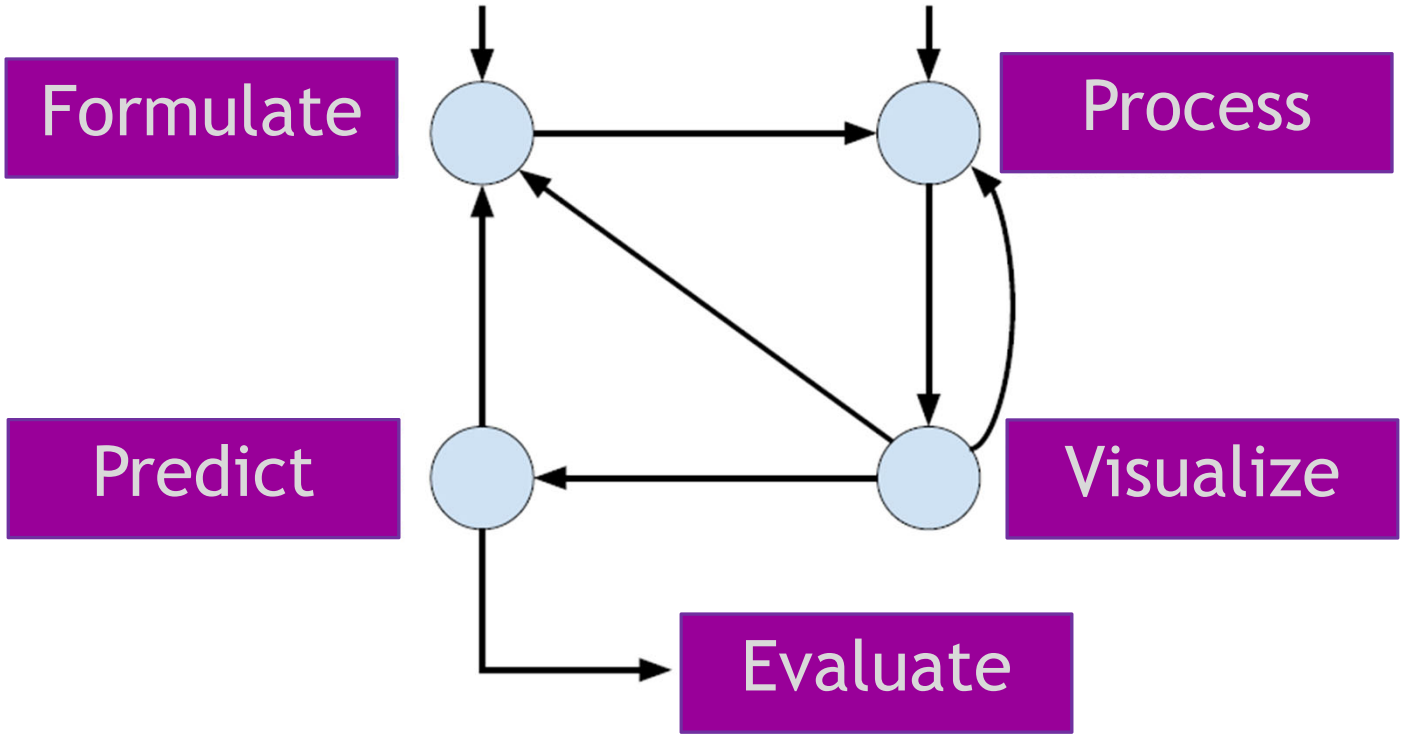
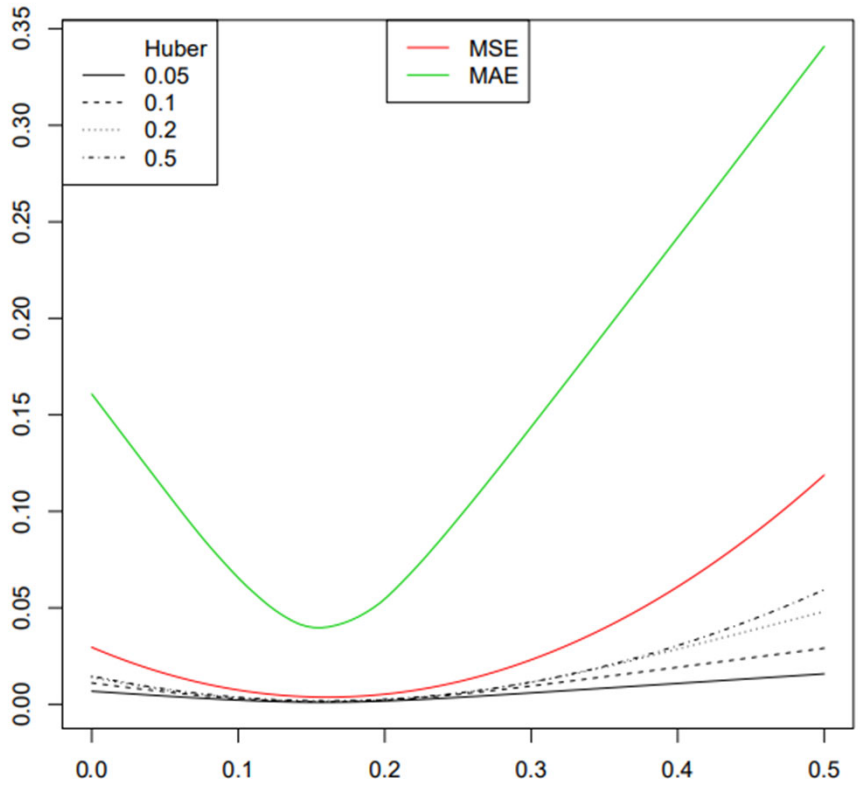
Final Exam Review



Loss Functions



- The mean square error has derivatives at all values of the function. Derivatives are helpful for finding minimum values.
- However, the mean square error has large output for large input. The function is not robust to outliers
- While the mean absolute error has a tricky derivative, the function does not have the same problem with outliers
- Huber Loss combines benefits of both loss functions

Risk

For example the square loss is
$L(\theta, X) = (\theta - X)^2$

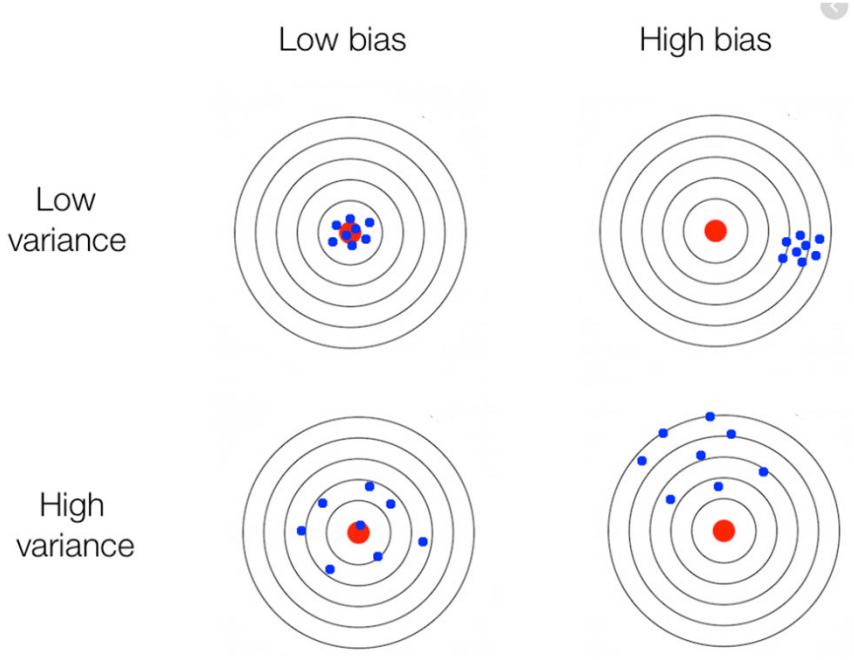
- Suppose we have a loss function L depending on dataset $x_{1,\ldots,}x_n$ and unknown quantity θ
- For fixed value of θ, we can compare the value of $L(\theta, x_1,\ldots,x_n)$ across different datasets
- We can take the datasets to correspond to different values of a random variable. If we repeatedly observe *n* values of a random variable X, then we can compute $L(\theta, x_1,\ldots,x_n)$ for each dataset

