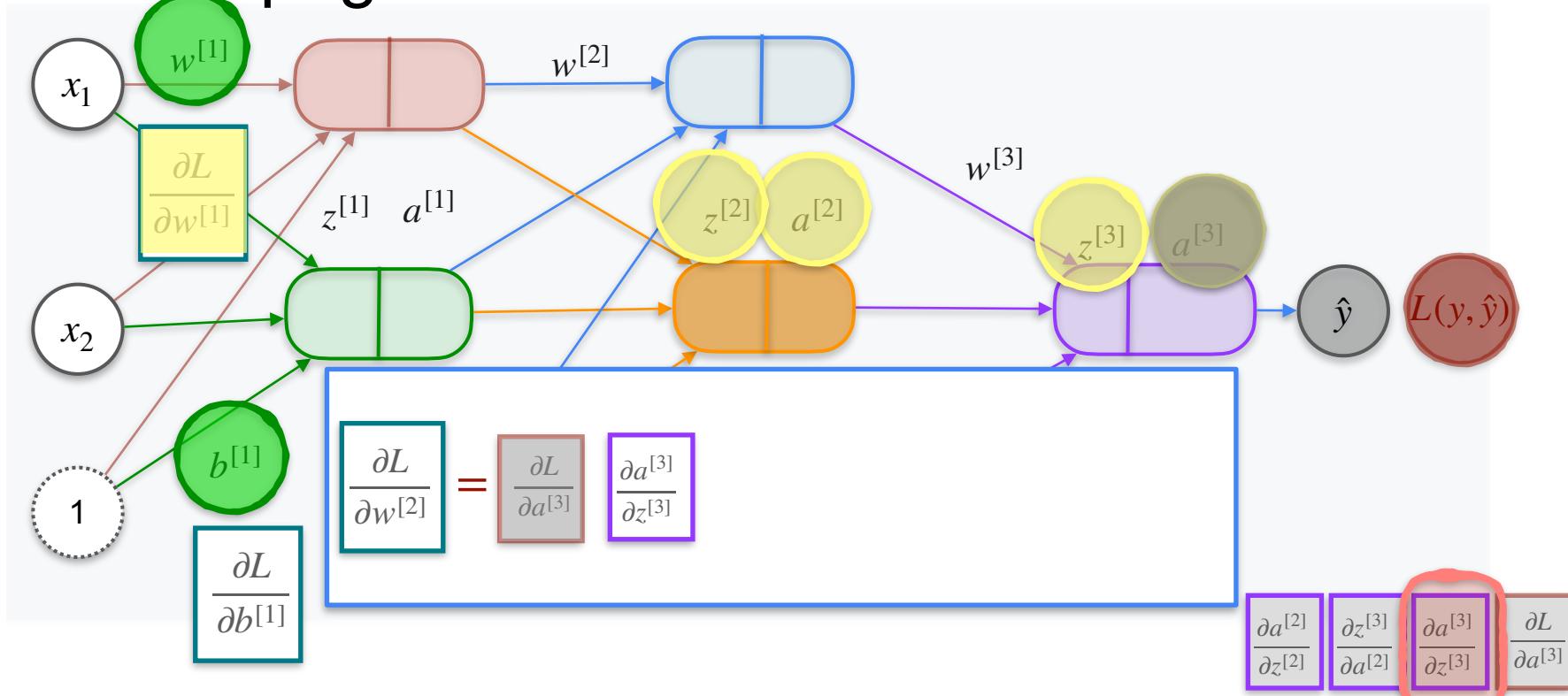
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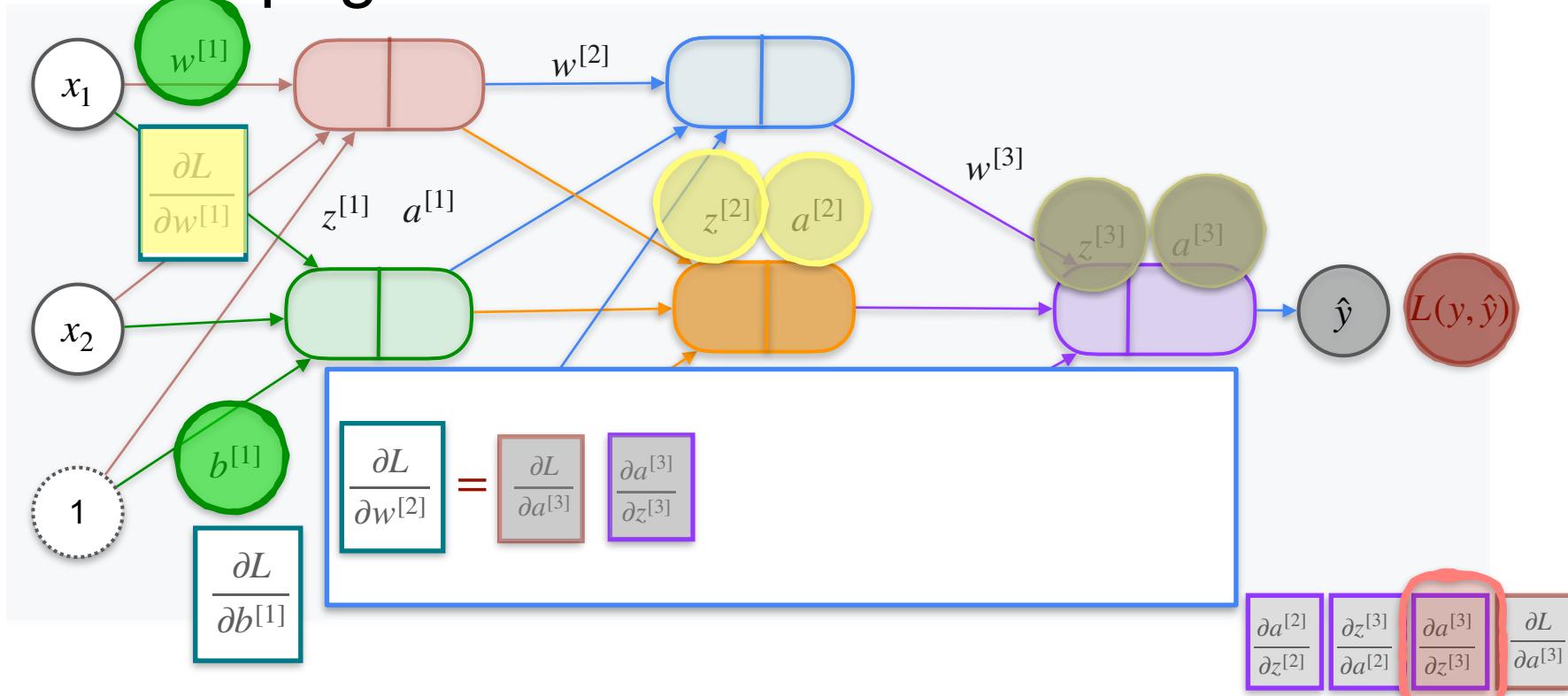
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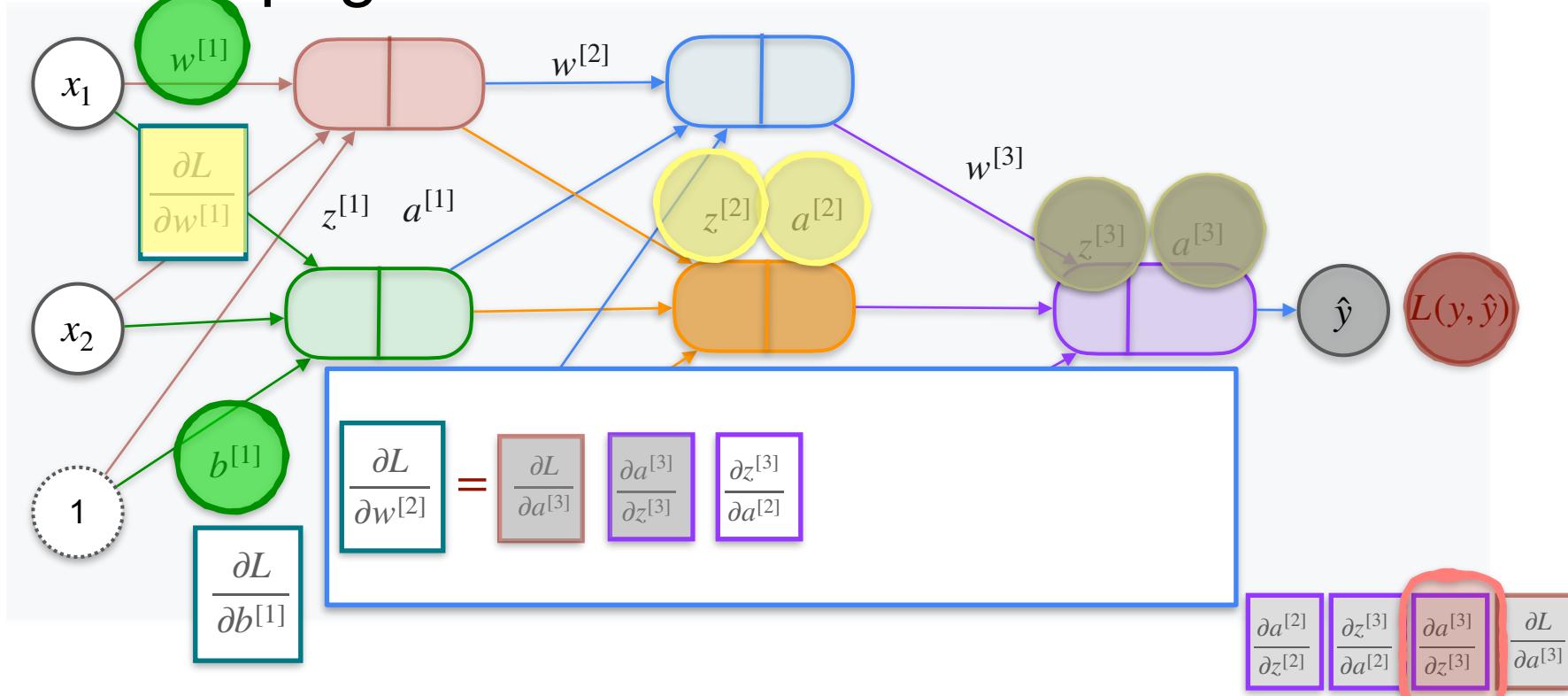
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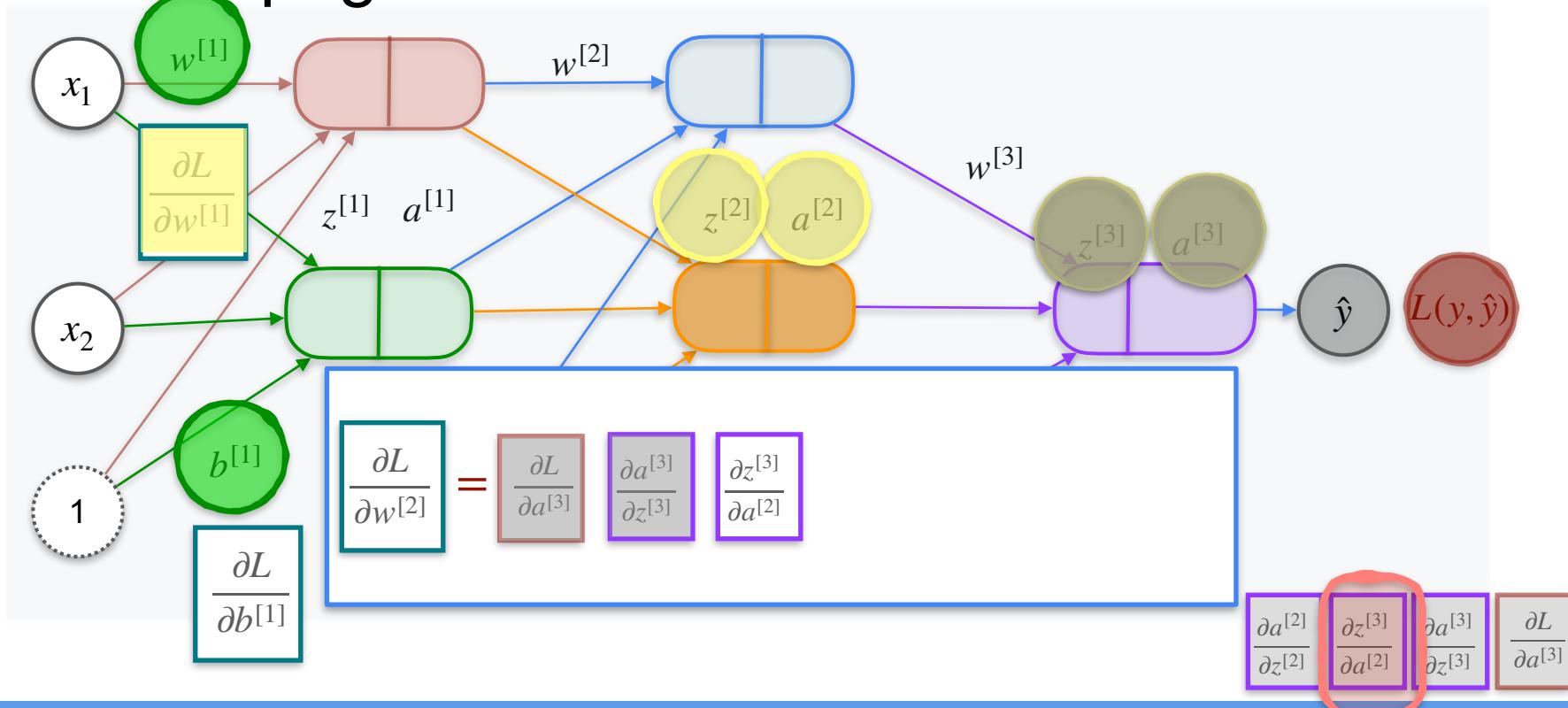
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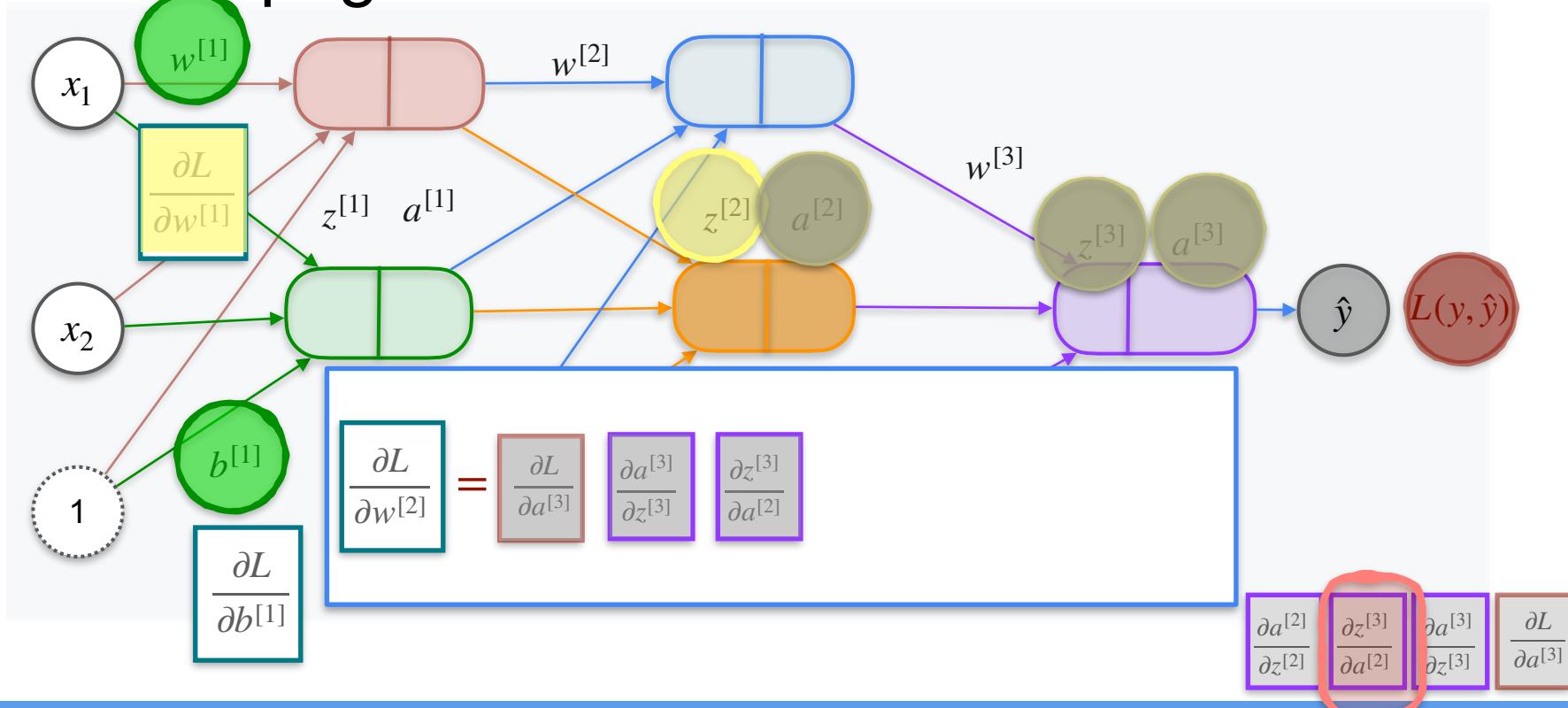
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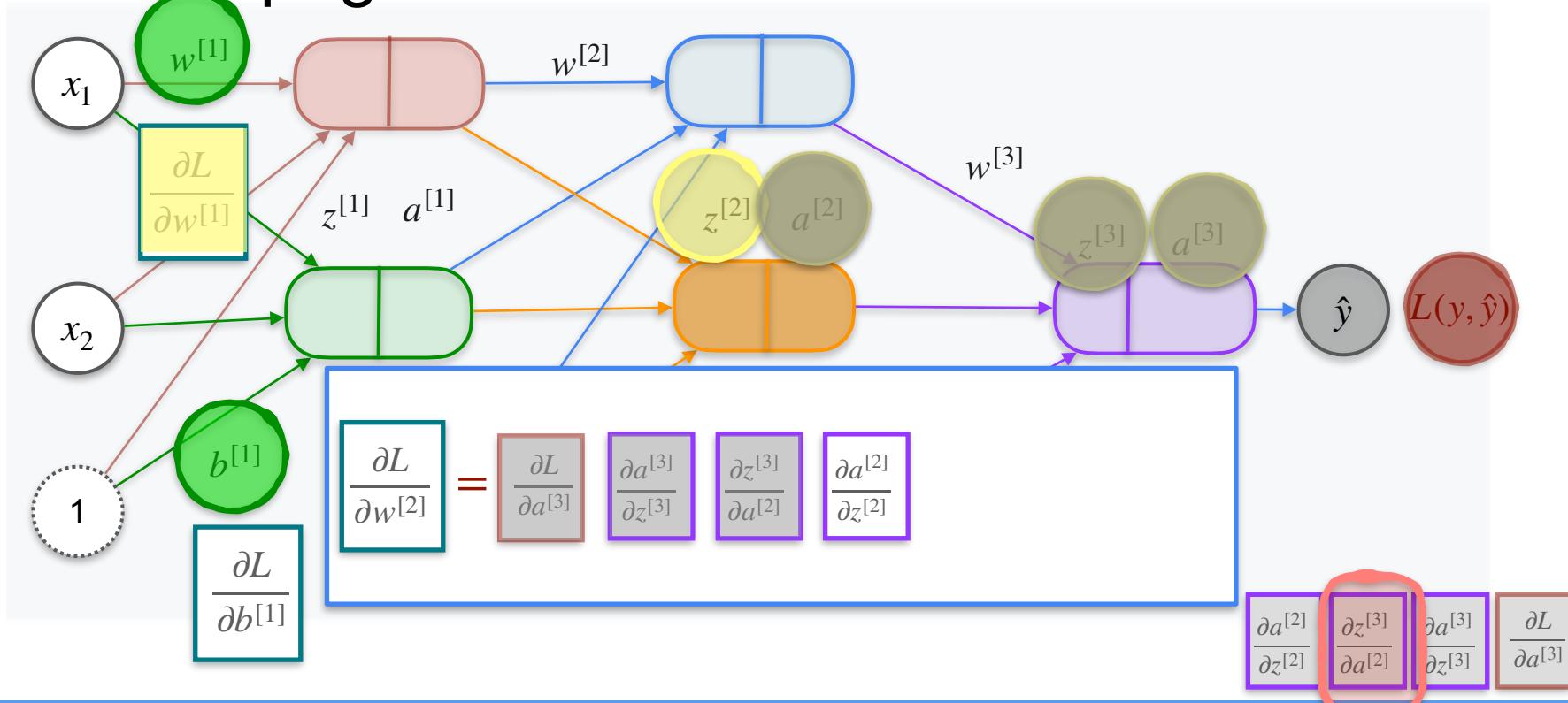
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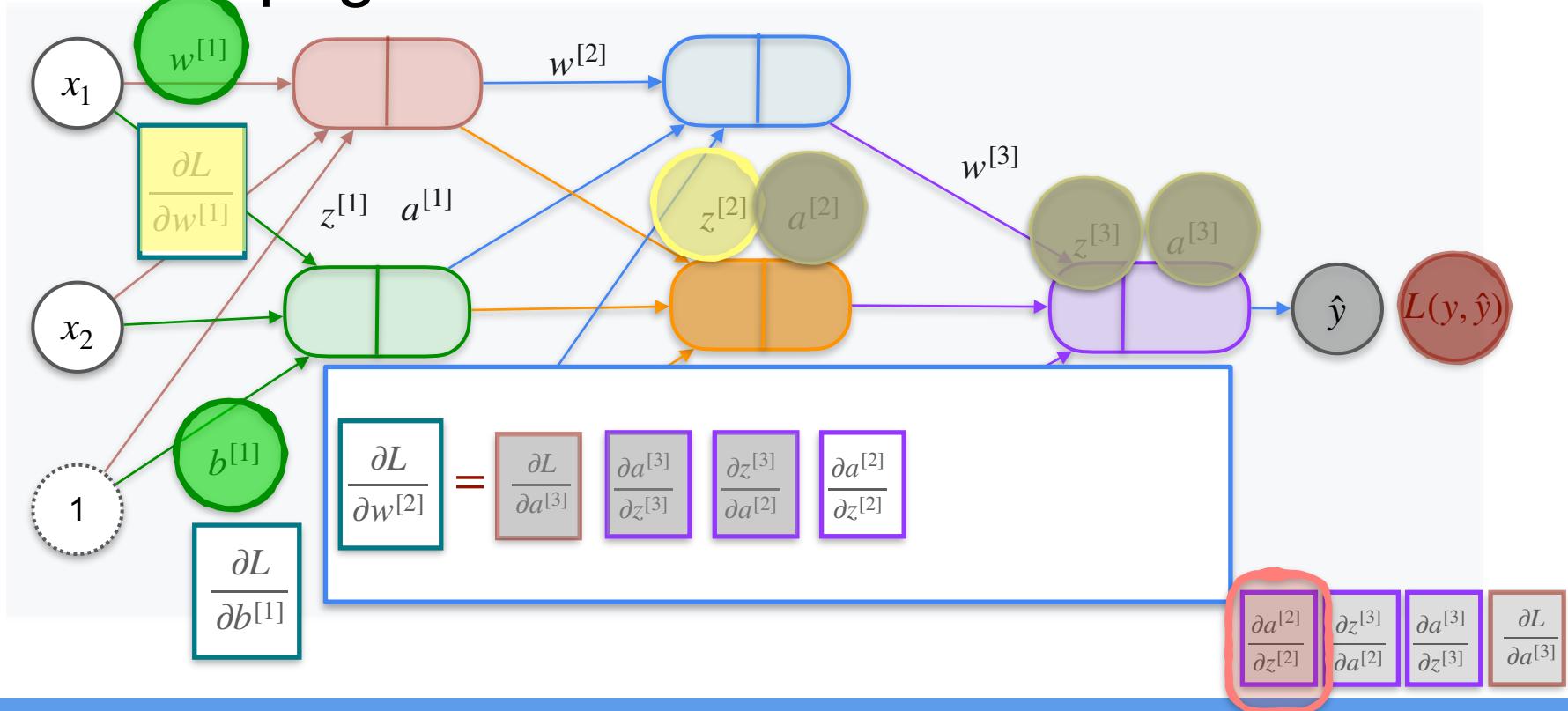
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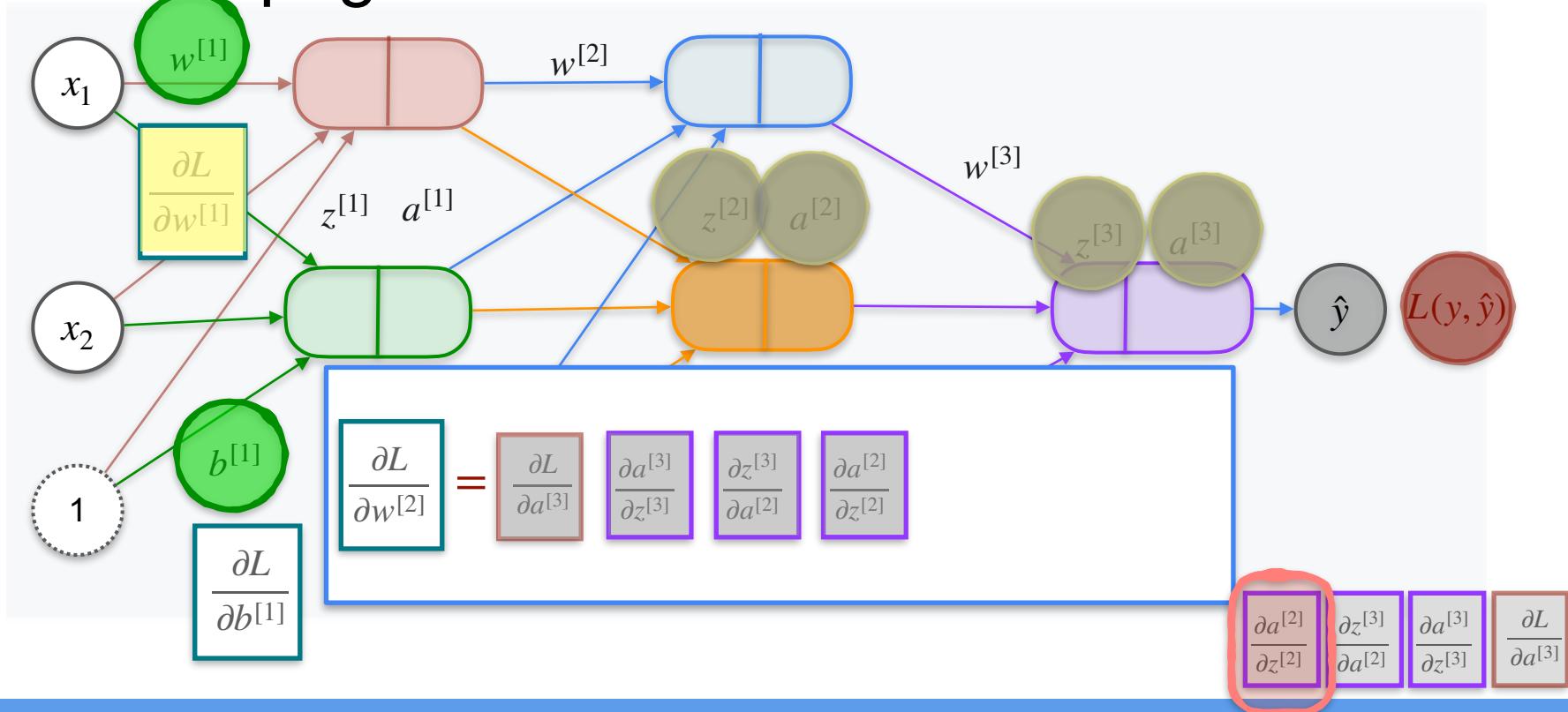
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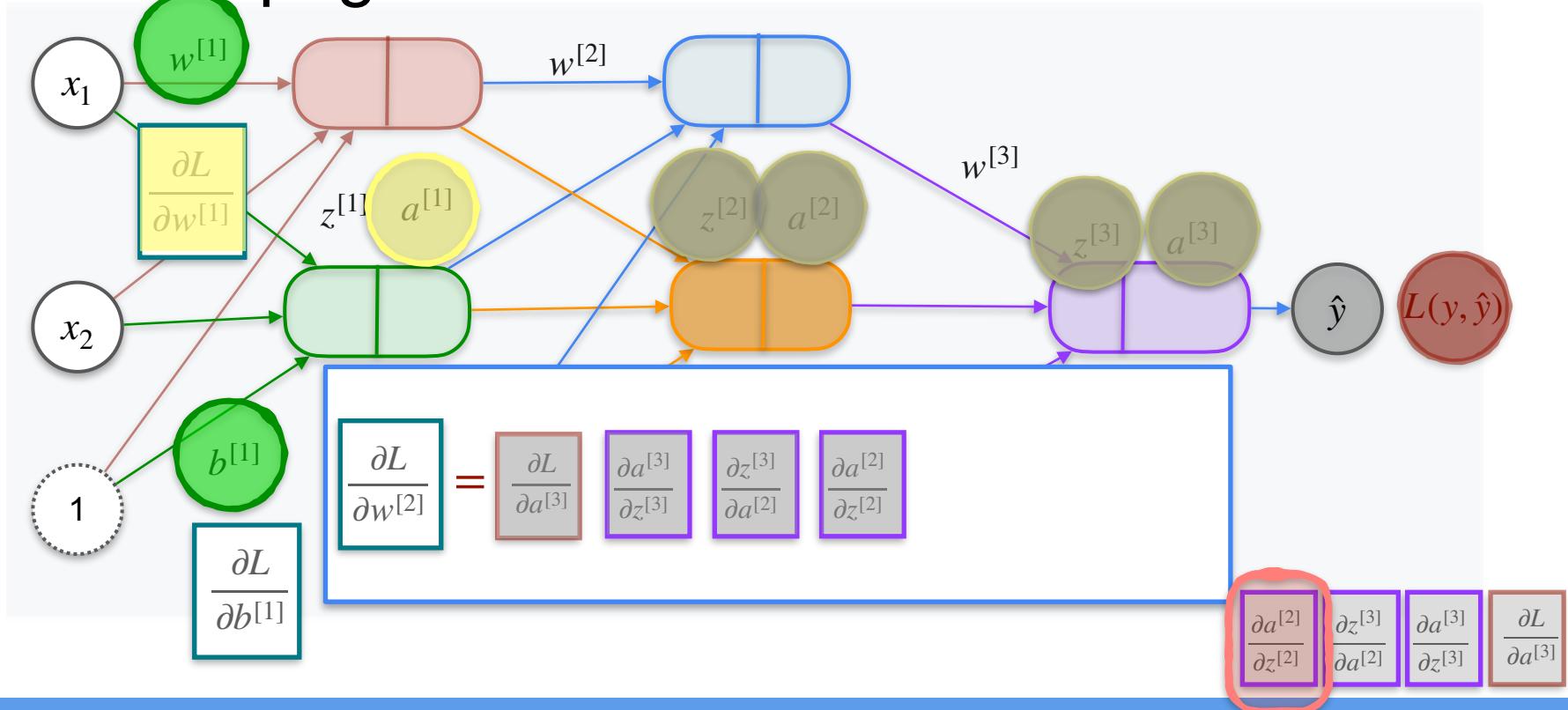
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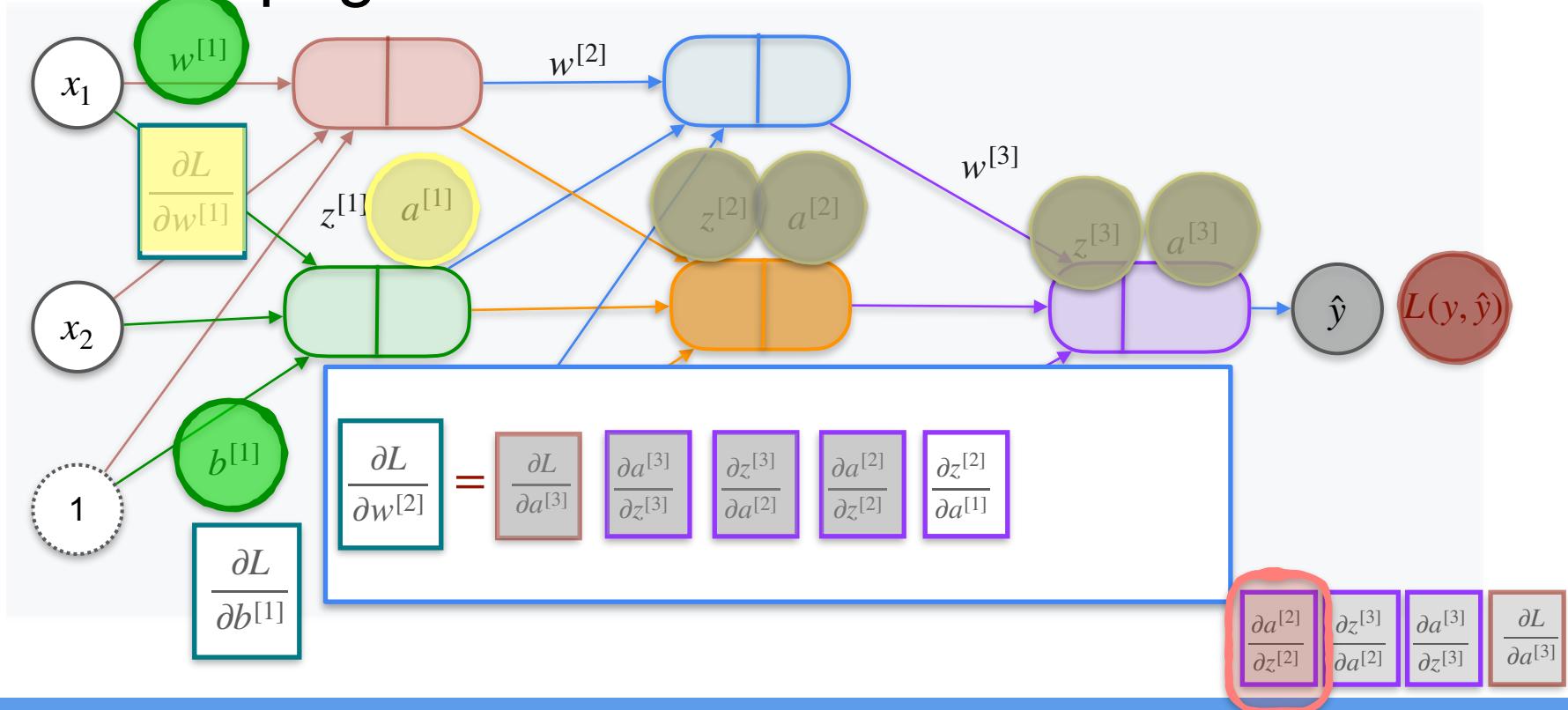
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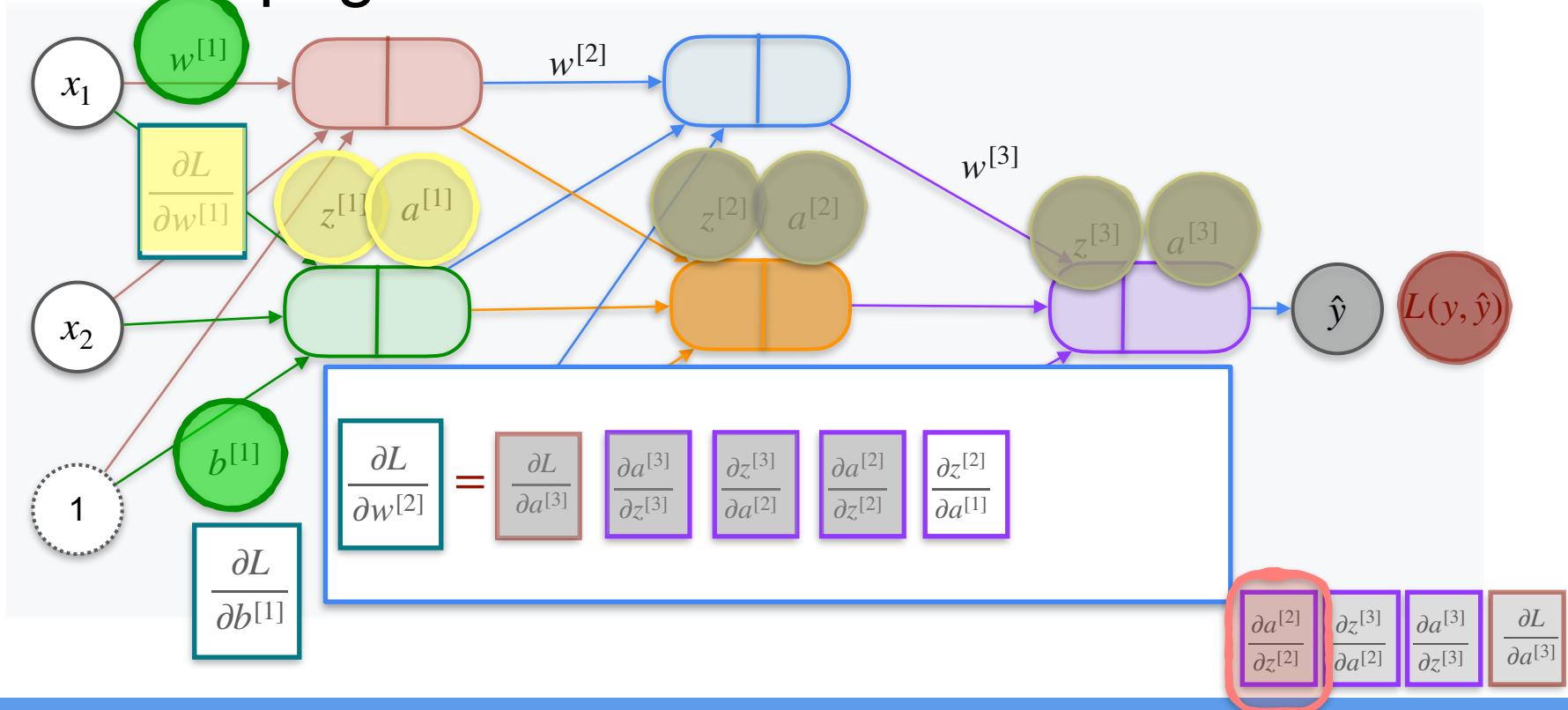
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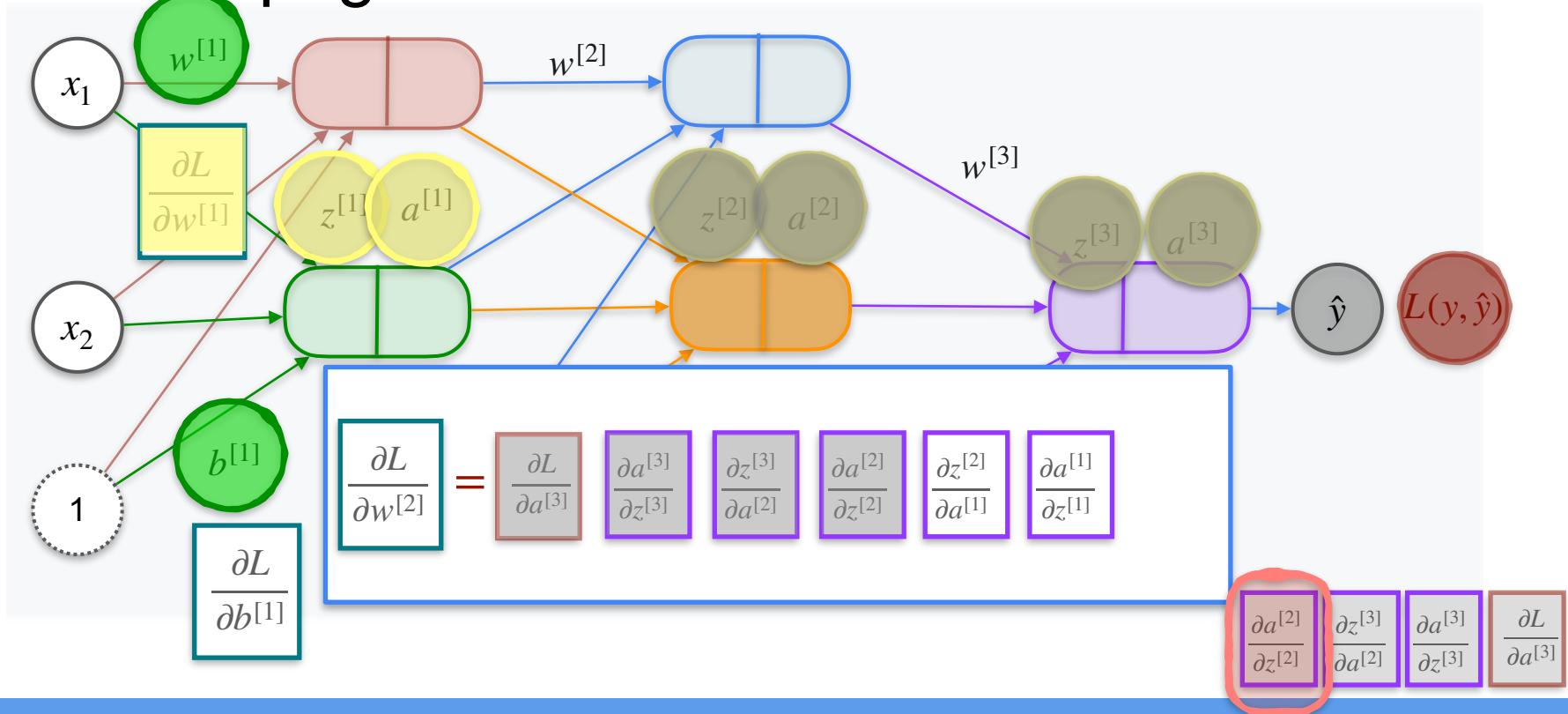
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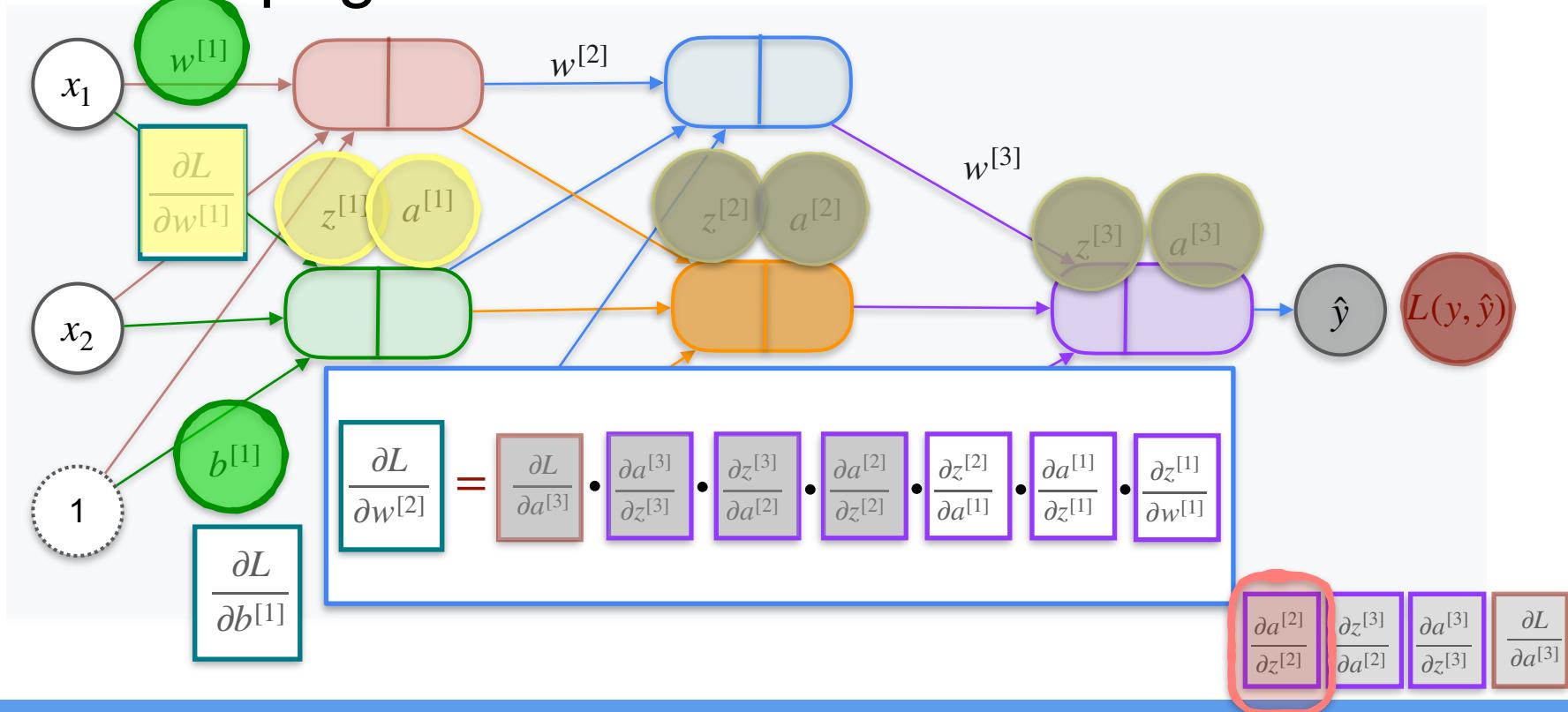
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Back Propagation Introduction



Back Propagation Introduction





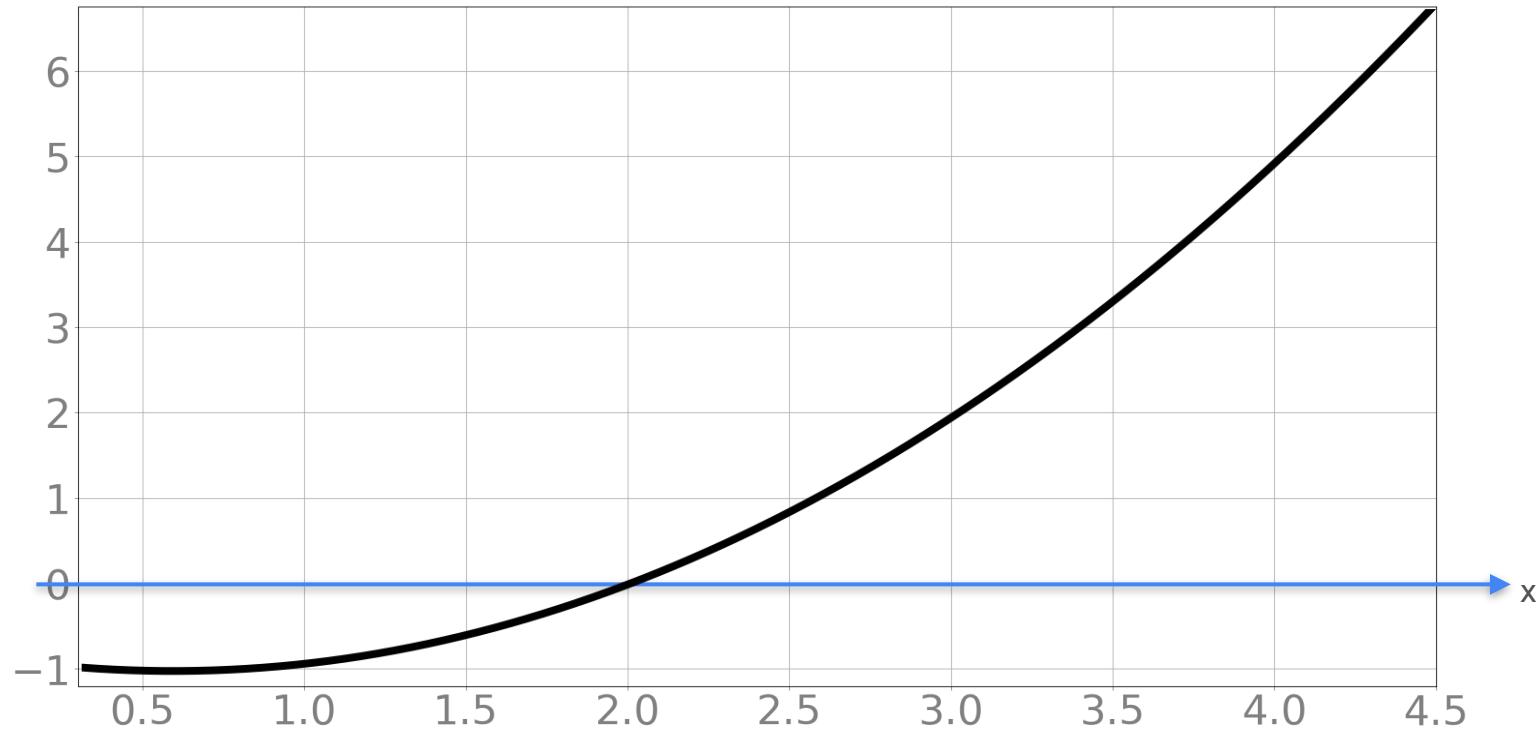
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

