Name:	Student ID:	
Week5-template		Math 563, Fall 2022

- **Q 1.** (Durrett 2.3.6.) Metric for convergence in probability. Show
- **Q 1.1.** (a) that $d(X,Y) = \mathbb{E}\left(\frac{|X-Y|}{1+|X-Y|}\right)$ defines a metric on the set of random variables, i.e.,
 - 1. d(X,Y) = 0 if and only if X = Y a.s.,
 - 2. d(X,Y) = d(Y,X),
 - 3. $d(X,Z) \le d(X,Y) + d(Y,Z)$

THE PROOF	
	QED
	QED.

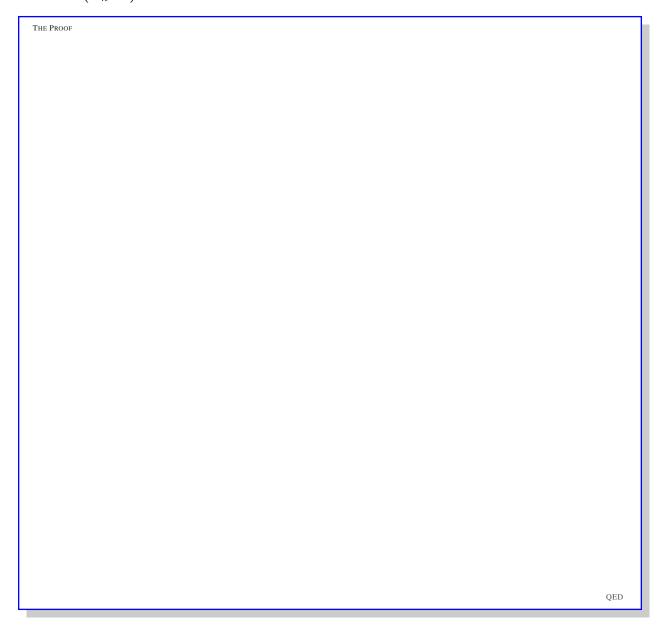
Q 1.2. and (b) that $d(X_n, X) \to 0$ as $n \to \infty$ if and only if $X_n \to X$ in probability.

THE PROOF	
	QED

Q 2.	(Durrett 2.3.10.) Kochen-Stone lemma.	Suppose $\sum_k \mathbb{P}(A_k) = \infty$.	Use Exercises	1.6.6 and 2.3.1 to
	show that if			

$$\limsup_{n\to\infty} \frac{\left(\sum_{k=1}^n \mathbb{P}(A_k)\right)^2}{\sum_{1\leq j,k\leq n} \mathbb{P}(A_j\cap A_k)} = \alpha > 0$$

then $\mathbb{P}(A_n \text{i.o.}) \ge \alpha$. The case $\alpha = 1$ contains Theorem 2.3.7.



Q 3.	(Durrett 2.3.14.)	Let X_1, X_2, \ldots	be independent.	Show that $\sup X_n$	< ∞	a.s.	if and onl	y if
	$\sum_n \mathbb{P}(X_n > A) <$	∞ for some A .						



Q 4.	(Durrett 2.3.15.) Let $X_1, X_2,$ be i.i.d.	with $\mathbb{P}(X_i > x) = e^{-x}$, le	et $M_n = \max_{1 \le m \le n} X_m$. Show
	that		

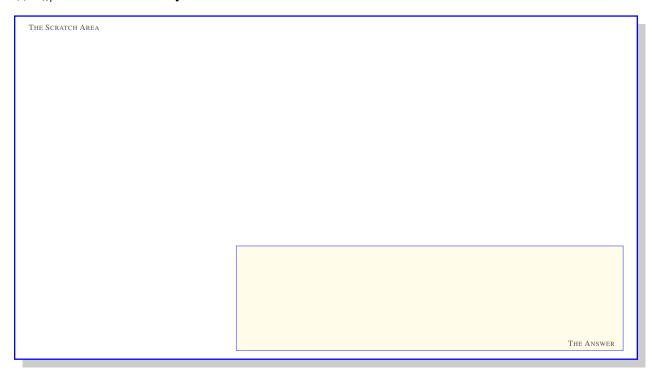
Q 4.1. (i) $\limsup_{n\to\infty} X_n/\log n = 1$ a.s.

THE PROOF	
	QED
	QLD.

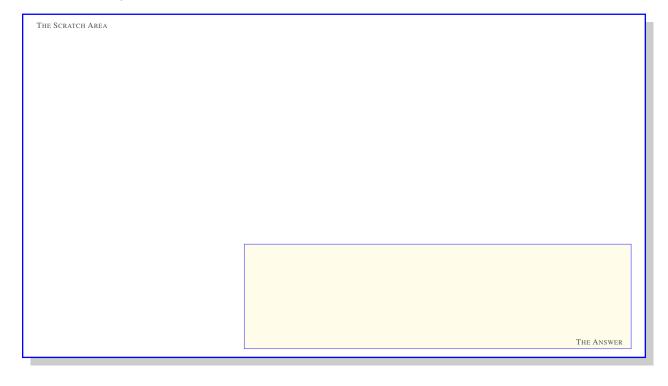
Q 4.2. and (ii) $M_n/\log n \to 1$ a.s.

The Proof	
	QED

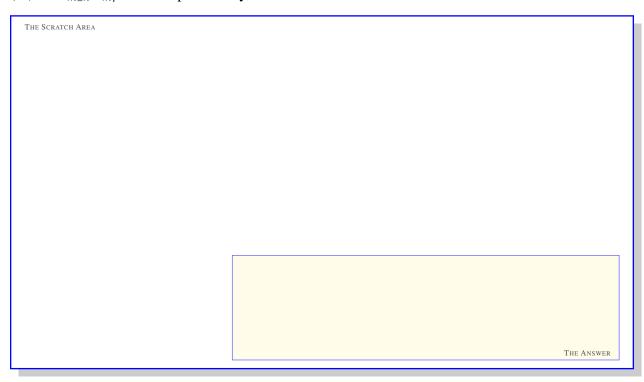
Q 5. (Durrett 2.3.17.) Let Y_1, Y_2, \ldots be i.i.d. Find necessary and sufficient conditions for **Q 5.1.** (i) $Y_n/n \to 0$ almost surely,



Q 5.2. (ii) $\max_{m \le n} Y_m/n \to 0$ almost surely,



Q 5.3. (iii) $\max_{m \le n} Y_m/n \to 0$ in probability,



Q 5.4. and (iv) $Y_n/n \to 0$ in probability.

