Contents

Dedicated to my mother, 1937 – 2024

Abstract / Summary

The goal of this project was the implementation, using the Python programming language, of a simple numerical, computer-based visualisation of gravitational waves emitted by two orbiting bodies. The simulation is for enhancing the conceptual understanding of certain aspects of general relativity in science teaching and for outreach purposes.

Overview

This documentation introduces some of the background physics for an elementary understanding of the emission of gravitational waves by a system of two bodies, such as a pair of neutron stars. It forms part of my Master's degree in Astrophysics at the University of Zürich.

A secondary goal of this document is to make as much of the theoretical framework and software development details accessible not just to specialists but to those knowing little about either computer science or astrophysics.

Main body of the Text

My text is constructed as follows:

- I begin by introducing the elastic sheet model, which is often used in pedagogical demonstrations of gravity.
- Then I explain the motivation for this project: Why is it interesting and important to use such an idea to model gravitational waves?
- Section 5 outlines the justification for the use of Python for implementing the project.
- I then explain, in the following section, Section 6 how simple physical ideas can be used for calculating the elements (oscillators) which, in the simulation, represent the undulating surface of the sheet.
- Section 7 highlights some numerical methods to animate the sheet surface. Initial parameter choices are touched upon in Section 8 and ways of improving the rendering speed (performance) are explained in the section that follows, Section 9
- Later on, I disucuss some of the more “computer science”-specific concepts, including the creation of GUIs and threading.
- Finally I show the need for using more powerful machines than simple laptops and show how remote computing can harness the power of GPUs to do a lot of the processing, finally providing the opportunity of smooth and fast rendering.
- I wanted Some ideas for stimulating further development of the project are listed in appendix [NOTE: add appendix letter]

On the Appendices

Appendix A – What can General Relativity Explain?
Since it is Einstein's general relativity that predicts the existence of gravitational waves, it may be useful for some readers to know a little about that theory. I describe some of its history and a few observational details, without going into the mathematical formalism, which is (mainly) beyond the scope of this work.

Appendix B – The Python Language
This provides the reader with the minimal knowledge of the Python language required for understanding the software implementation.

Appendix C – The Tkinter GUI
Here, I provide some details of the graphical user interface (GUI) used in the code, which allows control of the parameters needed for the run. I include a short explanation of the code used to create an object called a slider.

Thesis Style

This document adheres to the style guide of the *American Psychological Association (APA)*, Seventh Edition (2020)[1]. https://apastyle.apa.org/products/publication-manual-7th-edition

[1] Four of the other common styles for citing scientific papers, not used here, are MLA Style (Modern Language Association), Chicago Style, IEEE Style and Harvard Style.

1 Introduction

Some aspects of gravity (in particular, those of the general theory of relativity) can be represented by various models. A simple and common physical model that is used for gravity is the elastic sheet. This project is a software implementation of a numerical simulation that recreates such a physical model.

1.1 The Elastic Sheet Model[2]

Figure 1

Elastic sheet demonstration of relativistic gravity by Dan Burns, Los Gatos High School, California, United States

Note. Author's screen shot from the video, https://www.youtube.com/watch?v=MTY1Kje0yLg

In its physical form the elastic sheet model consists of a fabric surface, usually made of rubber or spandex[3], stretched upon a circular or rectangular frame. A metal sphere or an object like a billiard ball or snooker ball is then placed upon the surface and deforms the sheet because of its weight. If additional, smaller, spheres are placed on the surface and given an initial motion in any direction, their paths will become curved due to the distortion of the sheet from the heavier mass or masses. This often results in an (at least partial) "orbit" around the larger sphere. This setup can be used, for example, to approximately demonstrate the gravitational effects of our Sun upon one or more orbiting planets (as the video still of the demonstration, above, shows).

If the model is meant to refer, additionally, to general relativity, the sheet is supposed to represent not space but **spacetime** (three dimensions of space and one of time), the spheres representing (other, much larger!) masses within that spacetime.[4] The greater the mass of spheres used in the model, the greater the local sheet deformation (distortion) around them, hence the greater the masses being represented. The problem is that since the sheet is merely a two-dimensional object embedded in our three-dimensional space, it is limited to representing only two of the four spacetime dimensions!

[2] The technique for modelling a thin elastic sheet is also known as the **thin-shell material model** and is also used in the textile industry for the computer simulation of garments. The requirements for modelling an elastic sheet in my implementation are clearly different from textiles, however; for example, we do not need to simulate environmental factors such as viscosity and wind.

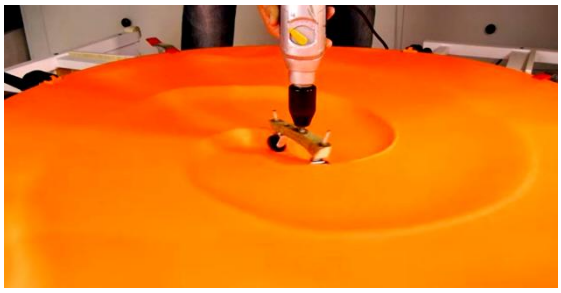
[3] Spandex is a synthetic fabric prized for its elasticity. The term refers to polyether-polyurea copolymer fabrics that have been made with a variety of production processes. The terms *spandex, elastane* and *Lycra* are synonymous. The only one of these, slightly surprisingly, that is a brand name, is *Lycra*.

[4] The Earth's and the Sun's masses are, to two significant figures, $6.0 \cdot 10^{24}$ kg and $2.0 \cdot 10^{30}$ kg, respectively.

Some physical elastic sheet models include features such as the gravitational wave production from two centrally orbiting bodies, for example, Mould (2016).

Figure 2

Mechanical simulation of gravitational waves



Note. Author's screen shot from the video, https://youtu.be/dw7U3BYMs4U

My work extends the use of real-world materials to replicate gravitational phenomena by people like Mould and others, by doing it all within the software. The additional motivation, apart from not needing to construct anything physical, was that my representation was intended to be

- closer to physical reality in several respects
- able to incorporate some useful parameters, alterable either at the beginning, or during the run itself.

Since my simulation was intended to be a representation of reality, it is worthwhile briefly discussing the use of models as a pedagogical tool in science.

1.2 On the use of Models

In physics, a model (or idealized model) is a simplified version of a physical situation that necessarily omits some characteristics that are present in reality. Writing about models used in science communication, Pössel (2018), elaborates: *A (teaching) model … is a concrete or abstract entity which represents* ***some*** [my emphasis] *of the structure … for the purpose of teaching…*

Every idealised model thus represents a **limited set of features** of a physical phenomenon. The skill is in knowing which aspects are "cut out" so that it

- is not a misleading representation
- represents the underlying reality in a useful way.

Models can exist in various forms, in several media. They can be

- physical (including static ones produced by modern 3D printing)
- mathematical
- phenomenological
- visualizations, in the form of a video or computer simulation.

There are several cognitive studies that attempt to reveal how we view models that represent scientific concepts. The age-range of the participants in this research area spans that of school-age children (for example, Baldy, 2007, Postiglione, 2021) to undergraduate students learning general relativity at university (Bandyopadhyay, 2010, Watkins, 2014). One important result of the study by Postiglione, concerning general relativity, was that young people tended to not always understand

that **all** masses deform spacetime, not just very large masses such as planets and stars. In principle, we ought to include everyday objects such as houses, cars, people, cats and the coffee cup sitting on the desk at which I'm writing!

Another important point is that a more sophisticated target audience might make the judgement that a particular model's main features are always more important than those aspects of reality that are under- or unrepresented by it (provided they happen to know, additionally, what those are). But it is important for the educationalist to keep in mind that such an interpretation may not always be valid.

1.3 Why are gravitation and general relativity of interest for a simulation?

Space-time tells matter how to move; matter tells space-time how to curve
John Wheeler

The general theory of relativity was published by Albert Einstein in 1915 (Einstein, 1915) and is a refinement of Newton's law of universal gravitation (Newton, 1687).
It provides a view of space and time that is conceptually and mathematically radically different, however, from the older Newtonian view. Contrary to our intuition, gravity is now regarded less as a force than a **geometric property** of space and time[5] (more accurately, of four-dimensional spacetime). It is the curvature or deformation of spacetime that governs how matter moves within it, as the aphorism by Wheeler quite accurately states.

General relativity is a term with which the public has become more and more familiar with over recent decades. One of the predictions of the theory is that of waves – gravitational waves – moving, at the speed of light, through empty space. The first confirmed observation of such waves was made only relatively recently, however, in 2015. It thus took a century before technology caught up with one of the many predictions contained in the theory and was able to conclusively test it.

1.4 Difficulties of the Theory

General relativity has a reputation for being very difficult;
I think the reason is... that it's very difficult.
Leonard Susskind[6]

Unfortunately, in spite of many decades of effort by practitioners and educators in making the theory accessible to a general audience, general relativity's daunting reputation has hardly diminished since its inception. Susskind has a point!

The obstacles that the theory presents are numerous and (mainly) still valid today:

- General relativity is **conceptually** challenging, featuring abstract concepts like the curvature of spacetime, geodesics and tensors. With the exception of the first term, these are not part of the vocabulary of most people!
- It uses **unfamiliar notation** (something called the Einstein summation convention), which has to be mastered early on.[7]

[5] Although this is hardly consoling for anyone falling to the ground from just a few metres!
[6] https://www.youtube.com/watch?v=cyW0LWEACfI

- It requires advanced undergraduate-level understanding of several branches of mathematics, in particular of **differential geometry.**
- Finally, once the theory has been grasped – by working through and understanding the derivations leading from a simple curved 2D surface all the way to the so-called **Einstein field equations** and to finally obtaining a useful equation called the **Schwarzschild metric** – there is a new learning curve. This occurs when we want to apply the theory to examine real astrophysical situations. At this "final" stage, one of the additional techniques to learn and apply is called "numerical relativity", which makes use of modern computing techniques to solve these types of problems.

At first glance these issues would appear insurmountable for science communicators wishing to explain the theory to a broad segment of the public. However, many physical features of general relativity can indeed be presented in a simple and phenomenological (qualitative or semi-quantitative) way. This is done by using models, which brings us full circle to the start of this section.

2 On the Choice of Programming Language

Having decided on the **choice of model**, the next stage of planning was to determine **which programming language** (or languages) the simulation would be implemented in. My own area of expertise has been mainframe computer languages used in business.[8] Mainframe programming languages such as COBOL and PL/1 are clearly unsuitable, however, not least because the implementation has to be able to run on modern devices such as laptops. The world of object-oriented programming seemed enticing but presents too steep a learning curve for an MSc project, for someone with no previous knowledge of such methods.

Python was designed in the late 1980s by Guido van Rossum at the CWI (Centrum Wiskunde & Informatica) and developed there by him and others. It was first released to the public in 1991. It is an open-source, high-level[9], object-oriented, programming language. However, as the previous paragraph makes clear, I do not explicitly use any of Python's Object-Oriented (OO) capabilities.

Additionally, Python is an interpreted language, which means that there is no compilation step: The advantage of this is a reduction of the timescale for each iteration of any edit-test-debug cycle.

[7] Other notational schemes exist, but do not necessarily make life easier for the student, as Susskind emphasises at the beginning of his recorded video lectures on general relativity.

[8] Before becoming a secondary school teacher, I used to work at some of the larger banks, both in London and in Zürich.

[9] A high level language is a programming language that enables a fairly straightforward implementation of a sequence of ideas (algorithm) into computer code. High level languages allow the writing of programs which are almost independent of the particular type of computer used to run them. Examples, apart from Python, include Fortran and Pascal. In contrast, a low level language is "machine-oriented" because the code must be specified in terms of the capabilities and specifications of the processor. Examples include assembly language and machine code.