- Risk is the expectation of a loss function for random variable

$$E[L(\theta, X)]$$

- We need to know the probability distribution of X to compute the expectation.
- If we can compute the expectation, then we better understand the value of the loss function across different random samples

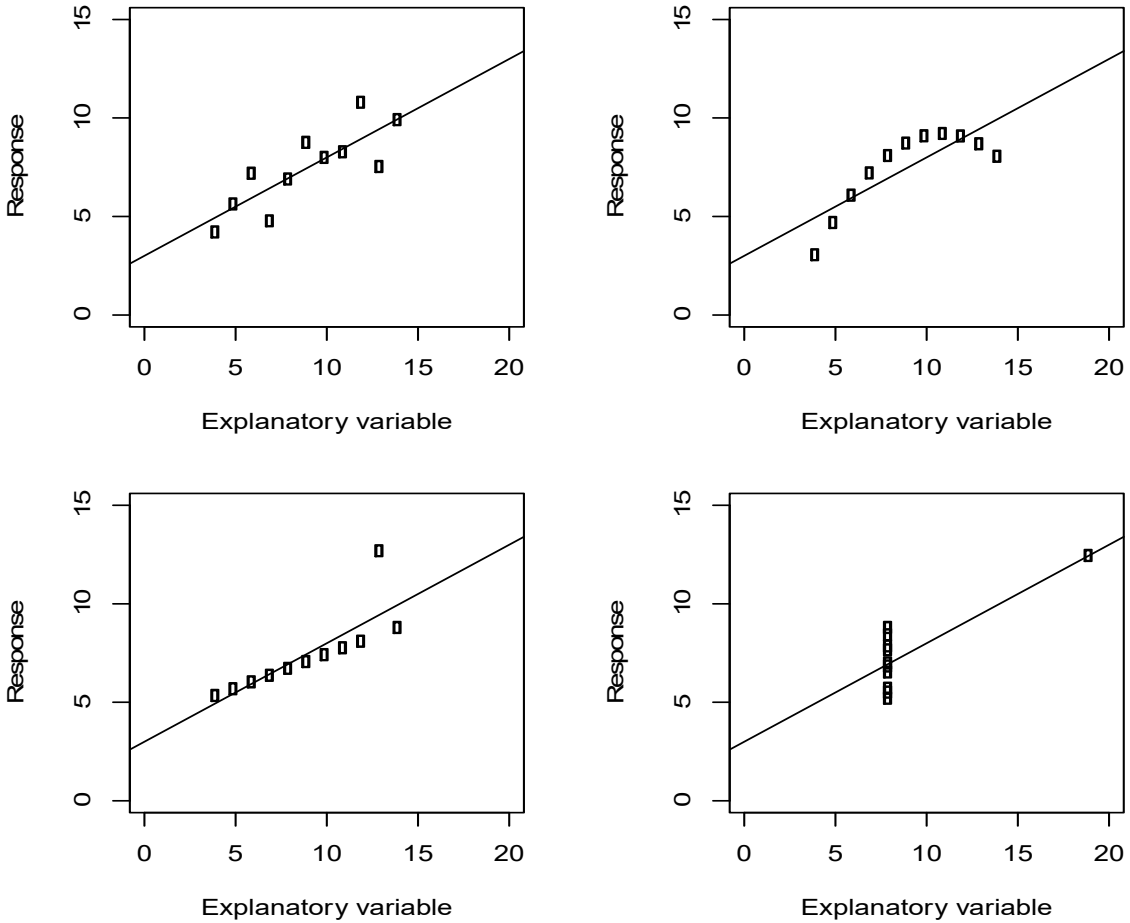
Bias and Variance



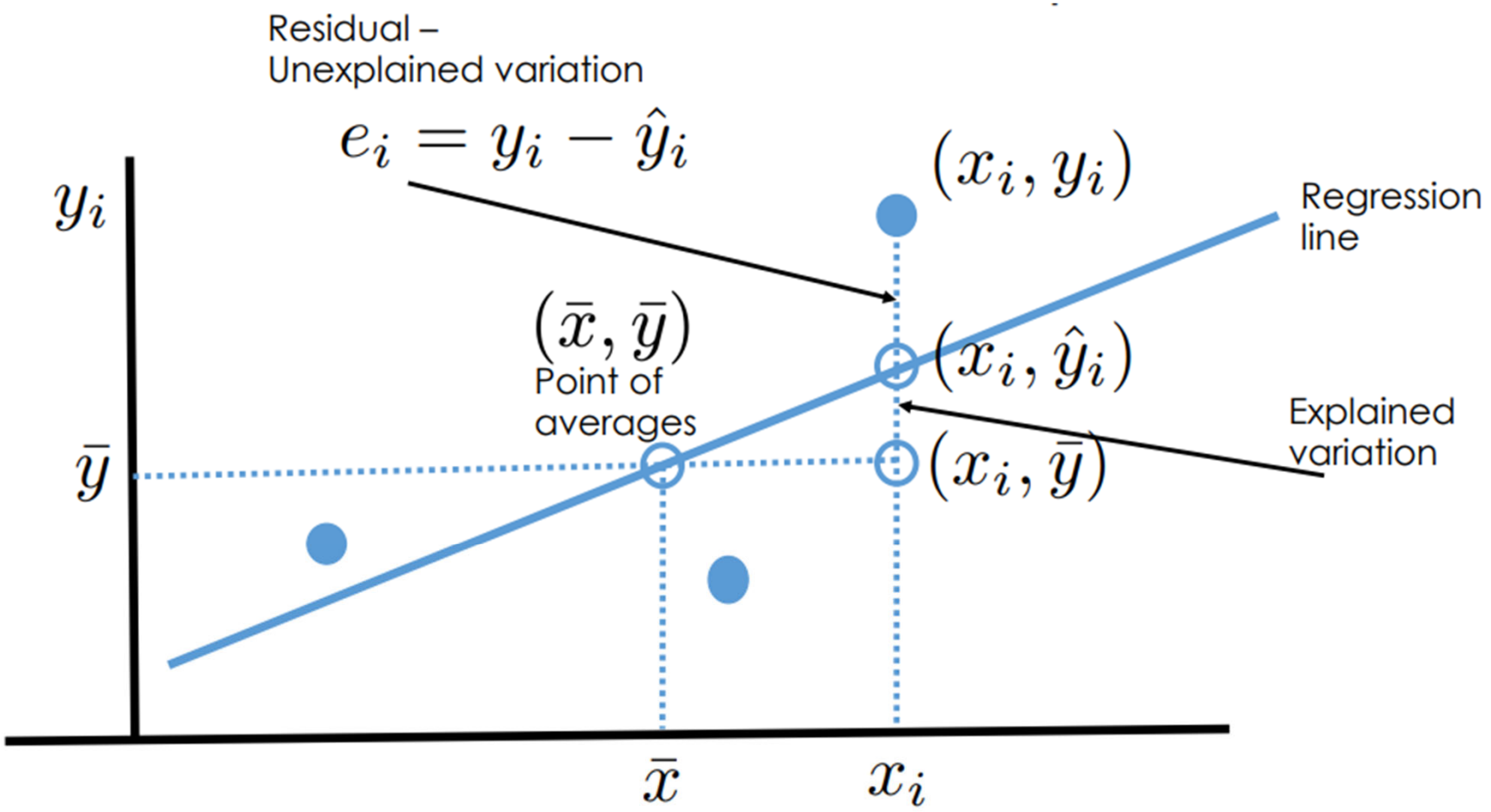
- We want to choose θ to make E[L(θ, X)] small. The choice of θ that makes the expectation smallest is $\hat{\theta}$. We can write $\hat{\theta}_n$ to remind us that the estimate from *n* samples.
- For the square loss, we can break the risk into two components
  - Bias measuring the accuracy of the estimator
  - Variance measuring the consistency of the estimator
- Here bias does not refer to a property of data but to a tendency of estimators

Correlation

- Correlation measures the concentration around a line in a scatter-plot of the independent and dependent variables.
- Correlation is a number between -1 and 1. Around 1 or -1 we have strong positive or negative correlation. Around 0 we have no correlation.
- Correlation measures association between variables. We cannot measure causation.