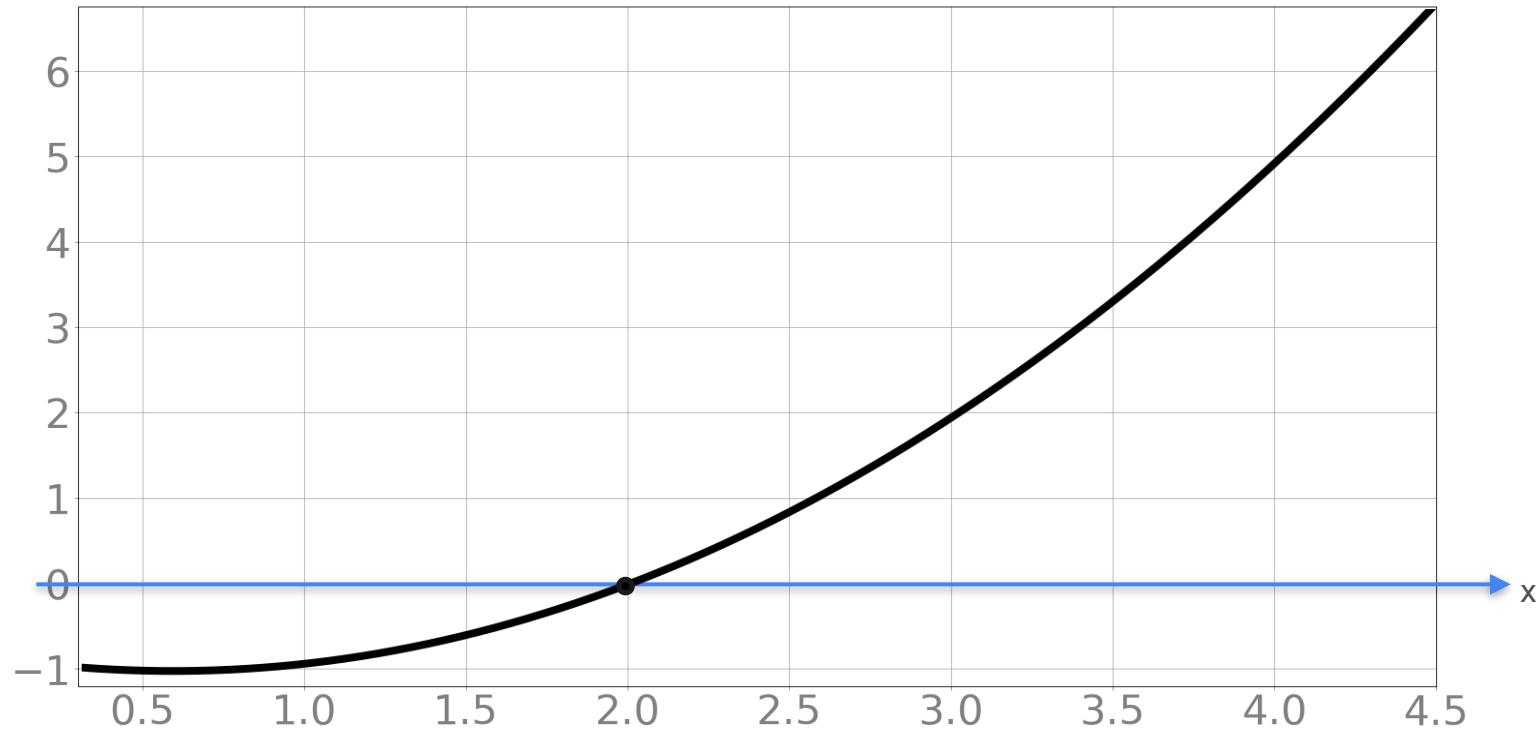
Newton's method

Newton's Method

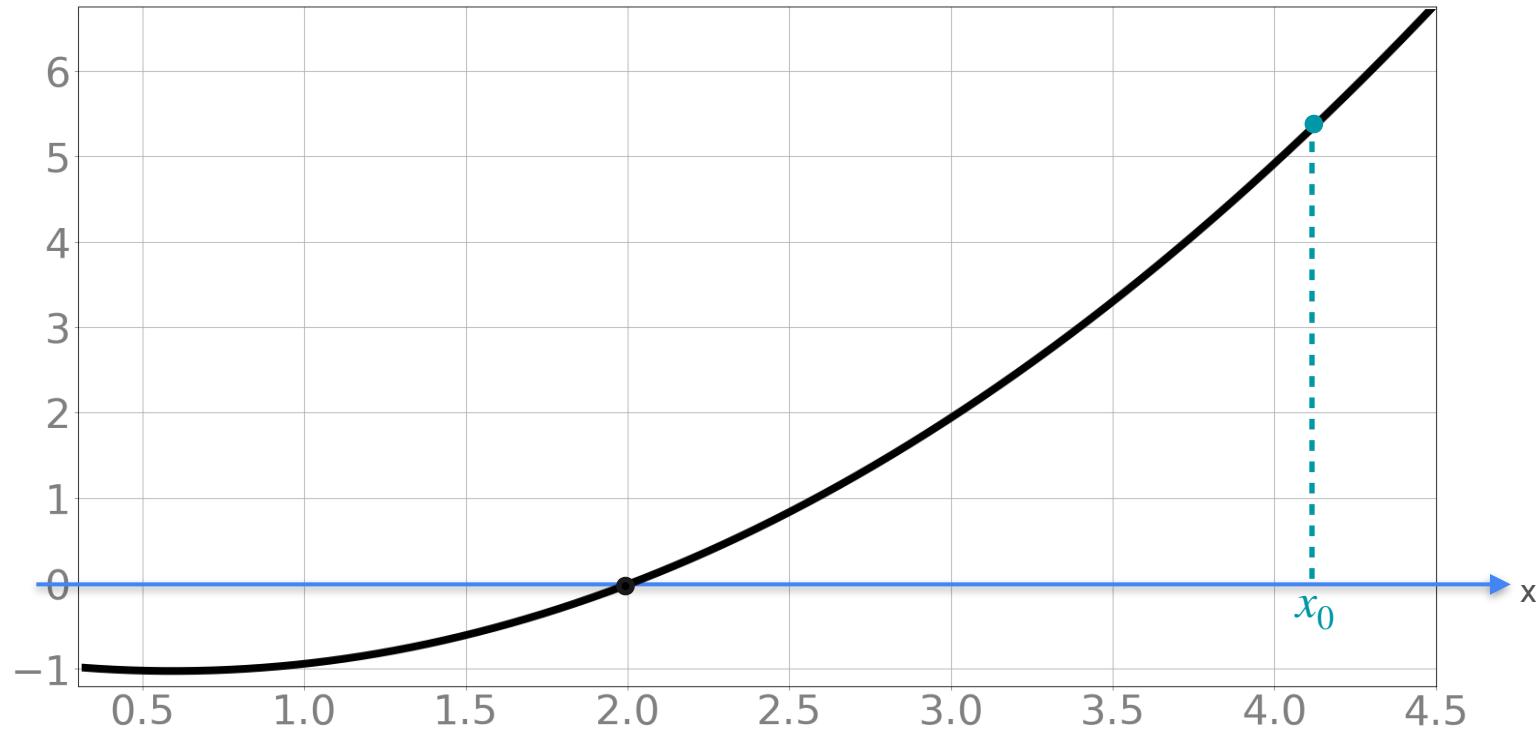
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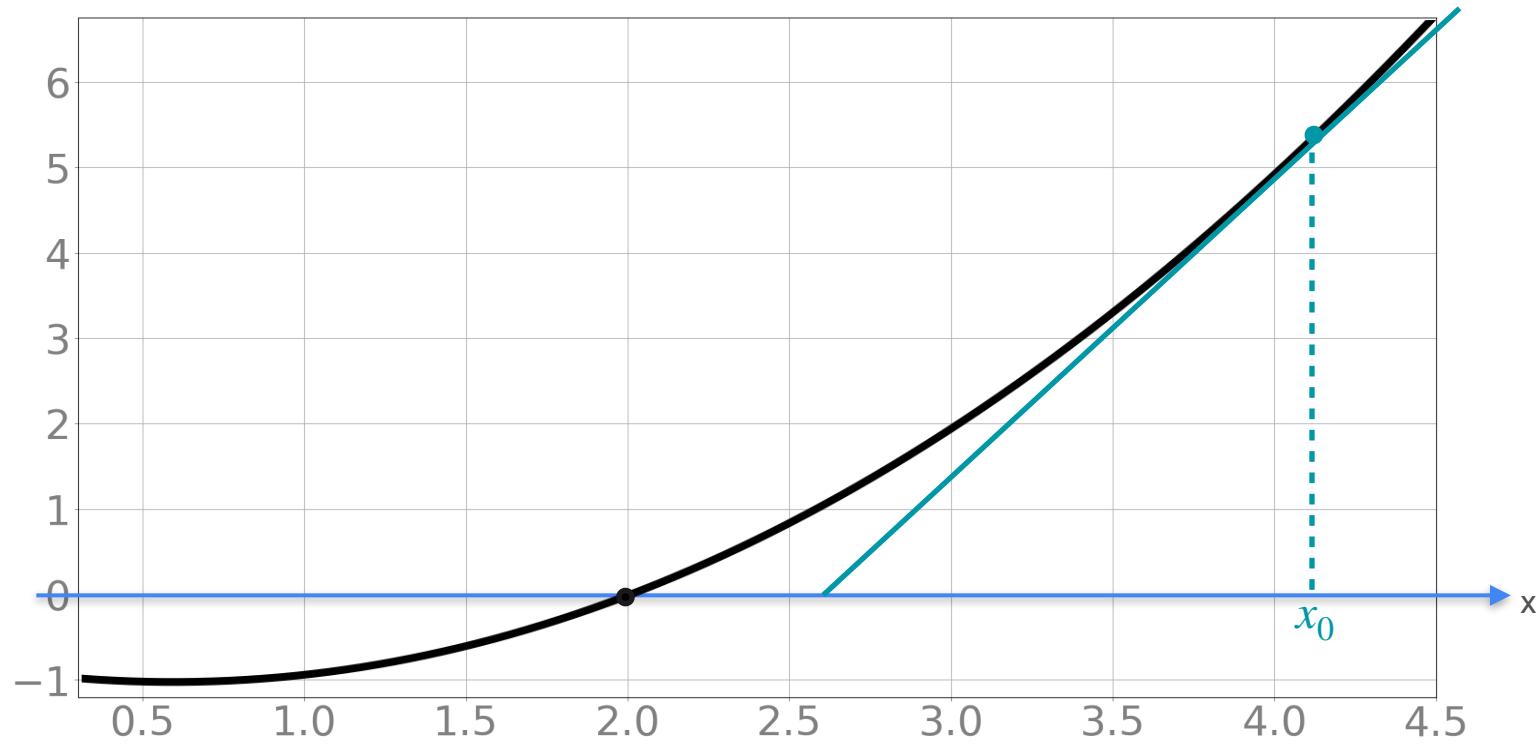
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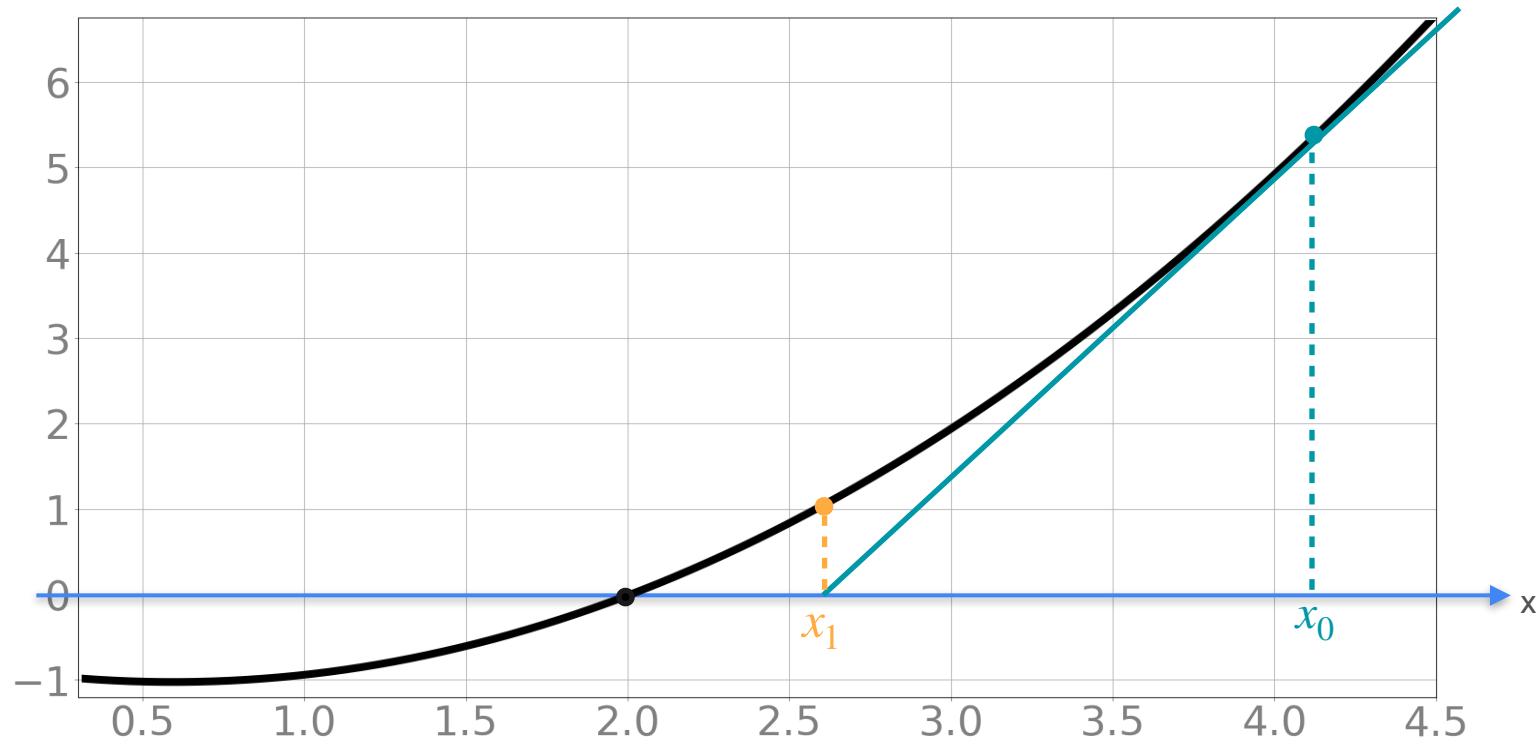
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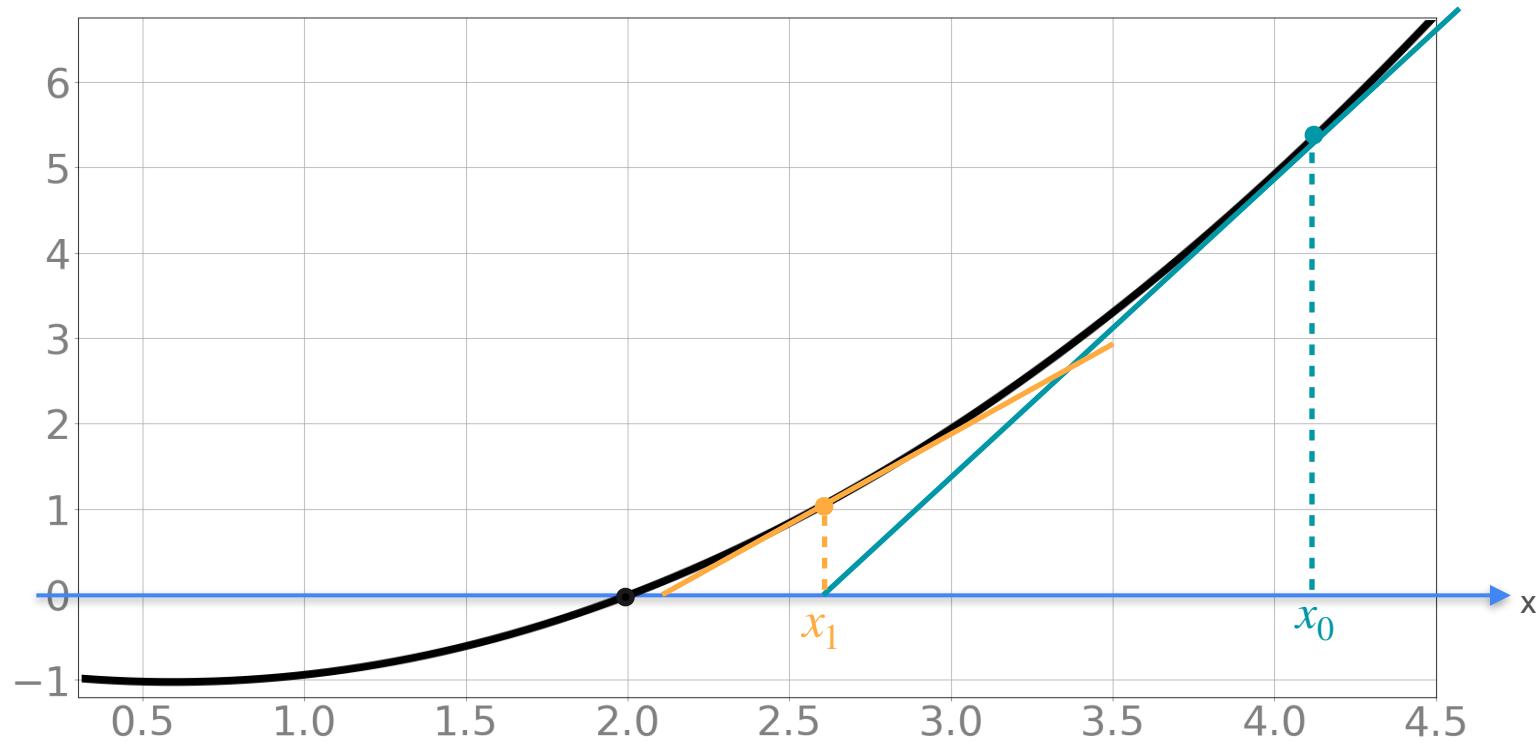
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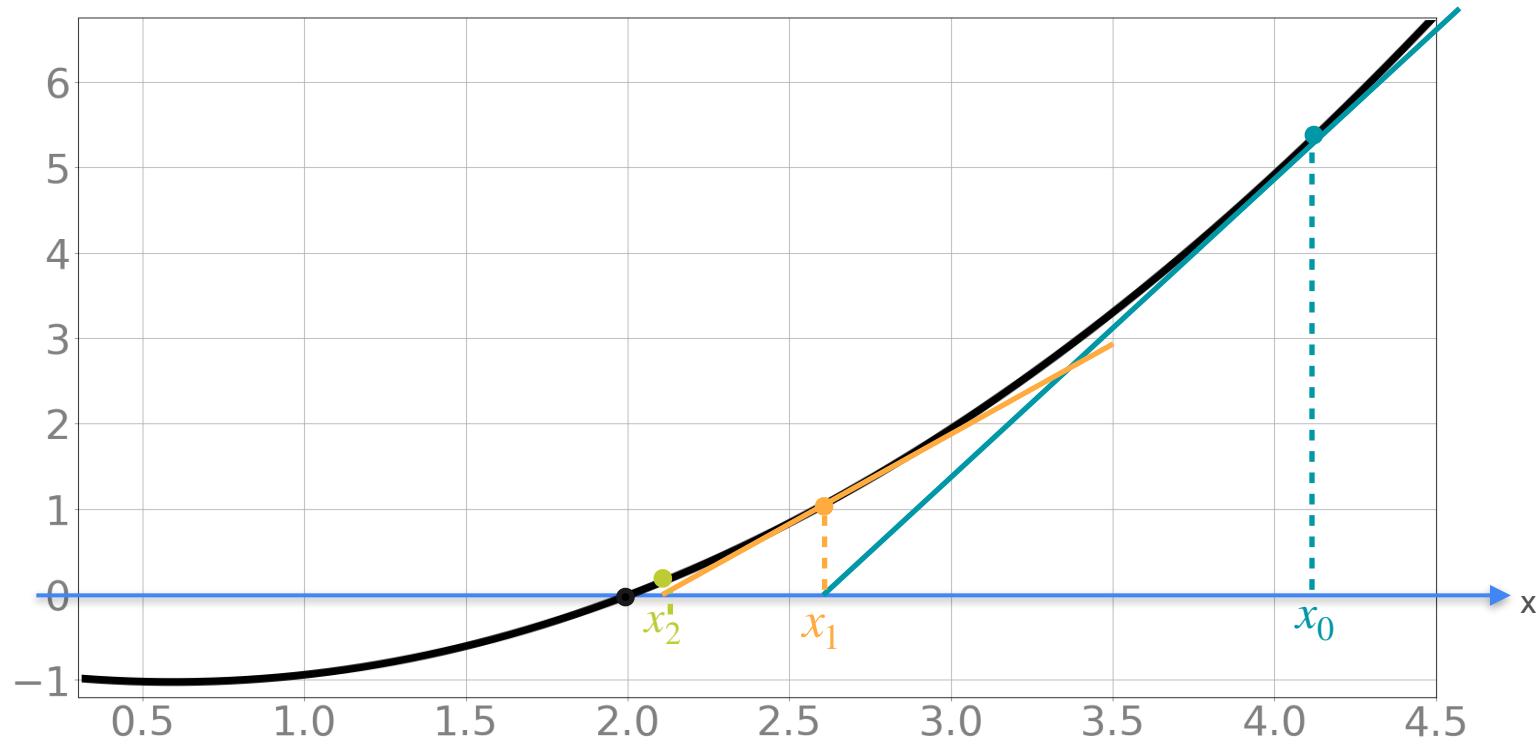
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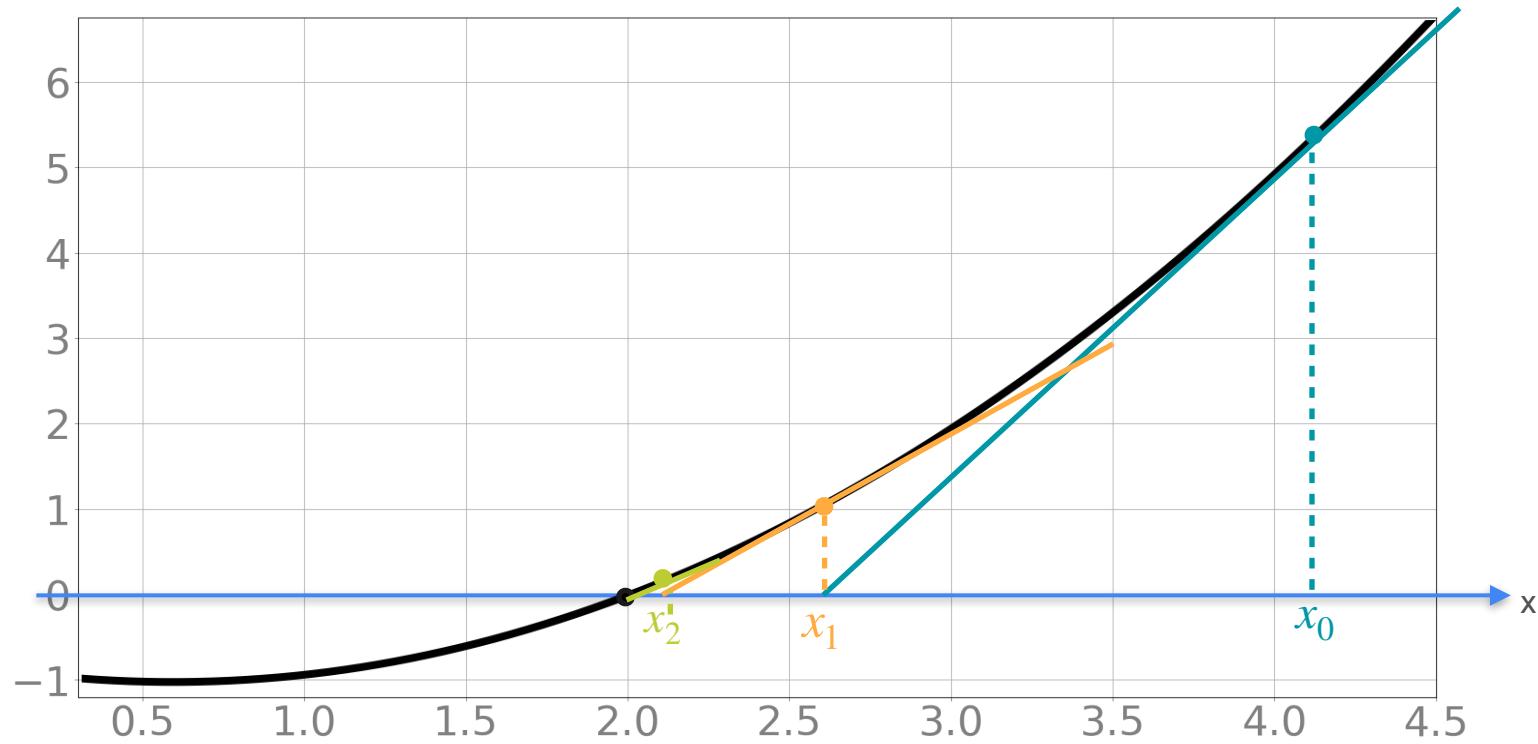
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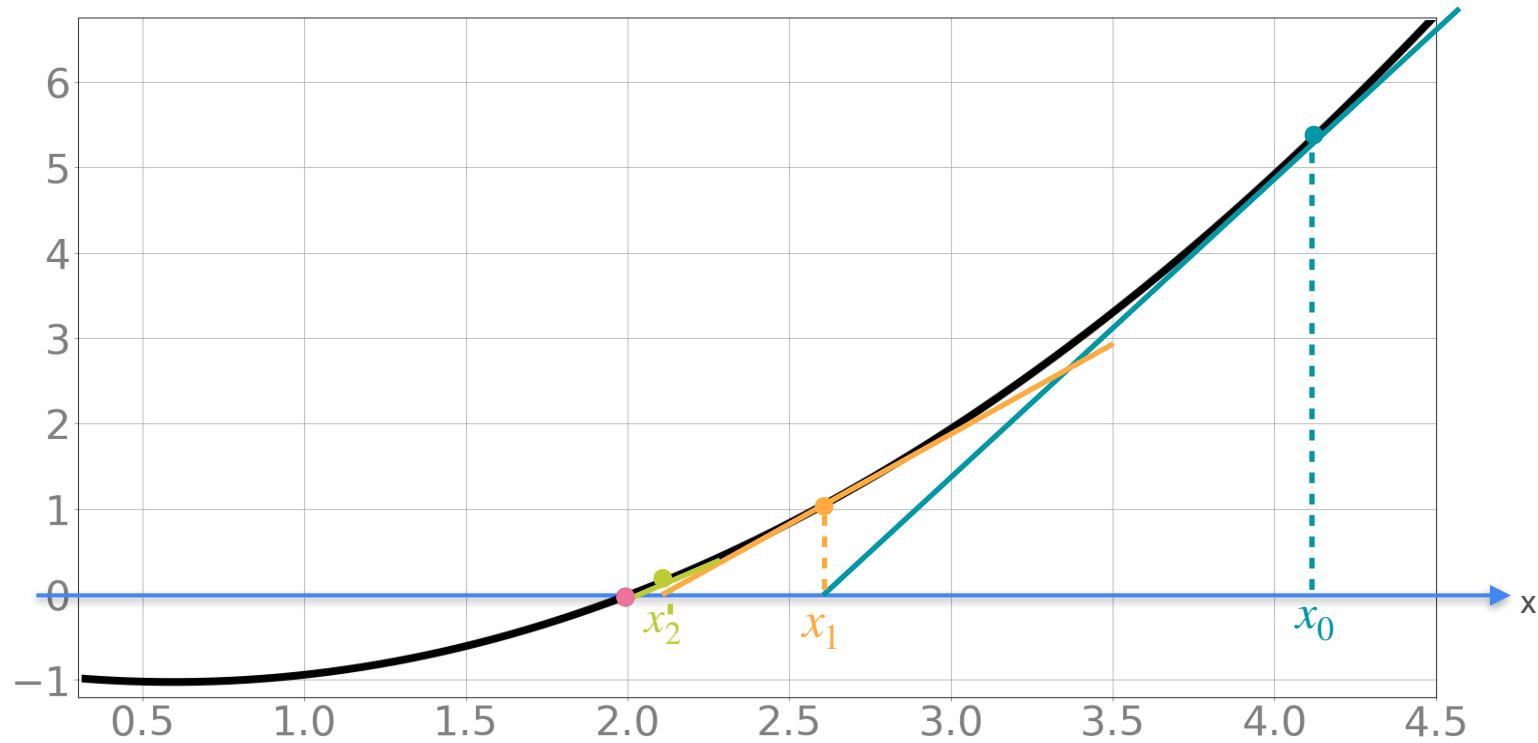
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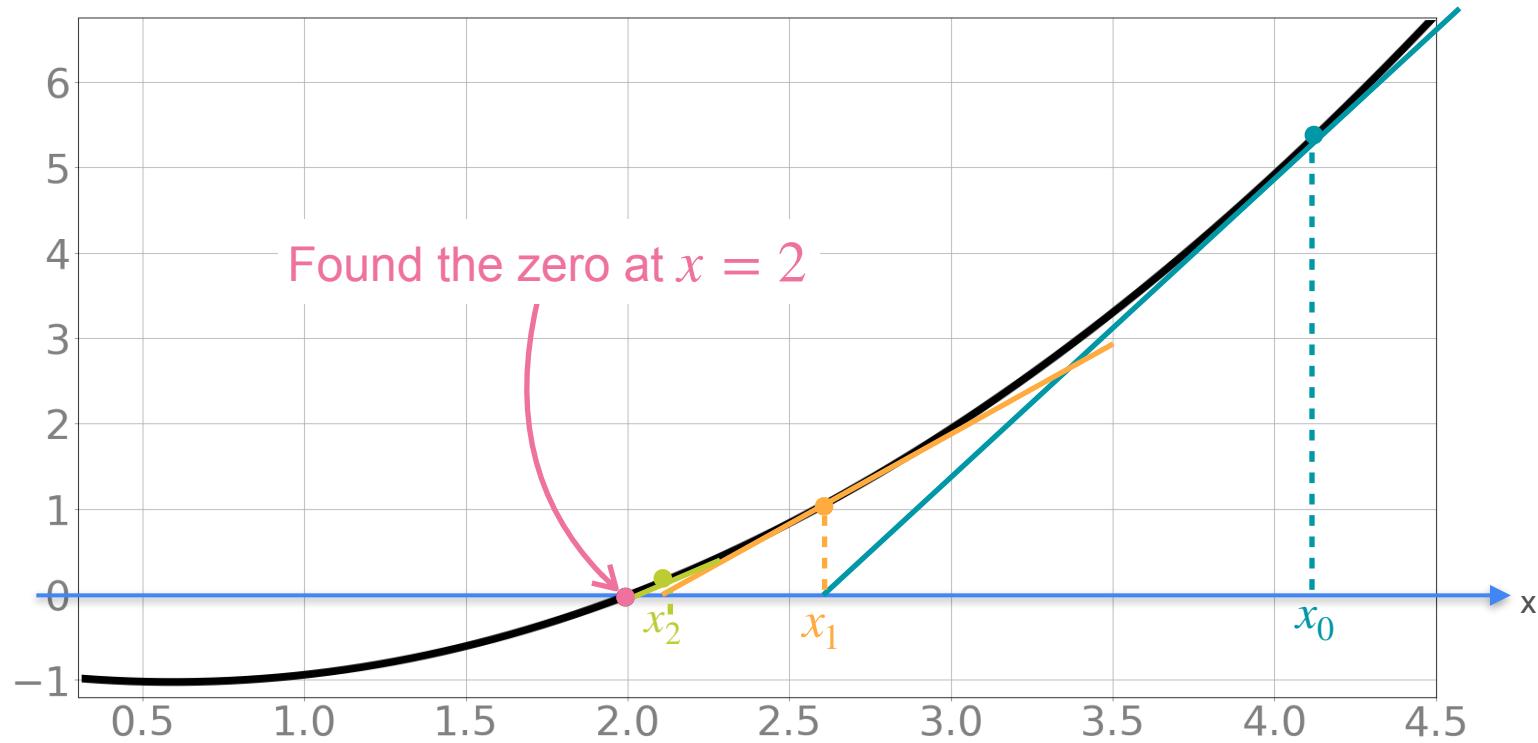
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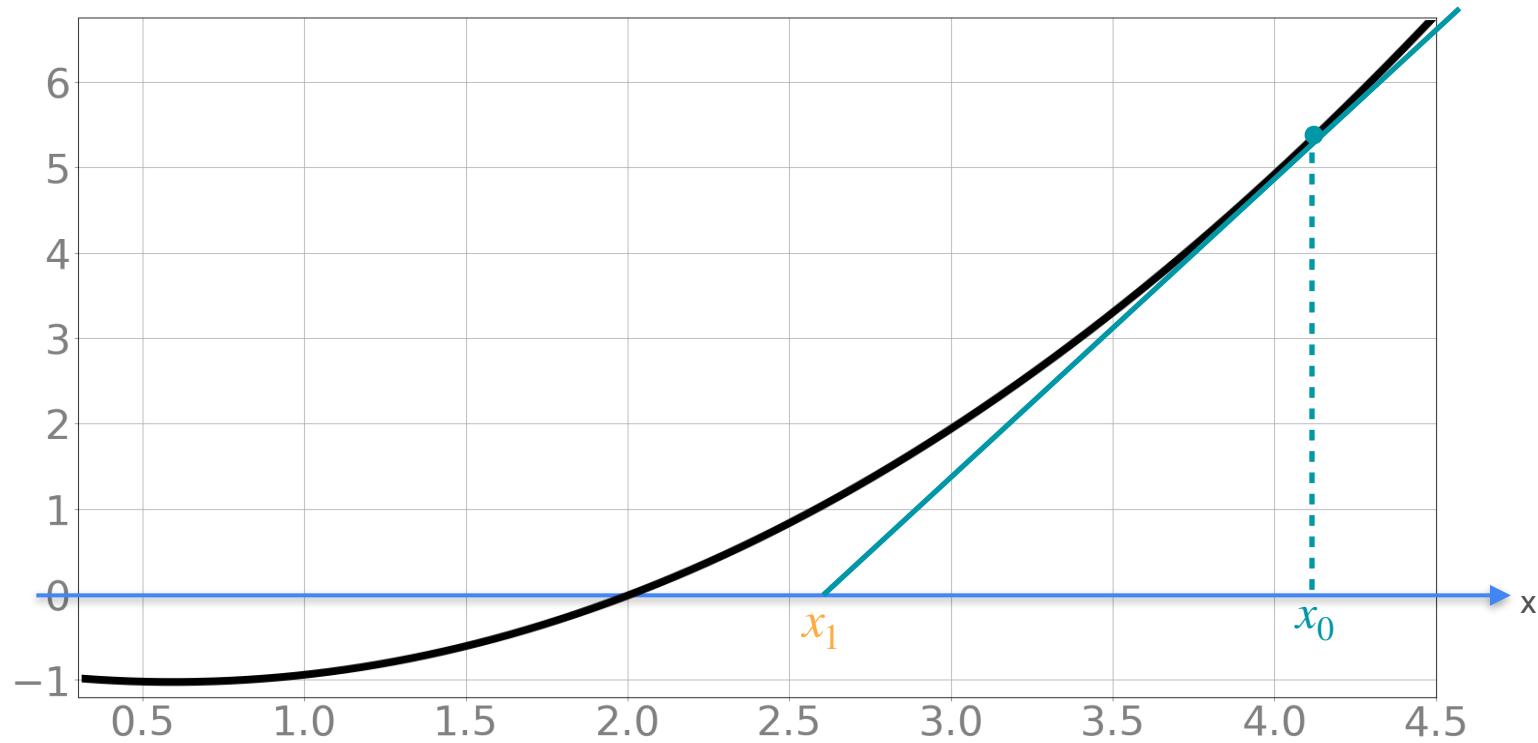
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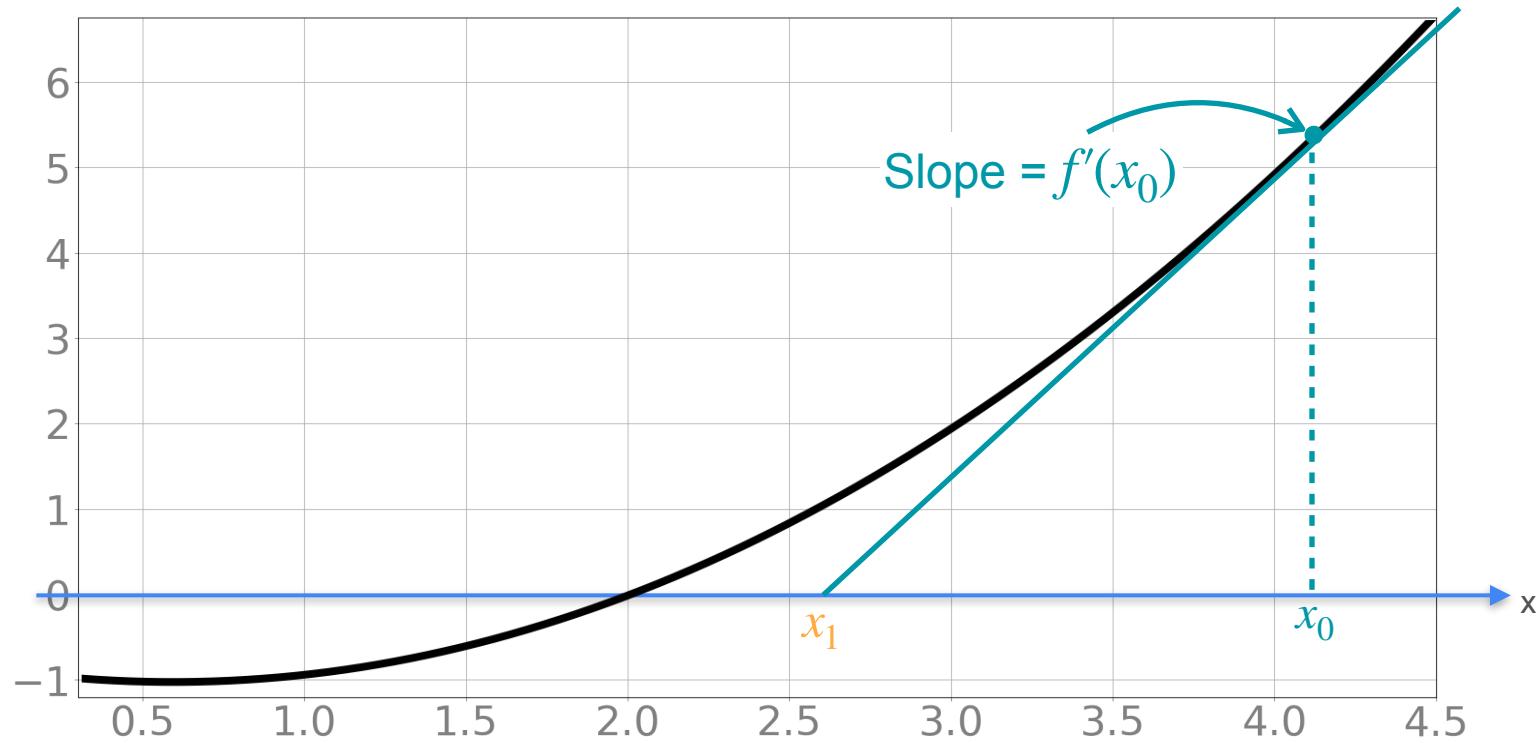
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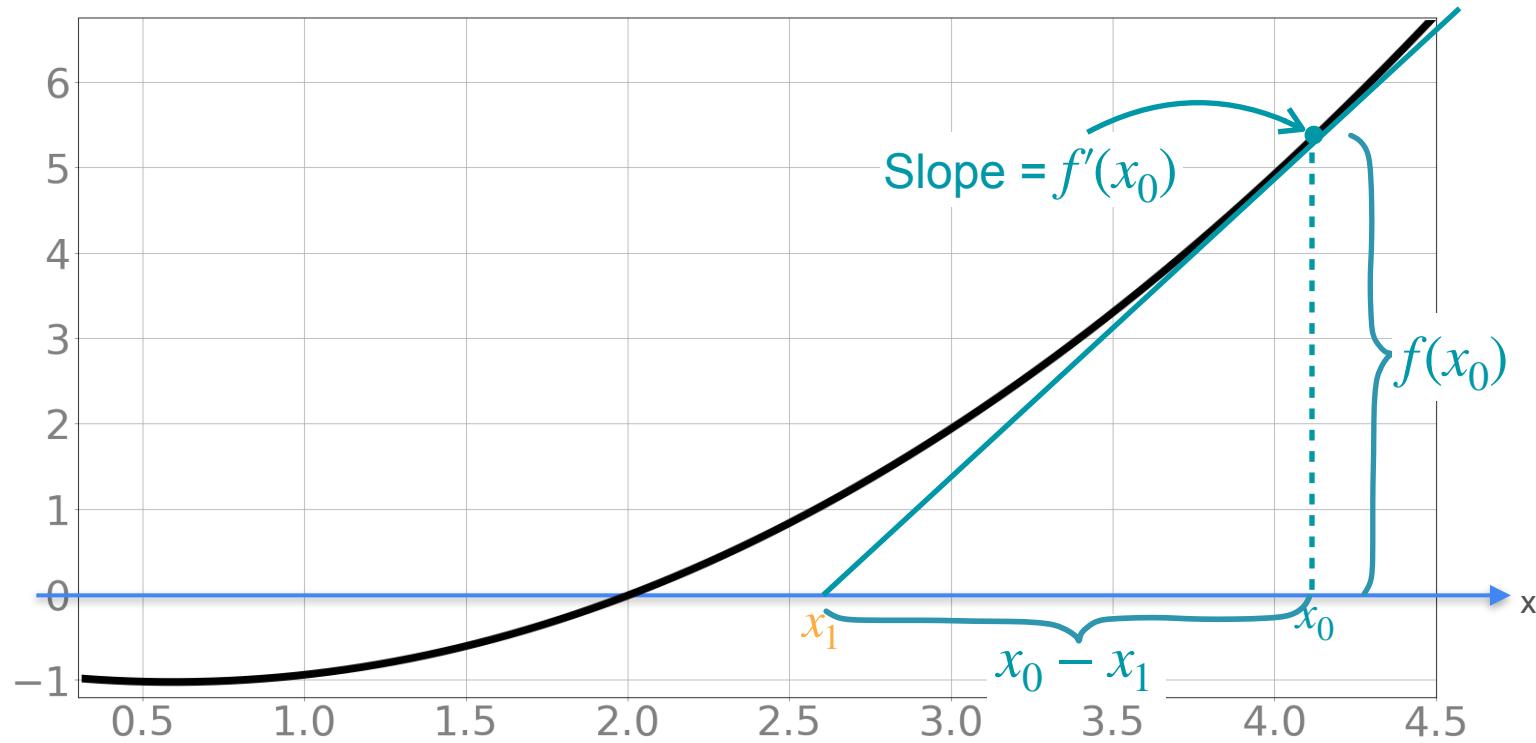
Update Approximation



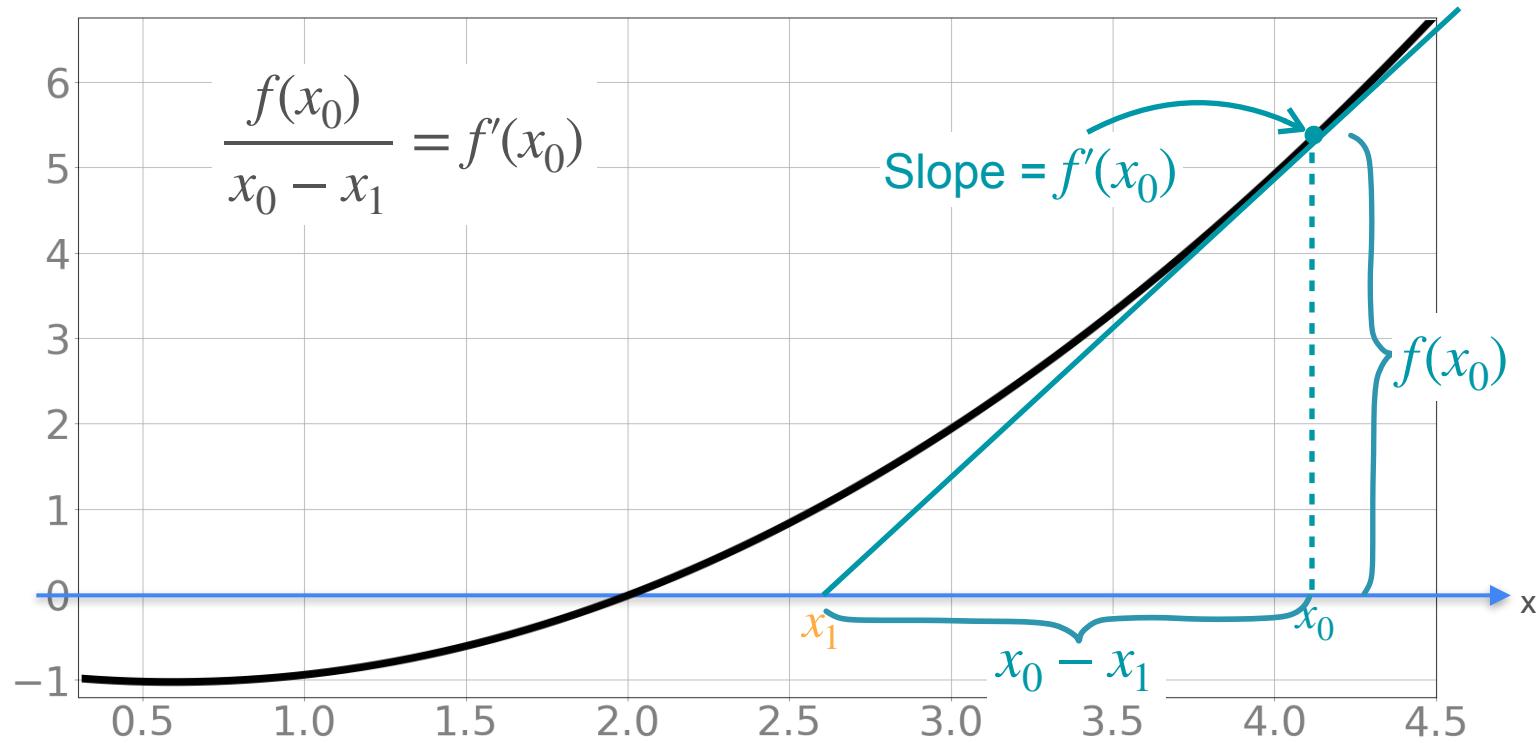
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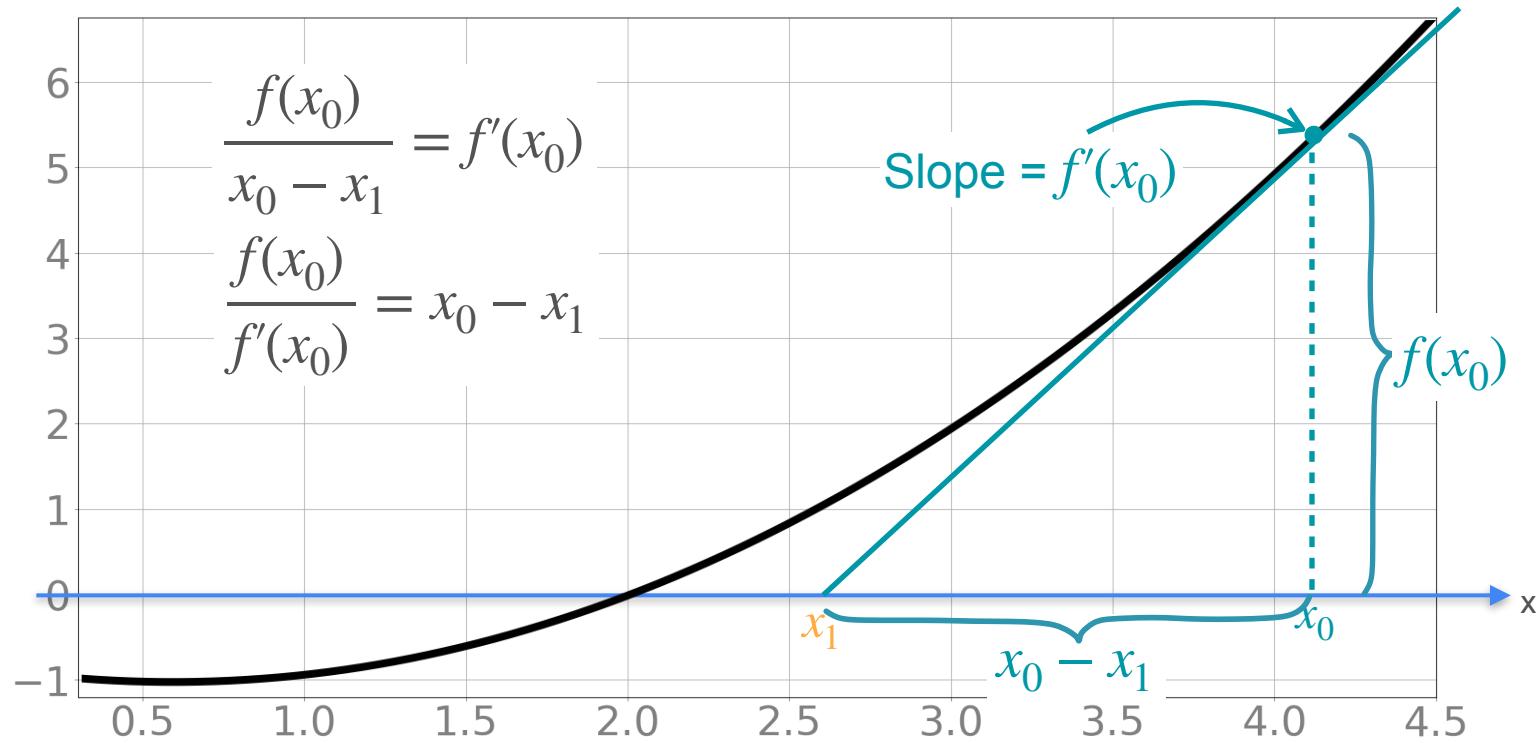
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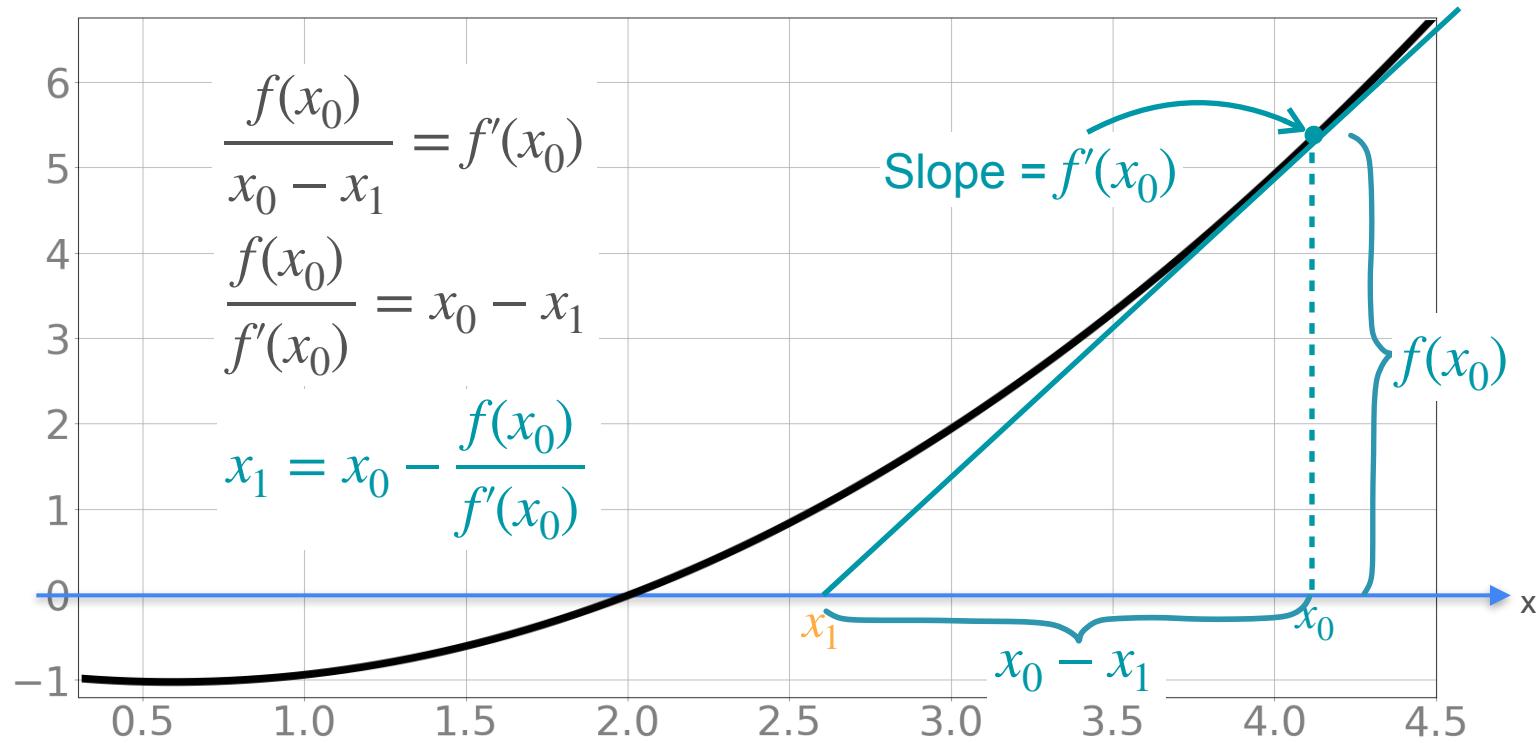
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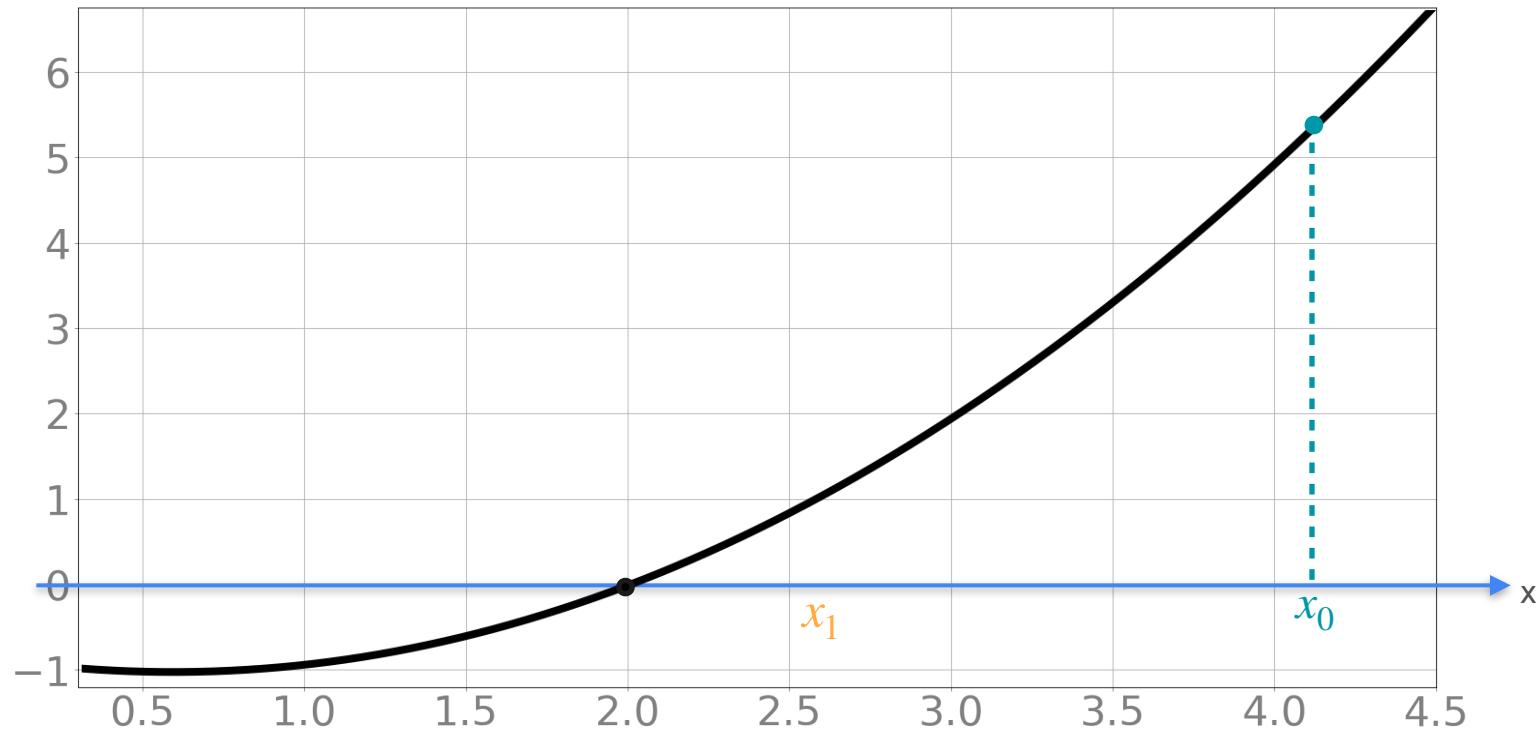
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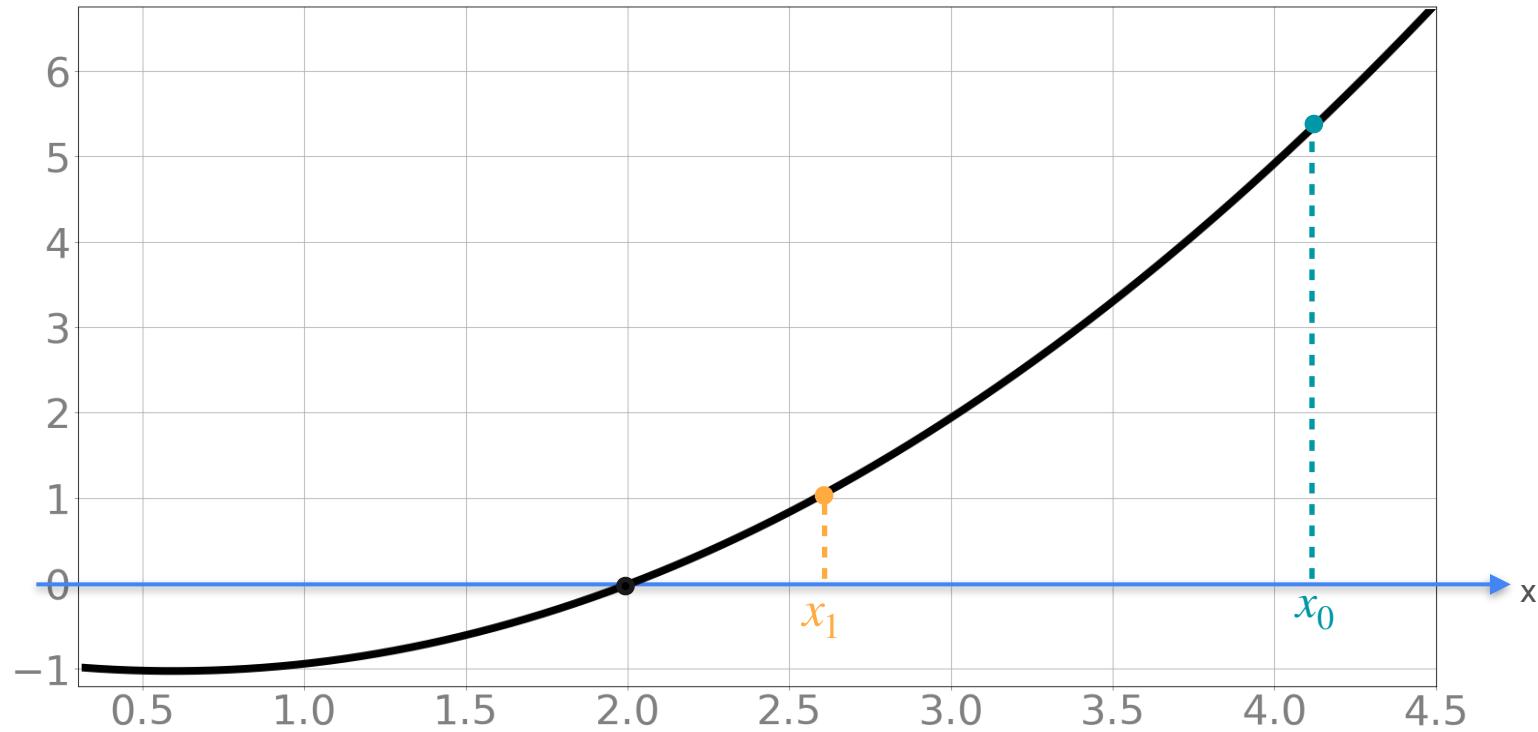
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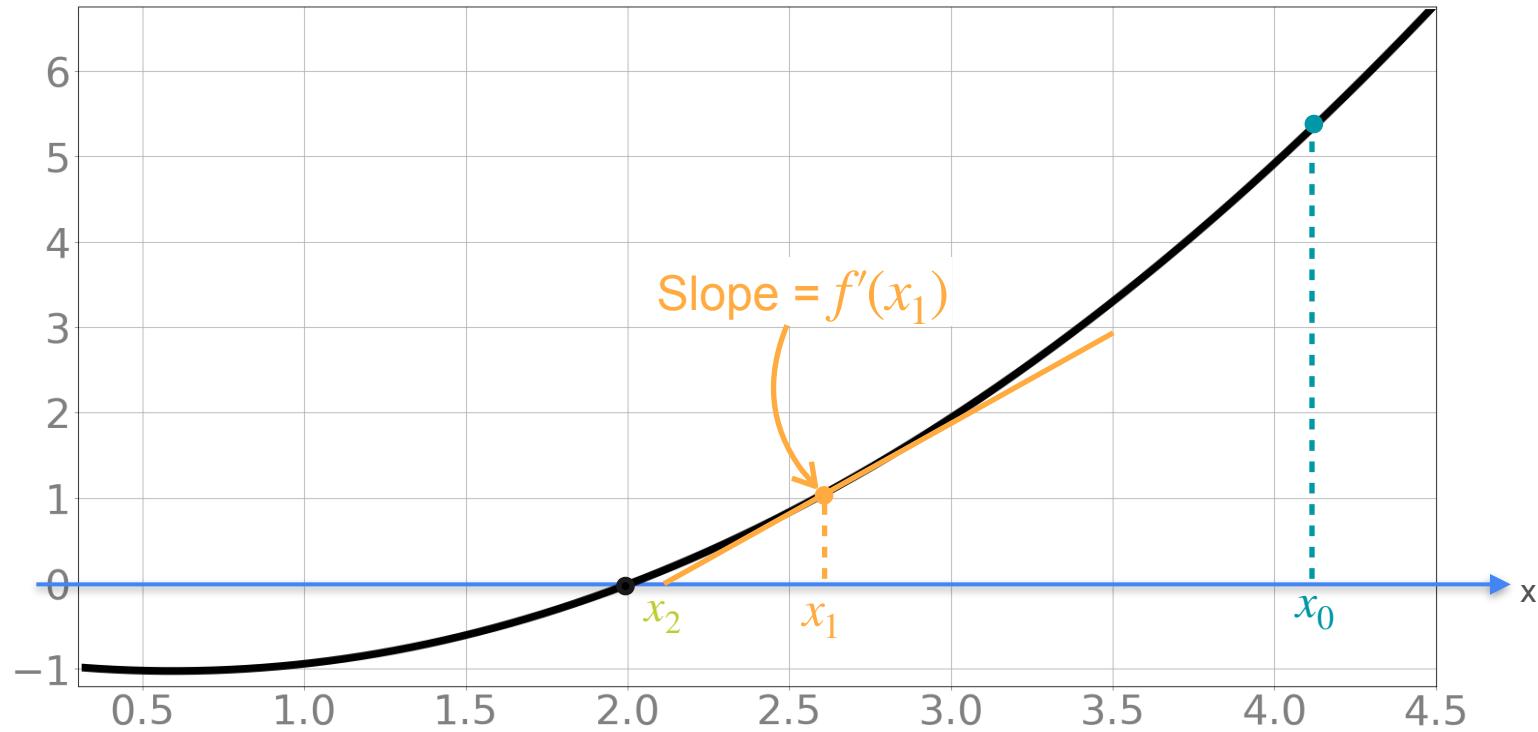
Update Approximation



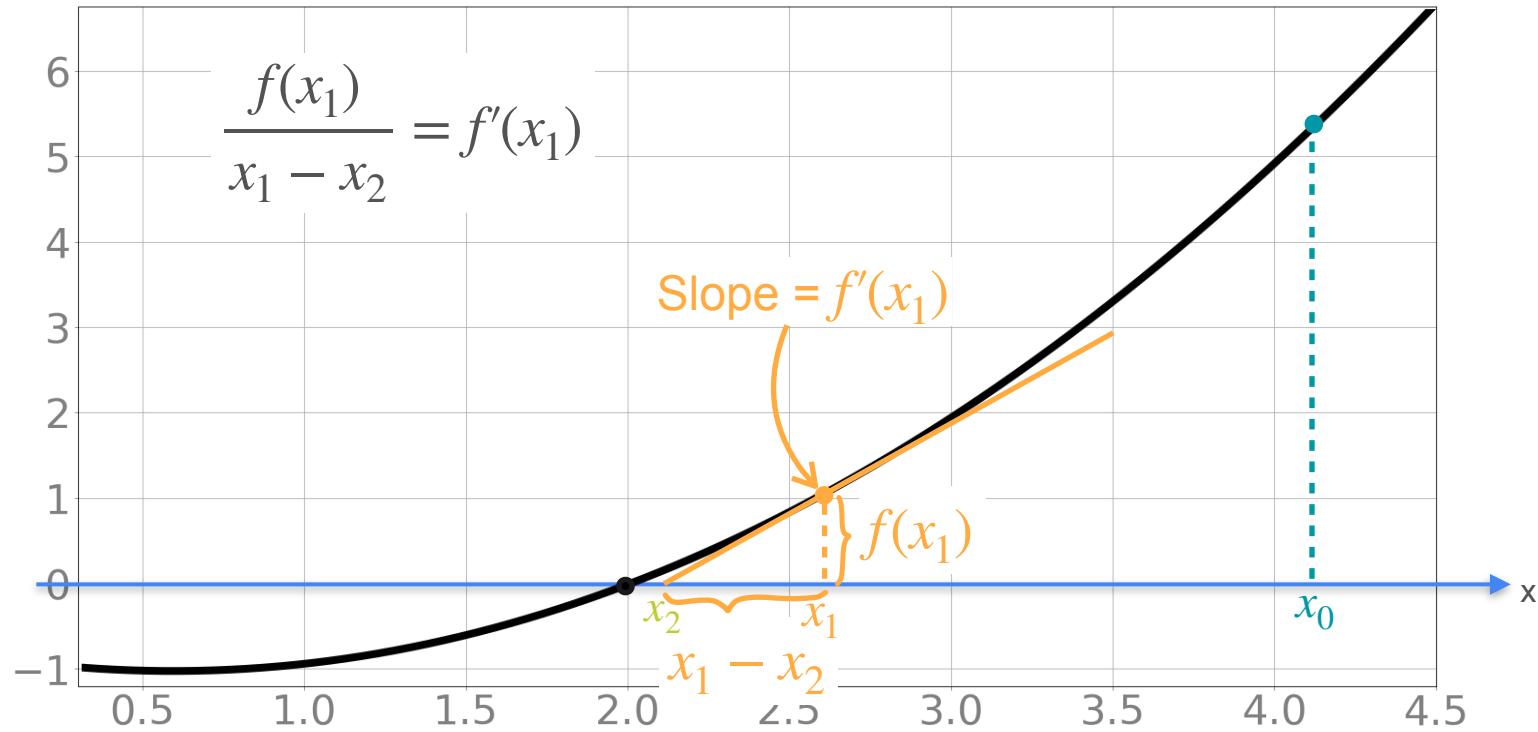
Update Approximation



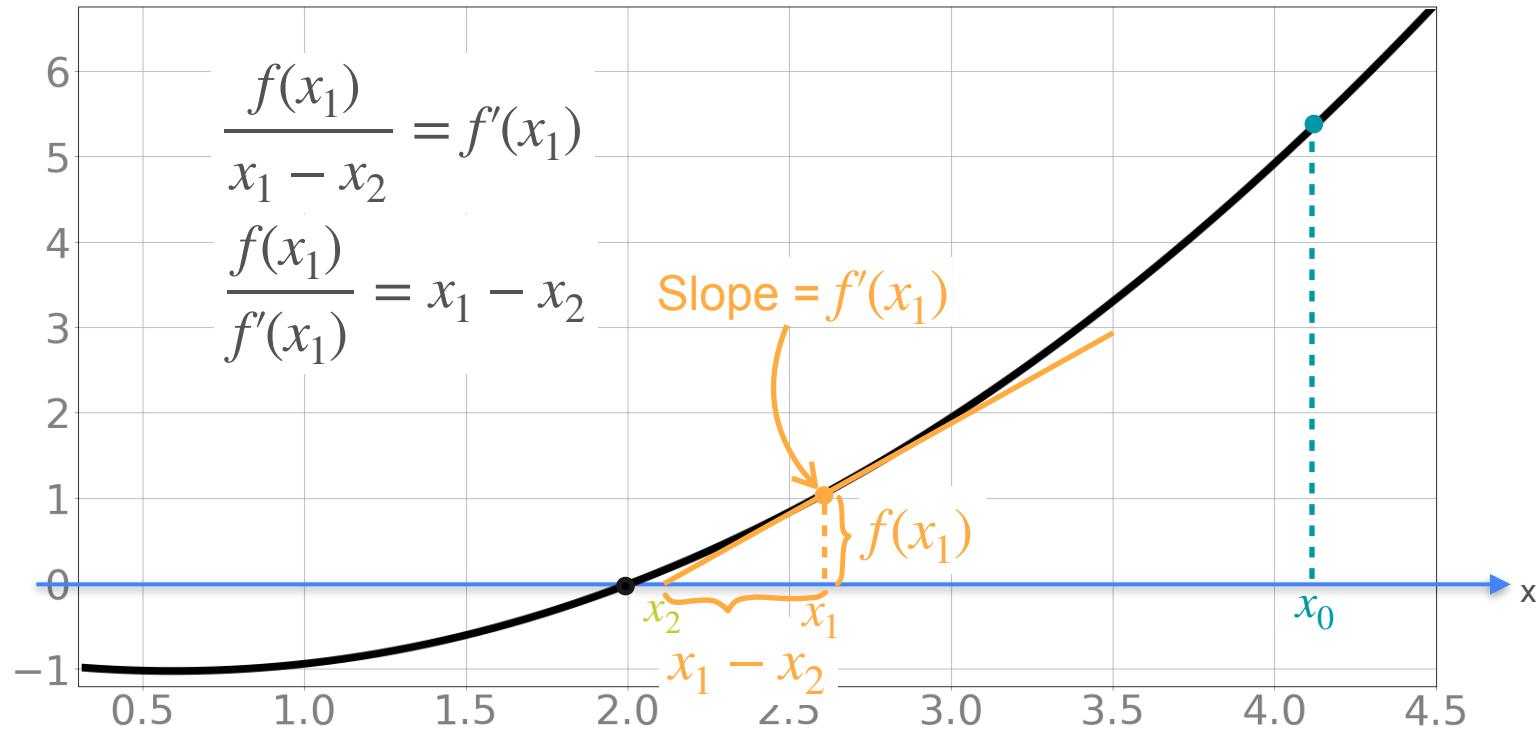
Update Approximation



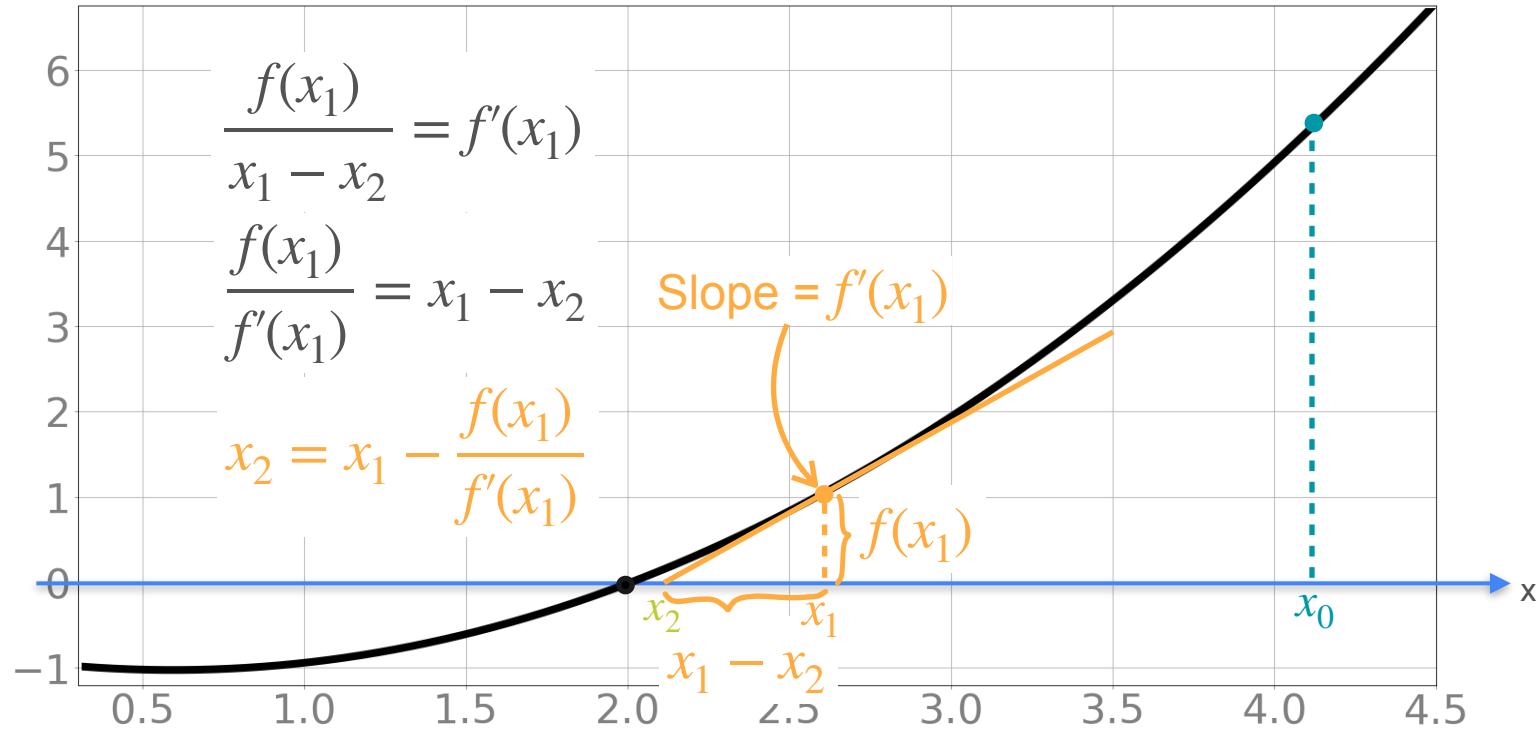
Update Approximation



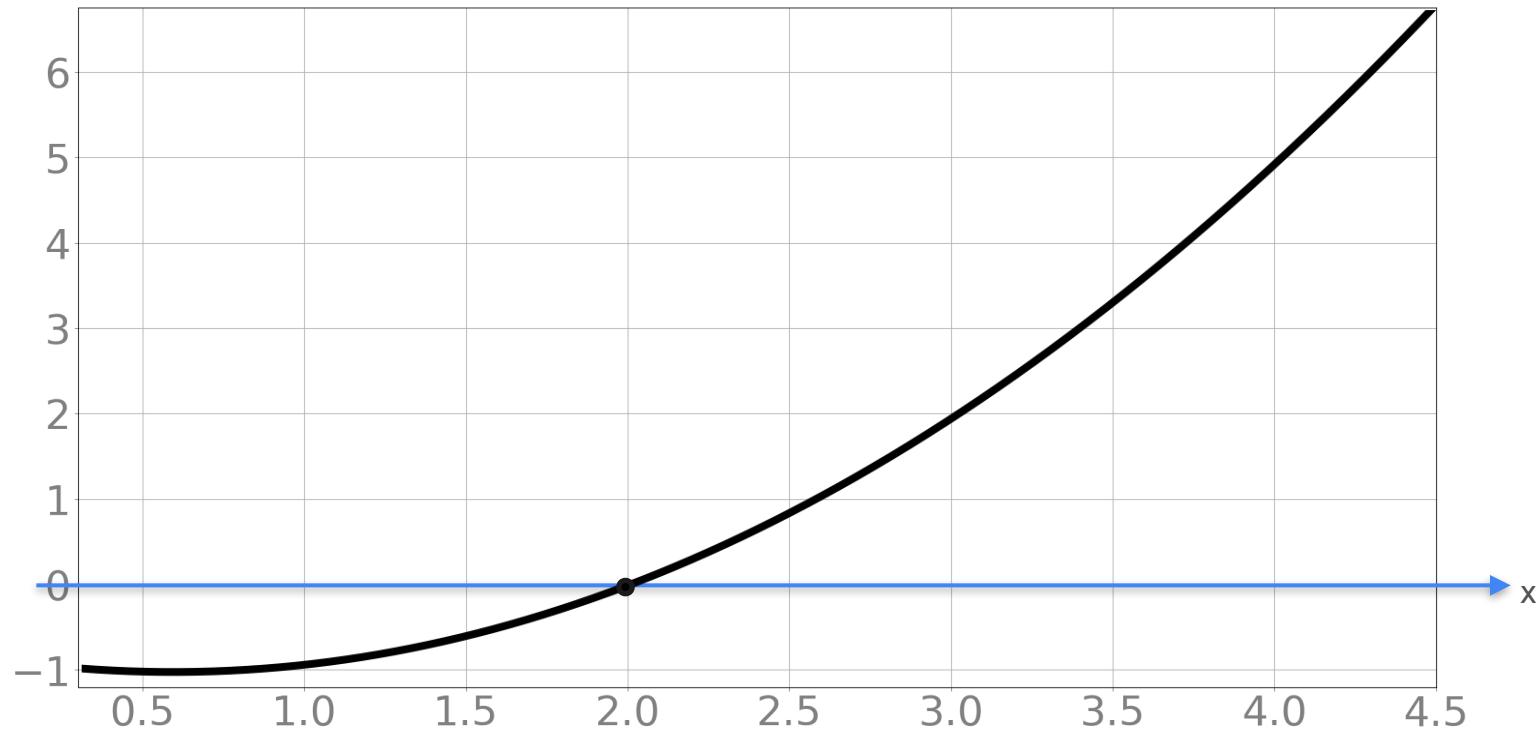
Update Approximation



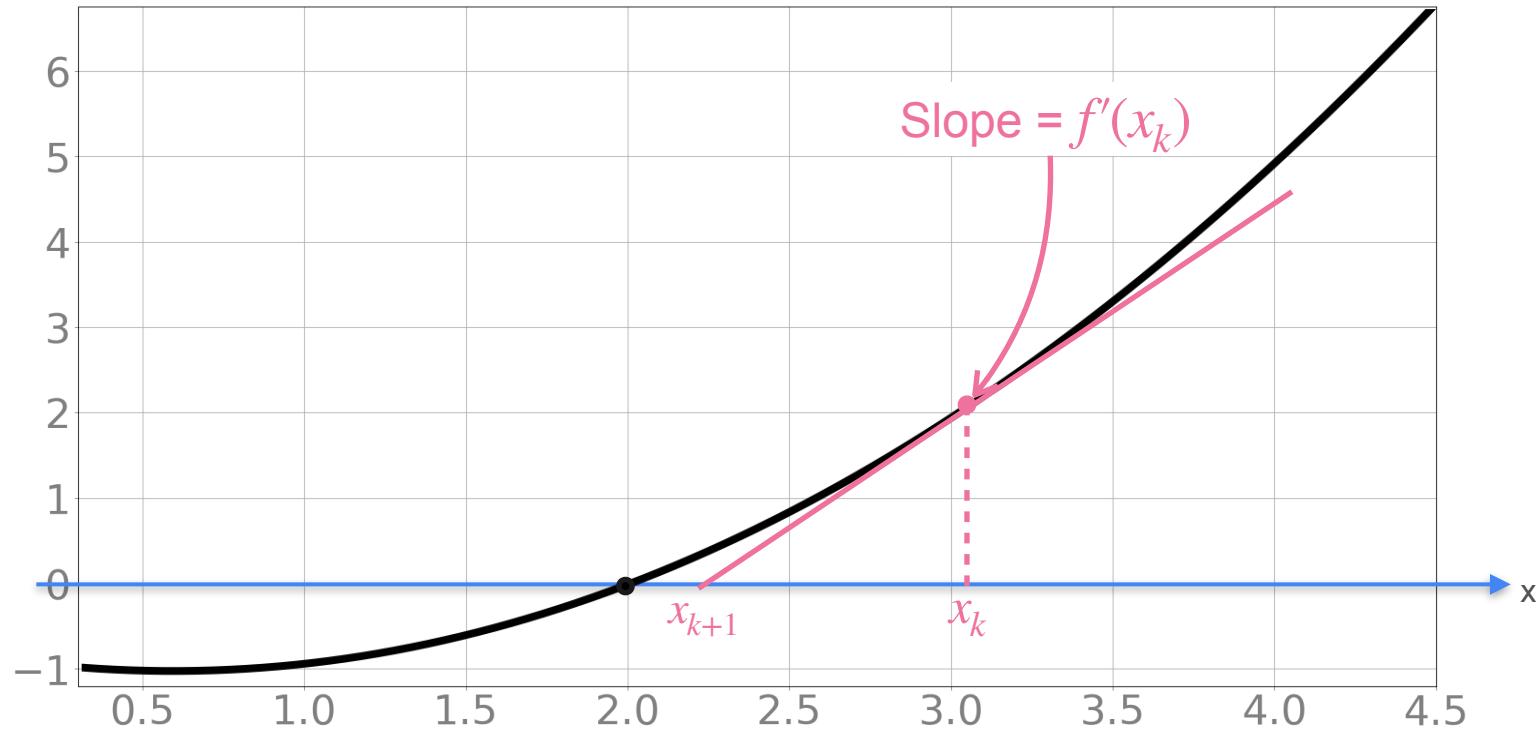
Update Approximation



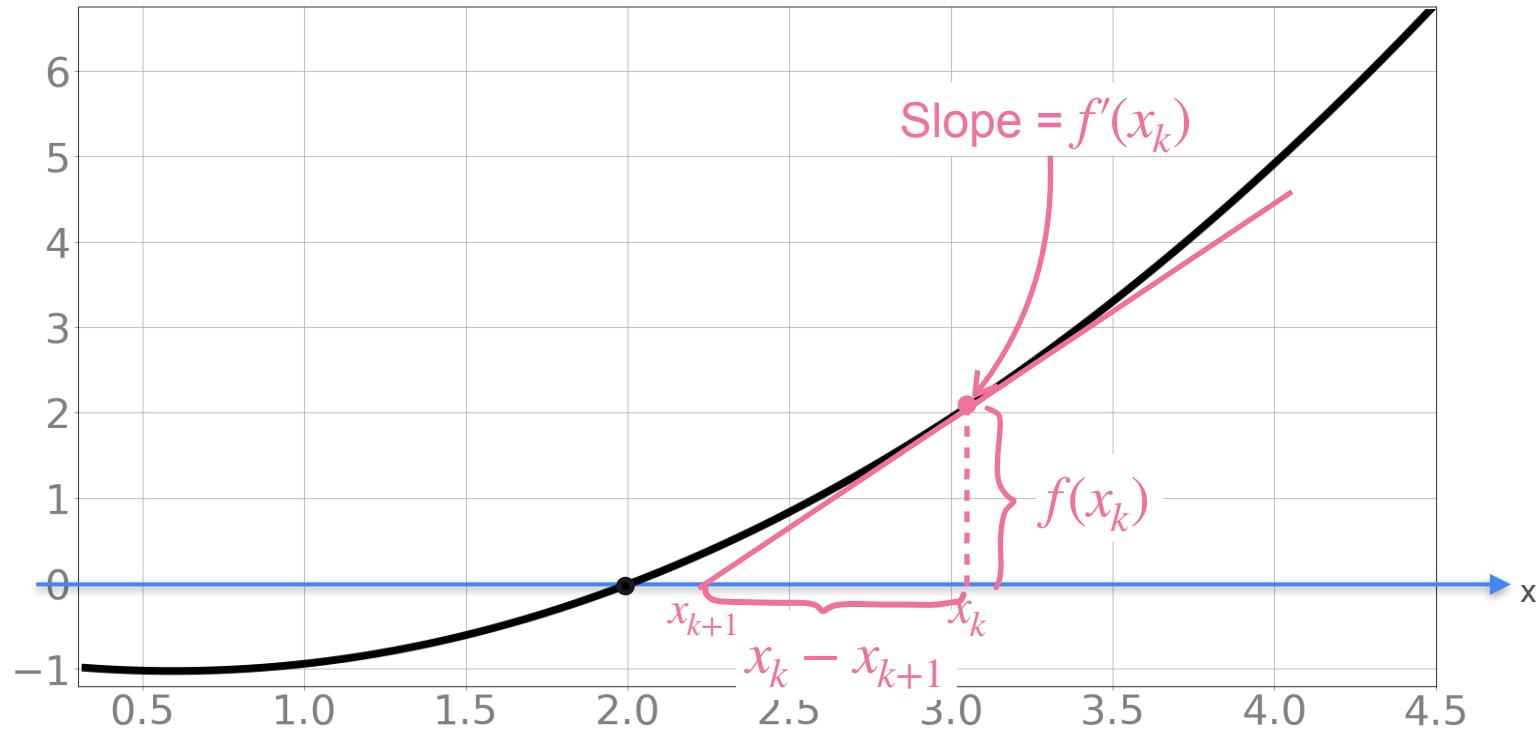
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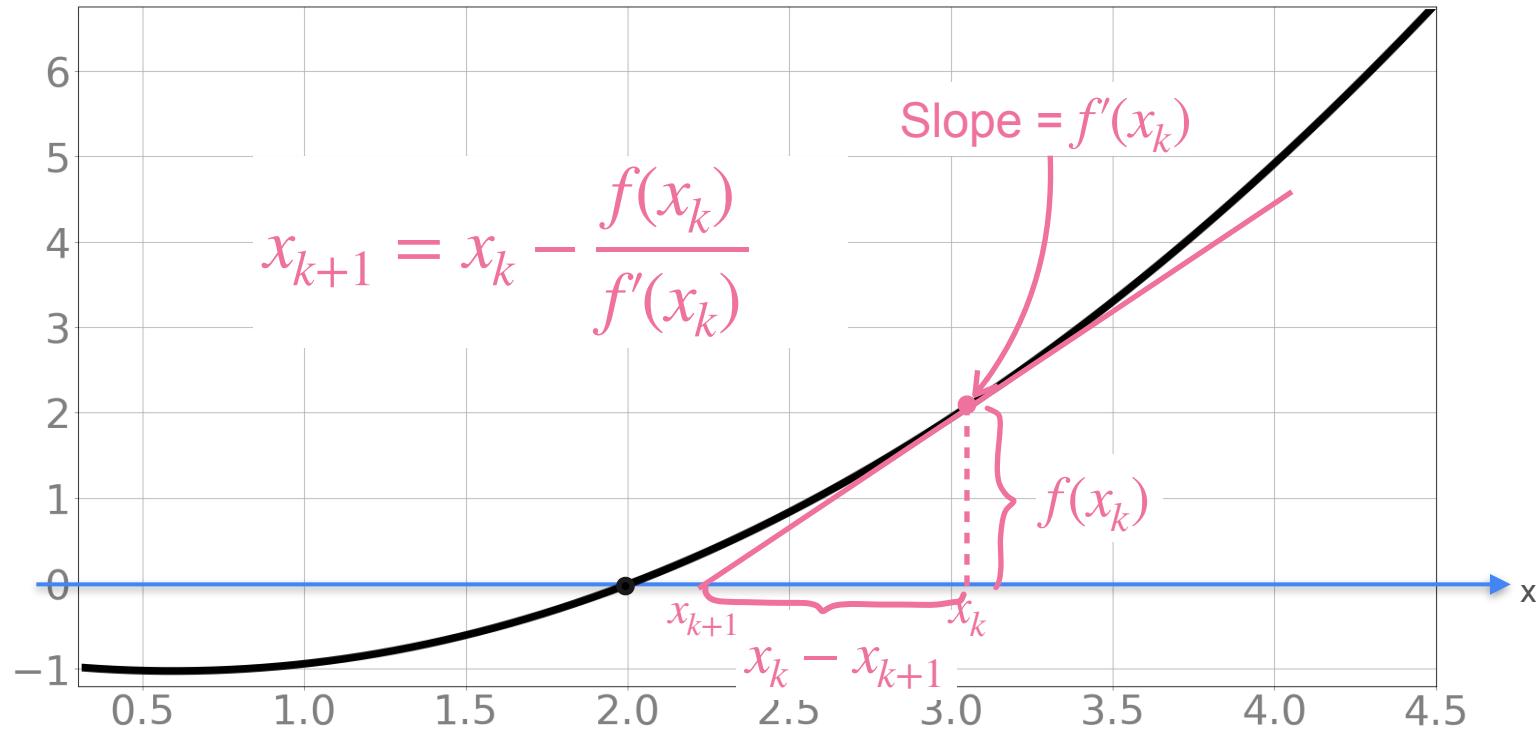
Update Approximation



Update Approximation



Update Approximation



Newton's Method for Optimization

Newton's Method for Optimization



Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$



Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$



NM for Optimization

Goal: minimize $g(x)$ find zeros of $g'(x)$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$



NM for Optimization

Goal: minimize $g(x)$ find zeros of $g'(x)$

$$f(x) \mapsto g'(x)$$

$$f'(x) \mapsto (g'(x))'$$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0



NM for Optimization

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Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

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2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$



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3) Repeat 2) until you find the root.



