Mechanics Of Materials

# DOM LUIS I

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# 1 Introduction

The Dom Luis I Bridge in Porto, Portugal, is a two-tiered steel arch bridge spanning the Douro River, which was constructed between 1881 and 1886, and it has become a symbol of the city's rich history. The bridge's upper level carries metro trains and pedestrians, while the lower level accommodates both vehicular and pedestrian traffic although it has been for the last years in maintenance and only pedestrian could go through it now.

Figure 1: Dom Luis I Bridge

**The goal** for this project is to understand the structural characteristics and loading conditions of the **Dom Luis I Bridge** by the design of a beam that meets the specified service and strength requirements. Then, analyze the impacts of support settlement and the environmental conditions that will confront.

# 2 Beam Design Approach

Firstly, to make our beam, we need to estimate one span for it so we will start with a guess of 55 meters span, 5 of width and 2 meters deep and its composition, out of the 2 options given, the *steel* would be the one chosen as it has better properties for the necessities that this would need to fulfill. Then for the supports, as we are talking about a bridge across a big river which may support big forces due to winds, but is near the ocean so no big temperature changes happens and it do need to move in any of the directions, so the fixed supports at the towers may be the best options.

To effectively design the beam, we need to gather relevant data about the bridge and the specific beam component which includes:

I. Bridge Dimensions: For the higher deck, the length is 385.5 meters and and span of 18 meters. For the arc's weight, we will simulate that there are two beams of this same material holding the upper deck, with a 4 width, 4 deep and height.

II. Dead Loads: The dead load includes the self-weight of the beam ($V_{\text{beam}} = 55m \cdot 5m \cdot 2m = 550m^3$), 2 beams (128 $m^3$) and upper deck (5770 $m^3$). In total, as **steel density** is 7750 kg/$m^3$, its weight is around 50964 tons ($\cdot 10^3$kg).

III. Live Loads: The live load represents the moving loads that the bridge carries, such as vehicular (each car has an average weight of 1800 kg), underground (28000 kg) and pedestrian (70.8 kg). To make it simpler, the live loads will be divided into 50 lb/$ft^2$ (psf) and the point of load in the middle of 72000 lbs, which will be only used to calculate the maximum shear force and so for the shear stress $\tau = V \cdot Q/It$.

# 3 Analysis and Result

## 3.1 Service Design

The governing differential equation for deflection applied to a beam with both fixed ends is given by:

$$EI\frac{d^2y}{dx^2} = -w\left(\frac{x-\frac{L}{2}}{2}\right)$$

Where the second derivative of deflection ($\frac{d^2y}{dx^2}$) represents the curvature of the beam and the negative sign indicates that the curvature is in the opposite direction to the applied load. As for the other factos, the length one, $\frac{x-\frac{L}{2}}{2}$, represents the distance from the center of the beam, normalized by half the span length and the other factors they are mainly influencing the curvature.

To obtain the moment of inertia, we have use $I_z = \dfrac{wh^3}{12}$ where the width and the height are already set. Then, for this part, only *dead-lifts* were taken into account. With all this information, a code in `python` was design [A.2][a] to check if the deflection was less than the requirements L/300 and also to take a look at the plot against the length. This are the results:

[a]Note that all the code can be found on my github

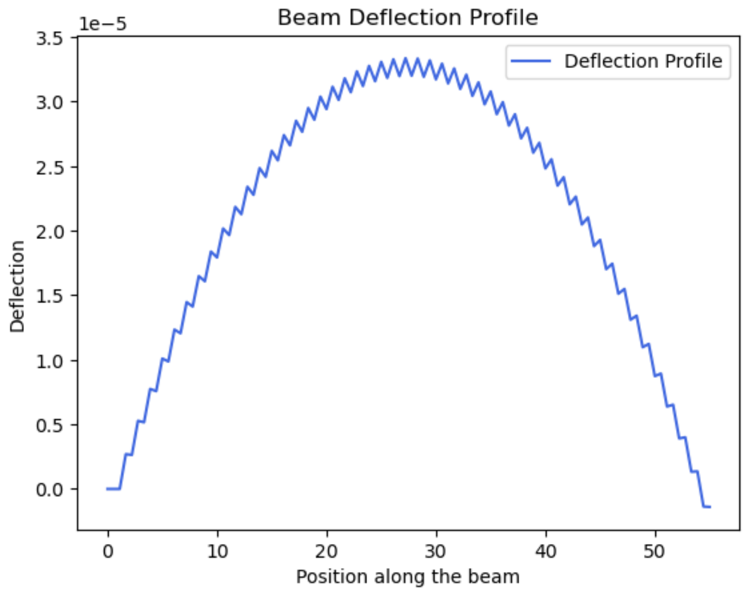


Figure 2: Deflection

## 3.2 Strength Design

To ensure the beam won't yield under applied loads, it will be necessary to compute the maximum shear stress and momentum which directly involves the use of the shear and moment diagrams using the boundary value approach.