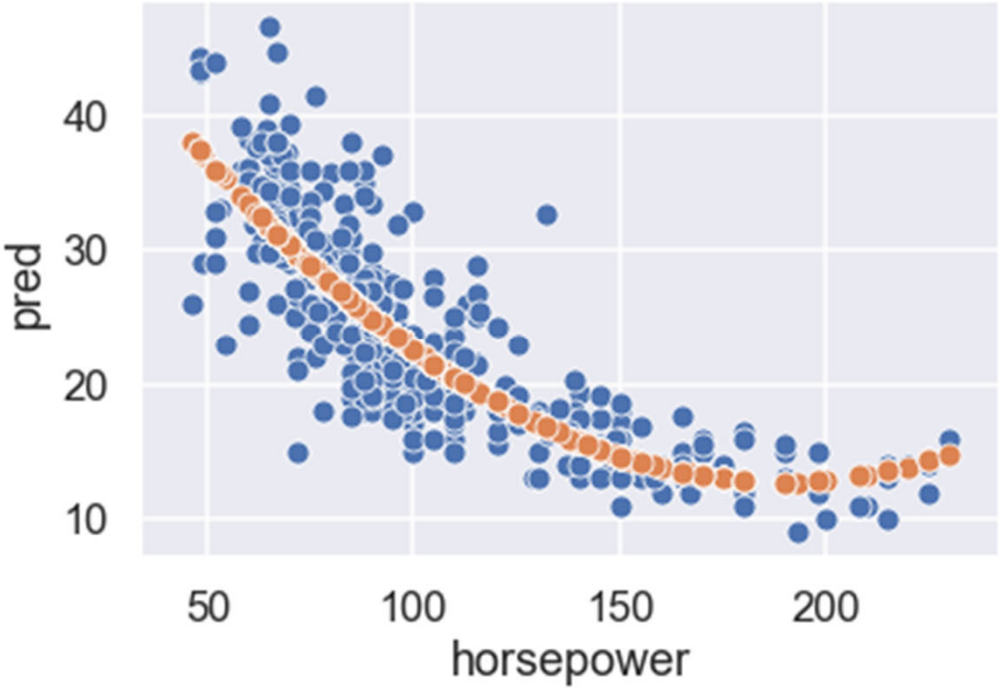


Linear Regression



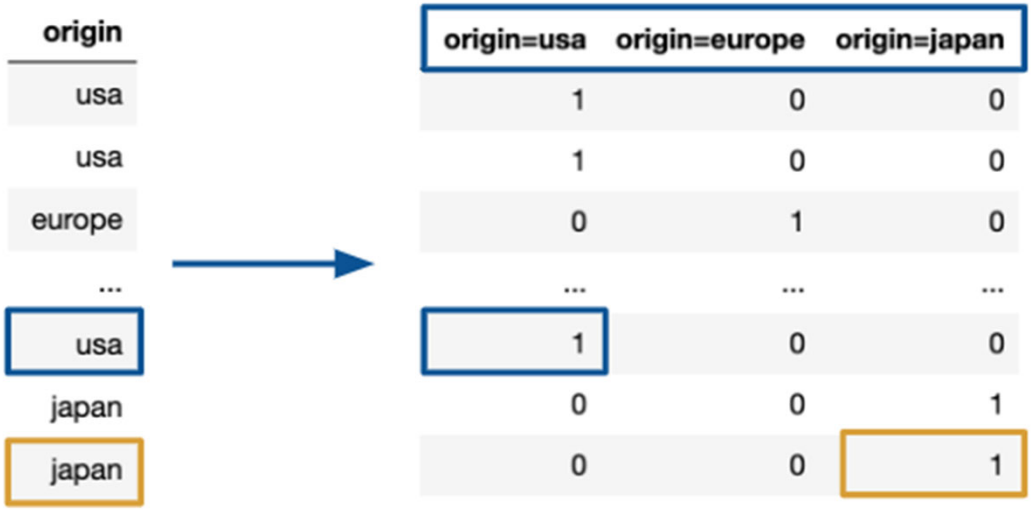
Polynomial Transformations

- If we replace an independent variable x with powers $1, x, x^2, x^3, \ldots$ then we have a polynomial transformation
- If we have multiple independent variables then we can multiply them to model interactions between the features.



One-Hot Encoding

- If we have qualitative data, then we must transform it to quantitative data. However we should be careful with the encoding of the categories.
- We can add another independent variable for each category. The additional variables take the value 0 or 1. We call it a one-hot encoding



origin
usa
usa
europe
...
usa
japan
japan

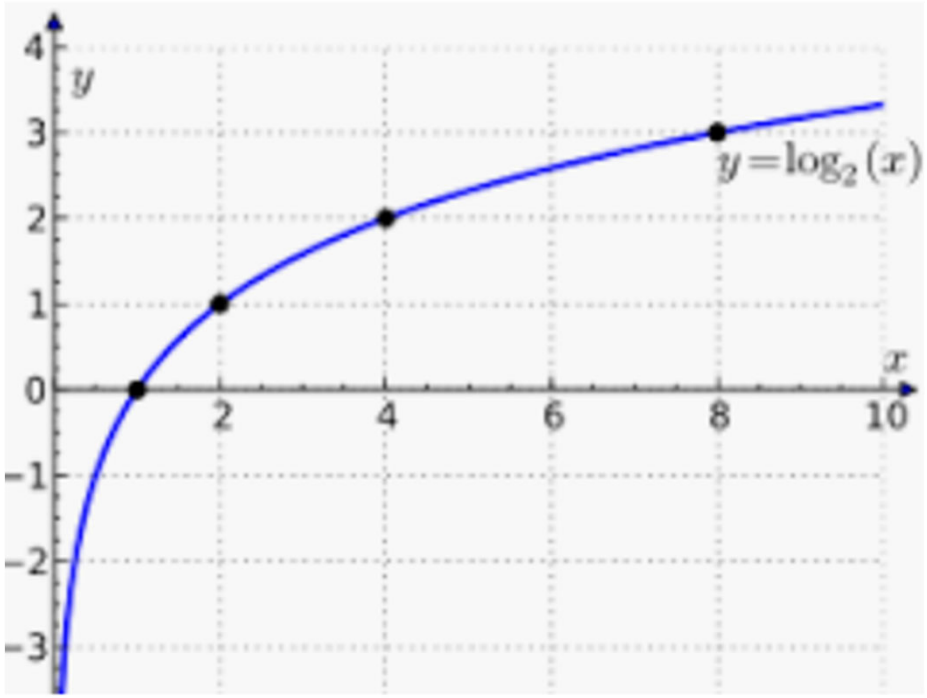
origin=usa	origin=europe	origin=japan
1	0	0
1	0	0
0	1	0
...	...	...
1	0	0
0	0	1
0	0	1

Logarithmic Transformations

- Remember that logarithmic transformations help us with visualization. We can transform a large range of numbers to a small range of numbers suitable for a chart.