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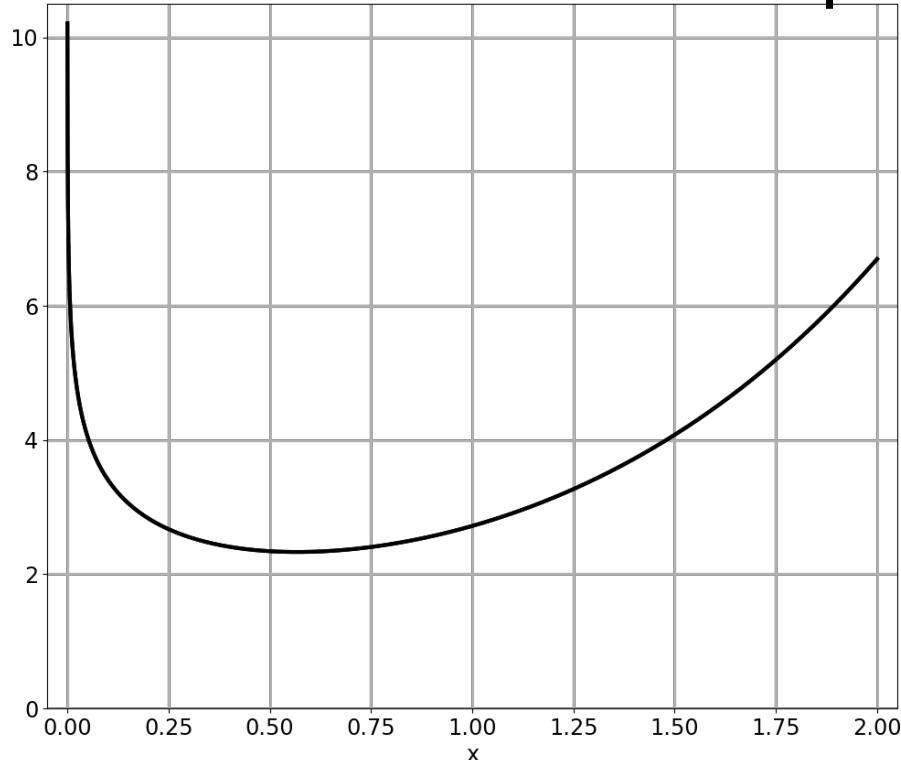
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

**Newton's method:
An example**

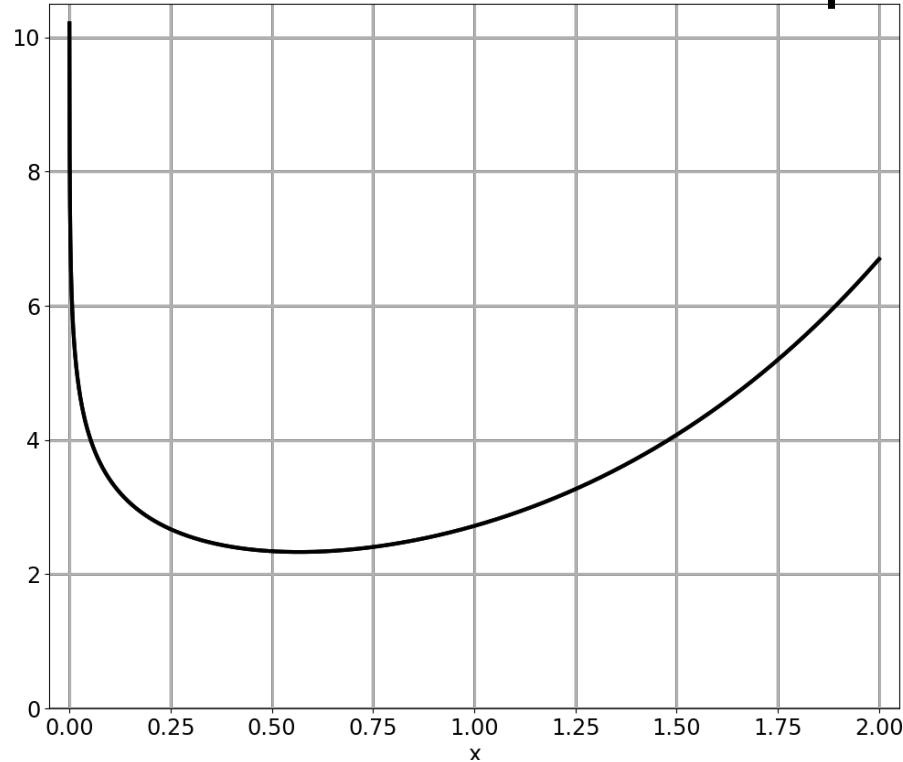
Newton's Method for Optimization

Newton's Method for Optimization



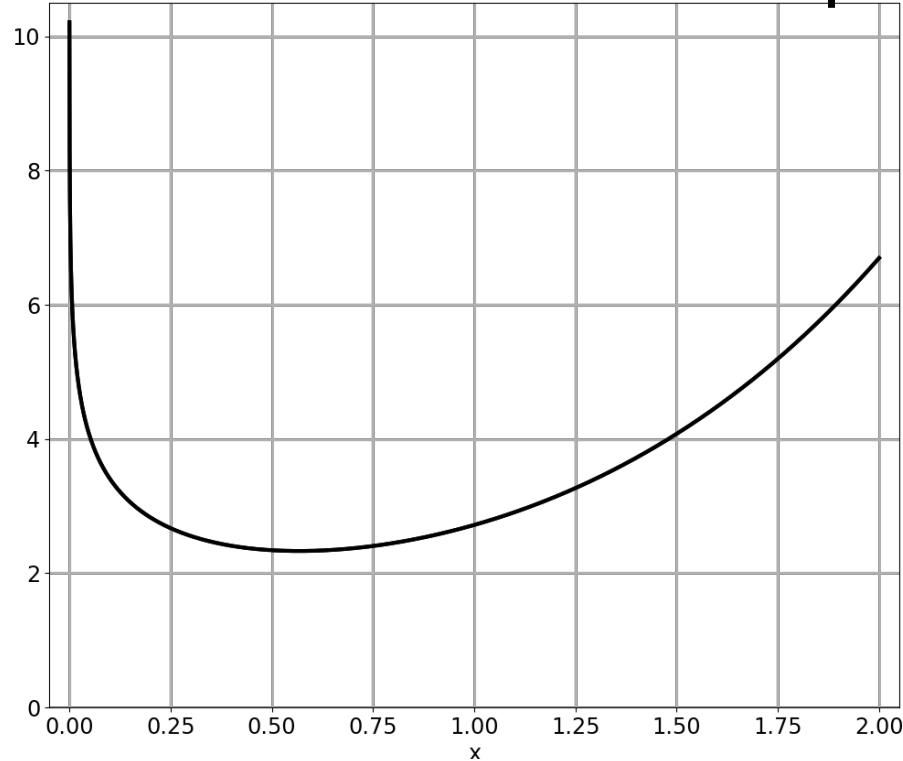
$$g(x) = e^x - \log(x)$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

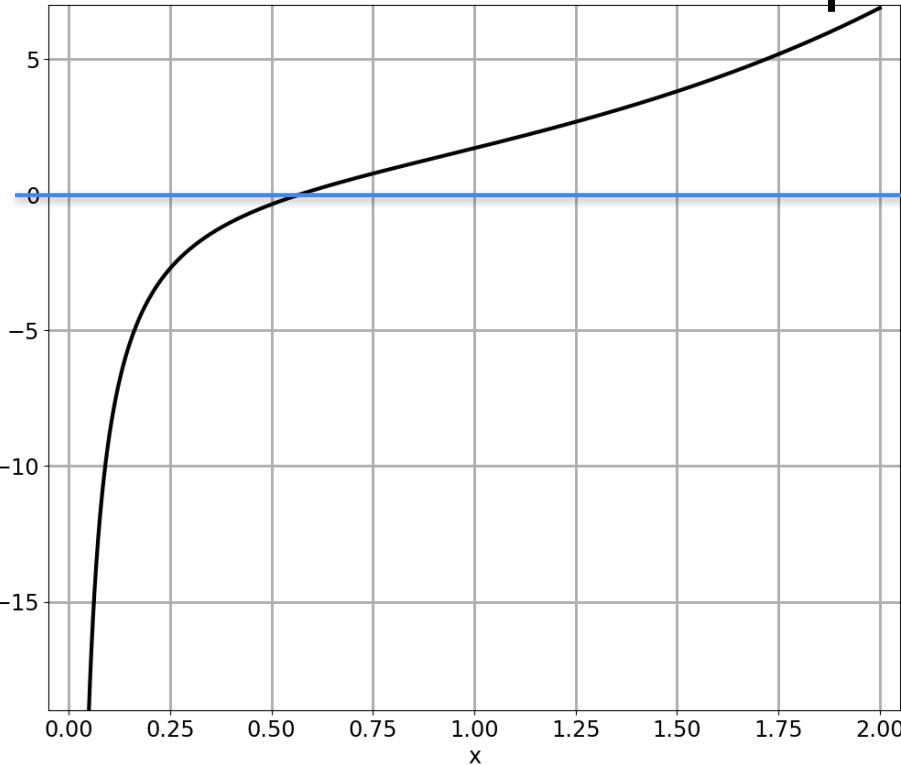
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

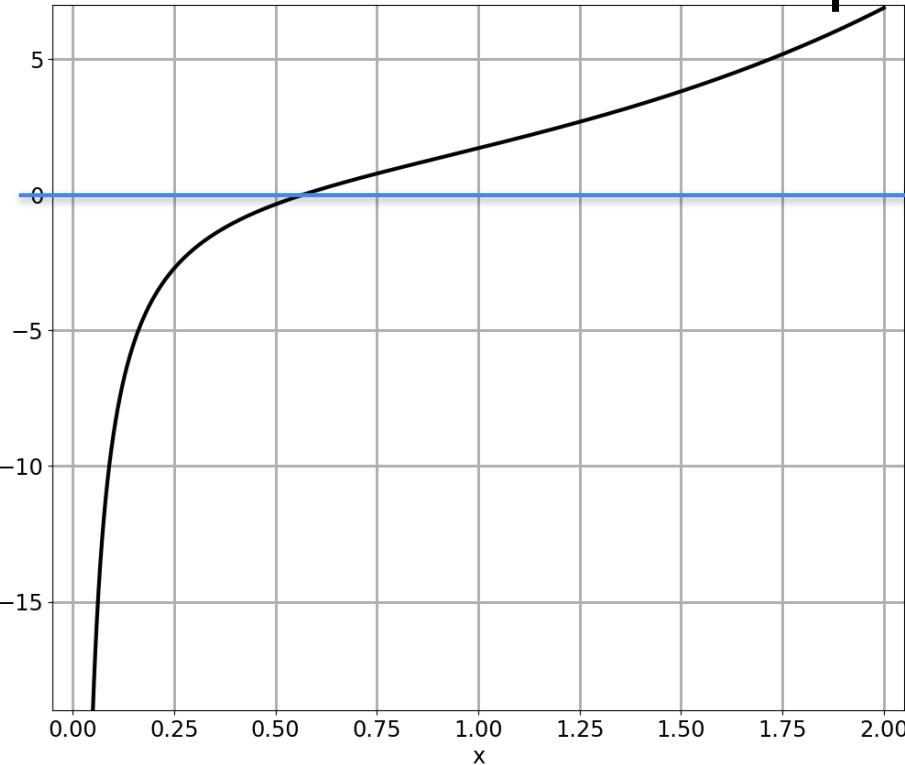
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

Newton's Method for Optimization

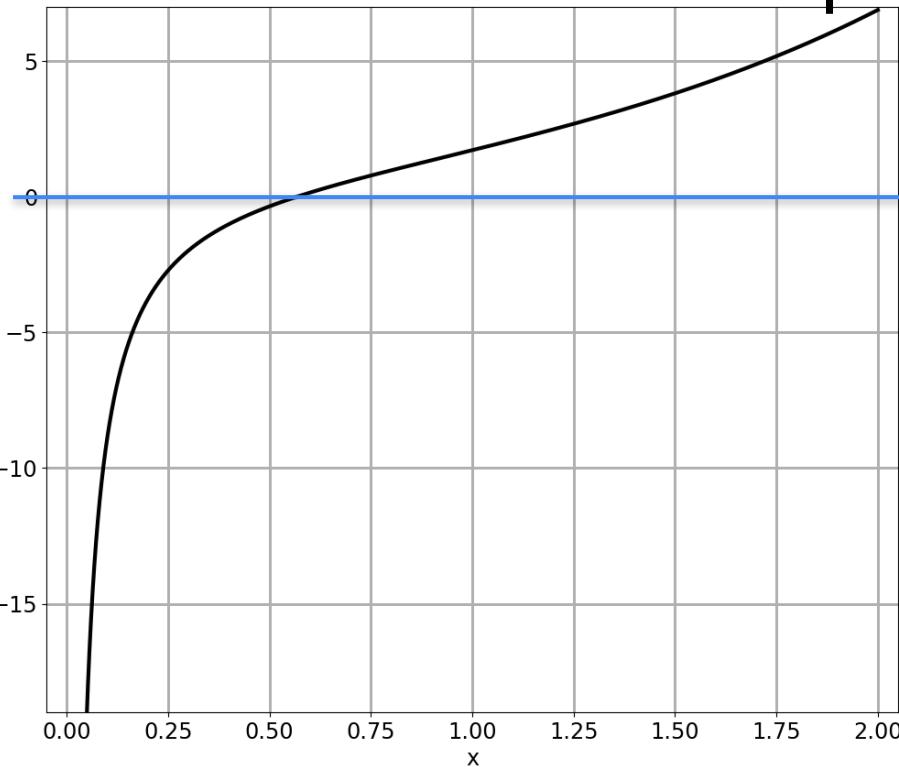


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

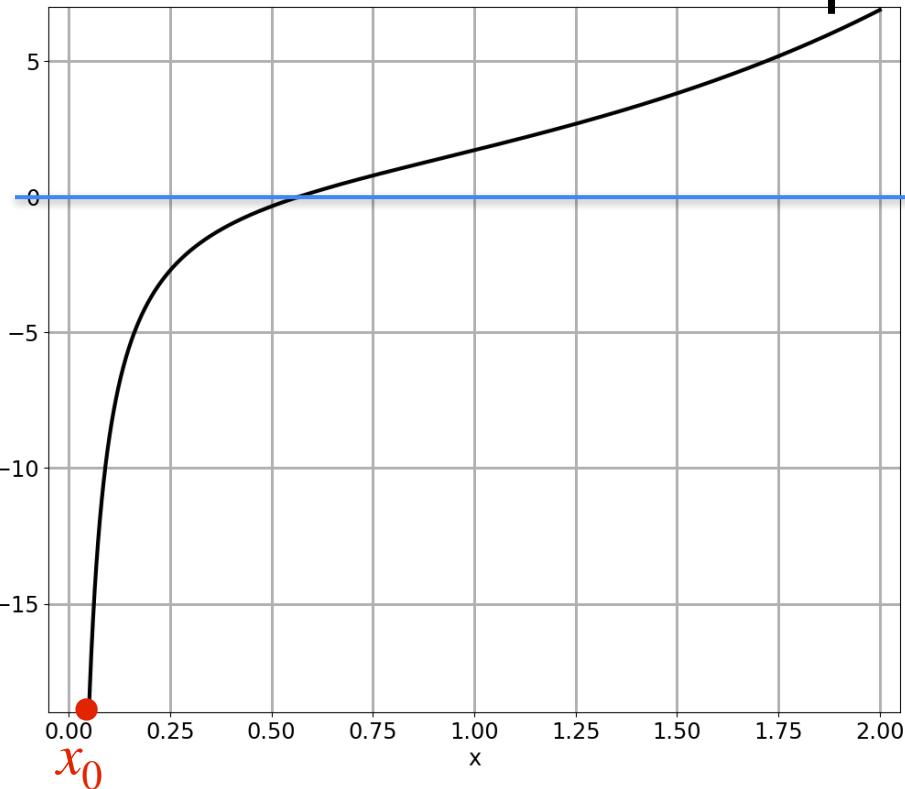


$$g(x) = e^x - \log(x) \quad \underbrace{g'(x) = e^x - 1/x}_{f(x)}$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2} \quad \underbrace{ f'(x)}_{\frac{1}{x^2}}$$

Newton's Method for Optimization

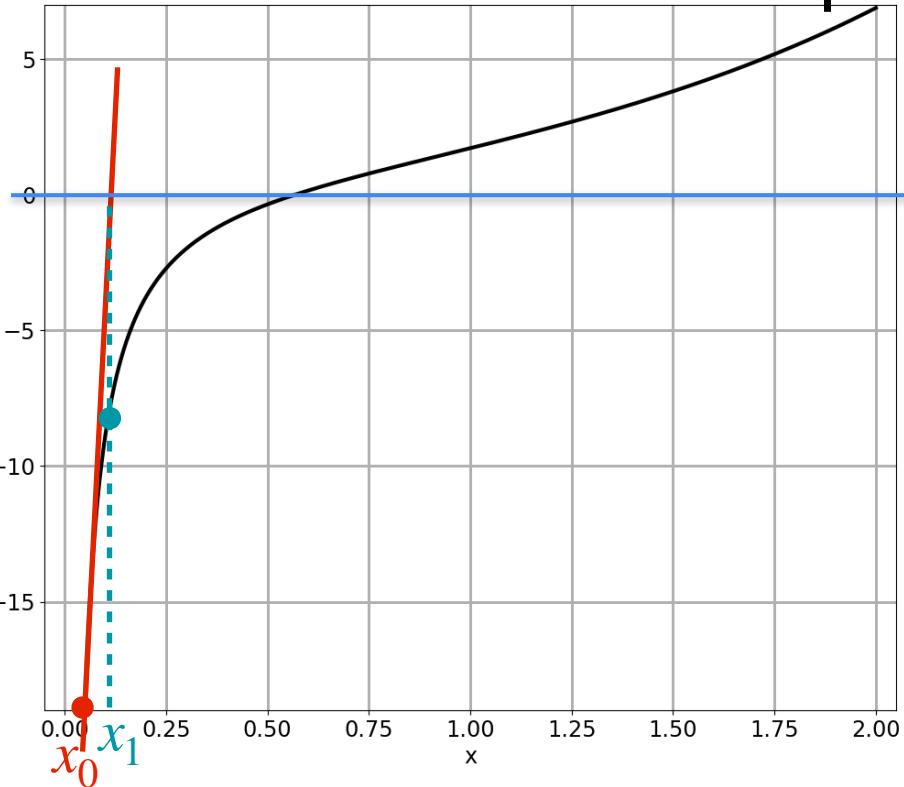


$$g(x) = e^x - \log(x)$$
$$g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$
$$x_0 = 0.05$$
$$f'(x)$$

Newton's Method for Optimization

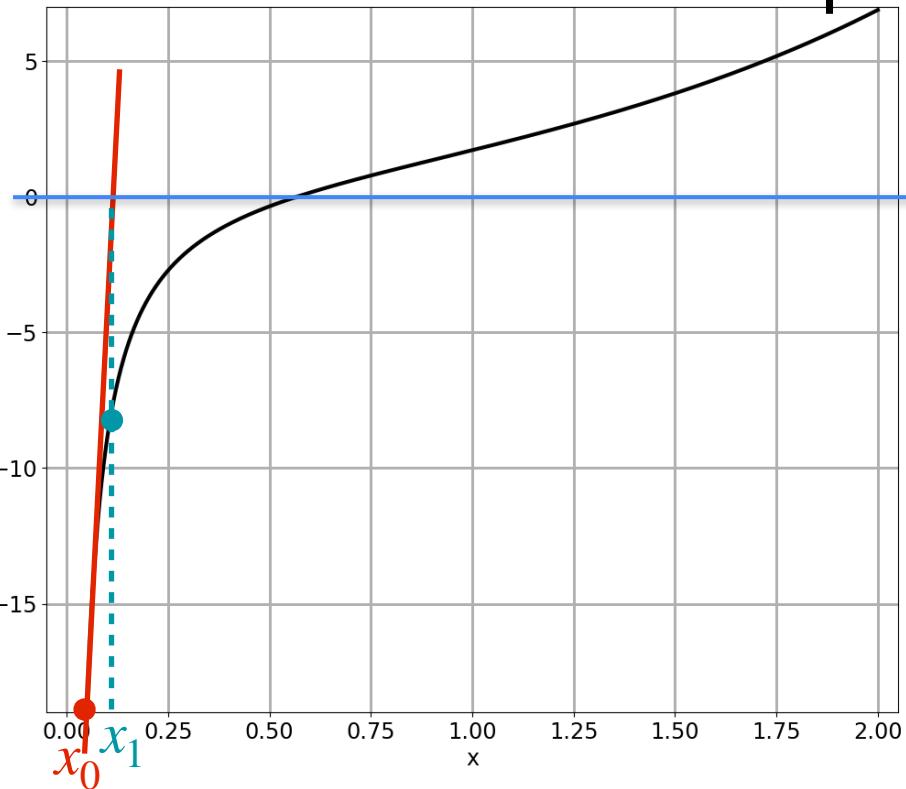


$$g(x) = e^x - \log(x)$$
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$$f'(x)$$
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Newton's Method for Optimization

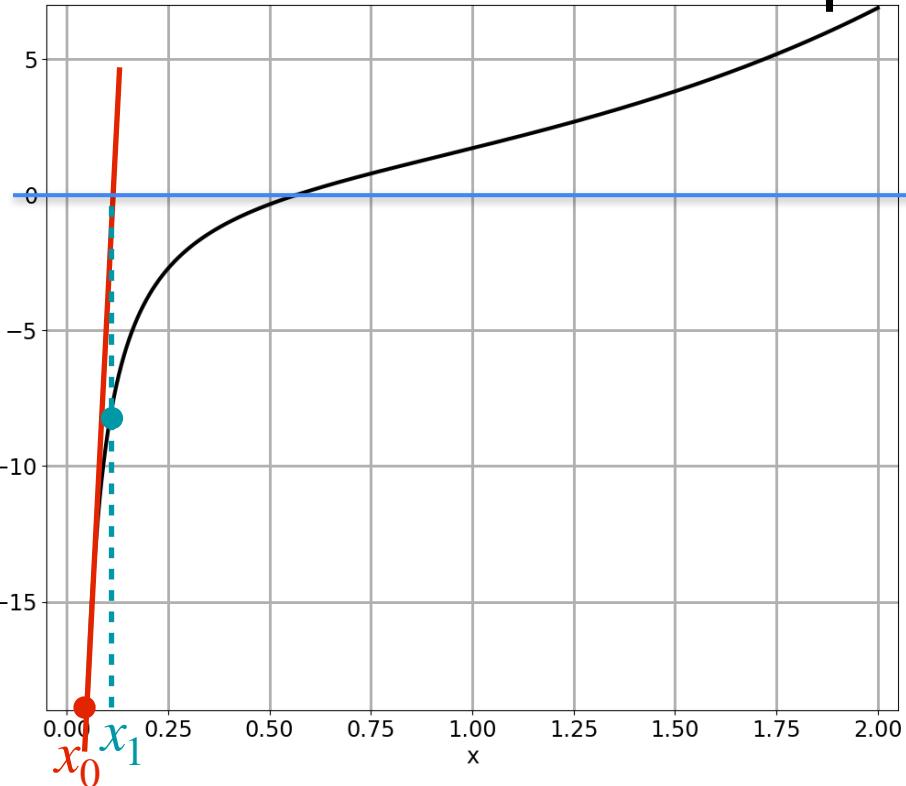


$$g(x) = e^x - \log(x) \quad \overbrace{g'(x)}^{f(x)} = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$
$$x_0 = 0.05$$
$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$
$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)}$$

Newton's Method for Optimization

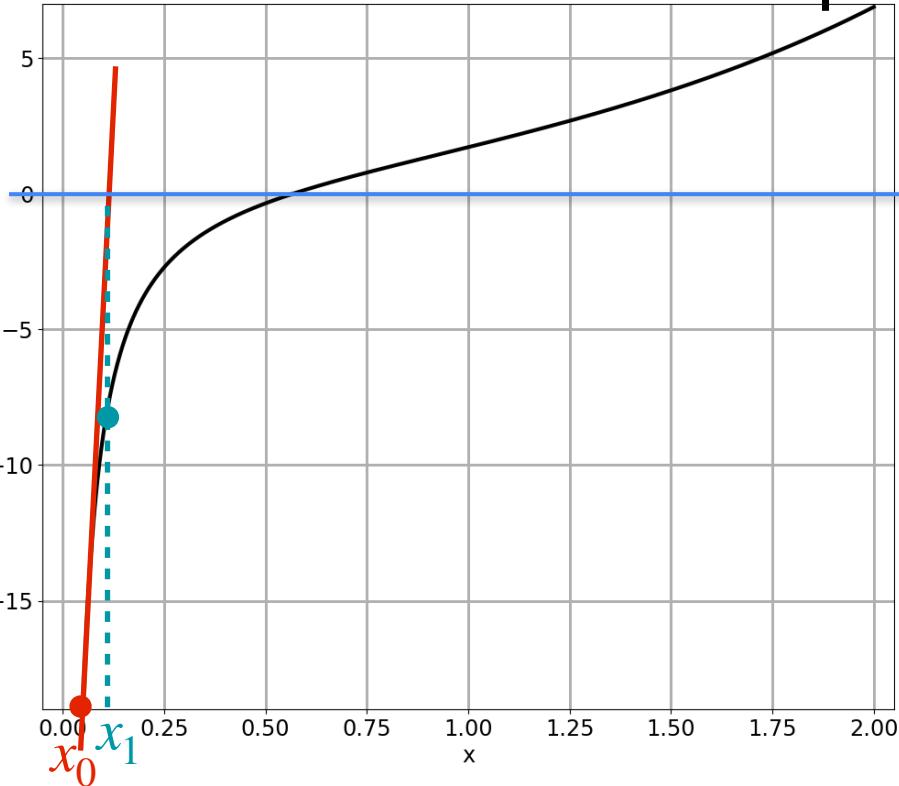


$$g(x) = e^x - \log(x) \quad \overbrace{g'(x)}^{f(x)} = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$
$$x_0 = 0.05$$
$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'} \quad \overbrace{f'(x)}^{g'(x)}$$
$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)} = 0.097$$

Newton's Method for Optimization

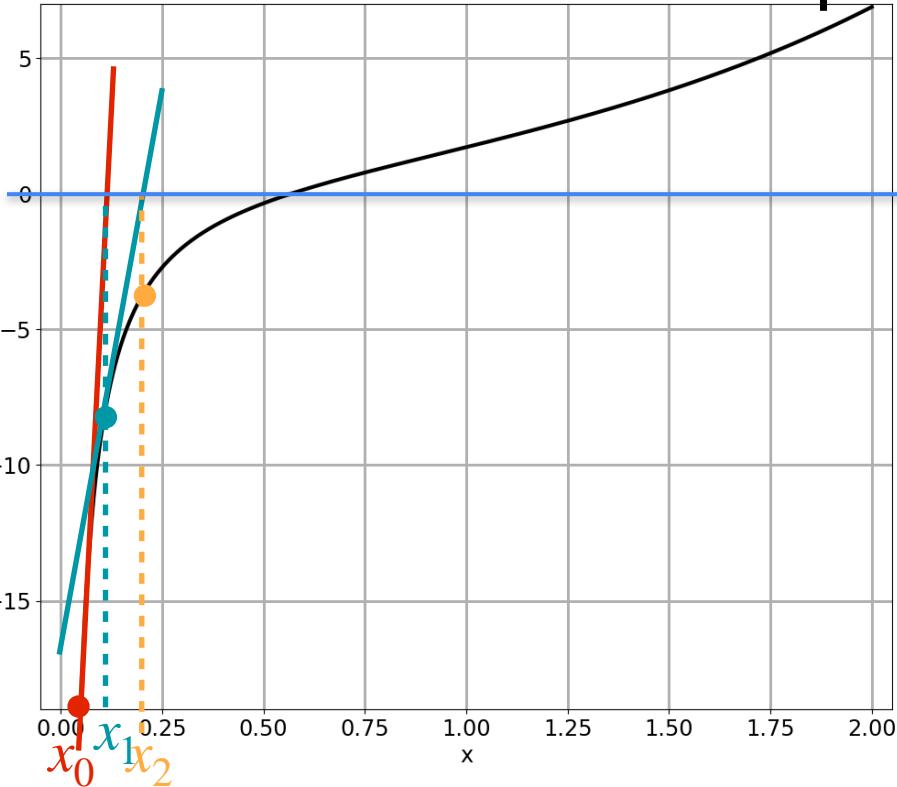


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



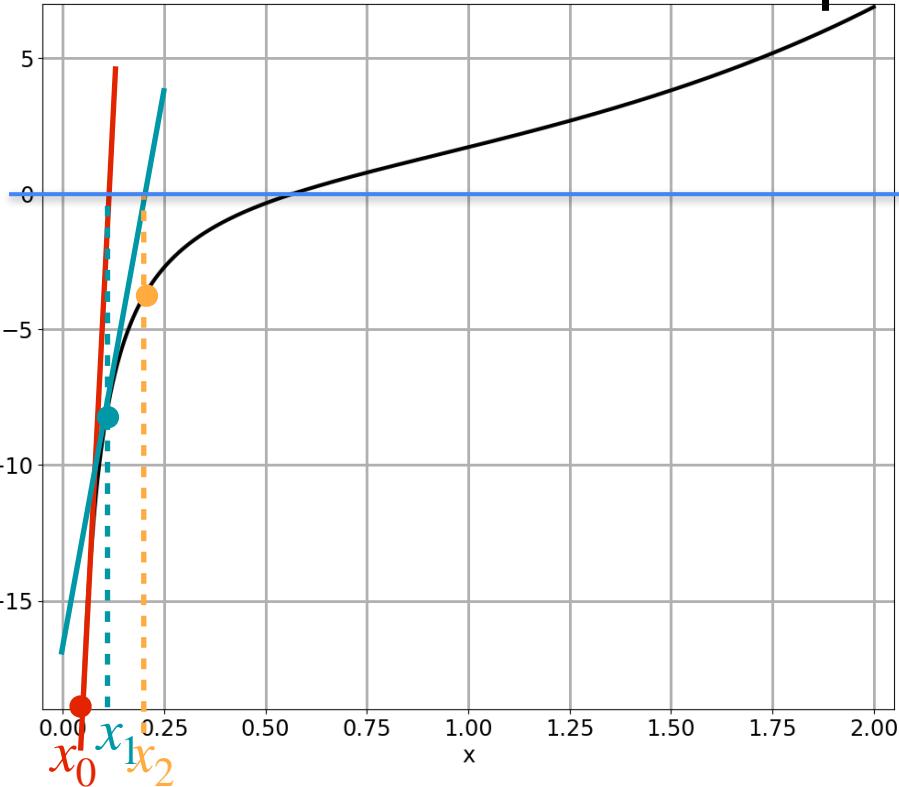
$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_1 = 0.097$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

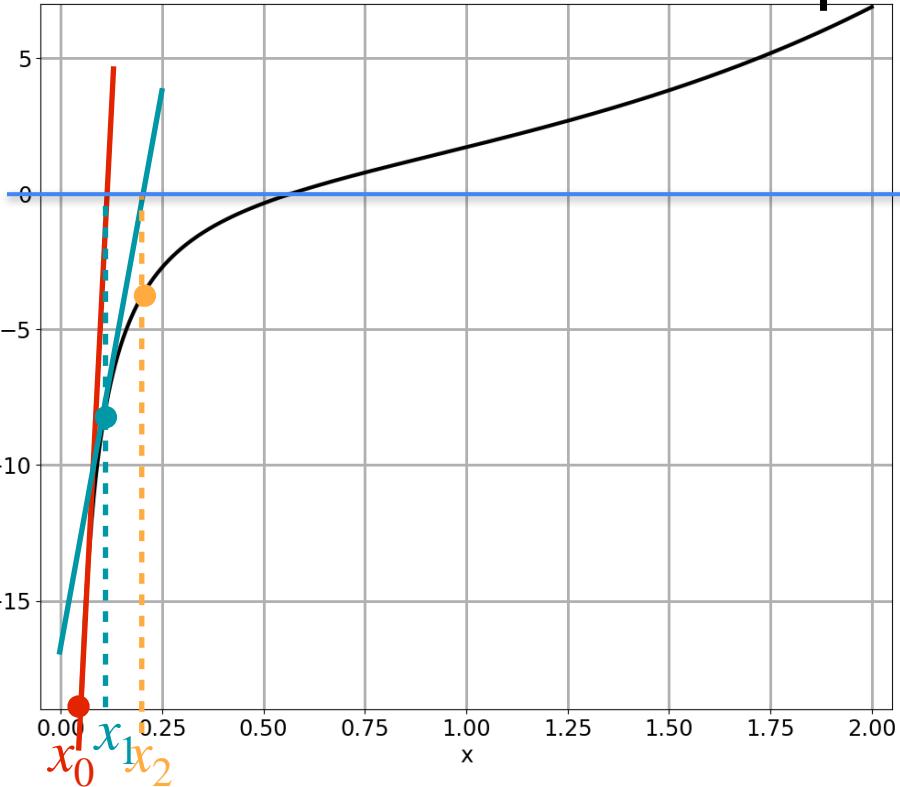
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_1 = 0.097$$

$$x_2 = x_1 - \frac{g'(x_1)}{(g'(x_1))'}$$

$$= 0.097 - \frac{\left(e^{0.097} - \frac{1}{0.097}\right)}{\left(e^{0.097} + \frac{1}{0.097^2}\right)} = 0.183$$

Newton's Method for Optimization

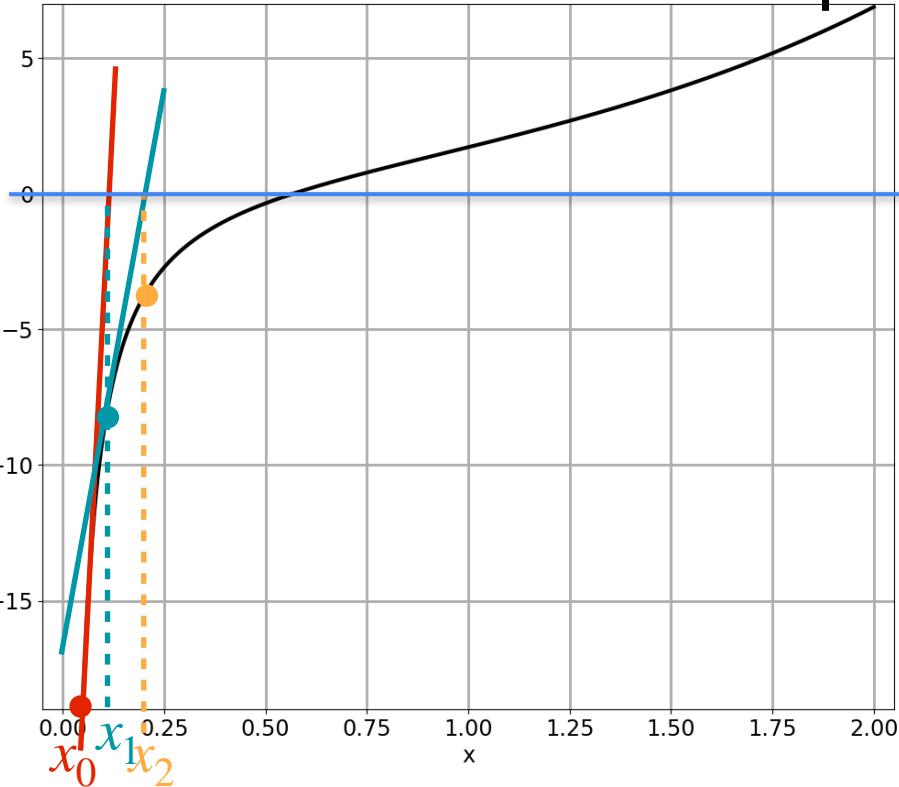


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



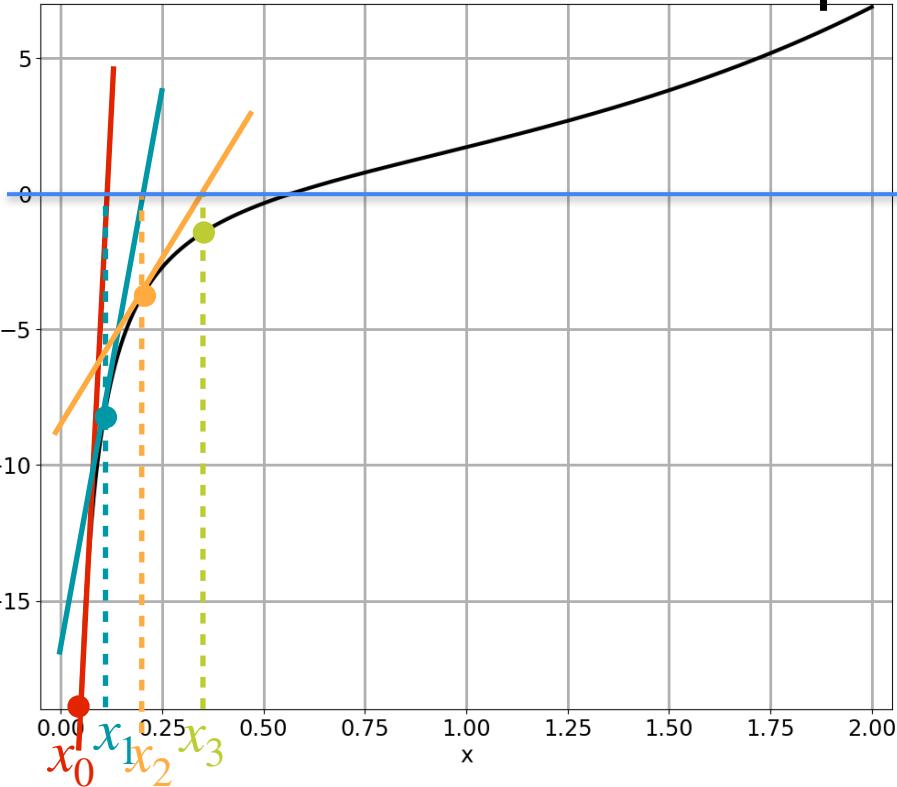
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$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

Newton's Method for Optimization



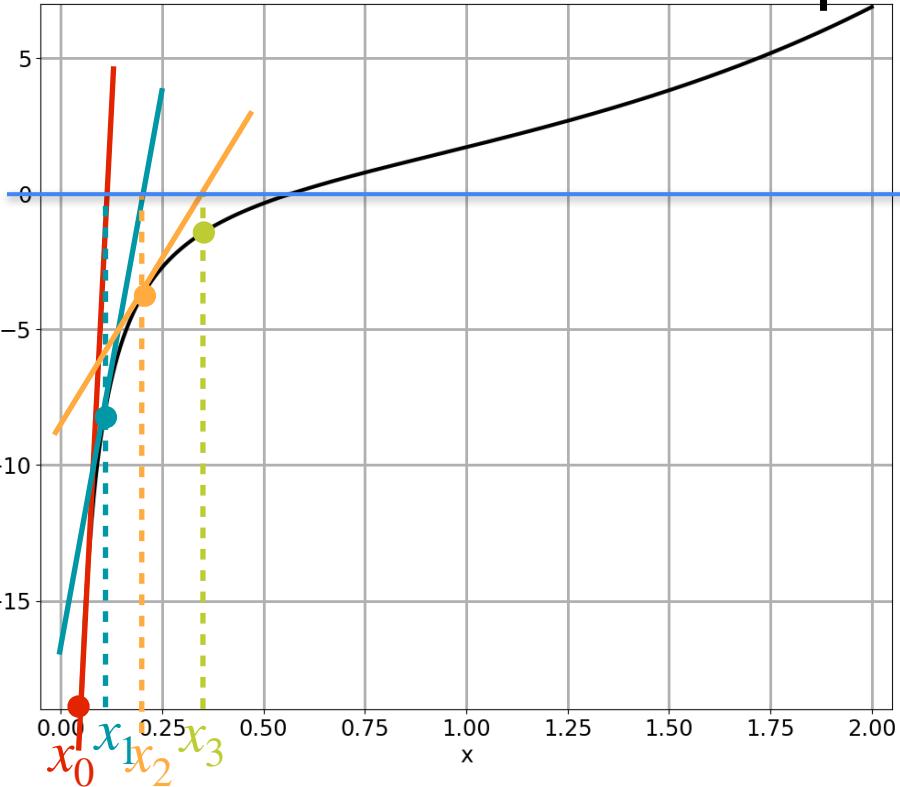
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Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

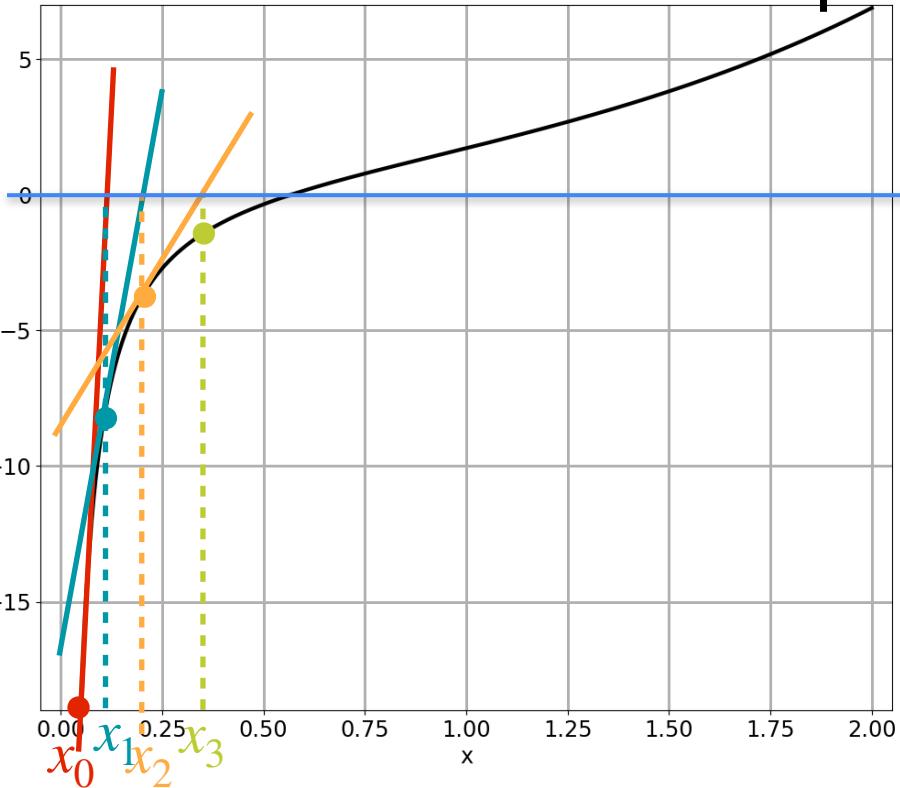
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

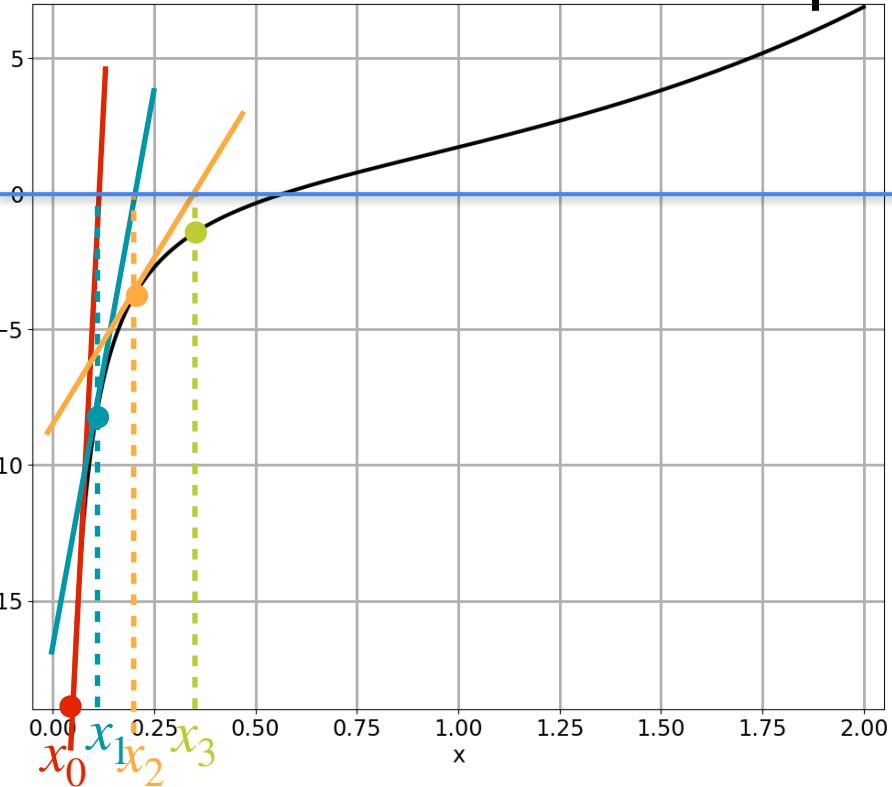
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)} = 0.320$$

Newton's Method for Optimization

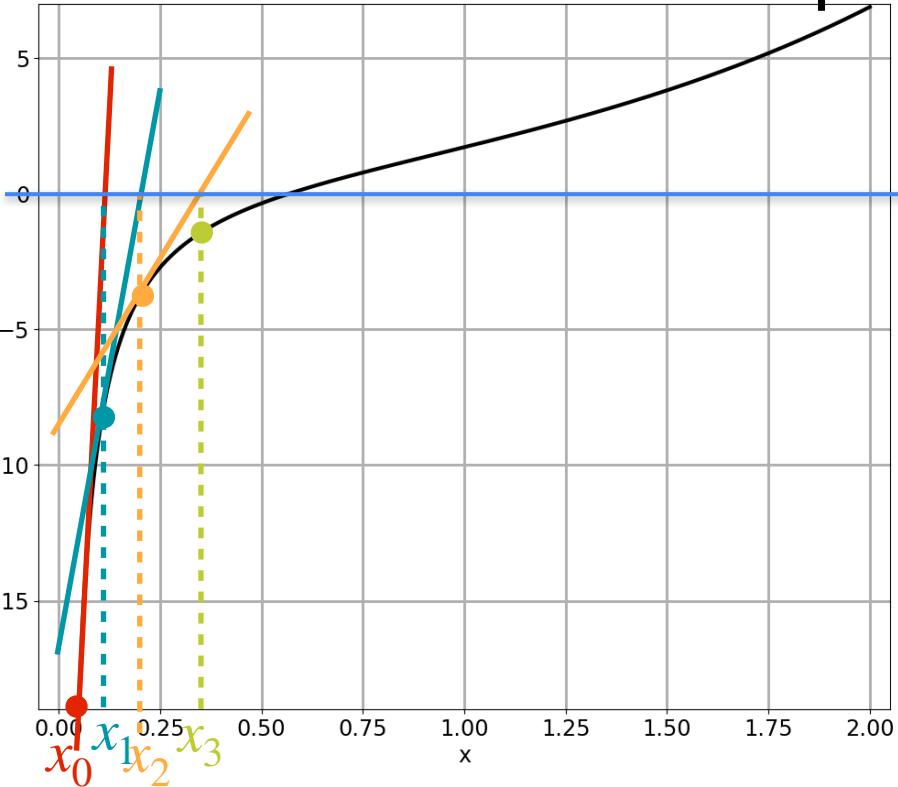


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



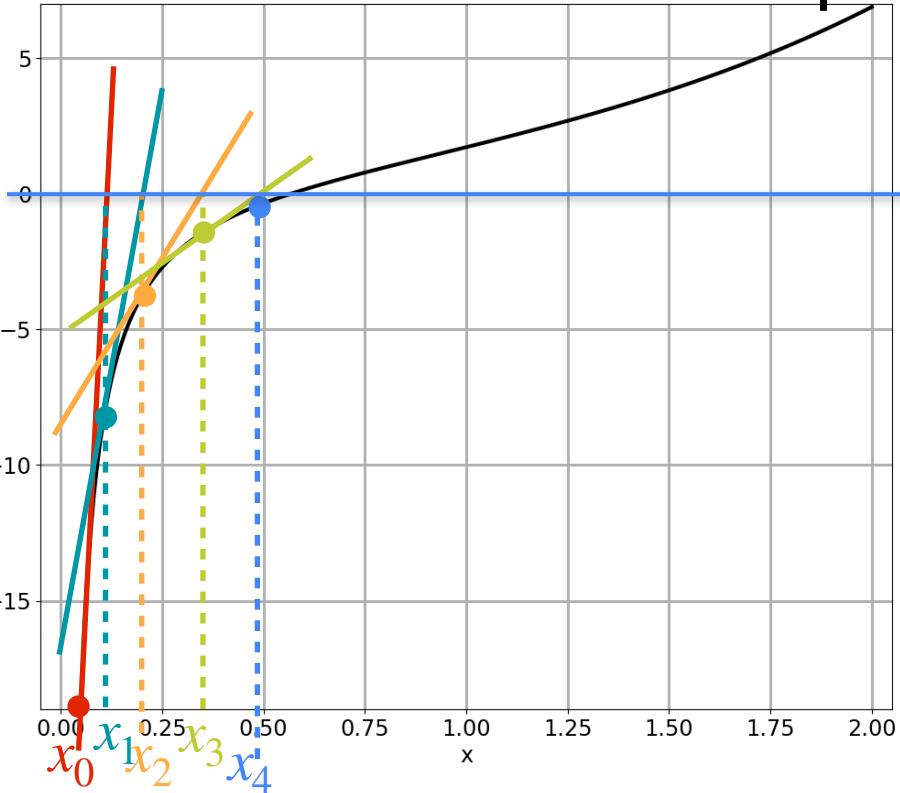
$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

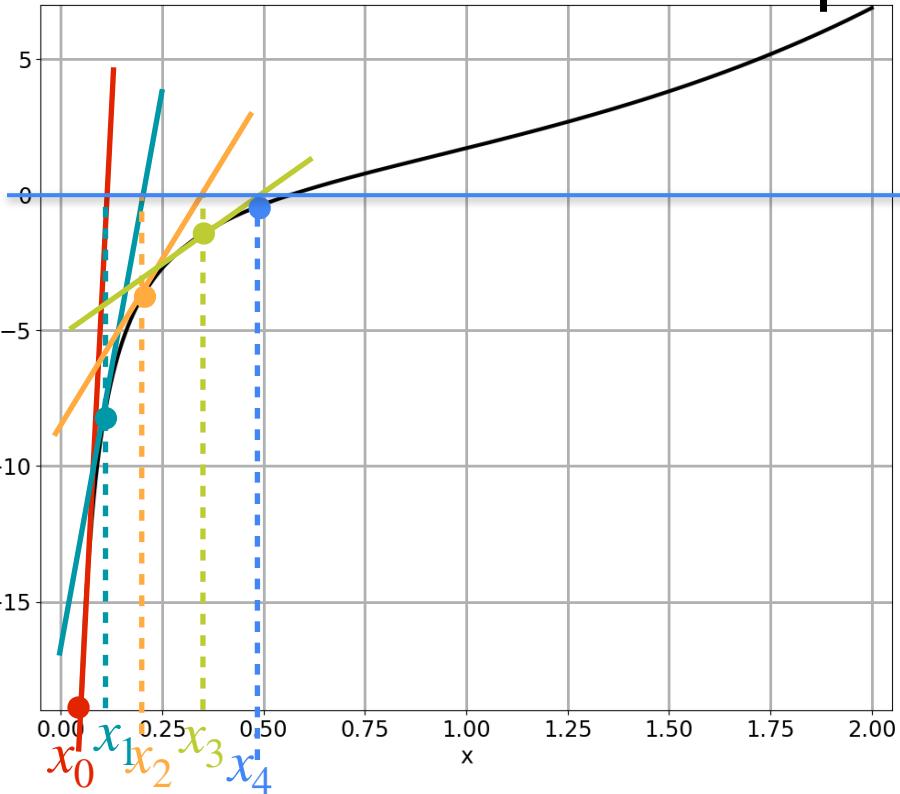
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

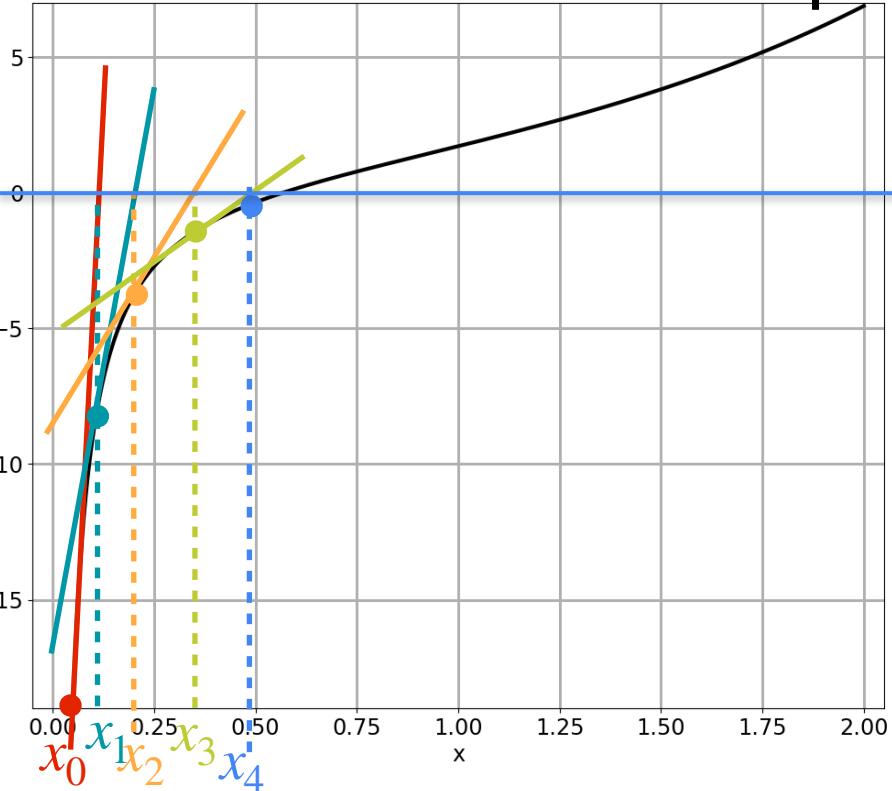
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)} = 0.477$$

Newton's Method for Optimization



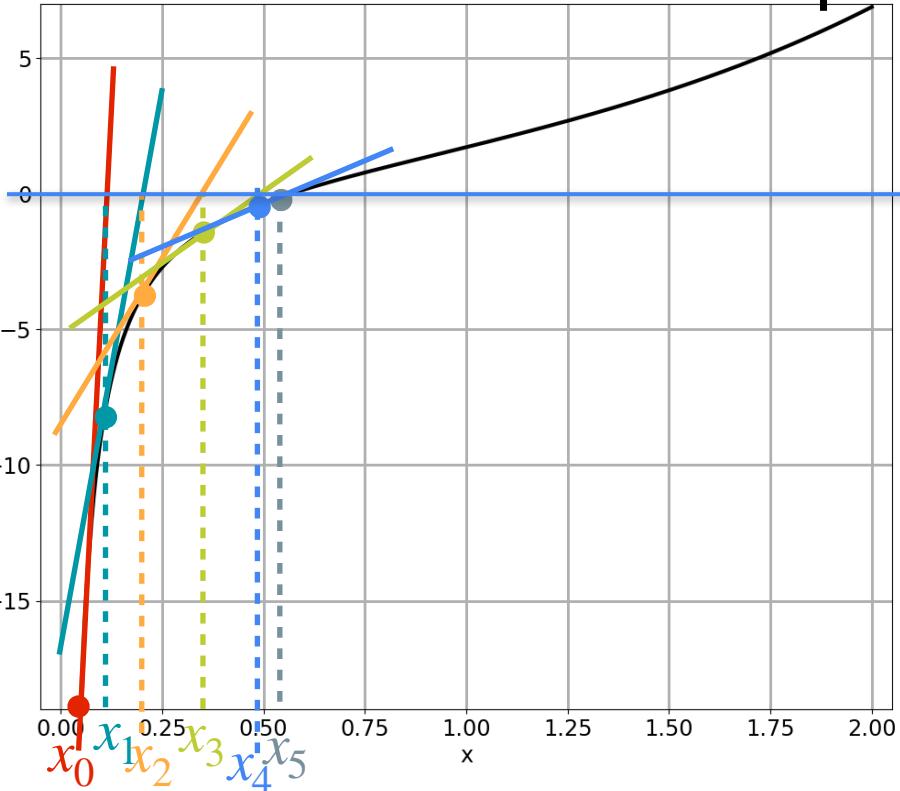
$$g(x) = e^x - \log(x)$$

$$g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.5671$$

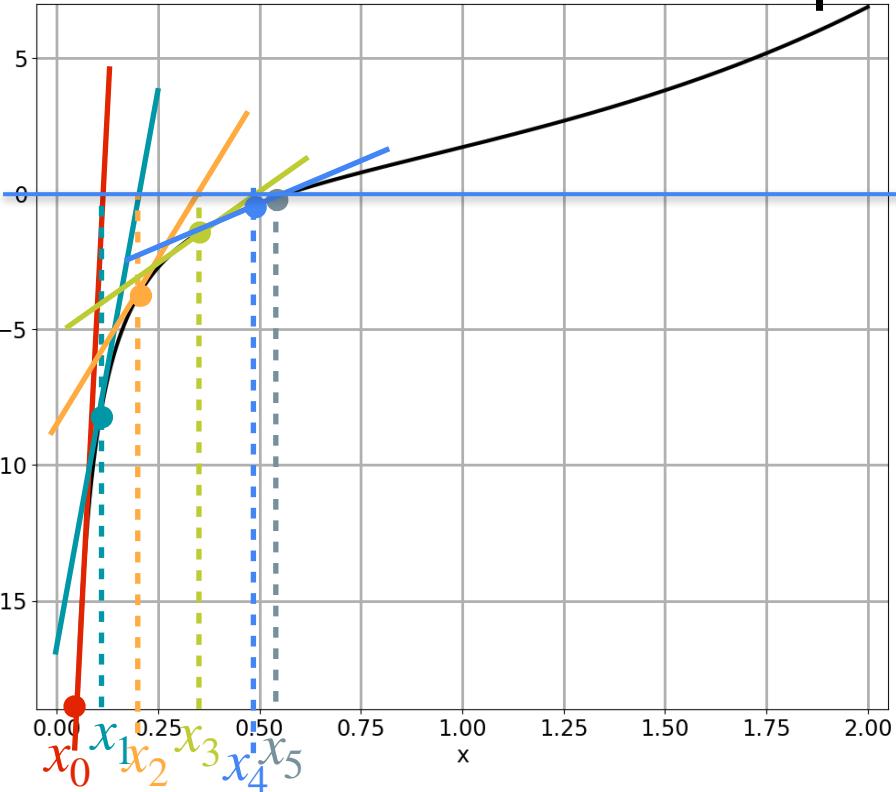
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_4 = 0.477$$

$$x_5 = x_4 - \frac{g'(x_4)}{(g'(x_4))'}$$

$$= 0.447 - \frac{\left(e^{0.447} - \frac{1}{0.447}\right)}{\left(e^{0.447} + \frac{1}{0.447^2}\right)} = 0.558$$

Newton's Method for Optimization

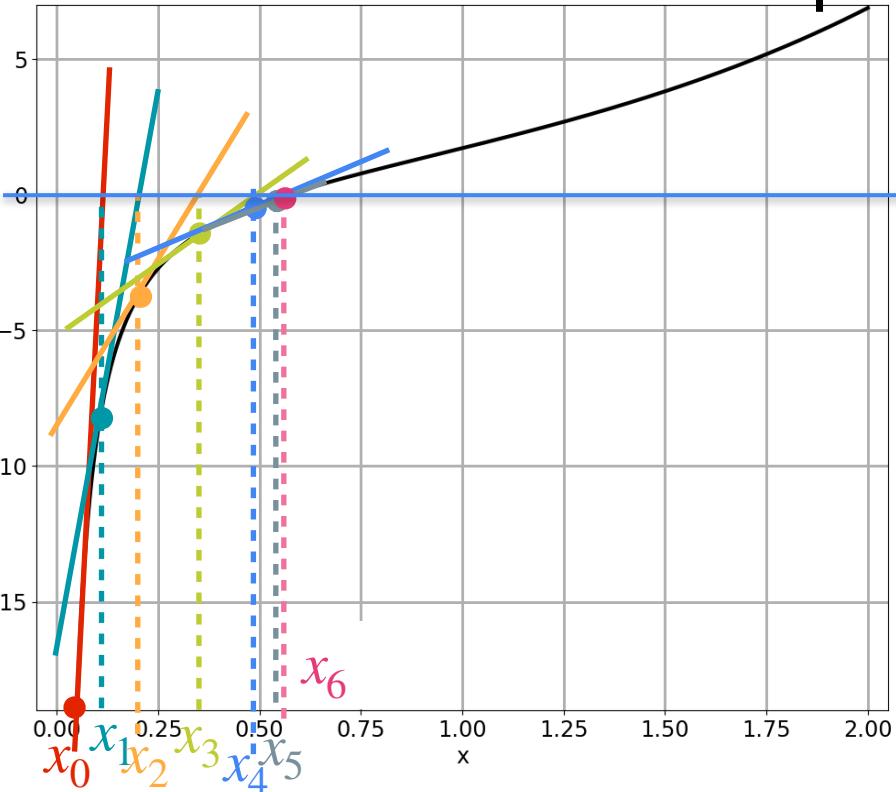


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

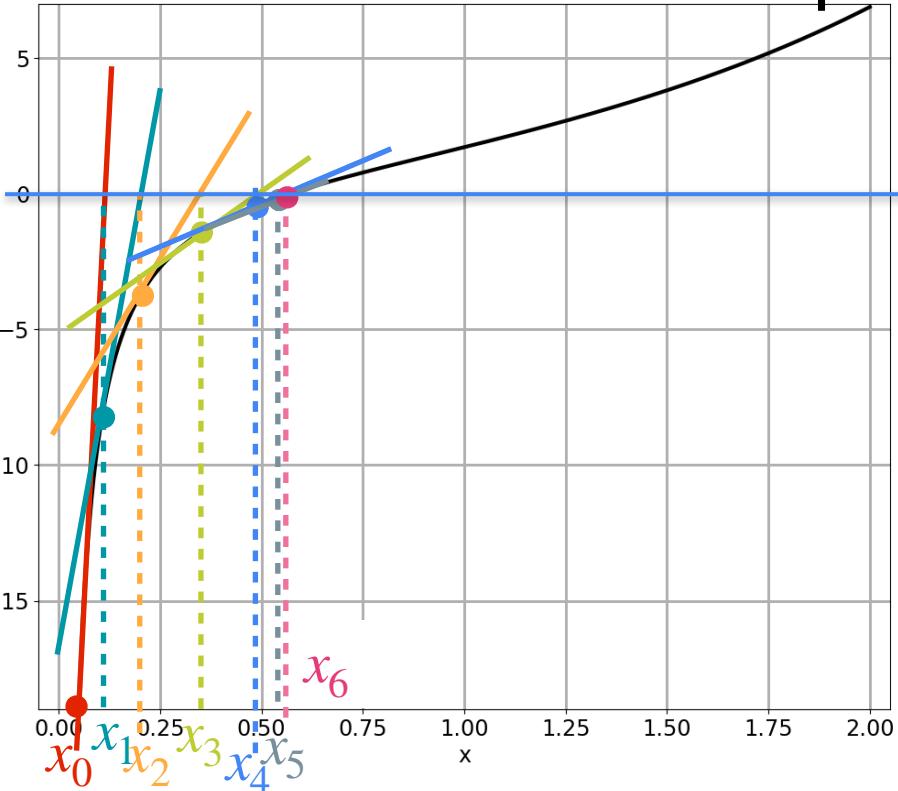


$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$
$$x_5 = 0.558$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

$$\text{Minimum: } x^* = 0.567$$

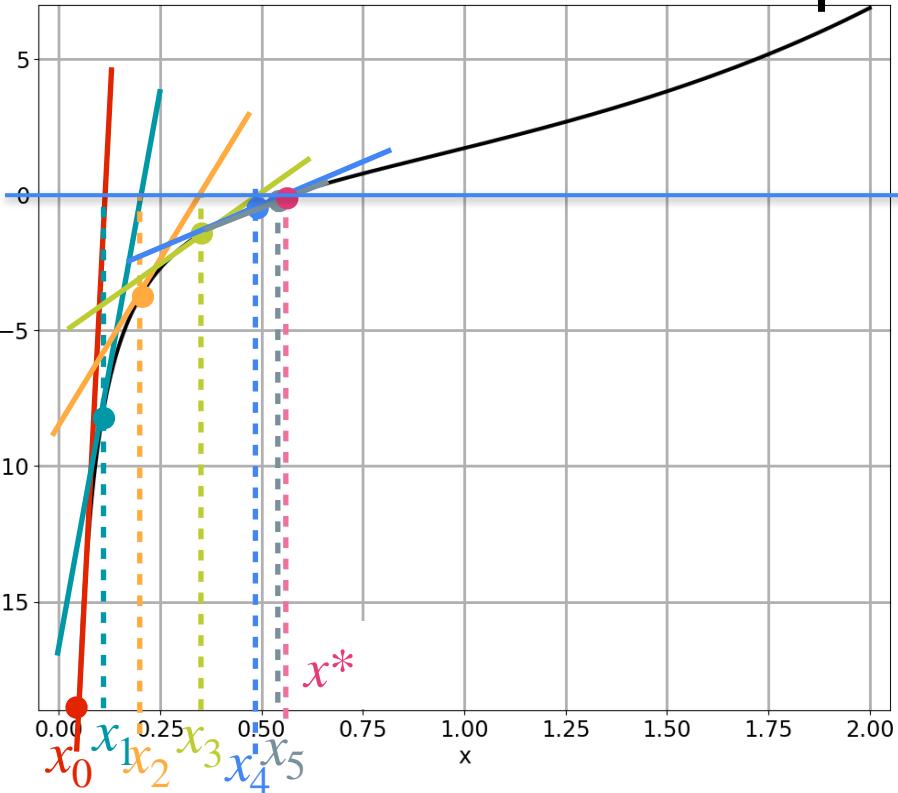
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x_6 = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \quad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x^* = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)} = 0.567$$



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

The second derivative

Second Derivative

Second Derivative

Newton's method:

Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$

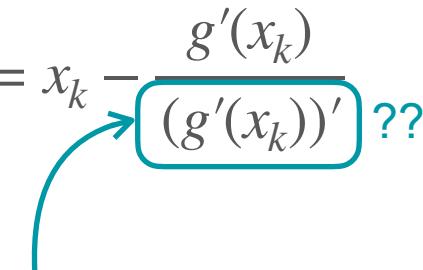
Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'} ??$

Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$??