It was decided then, provisionally, to implement the software using Python. The initial cost-benefit consideration was:

- Pros:
  - straightforward to learn
  - relatively uncomplicated to code and
  - simple to read (bearing a closer similarity to written English than most alternatives)
- Cons:
  - lack of speed (or the reputation thereof)

At this stage, the question as to whether Python alone would offer sufficient performance was left unanswered. Only experience, rather than calculation, could provide enlightenment.[10] Thus, alternative languages, either for all or part of the implementation, could not yet be ruled out at this exploratory stage.

In this work, I avoid using advanced Python features, in line with my philosophy of pedagogical simplicity. Concepts such as lambda functions [NOTE: check this], generator functions and Object-Oriented Programming (OOP) ideas such as classes, inheritance and polymorphisms are avoided.

Further details can be found in Appendix B.

3 Initial Development Ideas, using basic Physics

In secondary schools, one particular physical model sometimes used to visualise waves in physics lessons is called, simply, a “wave machine”. It consists of a series of connected rods, each of which is linked to a spring. The springs are coupled to allow the oscillations of one rod to influence adjacent rods, creating a chain reaction that illustrates wave propagation. Several modules can be linked serially, to produce a longer set of oscillators for the simulation.

Figure 3

A wave machine used at some secondary schools.

Note. Source https://www.leybold-shop.com/wave-machine-basic-module-1-40120.html

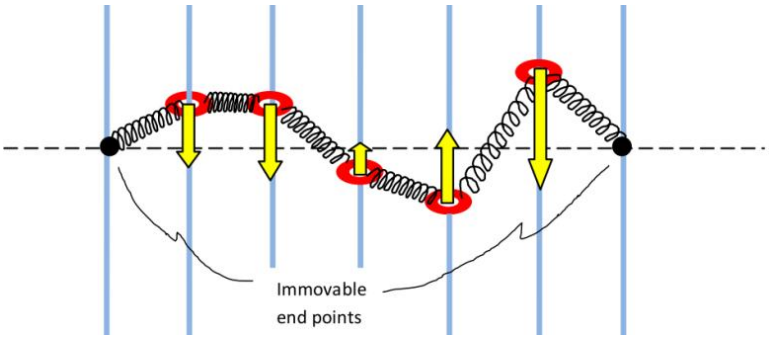
I wanted to create a software implementation of such a system of coupled oscillators, producing waves in one dimension, as proof of concept. My toy model (Gedankenmodell) consisted of several

[10] One idea at the time was to use Python for the front-end user interface and image dispay and the much faster Fortran language to deal with the more calculationally intensive aspects of the implementation. However, this became unnecessary once another solution had been found.

rings or beads able to move freely and frictionlessly[11], in the *y*-direction (up and down). These were connected by identical springs. The forces, qualitatively, are represented by the yellow arrows. See figure 4.

Figure 4

First concept of coupled oscillators, 1D.



Note. Author's figure

A quantitative result is then obtained by calculating the force on each ring/bead, by considering the sum of the two adjacent spring forces via Hooke's law. The force (along the y axis) is given by

$F_i = F_{i,i+1} + F_{i,i-1}$ **(1)**

where $F_{i,i+1}$ represents the force on bead/ring *i* due to the next one, *i + 1*, and $F_{i,i-1}$ that of the previous one.

Using Hooke's law then gives:

$D \cdot \Delta\ell_{\,i,i+1} \cos(\theta_{i+1}) - \Delta\ell_{\,i,i+1} \cos(\theta_{i-1})$, **(2)**

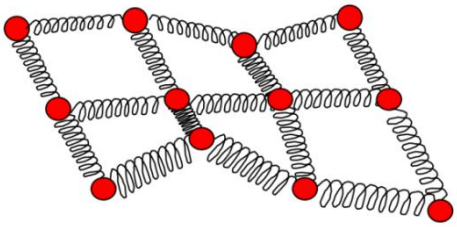
where:

- *D* is the spring constant,
- $\Delta\ell$ is the length extension or contraction of the spring between *i* and *i+1*, which is assumed to behave symmetrically in terms of the magnitude of force produced, regardless of whether the result of a contraction or expansion,
- θ is the angle between each spring and the vertical (*y*) axis.

Once this model had been coded (in Python) and tested, it was extended from one dimension to two. Conceptually, when "viewed from diagonally above", a small part of the elementary grid of coupled oscillators would resemble figure 5.

Figure 5

Concept of coupled oscillators, 2D



Note. Author's figure

[11] No other forces, including gravity!

By creating a two-dimensional grid of oscillators, I had now achieved, however rudimentarily, a software representation of the elastic sheet mentioned in section 1.1. The difference between the two was that my model was discretised (a mass concentrated at each coordinate point of the grid) rather than being a continuous, smooth entity. However, provided the sheet resolution (in effect, the grid size along each of the two axes) were sufficiently large, the elastic sheet would be well approximated in my representation also.

Since the number of force calculations, according to this discussion, is N_{grid}^2 we would, for a 200 * 200 grid, obtain 40 000. If we wished to render the movement of waves along our surface at, say, 25 fps (frames per second), we would require one million force calculations each second. This is no challenge for any modern computer (even my laptop machine). The problem now lies with the implementation language. As an interpreted language, Python runs significantly slower than all lower-level, compiled languages (like C/C++), mainly because of the "overhead" of its interpreter.

Calculating forces alone would not be sufficient to produce any meaningful movement of the oscillators making up the sheet surface. The additional information required about them is

- acceleration
- velocity and
- displacement.

3.1 Hooke and Newton[12]

The extension and movement of the oscillators at any particular instant in time can be calculated by using a combination of **Hooke's law** and **Newton's second law**. Hooke's law states that the force due to the deformation (change of extension) of any ideal (perfect) spring is given by

$$F = -k\Delta s, \qquad (3)$$

where k is the spring constant[13] and Δs is the distance the spring is stretched or compressed. The variable F here is called the restorative force. Its direction is always opposite to the direction of the spring's displacement Δs (hence the moniker, restorative).

If the sheet is required to be undulating in some fashion in order to produce transverse waves (like water waves), then the freedom of movement for the oscillators could be chosen to be, for example, exclusively in the vertical direction (along the z-axis). The basic equation relating acceleration and displacement for any oscillator would then be given by

$$dv_z/dt = -kz/m \qquad (4)$$

The combination of equations (3) and (4) for our coupled oscillators leads to a first order differential equation (ODE). The vast majority of these kinds of ODEs [NOTE: more specific] cannot be analytically solved ("solved by hand"); their implementation has to be done numerically, on a computer, which is, in effect, the main point of this project. The key idea for the calculations is to perform something called an **iterative temporal discretisation.** [NOTE: add to glossary and link] Unlike in pure mathematics, our computer code cannot truly use infinitesimals, so we have to replace ***dt*** with the interval ***Δt***.

[12] Ironically for the title of this subsection, Newton and Hooke, who were contemporaries and knew each other, didn't get along at all.

[13] This k is not to be confused with the same symbol used for the slopes in the Runge-Kutta method, which will be discussed shortly.

The aim is then to end up with as small a value for this time interval as possible, whilst ensuring that the animation still runs at a visually satisfying speed.

4 Initial Mathematics – Numerical Integration Methods

There is a choice of several different numerical methods that can be applied to our problem of coupled oscillators. Each method has its own strengths and weaknesses.

4.1 Euler (first order accuracy)

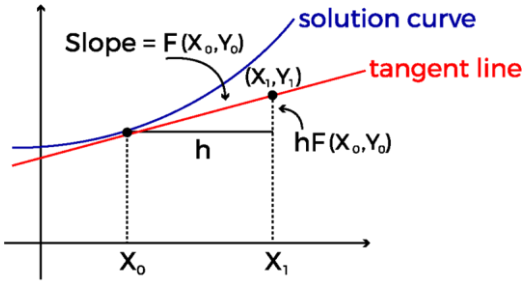
It is used to update both position and velocity (or other state variables) simultaneously.
Update of position: $x_{n+1} = x_n + v_n \Delta t$
Update of velocity: $v_{n+1} = v_n + a_n \Delta t$

Energy Conservation: The Euler method generally does not conserve energy well, it can lead to a drift in the total energy of the system over time.
Momentum Conservation: It also does not consistently conserve momentum.

The Euler method uses a first-order approximation for both position and velocity updates, which can lead to numerical errors accumulating over time.

Figure 6

Graphical representation of the Euler method



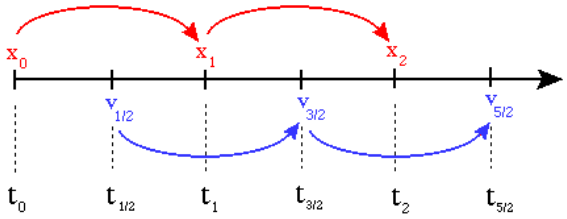
Note. Image source:
https://calcworkshop.com/first-order-differential-equations/eulers-method-table/

4.2 Leapfrog (second order accuracy)

The method involves calculating the position and velocity at discrete time steps with a "leapfrog" pattern:

Figure 7

Graphical representation of the Leapfrog method



Note. Image source:
http://cvarin.github.io/CSci-Survival-Guide/leapfrog.html

The steps are

1. Update of position: The position is updated at integer time steps using the (updated) velocity from the previous step.

$$x_{n+1} = x_n + v_{n+1/2}\Delta t$$

2. Update of velocity: The velocity is updated at half-integer time steps.

$$v_{n+3/2} = v_{n-1/2} + a_n \Delta t$$

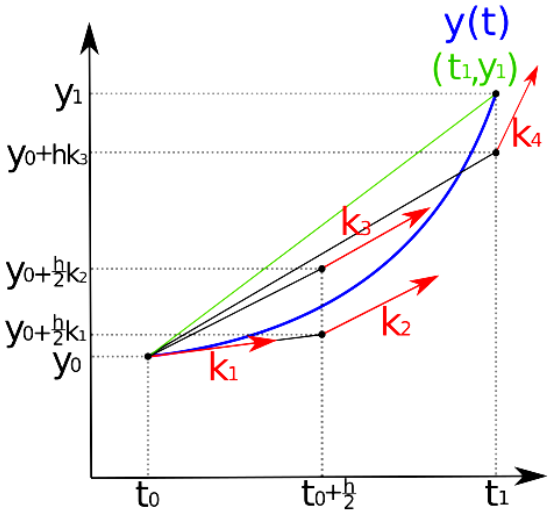
Energy Conservation: Leapfrog integration is known for conserving energy better than many other methods. This is because it symmetrically updates positions and velocities, viz.: Leapfrog integration maintains the time-reversal symmetry of the system. This helps in preserving the total energy over longer simulations, reducing the numerical drift in energy that is common in other methods.
Momentum: This is also well conserved.

4.3 Runge-Kutta 4th-Order Method (RK4)

This method produces solutions to ordinary differential equations using a fourth-order approximation.

Figure 8

Graphical representation of the RK4 method



Note. Image source:
https://lowebms.readthedocs.io/en/latest/_images/RK4.png

It computes four slopes, that are used as intermediate values. Generally, an indexed letter ***k*** is used (as referred to previously, in footnote 13):

- $k_1 = f(t_n, y_n)$
- $k_2 = f(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} k_1)$
- $k_3 = f(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} k_2)$
- $k_4 = f(t_n + \Delta t, y_n + \Delta t k_3)$

The solution is then calculated using:

$$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

We can see that the computational requirements for the RK4 method are significant: four function evaluations per time step compared to Leapfrog's two. Accuracy comes at a cost!

Energy conservation: RK4 does not inherently conserve energy.
Momentum conservation: Not fulfilled either.

