Results for Assignment 1 - Group xx

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1 Task 1

Information which was deduced from the text:
$P(D=0) = P(H) = 0.95$,
$P(D=1) = P(A) = 0.04$,
$P(D=2) = P(C) = 0.01$,
$P(T=1|C) = 0.98; P(T=0|C) = 1 - P(T=1|C) = 0.02$,
$P(T=1|H) = 0.01; P(T=0|H) = 1 - P(T=1|H) = 0.99$,
$P(T=1|A) = 0.20; P(T=0|A) = 1 - P(T=1|A) = 0.80$.

Table 1: Probability table.

T/D	H (D = 0)	A (D = 1)	C (D = 2)	p(T$_i$)
T = 1	0.0259	0.0011	0.0003	0.0273
T = 0	0.9241	0.0389	0.0097	0.9727
p(D_i)	0.95	0.04	0.01	1

At first calculate the marginal probability.
$P(T=1) = 0.01 * 0.95 + 0.2 * 0.04 + 0.98 * 0.01 = 0.0273$
$P(T=0) = 0.99 * 0.95 + 0.8 * 0.04 + 0.02 * 0.01 = 0.9727$

Then calculate all joint probabilities.
$P(H, T=1) = P(H) * P(T=1) = 0.95 * 0.0273 = 0.0259$
$P(A, T=1) = P(A) * P(T=1) = 0.04 * 0.0273 = 0.0011$
$P(C, T=1) = P(C) * P(T=1) = 0.01 * 0.0273 = 0.0003$

$P(H, T=0) = P(H) * P(T=0) = 0.95 * 0.9727 = 0.9241$
$P(A, T=0) = P(A) * P(T=0) = 0.04 * 0.9727 = 0.0389$
$P(C, T=0) = P(C) * P(T=0) = 0.01 * 0.9727 = 0.0097$

2 Task 2

Blabla explanation.
Blabla.

3 Task 3

Blabla explanation.
Blabla.

4 Task 4

4.1 Verify conditional mean.

$E_Y[Y|X = x] = \sum_{n=1}^{N} y_n * p(Y = y_n|X = x)$
$= \sum_{n=1}^{N} y_n * \frac{p(X=x|Y=y_n)*p(Y=y)}{p(X=x)}$,

where $p(X = x)$ is a constant and can be neglected and $p(Y = y_n) = 1$ because of the uniform distribution. Which brings the following equation:

$= \sum_{n=1}^{N} y_n * p(X = x|Y = y)$,

however, the equation needs to be normalized. Which is why the formular needs to be divided by $\sum_{n=1}^{N} p(X = x|Y = y)$, which brings following equation:

$E_Y[Y|X = x] = \frac{\sum_{n=1}^{N} y_n*p(X=x|Y=y)}{\sum_{n=1}^{N} p(X=x|Y=y)}$

4.2 Verify MAP.

$P(Y|X = x) = \frac{P(X=x|Y)*p(Y)}{p(X=x)}$
We seek value $y_n \in Y$ that maximises the posterior.

$\hat{y}_{MAX} = argmaxp(Y = y_n|X = x)$
$= argmax\frac{p(X=x|Y=y_n)*p(Y=y_n)}{p(X=x)}$,

where $p(X = x)$ is a constant and can be neglected and $p(Y = y_n) = 1$ because of the uniform distribution. Which brings the following equation:

$\hat{y}_{MAX} = argmaxp(X = x|Y = y_n)$

4.3 Plot results.

Here the plot containing all result images can be seen. (Sorry for the mess, latex is doing what it wants...)

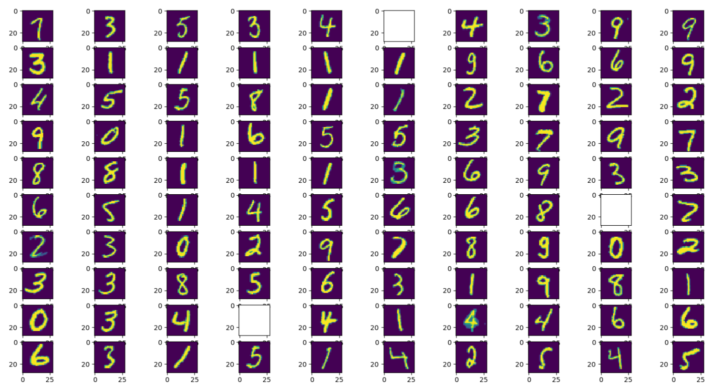

Figure 1: Result of CM algo with sigma 0.25.

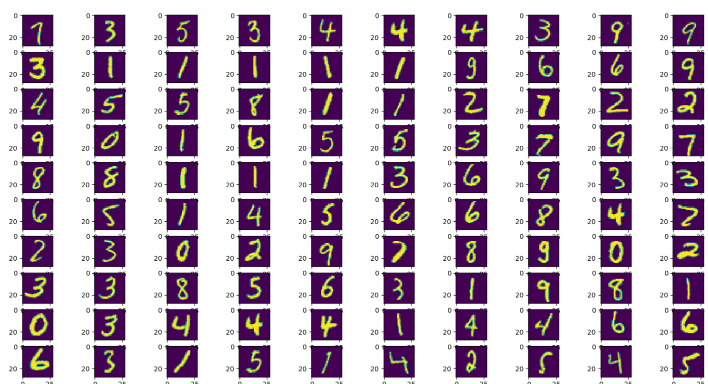

Figure 2: Result of MAP algo with sigma 0.25.