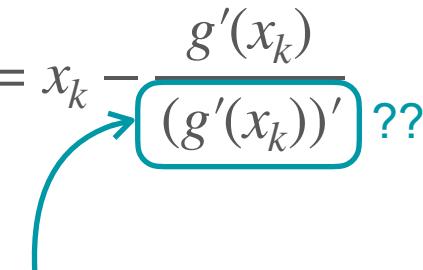
Second derivative



Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$??

Second derivative



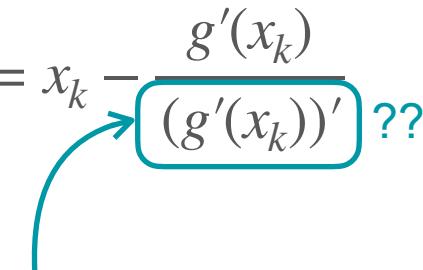
Leibniz notation:

$$\frac{d^2f(x)}{dx^2} = \frac{d}{dx} \left(\frac{df(x)}{dx} \right)$$

Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$??

Second derivative



Leibniz notation:

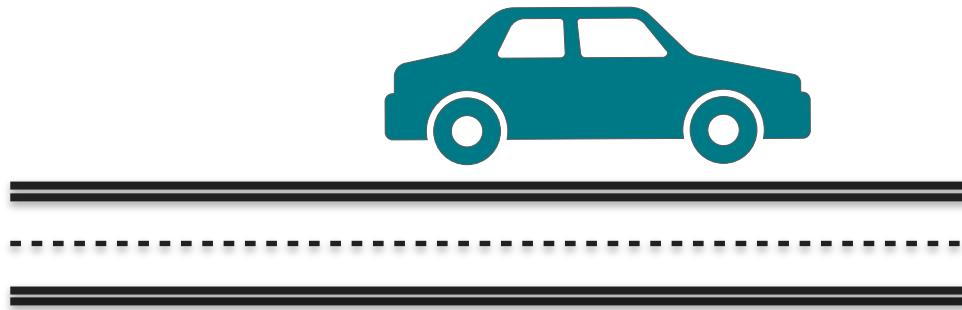
$$\frac{d^2f(x)}{dx^2} = \frac{d}{dx} \left(\frac{df(x)}{dx} \right)$$

Lagrange notation:

$$f''(x)$$

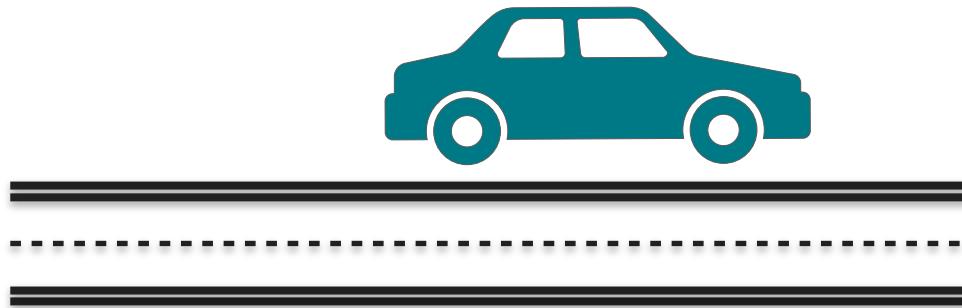
Understanding Second Derivative

Understanding Second Derivative

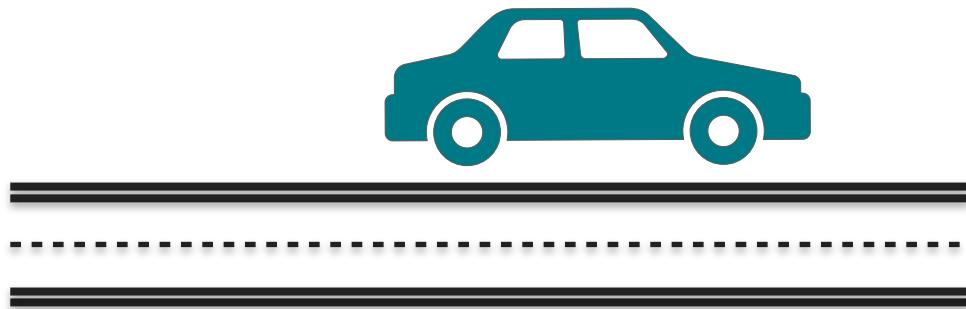


Understanding Second Derivative

x Distance



Understanding Second Derivative



x

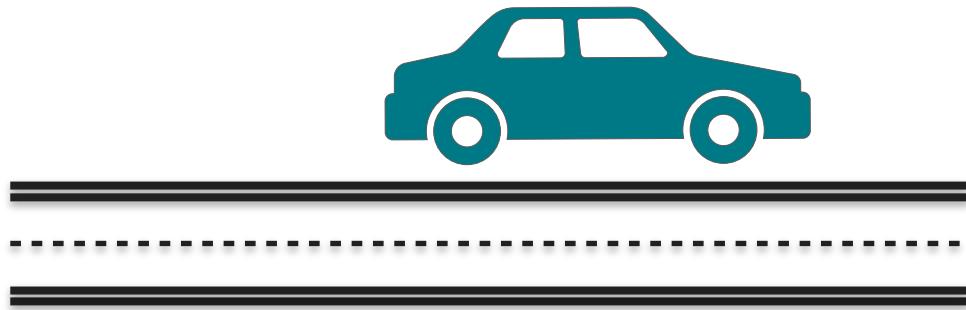
Distance

v

Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative



x Distance

v Velocity

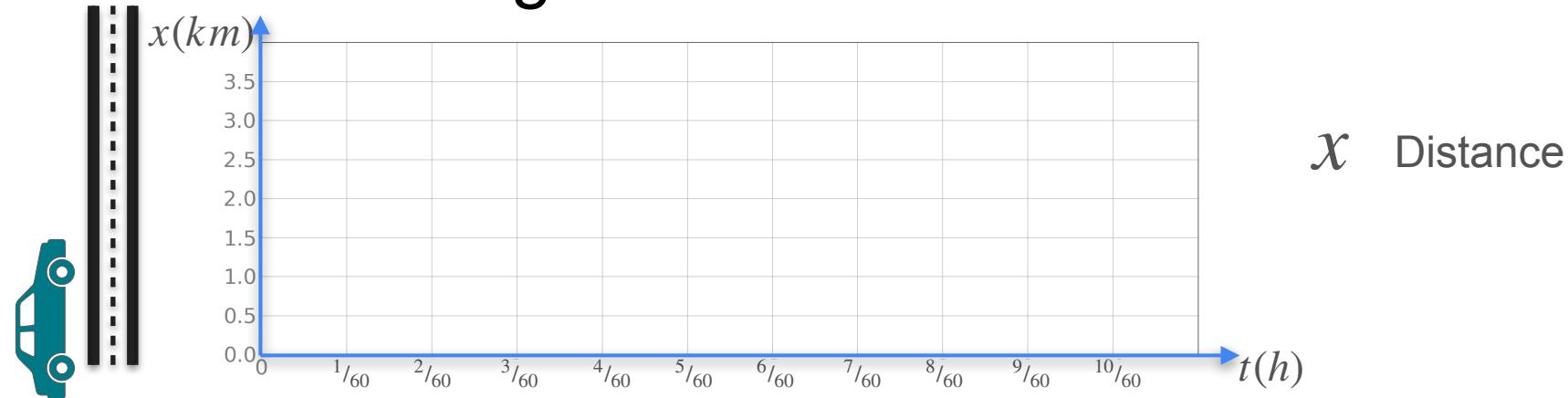
a Acceleration

$$\frac{dx}{dt}$$

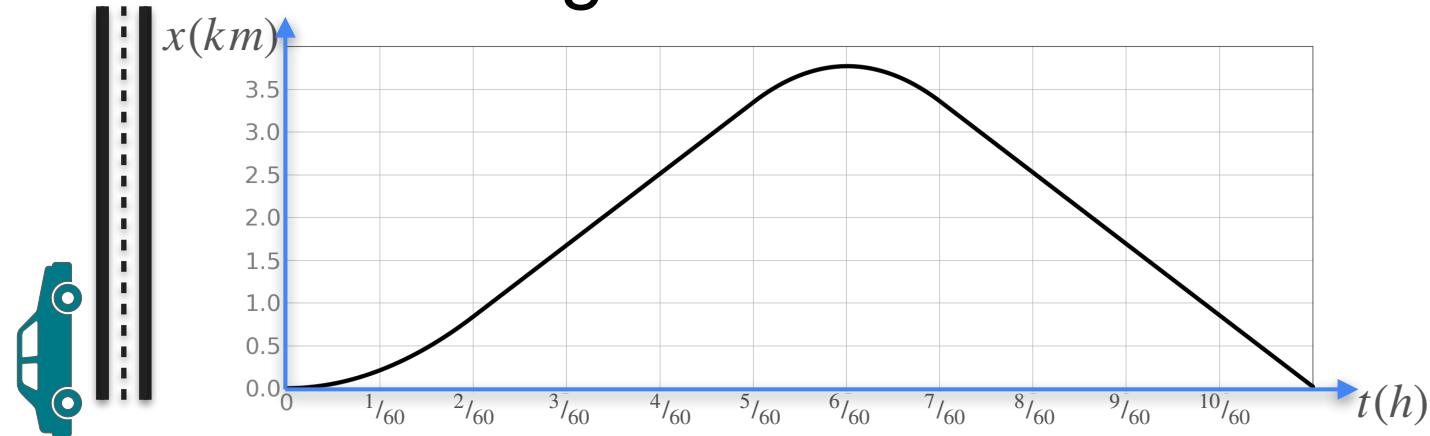
$$\frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Understanding Second Derivative

