Name:	Student ID:	
Week7-template		Math 563, Fall 2022

- **Q 1.** (Durrett 4.1.1.) **Bayes's Formula.** Let  $G \in \mathcal{G}$ .
- **Q 1.** (Durrett 4. **Q 1.1.** Show that

$$\mathbb{P}(G \mid A) = \frac{\int_{G} \mathbb{P}(A \mid \mathcal{G}) d\mathbb{P}}{\int_{\Omega} \mathbb{P}(A \mid \mathcal{G}) d\mathbb{P}}$$

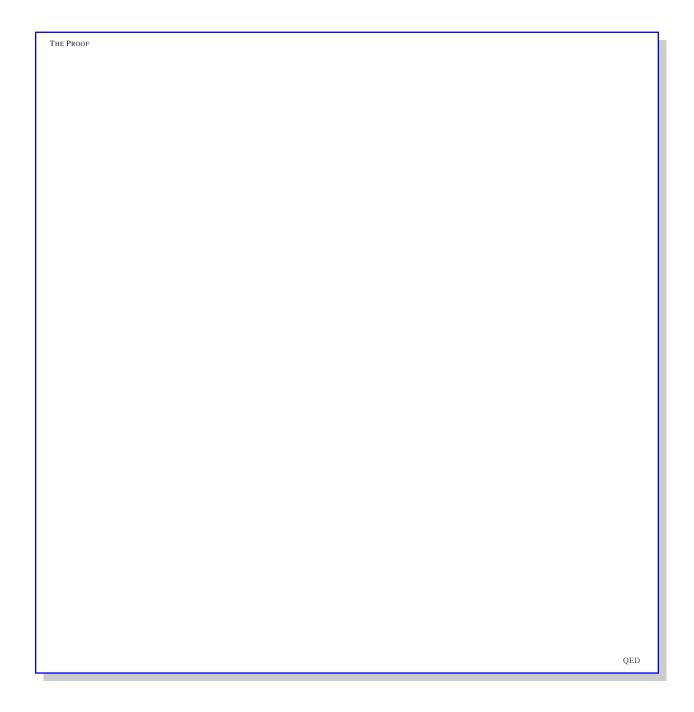
THE PROOF	
	QED

Q 1.2.	Show that when $\mathcal{G}$ is generated by a partition $\{G_1, G_2, \ldots\}$ , this reduces to the usual Bayes' for-
	mula: $\mathbb{P}(G_i \mid A) = \frac{\mathbb{P}(A \mid G_i) \mathbb{P}(G_i)}{\sum_j \mathbb{P}(A \mid G_j) \mathbb{P}(G_j)}.$

THE PROOF QED

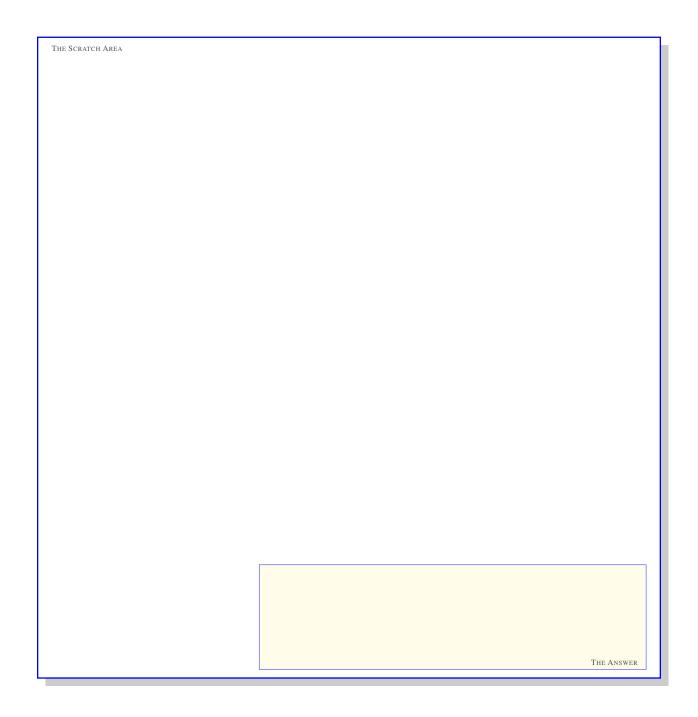
Q 2.	(Durrett 4.1.2.)	Prove Chebyshev's	sinequality. If $a > 0$ then
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$$\mathbb{P}(|X| \ge a \mid \mathcal{F}) \le a^{-2} \mathbb{E}(X^2 \mid \mathcal{F})$$

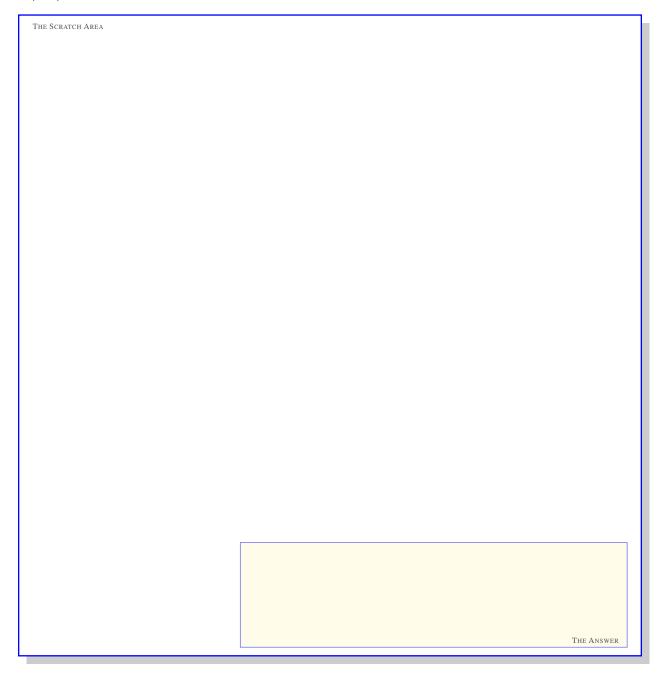


Q 3.	(Durrett 4.1.5.)	Give an example on	$\Omega = \{a,b,c\}$	in which:
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$$\mathbb{E}(\mathbb{E}(X \mid \mathcal{F}_1) \mid \mathcal{F}_2) \neq \mathbb{E}(\mathbb{E}(X \mid \mathcal{F}_2) \mid \mathcal{F}_1)$$



**Q 4.** (Durrett 4.1.9.) Show that if X and Y are random variables with  $\mathbb{E}(Y \mid \mathcal{G}) = X$  and  $\mathbb{E}(Y^2) = \mathbb{E}(X^2) < \infty$ , then X = Y a.s.



Q 5. (Durrett 4.1.10.) Bonus problem! The result of the last exercise implies that if  $\mathbb{E}Y^2 < \infty$  and  $\mathbb{E}(Y \mid \mathcal{G})$  has the same distribution as Y then  $\mathbb{E}(Y \mid \mathcal{G}) = Y$  a.s. Prove that under the assumption  $\mathbb{E}|Y| < \infty$ . Hint: The trick is to prove that  $\mathrm{sgn}(X) = \mathrm{sgn}(\mathbb{E}(X \mid \mathcal{G}))$  a.s., and then take X = Y - c to get the desired result.