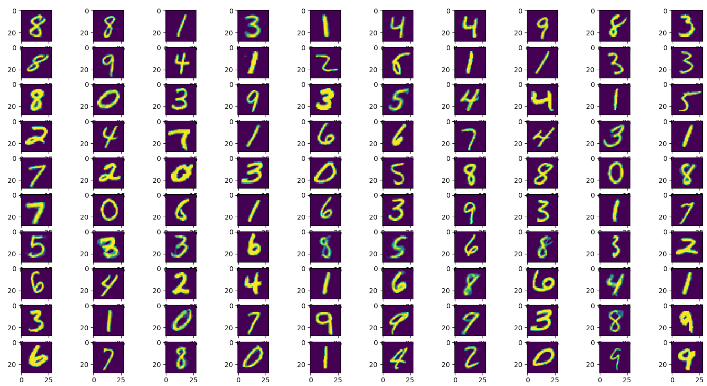

Figure 3: Result of CM algo with sigma 0.50.

4.4 Discuss results.

In our experiments we used three different values for $\sigma : 0.25, 0.5, 1$. The smaller the sigma, the smaller the amount of noise added to the testing images. The bigger the sigma, the bigger the amount of added noise. Generally speaking: the more noise an image has, the harder it is for an algorithm to remove the noise.

4.4.1 Sigma with 0.25

As can be seen in figure 1 the CM algorithm did a pretty good job at removing the noise. There are only a few images, where the noise was not removed completely (e.g. the 3 at position x=6,y=5, where the algorithm almost made an 8 out of the 3) The MAP algorithm, however, did an excellent job, as can be seen in figure 2

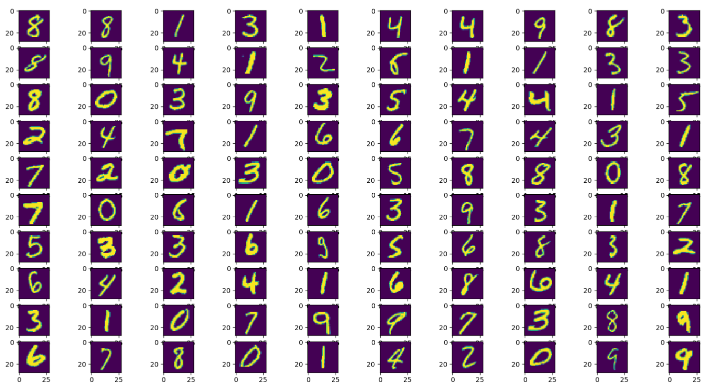

Figure 4: Result of MAP algo with sigma 0.50.

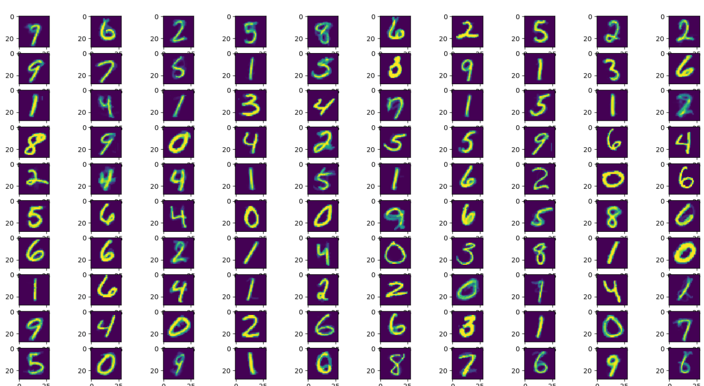

Figure 5: Result of CM algo with sigma 0.50.

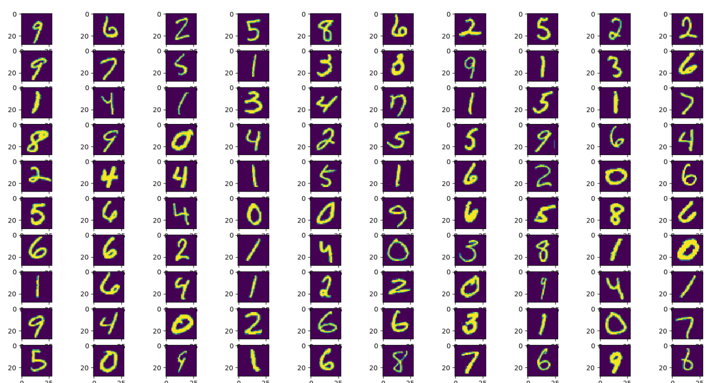

Figure 6: Result of MAP algo with sigma 1.0.