5 Optimal Parameter Choice

For this project it was important to obtain waves that could be accepted as being sufficiently "wave-like" in appearance. Several parameters needed to be arbitrarily "fine tuned" during the code development for this to be the case. The following parameters were of this type.

- Those required to be minimized:
  - Stepsize (integrative timestep).
- Those required to be maximized:
  - grid resolution size.
  - elastic constant for the coupled oscillators (same value for all).
  - initial elastic extension for the coupled oscillators (same value for all).

Although the point of the user interface (GUI) was to allow the user to change some parameters, because of the above restriction, the option of setting/altering them via the GUI was precluded; instead, they would be hard-coded.

Comparing the integration methods Leapfrog and RK4, it was found empirically that Leapfrog was more unstable (more often prone to growing instabilities – large "spikes" on parts of the grid surface growing without limit) than RK4 for

- large values of the elastic constant, of order 10^8 and above and/or
- values of the timestep of and above order $10^{-6.}$

These empirically-determined values were not universal, however; they only applied to this project.

6 Speeding Things Up – Initial Attempts

6.1 Vectorisation and Numpy

Since one of the key strengths of computer software is the ability to repeat tasks in the form of programmed loops, it was here that ongoing optimisation was searched for. A useful technique, for large datasets (data aggregates), is to perform operations on entire arrays instead of working with one element at a time. This means that fixed data types and contiguous (adjacent) memory allocation must be enforced (implicitly, "under the hood"). This technique allows the avoidance of explicit looping and thus enables faster processing. It is known as either **vectorization,** vector computing, or array computing.[14] **NumPy** is the extension module (library) for Python which provides just such functionality.

[14] Looping still takes place, but is relegated to optimized, pre-compiled C code which "lies beneath" the code seen by the programmer.

It was anticipated that the use of NumPy would thus address the performance issues that were already beginning to arise during the development-test cycles. Implementing and running Python with Numpy did help, at least for simple, relatively small arrays. However, it soon became clear that other methods, libraries or interfaces to other (faster) languages would need to be used in the next stage of development.

6.2 Mayavi

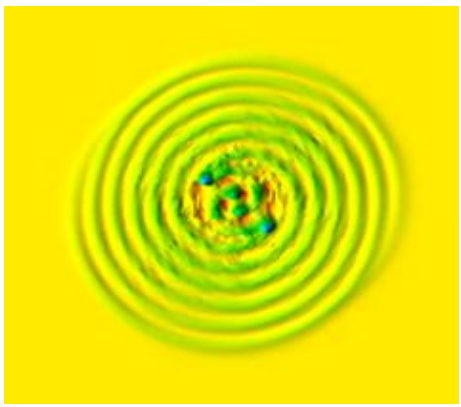
Mayavi is an open-source, cross-platform, 3D scientific data visualization package. It was created in 2001 by P. Ramachandran. Due to the fact that

- its interface can visualise the 3D data described by NumPy arrays[15] and
- since it uses openGL for its rendering[16],

Mayavi, working together with Python, seemed the obvious choice at this stage. However, although the results, visually, looked quite pleasing[17], as the example given by figure 6 shows, frame rates were too low to be useful for a good animation.

Figure 9

Early screenshot using Python with the Mayavi package.



Note. Author's image

7 Implementing a Graphical User Interface (GUI)

In order to allow more control of the animation, it was necessary to implement a Graphical User Interface. This would allow some parameters to be set by the user either at the beginning and/or during the run ("on the fly").

7.1 Tkinter

Tkinter[18] is Python's standard GUI and is probably the simplest way to develop visual elements for Python[19]. I chose it for this reason. Two other noteworthy GUIs for Python are called wxPython, and PyQt; these are not discussed further in this work.

[15] As, incidentally, can Matplotlib. Unfortunately, Matplotlib cannot access GPUs to increase its performance.

[16] This is because it uses something called the **Visualization Toolkit (VTK)** for its graphics, which itself uses openGL.

[17] A particularly nice feature of Mayavi is its sophisticated GUI which allows rotation of viewing field of view in both directions, as well as a zoom option.

[18] *Tkinter* comes from *Tk interface*. It was written by Steen Lumholt and Guido van Rossum (once again!).

[19] It is cross-platform, so the same code works on Windows, macOS, and Linux.

The interface provides a way to create GUI components (often referred to as widgets)[20] such as

- windows,
- sliders,
- buttons,
- labels and
- text boxes.

This project uses all of these.

The name "**Tkinter**" comes from "**Tk interface**", referring to the Tk GUI toolkit that Tkinter is based upon.

NOTE: update this section

The command **root = Tk()**
Tk() is a **class** in Tkinter that represents the main window of the GUI application.
The command root = Tk() initializes a Tkinter window, which serves as the main container for all other widgets (buttons, labels, frames, and so on).
Starting Tkinter
This is done by running the command root.mainloop(). This starts an infinite loop which is used to run the application and act upon user input, as and when it occurs. In computer science, something that indicates a user interacting with a program is called an **event**.
See **Appendix C** for further details, including one code example.

8 Threading in Python

8.1 Overview

In computer programs, threads allow the performing of multiple tasks concurrently within a single process, sharing resources such as memory and what are called file handles. Each thread is a separate flow of execution[21].

Multiple threads within a process share the same **data space** with the main thread and can therefore share information or communicate with each other more easily than if they were separate processes. Python threading enables different parts of the program to run concurrently: When we run a Python script, it starts an instance of the Python interpreter that runs our code in the main thread. The main thread is the default thread of a Python process.

A thread has a beginning, an execution sequence, and a conclusion. Three useful commands are

- threading.activeCount(), which returns the number of thread objects that are active.
- threading.currentThread(), returning the number of thread objects in the caller's thread control.
- threading.enumerate(), which gives a list of all thread objects that are currently active.

[20] A widget is also a device placed in an aluminium container of beer to manage the characteristics of the beer's head. The original widget was patented in Ireland by Guinness.

[21] However, the different threads do not actually execute at the same time: they merely appear to do so.

The Python standard library module, **Threading,** allows the creation and management of threads. Methods provided are

- run() gives the entry point for a thread.
- start() starts a thread by calling the run method.
- join([time]) waits for threads to terminate.
- isAlive() checks whether a thread is still executing.
- getName() returns the name of a thread.
- setName() sets the name of a thread.

8.2 Motivation

The reason that threading had to be explicitly programmed is because the tkinter GUI runs in an infinite loop, once started. However, once the GUI is visible (and parameters have been adjusted by the user, if required), we also need to start at least one other part of our script, dealing with:

- the repeated computing of the oscillator positions in time, and
- the rendering of the resulting surface as an animation on our computer screen.

The GUI remains displayed during the whole animation sequence, because some of its parameters are designed to be modifiable "on the fly" by the user. Sensible choices had to be made about the positions and relative sizes of the GUI root window as well as for a small supplementary display window [NOTE: edit this] (more on that later), especially as all three were designed to be displayed at the same time. An early decision was to set the code to keep both the GUI and the information windows fixed (non-moveable) and non-resizeable.

8.3 Daemon Threads

In Python, **daemon threads** are threads that run in the background and cannot block the program from ending even while they are still running. Any daemon threads will shut down immediately when the program exits. This project does not use them, but I mention this aspect of threading here for completeness.

8.3.1 Code Snippet Example

Code	Explanation
`from threading import (Thread,`	Allows the creation of Thread objects later in the code.
`Event,`	Allows synchronization between threads so that one or more threads can wait until they are notified that an event has occurred.
`…)`	
…	
`simulation_thread = Thread(target=main_simulation_loop)`	Create the thread object called simulation_thread.
`simulation_thread.start()`	Once a thread object is created, its activity must be started by calling

	the thread's start() method.
…	
`def main_simulation_loop():`	
…	…other lines of code that form part of the definition of the code block.

8.4 Sharing Data between Threads

An important feature of threads is that they can exchange data, but do not have to do so. Sharing of data between my running threads became a necessity, once the decision had been taken to implement an active, rather than a passive, GUI (i.e. responsive to any permitted user input, at any time during the animation run).

If the same fields of data is to be accessed by more than one thread, it needs to be ensured that a thread **race condition** does not occur**.** When two or more threads access shared data concurrently, the outcome of their execution may be unpredictable. The operating system can swap which thread is running at any time. Any updating of shared data resources is then not guaranteed to work because one thread could be reading a variable exactly at the same time as another is attempting to update it. In fact, the situation is worse than that because even a simple operation (such as x += 1) takes the processor several steps (separate processor instructions) to execute, and a thread could be "swapped out" between the execution of any of these instructions!

Python avoids the race condition by means of something called a **Lock**. The way this works is illustrated in the example code snippet.

8.4.1 Code Snippet Example

Code	Explanation
In the main code, global scope	
`from threading import (…,`	We now consider only the parameter Lock in the import statement.
`Lock)`	Used to ensure that only one thread accesses a shared resource at a time. This prevents multiple threads from interfering with one another (race condition).
…	
`shared_data = {`	The data that will be shared between two or more threads.
`'lock': Lock(),`	Used to ensure that only one thread accesses a shared resource at a time. This prevents multiple threads from interfering with one another (race conditions).
`'running': False` `}`	We set our default to Boolean false, so that it must be explicitly set to True in the code.

In the main loop	
`loop_end_time = time.time()`	We use the system time to compute the

`loop_duration = loop_end_time - loop_start_time`	loop run time.
`if loop_duration != 0:`	
`fps = 1/loop_duration`	Check for divide by zero.
`else:`	
`fps = 1`	
`loop_start_time = loop_end_time`	Reset the variable.
`...`	
`with shared_data['lock']:`	Only allow current thread access to the shared data.
`shared_data['running'] = True`	Check that the data is available.
`shared_data['fps'] = fps`	**Assign** the newly calculated variable to the shared data structure.

Function code block	
`def update_info_window(root):`	Function definition
`if shared_data['running']:`	Only allow current thread access to the shared data.
`with shared_data['lock']:`	Check that the data is available.
`elapsed_time = time.time() - shared_data['start_time']`	Do the calculation.
`fps = shared_data['fps']`	Take the value of the variable, which was updated in another thread, from the shared data.

9 Speeding Things Up 2

9.1 Taichi

This programming language[22] provides a way for Python to execute high-performance numerical and graphical computations. It was created by Yuanming Hu at Tsinghua University in Beijing in 2018. Taichi uses almost the same syntax as Python.

Taichi function blocks are differentiated from those of native Python by two types of decorator and everything within such code blocks then has a Taichi scope.

@ti.kernel

- Several kernels, unnested, may exist in the code: All kernels are mutually independent of each other and are compiled and executed in the same order as they are first called.
- They are called directly by Python code (i.e. from the Python **scope**) and serve as an entry point for Taichi.

[22] Not to be confused with *tai chi,* the ancient Chinese martial art in which practitioners perform a series of deliberate, flowing motions while focusing on deep, slow breaths.

- Kernels must take type-hinted arguments and return type-hinted values.

@ti.func

- Taichi functions are to be called by either kernels or other Taichi functions: nested functions are allowed.
- Functions do not have to take type-hinted arguments and return type-hinted values.

At the beginning of a Python program execution, Taichi performs a so-called just-in-time (JIT) compilation, which translates Taichi code into optimized machine code.

Taichi represents a mesh by two separate arrays, so that the triangle connections are managed independently of the actual vertex positions.

- an **Index Array** (or Connectivity Array). This holds sets of three indices which define sets of triangles
- a **Vertices Array**, holding the coordinates of the vertices in space in the form of a 3D vector.