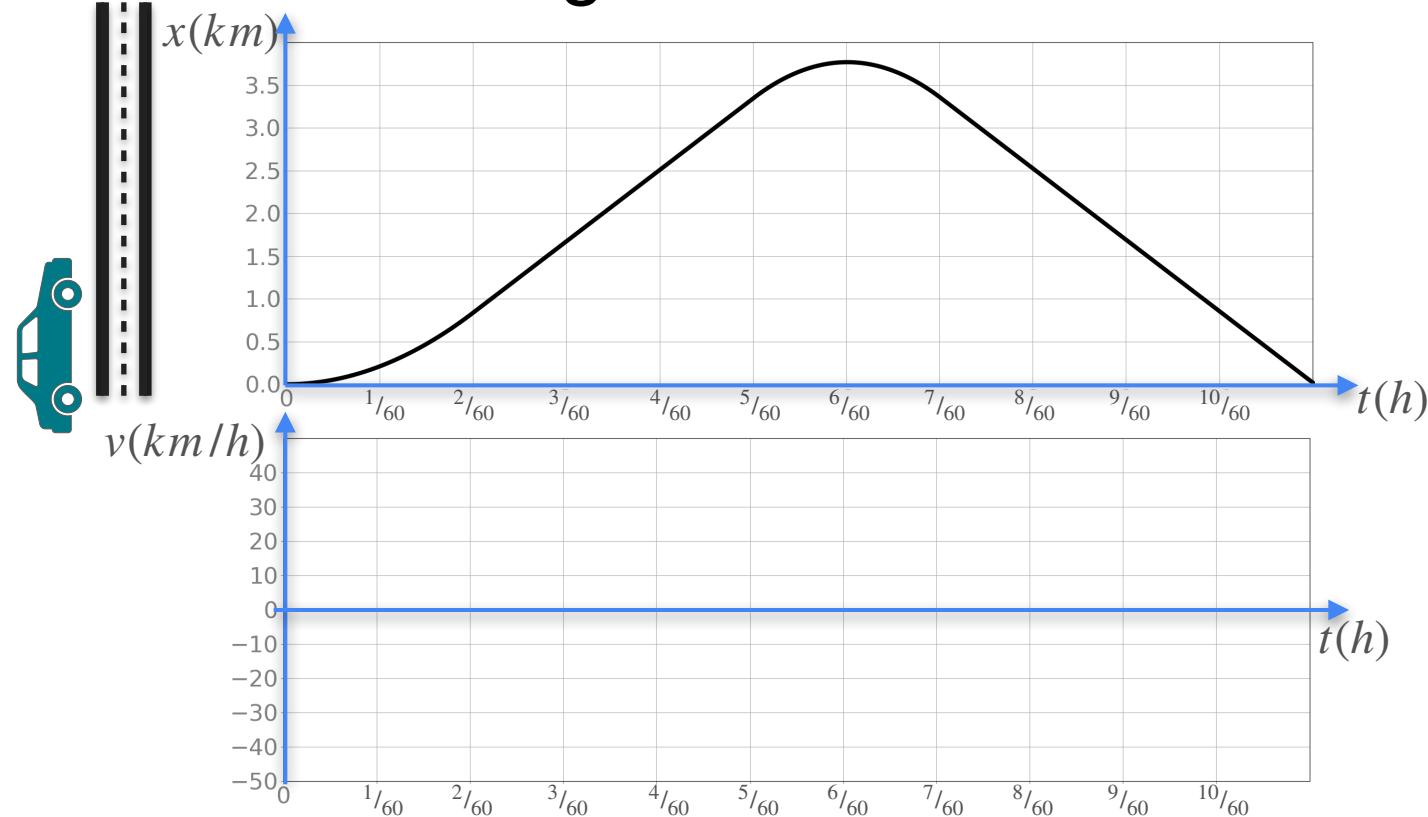
Understanding Second Derivative



Understanding Second Derivative



Understanding Second Derivative

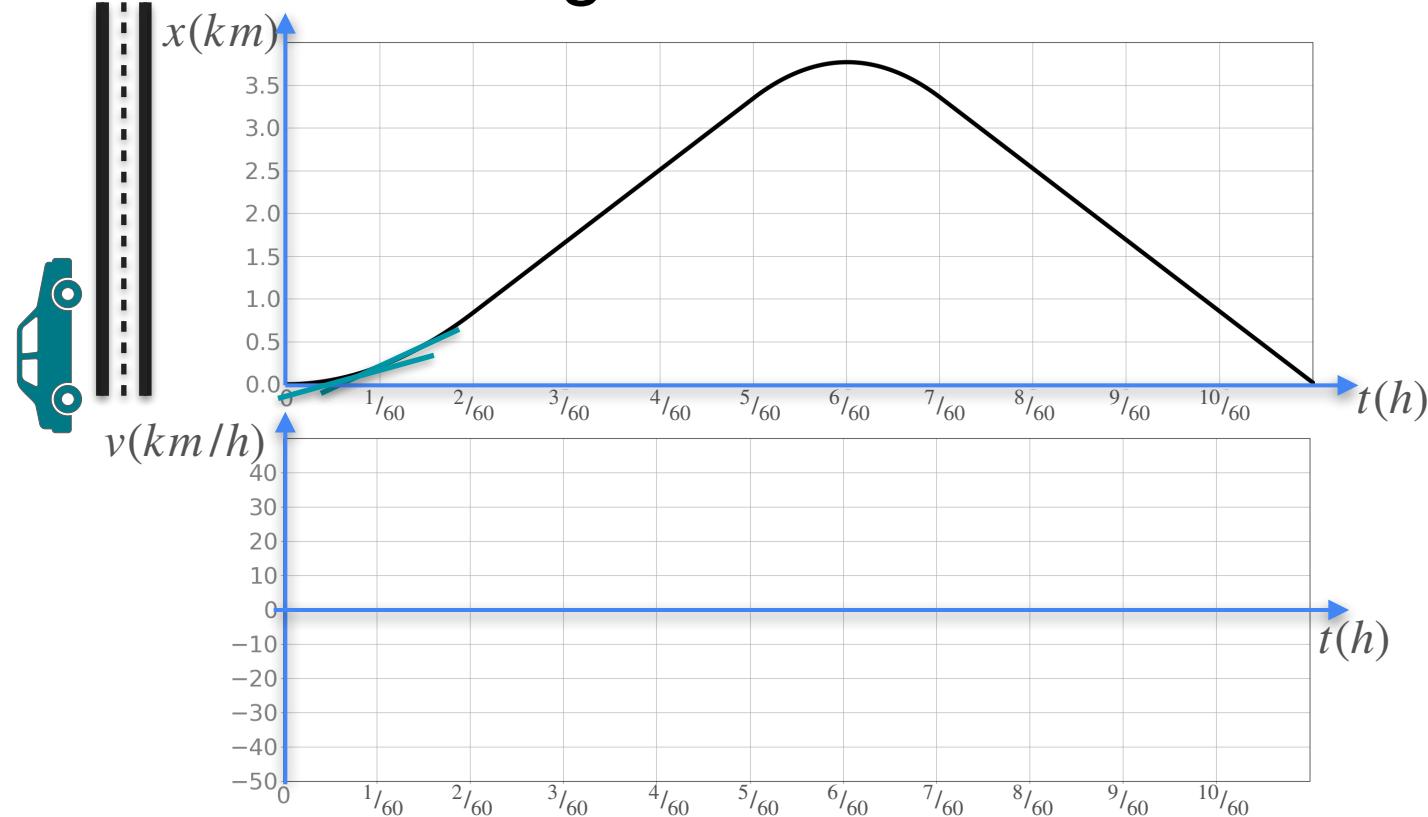


\mathcal{X} Distance

\mathcal{V} Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

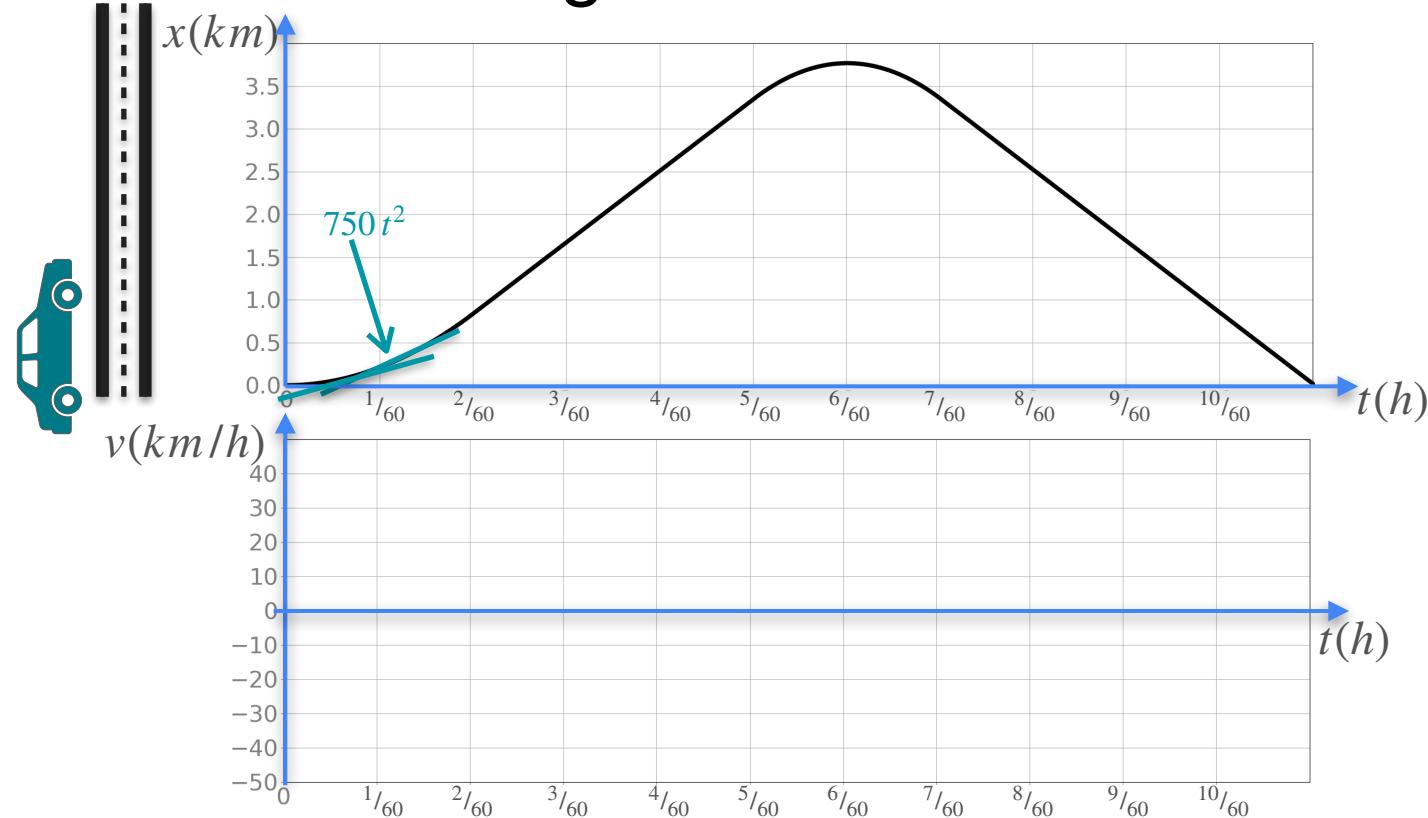


\mathcal{X} Distance

\mathcal{V} Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

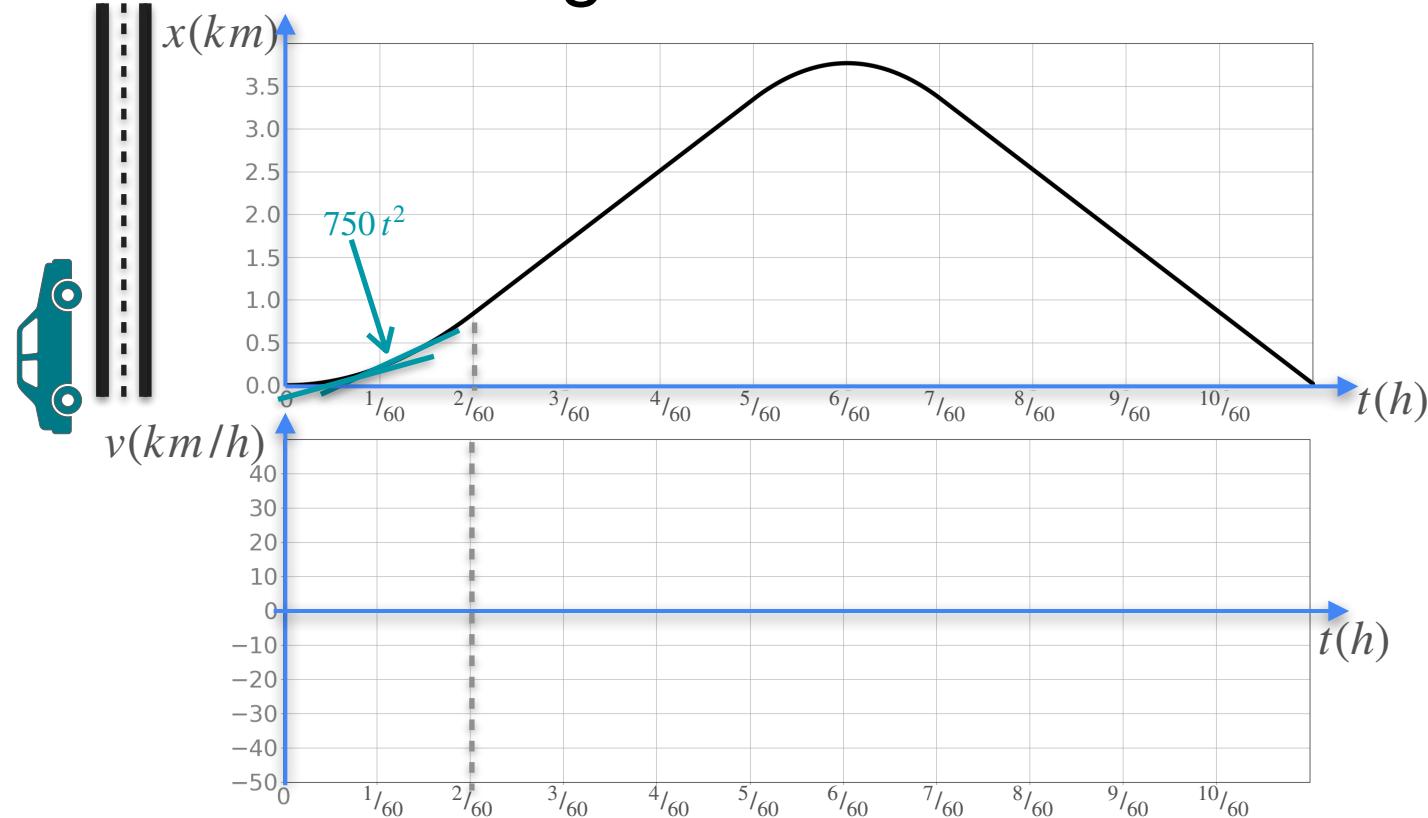


\mathcal{X} Distance

\mathcal{V} Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

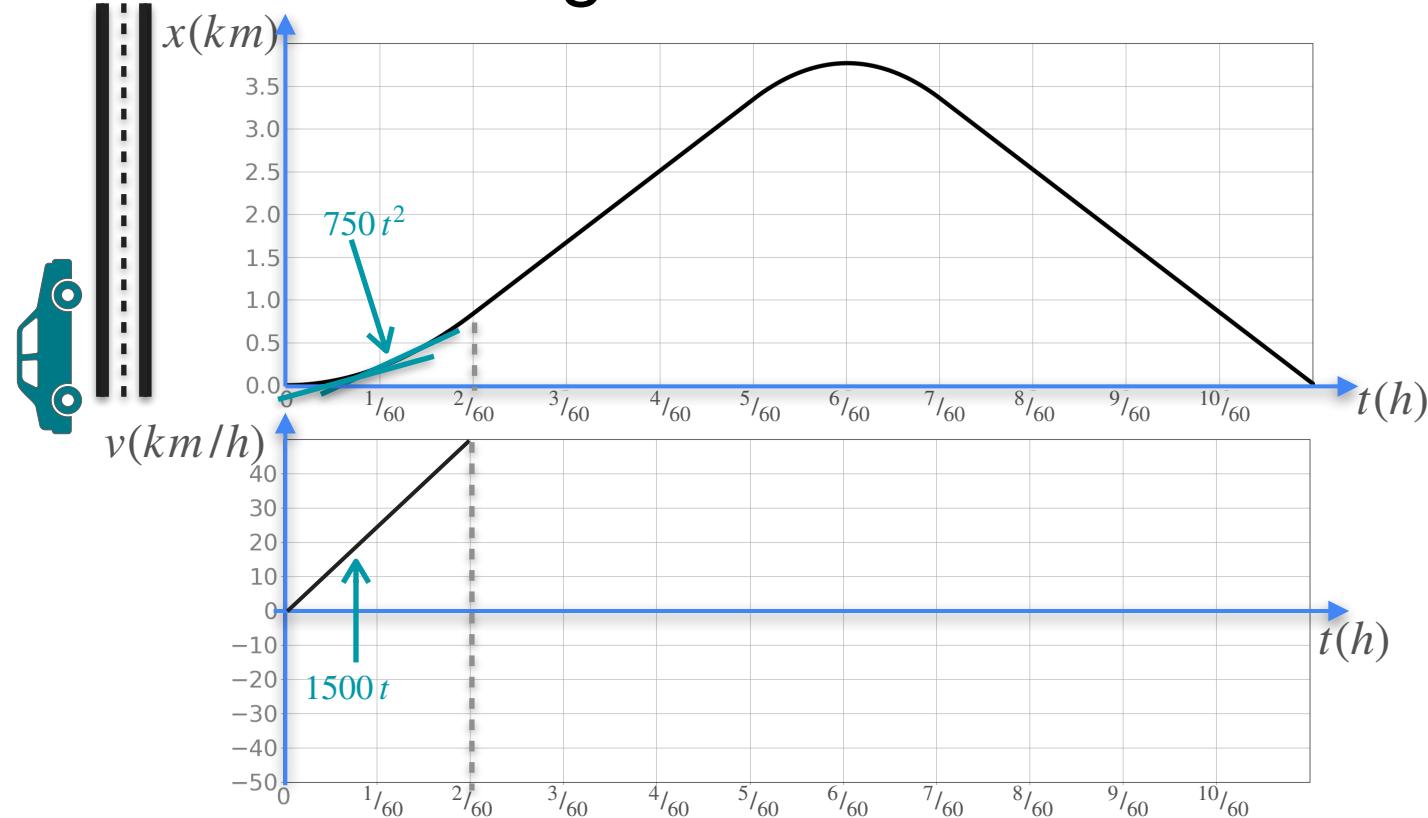


\mathcal{X} Distance

\mathcal{V} Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

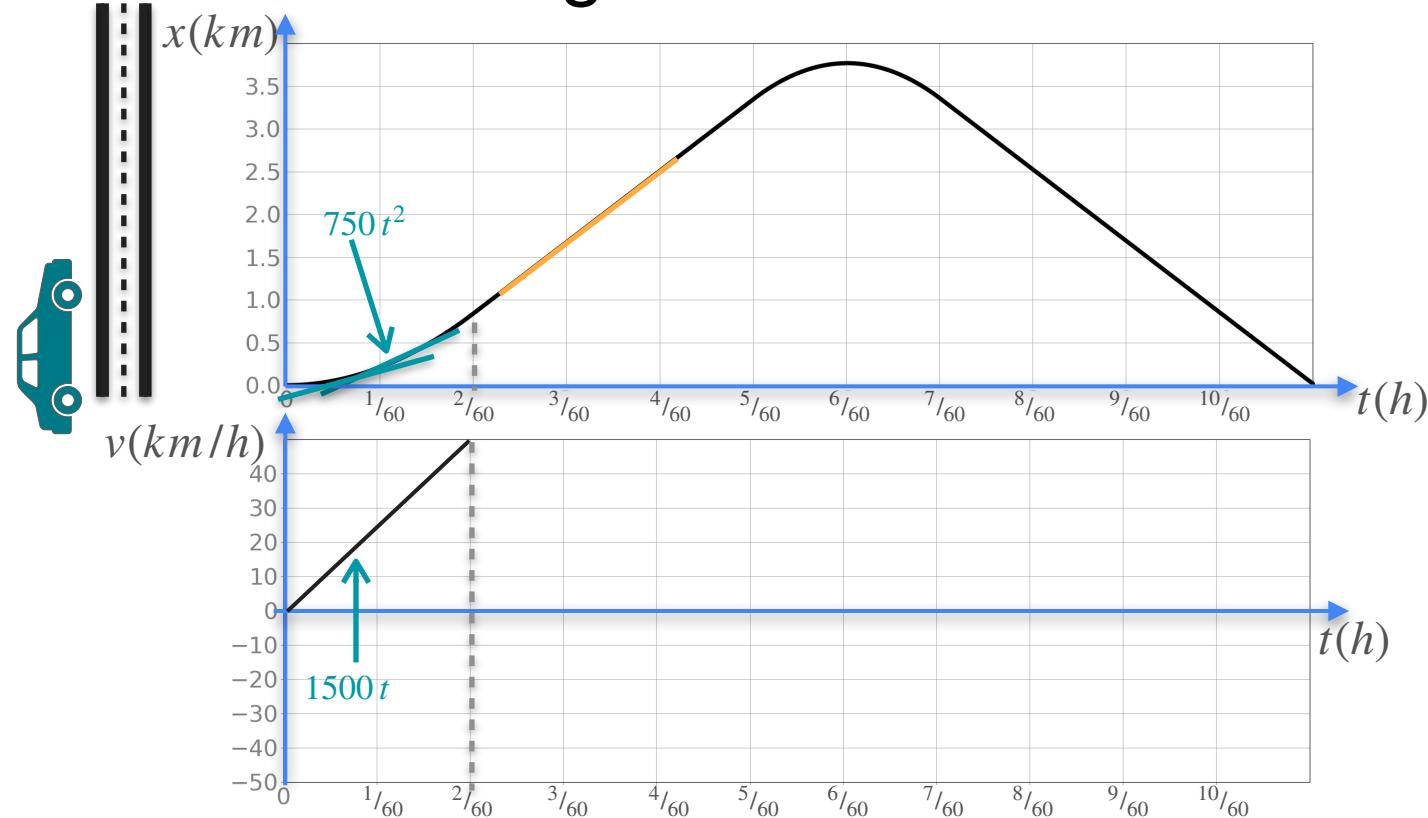


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

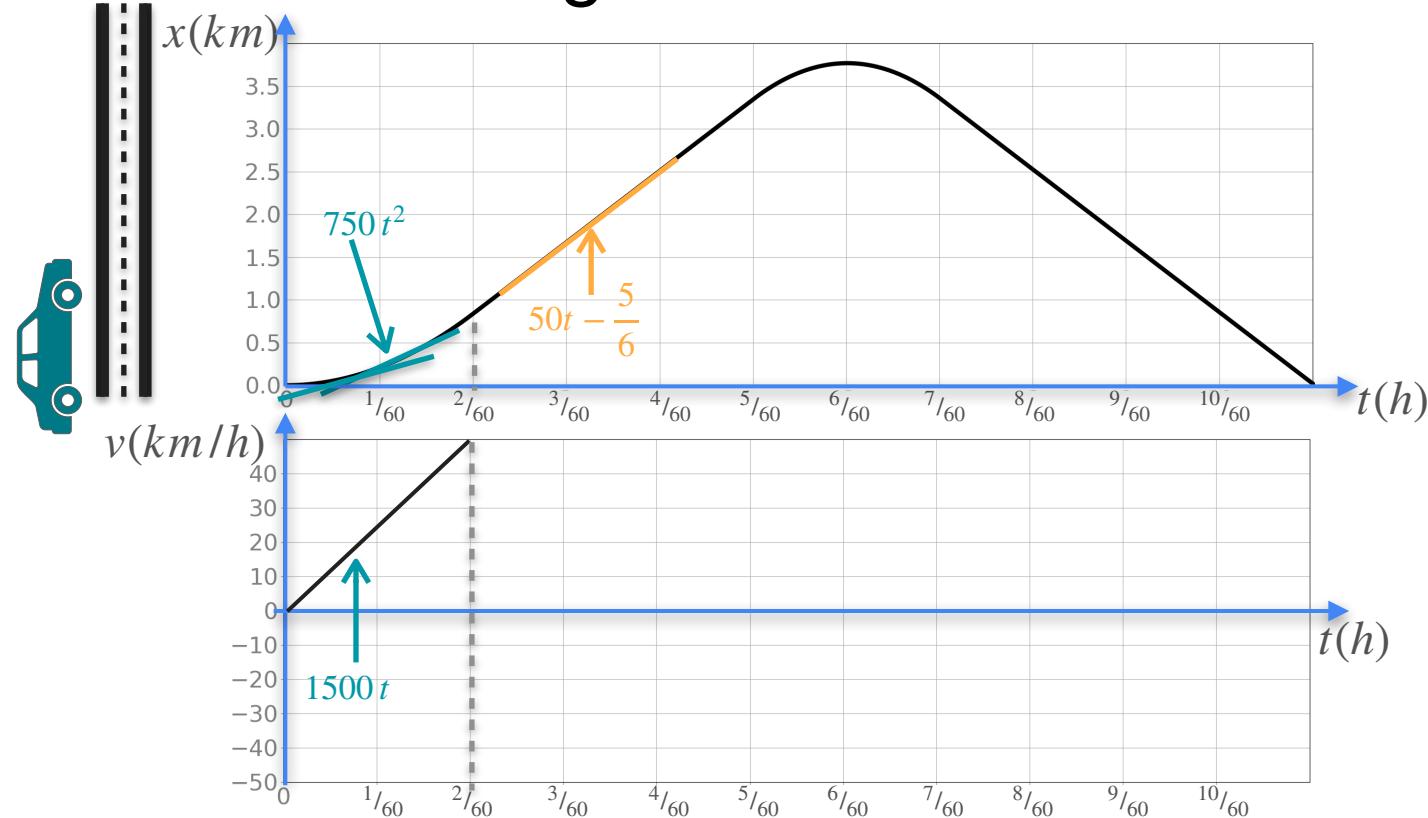


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

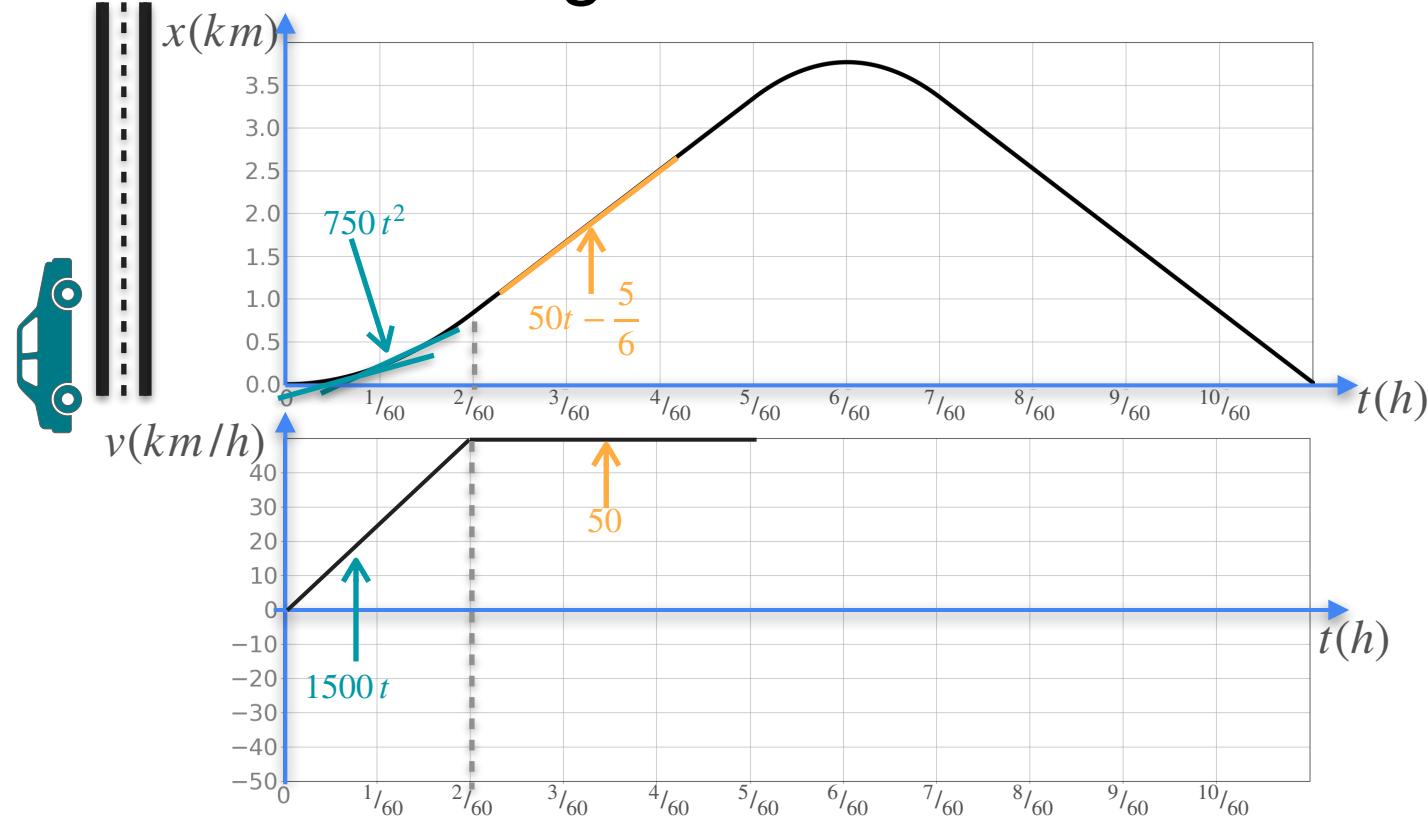


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

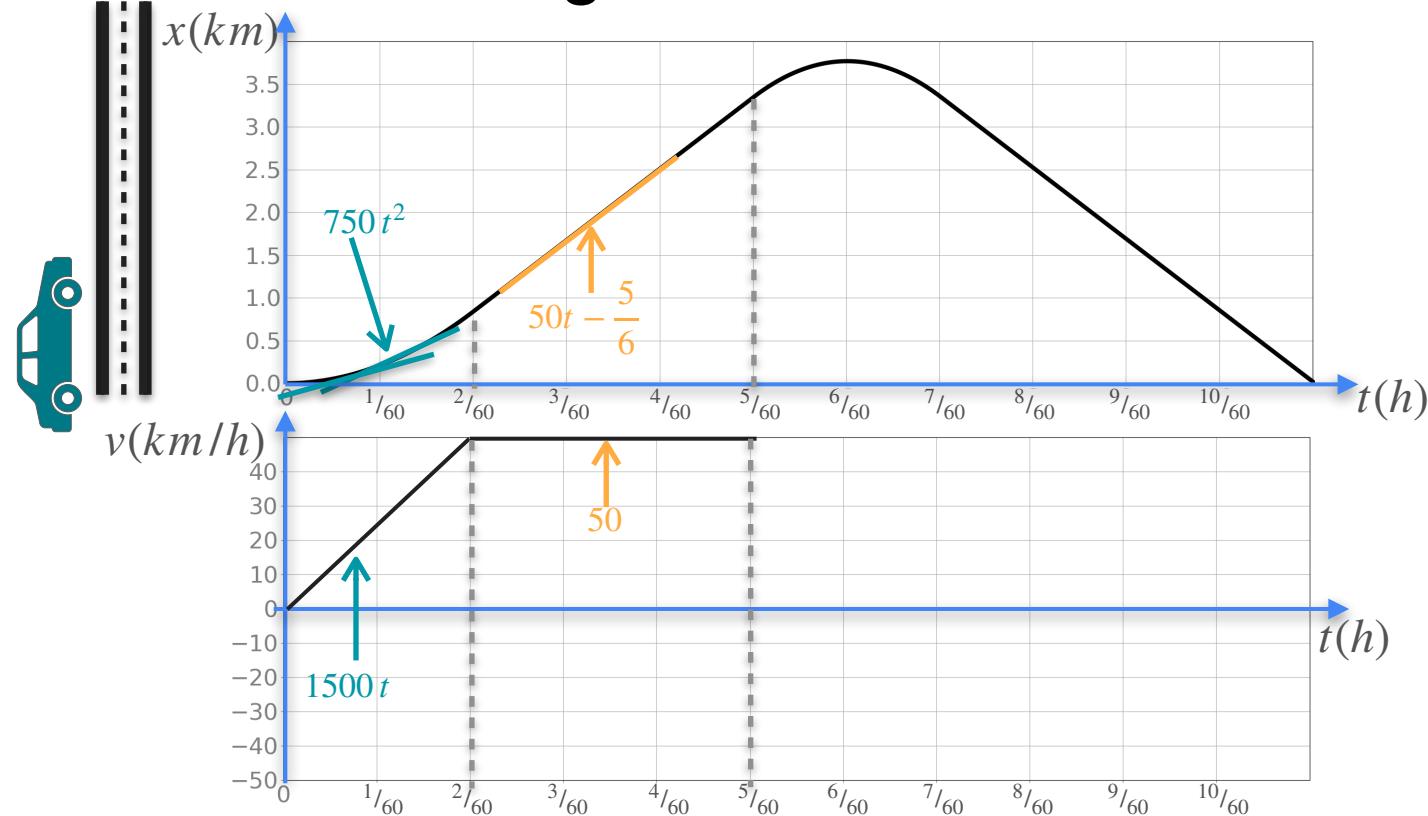


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

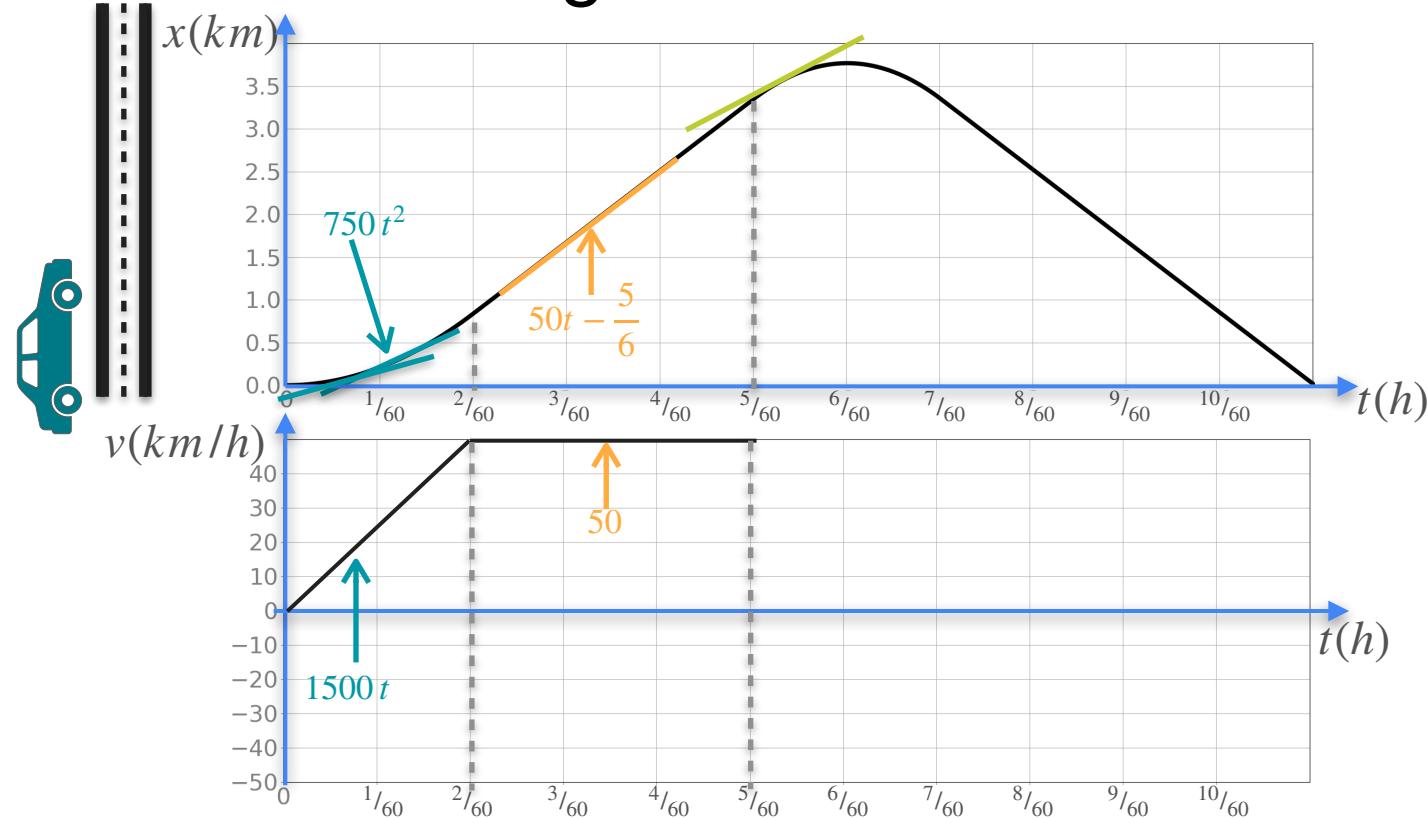


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

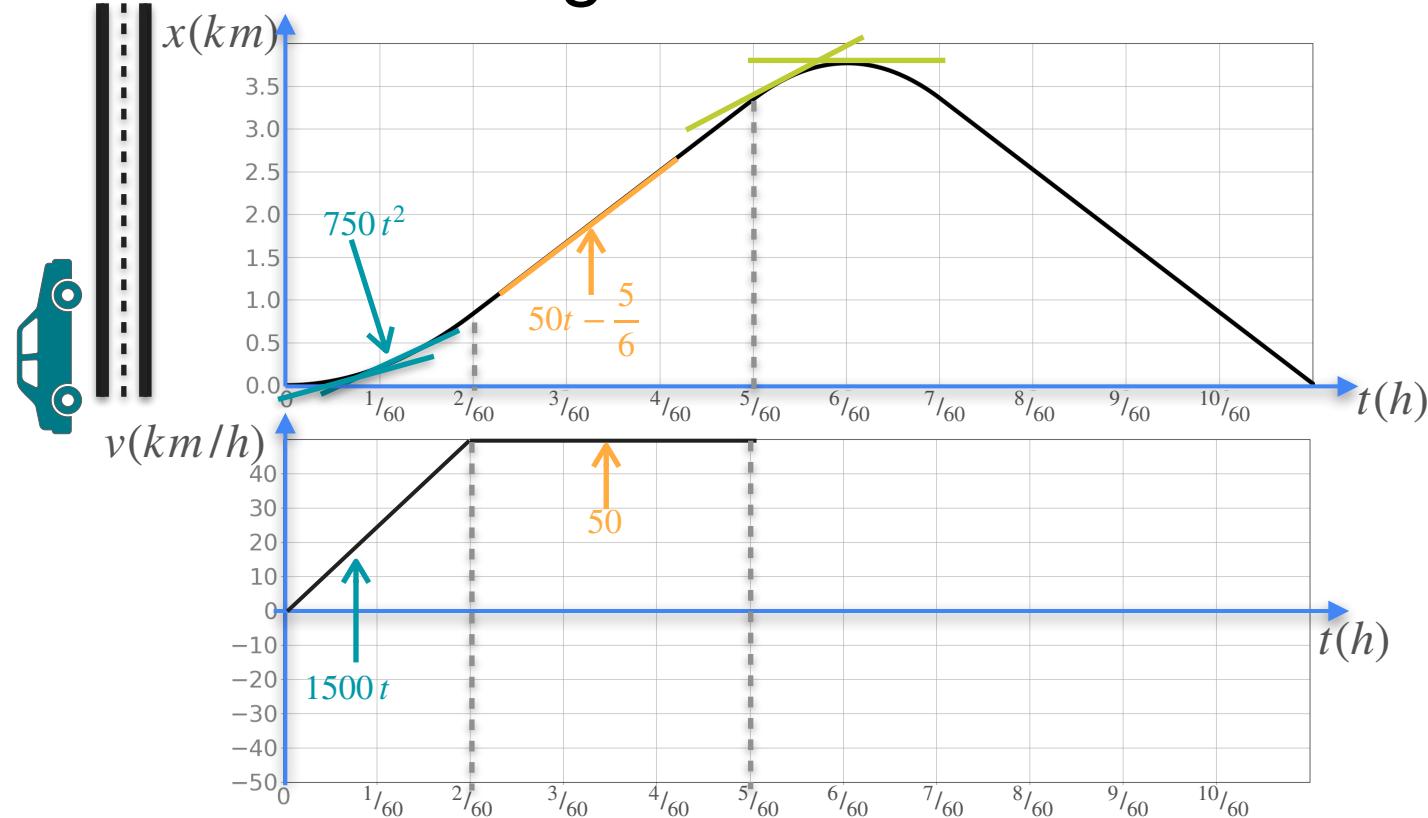


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

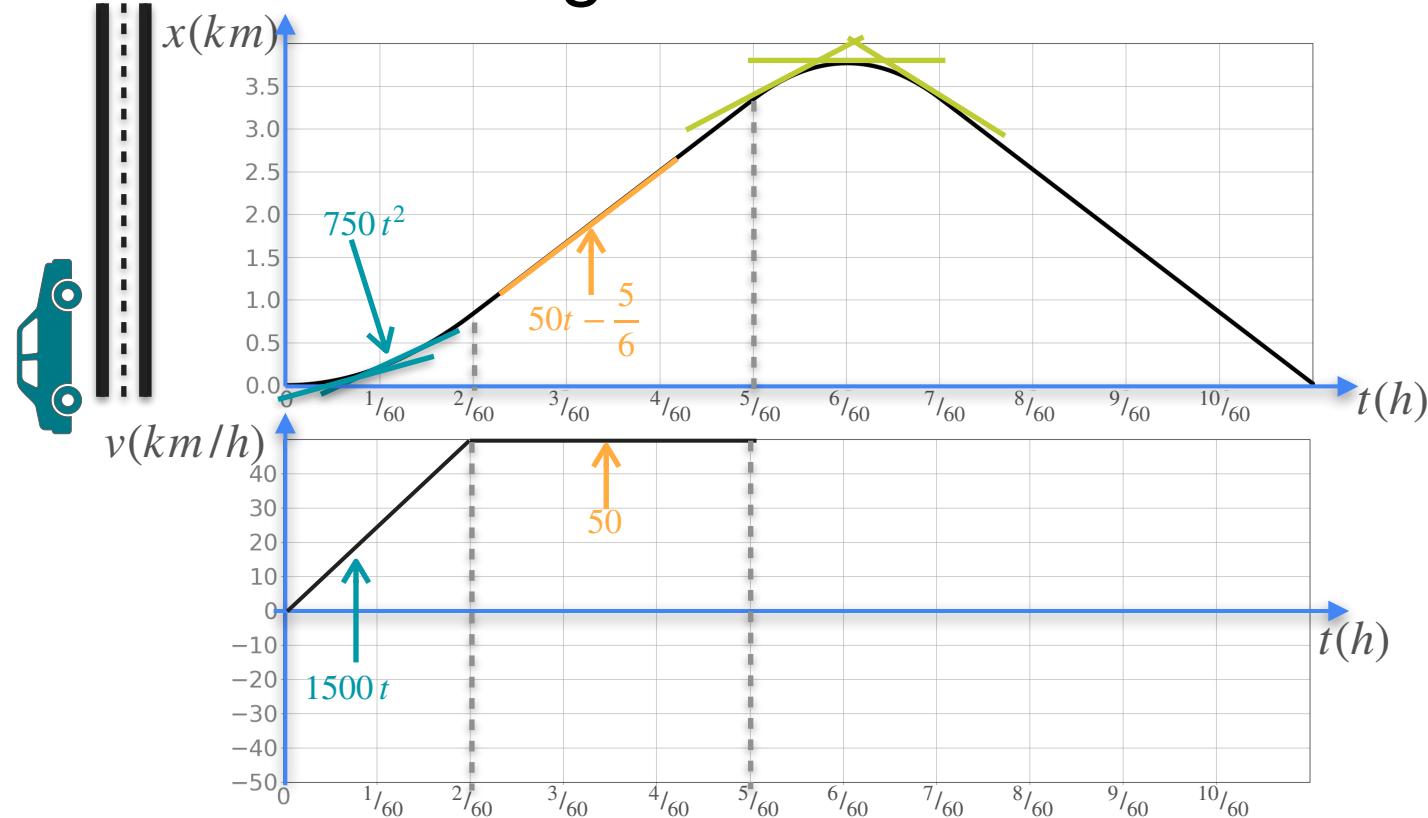


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

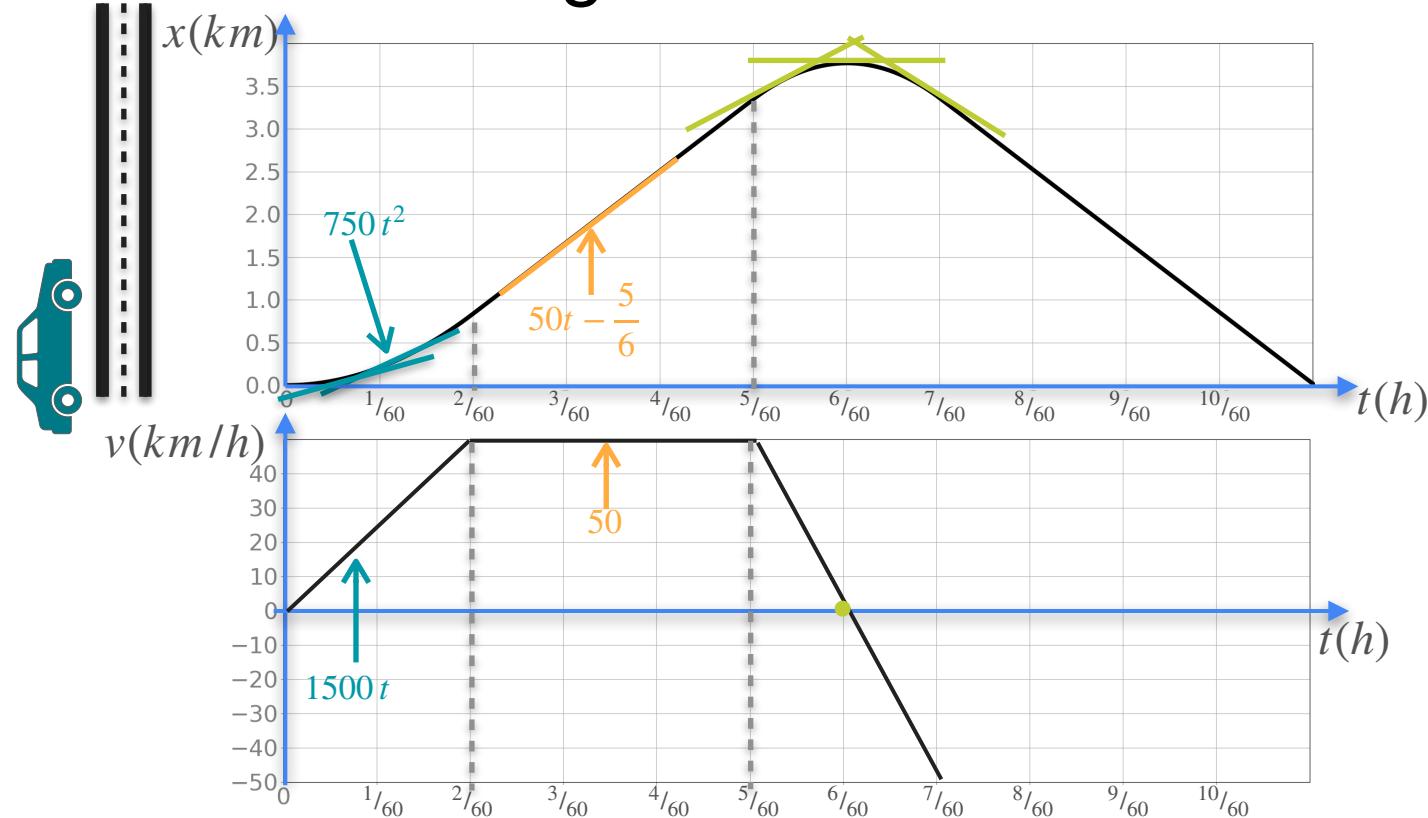


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

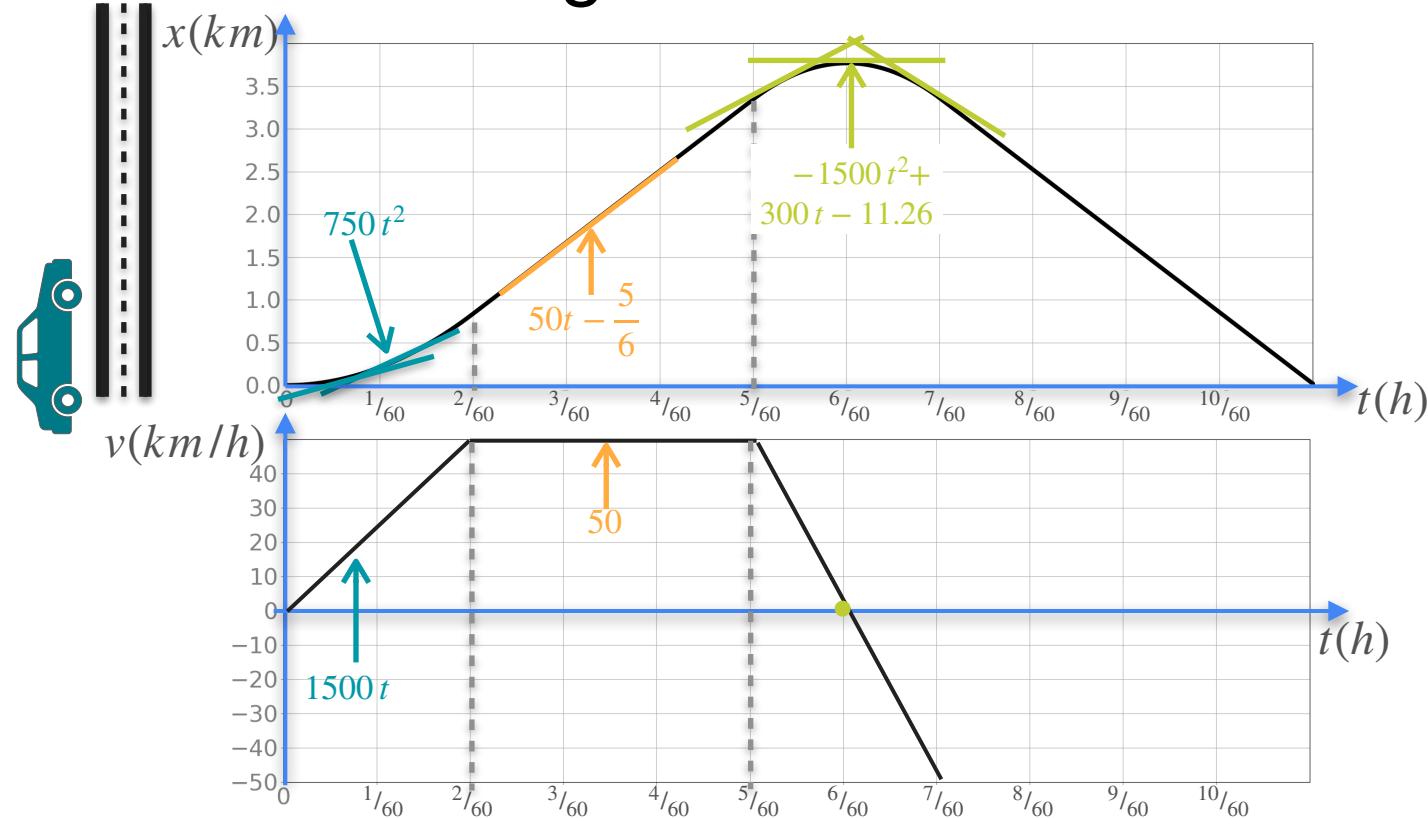


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

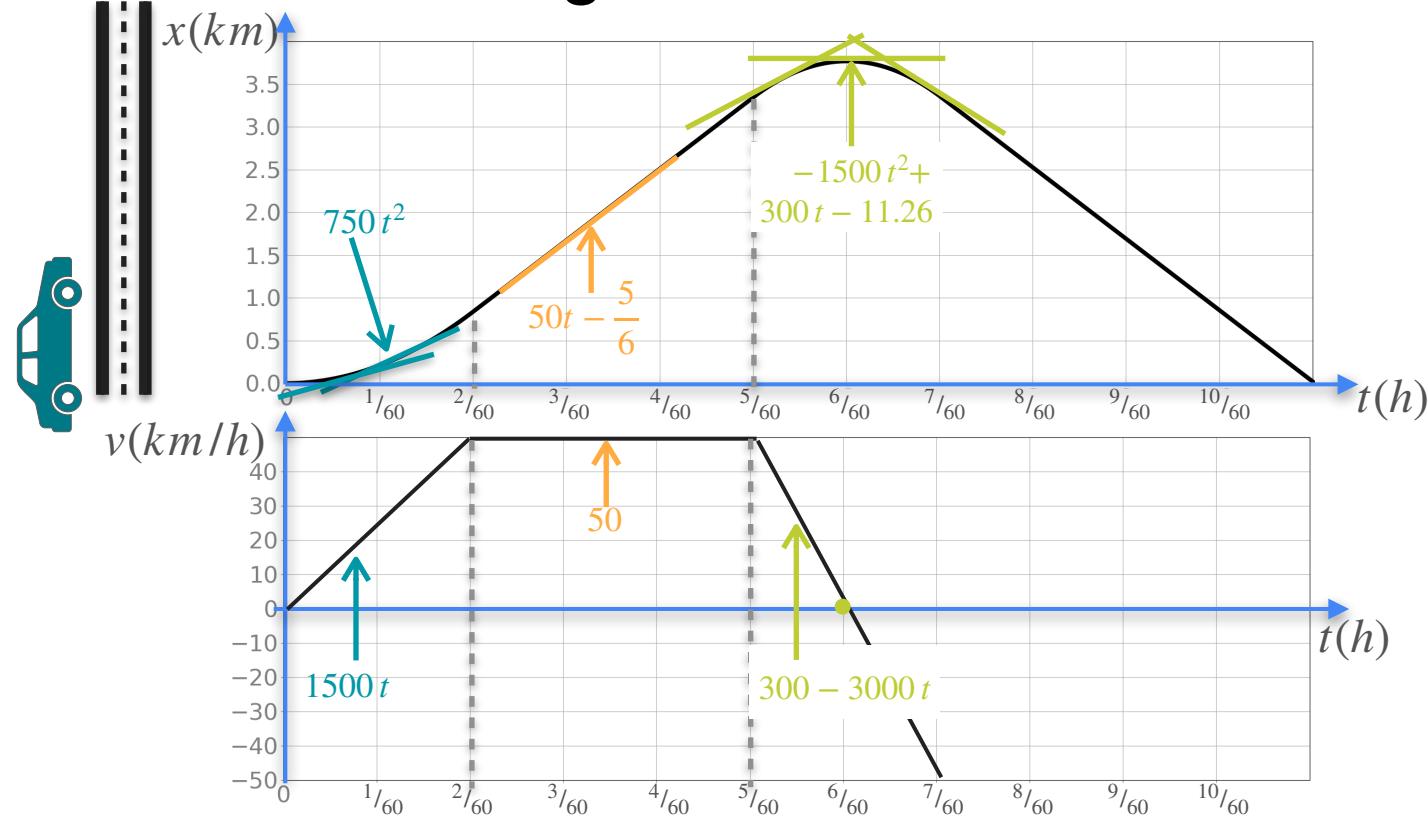


\mathcal{X} Distance

\mathcal{V} Velocity

$$\frac{d^2x}{dt^2}$$

Understanding Second Derivative

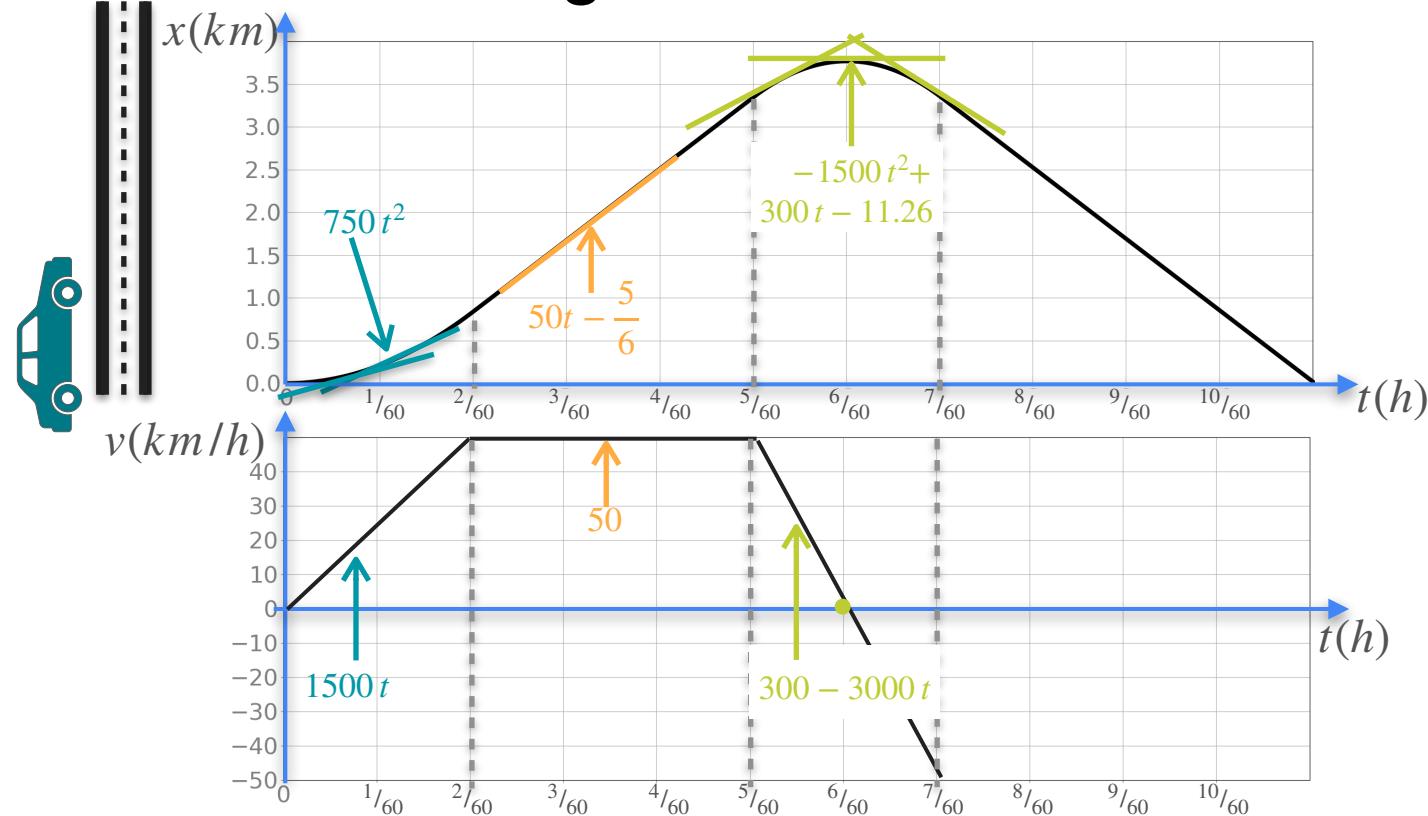


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

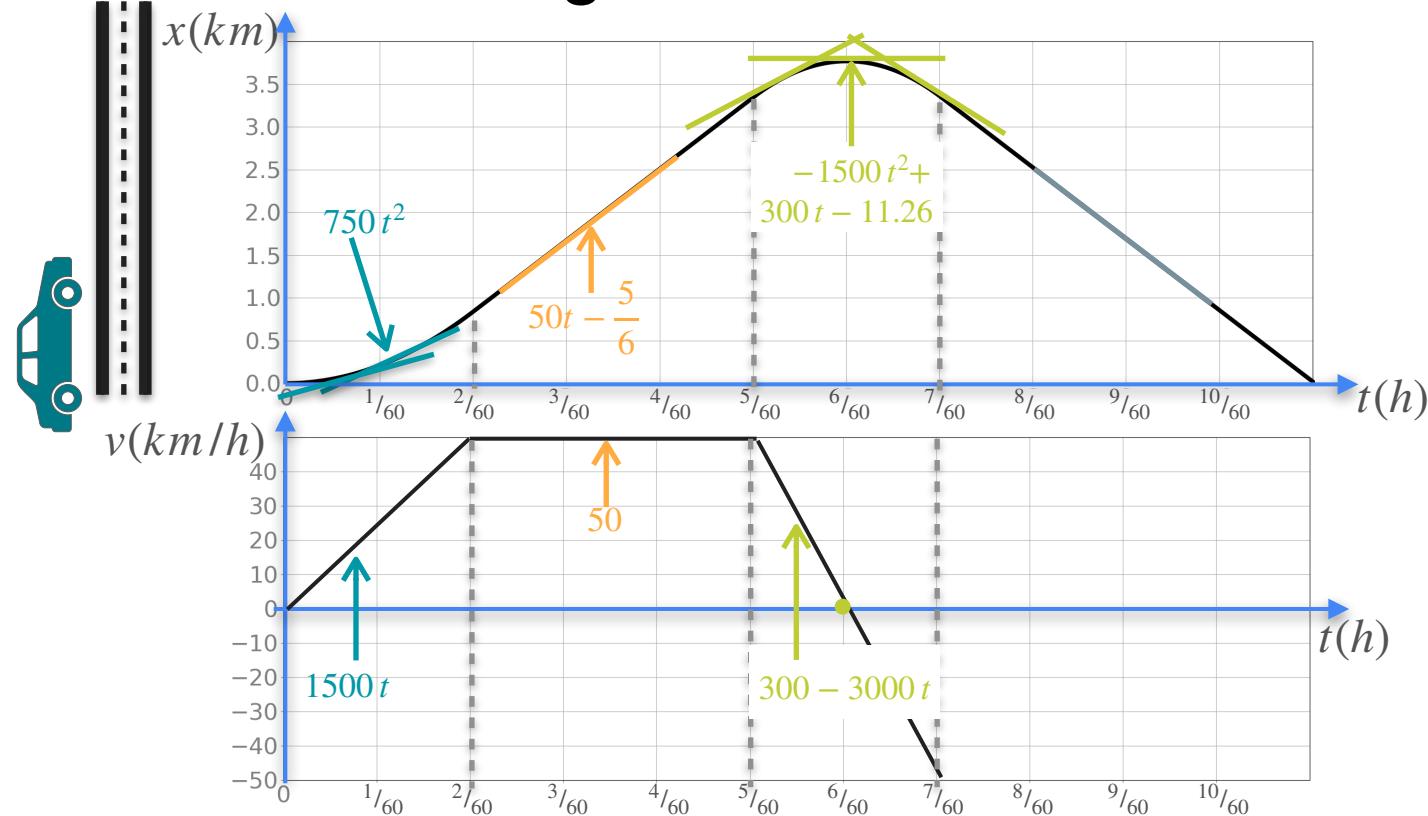


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

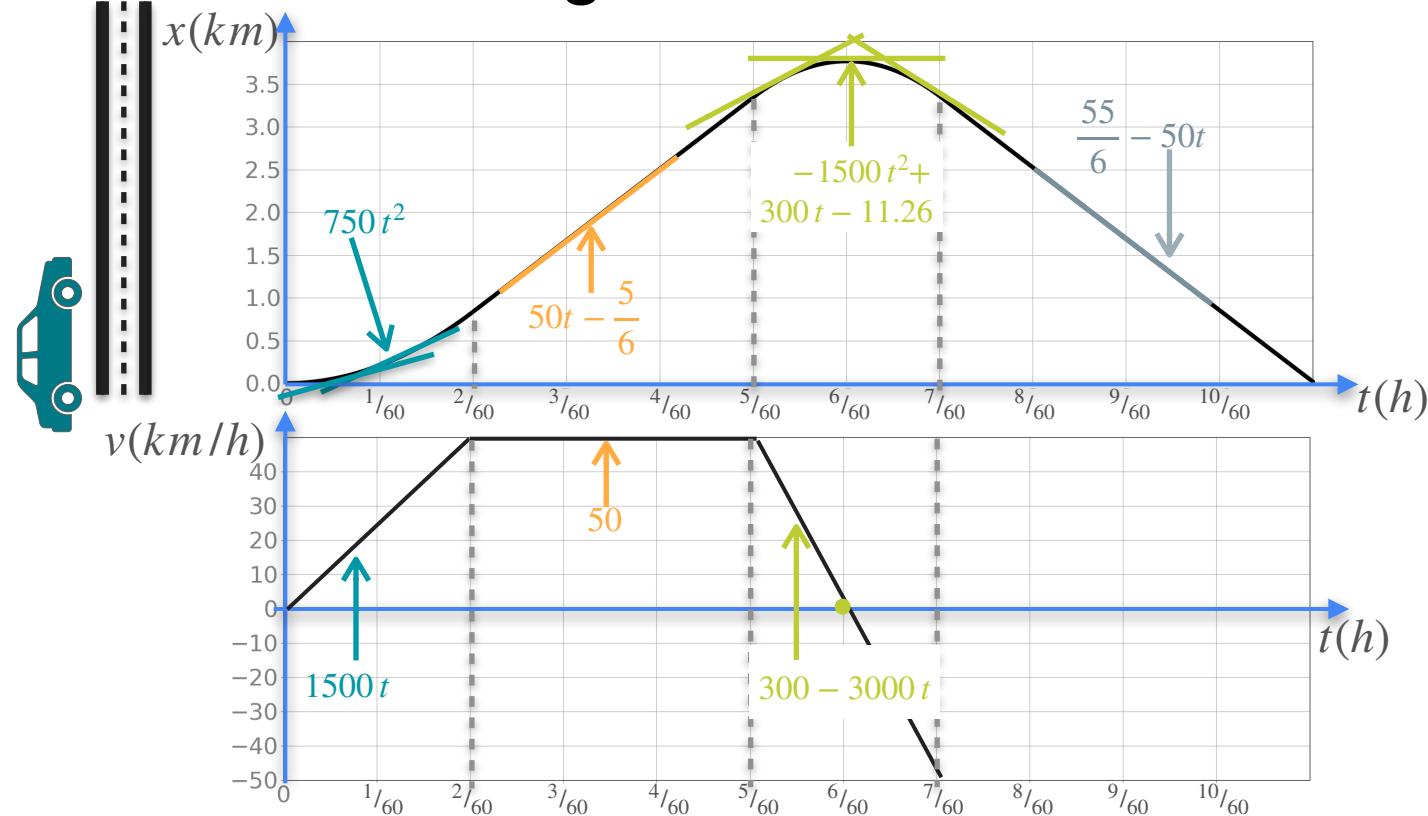


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

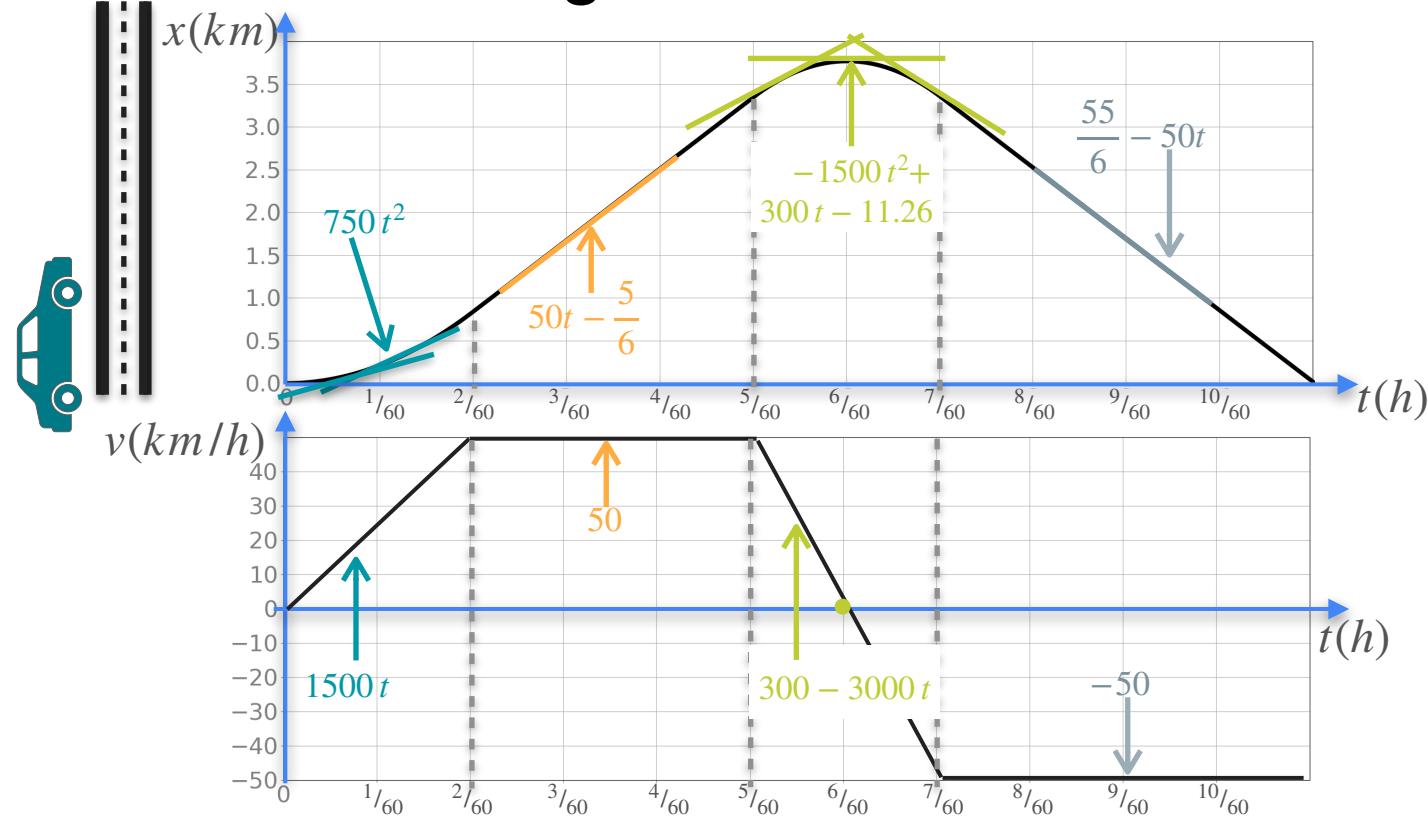


x Distance

v Velocity

$$\frac{dx}{dt}$$

Understanding Second Derivative

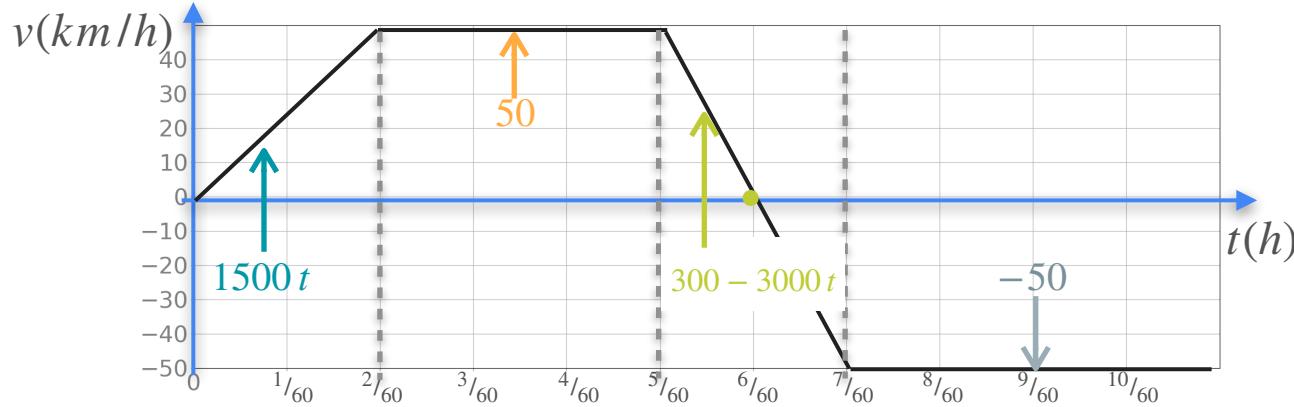


x Distance

v Velocity

$$\frac{dx}{dt}$$

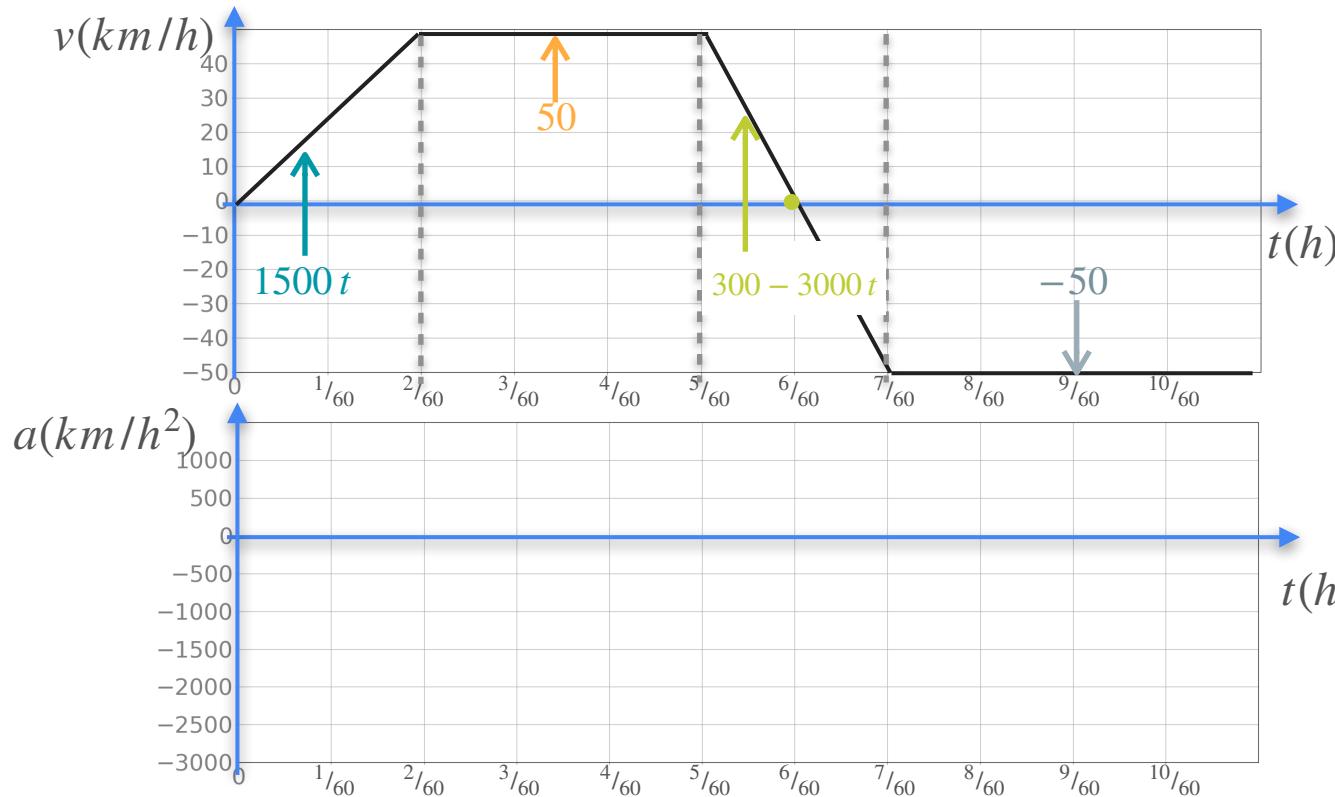
Understanding Second Derivative



v Velocity

$$\frac{dx}{dt}$$

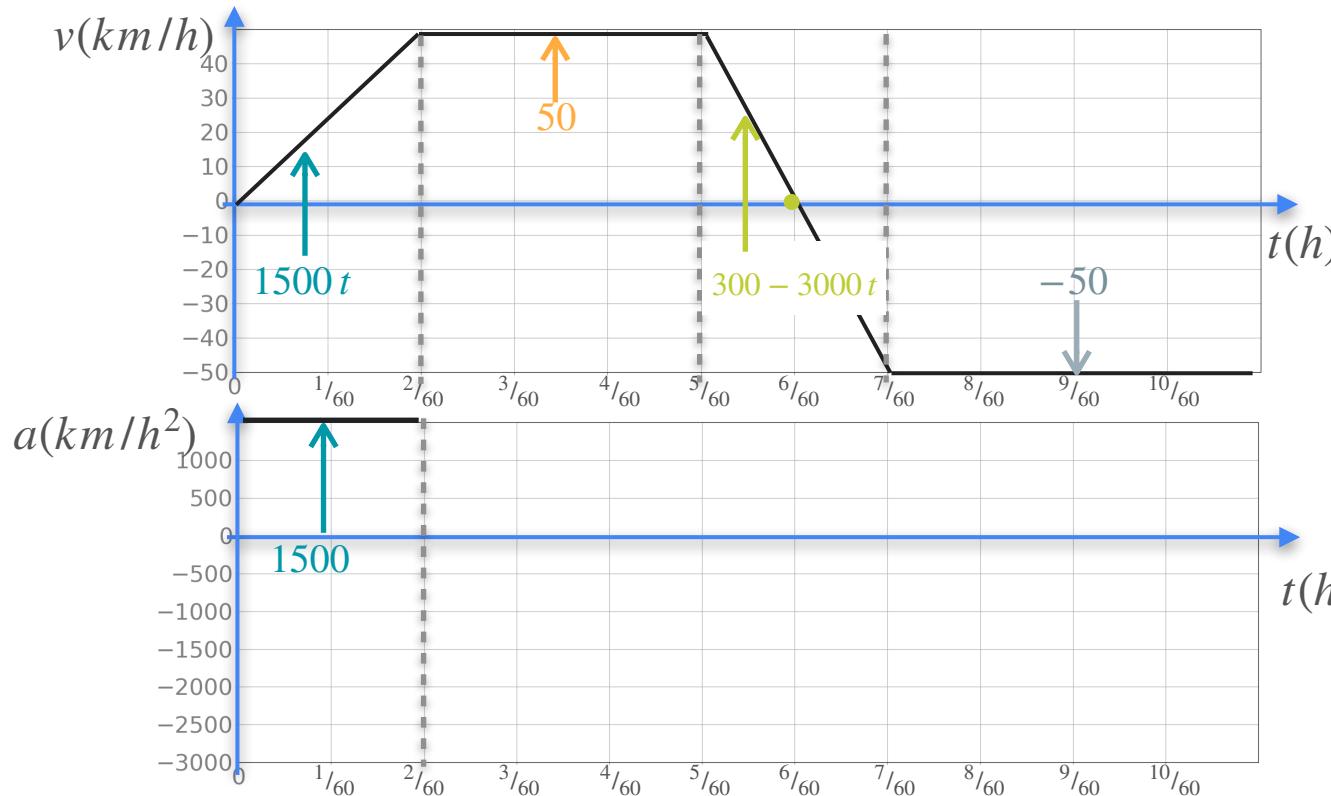
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

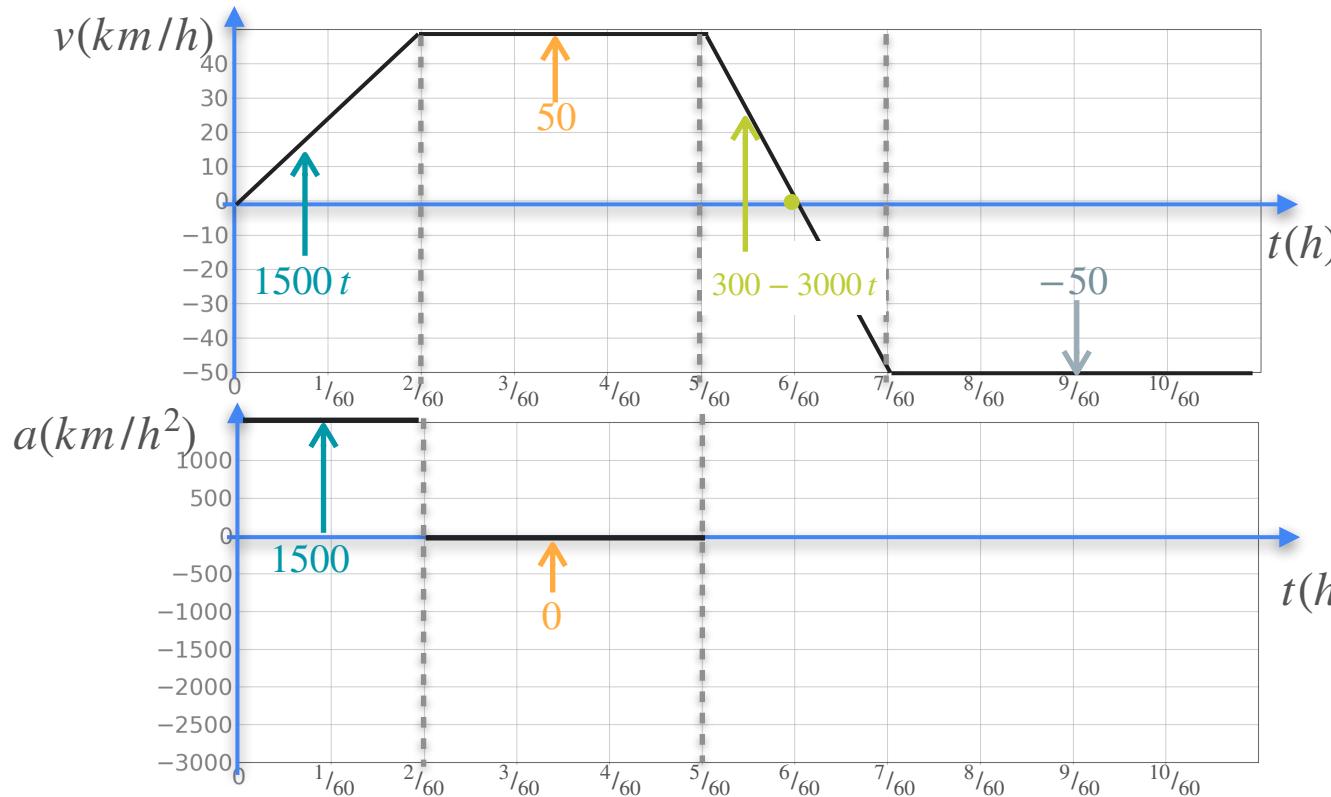
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

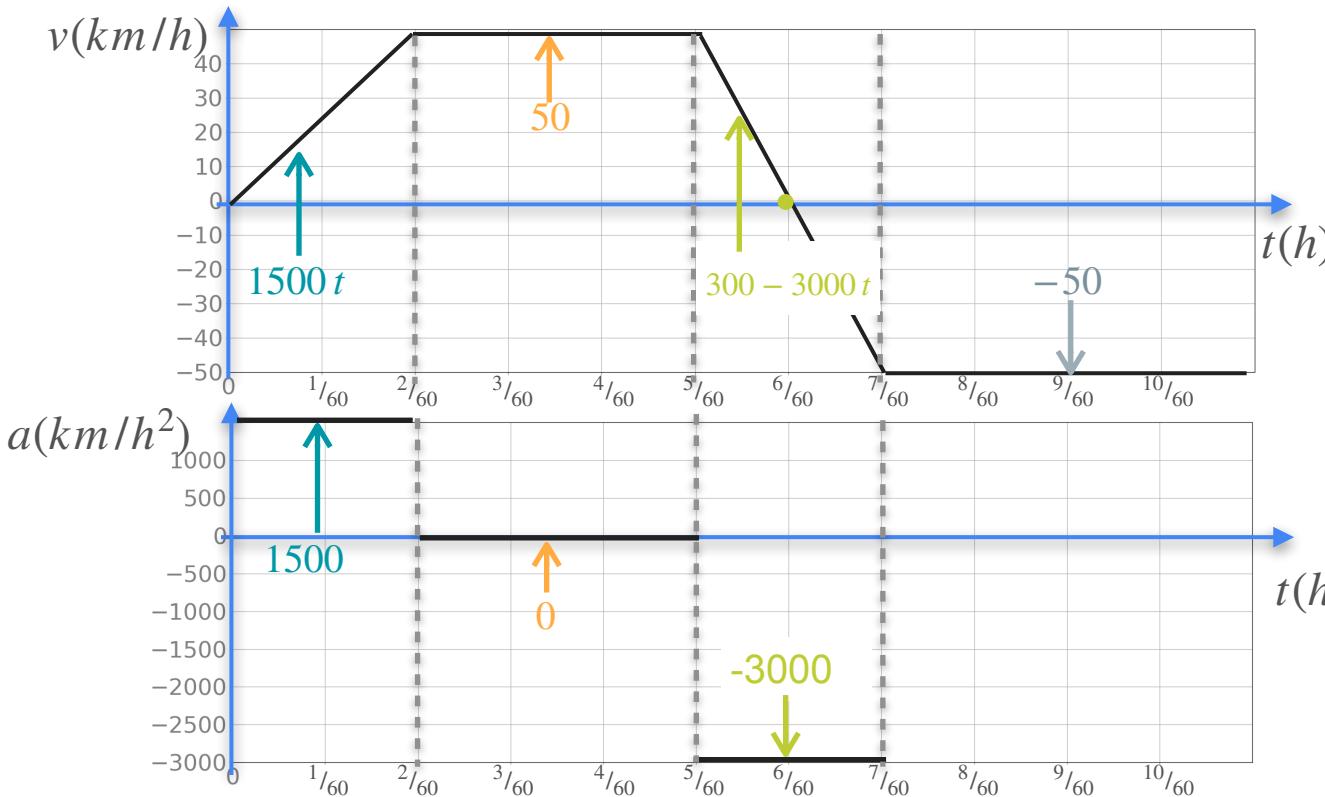
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

Understanding Second Derivative



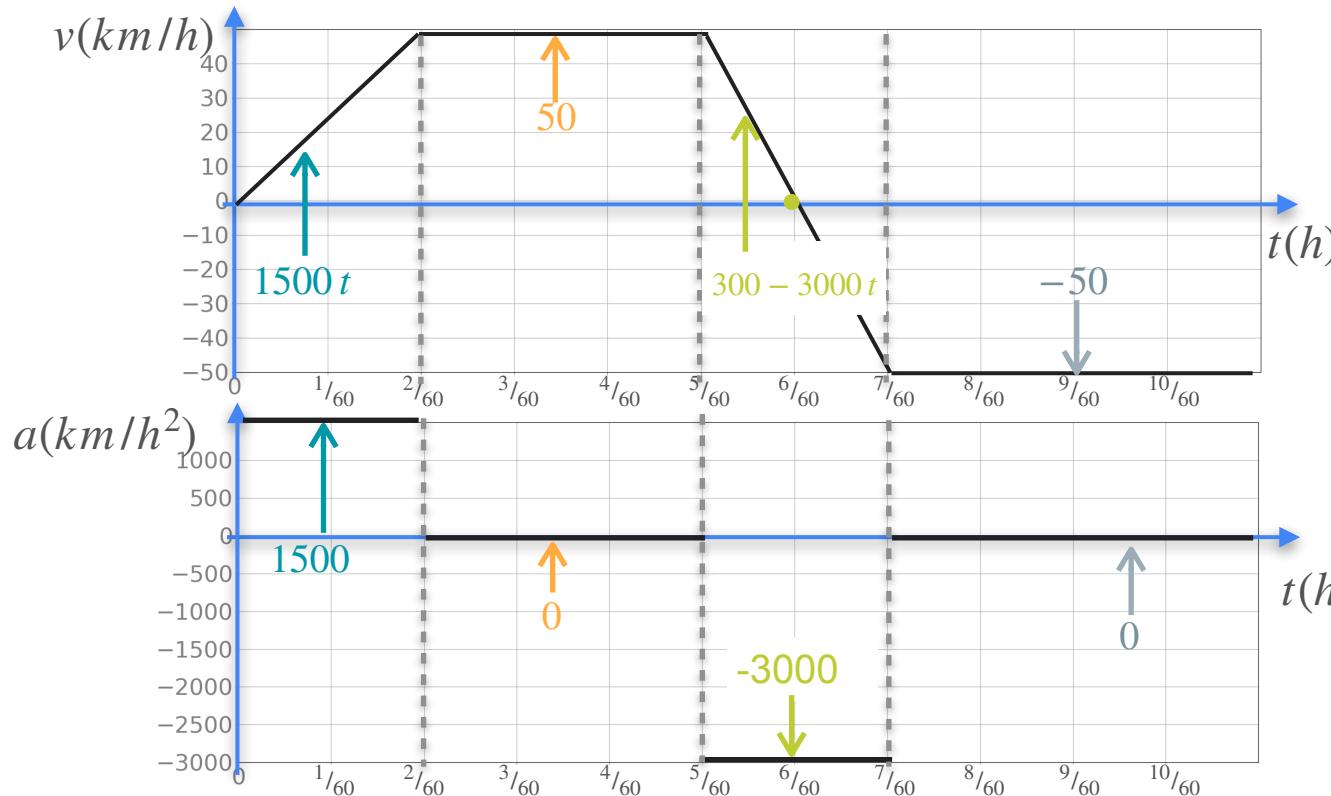
v Velocity

$$\frac{dx}{dt}$$

a Acceleration

$$\frac{dv}{dt}$$

Understanding Second Derivative



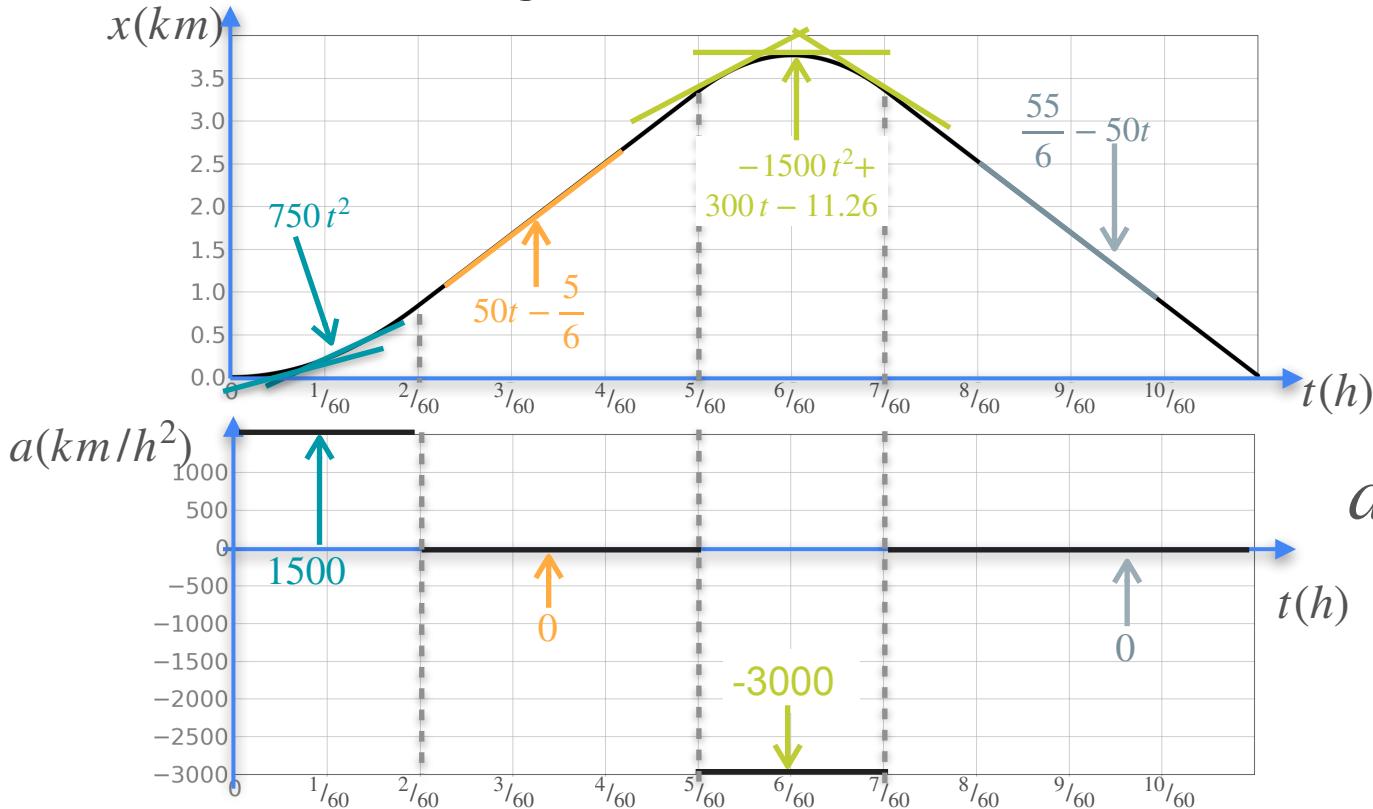
v Velocity

$$\frac{dx}{dt}$$

a Acceleration

$$\frac{dv}{dt}$$

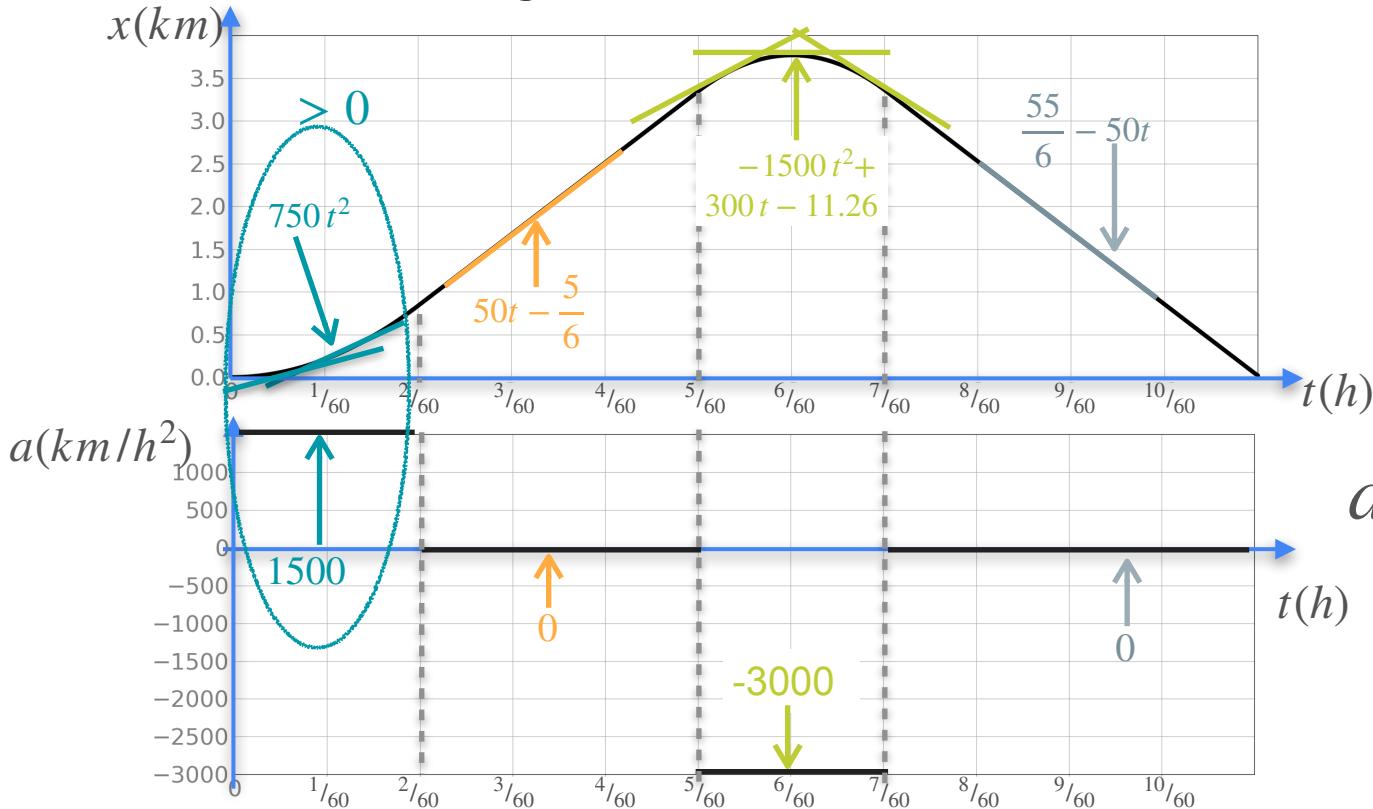
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

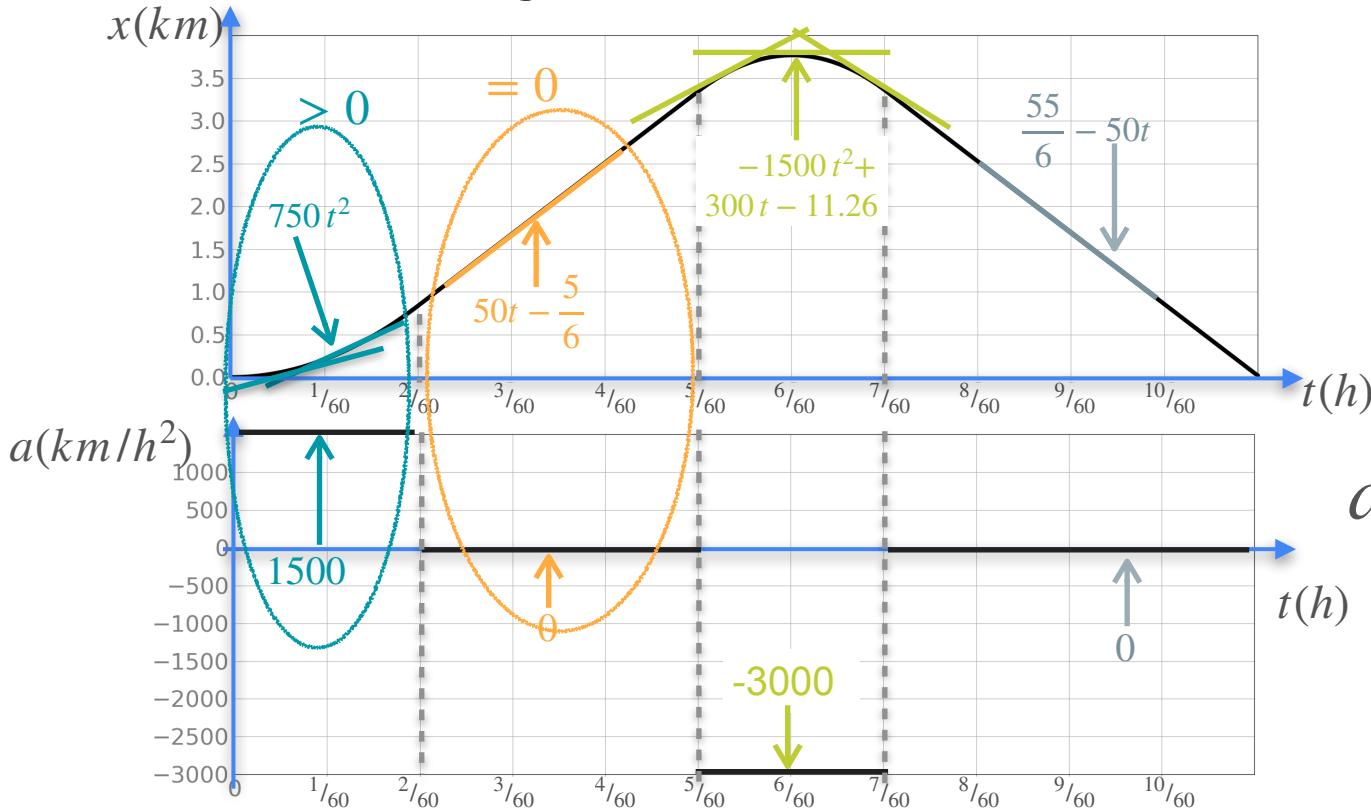
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

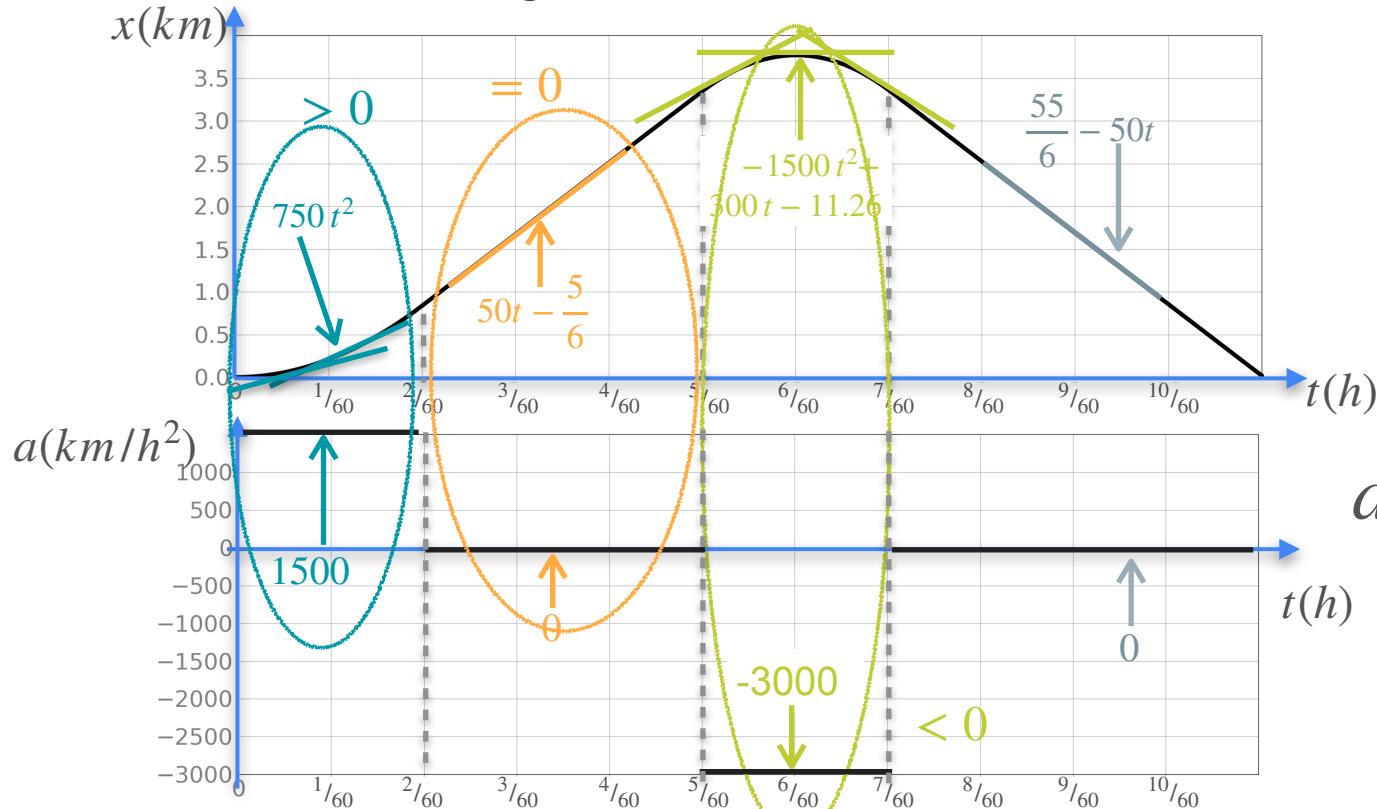
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

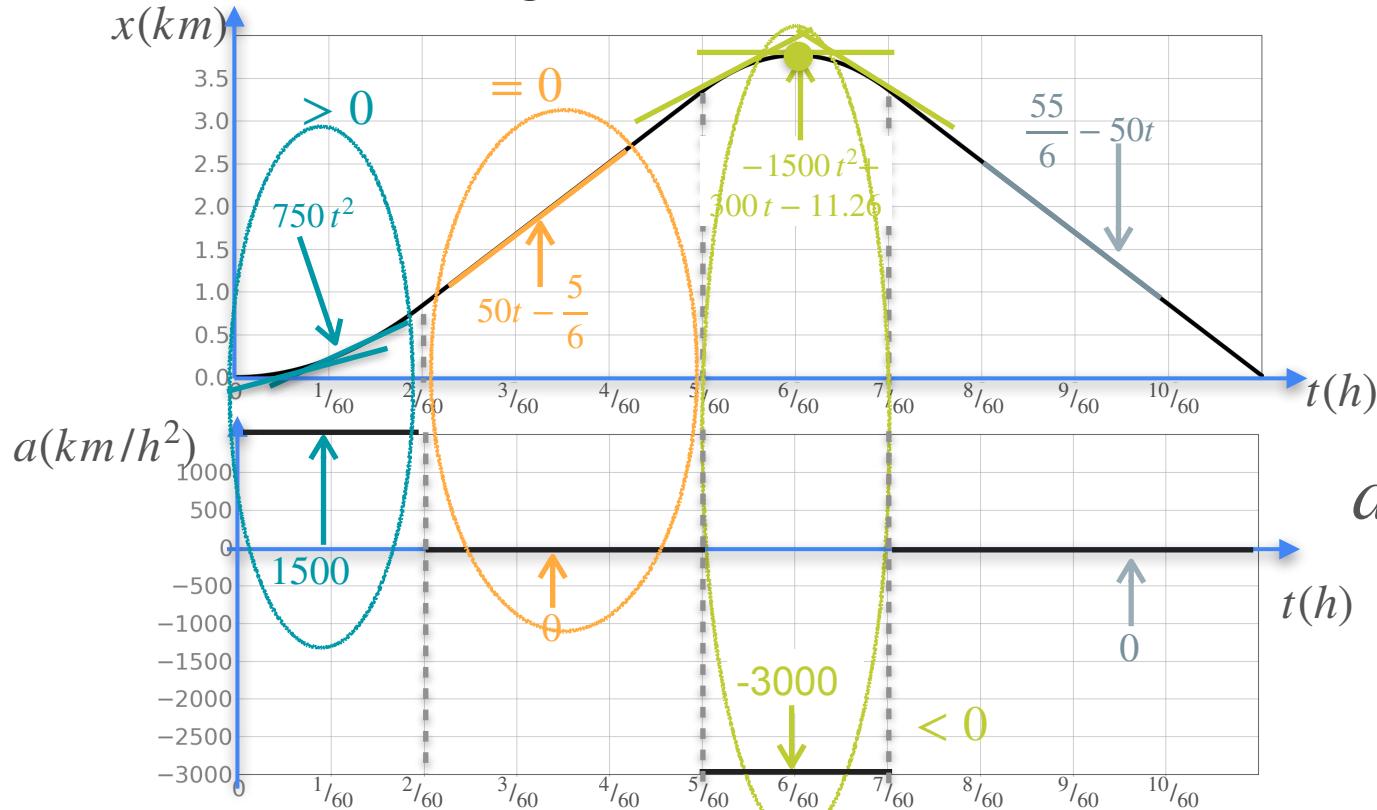
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

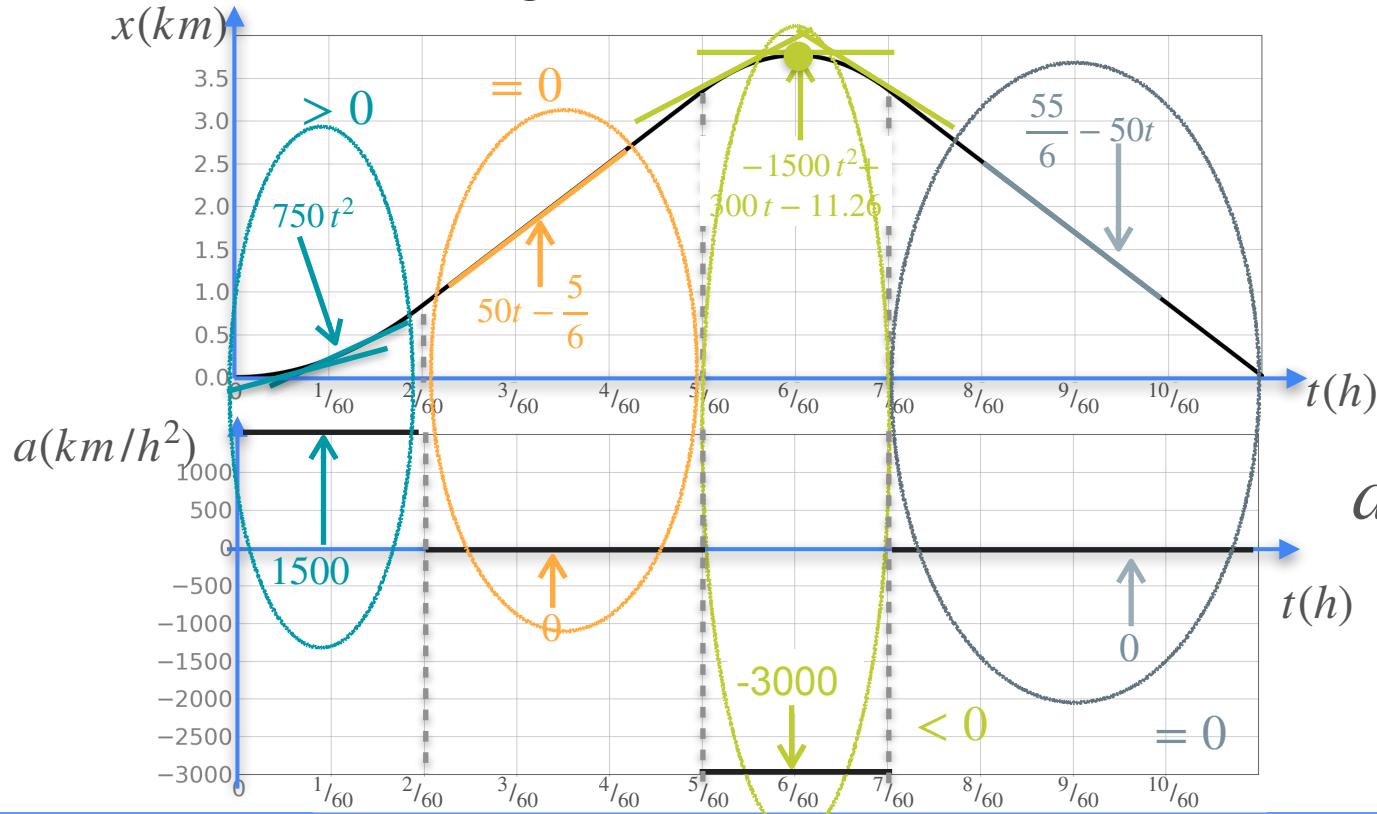
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

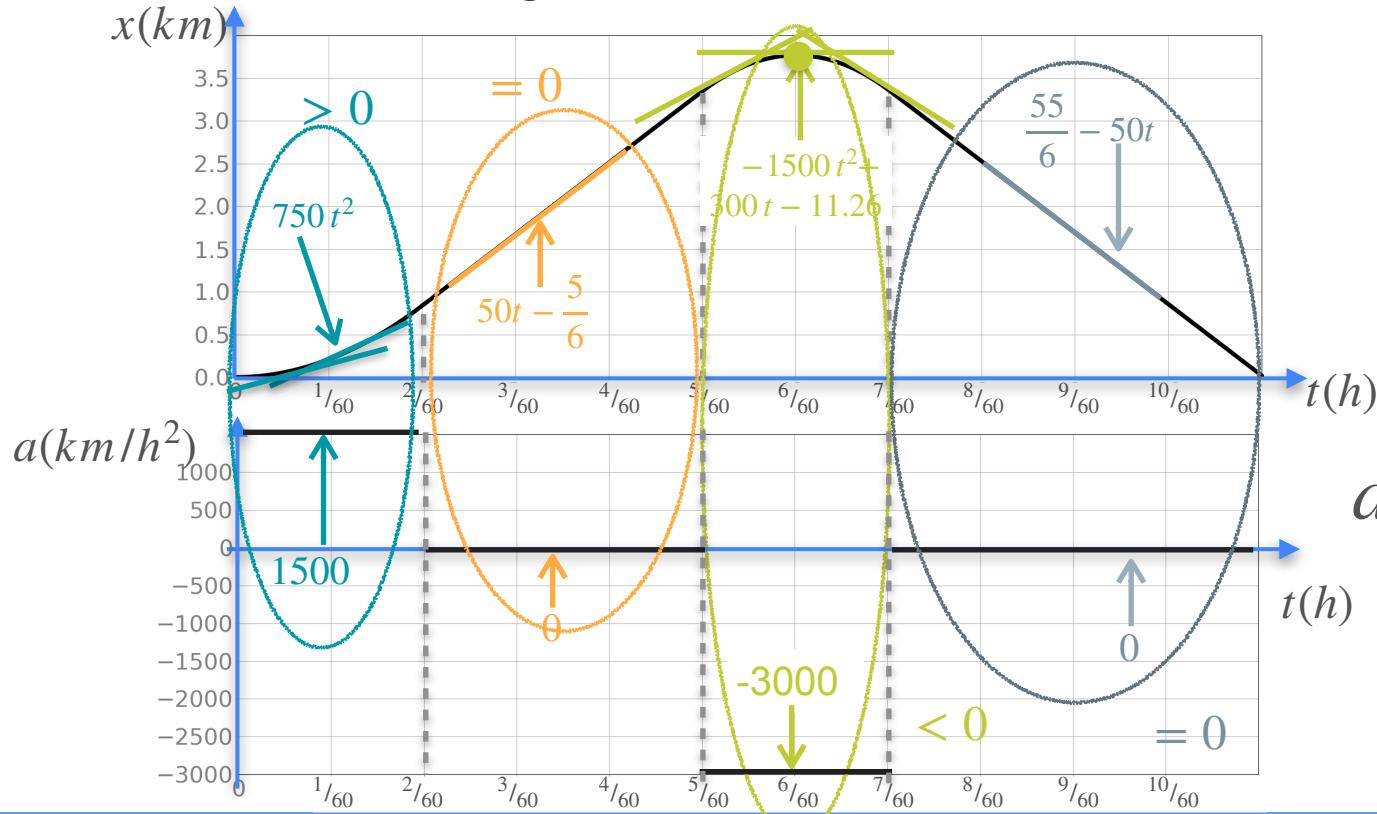
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2 x}{dt^2}$