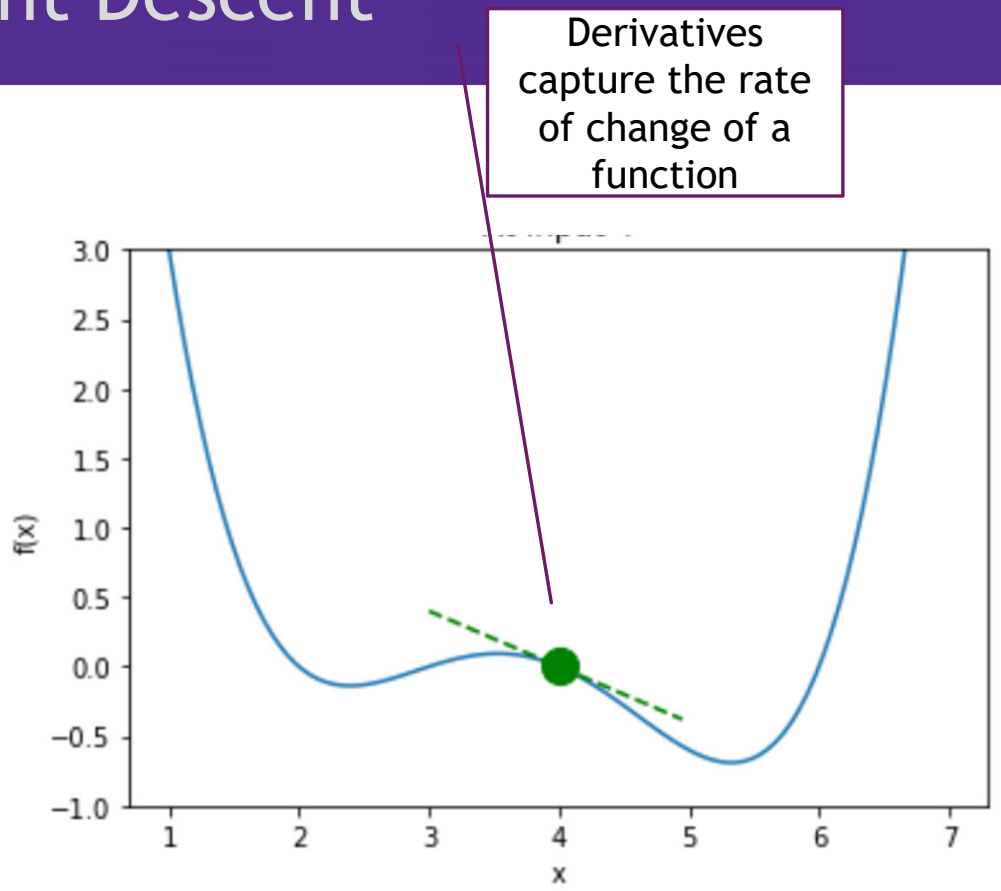
$$\log_b(a) = c \iff b^c = a$$

- If the independent variable and dependent variable have different scales, then we can apply logarithms to straighten out the data.



Gradient Descent

- By minimizing the average loss, we obtain parameters that fit the model to the data.
- We should not guess inputs and check outputs to minimize the function because that approach is inefficient and inaccurate.
- Instead we will just make one guess and use the derivative to update the guess. We call the approach gradient descent.

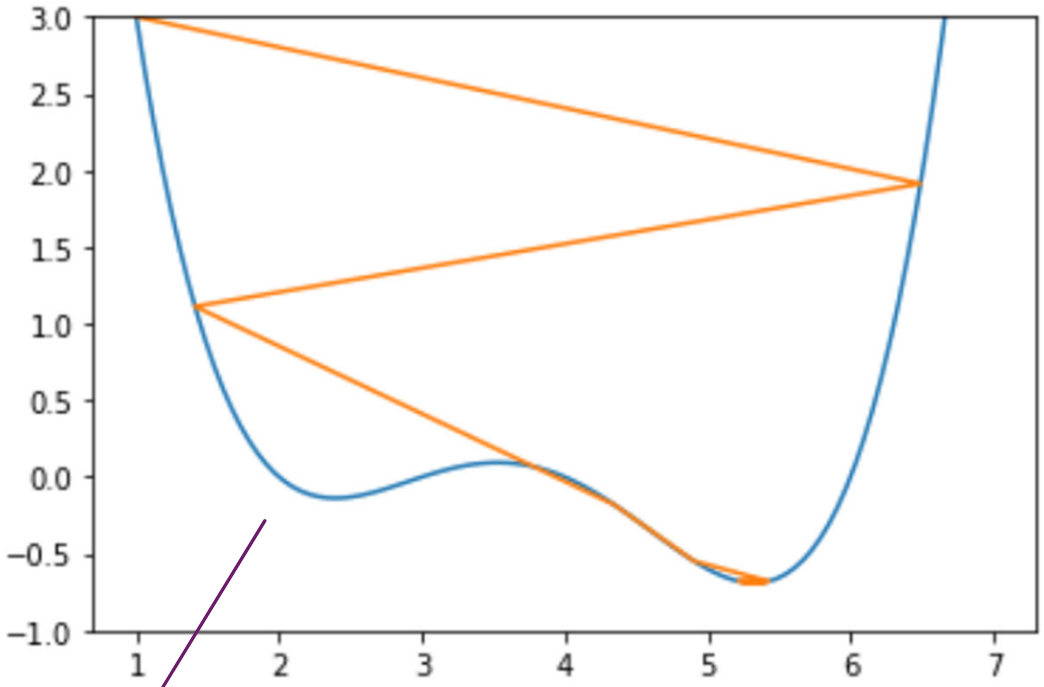


Gradient Descent

- Starting from an initial guess $x^{(0)}$ we update the guess with the formula

$$x^{(t+1)} = x^{(t)} - \alpha \frac{d}{dx} f(x)$$

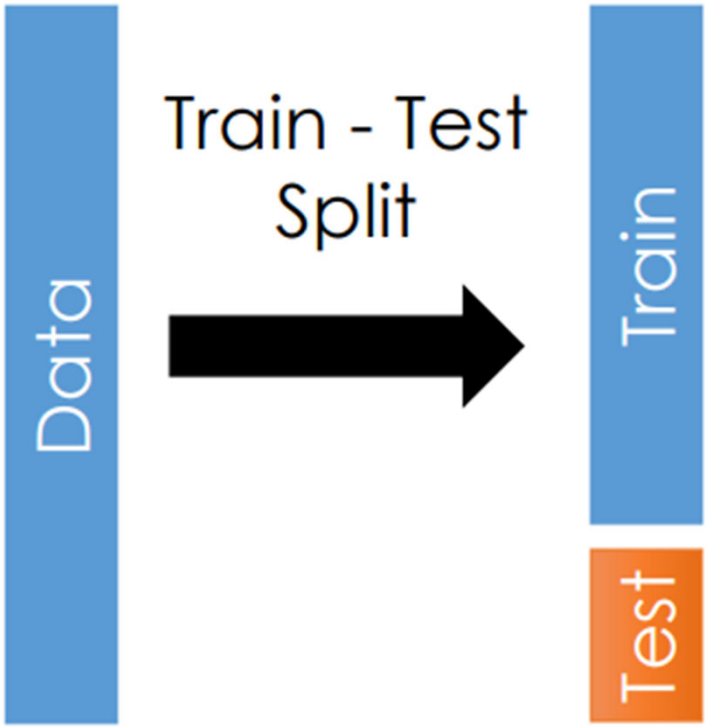
- Here α denotes the learning rate. If α is large, then guesses can change a lot between iterations. If α is small, then guesses can change a little between iterations



If the learning rate is too large then gradient descent might diverge from the minimum