10 Further Physics – The Orbiting Bodies

10.1 Orbital Angular Speed of two Orbiting Masses

We know from Newton's universal law of gravitation, that the force between two masses M_1 and M_2 separated by a distance r is given by:

$$F_G = \frac{G M_1 M_2}{r^2} \qquad (1)$$

where G is the gravitational constant (Newton's constant). The centripetal force between them is

$$F_C = \frac{M_1 v_1^2}{r_1} = M_1 \omega_1^2 r_1 = M_2 \omega_2^2 r_2 \qquad (2)$$

The centre of mass is defined as the point where the weighted relative position of the masses balances,

$$x = \frac{\sum (m_i x_i)}{\sum m_i}$$

So $x_{BARYCENTRE} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$ becomes, on moving the pivot point to the centre of M_1,

$r_1 = \frac{M_2 a}{M_1 + M_2}$, where a is now the distance between the two bodies.

Using this, $M_1 \omega^2 r_1$ becomes:

$M_1 \omega^2 \frac{M_2 a}{M_1 + M_2}$, which we know from (1) and (2) also equals $\frac{G M_1 M_2}{r^2}$

Hence,

$$\omega^2 = \frac{G M_1 M_2}{r^2} \cdot \frac{(M_1 + M_2)}{M_2 a M_1} = \frac{G(M_1 + M_2)}{a^3} \qquad (3)$$

We can, in our animation, now incorporate what I shall henceforth call the „astrophysical omega" in our information display window. This omega is based upon the scaled-up (from the grid „toy model universe" to reality) values of the two bodies' masses and their physical separation distance. The value of G used in the software, is, of course, the real value of Newton's constant as determined empirically.

10.2 Power of the Gravitational Waves radiated by a Binary System

$$P = \frac{G^4}{c^5} * \frac{32}{5} \left(\frac{M_1 M_2}{M_1 + M_2} \right)^2 a^4 \Omega^6.$$

This can also be regarded as the "gravitational luminosity.[23]

[23] An equivalent formulation is

$$P = \frac{G^4(m_1 m_2)^2(m_1+m_2)\omega^6 r^6}{c^5}$$

10.3 Orbital Decay

The equation for this, without derivation, is

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{(m_1 m_2)(m_1 + m_2)}{r^3}$$

where r is the separation between the bodies, t time, G the gravitational constant, c the speed of light, and m_1 and m_2 the masses of the bodies.

$$\dot{a} = -\frac{64}{5}\frac{M_1 M_2 (M_1 + M_2)}{a^3}$$

10.4 Reducing Reflections

In any numerical simulation of waves, the behaviour of the oscillators at and near the grid boundary is problematic. This is because, in general, waves are reflected from all such borders.[24] This behaviour is usually undesirable (unless reflections are intended to be modelled also). In this work, such artifacts are especially undesirable, as it is physically unrealistic to expect reflections of waves (gravitational or otherwise) to be observed in empty space!

One simple technique to reduce border reflections in computer simulations is to code some form of viscous damping.

10.5 Viscous Damping (Dashpot Damping)

In real-life mechanical systems a dashpot is a device that provides damping by creating resistance to motion via a fluid, typically oil, using a piston or cylinder. Since the resistance is proportional to the velocity of the moving component, the damping force increases with the speed of motion. This was the idea for my coded dashpot damping.

We extend the equation of motion like this:

$$m \frac{d^2x}{dt^2} = -kx \boxed{- c \frac{dx}{dt}}$$

where, as usual,
m is the mass,
k is the spring constant,
x is the displacement and, now, additionally,
c is the damping coefficient (related to the dashpot).

My implementation then imposes a reduction of both the velocities and vertical (height) positions of the oscillators inside the specified boundary regions, with the intensity of damping increasing as each of the four edge boundaries of the grid is approached.

The mathematics for attempting to remove reflections at simulation boundaries is (at least, for me) quite formidable. I provide three references merely to provide a flavour of the mathematics and style of argumentation. Upon researching this topic, I decided that an intuitive, empirical approach was the more straightforward – and, it turns out, still quite effective – way to proceed.

The boundary damping factor found to be optimal in the implementation, assuming a number of damping layers proportional to the grid size (integer division: grid_size//20) turned out to be around 0.05. Clearly, a larger fraction of the grid could always be used for this, but the value chosen (in effect, one-tenth of the grid extent along each of the two axes) does not seem excessive. Utilising too many layers to reduce the reflected waves would negatively impinge upon the model's overall appearance. (We were not attempting to show an animation of attenuated gravitational waves!).

[24] This spurious reflection, caused by inaccurate treatment of the artificial boundaries, is not caused by the finite precision of the simulation (unlike the discretization errors in general).

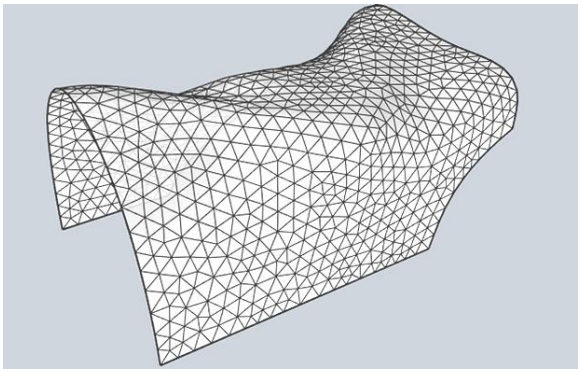
The same damping factor and depth of layers was used for reducing both the velocity and the position vectors of the oscillators near the borders.

11 Triangle Meshes

A mesh of connected triangles, like in **Figure 7**, can be used to approximate any surface. Such a mesh is used to represent and render the virtual elastic displayed by my simulation.

Figure 7

A triangle mesh, representing an arbitrary 2D surface.



Note. Source https://discourse.mcneel.com/t/convert-mesh-into-equilateral-triangles/107141/18

Rendering is done by calculating the vertices of the triangles and then shading the surface obtained.

12 Final Remarks

[NOTE: Summary of work,
goals achieved, not achieved,
future enhancements,
overall difficulty or ease of the project.]

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Appendix A – What can General Relativity Explain?

In this section, I provide an outline of general relativity and an overview of some of the observations in astronomy and cosmology for which the theory provides explanations. The experimental verification of gravitational waves in 2015 has now led to a field called **gravitational wave astronomy**, providing a new "window of information" about our cosmos.

What is General Relativity

General relativity describes how mass concentrations distort the so-called spacetime (our three dimensions of space plus the single dimension of time) around them – and how this distortion affects other objects in their neighbourhood.

Due to the immense scales involved, far beyond those of our everyday experiences on Earth, general relativity excels at explaining large-scale astronomical phenomena. One example is an effect known as **gravitational lensing** by (and of) galaxies and groups of galaxies.

Even at the comparatively smaller scale of our Solar System, it outperforms its "competitor," Newton's theory of gravity, in explaining certain phenomena. An example is the deflection (bending) of starlight passing close to our Sun, as viewed from Earth. This was first confirmed by observations of a solar eclipse in 1919 at made at Sobral, Brazil, and Principe, West Africa[25] .

Figure A1

*An **Einstein ring** created by gravitational lens LRG 3-757*

Note. Source: https://apod.nasa.gov/apod/ap111221.html

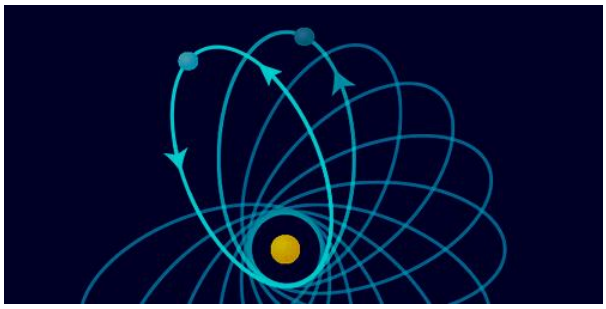
Another observation within our Solar System is the precession of the elliptic orbital shape of the planet Mercury around our Sun and (some of) the physics of black holes, where a massive star is

[25] The 1919 expeditions (Dyson et al., 1920) are further discussed in Coles (2001) and Gilmore et al. (2020): The latter study reconfirms both the integrity and scientific objectivity of Eddington and the validity of the data obtained at that time. It is unfortunate that Stephen Hawking erroneously states in his 1988 bestseller *A Brief History of Time* (Hawking, 1988) that " … later examination of the photographs taken on [the 1919 solar eclipse] expedition showed the errors were as great as the effect they were trying to measure. Their measurement had been sheer luck … ". This statement is untrue.

collapsing “forever” inward and no longer allows anything, not even light, to escape from its "surface". Indeed, the very idea of a surface has to be carefully defined for such a system.

Figure A2

Cartoon depiction of the precession of the orbit of the planet Mercury around the Sun



Note. Not to scale; distances and angles highly exaggerated.

On the largest scales of all – cosmological scales – general relativity **predicts** the expansion of the Universe itself. This was not the prevailing view when Einstein published his paper in 1915, as there was no observational evidence to support it at the time. Furthermore, the incorrect static model would have been favoured by scientists on the additional usual and useful grounds of simplicity and elegance, compared to any "dynamical" alternative (expansion or contraction). This erroneous thinking led Einstein, in 1917, to insert an additional term into his theory called the **cosmological constant**, to allow for just such a static Universe. When, a few years later, it was observationally confirmed by Edwin Hubble (1929) that the Universe is expanding after all[26], Einstein called this addition his “biggest blunder”. It appears that this story is not apocryphal but true (O’Raifeartaigh et al. 2018); one can easily imagine the great physicist’s frustration upon realising that he had missed the opportunity of using the power of his own theory to correctly predict an as yet unconfirmed feature of the cosmos!

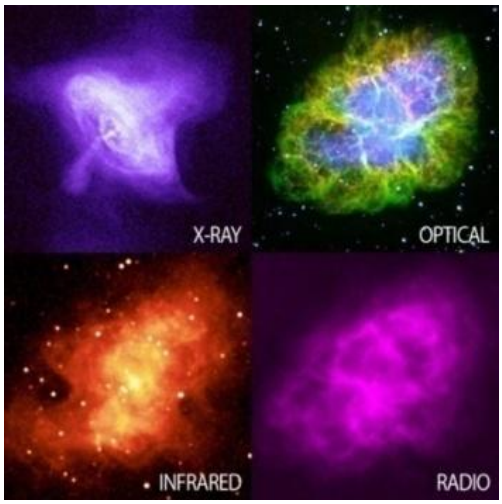
Since the invention of the telescope in the Netherlands, probably in 1608, and Galileo’s famous use of his own improved versions in Italy over the following years (more primitive, by today’s standards, than many children’s telescopes), astronomers have been able to more and more effectively to make use of the information contained in the electromagnetic radiation emitted by sources in the sky. Indeed, the twentieth century saw the construction, for the first time, of telescopes which could produce images using wavelengths other than the visible part of the spectrum, beginning with the longer wavelengths like radio waves – in the 1930s – and subsequently using the infrared, ultraviolet, X-ray and gamma ray regions aswell. However, since the Earth’s atmosphere prevents nearly all wavelengths shorter than visible light arriving at its surface, observations in these wavebands cannot be made from ground-based telescopes but only from space. For these observations the equipment is placed aboard rockets or on Earth-orbiting satellites.

[26] Swedish astronomer Knut Lundmark appears to have been the first person to find observational evidence for the expansion, in 1924. Three years later, Belgian priest and cosmologist Georges Lemaître presented somewhat more exact observational evidence, correctly stating that the relationship between distance to galaxies and their recessional velocity is a linear one. Edwin Hubble confirmed their findings in 1929 and it is generally he who is credited with the observations confirming the Universe’s expansion. (Modified from Wikipedia.)