The way it will be checked if it would fail under stress is with Von Mises' criteria, which is a measure of the combined effect of normal and shear stresses and this parameter that we obtain will be compare with a safety factor of 2. The calculation for this process will be:

First, compute the shear and moment diagrams using the boundary value approach, with the reactions, taking into account the live loads too (point of load start in the V(x) ). Then, identify the maximum moments ($M_{\text{max}}$) and maximum shears ($V_{\text{max}}$) along the beam and calculate the Von Mises' stress ($\sigma_{\text{VM}}$) with these values using the formula:

$$\sigma_{\text{VM}} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \cdot \sigma_2}$$

where $\sigma_{1,2}$ are the principal stresses.

Lastly, compare $\sigma_{\text{VM}}$ with the maximum allowable stress (50 ksi $\approx$ 344.74 MPa). If $\sigma_{\text{VM}}$ is less than or equal to the maximum allowable stress, the design would fulfill the requirement needed and then will work. For all this, a code [A.3] was implemented so it is easier to iterate. This is the result:

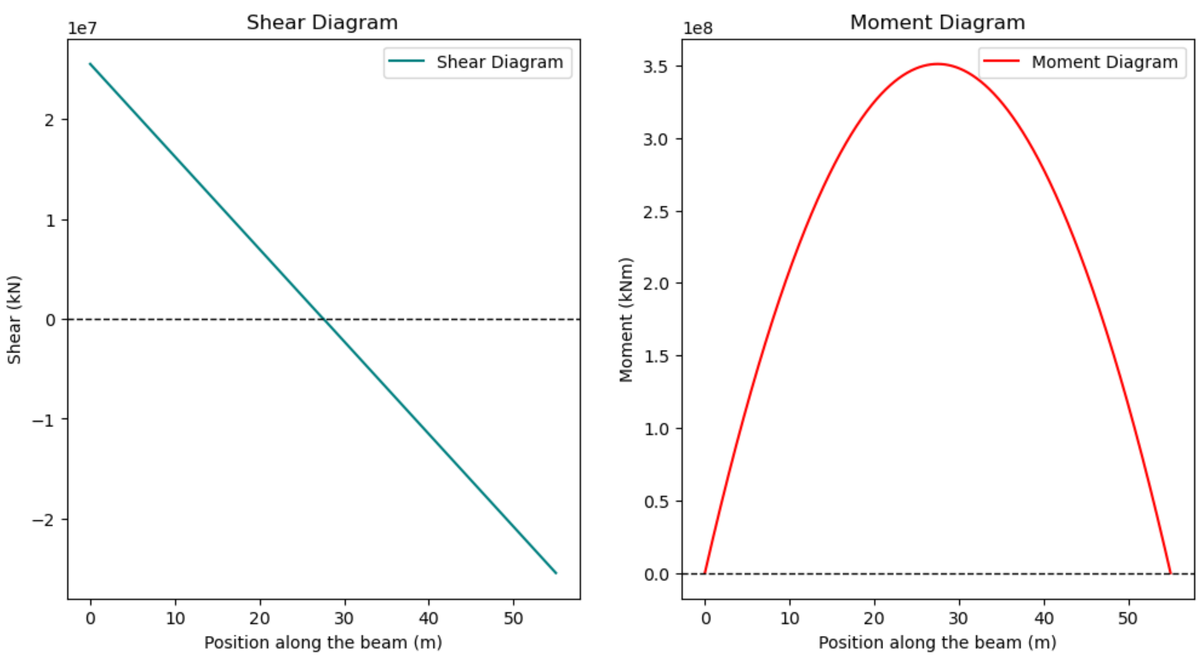


Figure 3: Deflection

As it can be noticed, the point load is really small in comparison with the dead load, the diagrams are not really affect, one needs to zoom in to note the change.

To make it more effective, instead of doing a rectangular beam, we will need to make it lighter by changing the design of a rectangular beam to a double T-shape beam or any others that are not this tough. Then, we will need to iterate in the code to see if it matches the desired restrictions

# 4 Post-Design Scenario: Temperature

What happens if the beam is accidentally thermally restrained and experiences a temperature swing of 30 degrees F? If the beam experiences a temperature swing of 30 F as it is thermally restricted, there are that changes can indeed affect the mechanical properties of materials, including the yield strength. This phenomenon is typically accounted for through the concept of thermal stress and the coefficient of thermal expansion (CTE).