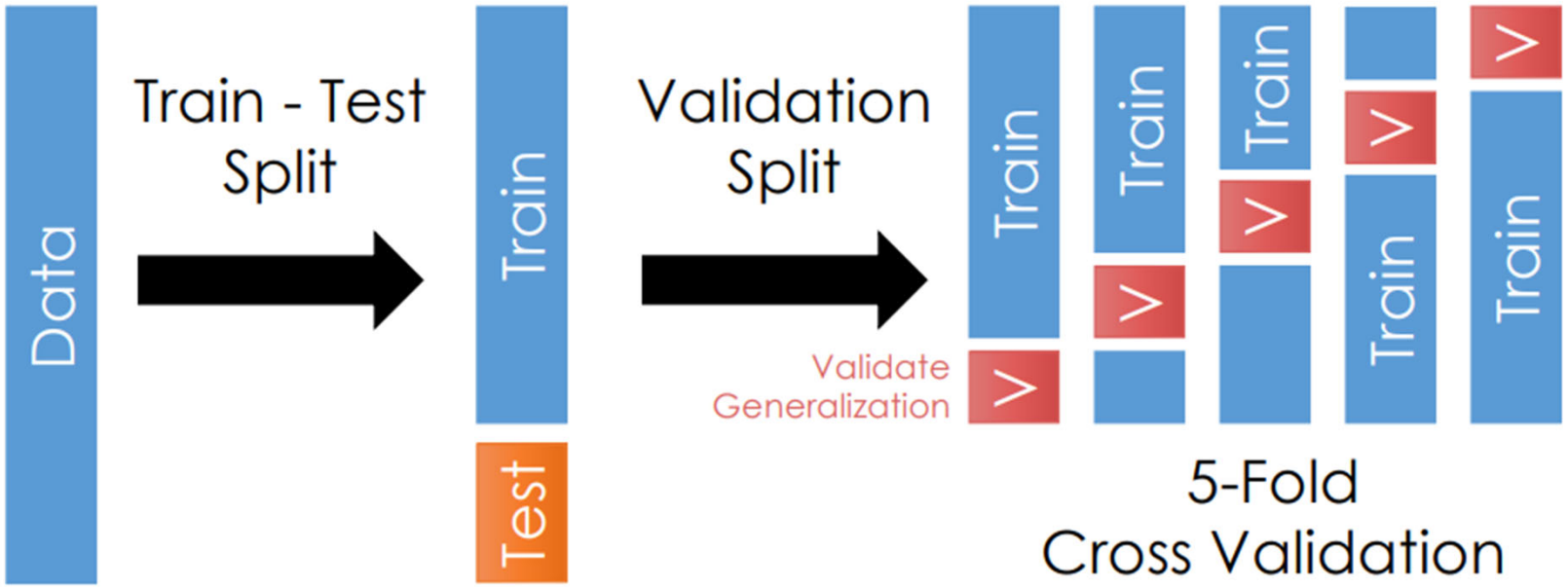
Testing Set

- Instead of studying one sample, we need to study two samples
  - training set
  - testing set
- We will fit the model to the data in the training set. We will check the accuracy of the predictions on the testing set.
- Usually we take 80% of the data for the training set and 20% of the data for testing set.



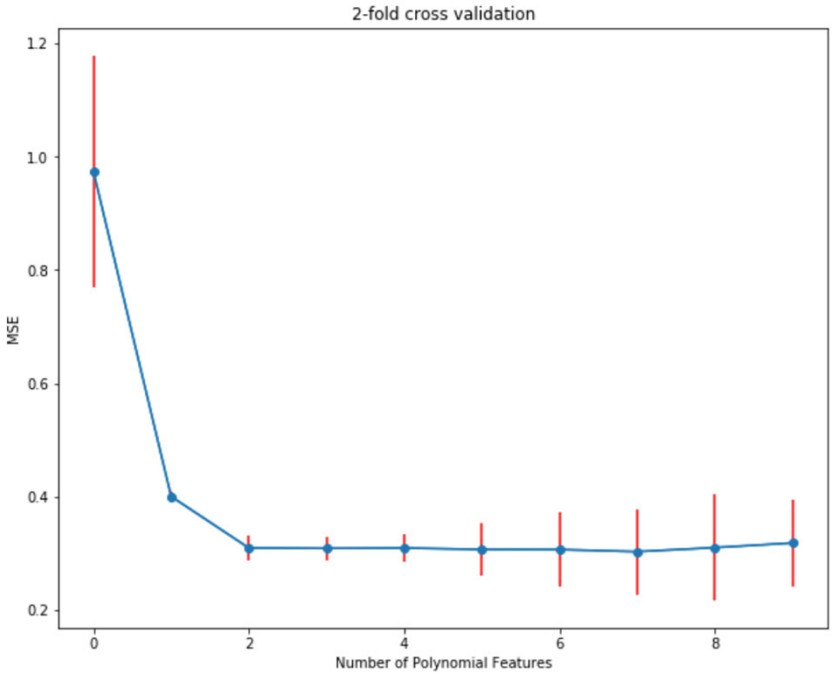
Validation Set

- We split one sample into two sample to generate the training set and testing set. Next we split the training set into k folds. Each fold has a training set and a validation set



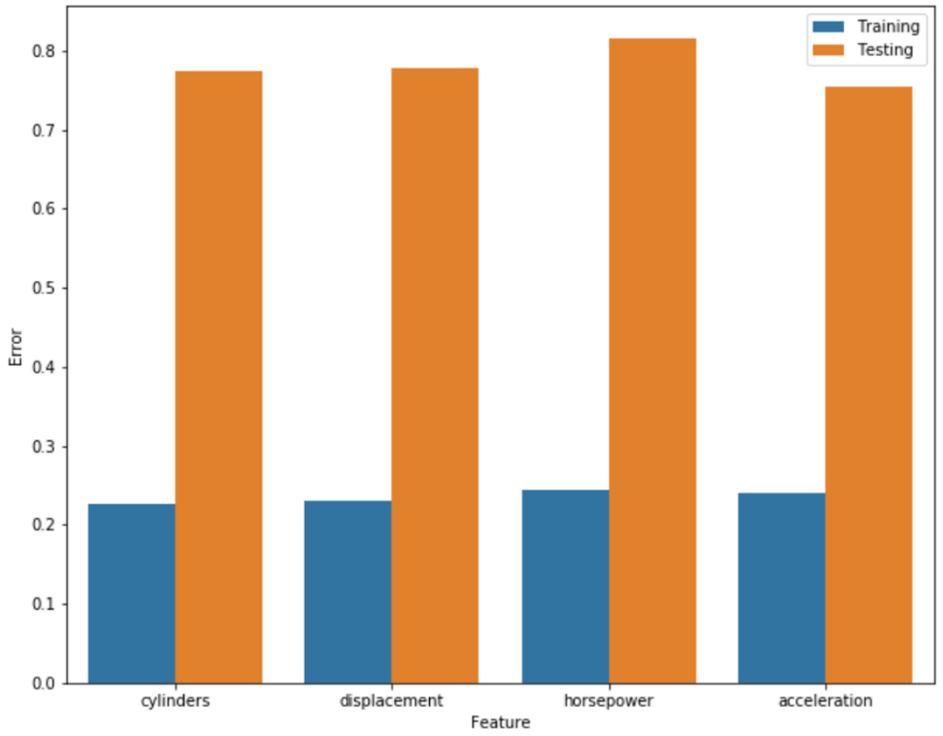
Cross Validation

- We want to choose models that are both accurate and consistent.
- With cross validation we measure the difference between predictions and observations on many datasets.
  - Small errors give us accuracy
  - Similarity between errors give us consistency
- We can visualize both the accuracy and consistency through a line chart with error bars



Feature Selection

- If we want to remove features to prevent against overfitting, then we could try to assess the effect of dropping combinations of features.
- In backward feature selection we
  - select a feature
  - drop it from the table
  - fit a model to the data
  - calculate average loss
- The feature that led to the smallest increase in loss should be excluded from predictions