A well-known example of an astronomical object imaged at different wavelengths is the Crab nebula, the remains of a supernova explosion in our Galaxy recorded by Chinese astronomers in 1054.[27] It can no longer be seen with the naked eye, as its apparent brightness in the sky is around nine times too faint, but it can be seen as a small faint patch of light using a good pair of binoculars.[28] Each of the four images in the figure below contains different information about the nebula. Since humans cannot, in principle, perceive colours beyond the boundaries of wavelengths visible to our eyes, non-optical images in astronomy usually employ "false colours", to represent signal intensity (and to look aesthetically pleasing).

Figure A3

The Crab nebula in different energy regimes (wavelengths)



Note. Source https://ecuip.lib.uchicago.edu/multiwavelength-astronomy/x-ray/impact/07.html

Gravitational Waves

All masses have a constant gravitational field in the region around them. According to general relativity, however, masses that are accelerating produce something in addition: the emission of something called gravitational waves.[29] Accelerated masses produce these waves in a similar way to **accelerating charges** producing **electromagnetic** waves. For example, electrons rapidly oscillating (moving back and forth) along the length of an antenna emit electromagnetic radiation – photons – with wavelengths in the radio band of the spectrum; a radio transmitter.

Both types of energy, electromagnetic and gravitational, can travel through the vacuum of empty space. They do so at the speed of light ($3{,}00 \cdot 10^8$ m/s).

[27] There is no recorded evidence that it was observed from other parts of the world, not even in Europe.

[28] Its so-called **apparent magnitude** in the sky is 8.4. Since the astronomical magnitude scale is **reverse logarithmic** (the brighter an object is, the lower its magnitude number; a difference of 1.0 in magnitude corresponds to a brightness ratio of 2.512) and assuming that the limit of naked eye perception is (in best nighttime observing conditions) magnitude 6.0, the difference of 2.4 magnitudes works out as a brightness factor difference of around $2.512^{8.4 - 6} = 9.12$. This means that the Crab nebula is nine times too faint to be seen with the unaided eye.

[29] Provided that this acceleration is not perfectly spherically or cylindrically symmetric.

As it turns out, general relativity is not the only theory of gravity that predicts gravitational waves; any relativistic theory of gravity will predict them.[30]

On the difficulty of their detection

There are several reasons why gravitational waves (as opposed to gravity itself) are extremely difficult to detect (let alone, measure):

- Their interaction with matter is very weak. The gravitational waves emitted by the movement of everyday objects on the Earth (people, cars, planes) are far too small to be measurable. Indeed, the extreme weakness of gravitational waves makes detection difficult even for sources that are large and massive because the amount of energy emitted in this form by objects is generally extremely small. For example, the power of gravitational waves emitted by a pair of masses m_1 and m_2, orbiting around their common centre of gravity (barycentre) in circular paths of radius r is given (here without derivation) by the following equation:

$$P = \frac{-32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5}$$

  Taking numerical values of Newton's constant G and the speed of light c, to three significant figures, we have the values:
  $G^4 = (6.67\cdot10^{-11})^4 = 2.02\cdot10^{-41}$ and
  $c^5 = 2.43\cdot10^{42}$.
  Combining these gives us an order of magnitude of $\mathbf{10^{-83}}$ for the proportionality.

  In order for any significant power to be produced by the emitted waves, the equation tells us that we need truly enormous masses, together with a relatively small orbital radius, in order to "compensate" for this extremely small parameter.

- Gravitational wave detectors measure the "distortion" of space (more precisely, spacetime), but significant energies in the sources are required to produce such distortions in their vicinity. This is illustrated as follows. The "stiffness of spacetime", expressed as the Young's modulus, can be determined, classically, using the equation

$$\frac{F}{A} = \frac{Y\Delta\ell}{\ell}$$

$$[Y] = \frac{F}{A}$$

  Again using G and c, but this time using dimensional analysis rather than taking their numerical values, and additionally incorporating frequency, f, we obtain

$$[Y] = \frac{F}{A} = \frac{N}{m^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} = \frac{(LT^{-1})^2 \cdot T^{-2}}{M^{-1}L^3T^{-2}}$$

[30] Several attempts were made, prior to 1915, of constructing such theories, notably by Heaviside, Poincaré and Nordstrom. However, none of these alternatives was as fully worked out as the version by Einstein.

It can be shown that the dimensionality of the expression $\frac{c^2 \cdot f^2}{G}$, for example, satisfies the above.

If we make the assumption that the Young's modulus is frequency-dependent, we can thus write

$Y_{SPACETIME} \sim \frac{c^2 f^2}{G} = 4.5 \cdot 10^{27} \boldsymbol{f^2}$.

For a gravitational wave of frequency 100 Hz, which is quite realistic, we finally obtain

$Y_{SPACETIME} \sim 10^{20} Y_{STEEL}$ ($\sim 10^{31}$ Pa).

It therefore takes vast energies for an accelerating mass to distort spacetime.

- Third, as for all waves, their flux (power over area) is inversely proportional to the square of the distance to the wave source (r^{-2}). Since the energy of a wave is proportional to its amplitude squared, $E \propto A^2$, the measured amplitude of the waves will go as $1/r$. Thus, as expected, the farther out the event creating the waves, the smaller the observed effect on spacetime that can be measured locally, here on Earth.

Bearing in mind the above three points, we see that it would require events of great violence to produce measurable signals in the best (most sensitive) terrestrial detectors currently available.

Any system producing the gravitational waves would have to be

- very massive (by everyday Earthly standards): at least the mass of a Sun-sized star, $m \sim 10^{30}$ kg, and
- accelerating
  - strongly and
  - in some non-symmetrical fashion.

Events of this kind only occur in places far beyond our Solar system (our local region is "boringly average", luckily for life on Earth).

What are the candidates for producing measurable gravitational waves?

Supernovae

The supernova explosion of a star, if perfectly symmetrical, would not produce gravitational waves, in spite of the violence of the event; but any detection of a gravitational wave associated with one might provide interesting information about the asymmetry, should it exist, of such an explosion. This information could then be "fed into" the supernova models we currently have.[31] (For example, such information might conceivably help with "producing the bang" in the first place, which is currently theoretically problematic).

[31] Unfortunately and obviously, there were no gravitational wave detectors on Earth in the year 1054.

Two orbiting bodies

Two stars orbiting one another (since all bodies in an orbit are being accelerated[32]) would also fulfill the above two requirements, at least in principle. Furthermore, the closer the objects to one another, the greater the gravitational radiation produced. Very dense bodies would allow closer proximity without touching, than less dense objects, so a high density is another desirable criterion. The densest objects in the Universe are believed to be neutron stars (or, denser still, black holes, provided these are not of the supermassive variety). Thus the most likely detectable sources of gravitational waves would be two closely orbiting black holes, or a black hole with a neutron star companion. These are indeed what has been observed since the experimental confirmation of gravitational waves in 2015.

In many cases one would expect a merger of the two bodies with one another very soon after first observing the gravitational waves emitted by such a system. This is because our observations selectively favour those events that are already the most dramatic; and we know that the gravitational wave radiation produced by two orbiting bodies is at a maximum when those bodies are spiralling inward and are closest to one another, just prior to their collision and merger. It turns out that the duration of most observations ranges from around a minute to only a fraction of a second.

The reason for a spiral orbit is that the combined kinetic and gravitational potential energy of the binary is continually being converted into gravitational wave energy which is radiated outwards. This constant loss of energy means constantly shrinking orbits.

We can verify the claim of shrinking orbits by using simple classical (Newtonian) physics, together with the (not very realistic) assumption that the orbits are circular.

The force on body A due to B is given by Newton's law of universal gravitation,

$$F_A = \frac{G m_A m_B}{r^2}, \qquad \text{(A.1)}$$

where G is Newton's constant and r is the distance between the two masses.

Newton's second law of motion, $F = ma$, then gives us

$$F_A = \frac{G m_A m_B}{r^2} = \frac{m_A v_A^2}{r_A} \qquad \text{(A.2)}$$

Considering the kinetic energy of each body, we have

$$\tfrac{1}{2} m_A v_A^2 = \frac{r_A \tfrac{1}{2} m_A v_A^2}{r_A} = \frac{r_A \tfrac{1}{2} G m_A m_B}{r^2}, \quad \text{using (A.2).}$$

Similarly,

$$\tfrac{1}{2} m_B v_B^2 = \frac{r_B \tfrac{1}{2} G m_A m_B}{r^2}$$

Thus for the sum of the kinetic energy of both bodies, we have

[32] Although their speed (the magnitude of their velocity) may not be changing, their direction of movement is. This is sufficient, for only one of these two properties need change for acceleration to be occurring.

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \frac{(r_A + r_B)\, \tfrac{1}{2} G m_A m_B}{r^2}$$

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \frac{G m_A m_B}{2r} \qquad \text{(A.3)}$$

The total mechanical energy for the system is then given by this kinetic energy plus the **negative** gravitational potential energy:

$$E = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 - \frac{G m_A m_B}{r}$$

which, upon using (A.3) for the kinetic energy, becomes:

$$E = \frac{G m_A m_B}{2r^2} - \frac{G\, m_A m_B}{r}$$

$$E = -\frac{G\, m_A m_B}{2r} \qquad \text{(A.4)}$$

This negative energy reflects the stability of the bound orbit and the fact that work would need to be done (energy added) to separate the objects "completely" (technically, separated at an infinite distance from one another).

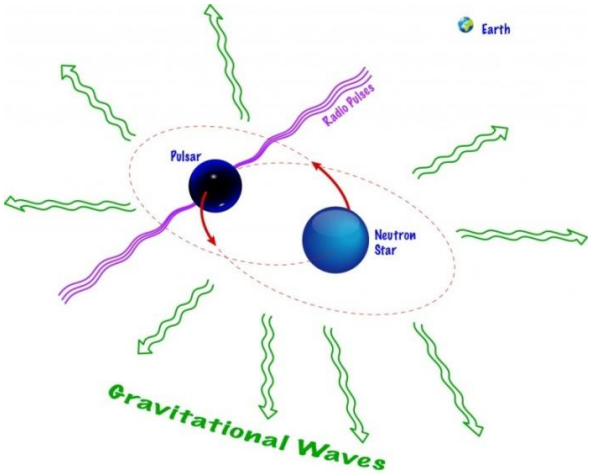
The radiation of gravitational energy of the binary clearly implies a decrease in the total energy of the system of radiating bodies. A decrease on the right hand side of (A.4) therefore means that the energy term becomes even **more negative**. Thus *r* must be getting smaller, which in turn means that the distance between the two masses must decrease over time. This corresponds with our physical intuition.

Gravitational-Wave Astronomy

The first gravitational wave detection was an indirect one, via the **electromagnetic** emission, rather than any gravitational emission, of the source. The Hulse-Taylor binary neutron star system consists of two neutron stars, one of which is rotating on its axis 17 times per second and emitting regular pulses of radio waves at that frequency (rather like the sweeping light beam emitted by a lighthouse).

Figure A4

The Hulse/Taylor neutron star binary system. As they orbit one another, both bodies emit gravitational waves, causing a gradual shrinking of their *orbit.*



Note. Image: Shane L. Larson

These pulsed radio beams are, by chance, lined up with Earth and their period is known to be changing in a periodic way. From these observations it can be calculated that the separation between the two neutron stars is slowly decreasing: a shrinking orbit. This can only be explained by the binary continually losing energy[33] in the form of gravitational waves: There is no other mechnism to explain the energy loss. (The radio waves themselves cannot cannot carry away enough energy to account for the orbital shrinkage and friction/drag is nonexistent, since the system is not embedded in a gas or dust cloud).

