Exercise 5

Submitted by: Yaniv Tal - 031431166

Part A – Theoretical part

1. Joint probability based on graph:

$$P(A,B,C,D,E,F) = P(A)P(B|A)P(C|B)P(D|A)P(E)P(F|B,D,E)$$

2. We need a total of **16 parameters** to define the network:
   a. P(A) - 1
   b. P(B|A) – 2
   c. P(C|B) – 2
   d. P(D|A) – 2
   e. P(E) – 1
   f. P(F|B, D, E) – 8

3. To define the full distribution P(A, B, C, D, E, F) we would need $2^6 = 64$ parameters

4. As following:
   a. $D \perp\!\!\!\perp E \mid \phi$ - **True -** No data flow on D-F-E because F is unknown
   b. $C \perp\!\!\!\perp E \mid F$ – **False** –Information flows on the path E-F-B-C
   c. $C \perp\!\!\!\perp E \mid A$ – **True - T**he path E-F-B-C is closed because no data if F
   d. $A \perp\!\!\!\perp F \mid \{B, D\}$ – **True** - Both paths A-B-F and A-D-F are closed by the evidence in B, D respectively

5. **Yes, the table suits the graph.** We can see from the covariances of the marginal distributions that:
   a. $A \perp B \mid \phi$
   b. $A \not\perp\!\!\!\perp C$, $B \not\perp\!\!\!\perp C$

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Marginal for P(a):
[0.5 0.5]
Marginal for P(b):
[0.25 0.75]
Marginal for P(c):
[0.4625 0.5375]

Marginals for P(('a', 'b')):
[[0.125 0.125]
 [0.375 0.375]]

Covariance matrix for P(('a', 'b')):
[[0. 0.]
 [0. 0.]]

Marginals for P(('b', 'c')):
[[0.34375 0.11875]
 [0.15625 0.38125]]

Covariance matrix for P(('b', 'c')):
[[ 0.0253125 -0.0253125]
 [-0.0253125  0.0253125]]

Marginals for P(('c', 'a')):
[[0.11875 0.34375]
 [0.13125 0.40625]]

Covariance matrix for P(('c', 'a')):
[[0.0253125 0.0309375]
 [0.0309375 0.0378125]]
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