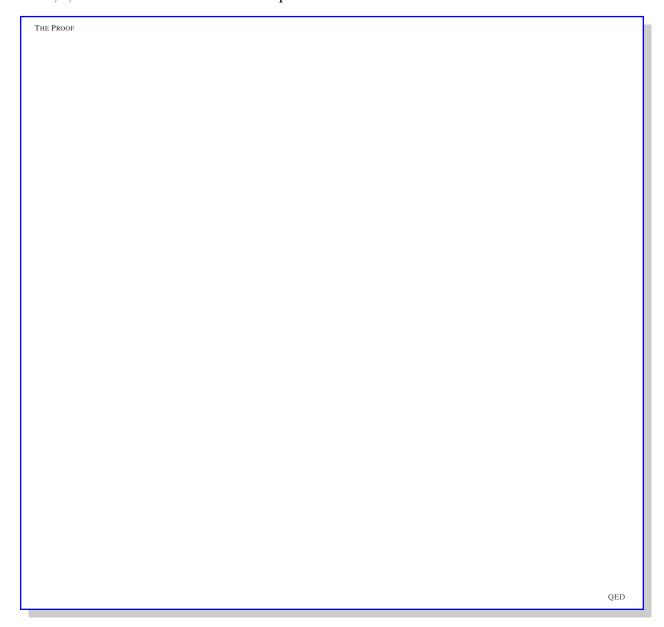
Name:	_ Student ID:	
Week4-template		Math 563, Fall 2022
(Durrett 2.1.3.) Let $\rho(x,y)$ be a metric. (i) Suppose h is differentiable with $h(0) = 0$, $h'(x)$ Then $h(\rho(x,y))$ is a metric	> 0 for x > 0 and h'(x)	decreasing on $[0, \infty)$.
THE PROOF		
		QED
h(x) = x/(x+1) satisfies the hypothesis in (i)		
THE PROOF		

Q 1.2.

Q 1. Q 1.1.

THE PROOF	
	QED

Q 2. (Durrett 2.1.4.) Let $\Omega = (0,1)$, $\mathcal{F} = \text{Borel sets}$, $\mathbb{P} = \text{Lebesgue measure}$. $X_n(\omega) = \sin(2\pi n\omega)$, $n = 1, 2, \ldots$ are uncorrelated but not independent.



Q 3.	(Durrett 2.1.7.) Let $K \ge 3$ be a prime and let X and Y be independent random variables that are
	uniformly distributed on $\{0, 1, \dots, K-1\}$. For $0 \le n < K$, let $Z_n = X + nY \mod K$.

Q 3.1. Show that Z_0, Z_1, \dots, Z_{K-1} are pairwise independent, i.e., each pair is independe	Q	3.	1.	Show that	Z_0, Z	Z_1,\ldots,Z_{I}	ζ_{-1} are	pairwise	inde	pendent,	i.e.,	each	pair i	s inde	pende	nt.
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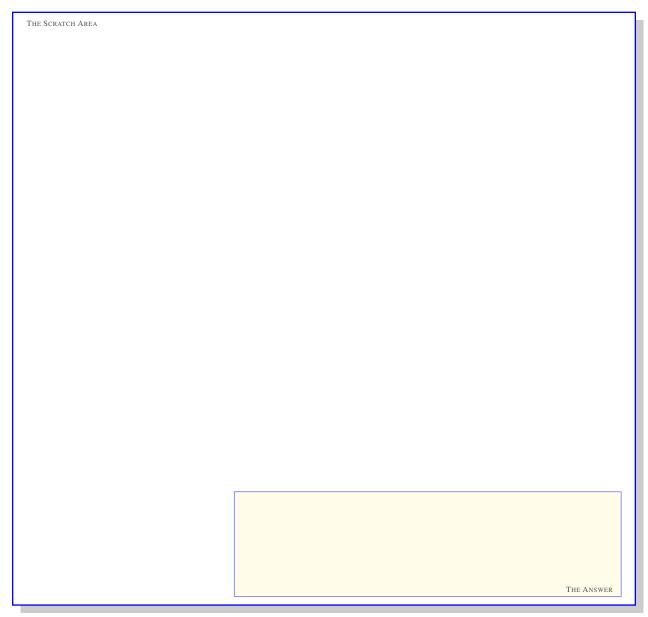


Q 3.2. They are not independent because if we know the values of two of the variables then we know the values of all the variables.

(NOTE: Prove that the value of two determine the values of all)



Q 4.	(Durrett 2.1.14.) Let $X, Y \ge 0$ be independent with distribution functions F and G . Fir	d the
	distribution function of XY .	



Q 5.	(Durrett 2.1.15.) If we want an infinite sequence of coin tossings, we do not have to use Kol-
	mogorov's theorem. Let Ω be the unit interval $(0,1)$ equipped with the Borel sets $\mathcal F$ and Lebesgue
	measure \mathbb{P} . Let $Y_n(\omega) = 1$ if $\lfloor 2^n \omega \rfloor$ is odd and 0 if $\lfloor 2^n \omega \rfloor$ is even. Show that Y_1, Y_2, \ldots are independent
	dent with $\mathbb{P}(Y_k = 0) = P(Y_k = 1) = 1/2$.