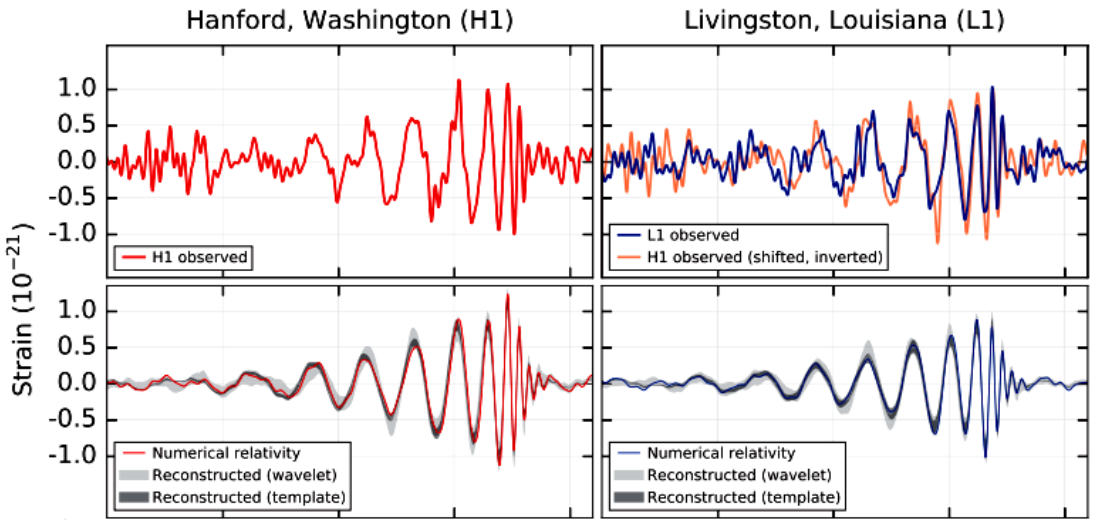
The first **direct** detection of a gravitational wave took place in the autumn of 2015 when the two LIGO ground-based detectors (at Hanford, Washington and Livingston, Louisiana, United States) measured the signal of two coalescing (colliding) black holes (Abbott et al. 2016). The signal was named **GW**150914 – from 'Gravitational Wave' and the date of observation (in the format YYMMDD).

So far, the most massive and the least massive object detected in a merger were found in the events called, respectively, GW190521 (two black holes, 85 and 66 solar masses) and GW190814 (one black hole of 23 solar masses with an object of 2.6 solar masses). Both observations were made in 2019.

The data analysis for gravitational wave detection is complicated. Part of the problem is that of other, unwanted, terrestrial signals of comparable levels, such as seismic vibrations and road traffic. Some of the several techniques used are known as matched filtering, together with Bayesian inference (Bayes, 1764).

Figure A5

Signals of the first confirmed discovery of a gravitational wave (autumn 2015) observed by the LIGO detectors at Hanford and Livingston, United States.



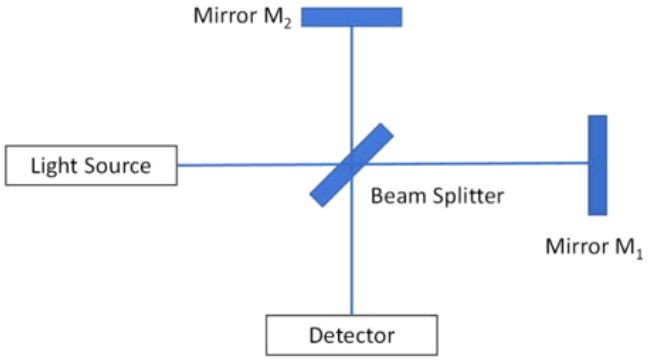
Note. Top row: observations at the two sites; bottom row: a simulation for a system with the parameters of GW150914 using numerical relativity. GW150914 arrived first at Livingston and 6.9 ms later at Hanford.

[33] The waves carry not only energy but **information** about their source, such as the mass and spin of the objects; hence the possibility of the discussion in this paragraph.

The principle of gravitational wave detectors is that of the **Michelson interferometer**, which uses the interference properties of light.

Figure A6

Schematic of a Michelson interferometer



Note. Image and text modified from Hilborn (2018).

Light from a source (usually a laser) hits a so-called beam splitter (for example, a half-silvered mirror). Part of the light is reflected from the beam splitter (here, upward) and is later reflected by mirror M_2. The other part of the beam goes through the beam splitter (to the right), then reflects from mirror M_1.

After hitting the beam splitter again, parts of the two beams combine and hit the detector, where an interference pattern is observed. In the case of LIGO a laser beam is sent along each of two several-kilometre long "arms" oriented at 90° to one another, and reflected back by mirrors. If there is a phase shift of the two beams relative to one another, this may indicate a spatial distortion that could be evidence of a gravitational wave passing through the detector at that moment. The gravitational waves that can currently be detected by such instruments have frequencies between 10 Hz to a few kHz.

Figure A7

Aerial view of the LIGO detector in Livingston, Louisiana, United States. The vast length of one of the two arms is clearly visible.

Note. Image source: https://www.ligo.caltech.edu/image/ligo20150731c

Since 2017, there has been a collaboration between the United States-based LIGO detectors and the Virgo gravitational wave detector near Pisa, Italy. With the additional data from this third detector, researchers can better localize the sources of the gravitational waves using **triangulation techniques**.

Figure A8

Aerial view of the Virgo gravitational wave detector near Pisa, Italy

Note. Image source: https://www.aps.org/publications/apsnews/updates/ligo-virgo.cfm

Since the first discovery, the LIGO-Virgo collaboration has made over ninety confirmed detections of gravitational waves[34], mainly originating from the merger of two black holes. This direct observation of gravitational waves has thus opened a "new window" on the cosmos, since the information obtained from them about their sources is different from, and complementary to, that of electromagnetic waves. Gravitational-wave astronomy has thus become a new tool for observing the Universe.

[34] Results of the third Gravitational-Wave Transient Catalog (GWTC-3) on November 7, 2021, including events released in prior observing runs.

Appendix B – The Python Language

If the implementation is hard to explain, it's a bad idea.
*If the implementation is easy to explain, it **may**[35] be a good idea.*
Tim Peters, „PEP 20 - The Zen of Python"

In this section, in order to distinguish my comments and explanations from the Python code itself, I have highlighted all statements that can be a part of a Python program in blue.

Importing Modules to the Code

To use the functionality present in external modules, the Python keyword command import is required in the code, together with the external module name. When the interpreter parses the import statement, it "pulls" the module to the current program. The grouping of imports should be in the following order:

1. Standard library imports.
2. Related third party imports.
3. Local application/library specific imports.

Different Import types

Full Module Import with alias

```
import taichi as ti
```

Direct Import (Selective Import)

```
from pyautogui import size
```

Multiple Import (Explicit Import)

```
import (Thread,
        Event,
        Lock)
```

Wildcard imports (from <module> import *) should be avoided, as they make it unclear which names are present in the namespace.

Comments

Comments start either with a hash or begin and end with three quote marks. They should be complete sentences, the first word being capitalized (unless it is an identifier that begins with a lower case letter). A comment can be written entirely on its own line, next to a statement of code, or as a multi-line comment block.

Examples

```
# This line is a comment.

a = 6.1 # Everything after the hash is a comment.
```

[35] My emphasis.

```
“”” This is a
multiple line block
of comment. “””
```

Variable Naming

Variable names are case-sensitive.

Multiple-word variable names

Variable names consisting of more than a single word can be difficult to read. There are several options for improving their readability:

- **Pascal Case**
  Each word starts with a capital letter: MyVariableName
- **Camel(back) Case**
  Each word, except the first, starts with a capital letter: myVariableName
- **Snake Case**
  Each word is separated by an underscore character: my_variable_name
  I use this third option in all code presented in this document.

Dynamic Typing

Python is a **dynamically typed** language[36]: it doesn't "know" about its variable types until the code is run. Indeed, there is no command for declaring a variable! This implies that variables, parameters, and return values of a function can be any type and are set during the run of the program.

Python has the following data types built-in by default:

Text Type	str
Numeric Types	int, float, complex
Sequence Types	list, tuple, range
Mapping Type	dict
Set Types	set, frozenset
Boolean Type	bool
Binary Types	bytes, bytearray, memoryview
None Type	NoneType

Examples

Python statement	**Data Type**
x = "Hello World"	String (str)
x = 42	Integer (int)
x = 1.1	Floating Point (float)

Furthermore, the types of variables can **change while the program is running.** The (current) data type is obtained with the type() function:

```
print (type(x))
```

Evaluation Order of an Expression

When Python evaluates an assignment, the source (right-hand side) is evaluated before the target (left-hand side), each side in left to right order. For any subscript in the target, the container is evaluated before the subscript.

[36] Other such languages include: JavaScript, PHP, Lisp and Ruby.

Data Collections (Aggregate Data Types)

There are four data types used to store **collections** of data (multiple items) "inside" a single variable (container):

- list
- tuple
- set (and frozenset)
- dictionary.

The differences between them have to do with whether they

- are **ordered** (and can therefore be indexed)
- allow **duplicate members** (multiple elements with identical contents)

are **mutable** (their values are changeable during execution).

List

Lists can hold items of different data types, including other lists. They are defined by enclosing the items in square brackets [], with each item separated by a comma.

Examples

Assignment

```
my_empty_list = []              # empty list
my_list = [1, "Hello", 3.4]     # list with mixed data types
```

Access

```
my_list [0]
```

Output

1

Access

```
my_list [-1]
```

Output

3.4

Tuple

The contents of a tuple are immutable (unchangeable during execution): This immutability allows faster execution; iteration through a tuple is faster than for a list (which is mutable). Referencing is with an integer index (which may be negative). Parentheses are optional.

Examples

Assignment

```
this_tuple = 0, 0
that_tuple = "Monty", "Python's", "Flying", "Circus"
```

the_other_tuple = ("apple", "banana", "cherry")

Access
that_tuple[1]

Output
Python's

The various data types and the subset of them that are **aggregate** data types are listed in Tables **B1** & **B2**, respectively.

Table B1

The Python Built-in Data Types

Data type	Name	Example	Comments
Text	str	a = "Norwegian Blue"	str() - constructs a string from a wide variety of data types, including strings, integer literals and float literals.
Numeric	int	x = 42	int() - constructs an integer number from an integer literal, a float literal (by removing all decimals), or a string literal (providing the string represents a whole number).
	float	y = 4.2	Numeric literals containing a decimal point or an exponent sign yield floating point numbers, which are represented as 64-bit double-precision values in Python. The maximum value any such floating-point number can have is approx. 1.8 x 10308. Any number greater than this will be indicated by the string inf.
			float() - constructs a floating point number from an integer literal, a float literal or a string literal (providing the string represents a float or an integer).
	complex	z = 1j	Appending 'j' or 'J' to a numeric literal yields an imaginary number (a complex number with a zero real part) which you can add to an integer or float to get a complex number with real and imaginary parts.
			complex() can be used to produce complex numbers. Both the real and imaginary parts are a floating point number. z.real and z.imag allow the extraction of these from a complex number z.
Sequence	list	x = ["Cleese", "Palin", "Jones"]	
	tuple	x = ("Gilliam", "Idle", "Chapman")	
	range	x = range(6)	
Mapping	dict	x = {"name" : "Eric", "age" : 81}	
Set	set	x = {"Dead", "Parrot", "Sketch"}	

	frozenset	x = frozenset({"I'm", "a", "lumberjack"})	
Boolean	bool	x = True	Almost any value is evaluated to True if it has some sort of content. Any string is True, except empty strings. Any number is True, except 0. Any list, tuple, set, and dictionary are True, except empty ones.
Binary Types	bytes	x = b"Hello"	
	bytearray	x = bytearray(5)	
	memoryview	x = memoryview(bytes(5))	

Table B2

The Python Aggregate built-in Data Types

Data type	**Name**	**Properties for collections of elements, where applicable**
Text Type	str	
Numeric Types	int	
	float	
	complex	
Sequence Types	list	ordered, duplicates allowed, mutable
	tuple	ordered, duplicates allowed, immutable
	range	
Mapping Type	dict	
Set Types	set	unordered, duplicates not allowed, mutable
	frozenset	unordered, duplicates not allowed, immutable (hence the name!)
Boolean Type	bool	
Binary Types	bytes	
	bytearray	
	memoryview	

Constructors

For numeric types, the **constructors**

- int()
- float() and
- complex()

can be used to produce numbers of a specific type.