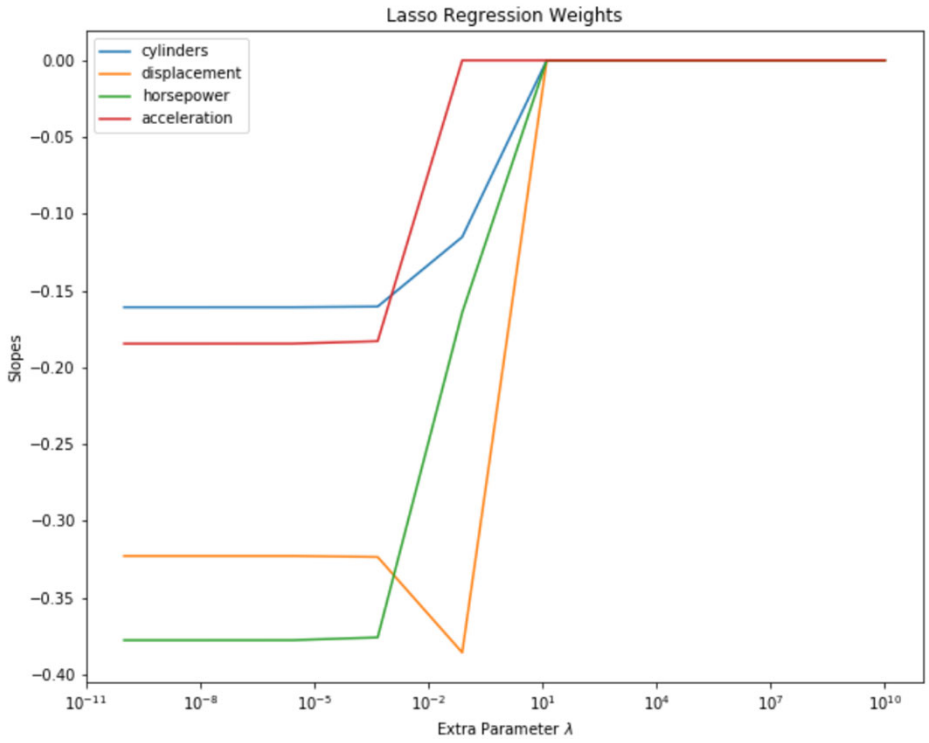
Lasso Regression

- We can replace the average loss for linear regression

$$\frac{1}{n}\sum_{i=1}^{n}(a+bx_i-y_i)^2$$

with regularized average loss for lasso regression

$$\lambda(|a|+|b|)+\frac{1}{n}\sum_{i=1}^{n}(a+bx_i-y_i)^2$$



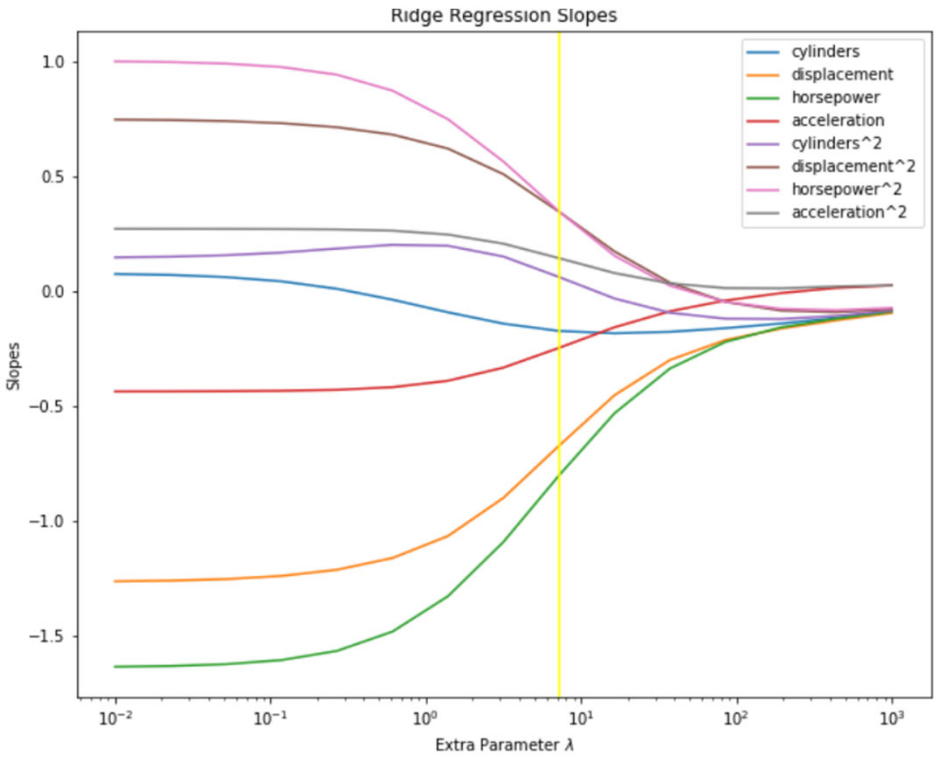
Ridge Regression

- We can replace the average loss from linear regression

$$\frac{1}{n}\sum_{i=1}^{n}(a+bx_i-y_i)^2$$

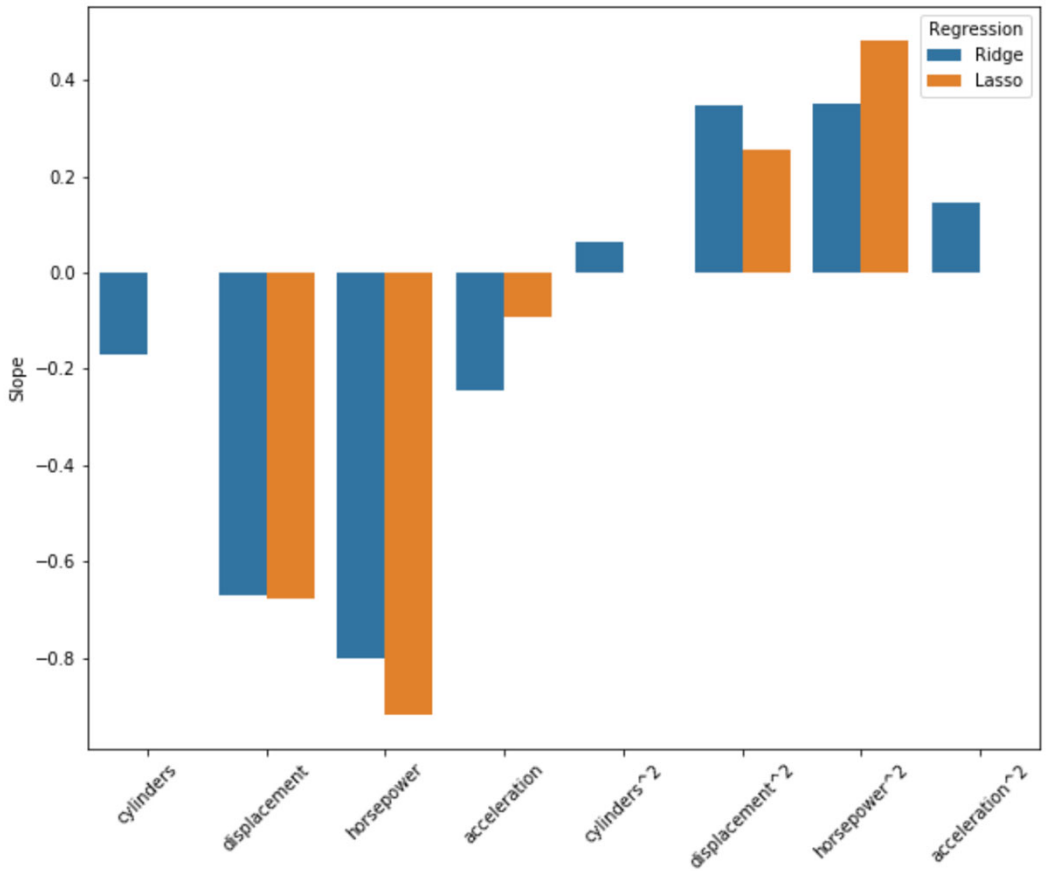
with regularized average loss for ridge regression