As temperature increases, materials tend to expand, and as it decreases, they contract, and this movement is what actually induce thermal stresses within the material and are reflected to the beam. So this swing temperature, in our design, as it is of huge dimensions, probably it wont be notice that match, however, if the change was bigger, such as 60 degrees, as the change is the young modulus is almost exponential, it will suffer a large variation which may provoke the collapse of the beam as the service design would variate.

# Appendix A

## A.1 Packages and Constants

```
from qiskit import QuantumCircuit, QuantumRegister, execute, Aer
import numpy as np
```

## A.2 Service Design

```
def compute_deflections(len, uniform_load, youngs_modulus, moment_of_inertia):

    num_intervals = 100
    x_values = np.linspace(0, len, num_intervals)
    deflection = np.zeros_like(x_values)
    curvature = np.zeros_like(x_values)
    for i in range(2, num_intervals): #inifinte approximation
        dx = x_values[i] - x_values[i - 1]
        curvature[i] = -uniform_load / (youngs_modulus * moment_of_inertia) * (x_values[i]-len/2)/2
        deflection[i] = deflection[i - 2] + curvature[i - 1] * dx**2 / 2

    return x_values, deflection

#SI units
mass=50964000 #kg
l= 55
w=5
h=2
uniform_load = mass/l
y_mod = 2.1*10**(11)
I_m = w*h**3/12

x_values, deflection_values = compute_deflections(l, uniform_load, y_mod, I_m)

# Check that the maximum deflection is less than the allowable deflection
assert max(deflection_values) < l/300, "Deflection is too high!"
# Plot
plt.plot(x_values, deflection_values, label="Deflection Profile", color="royalblue")
plt.xlabel("Position along the beam")
plt.ylabel("Deflection")
plt.title("Beam Deflection Profile")
```

```
plt.legend()
plt.show()
```

## A.3 Strength design

```
def calculate_shear_moment(len, uniform_load, p_loadxn, p_load):
    num_intervals = 1000
    x_values = np.linspace(0, len, num_intervals)
    shear = np.zeros_like(x_values)
    moment = np.zeros_like(x_values)

    tot_uload = uniform_load * len
    p_load_moment = p_load * (len - p_loadxn)
    total_moment = p_load_moment
    R_a = (tot_uload + p_load) / 2
    R_b = tot_uload + p_load - R_a

    for i, x in enumerate(x_values):
        shear[i] = R_a - uniform_load * x
        if x >= p_loadxn:
            shear[i] += p_load
        moment[i] = R_a * x - 0.5 * uniform_load * x**2
        if x >= p_loadxn:
            moment[i] -= p_load * (x - p_loadxn)

    return x_values, shear, moment

def check_strength_design(moment_values, shear_values, yield_strength, factor_of_safety):
    normal_stress = moment_values / I_m
    shear_stress = shear_values / (w * h)
    von_mises = np.sqrt(normal_stress**2 + 3 * shear_stress**2)

    max_stress = yield_strength / factor_of_safety

    #assertition; if true, nothing happens, if false, error
    design_satisfactory = np.all(von_mises <= max_stress)

    return design_satisfactory


#SI units
uload_new = uniform_load+ 244.121 #50 lb/ft^2 to kN/m^2
p_loadxn = l/2
```

```python
p_load = 32658.651
yield_strength = 344.73786466 *10**6 #50 ksi to Pa
factor_of_safety = 2

x_values, shear_values, moment_values = calculate_shear_moment(l, uload_new, p_loadxn, p_load)
design_satisfactory = check_strength_design(moment_values, shear_values, yield_strength, factor_of_safe

if design_satisfactory == True:
    print("The design meets the safty factors.")
else:
    print("Adjustments needs to be done.")

plt.figure(figsize=(12, 6))
plt.subplot(1, 2, 1)
plt.plot(x_values, shear_values, label="Shear Diagram", color='teal')
plt.axhline(0, color='black', linestyle='--', linewidth=1)
plt.xlabel("Position along the beam (m)")
plt.ylabel("Shear (kN)")
plt.title("Shear Diagram")
plt.legend()

plt.subplot(1, 2, 2)
plt.plot(x_values, moment_values, label="Moment Diagram", color='red')
plt.axhline(0, color='black', linestyle='--', linewidth=1)
plt.xlabel("Position along the beam (m)")
plt.ylabel("Moment (kNm)")
plt.title("Moment Diagram")
plt.legend()
plt.show()
```

# Bibliography

[1] Dom Luís I Bridge. Structurae. (2023). Retrieved from:
https://structurae.net/en/structures/dom-luis-i-bridge