Operators

Python divides operators into the following groups:

- Arithmetic operators
- Assignment operators
- Bitwise operators
- Comparison operators
- Identity operators

- Logical operators
- Membership operators

Assignment Operators

Operator	Example	Same As
=	x = 5	x = 5
+=	x += 3	x = x + 3
-=	x -= 3	x = x - 3
*=	x *= 3	x = x * 3
/=	x /= 3	x = x / 3
%=	x %= 3	x = x % 3
//=	x //= 3	x = x // 3
**=	x **= 3	x = x ** 3
&=	x &= 3	x = x & 3
|=	x |= 3	x = x | 3
^=	x ^= 3	x = x ^ 3
>>=	x >>= 3	x = x >> 3
<<=	x <<= 3	x = x << 3

Arithmetic Operators

Operator	Name	Example
+	Addition	x + y
-	Subtraction	x - y
*	Multiplication	x * y
/	Division	x / y
%	Modulus	x % y
**	Exponentiation	x ** y
//	Floor division	x // y

Bitwise Operators

Operator	Name	Description
&	AND	Sets each bit to 1 if both bits are 1
|	OR	Sets each bit to 1 if one of two bits is 1
^	XOR	Sets each bit to 1 if only one of two bits is 1
~	NOT	Inverts all the bits
<<	Zero fill left shift	Shift left by pushing zeros in from the right and let the leftmost bits fall off
>>	Signed right shift	Shift right by pushing copies of the leftmost bit in from the left, and let the rightmost bits fall off

Comparison Operators

Operator	Name	Example
==	Equal	x == y
!=	Not equal	x != y
>	Greater than	x > y

<	Less than	x < y
>=	Greater than or equal to	x >= y
<=	Less than or equal to	x <= y

Logical Operators

Operator	Description	Example
and	Returns True if both statements are true	x < 5 and x < 10
or	Returns True if one of the statements is true	x < 5 or x < 4
not	Reverse the result, returns False if the result is true	not(x < 5 and x < 10)

Identity Operators

Operator	Description	Example
is	Returns True if both variables are the same object	x is y
is not	Returns True if both variables are not the same object	x is not y

Membership Operations

Operator	Description	Example
in	Returns True if a sequence with the specified value is present in the object	x in y
not in	Returns True if a sequence with the specified value is not present in the object	x not in y

Source: https://www.w3schools.com/python/python_operators.asp

Functions

A function allows the definition of a reusable block of code that can be executed many times, which is essential to **modular programming**.

The Python language passes arguments neither by reference nor by value, but **by assignment**. It uses a mechanism known as **Call-by-Object**, or **Call by Object Reference** (less commonly termed **Call by Sharing**). This is equivalent to passing a **pointer** in other languages.

Default argument passing

Sometimes the user cannot or does not wish to provide a value for one or more parameters that are taken by a function. For those arguments for which no value is passed during the function call, **default values** are used. These are provided in the parameters list, where the assignment operator '=' is used for them.

Example

```
def sum(a=5, b=7): # function with default argument
 """ This function will print the sum of two numbers
    if the arguments are not supplied
    it will add the default value """
 print (a+b)
```

The return statement

The Python style guide (called *PEP8*) is noncommittal towards whether the return statement should contain one (or more) variables vs. returning a value without assignment in the function.

Examples

```
def my_function (argument1, ...):
   a = value_calculated_from_the_arguments
   return a

def function2(argument1, ...):
  return value_calculated_from_the_arguments
```

Checking the Code

A bit of etymology...

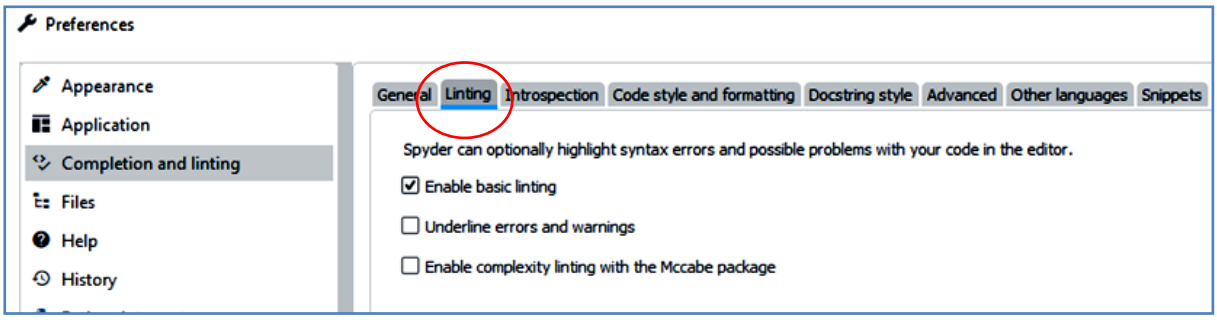
Lint is the common name for visible accumulations of textile fibres that can detach from clothing. A modern tumble dryer uses something called a lint screen to collect these remnants. Thus, linting, used as a verb, is the "cleaning of tiny bits of fluff". Python code requires such initial "linting", in the figurative sense, to help the programmer find simple common errors in the source code (which is particularly important because of Python's dynamic typing and the "missing" compilation step which would normally do most of this work). The **linter** thus provides warnings about errors in syntax and formatting, as well as the use of locally unspecified variables. It performs **static analysis** of source code and can be regarded as a simple **debugger**.

Examples

- **Pyflakes.** The Spyder IDE (integrated development environment) uses the simple parser called Pyflakes, available at https://pypi.org/project/pyflakes/
  Pyflakes is faster than **Pylint** (primarily because the former only examines the syntax tree of each file individually). The cost/drawback is that Pyflakes is thus more limited in the types of things it can check.
- **Pylint**, whose name explicitly refers to its function, is available - although not used in this project - at https://pypi.org/project/pylint/

Figure B1

Example of a linting option for the Spyder IDE (integrated development environment)



Note. This can be found in: *Spyder -> Tools -> Preferences -> Completion and Linting*

Figure B2

Another tool for linting!

Figure B3

A tumble dryer

Appendix C – The Tkinter GUI

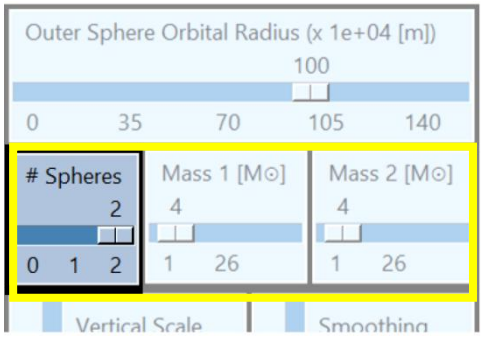
Brief code example and explanation

Creation of a Frame (highlighted here, for emphasis, by the yellow box)

Frame widgets are used to group/organise other widgets on the screen. The Frame controls the positions of the individual widgets that fit inside it using a geometry manager. Tkinter has three such manager methods for organizing layout: **pack()**, **grid()** and **place()**. After some experimentation with all three, I decided to use the first of these, as it was, for me, the easiest and simplest.
In the example here, the **Frame** widget is instantiated with a black background (**bg="black"**), then attempted to be placed at the top of the main GUI window. However, because another frame has already been specified first, the new frame sits directly underneath it.

Figure C1

Part of the GUI (Graphical User Interface)



Note. Author's image

The **pack** method places the frame using (**side=TOP**) and makes it expand horizontally (**fill=X**) to take up the full width of the GUI window:

```
padx, pady = 2, 2
horiz_slider_arguments = {
   "bg": "light steel blue",
   "troughcolor": "steel blue",
   "orient": "horizontal",
}
…
frame = Frame(root, bg="black")
frame.pack(side=TOP, fill=X, padx=padx, pady=pady)  # Expand frame to full width but not height
```

The root variable is used as the parent window for all other widgets.

This frame will contain three widgets, placed in a row: all of them here are of the type **slider**.
The code provided shows only the first, left-hand-side one. This slider, labeled "# Spheres", is associated with a Tkinter variable called `tkinter_spheres_to_display`. The slider allows users to select a value from 0 to 2, with tick marks at intervals of 1. Some padding, defined by padx and pady is placed around each widget inside the frame, for aesthetic reasons:

```
slider_spheres_to_display = Scale(frame,
```

```
                                label="# Spheres",
                                variable=tkinter_spheres_to_display,
                                from_=0, to=2,
                                tickinterval=1,
                                **horiz_slider_arguments)
slider_spheres_to_display.pack(side=LEFT, padx=padx, pady=pady, fill=X)
slider_spheres_to_display.set(2)
```

Colours with Tkinter

These are represented in one of two ways:

- As a string specifiying the proportion of red, green and blue, in hexadecimal digits. For example, "#fff" (white), "#000000" (black), "#000fff000" (green).
- Using defined standard colour names. Examples are: "white", "black", "red", "green", "blue", "cyan", "yellow", and "magenta".

I use only the latter.

Examples

"bg": "light steel blue": light steel blue background

"troughcolor": "steel blue": steel blue slider trough colour

Colour options

Name	Function
activebackground	Background for the widget when the widget is active.
activeforeground	Foreground for the widget when the widget is active.
background	Background for the widget. This can also be represented as bg.
disabledforeground	Foreground for the widget when the widget is disabled.
foreground	Foreground for the widget. This can also be represented as fg.
highlightbackground	Background of the highlight region when the widget has focus.
highlightcolor	Foreground of the highlight region when the widget has focus.
selectbackground	Background for the selected items of the widget.
selectforeground	Foreground for the selected items of the widget.

An exhaustive list of Tkinter named colours is shown here.