$$\lambda\left(a^2+b^2\right)+\frac{1}{n}\sum_{i=1}^{n}(a+bx_i-y_i)^2$$



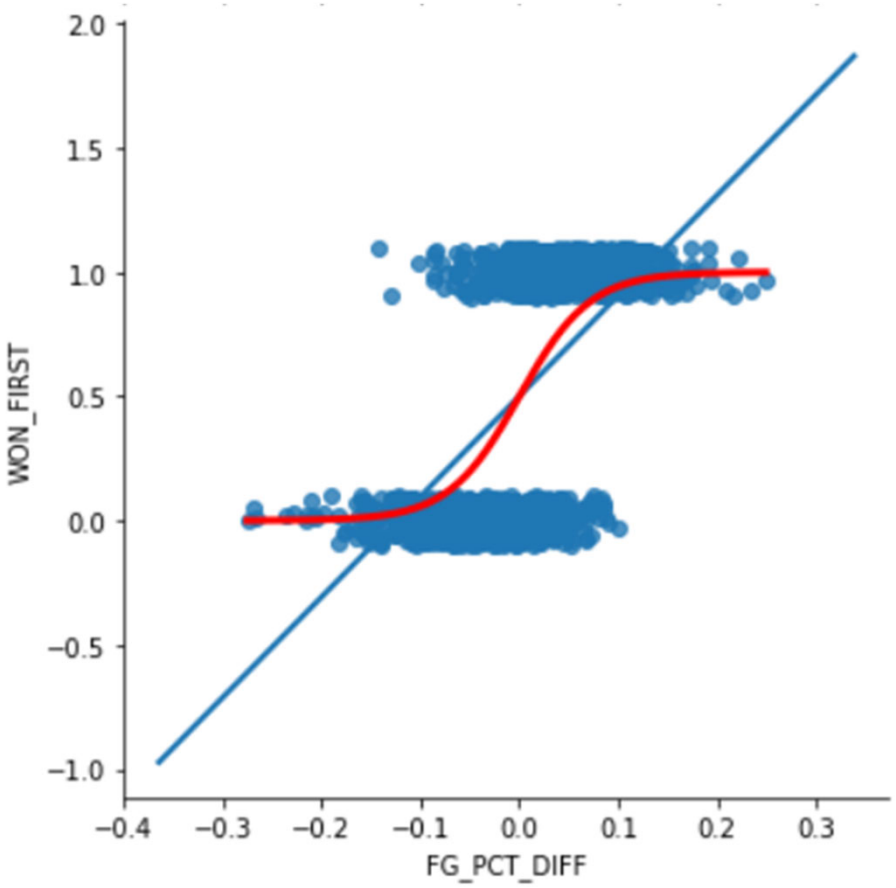
Shrinking Parameters

- Lasso Regression tends to shrink parameters down to zero.
  - Helpful to eliminate features from the model
  - Erratically chooses between associated features
- Ridge Regression tends to shrink parameters close to zero.
  - Helpful to average out the values of the parameters among associated features
  - Cannot eliminate features from the model



Logistic Regression

- We use linear regression to predict a quantitative response variable.
- We use logistic regression to predict a qualitative response variable.
- Usually we encode the categories with numbers like 0 and 1

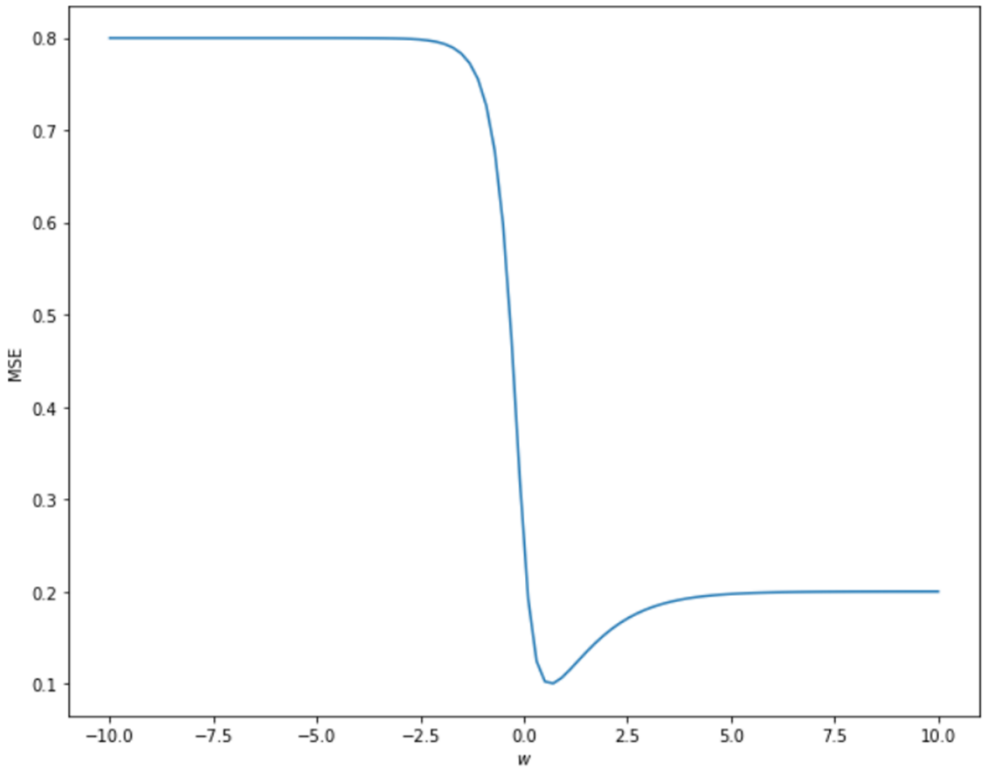


Logistic Loss

- Instead of the square loss we should take the logistic loss.

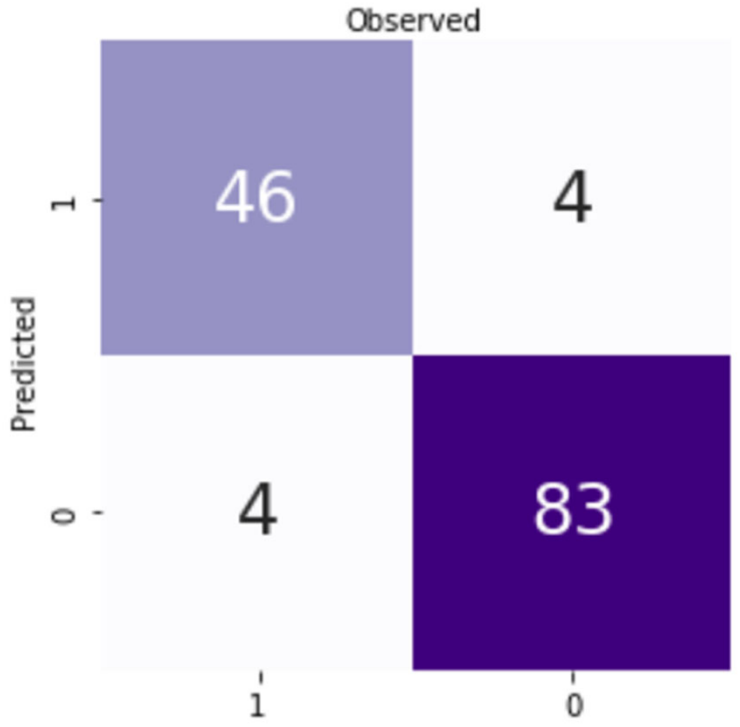
$$-y \log \hat{y} - (1 - y) \log(1 - \hat{y})$$

- Unlike the square loss, the logistic loss does not generate flat regions that prevent gradient descent from finding the minimum.



