Name: ____________________ ____________________ **Student ID:** ____________________

Week4-template **Math 563, Fall 2022**

Q 1. (Durrett 2.1.3.) Let $\rho(x, y)$ be a metric.

Q 1.1. (i) Suppose h is differentiable with $h(0) = 0$, $h'(x) > 0$ for $x > 0$ and $h'(x)$ decreasing on $[0, \infty)$. Then $h(\rho(x, y))$ is a metric

THE PROOF

QED

Q 1.2. $h(x) = x/(x + 1)$ satisfies the hypothesis in (i)

THE PROOF

QED

Q 2. (Durrett 2.1.4.) Let $\Omega = (0,1)$, $\mathcal{F}$ = Borel sets, $\mathbb{P}$ = Lebesgue measure. $X_n(\omega) = \sin(2\pi n\omega)$, $n = 1, 2, \ldots$ are uncorrelated but not independent.

THE PROOF

QED

Q 3. (Durrett 2.1.7.) Let $K \geq 3$ be a prime and let X and Y be independent random variables that are uniformly distributed on $\{0, 1, \ldots, K-1\}$. For $0 \leq n < K$, let $Z_n = X + nY \mod K$.

Q 3.1. Show that $Z_0, Z_1, \ldots, Z_{K-1}$ are pairwise independent, i.e., each pair is independent.

THE PROOF

QED

Q 3.2. They are not independent because if we know the values of two of the variables then we know the values of all the variables.

(NOTE: Prove that the value of two determine the values of all)

THE PROOF

QED

Q 4. (Durrett 2.1.14.) Let $X, Y \geq 0$ be independent with distribution functions F and G. Find the distribution function of XY.

THE SCRATCH AREA

THE ANSWER

Q 5. (Durrett 2.1.15.) If we want an infinite sequence of coin tossings, we do not have to use Kolmogorov's theorem. Let Ω be the unit interval $(0,1)$ equipped with the Borel sets $\mathcal{F}$ and Lebesgue measure $\mathbb{P}$. Let $Y_n(\omega) = 1$ if $\lfloor 2^n\omega \rfloor$ is odd and 0 if $\lfloor 2^n\omega \rfloor$ is even. Show that $Y_1, Y_2, \ldots$ are independent with $\mathbb{P}(Y_k = 0) = P(Y_k = 1) = 1/2$.

THE PROOF

QED