Named colour chart
snow ghost white white smoke gainsboro floral white old lace linen antique white papaya whip blanched almond bisque peach puff navajo white lemon chiffon mint cream azure alice blue lavender lavender blush misty rose dark slate gray dim gray slate gray light slate gray gray light grey midnight blue navy cornflower blue dark slate blue slate blue medium slate blue light slate blue medium blue royal blue blue dodger blue
deep sky blue sky blue light sky blue steel blue light steel blue light blue powder blue pale turquoise dark turquoise medium turquoise turquoise cyan light cyan cadet blue medium aquamarine aquamarine dark green dark olive green dark sea green sea green medium sea green light sea green pale green spring green lawn green medium spring green green yellow lime green yellow green forest green olive drab dark khaki khaki pale goldenrod light goldenrod yellow light yellow yellow
gold light goldenrod goldenrod dark goldenrod rosy brown indian red saddle brown sandy brown dark salmon salmon light salmon orange dark orange coral light coral tomato orange red red hot pink deep pink pink light pink pale violet red maroon medium violet red violet red medium orchid dark orchid dark violet blue violet purple medium purple thistle snow2 snow3 snow4 seashell2
seashell3 seashell4 AntiqueWhite1 AntiqueWhite2 AntiqueWhite3 AntiqueWhite4 bisque2 bisque3 bisque4 PeachPuff2 PeachPuff3 PeachPuff4 NavajoWhite2 NavajoWhite3 NavajoWhite4 LemonChiffon2 LemonChiffon3 LemonChiffon4 cornsilk2 cornsilk3 cornsilk4 ivory2 ivory3 ivory4 honeydew2 honeydew3 honeydew4 LavenderBlush2 LavenderBlush3 LavenderBlush4 MistyRose2 MistyRose3 MistyRose4 azure2 azure3 azure4 SlateBlue1
SlateBlue2 SlateBlue3 SlateBlue4 RoyalBlue1 RoyalBlue2 RoyalBlue3 RoyalBlue4 DodgerBlue2 DodgerBlue3 DodgerBlue4 SteelBlue1 SteelBlue2 SteelBlue3 SteelBlue4 DeepSkyBlue2 DeepSkyBlue3 DeepSkyBlue4 SkyBlue1 SkyBlue2 SkyBlue3 SkyBlue4 LightSkyBlue1 LightSkyBlue2 LightSkyBlue3 LightSkyBlue4 SlateGray1 SlateGray2 SlateGray3 SlateGray4 LightSteelBlue1 LightSteelBlue2 LightSteelBlue3 LightSteelBlue4 LightBlue1 LightBlue2
LightBlue3 LightBlue4 LightCyan2 LightCyan3 LightCyan4 PaleTurquoise1 PaleTurquoise2 PaleTurquoise3 PaleTurquoise4 CadetBlue1 CadetBlue2 CadetBlue3 CadetBlue4 turquoise1 turquoise2 turquoise3 turquoise4 cyan2 cyan3 cyan4 DarkSlateGray1 DarkSlateGray2 DarkSlateGray3 DarkSlateGray4 aquamarine2 aquamarine4 DarkSeaGreen1 DarkSeaGreen2 DarkSeaGreen3 DarkSeaGreen4 SeaGreen1 SeaGreen2 SeaGreen3 PaleGreen1 PaleGreen2 PaleGreen3 PaleGreen4
SpringGreen2 SpringGreen3 SpringGreen4 green2 green3 green4 chartreuse2 chartreuse3 chartreuse4 OliveDrab1 OliveDrab2 OliveDrab4 DarkOliveGreen1 DarkOliveGreen2 DarkOliveGreen3 DarkOliveGreen4 khaki1 khaki2 khaki3 khaki4 LightGoldenrod1 LightGoldenrod2 LightGoldenrod3 LightGoldenrod4 LightYellow2 LightYellow3 LightYellow4 yellow2 yellow3 yellow4 gold2 gold3 gold4 goldenrod1 goldenrod2 goldenrod3 goldenrod4
DarkGoldenrod1 DarkGoldenrod2 DarkGoldenrod3 DarkGoldenrod4 RosyBrown1 RosyBrown2 RosyBrown3 RosyBrown4 IndianRed1 IndianRed2 IndianRed3 IndianRed4 sienna1 sienna2 sienna3 sienna4 burlywood1 burlywood2 burlywood3 burlywood4 wheat1 wheat2 wheat3 wheat4 tan1 tan2 tan4 chocolate1 chocolate2 chocolate3 firebrick1 firebrick2 firebrick3 firebrick4 brown1 brown2 brown3
brown4 salmon1 salmon2 salmon3 salmon4 LightSalmon2 LightSalmon3 LightSalmon4 orange2 orange3 orange4 DarkOrange1 DarkOrange2 DarkOrange3 DarkOrange4 coral1 coral2 coral3 coral4 tomato2 tomato3 tomato4 OrangeRed2 OrangeRed3 OrangeRed4 red2 red3 red4 DeepPink2 DeepPink3 DeepPink4 HotPink1 HotPink2 HotPink3 HotPink4 pink1 pink2
pink3 pink4 LightPink1 LightPink2 LightPink3 LightPink4 PaleVioletRed1 PaleVioletRed2 PaleVioletRed3 PaleVioletRed4 VioletRed1 VioletRed2 VioletRed3 VioletRed4 magenta2 magenta3 magenta4 orchid1 orchid2 orchid3 orchid4 plum1 plum2 plum3 plum4 MediumOrchid1 MediumOrchid2 MediumOrchid3 MediumOrchid4 DarkOrchid1 DarkOrchid2 DarkOrchid3 DarkOrchid4
purple1 purple2 purple3 purple4 thistle1 thistle2 thistle3 thistle4
gray26 gray27 gray28 gray29 gray30 gray31 gray32 gray33 gray34 gray35 gray36 gray37 gray38 gray39 gray40 gray41 gray42 gray43 gray44 gray45 gray46 gray47 gray48 gray49 gray50 gray51 gray52 gray53 gray54 gray55 gray56 gray57 gray58 gray59 gray60 gray61 gray62 gray63
gray64 gray65 gray66 gray67 gray68 gray69 gray70 gray71 gray72 gray73 gray74 gray75 gray76 gray77 gray78 gray79 gray80 gray81 gray82 gray83 gray84 gray85 gray86 gray87 gray88 gray89 gray90 gray91 gray92 gray93 gray94 gray95 gray97 gray98 gray99

Partial Glossary

These terms are those that are referred to, but not explicitly expanded upon, in the main body of the text. I include those that may be unfamiliar to some readers, terms, particularly from computer science. Some entries have been taken and modified from the following sources:
https://en.wikipedia.org/wiki/Glossary_of_computer_science
https://www.computerhope.com/jargon.htm
https://www.ibm.com/ibm/history/documents/pdf/glossary.pdf

The remaining entries are my own text.

Application Programming Interface (API)

A software interface that enables applications to communicate with each other. An API is the set of programming language constructs or statements that can be coded in an application program to obtain the specific functions and services provided by an underlying operating system.

Argument

A data value passed to a function. Arguments are assigned to the named local variables in a function body. There are two kinds of argument in a function call:

- **keyword** argument: an argument preceded by a name label (an identifier, e.g. *name=*)
- **positional** argument: an argument that is not preceded by an identifier and whose position in the call therefore matters (usually to be placed before all keyword arguments).

Array Slicing

Array slicing is an operation that extracts a subset of elements from an array and packages them as another array, possibly in a different dimensionality from the original.

Callback

A Python function that takes no arguments.

Class

In an object-oriented language, a construct that encapsulates a group of variables and methods.

Code Comment

An annotation or explanation in the source code of a program. Its purpose is to make the code easier to understand. Comments are ignored by interpreters and compilers.

Core

Synonym: **CPU core**

A core receives instructions, and performs calculations, or operations, to satisfy those instructions. A single CPU can have multiple cores inside it. Each core can perform operations independently of the others. The majority of consumer CPUs currently feature between about two and twelve cores.

CUDA

Originally: Compute Unified Device Architecture, acronym discontinued.

CUDA is an architecture for GPUs developed by NVIDIA, introduced in 2007. It improves the performance of those computing tasks which benefit from parallel processing. CUDA GPUs feature several thousand CUDA cores, integrated onto a single video card. Software must be written specifically for the architecture. Although the native programming language is C++, so-called wrappers are written for other languages.

Decorator

A decorator is a callable function (or, sometimes, a class) that accepts either a function or a class and returns a new one that "wraps" around the original one. A function or a class in Python is decorated with the @ symbol followed by the decorator name on the same line. The body of the function or class then starts on the following line.

Duck Typing

Duck Typing is a concept related to dynamic typing, where the type or the class of an object is less important than the method it defines. Using duck typing, types are not checked at all. Instead, there is a check for the presence of a given method or attribute. (The reason for the name: "If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck".)

Dynamic Typing

The term means that a compiler or an interpreter assigns a type to all the variables at run-time, the types being decided based on their values.

Function

A reusable block of code designed to perform a specific task or computation. Functions take inputs, process them, and return an output.

Garbage Collection

The process of freeing memory when it is not used anymore. Garbage collection automates this process using a garbage collector to search for previously allocated memory that is no longer being used by a program. In some programming languages, this freeing of memory must be done manually. Python does it automatically and allows additional control via something called the *gc module*.

Generator Function

A function which returns a generator iterator. It looks like a normal function except that it contains **yield** expressions for producing a series of values usable in a for-loop or that can be retrieved one at a time, using the **next()** function. Each yield action temporarily suspends processing, remembering the location execution state (including local variables).

Graphics Processing Unit (GPU)

The GPU is an electronic circuit used to speed up the creation of both 2D and 3D images. GPUs can either be

- **integrated** (built into the computer's CPU or motherboard), or
- **dedicated**, as a separate piece of hardware known as a video card.

By having its own, separate, processor, the GPU allows the computer's CPU resources to be used for other tasks.

Immutable Object

In Python, an object with a fixed value which cannot be altered during program execution. Immutable objects include numbers, strings and tuples. (If the object is immutable when we update the variable, we actually point it to another object, and Python's garbage collection will recycle the original object if it is no longer used.)

Interpreted Language

An interpreted computer language is a type of programming language in which most of the instructions are executed directly by an interpreter rather than being compiled into machine code beforehand. The interpreter processes the instructions on-the-fly.

Iterator

Synonym: **Iterable**

An iterable is any object that can return its members one at a time as a stream of data, by repeated calls.

Python has two commonly used types of iterables:

- Sequences (element access using integer indices)
- Generators

Just-in-time (JIT) compilation

Synonyms: **Dynamic translation**, **run-time compilation**.

This is a way of executing computer code that involves compilation during execution of a program (at run time) rather than as an independent step before execution.

Namespace

The place where a variable is stored. Namespaces support modularity by preventing naming conflicts and they aid readability and maintainability by making it clear which module implements a function.

Object

Member or instance of a particular class. It contains real values instead of variables.

Object-oriented programming

Synonyms: **OOP, OO programming.**

A programming language paradigm in which the code can be structured as reusable components, some of which may share properties or behaviours. Its basic "building blocks" are classes and objects.

Passing by reference

When passing by reference is used, the memory address of an argument is passed into the function. This means that the function can change the current value of the argument.

Passing by value

When passing by value is used, a copy of the argument's value is passed into the function. This means that the function cannot change the original value of the argument.

PEP (Python Enhancement Proposal)

A PEP is a design document providing information to the Python community, or describing a new feature for the Python language. PEPs are intended to be the primary mechanisms for proposing major new features, for collecting community input on an issue, and for documenting the design decisions that have gone, and continue to go, into Python.

Pythonic

An idea or piece of code which closely follows the most common idioms of the Python language, rather than implementing code using concepts common to other languages.

Return Value

The value(s) that a function returns when it completes.

Scope

Synonym: **Visibility**

The reference to an entity (in Python, any object) may not be possible from all places in the code. If this is the case, then in some parts of the program the name may refer to either a different entity, or to nothing at all. The region(s) of the code where a correct and error-free reference can be made to the object is called that object's scope.

Static Typing

A **statically-typed** language is a language (such as Java, C, or C++) where variable types are known at compile time. Although Python is a **dynamically typed** language, it is possible to code so-called **type hints**, which make it possible to perform type checking of Python code. In contrast to genuinely statically-typed languages, however, type hinting does not cause Python to enforce the types given and can thus be regarded as code comments.

System Bus

Synonyms: **FSB** (front-side bus), **processor bus**, **memory bus**

Connects the CPU (chipset) with the main memory and cache.

Type Hinting

An indication, as part of the code, of the data type of a variable, in order to make the code more self-explanatory. This is usually done within the relevant function.

Python Example

```
def sum(num1: int, num2: int) -> int:
    return num1 + num2
```

Wrapper

A wrapper describes an intermediate set of functions that allow one piece of software to be accessed directly by other software, without additional computation. For instance, compiled

software written in the C++ programming language may offer a wrapper that allows it to be used directly by programs written in Python.

Yield

A keyword used to return from a function without destroying the states of its local variables. When the function is called, the execution starts from the last yield statement. Any function that contains a yield keyword is termed a **generator.**

Zen of Python

Listing of Python design principles and philosophies that are helpful in understanding and using the language. The listing can be found by typing “import this” at the interactive prompt.