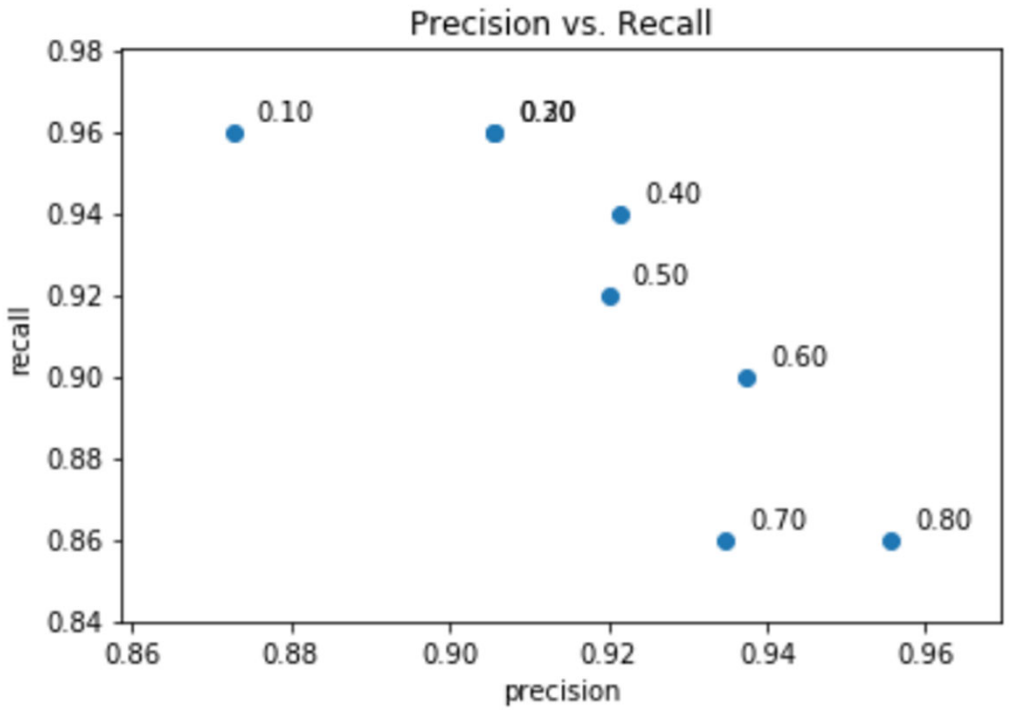
Confusion Matrix

- The observation take the value 1 or 0. The predictions take the value 1 or 0. So we have four possibilities
  - True Positive
  - False Positive
  - False Negative
  - True Negative
- We can visualize the number of each possibility for a dataset with a confusion matrix



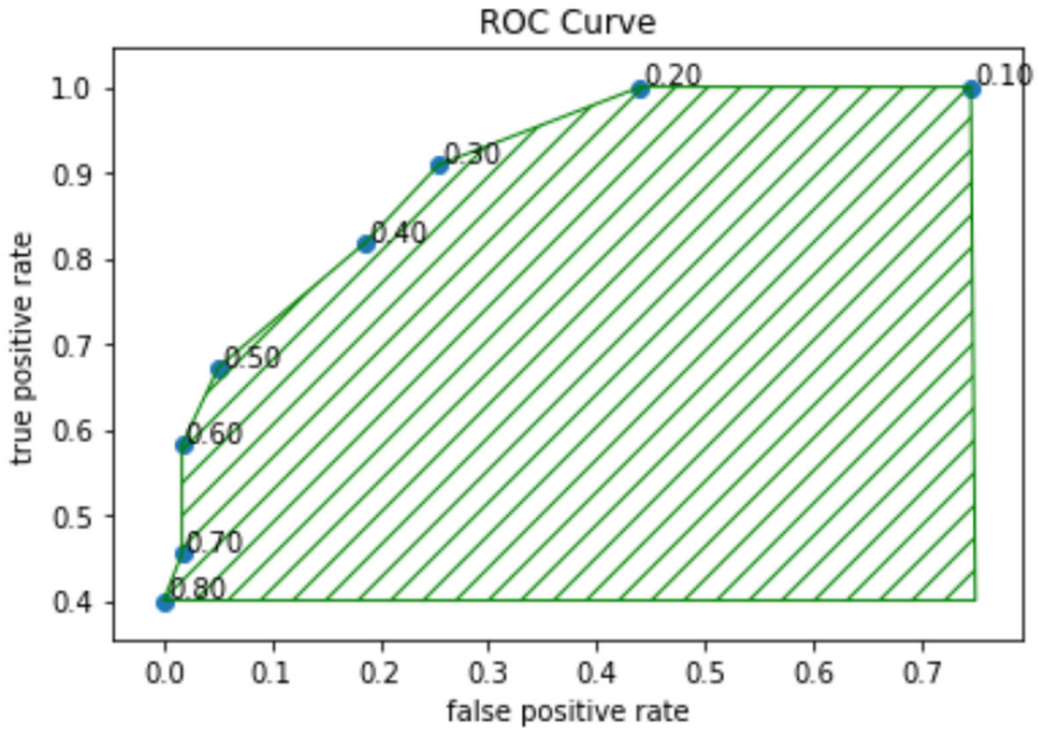
Precision Recall Curve

- Accuracy might not capture the differences between observations and prediction with an imbalance between categories
  - Precision penalizes false positives
  - Recall penalizes false negative
- We can visualize the trade-off between recall and precision through a precision-recall curve



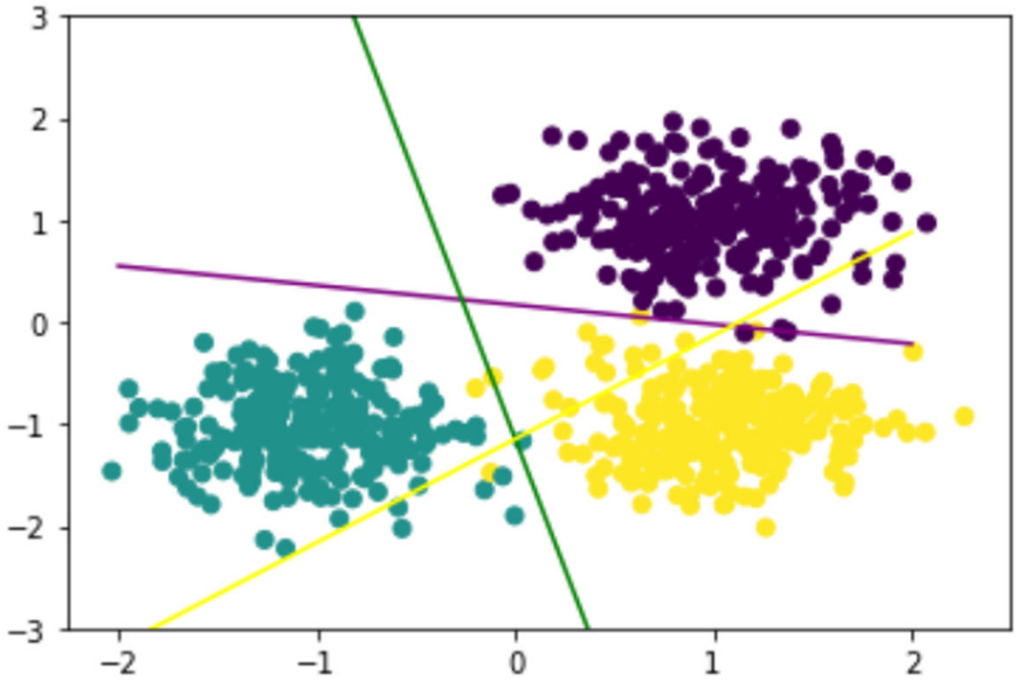
ROC Curve

- A ROC curve plots the true positive rate and the false positive rate
- The acronym ROC stands for Receiver Operating Characteristic.
- We can summarize the ROC curve with the area under the curve. We abbreviate the area under the curve as AUC.



Multiple Categories

- If we have three or more categories, then we can split the classification problem into multiple problems with two categories.
- Each problem try to classify one category versus the other categories. We call the approach One-versus-Rest.



Nearest Neighbors

- We determine the category of the unlabeled records from the categories of the nearest labeled records.
- If we predict categories for many unlabeled records then we can determine the decision boundary