Understanding Second Derivative



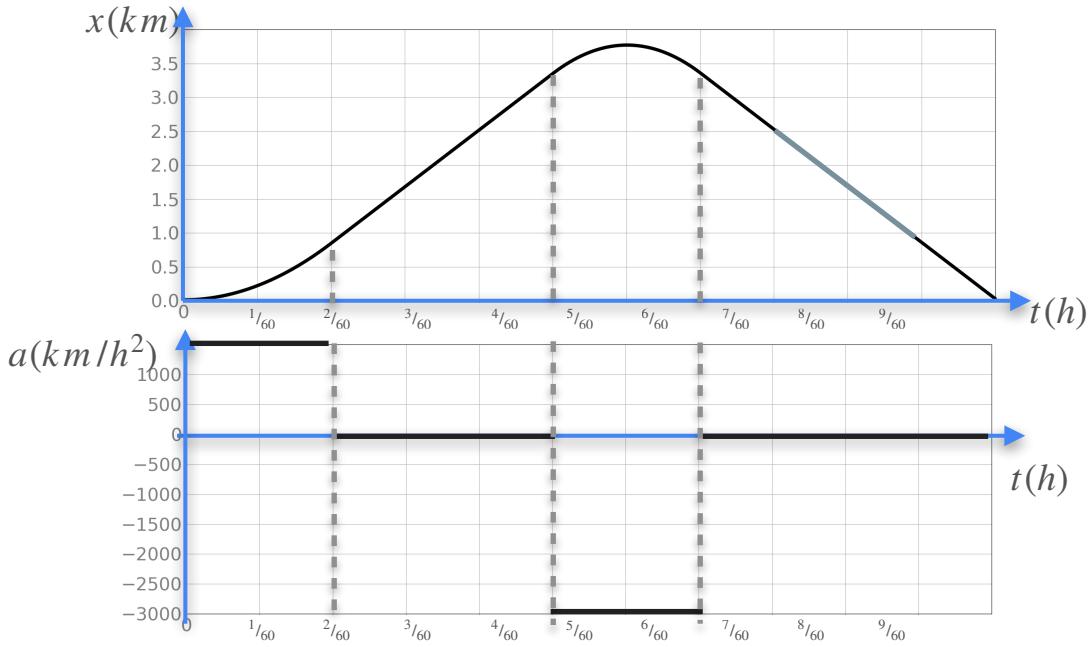
x Distance

Second derivative tells us about the curvature

a Acceleration $\frac{d^2x}{dt^2}$

Curvature

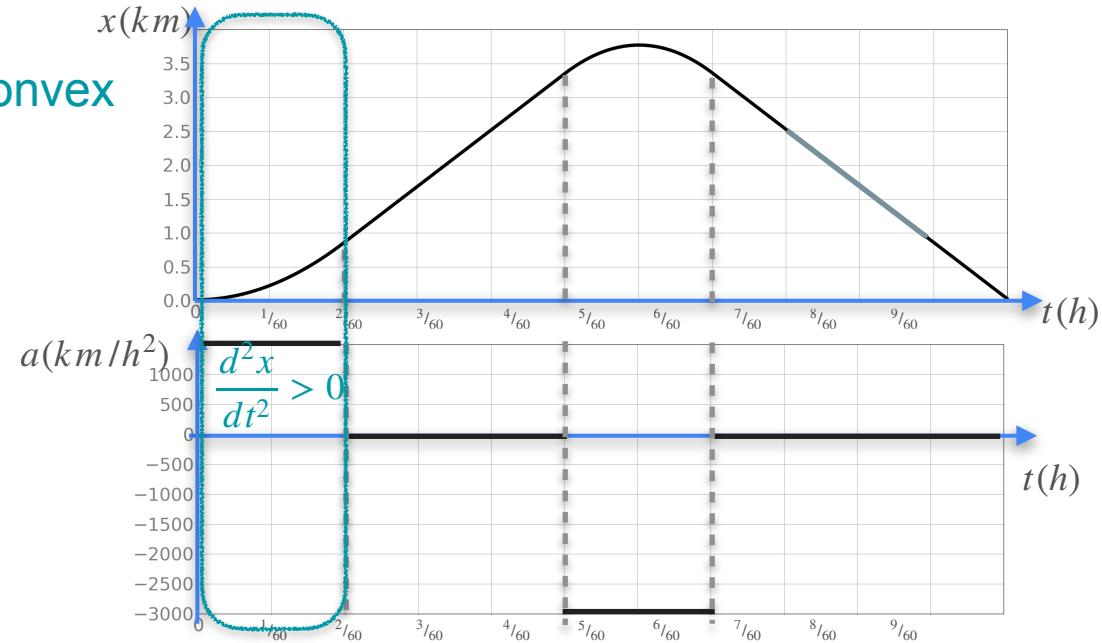
Curvature



Curvature

$$\frac{d^2x}{dt^2} > 0$$

Concave up or convex



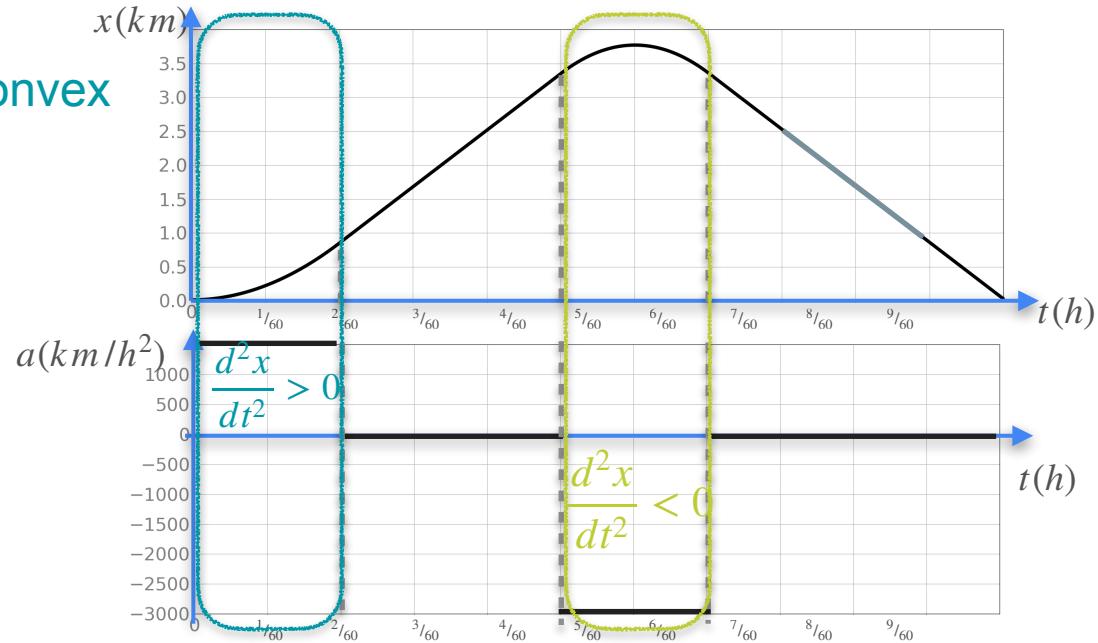
Curvature

$$\frac{d^2x}{dt^2} > 0$$

Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$

Concave down



Curvature

$$\frac{d^2x}{dt^2} > 0$$

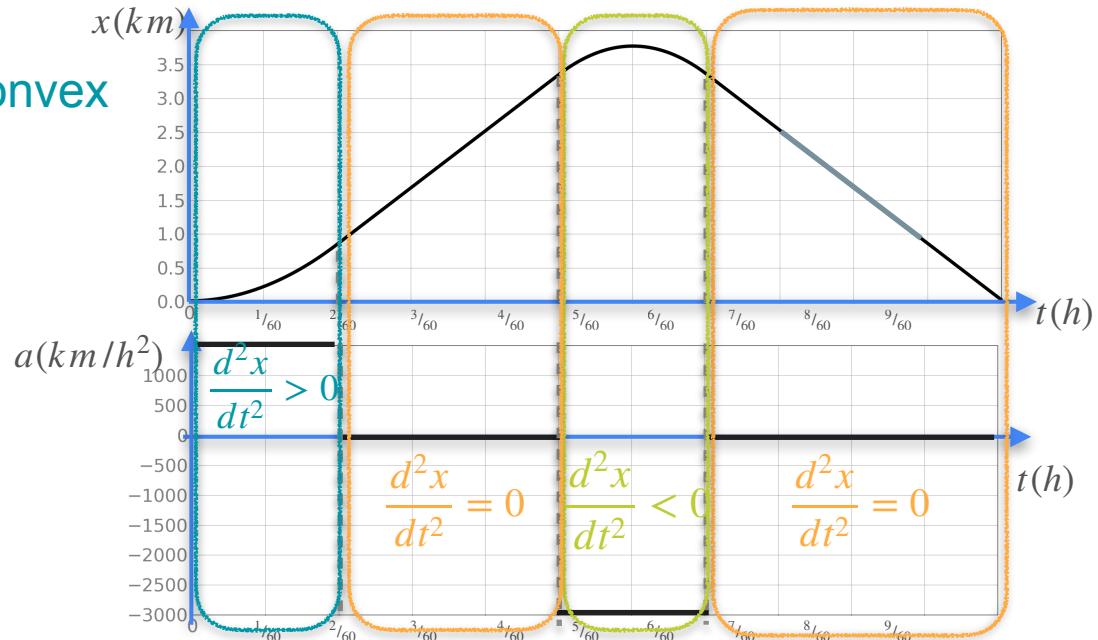
Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$

Concave down

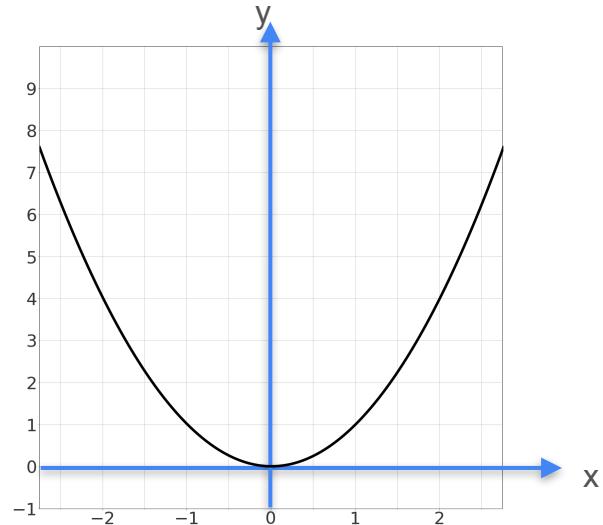
$$\frac{d^2x}{dt^2} = 0$$

Need more information

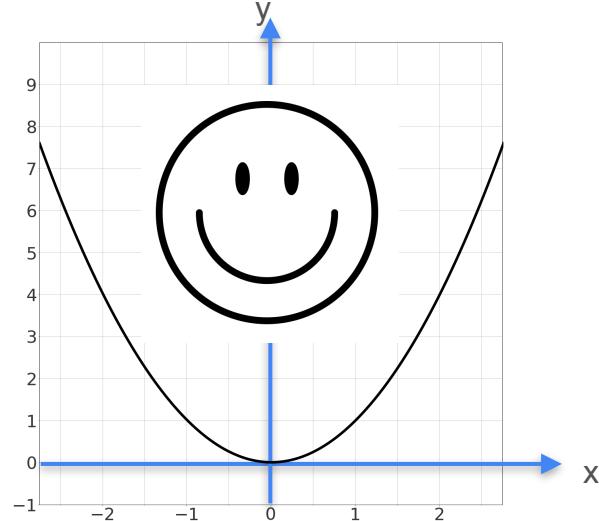


Curvature

Curvature



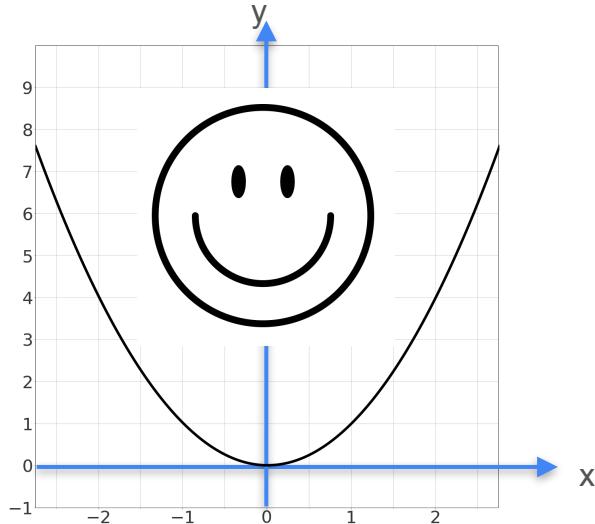
Curvature



Concave up or convex

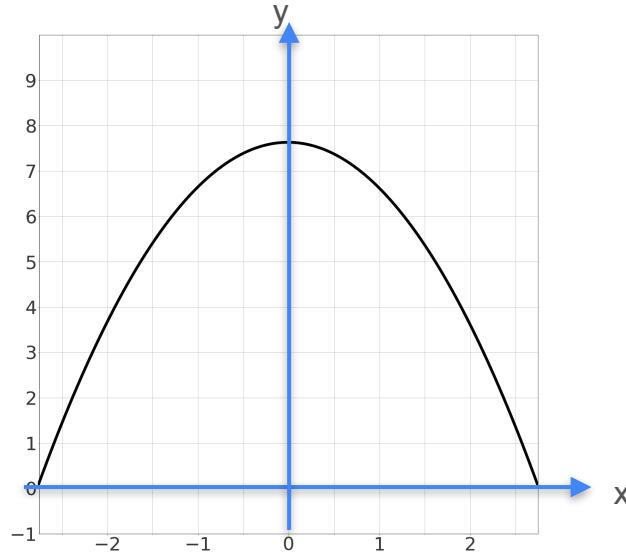
$$f''(0) > 0$$

Curvature

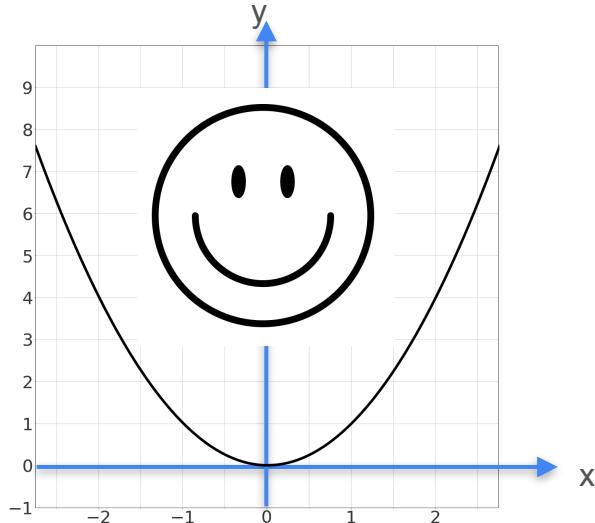


Concave up or convex

$$f''(0) > 0$$

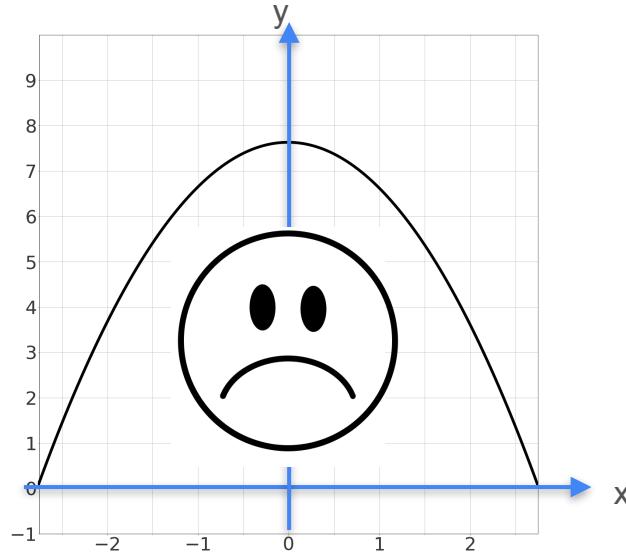


Curvature



Concave up or convex

$$f''(0) > 0$$



Concave down

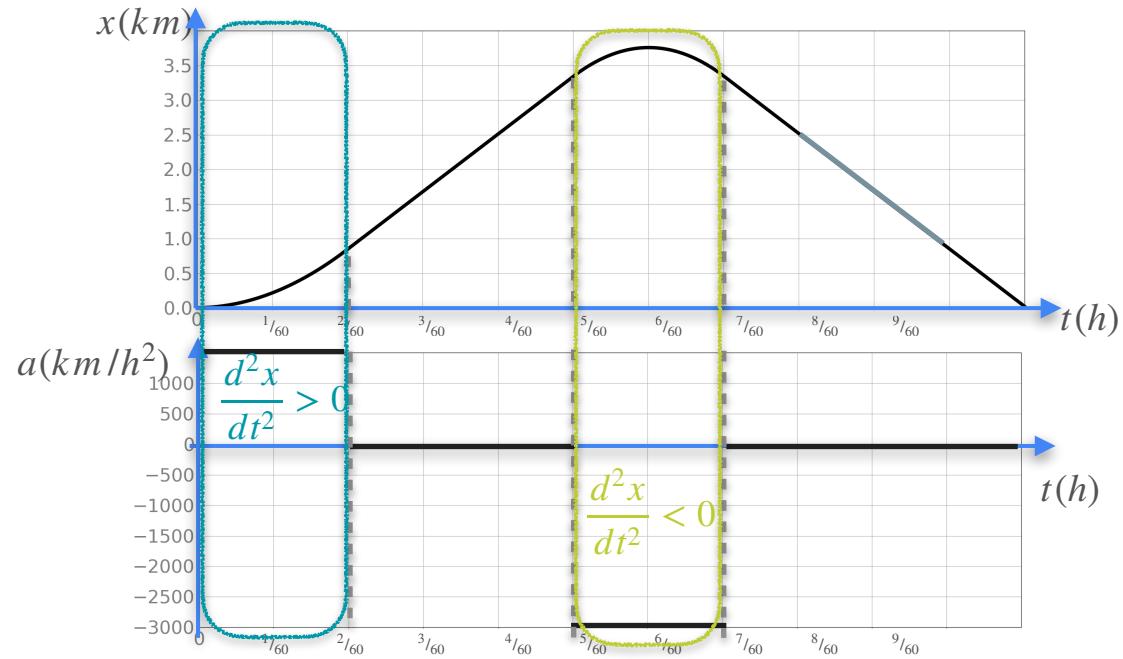
$$f''(0) < 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

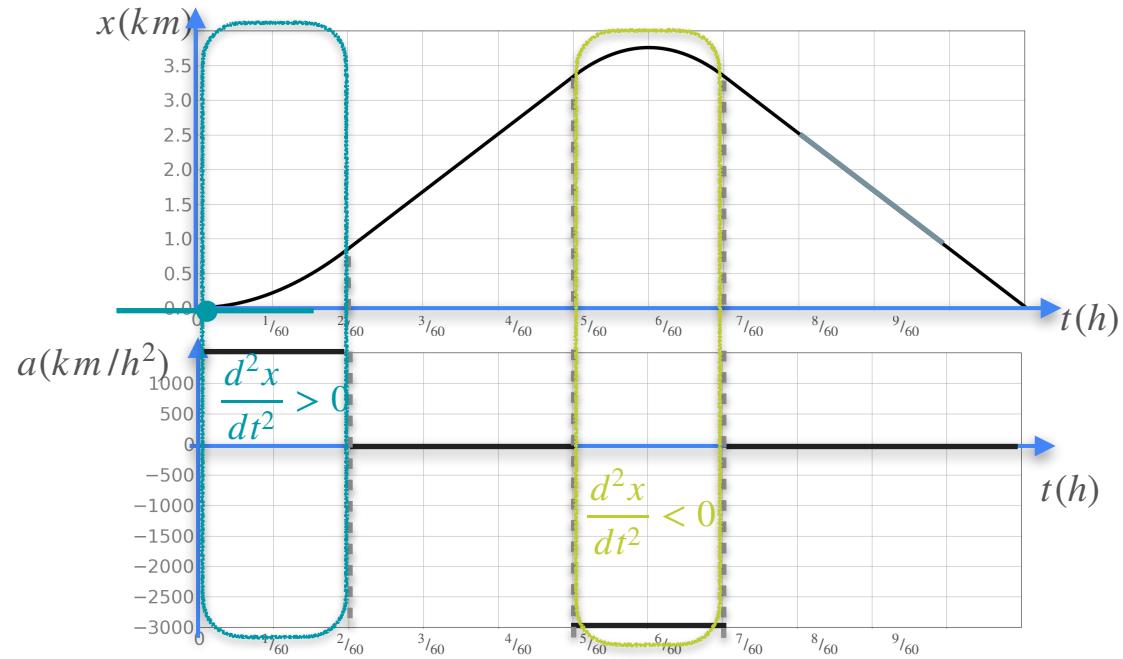


Second Derivative and Optimization

$\frac{d^2x}{dt^2} > 0$ (Local) Minimum

$\frac{d^2x}{dt^2} < 0$

$\frac{d^2x}{dt^2} = 0$

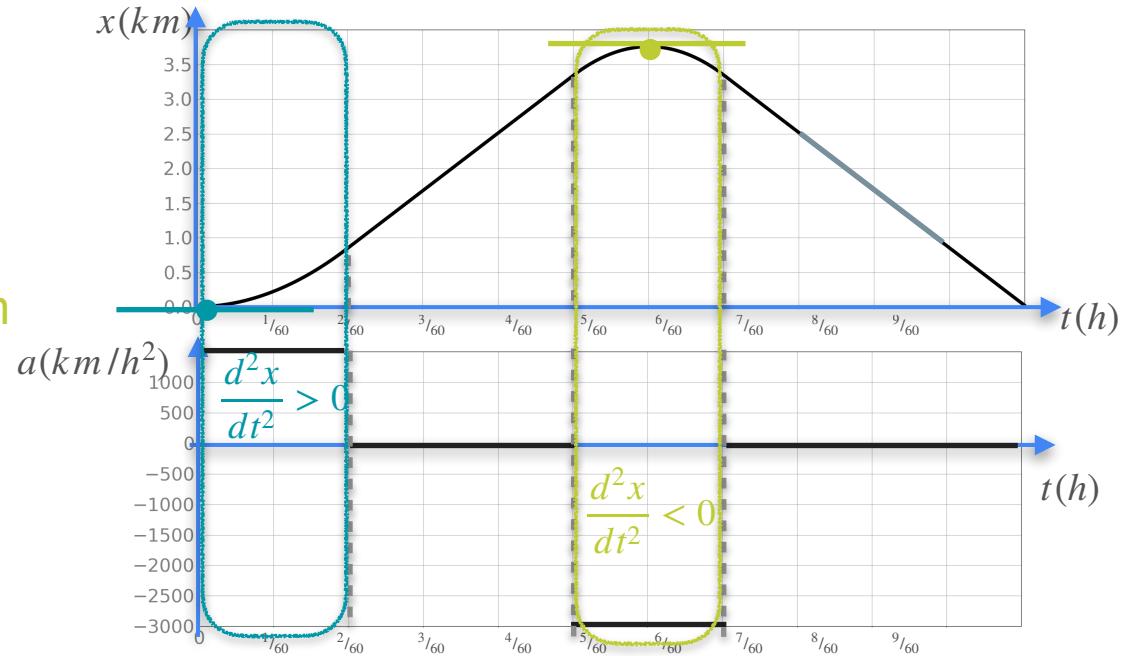


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0 \quad (\text{Local}) \text{ Minimum}$$

$$\frac{d^2x}{dt^2} < 0 \quad (\text{Local}) \text{ maximum}$$

$$\frac{d^2x}{dt^2} = 0$$

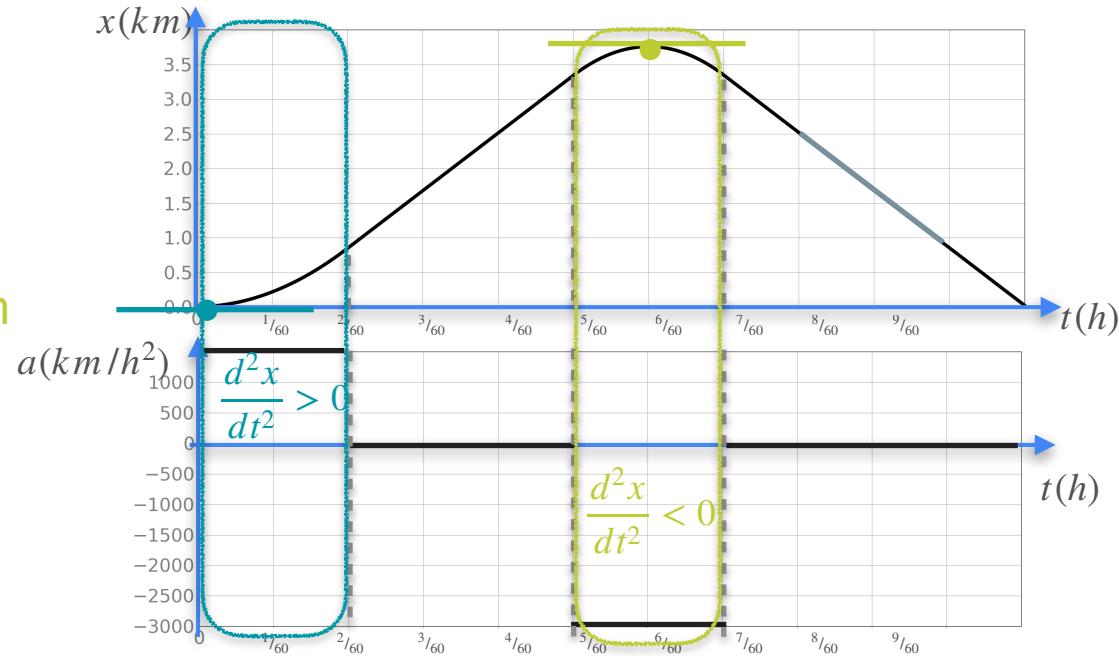


Second Derivative and Optimization

$\frac{d^2x}{dt^2} > 0$ (Local) Minimum

$\frac{d^2x}{dt^2} < 0$ (Local) maximum

$\frac{d^2x}{dt^2} = 0$ Inconclusive



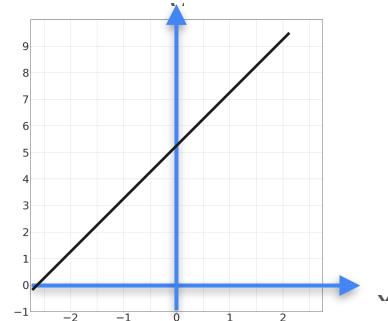
Curvature

First derivative

Second derivative

Curvature

First derivative



Increasing

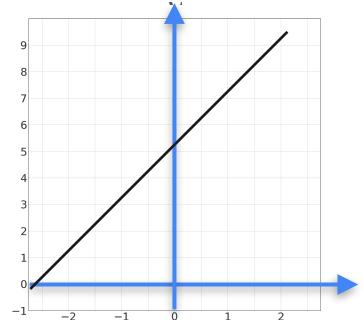
$$f'(0) > 0$$

Second derivative



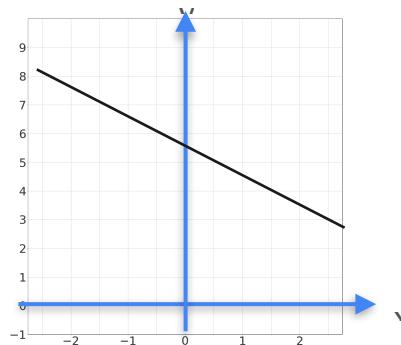
Curvature

First derivative



Increasing

$$f'(0) > 0$$



Decreasing

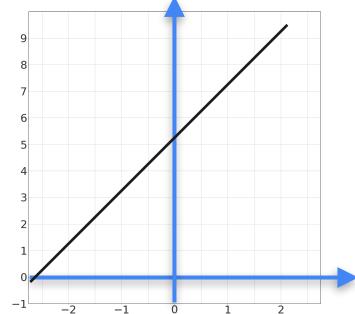
$$f'(0) < 0$$

Second derivative



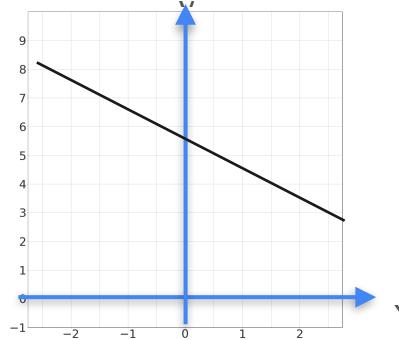
Curvature

First derivative



Increasing

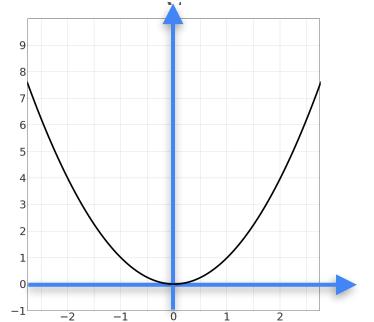
$$f'(0) > 0$$



Decreasing

$$f'(0) < 0$$

Second derivative

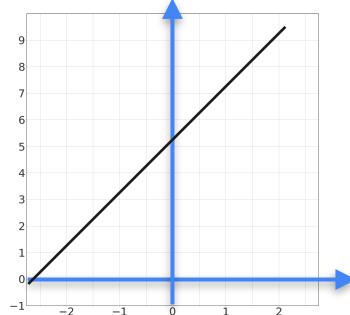


Concave up

$$f''(0) > 0$$

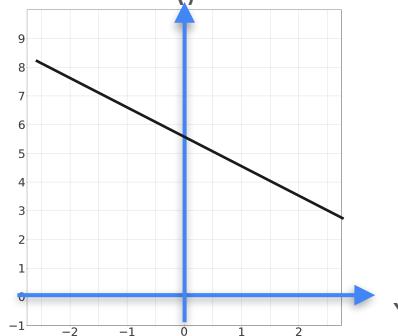
Curvature

First derivative



Increasing

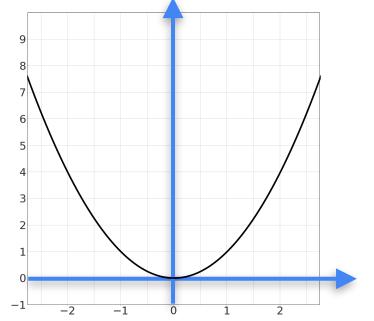
$$f'(0) > 0$$



Decreasing

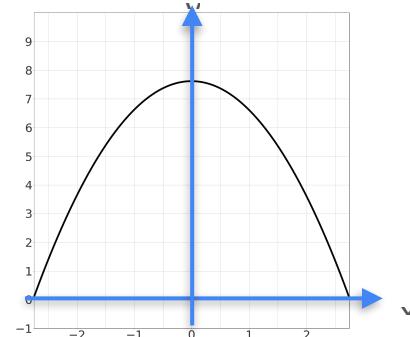
$$f'(0) < 0$$

Second derivative



Concave up

$$f''(0) > 0$$



Concave down

$$f''(0) < 0$$



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

The Hessian

Second Derivative

Second Derivative

1 variable

2 variables

Second Derivative

	1 variable	2 variables
Function	$f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$ $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$

Second Derivative

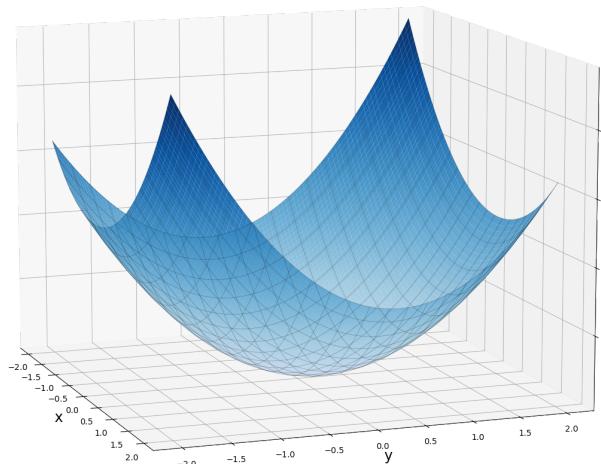
	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$
Second derivative	$f''(x)$ Rate of change of the rate of change of $f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$
Second derivative	$f''(x)$ Rate of change of the rate of change of $f(x)$???

Second Derivative

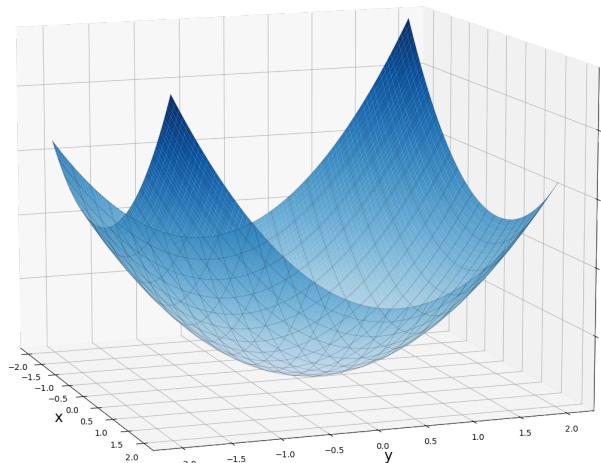
Second Derivative



$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

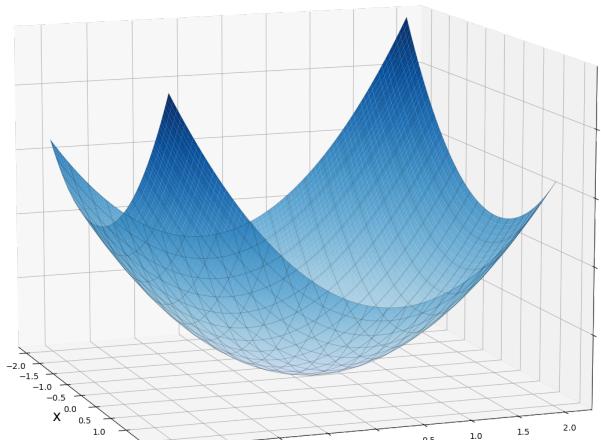
Second Derivative



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\begin{matrix} 4x - y \\ x \end{matrix}$$

Second Derivative

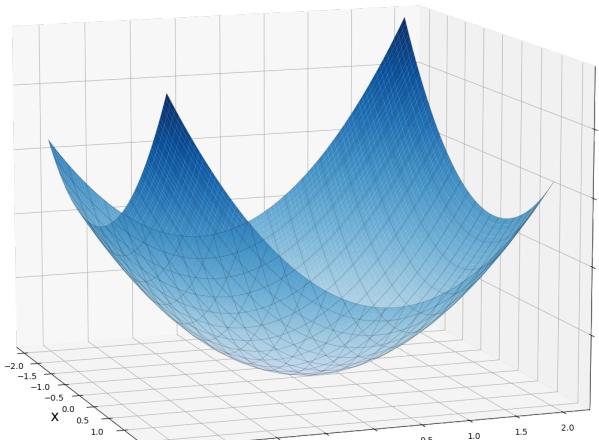


$$f(x, y) = 2x^2 + 3y^2 - xy$$

A diagram illustrating the second derivatives of the function $f(x, y) = 2x^2 + 3y^2 - xy$. A central point is connected by three arrows to the terms $4x - y$, x , and $6y - x$.

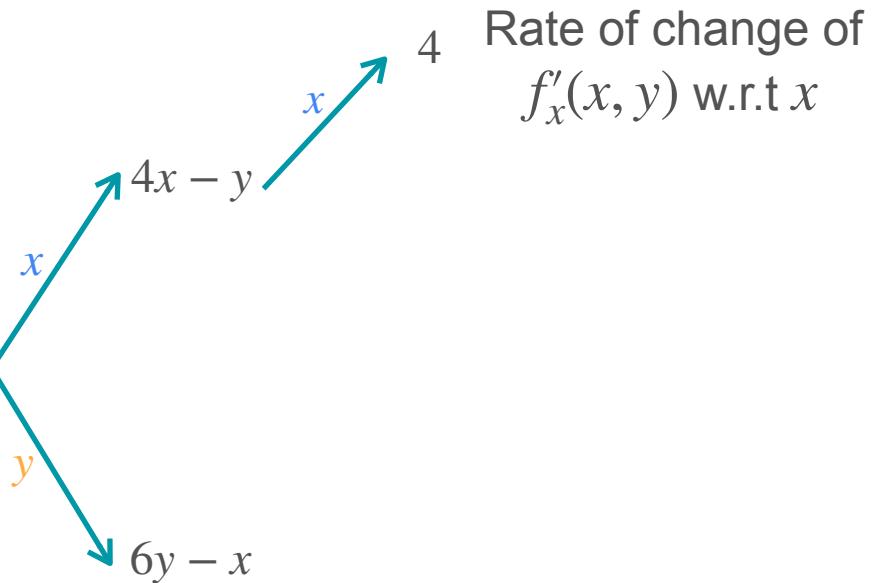
The term $4x - y$ is associated with the x -axis direction, x is associated with the central point, and $6y - x$ is associated with the y -axis direction.

Second Derivative

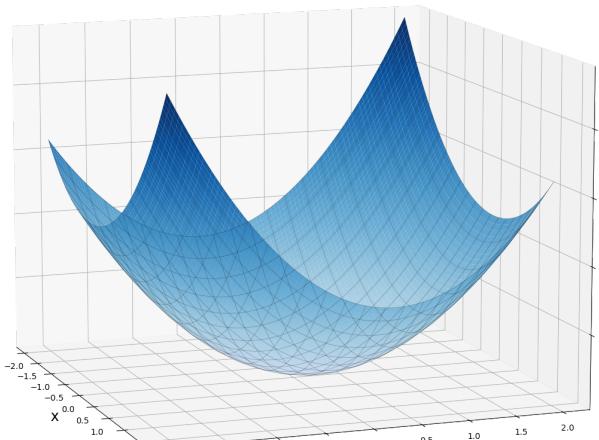


$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

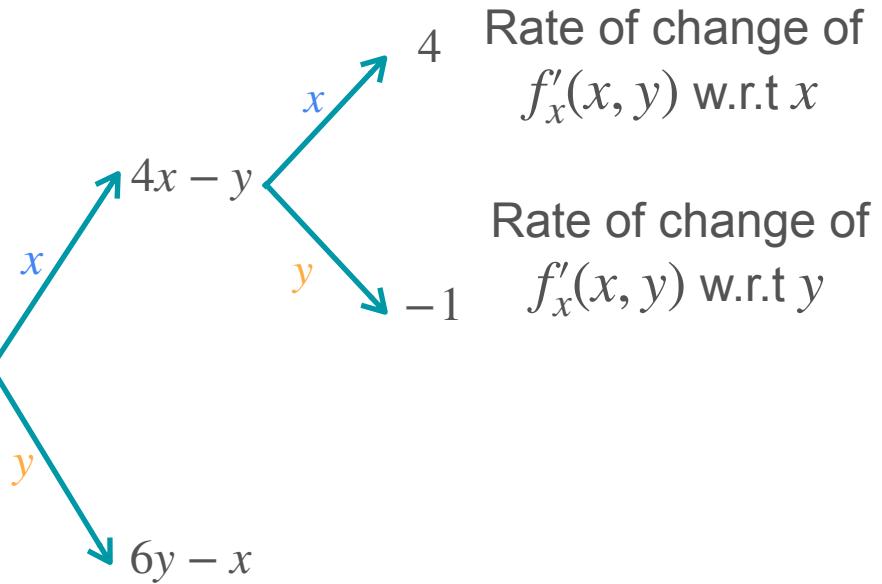


Second Derivative

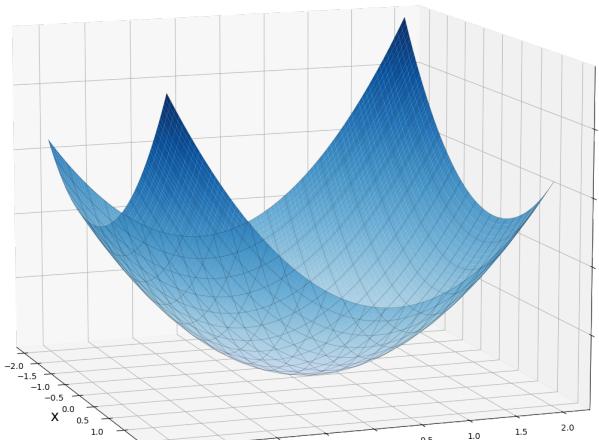


$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

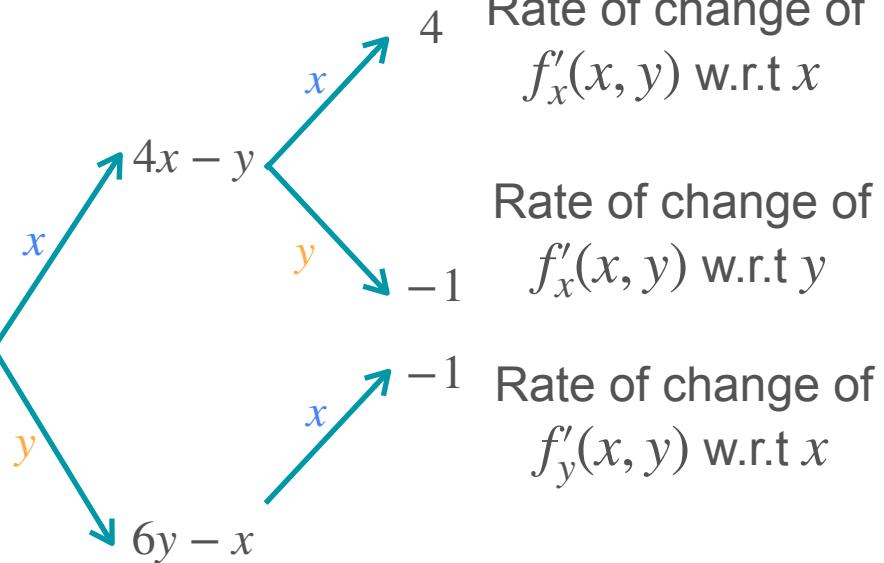


Second Derivative

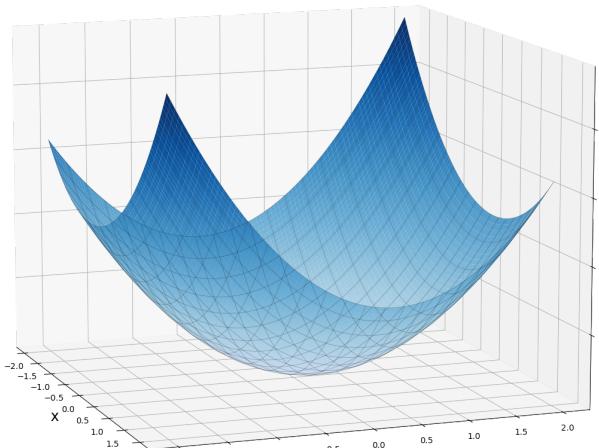


$$f(x, y) =$$

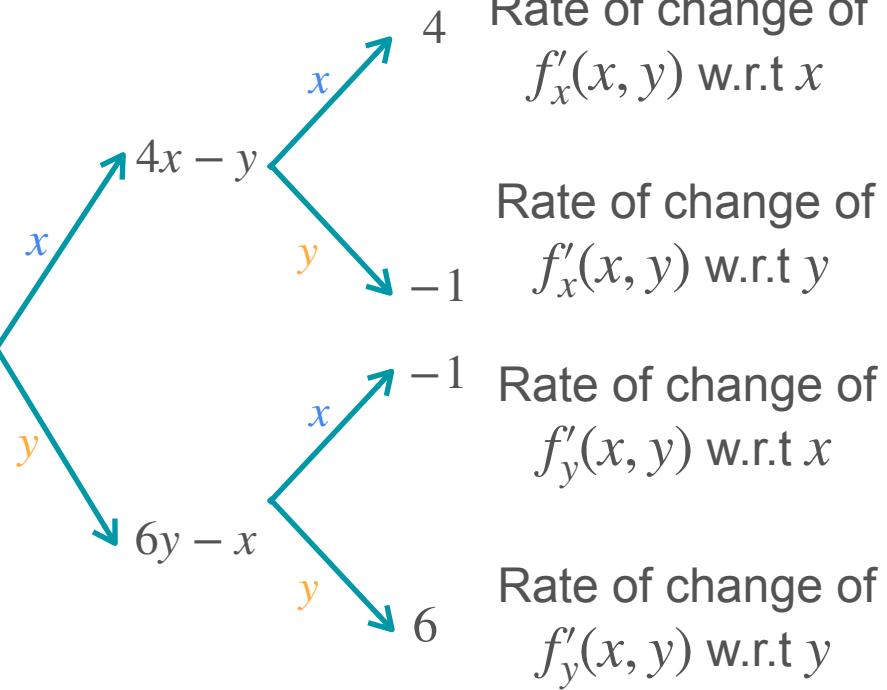
$$2x^2 + 3y^2 - xy$$



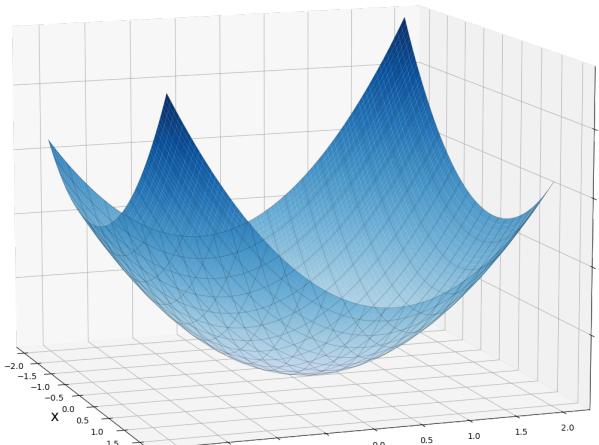
Second Derivative



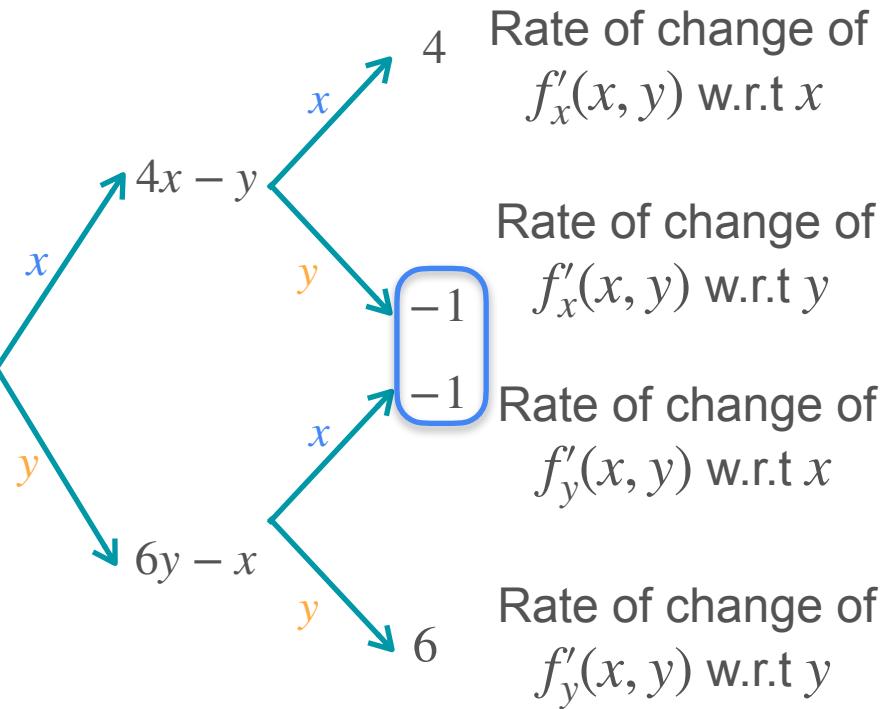
$$f(x, y) =$$
$$2x^2 + 3y^2 - xy$$



Second Derivative



$$f(x, y) =$$
$$2x^2 + 3y^2 - xy$$



What Do These Mean?

Rate of change of
 $f_x(x, y)$ w.r.t x

Rate of change of
 $f_y(x, y)$ w.r.t y

Rate of change of
 $f_x(x, y)$ w.r.t y

Rate of change of
 $f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of
 $f_x(x, y)$ w.r.t x

Rate of change of
 $f_y(x, y)$ w.r.t y

Change in the change in the function
w.r.t tiny changes in x and y

Rate of change of
 $f_x(x, y)$ w.r.t y

Rate of change of
 $f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of

$f_x(x, y)$ w.r.t x

Rate of change of

$f_y(x, y)$ w.r.t y

Rate of change of

$f_x(x, y)$ w.r.t y

Rate of change of

$f_y(x, y)$ w.r.t x

Change in the change in the function
w.r.t tiny changes in x and y

Same idea as
with one
variable!

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

What Do These Mean?

Rate of change of
 $f_x(x, y)$ w.r.t x

Rate of change of
 $f_y(x, y)$ w.r.t y

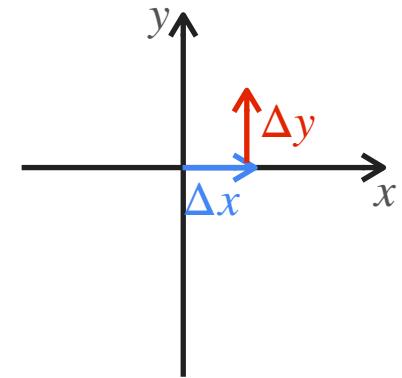
Change in the change in the function
w.r.t tiny changes in x and y

Rate of change of
 $f_x(x, y)$ w.r.t y

Rate of change of
 $f_y(x, y)$ w.r.t x

Same idea as
with one
variable!

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis



What Do These Mean?

Rate of change of

$f_x(x, y)$ w.r.t x

Rate of change of

$f_y(x, y)$ w.r.t y

Rate of change of

$f_x(x, y)$ w.r.t y

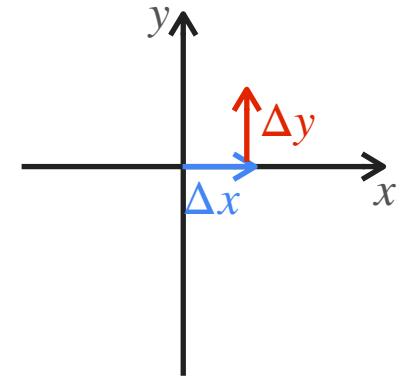
Rate of change of

$f_y(x, y)$ w.r.t x

Change in the change in the function
w.r.t tiny changes in x and y

Same idea as
with one
variable!

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis
2. They are the same!



What Do These Mean?

Rate of change of
 $f_x(x, y)$ w.r.t x

Rate of change of
 $f_y(x, y)$ w.r.t y

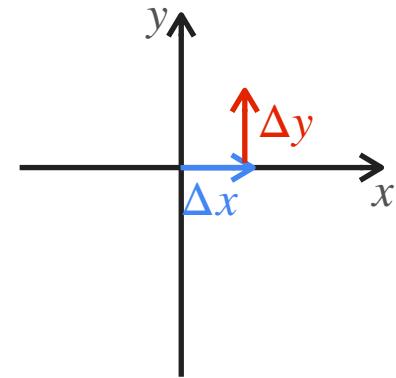
Change in the change in the function
w.r.t tiny changes in x and y

Rate of change of
 $f_x(x, y)$ w.r.t y

Rate of change of
 $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis
2. They are the same!
(In most cases)

Same idea as
with one
variable!



Notation

Rate of change of

$$f'_x(x, y) \text{ w.r.t } x$$

Rate of change of

$$f'_y(x, y) \text{ w.r.t } y$$

Rate of change of

$$f'_x(x, y) \text{ w.r.t } y$$

Rate of change of

$$f'_y(x, y) \text{ w.r.t } x$$

Notation

Leibniz's notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

Rate of change of
 $f'_y(x, y)$ w.r.t y

Rate of change of
 $f'_x(x, y)$ w.r.t y

Rate of change of
 $f'_y(x, y)$ w.r.t x

Notation

Leibniz's notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

$$\frac{\partial^2 f}{\partial x^2}$$

Rate of change of
 $f'_y(x, y)$ w.r.t y

$$\frac{\partial^2 f}{\partial y^2}$$

Rate of change of
 $f'_x(x, y)$ w.r.t y

Rate of change of
 $f'_y(x, y)$ w.r.t x

Notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

Rate of change of
 $f'_y(x, y)$ w.r.t y

Rate of change of
 $f'_x(x, y)$ w.r.t y

Rate of change of
 $f'_y(x, y)$ w.r.t x

Leibniz's notation

$$\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial y \partial x}$$

Notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

Rate of change of
 $f'_y(x, y)$ w.r.t y

Rate of change of
 $f'_x(x, y)$ w.r.t y

Rate of change of
 $f'_y(x, y)$ w.r.t x

Leibniz's notation

$$\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial y \partial x}$$

Lagrange's notation

Notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

Rate of change of
 $f'_y(x, y)$ w.r.t y

Rate of change of
 $f''_x(x, y)$ w.r.t y

Rate of change of
 $f''_y(x, y)$ w.r.t x

Leibniz's notation

$$\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial y \partial x}$$

Lagrange's notation

$$f_{xx}(x, y)$$

$$f_{yy}(x, y)$$

Notation

Rate of change of
 $f'_x(x, y)$ w.r.t x

Rate of change of
 $f'_y(x, y)$ w.r.t y

Rate of change of
 $f''_x(x, y)$ w.r.t y

Rate of change of
 $f''_y(x, y)$ w.r.t x

Leibniz's notation

$$\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial y \partial x}$$

Lagrange's notation

$$f_{xx}(x, y)$$

$$f_{yy}(x, y)$$

$$f_{xy}(x, y)$$

$$f_{yx}(x, y)$$

Hessian Matrix

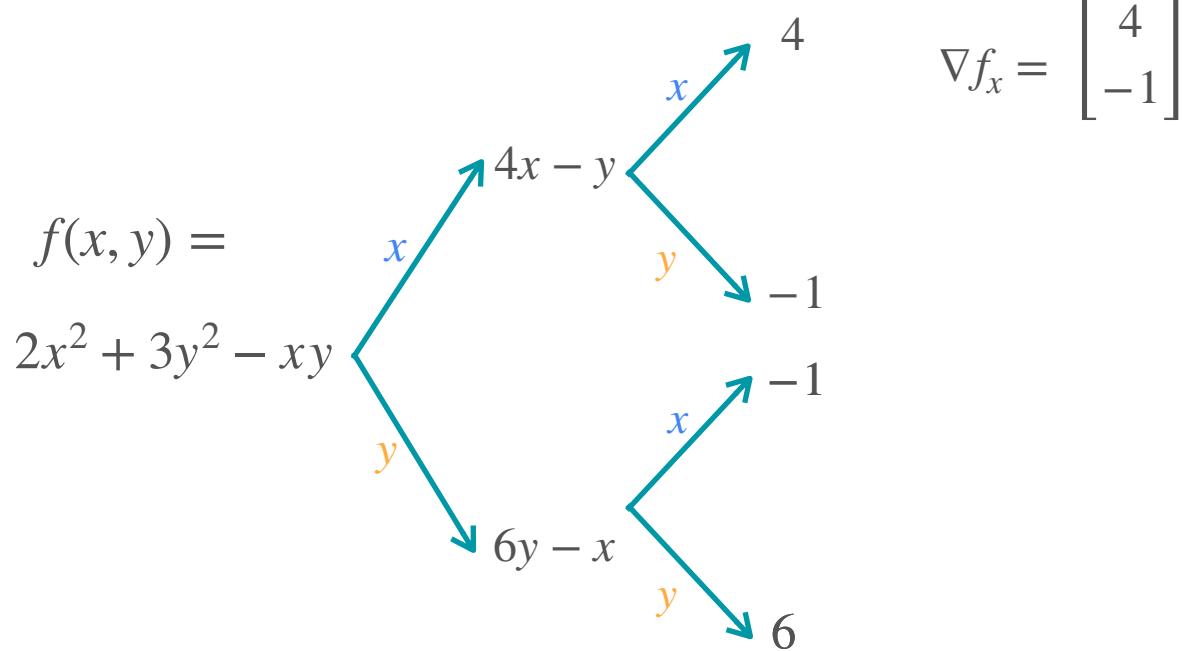
Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$
$$\begin{matrix} & \begin{matrix} 4 & \\ & -1 \end{matrix} \\ \begin{matrix} x \\ y \end{matrix} & \begin{pmatrix} 4x - y & \\ & 6y - x \end{pmatrix} \\ & \begin{matrix} -1 & \\ & 6 \end{matrix} \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$
$$\begin{matrix} 4 & -1 \\ -1 & 6 \end{matrix}$$

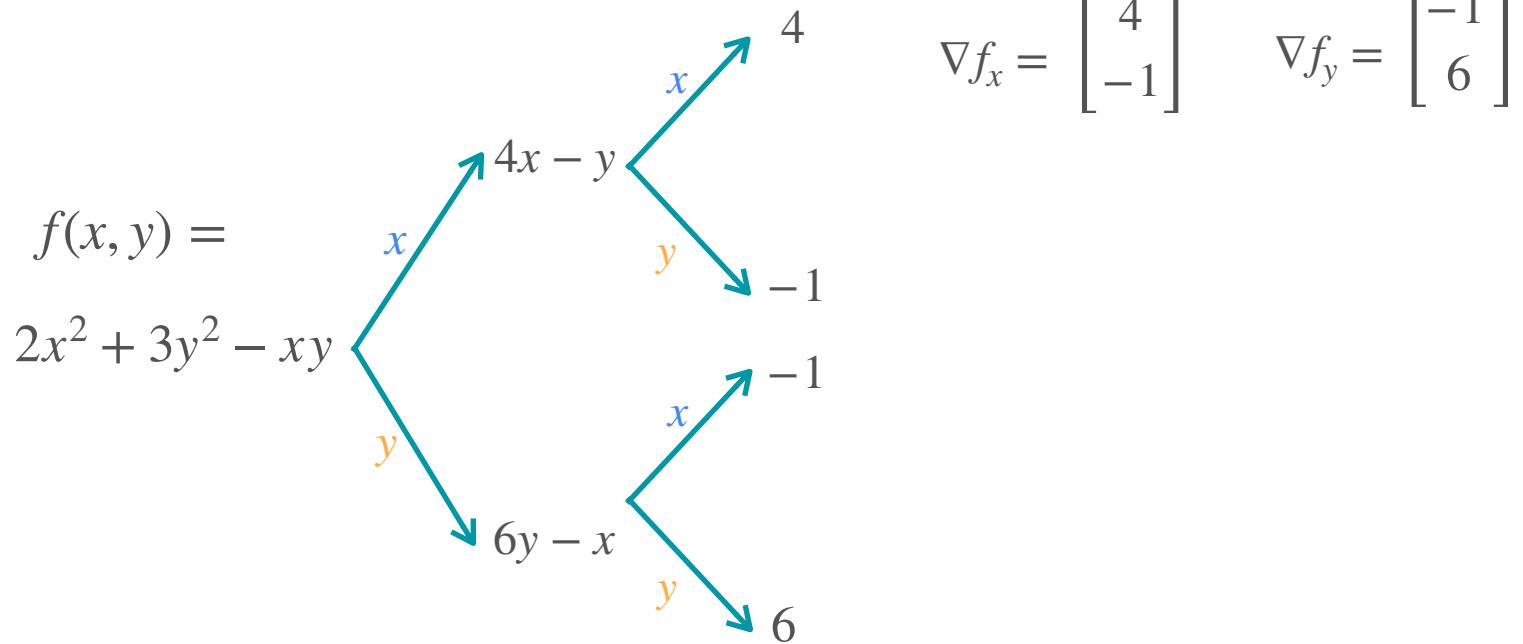
Hessian Matrix



Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$
$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$
$$\begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

Hessian Matrix



Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$
$$\begin{matrix} 4 & -1 \\ -1 & 6 \end{matrix}$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \quad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$
$$\begin{matrix} & \begin{matrix} 4x - y & 4 \\ 6y - x & 6 \end{matrix} \\ \begin{matrix} x \\ y \end{matrix} & \end{matrix}$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \quad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x^T \\ \nabla f_y^T \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Diagram illustrating the second partial derivatives of the function $f(x, y) = 2x^2 + 3y^2 - xy$. The function value is at the center. Four arrows point outwards from the center, each labeled with a second derivative:

- Top-right arrow: 4 (blue)
- Top-left arrow: -1 (orange)
- Bottom-left arrow: -1 (blue)
- Bottom-right arrow: 6 (blue)

The labels $4x - y$ and $6y - x$ are also present near the arrows.

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \quad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

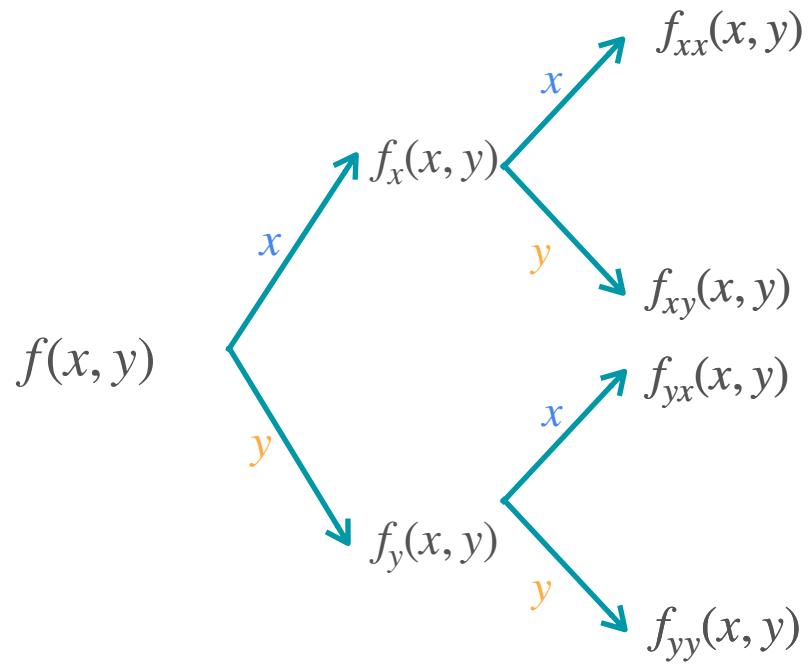
$$H = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x^T \\ \nabla f_y^T \end{bmatrix}$$

**Hessian
matrix**

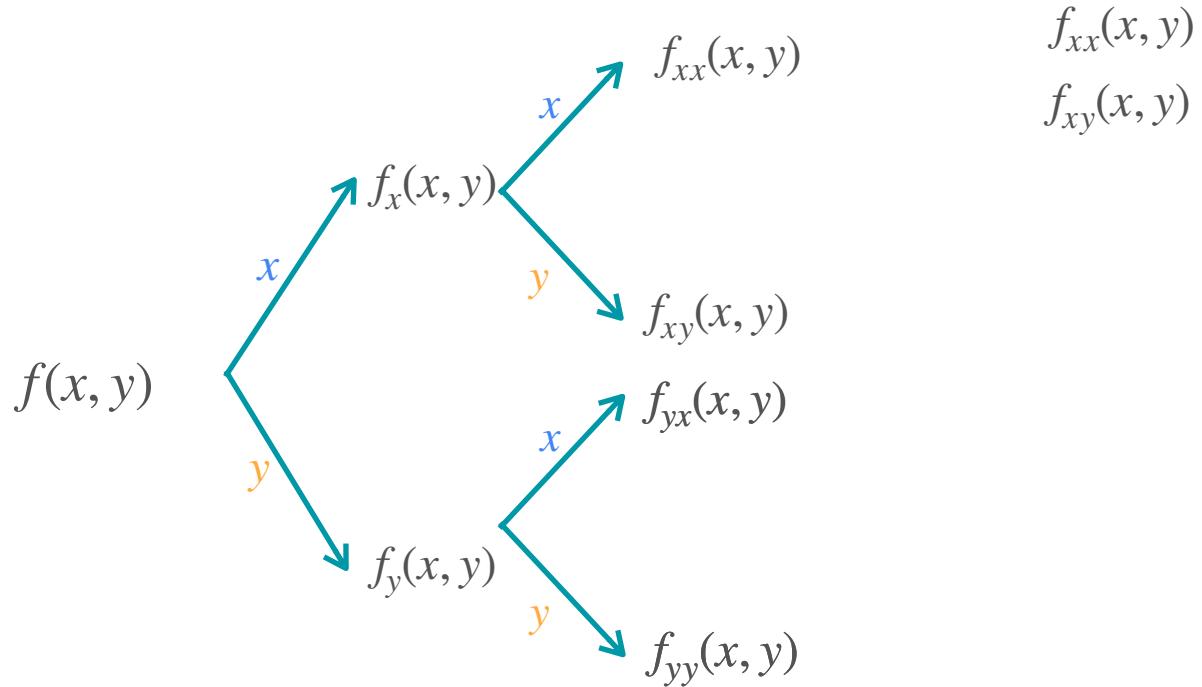
All information
about second
derivatives

Hessian Matrix - General Case

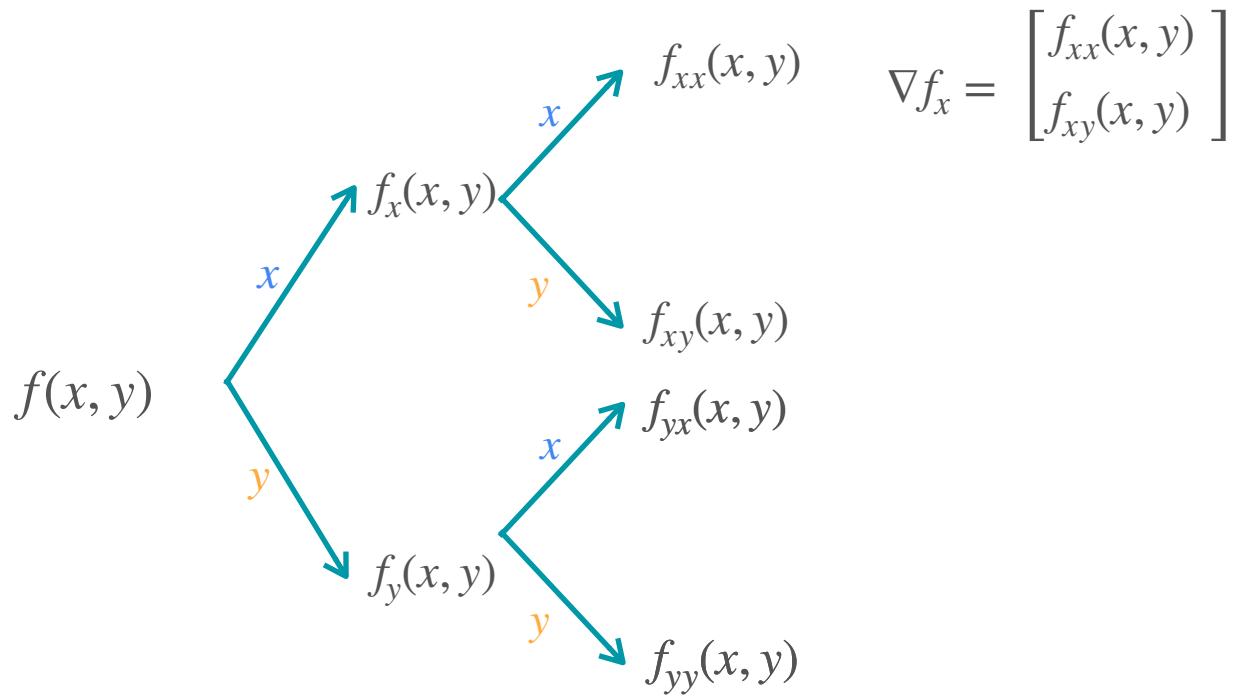
Hessian Matrix - General Case



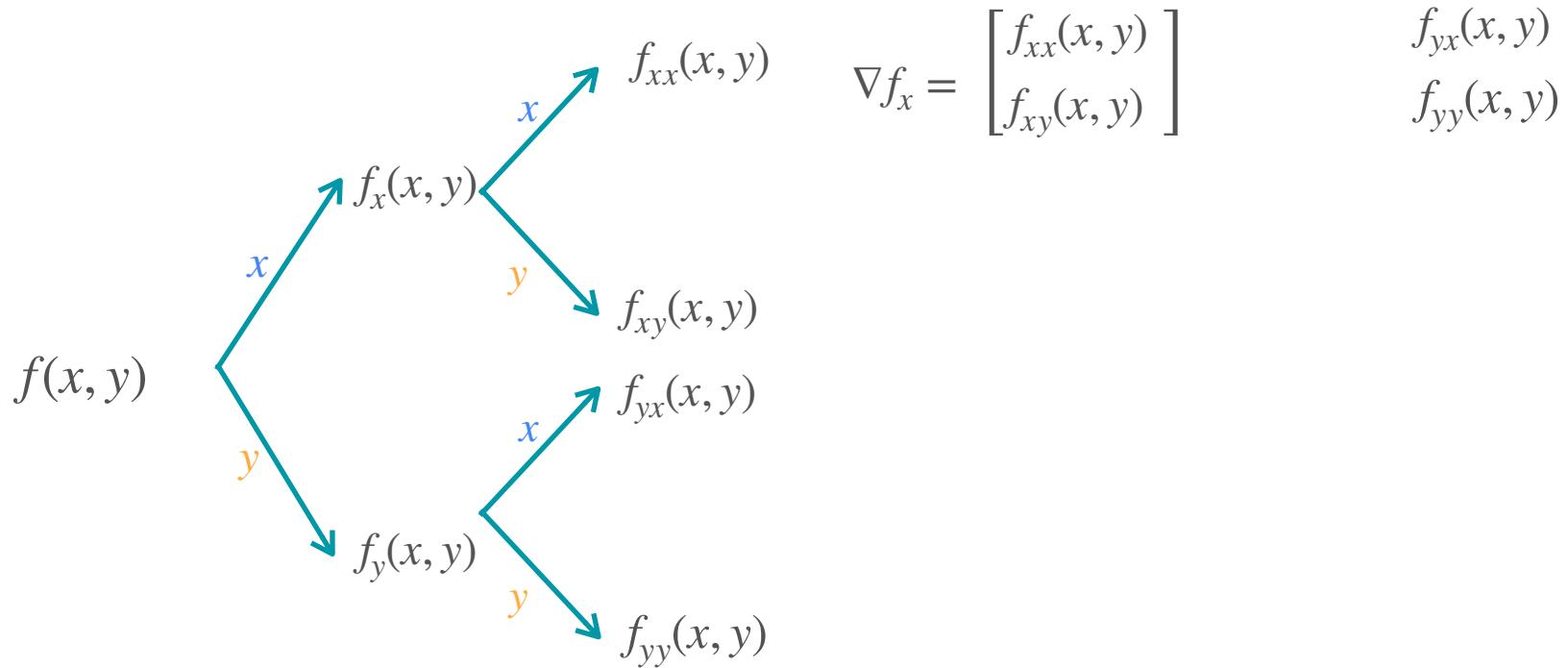
Hessian Matrix - General Case



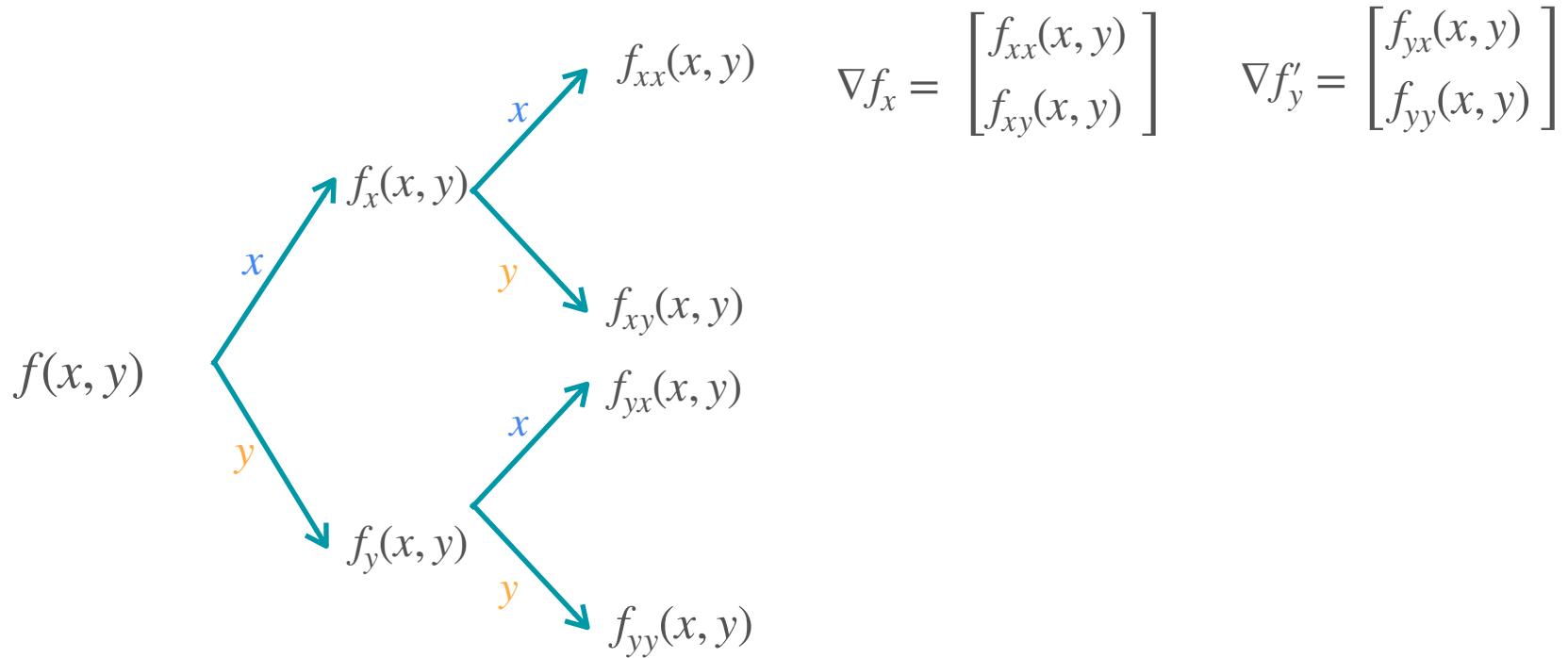
Hessian Matrix - General Case



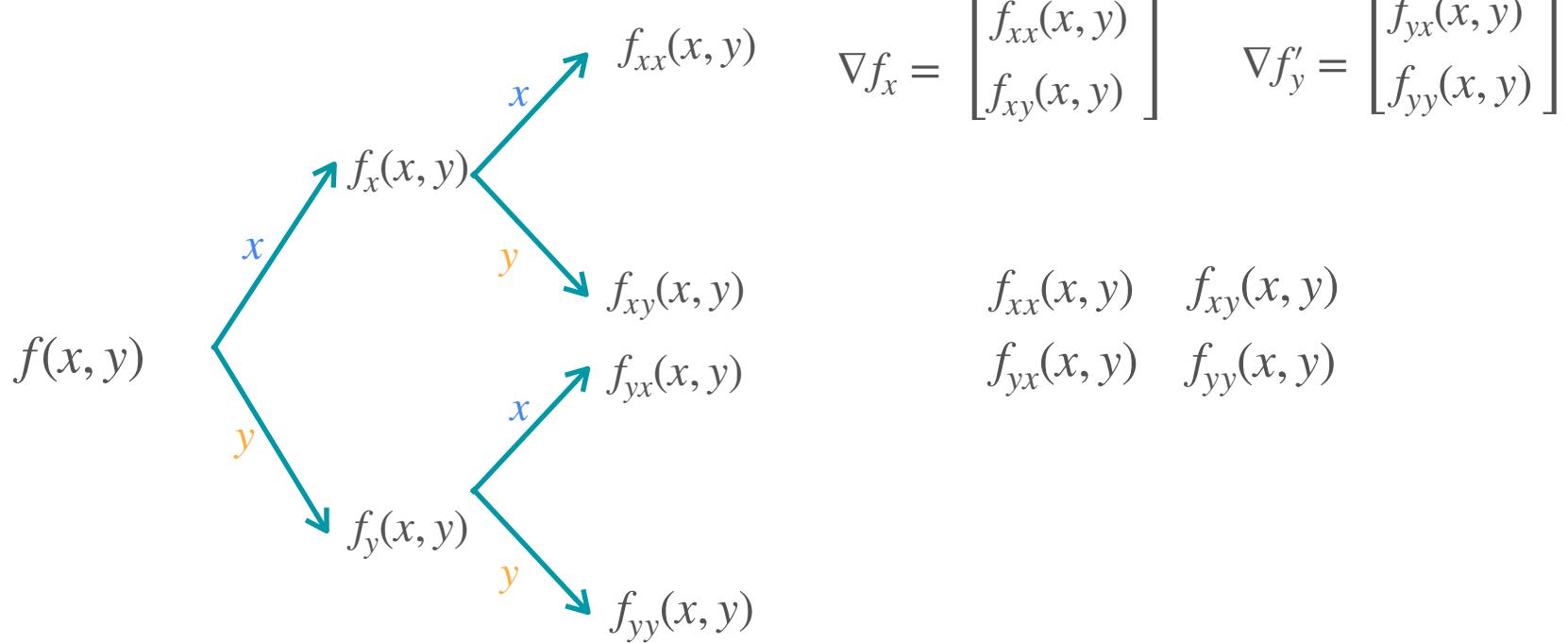
Hessian Matrix - General Case



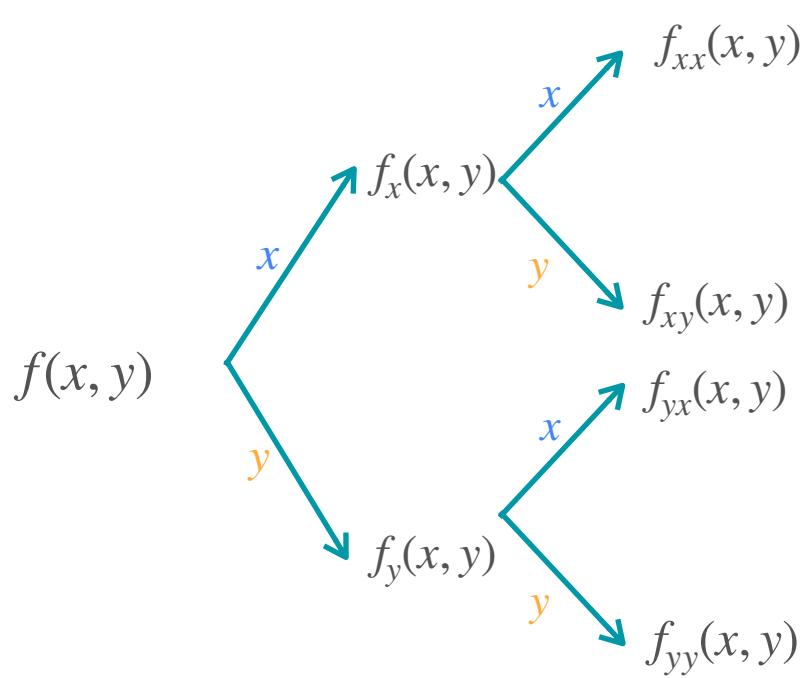
Hessian Matrix - General Case



Hessian Matrix - General Case



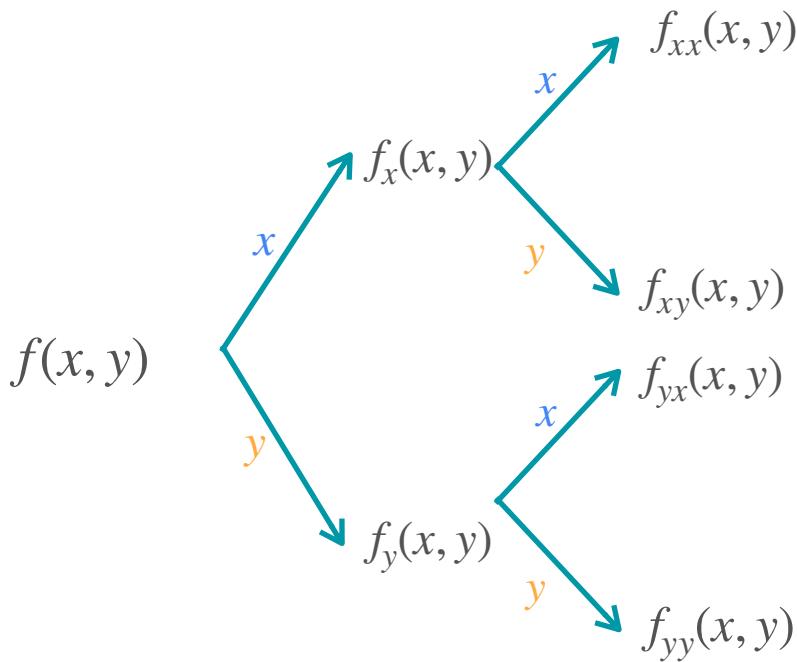
Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \quad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$\begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix} = \begin{bmatrix} \nabla f_x^T \\ \nabla f_y^T \end{bmatrix}$$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \quad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$H = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix} = \begin{bmatrix} \nabla f_x^T \\ \nabla f_y^T \end{bmatrix}$$

**Hessian
matrix**

All information
about second
derivatives

Second Derivative

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$
Second derivative	$f''(x)$ Rate of change of the rate of change of $f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$
Second derivative	$f''(x)$ Rate of change of the rate of change of $f(x)$	$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$



DeepLearning.AI

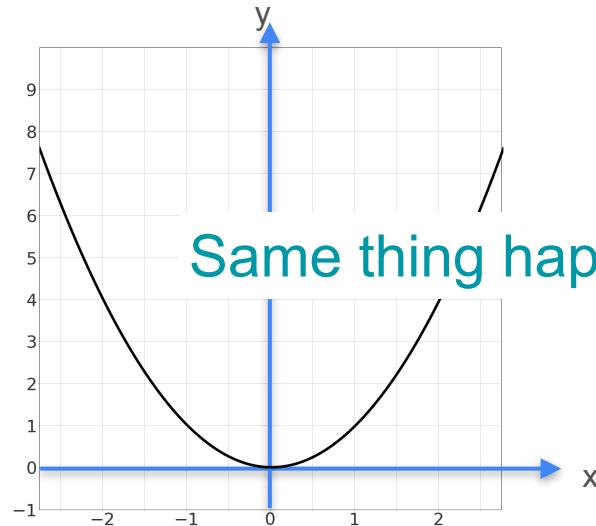
Optimization in Neural Networks and Newton's Method

Hessians and concavity

Remember...

Same thing happens for many variables!

Remember...

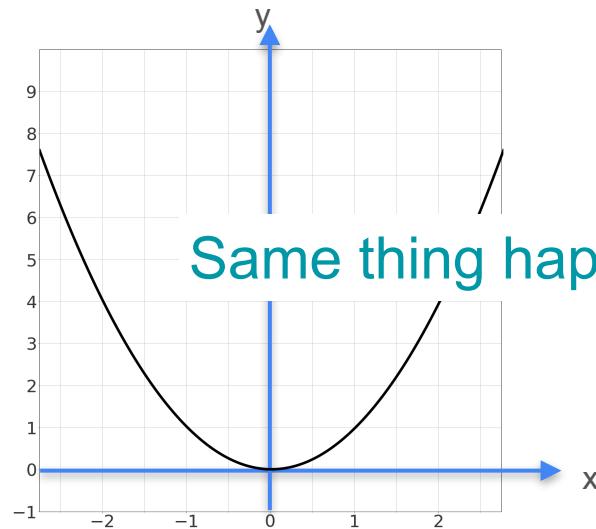


Same thing happens for many variables!

Concave up or convex

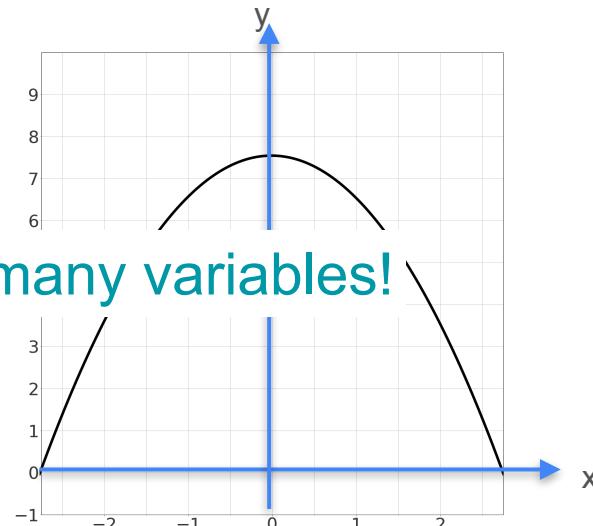
$$f''(0) > 0$$

Remember...



Concave up or convex

$$f''(0) > 0$$

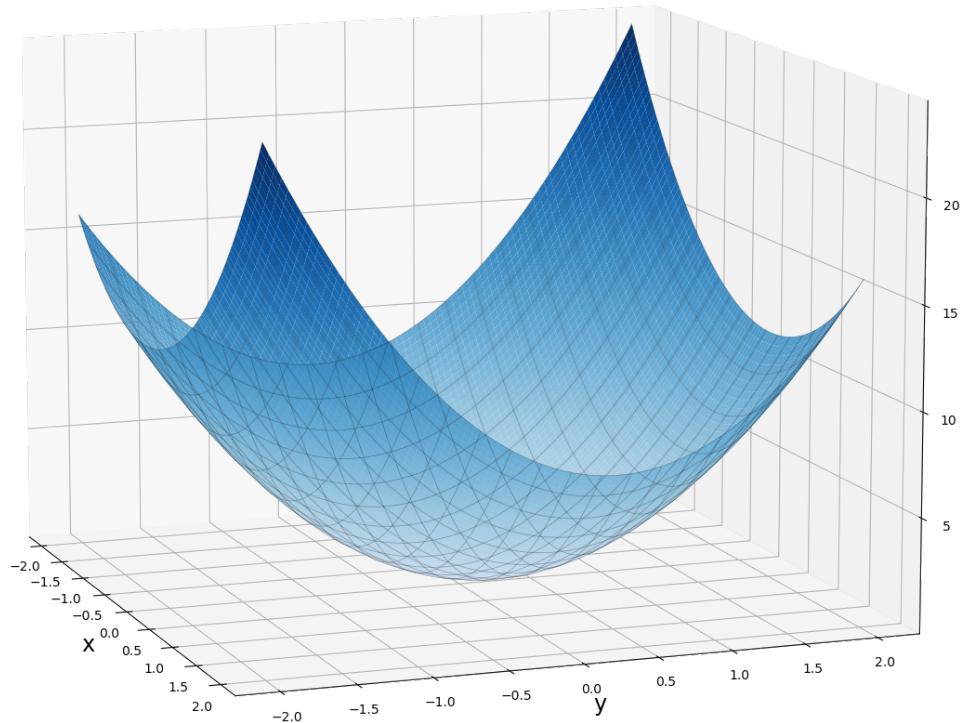


Concave down

$$f''(0) < 0$$

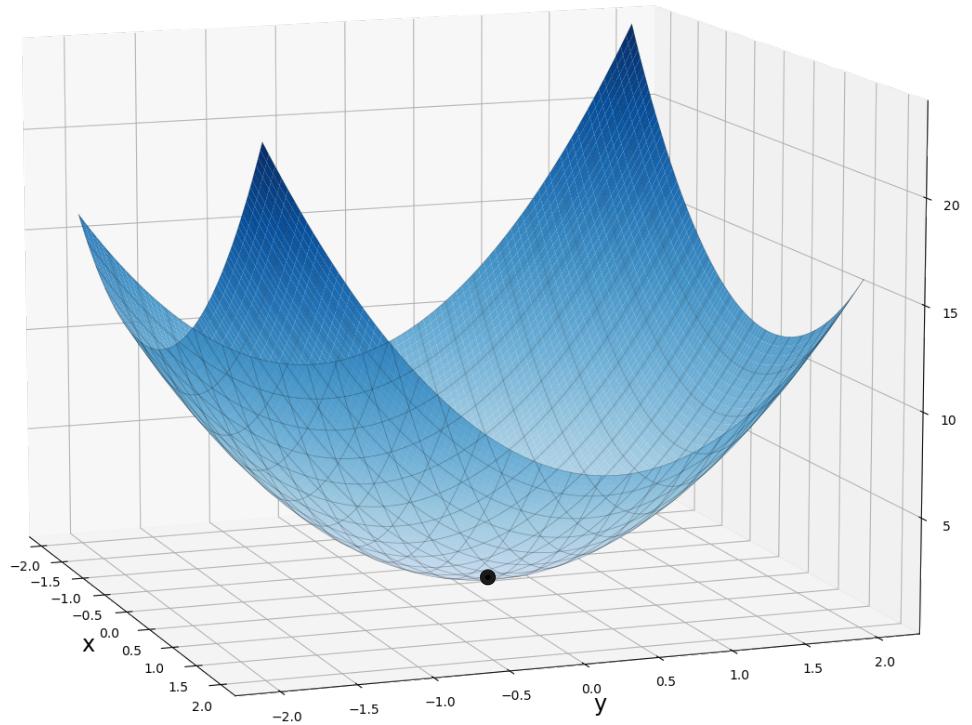
Concave Up

Concave Up



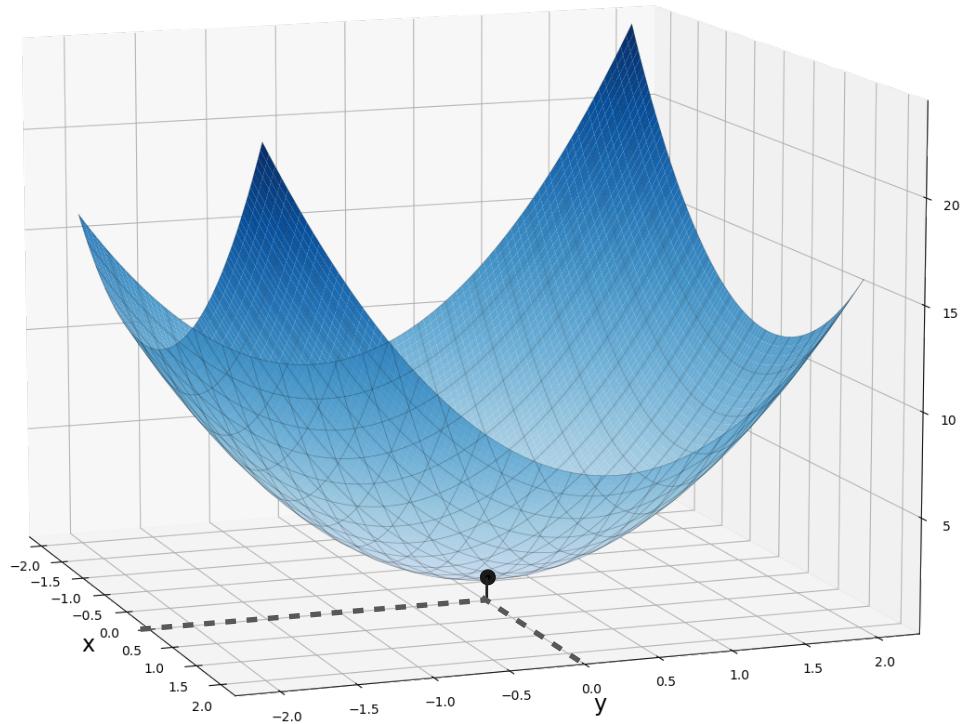
$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up



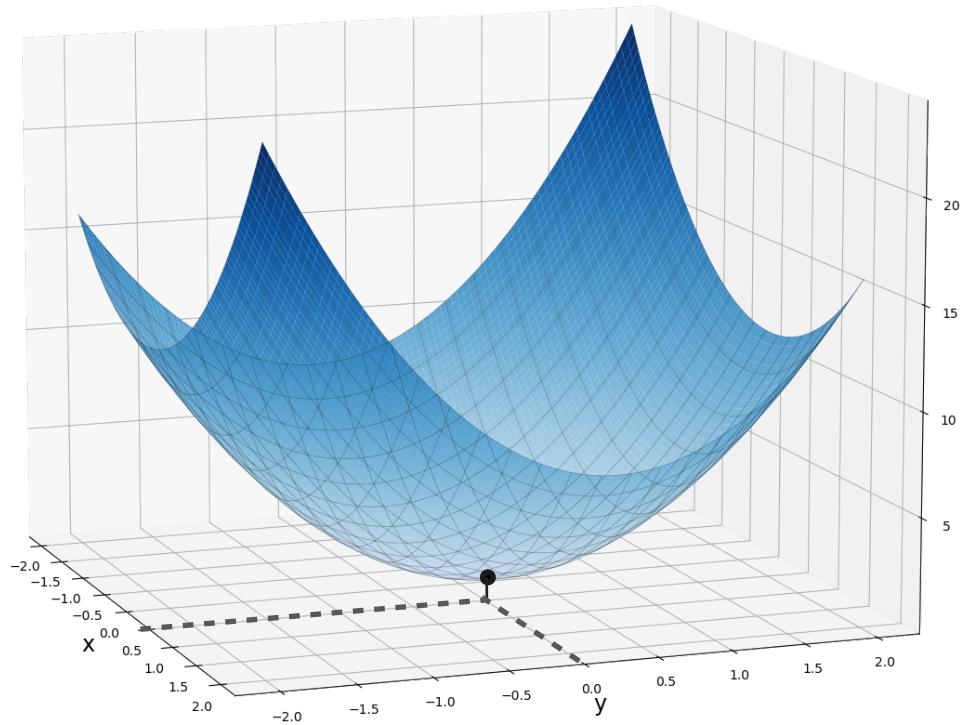
$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

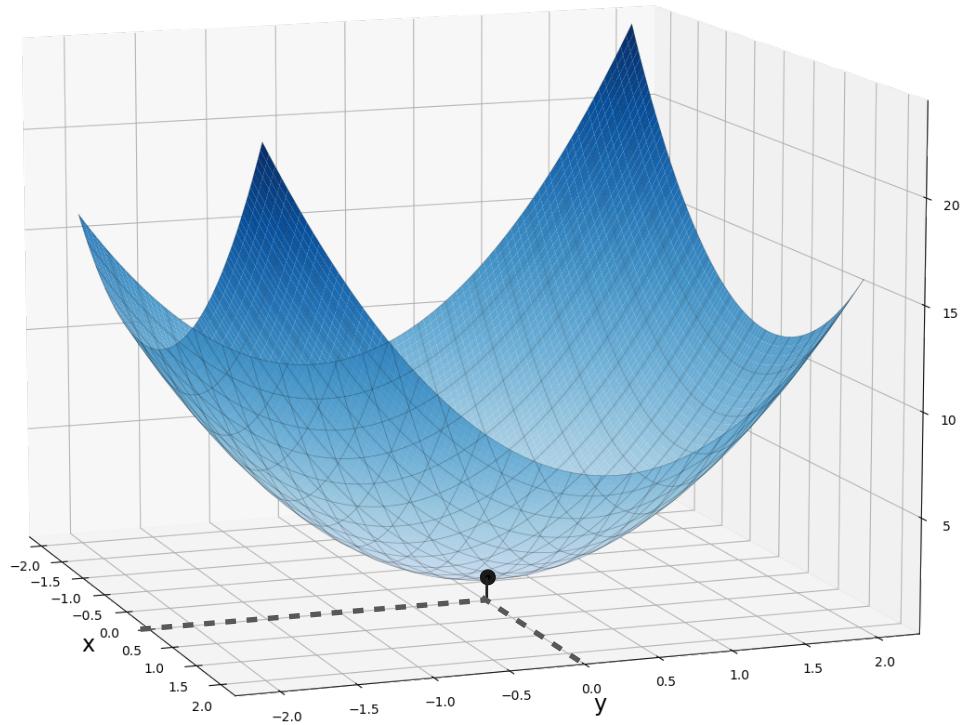
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

Concave Up

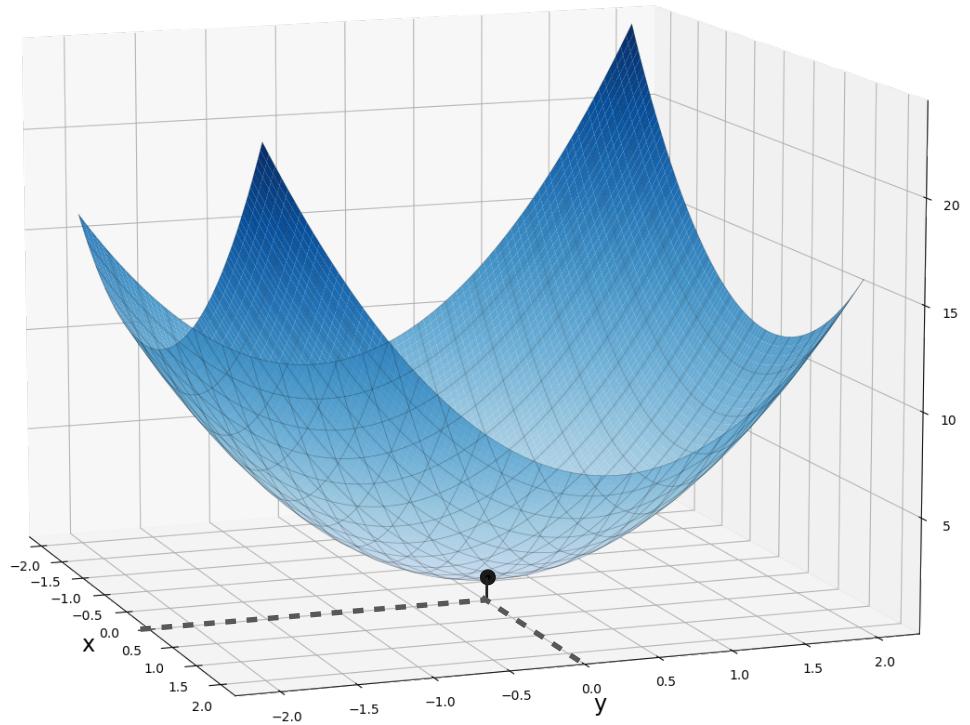


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Concave Up

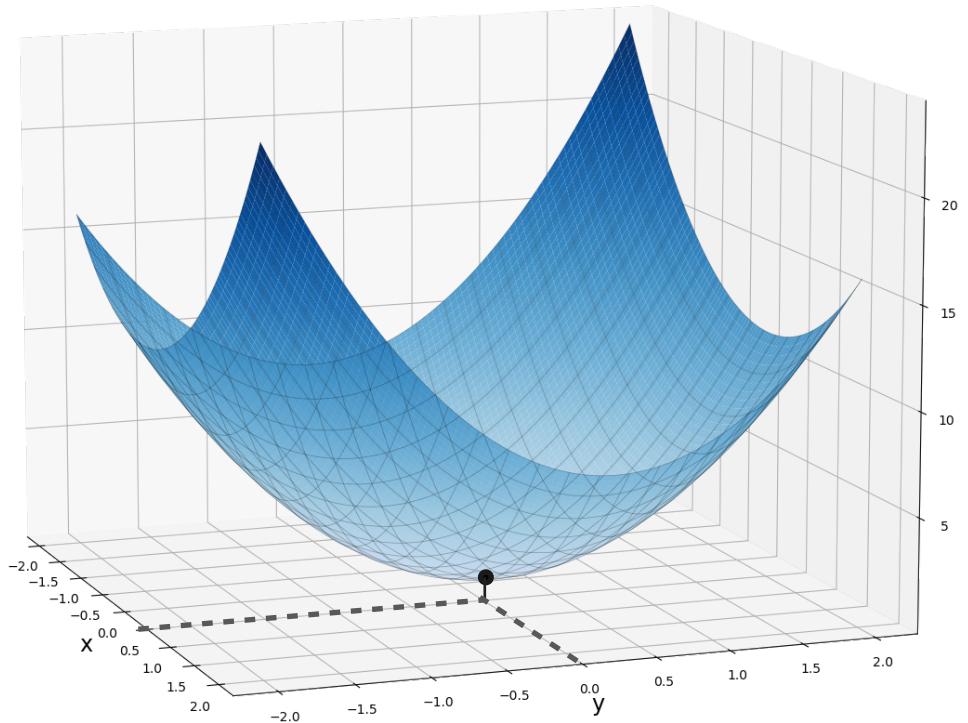


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

Concave Up

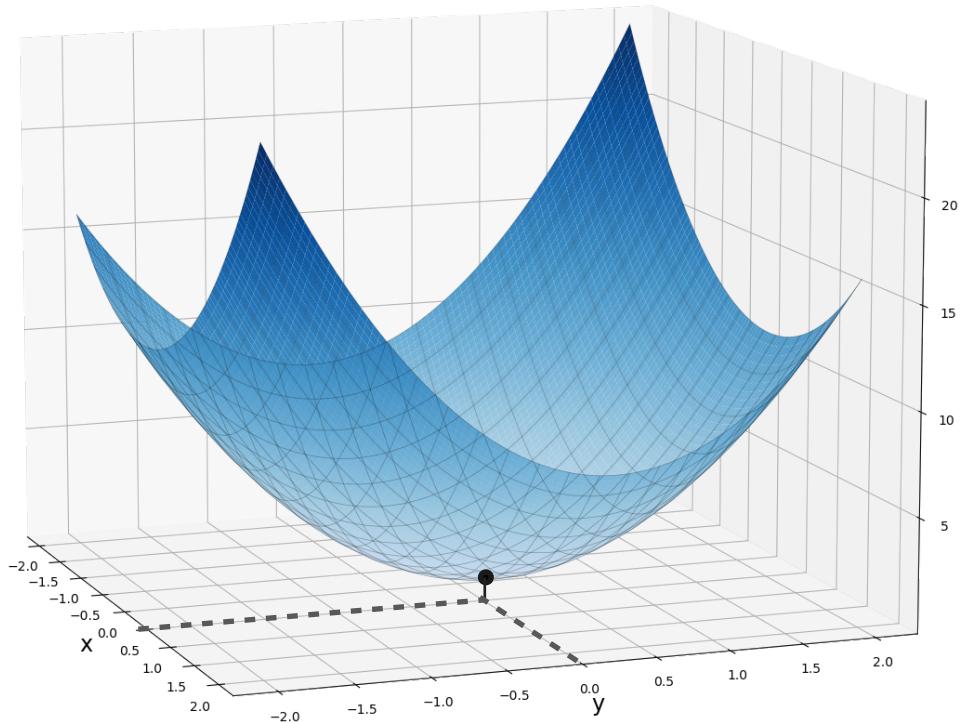


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$
$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

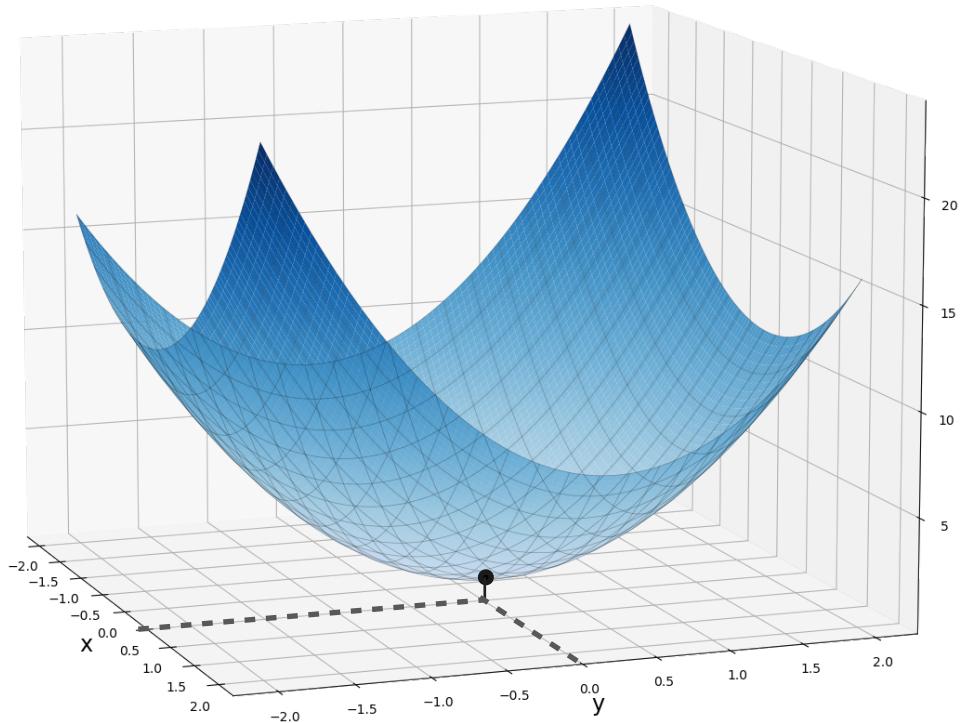
$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

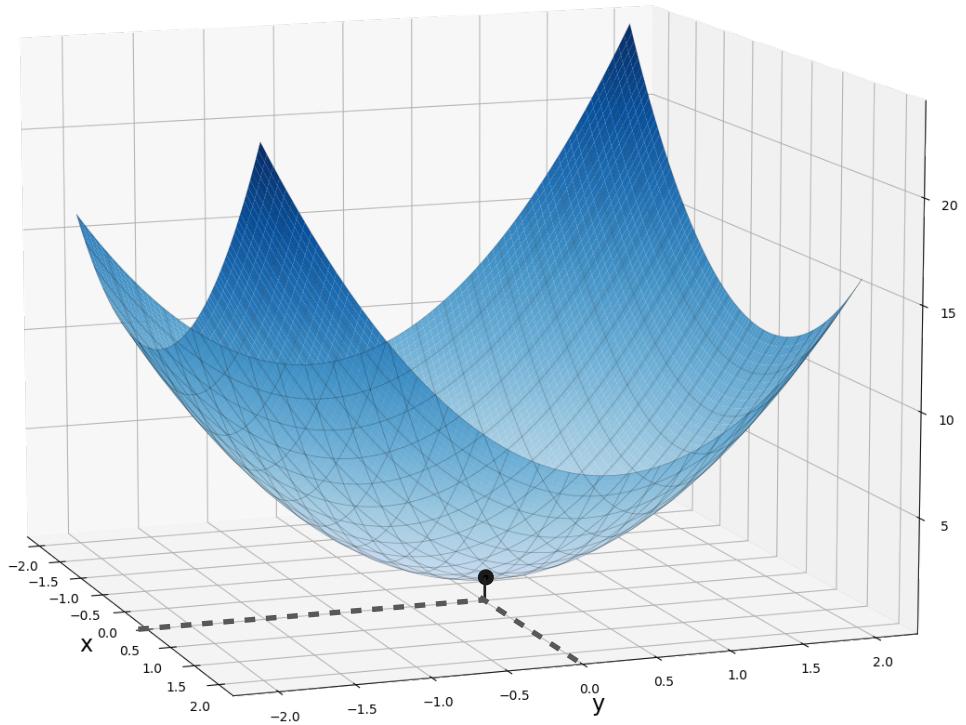
$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23 \rightarrow \lambda_1 = 6.41$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

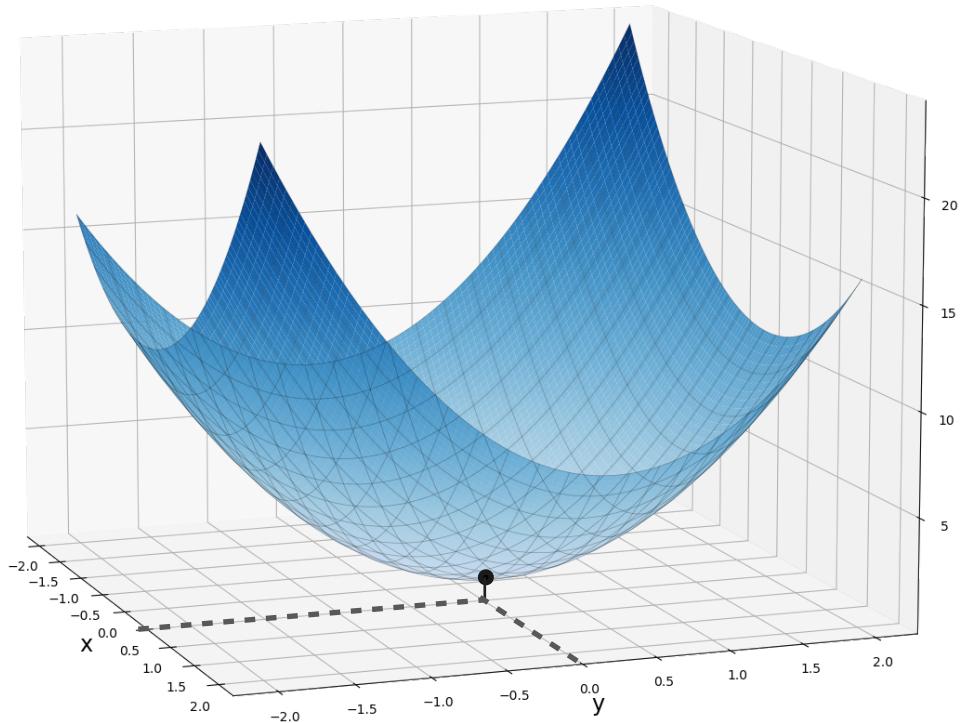
$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\begin{array}{l} \xrightarrow{\text{blue}} \lambda_1 = 6.41 \\ \xrightarrow{\text{blue}} \lambda_2 = 3.59 \end{array}$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

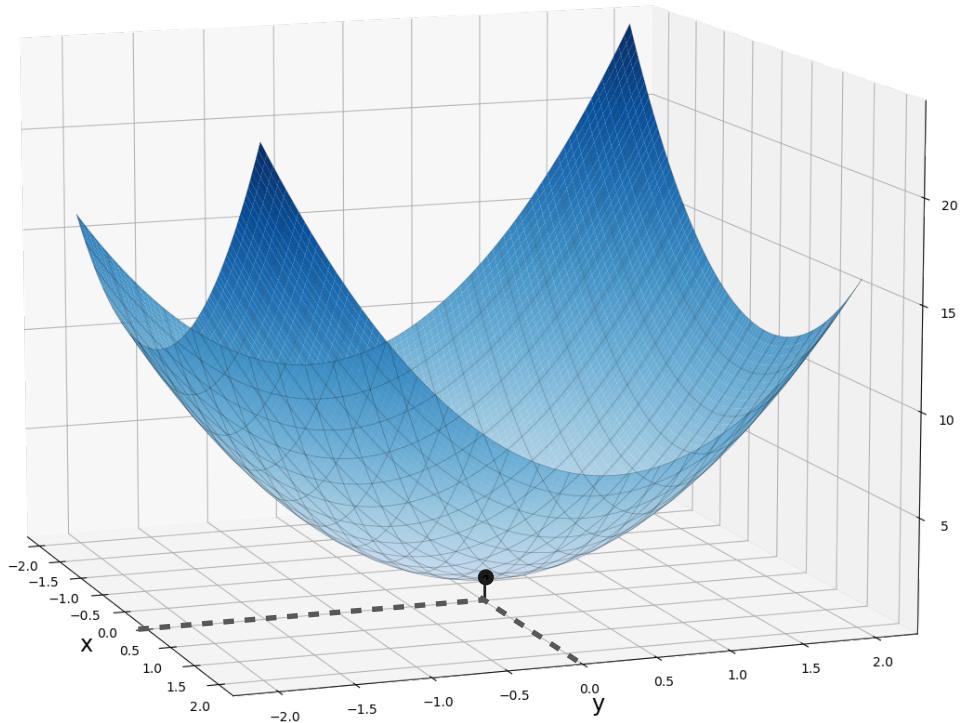
$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\lambda_1 = 6.41$$
$$\lambda_2 = 3.59$$

> 0

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det \left(\begin{bmatrix} 4 - \lambda & -1 \\ -1 & 6 - \lambda \end{bmatrix} \right)$$

$$= (4 - \lambda)(6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

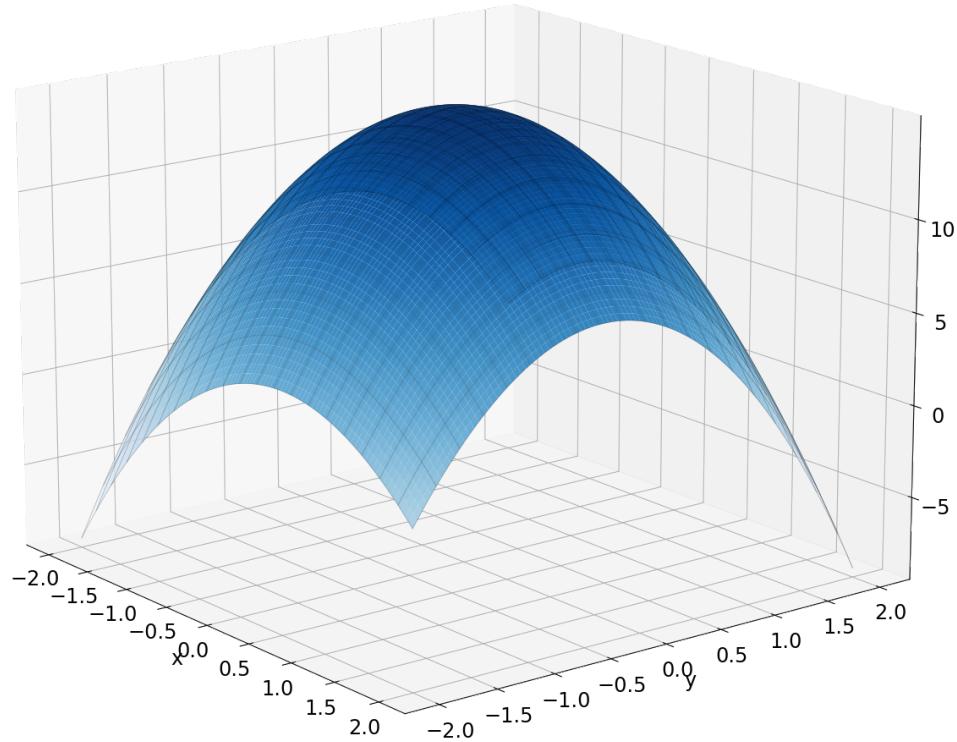
$$\lambda_1 = 6.41$$
$$\lambda_2 = 3.59$$

(0,0) is a minimum!

> 0

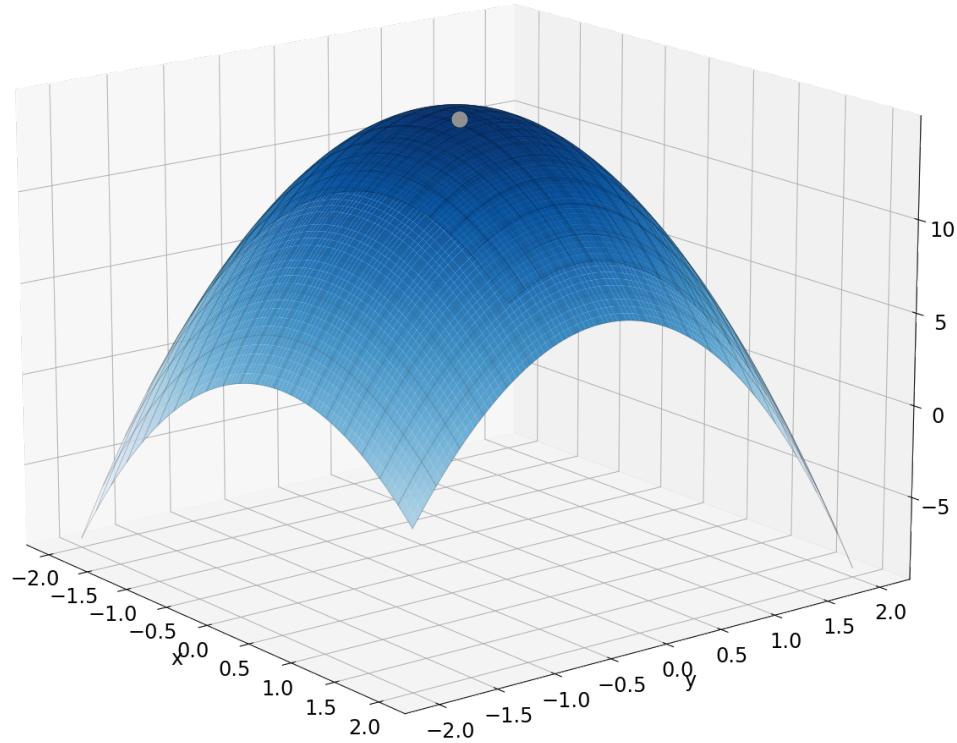
Concave Down

Concave Down



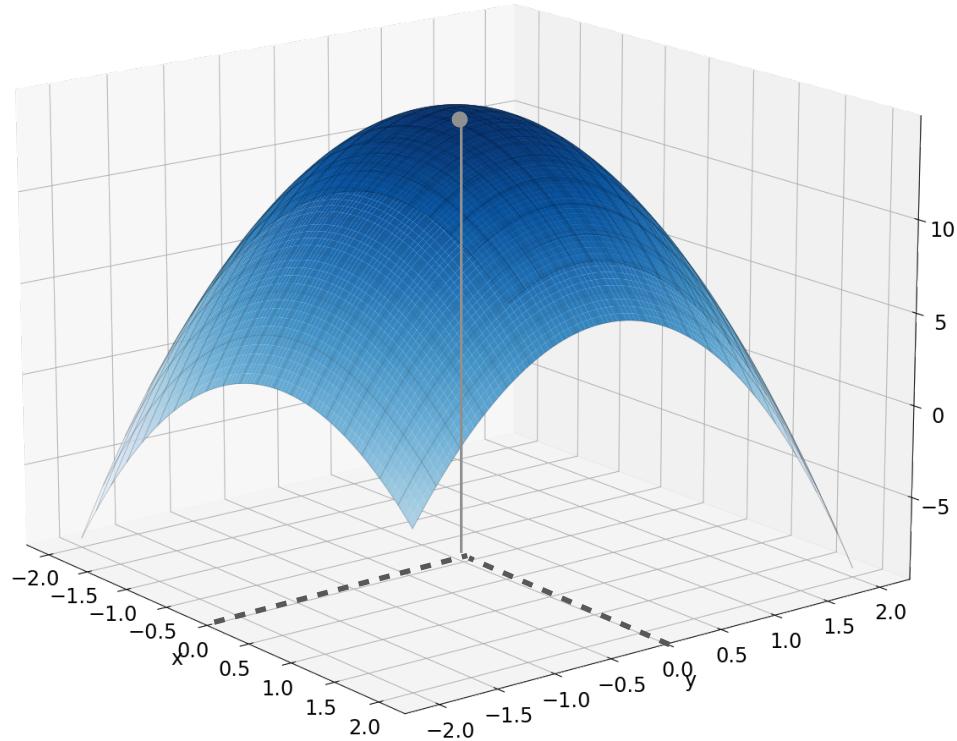
$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

Concave Down



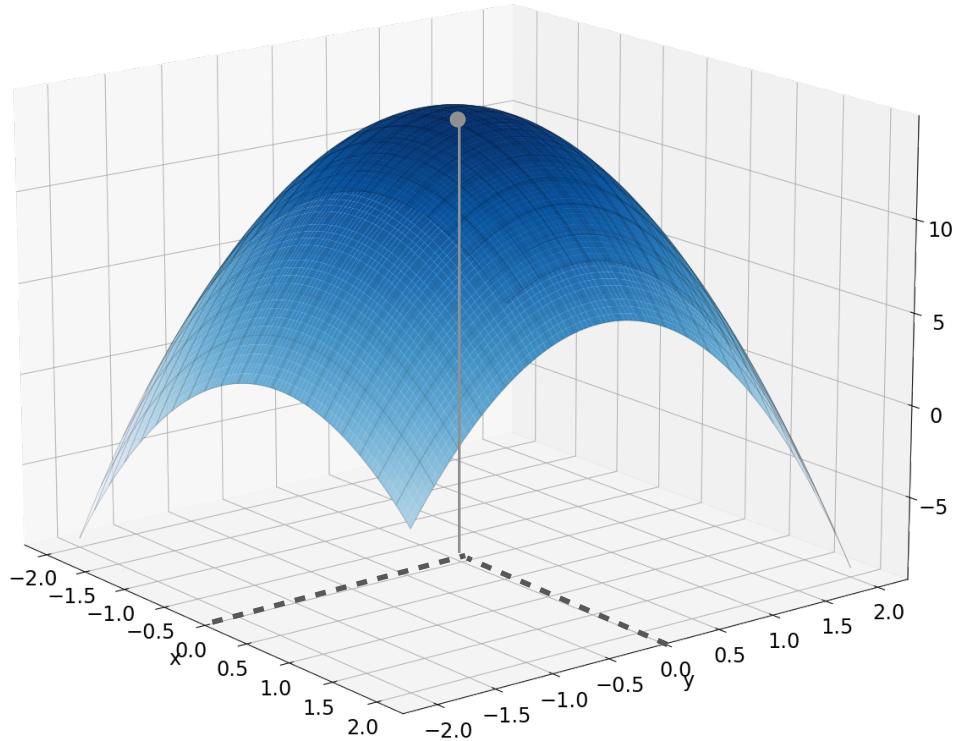
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Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

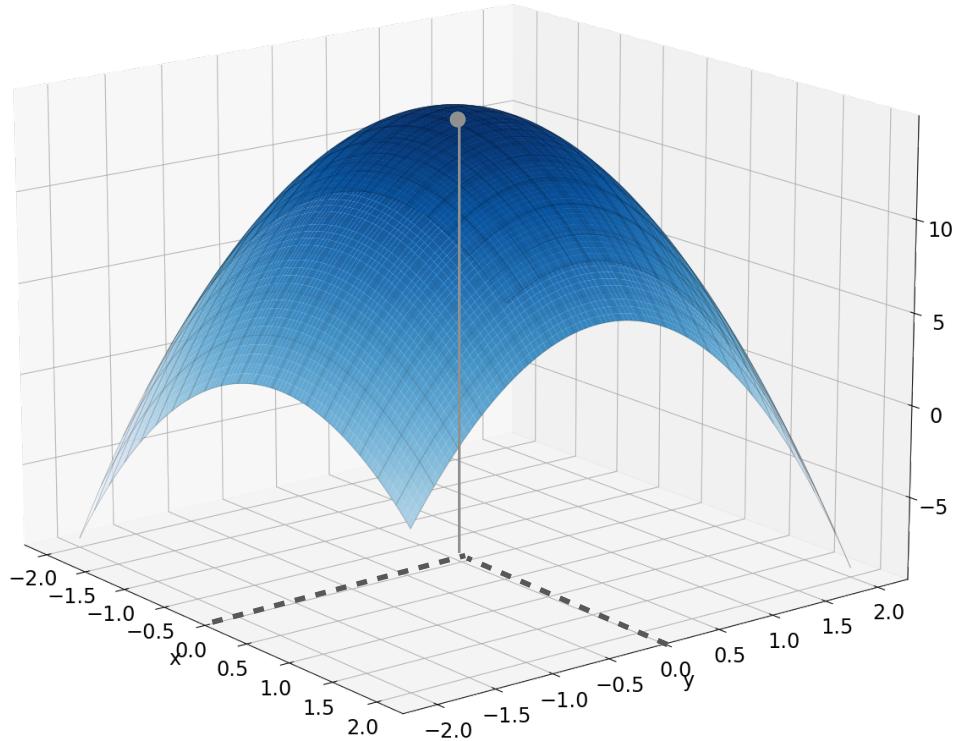
Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

Concave Down

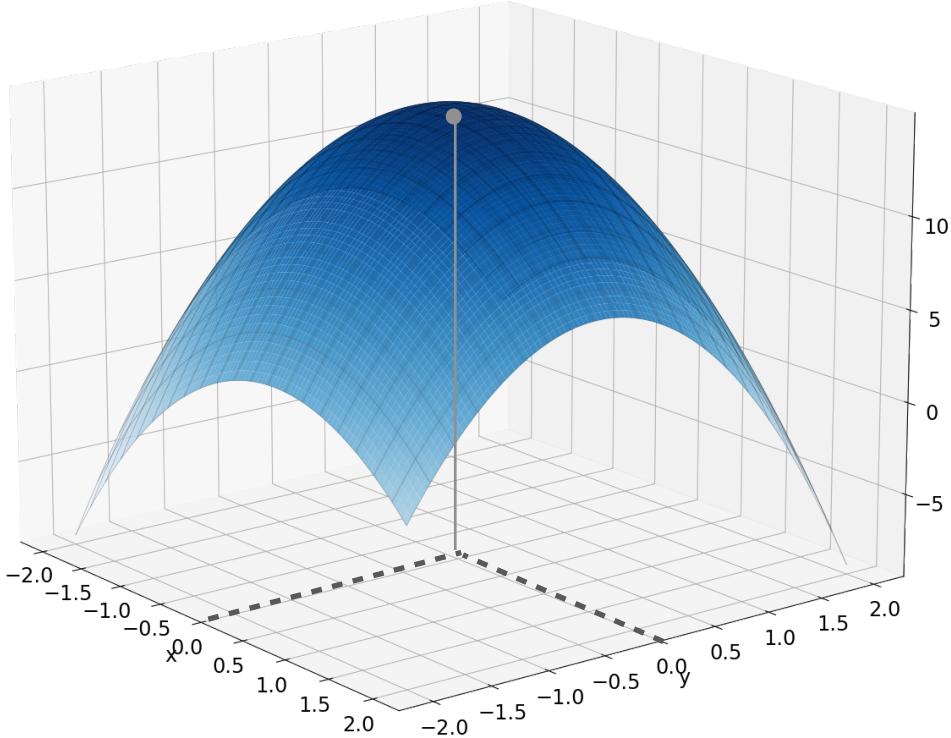


$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

Concave Down



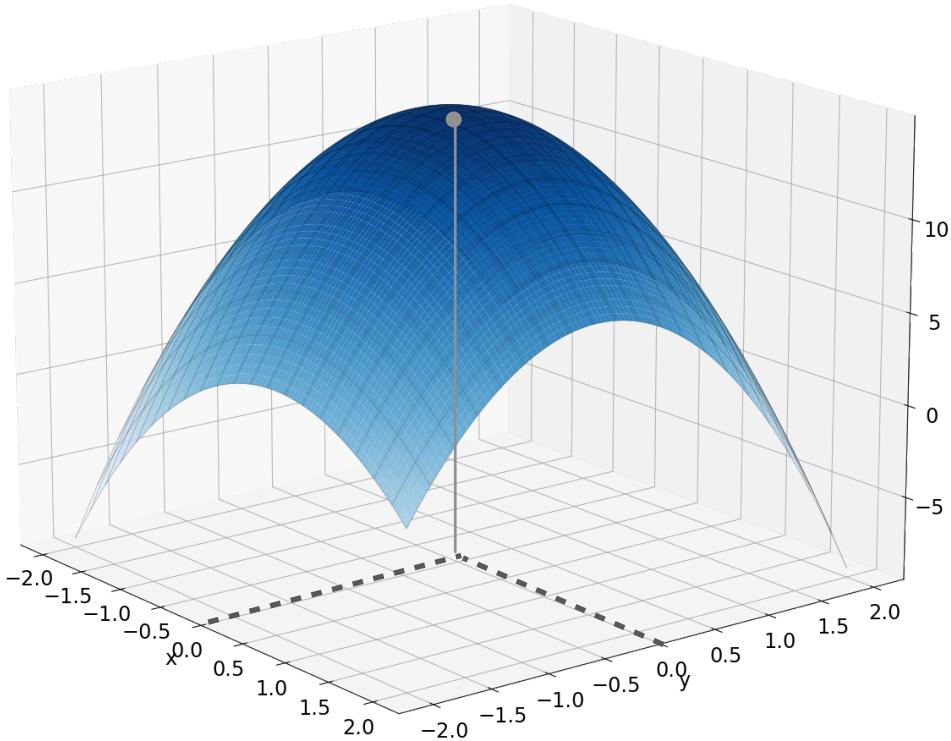
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$$\det(H(0,0) - \lambda I) =$$

Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

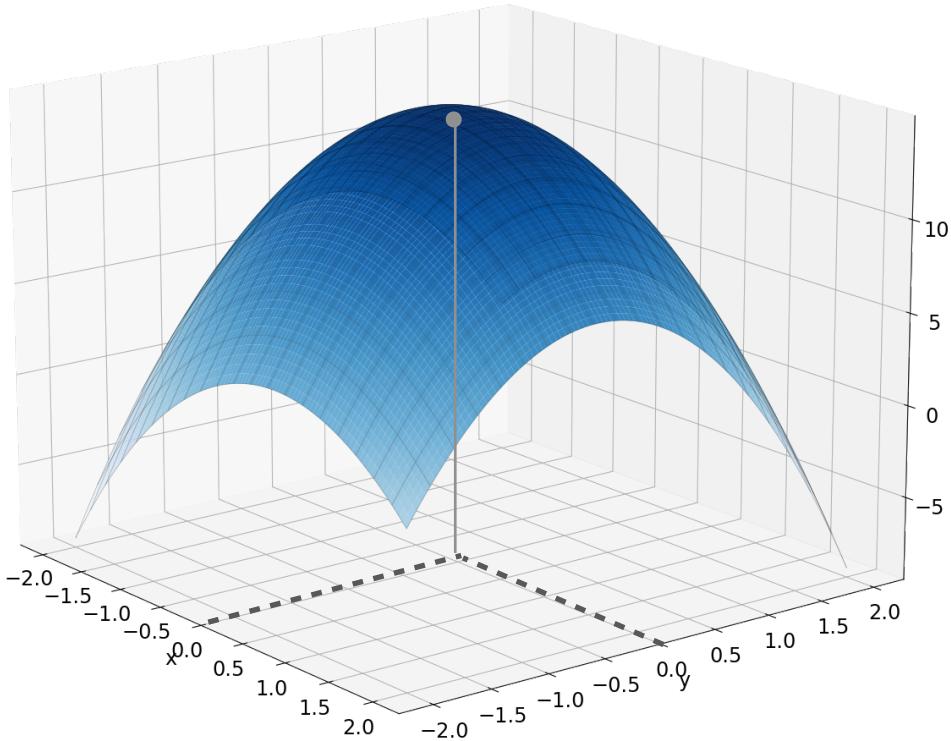
$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

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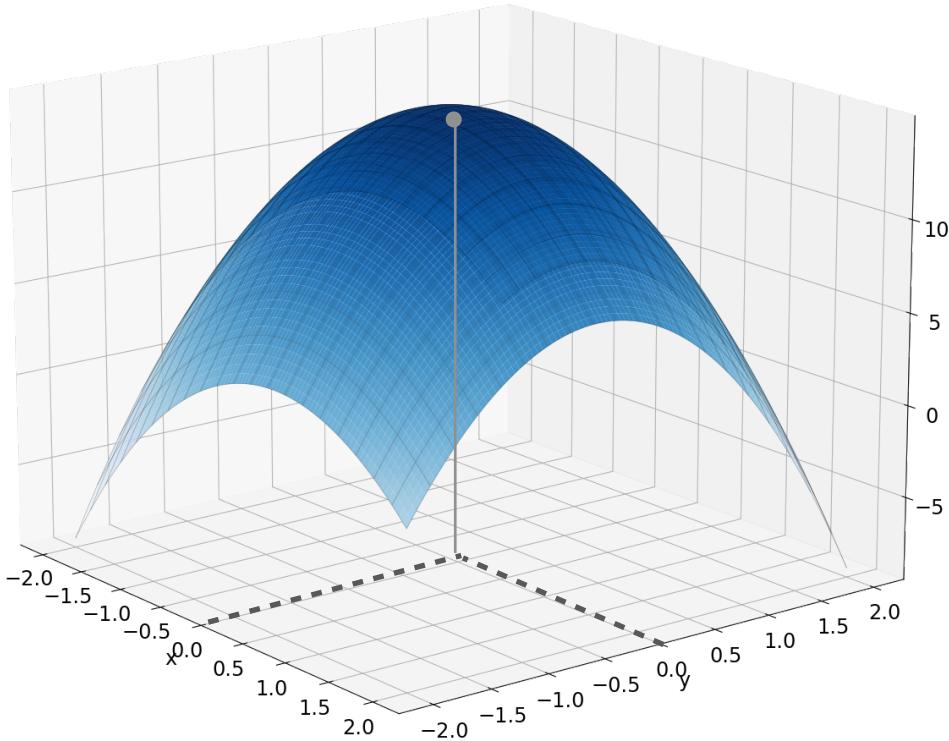
$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

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$$= \lambda^2 + 10\lambda + 23$$

Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

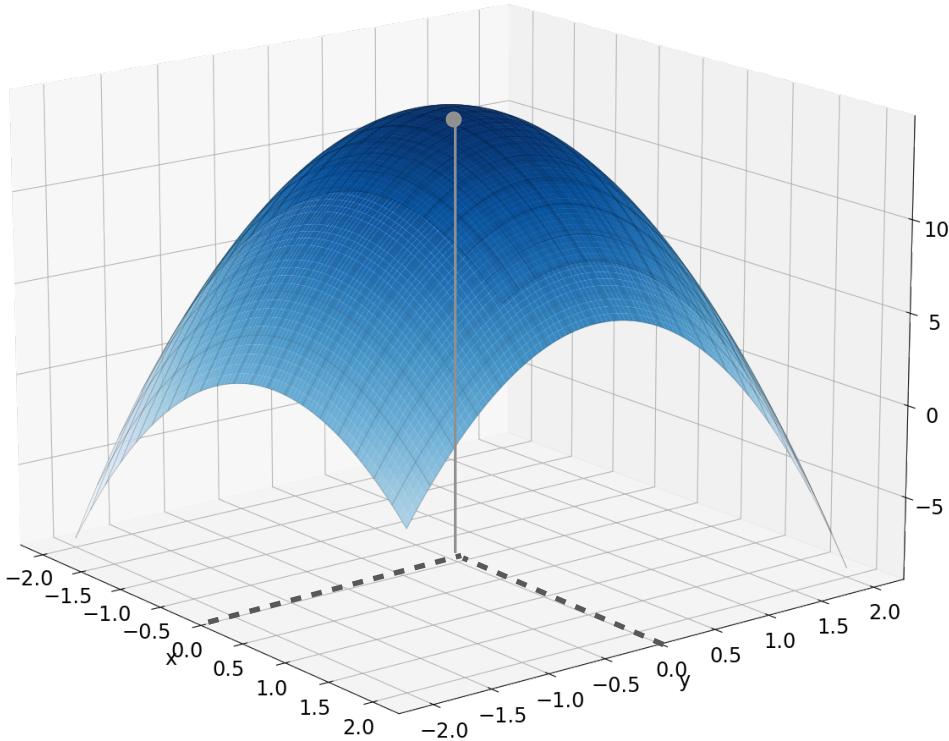
$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23 \rightarrow \lambda_1 = -3.49$$

Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

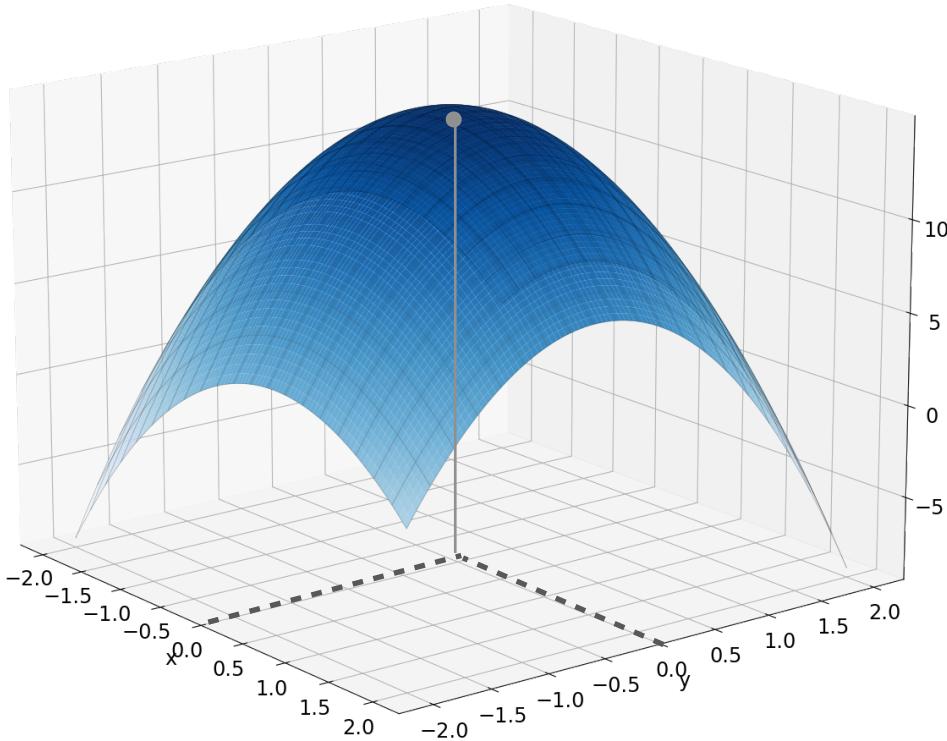
$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

$\lambda_1 = -3.49$
 $\lambda_2 = -6.41$

Concave Down



$$f(x, y) = -2x^2 - 3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

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$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

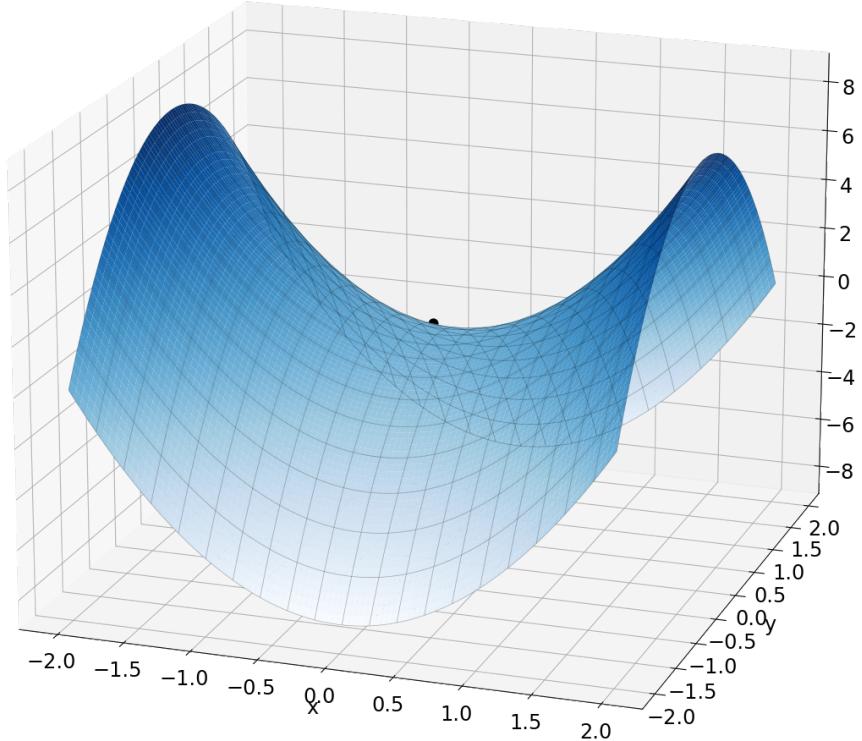
$$\lambda_1 = -3.49$$
$$\lambda_2 = -6.41$$

(0,0) is a maximum!

< 0

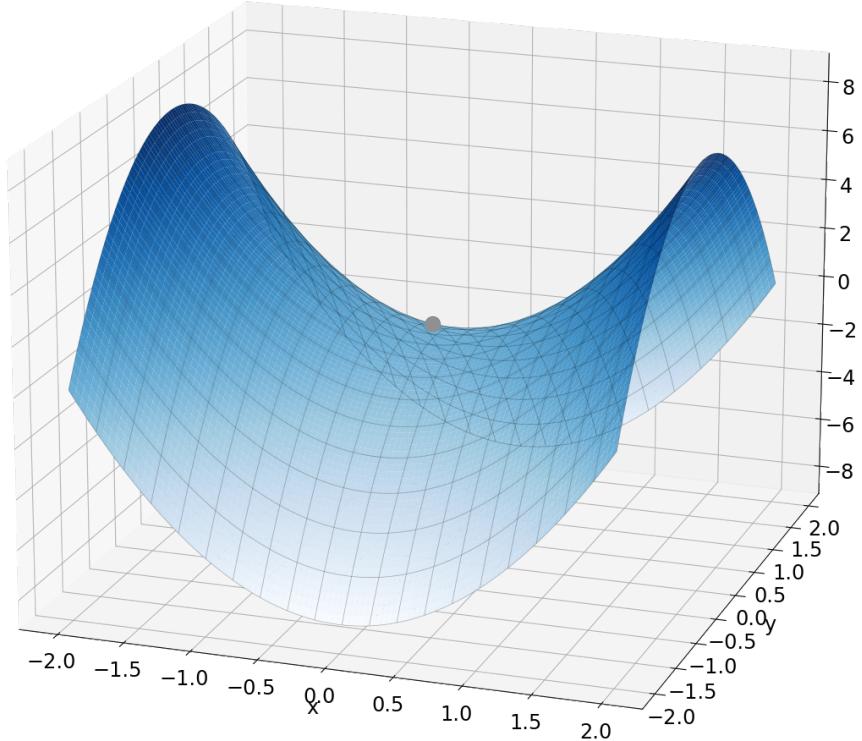
Saddle Point

Saddle Point



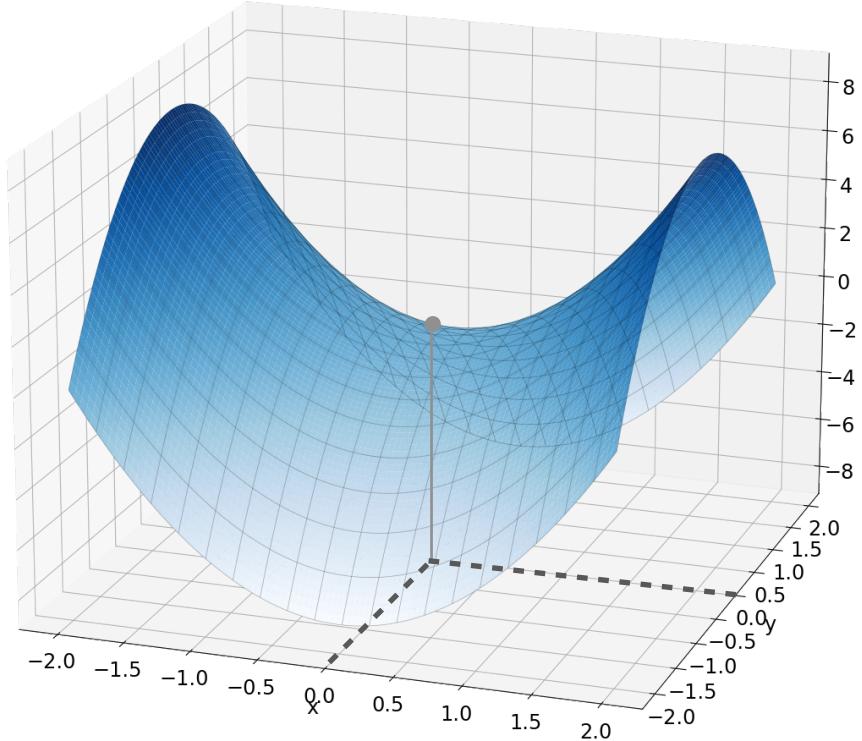
$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point



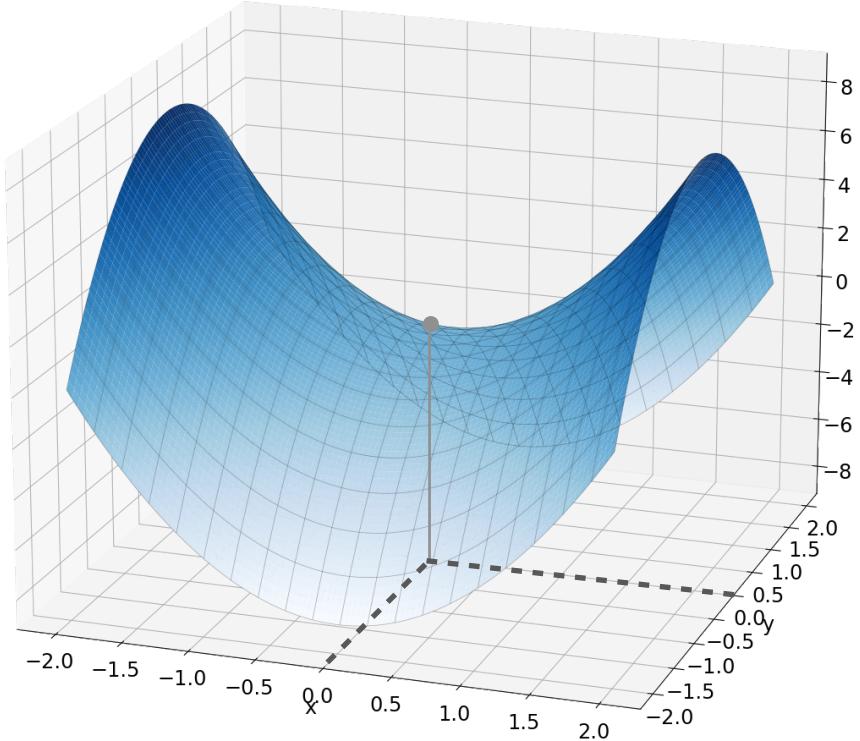
$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

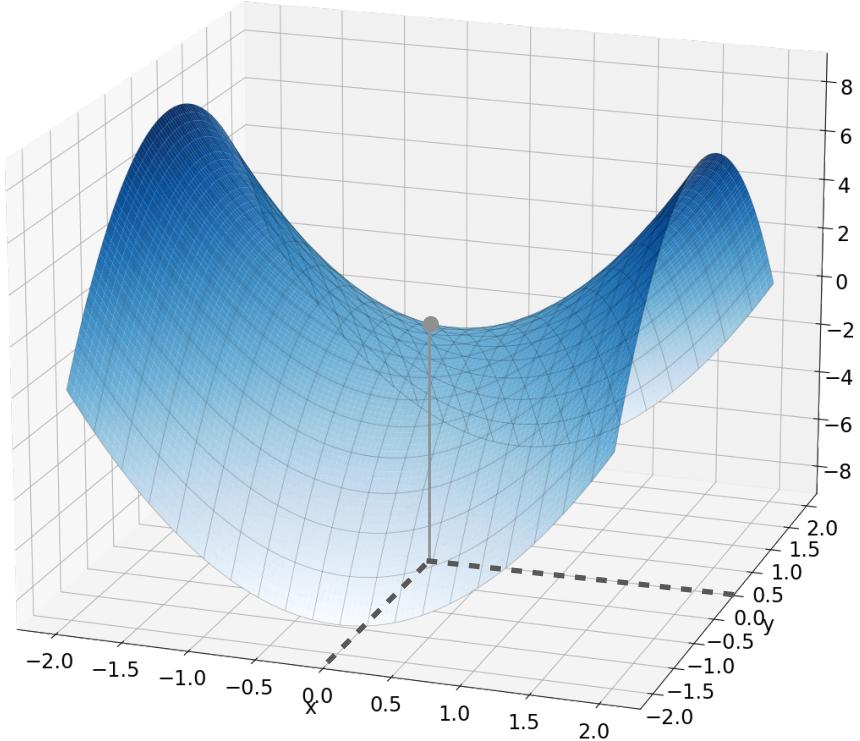
Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

Saddle Point

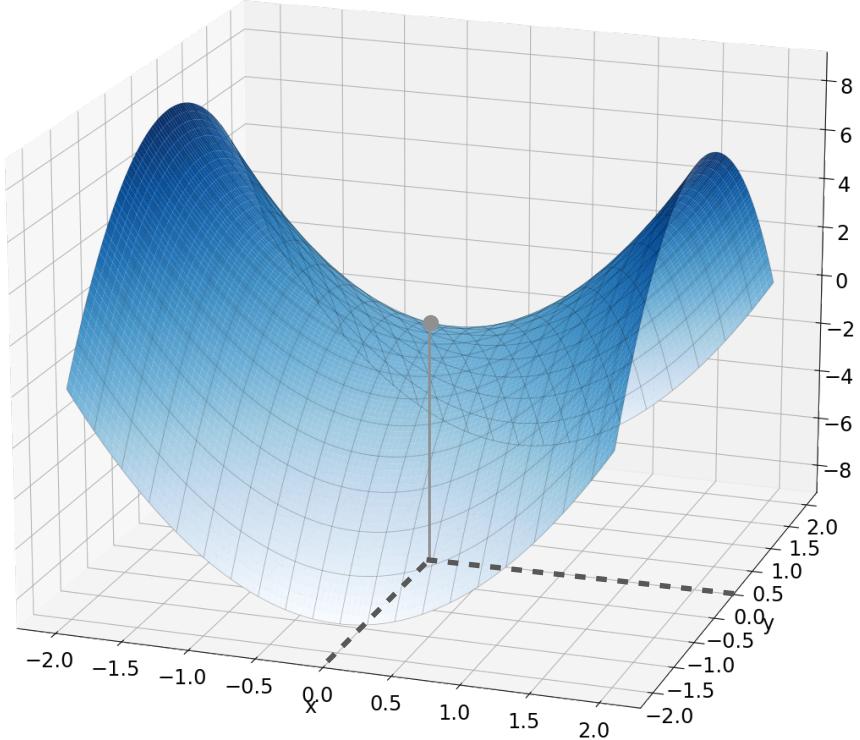


$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

Saddle Point



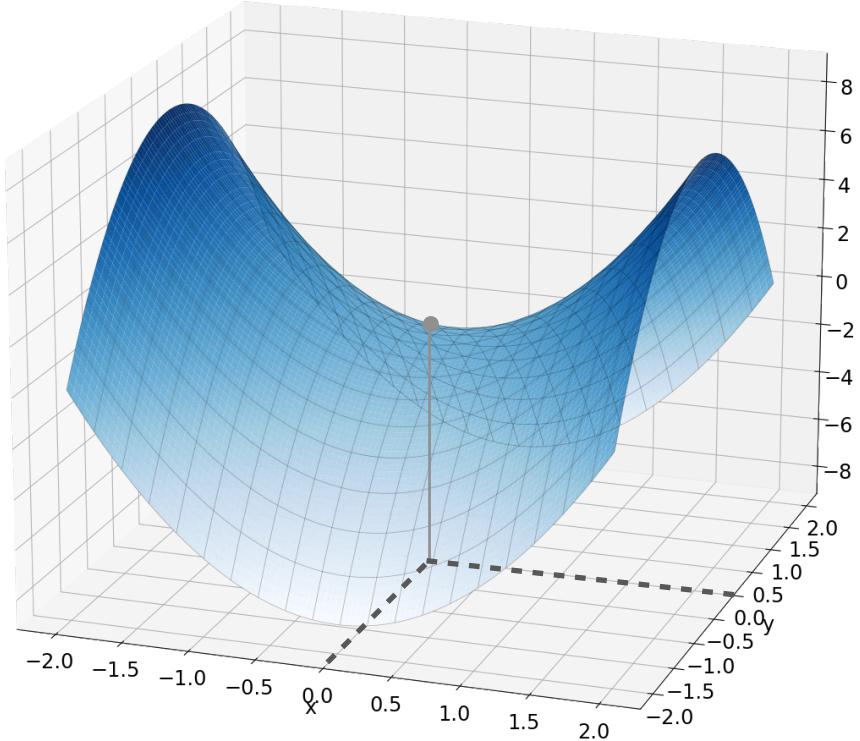
$$f(x, y) = 2x^2 - 2y^2$$

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$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

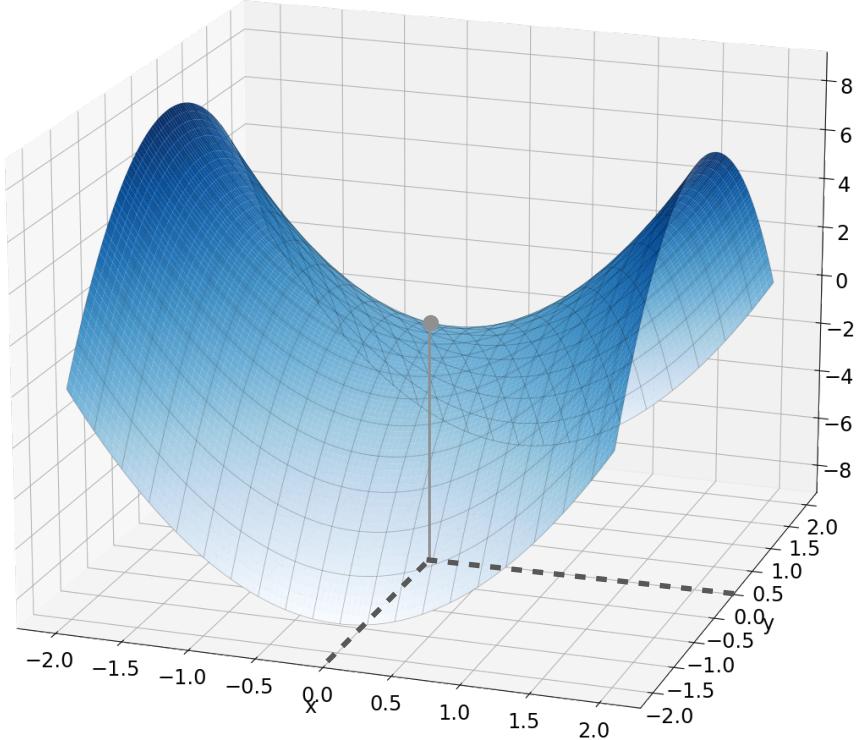
$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

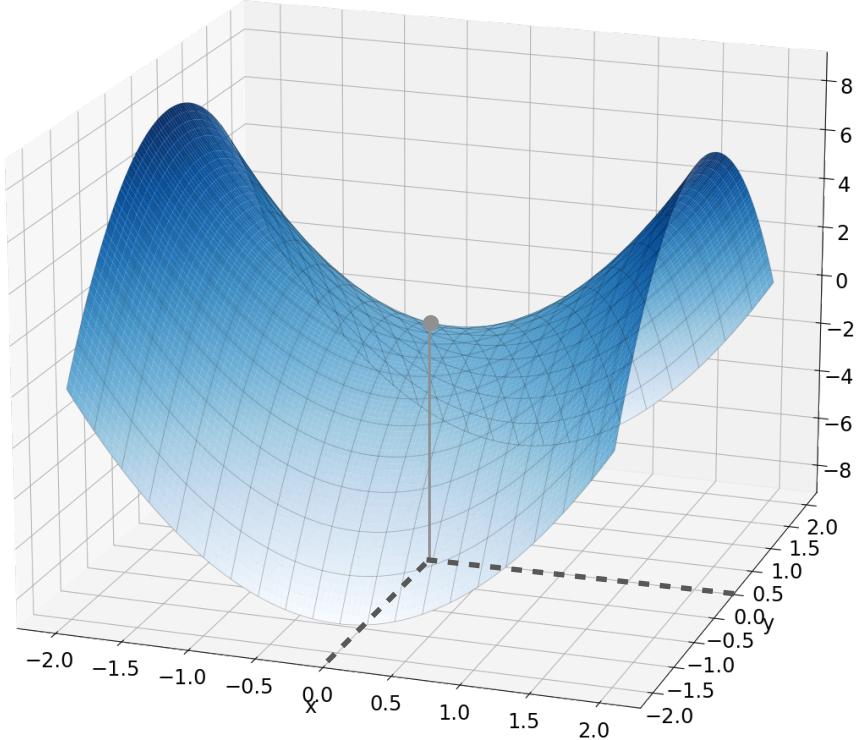
$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\begin{array}{l} \xrightarrow{\hspace{1cm}} \lambda_1 = -4 \\ \xrightarrow{\hspace{1cm}} \lambda_2 = 4 \end{array}$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

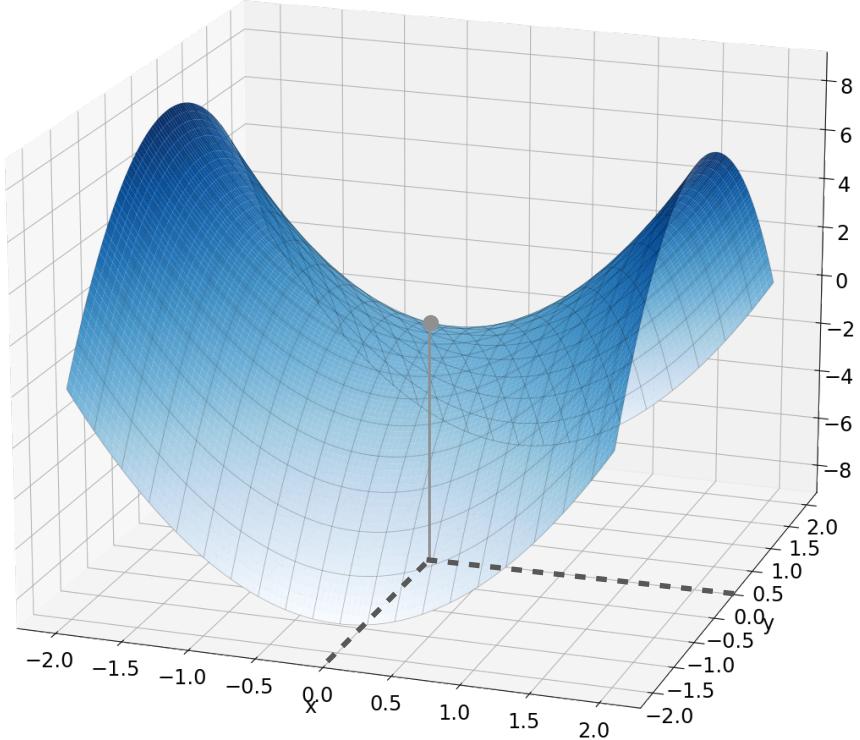
$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0 < 0$$

$\lambda_1 = -4$
 $\lambda_2 = 4$

(0,0) is saddle point

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0 < 0$$

$\lambda_1 = -4$

$$> 0$$

$\lambda_2 = 4$

(0,0) is saddle point

Summary

Summary

1 variable
 $f(x)$

2 variables
 $f(x, y)$

More variables
 $f(x_1, x_2, \dots, x_n)$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \text{ & } \lambda_2 < 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \text{ & } \lambda_2 < 0$	All $\lambda_i < 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
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Need more information	$f''(x) = 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
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Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \text{ & } \lambda_2 < 0$ $\lambda_1 < 0 \text{ & } \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \text{ & } \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \text{ & } \lambda_2 < 0$ $\lambda_1 < 0 \text{ & } \lambda_2 > 0$ Or some $\lambda_i = 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \dots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \text{ & } \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \text{ & } \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \text{ & } \lambda_2 < 0$ $\lambda_1 < 0 \text{ & } \lambda_2 > 0$ Or some $\lambda_i = 0$	Some $\lambda_i > 0$ and some $\lambda_j < 0$ OR At least one $\lambda_i = 0$



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

**Newton's method for two
variables**

Newton's Method

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

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Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} -$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \textcolor{orange}{H^{-1}(x_k, y_k)} \quad \textcolor{teal}{\nabla f(x_k, y_k)}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \nabla f(x_k, y_k) \quad H^{-1}(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \nabla f(x_k, y_k)$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \nabla f(x_k, y_k) \cancel{H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{\mathbf{H}^{-1}(x_k, y_k)}_{2 \times 2} \underbrace{\nabla f(x_k, y_k)}_{2 \times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \nabla f(x_k, y_k) \cancel{- \mathbf{H}^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

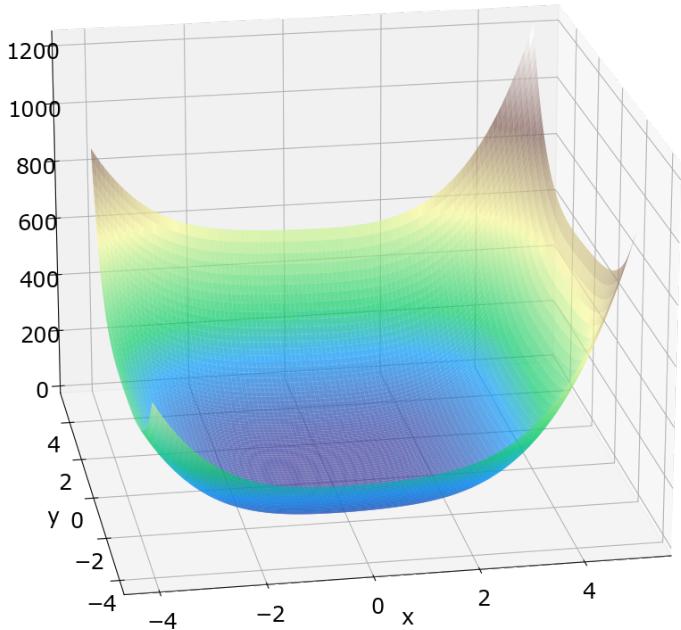
$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2 \times 2} \underbrace{\nabla f(x_k, y_k)}_{2 \times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \nabla f(x_k, y_k) \cancel{- H^{-1}(x_k, y_k)}$$

When working with 2 variables the order is crucial!

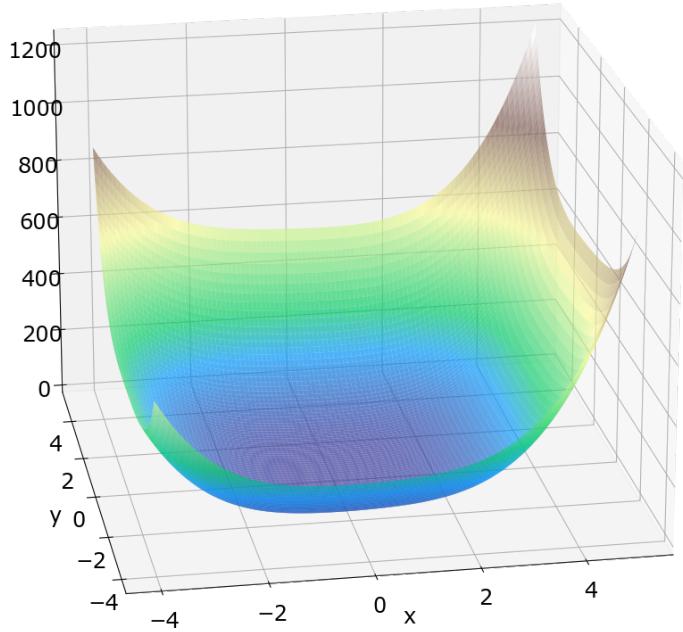
An Example

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

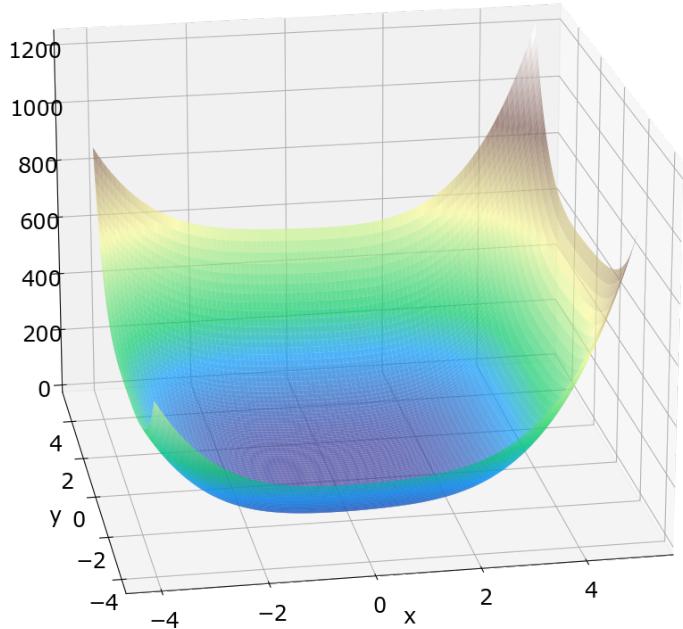
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x, y) \rightarrow 4x^3 + 8x - y - 0.4xy$$

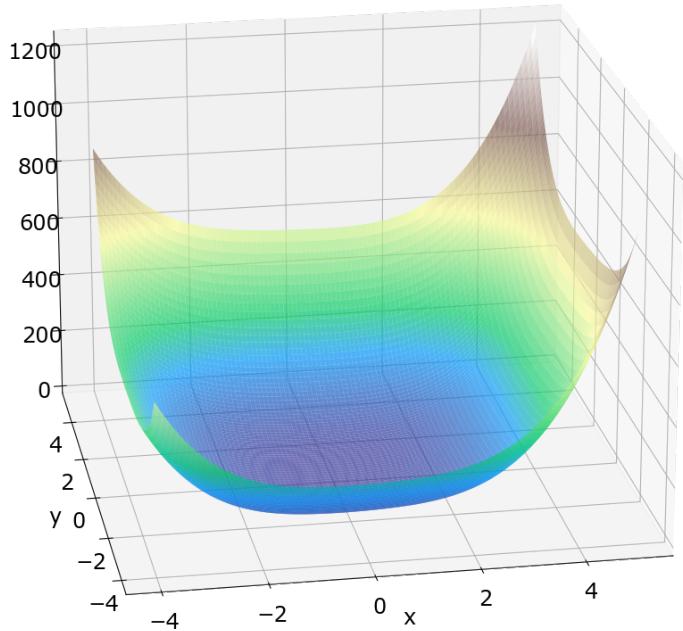
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\begin{array}{l} \textcolor{blue}{x} \nearrow 4x^3 + 8x - y - 0.4xy \\ f(x, y) \\ \textcolor{orange}{y} \searrow 3.2y^3 + 4y - x - 0.2x^2 \end{array}$$

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

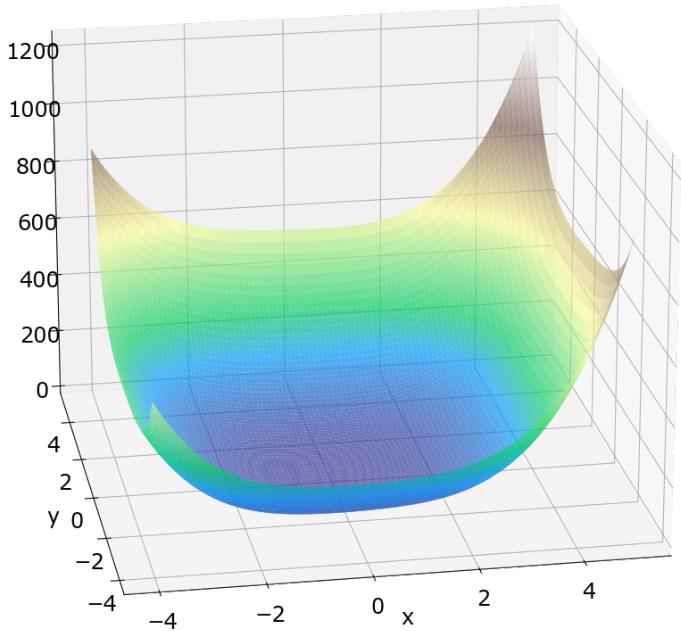
$f(x, y)$

x $12x^2 + 8 - 0.4y$

y $4x^3 + 8x - y - 0.4xy$

$3.2y^3 + 4y - x - 0.2x^2$

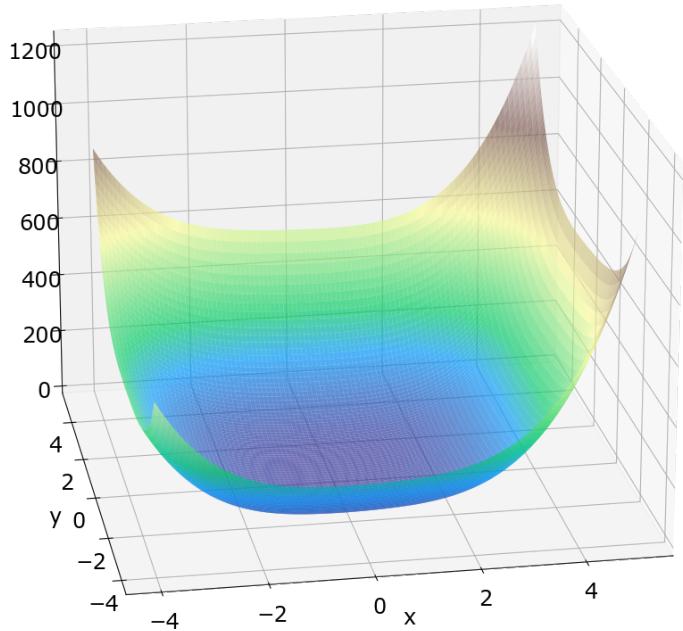
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

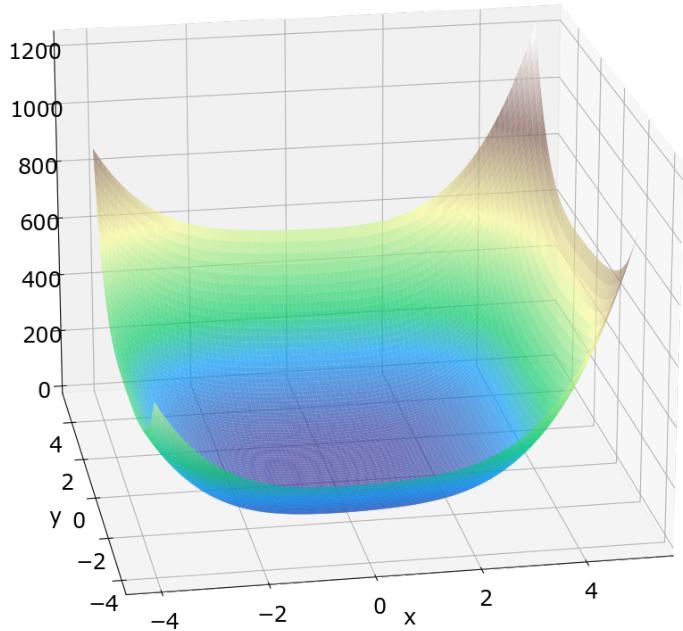
$$f(x, y) \begin{matrix} \nearrow x \\ \searrow y \end{matrix} \begin{array}{l} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \\ 12x^2 + 8 - 0.4y \\ -1 - 0.4x \end{array}$$

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$
$$\begin{aligned} f(x, y) &\xrightarrow{x} 4x^3 + 8x - y - 0.4xy & \xrightarrow{y} 12x^2 + 8 - 0.4y \\ &\xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2 & \xrightarrow{x} -1 - 0.4x \end{aligned}$$

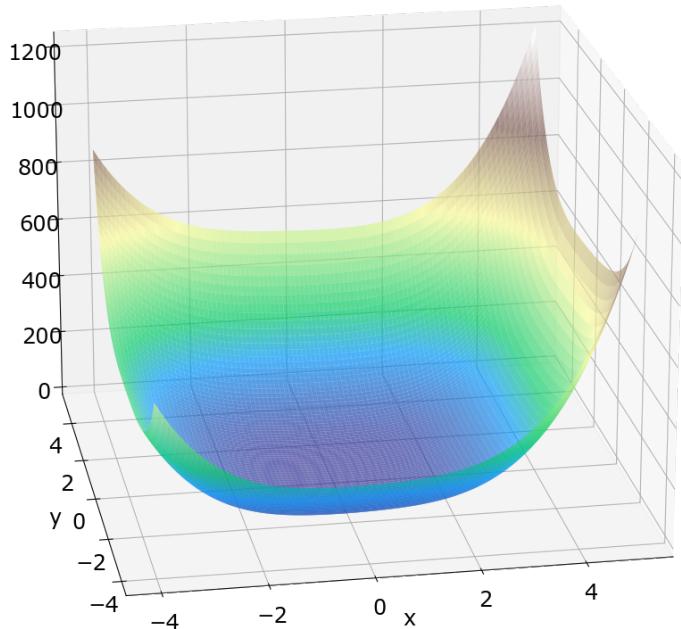
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

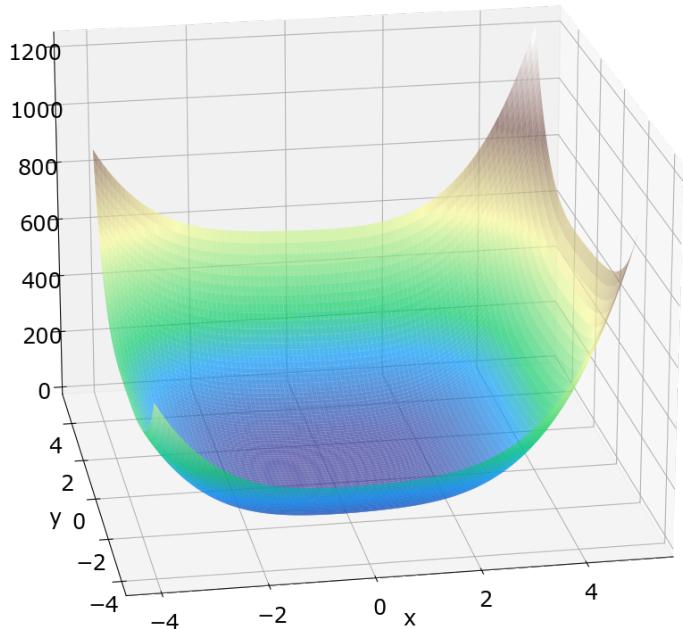
$$\begin{aligned} f(x, y) &\quad \begin{array}{l} \nearrow x \\ \searrow y \end{array} \\ &= 4x^3 + 8x - y - 0.4xy && \begin{array}{l} \nearrow x \\ \searrow y \end{array} & 12x^2 + 8 - 0.4y \\ &\quad \begin{array}{l} \nearrow x \\ \searrow y \end{array} && & -1 - 0.4x \\ &= 3.2y^3 + 4y - x - 0.2x^2 && \begin{array}{l} \nearrow x \\ \searrow y \end{array} & -1 - 0.4x \\ &\quad \begin{array}{l} \nearrow x \\ \searrow y \end{array} && & 9.6y^2 + 4 \end{aligned}$$

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

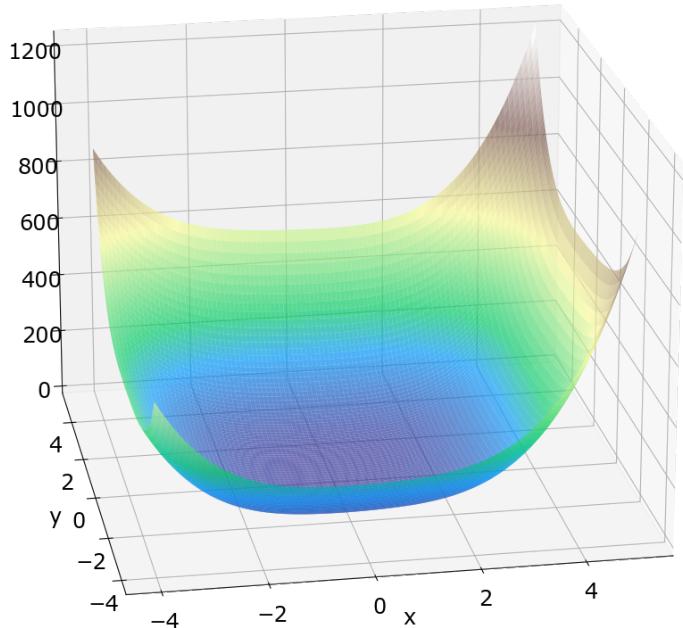
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

An Example

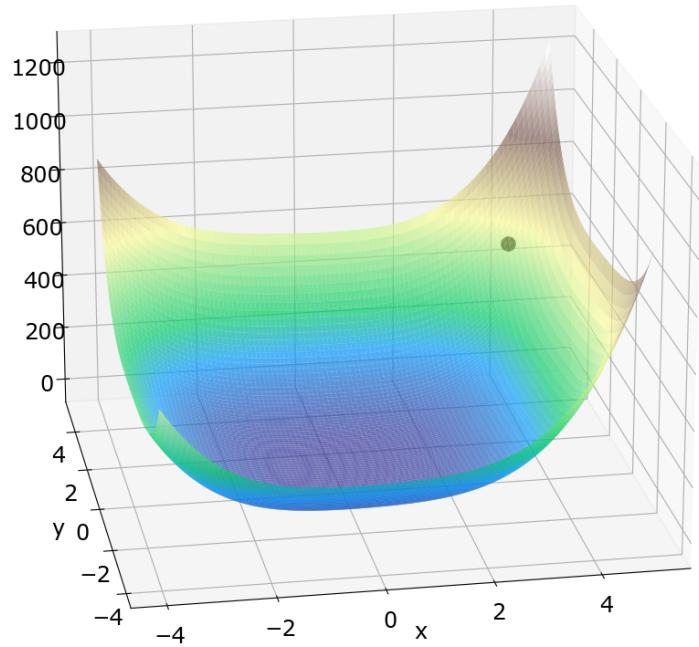


$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

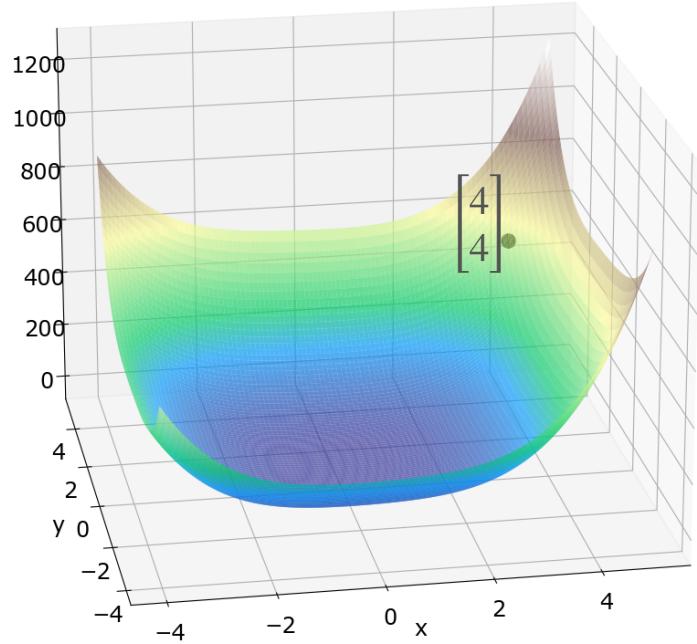
$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

$$H(x, y) = \begin{bmatrix} 12x^2 + 8 - 0.4y & -1 - 0.4x \\ -1 - 0.4x & 9.6y^2 + 4 \end{bmatrix}$$

An Example

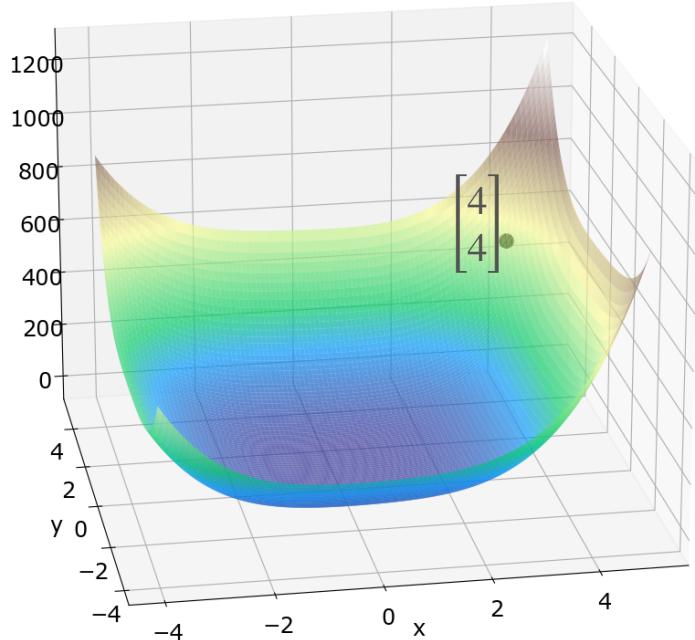


An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

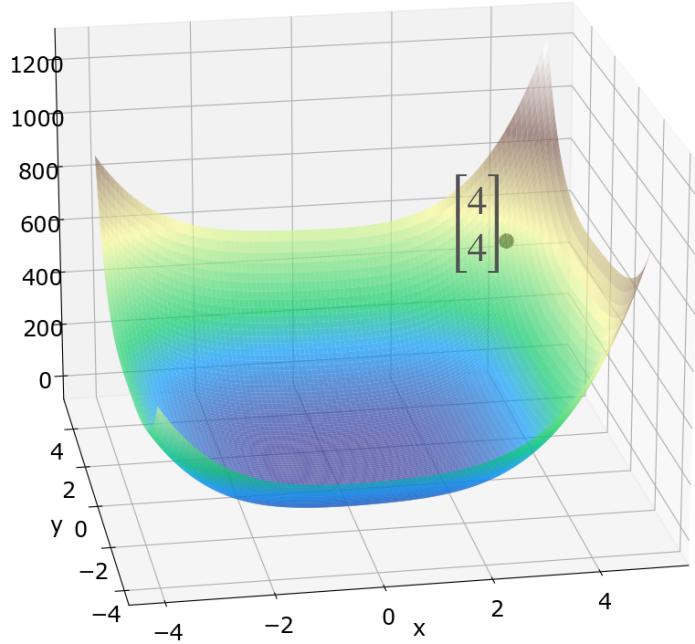
An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix}$$

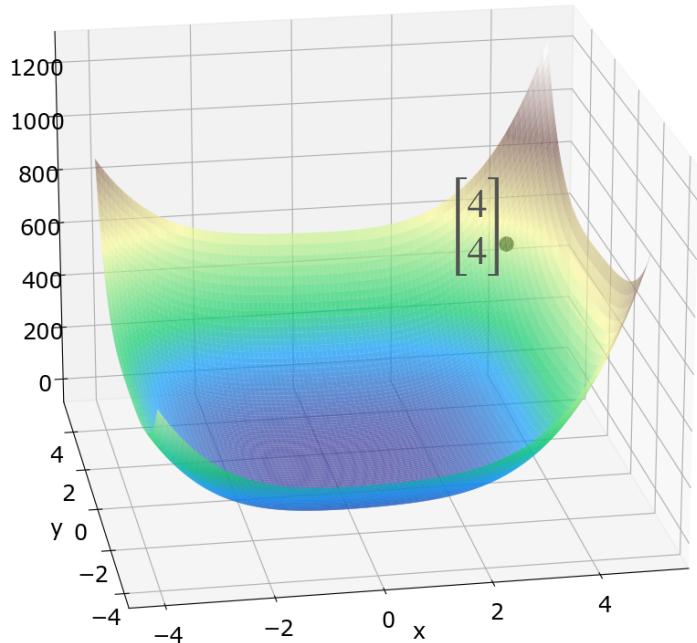
An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \quad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

An Example

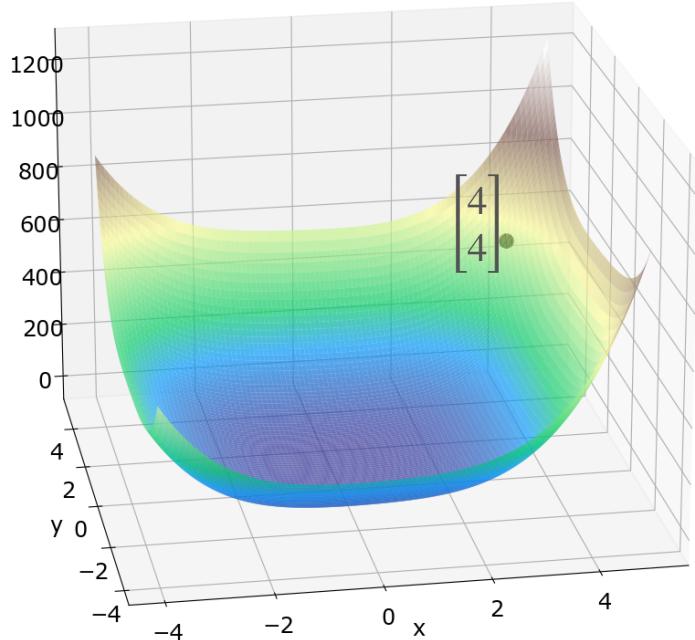


Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \quad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} -$$

An Example

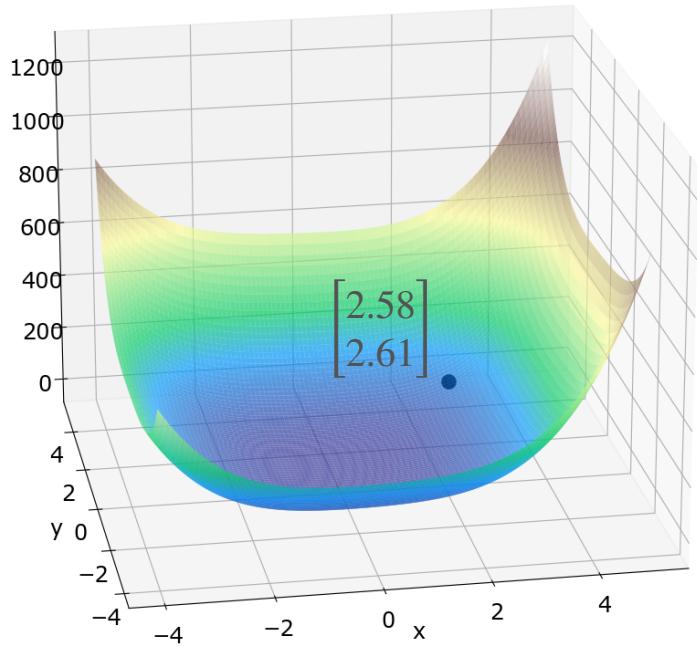


Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \quad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}^{-1} \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix}$$

An Example



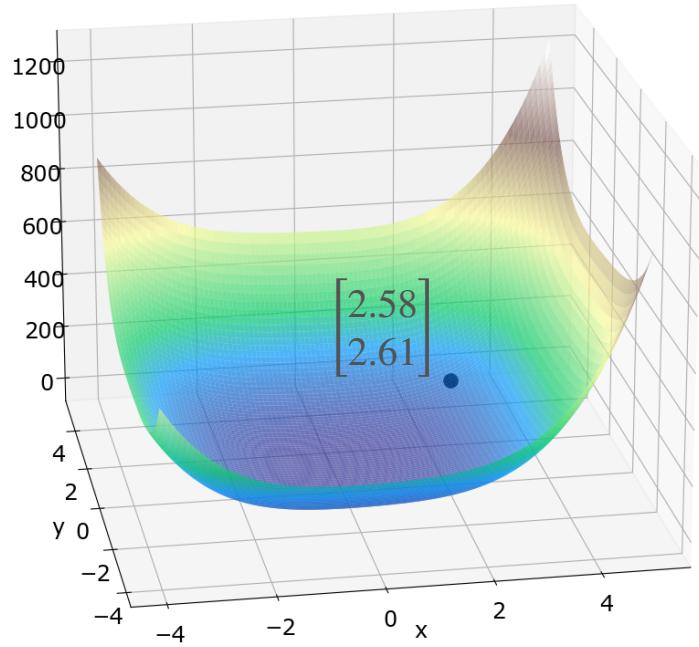
Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \quad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

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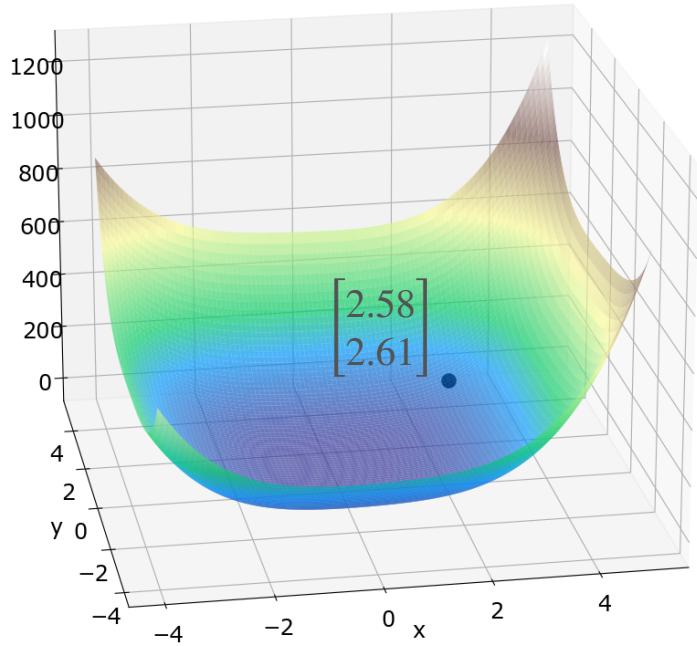
$$= \begin{bmatrix} 2.58 \\ 2.62 \end{bmatrix}$$

An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

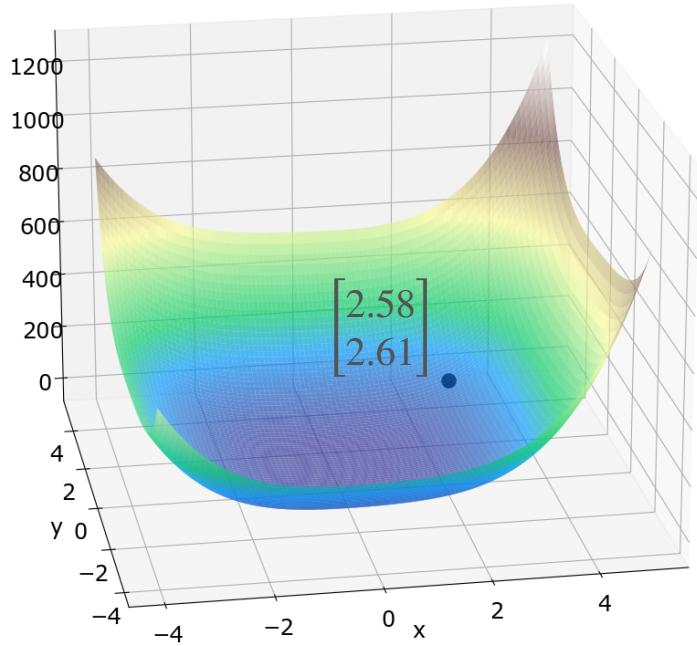
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

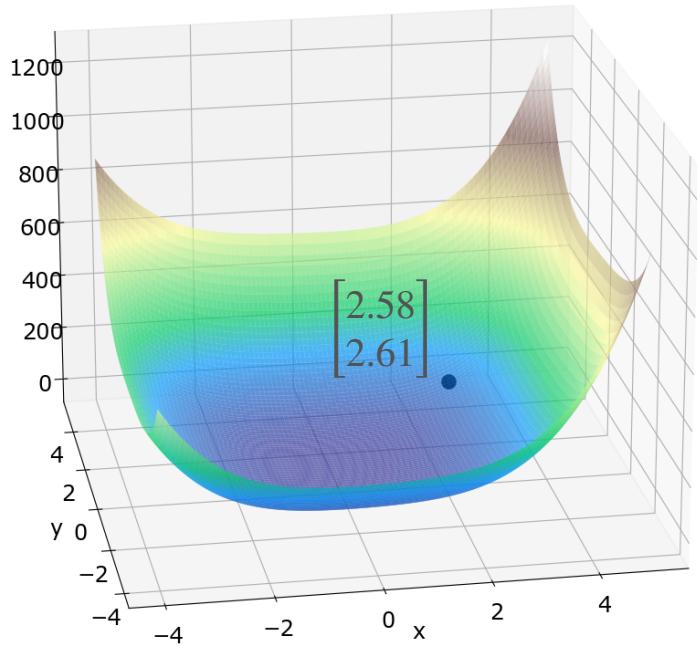
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

An Example

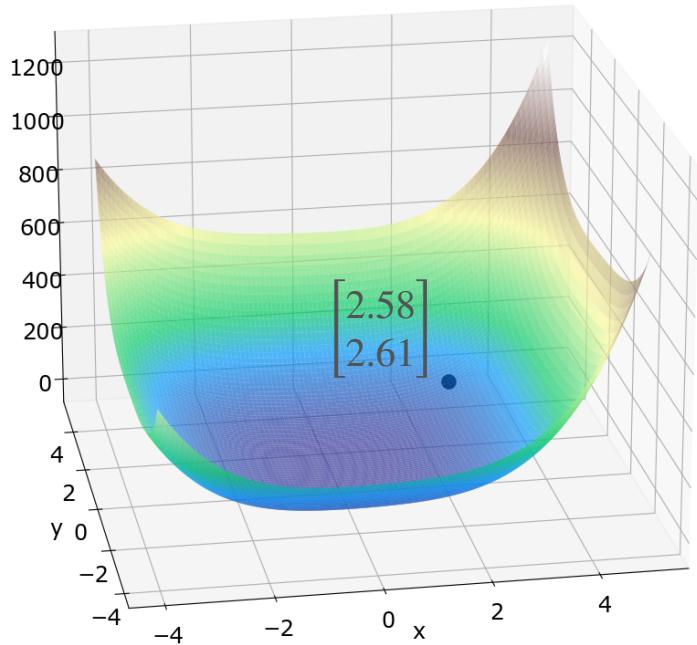


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} -$$

An Example

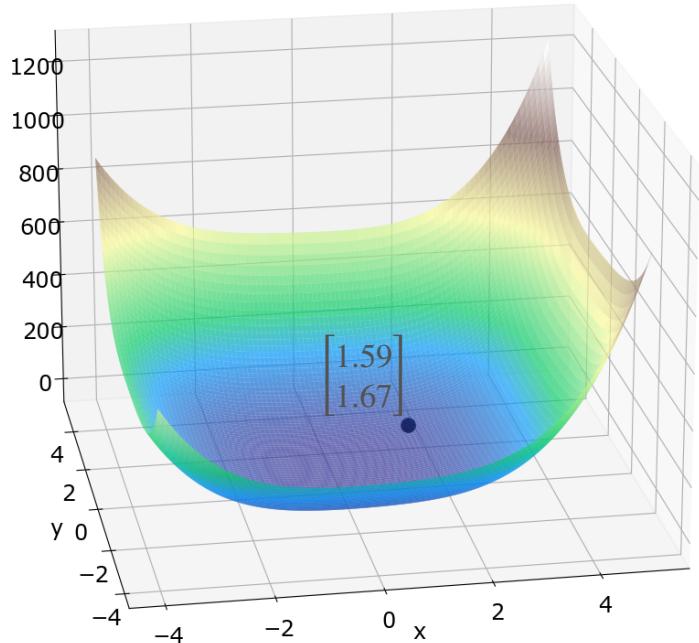


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} - \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}^{-1} \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

An Example

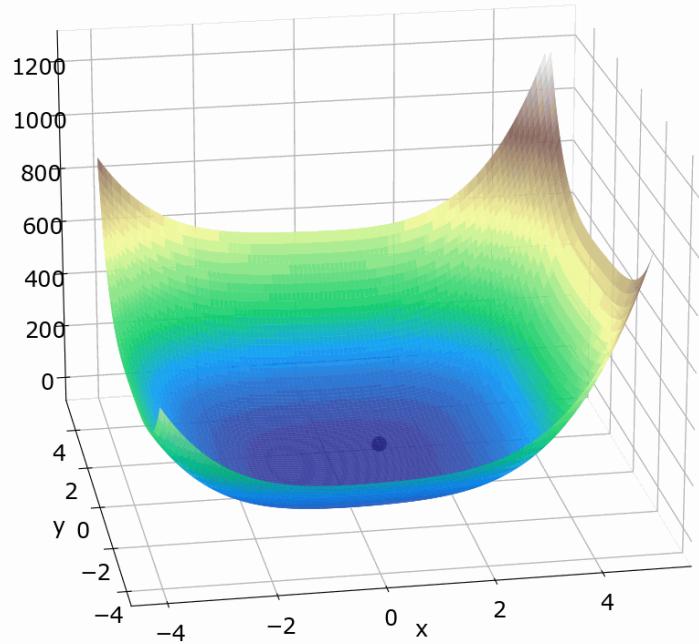


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

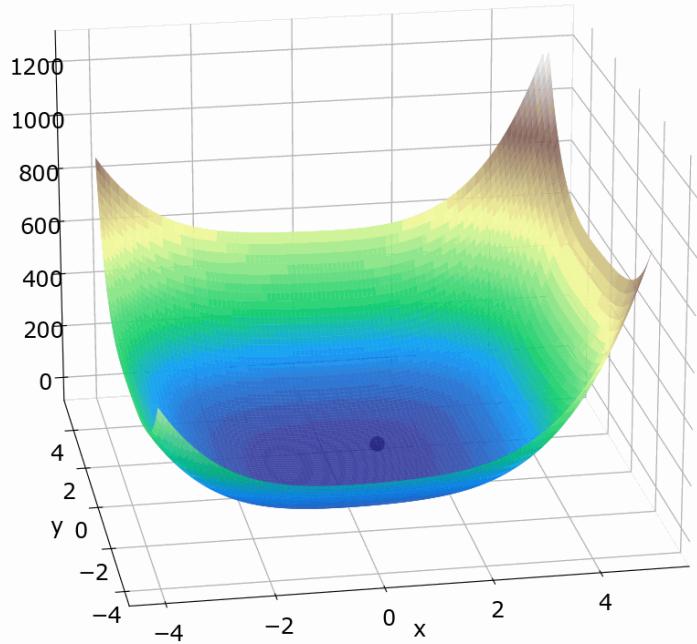
$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} - \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}^{-1} \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$
$$= \begin{bmatrix} 1.59 \\ 1.67 \end{bmatrix}$$

An Example

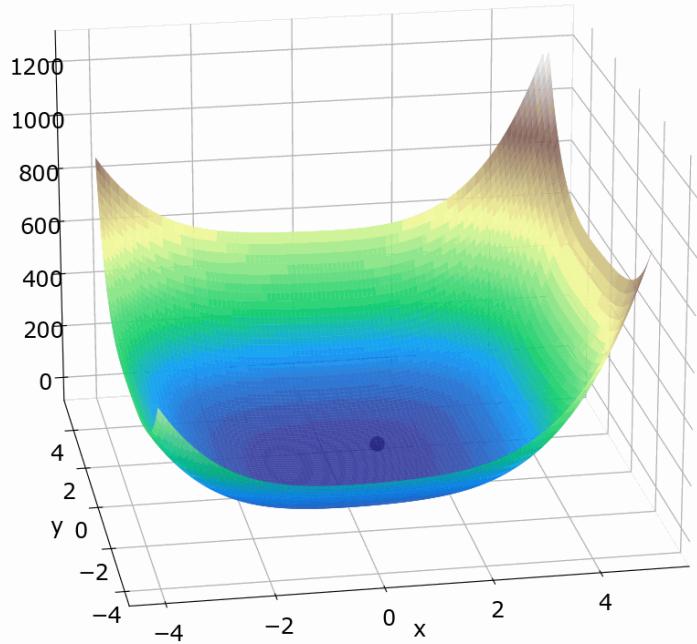


An Example



Repeat until you are close enough to the actual zero!

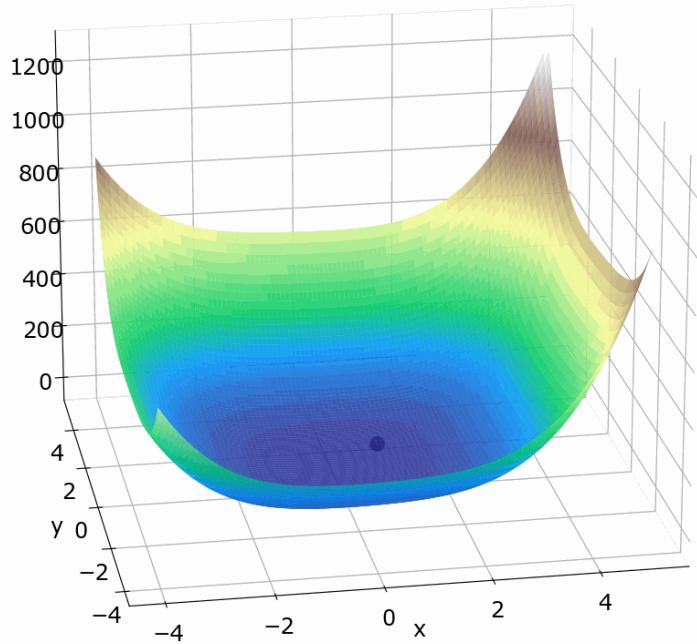
An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

An Example

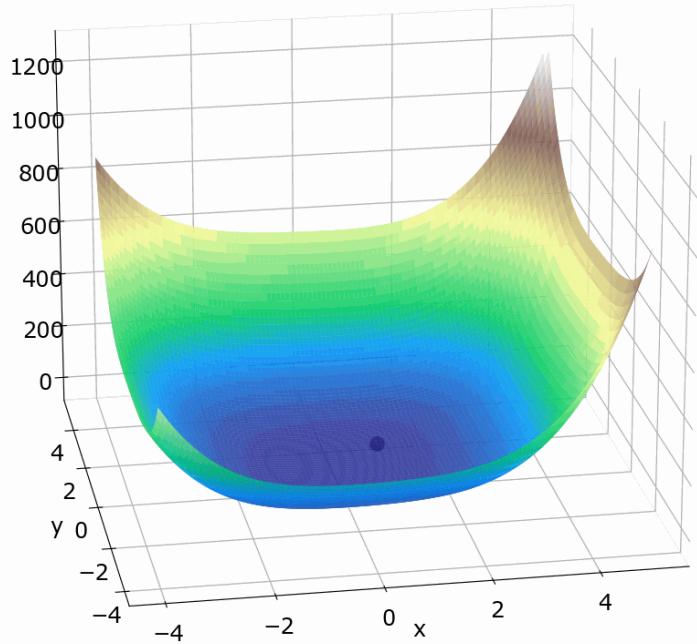


Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15 \cdot 10^{-17} \\ -2.05 \cdot 10^{-17} \end{bmatrix}$$

An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15 \cdot 10^{-17} \\ -2.05 \cdot 10^{-17} \end{bmatrix}$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Conclusion