Name:	Student ID:	_
Week1-template		Math 563, Fall 2022

- (Analysis, Set Theory Review)
 Carefully state the **Axiom of Choice**. Use proper notation, consistent with the book. Q 1. Q 1.1.

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	THE ANSWER
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Q 1.2.	Let x_i , $i \in I$, be an uncountable family of positive real numbers. Let $ J $ denote the cardinality of a
	subset $J \subseteq I$. Prove that

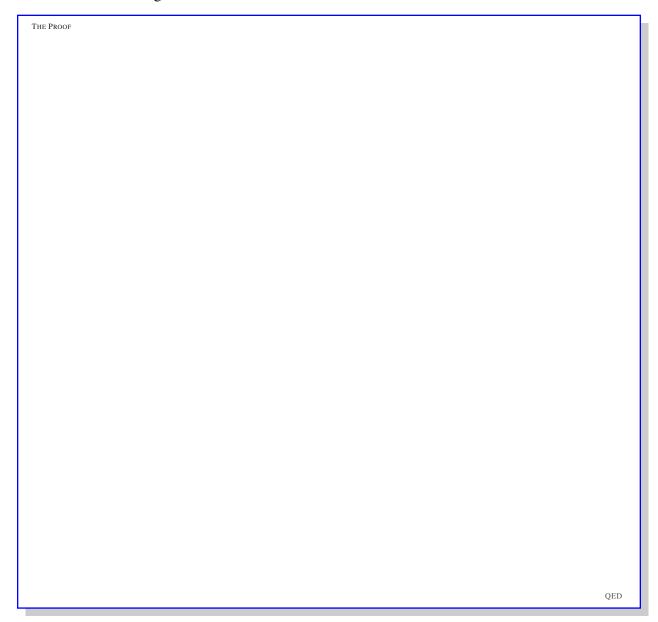
$$\sup_{J \in I, \, |J| < \infty} \sum_{i \in J} x_i = \infty.$$

Make sure to show the step which relies upon the Axiom of Choice.

THE PROOF	
	QED

- **Q 2.** (Durrett, Problem 1.1.1) Let $\Omega = \mathbb{R}$, $\mathcal{F} =$ all subsets so that A or A^c is countable, $\mathbb{P}(A) = 0$ in the first case and = 1 in the second. Show that $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space.
- **Q 2.1.** Which are the **axioms** of a σ -algebra? (When in doubt, follow the book.)
 - (A) closed under complement
 - (B) closed under finite intersection
 - (C) closed under countable union
 - (D) contains empty set Ø
 - (E) contains Ω
 - (F) contains all subsets of Ω

Q 2.2. Prove that \mathcal{F} is a σ -algebra.



- **Q 2.3.** Which are **properties** (including the axioms) of \mathbb{P} , a general probability measure? (When in doubt, follow the book.)
 - A The domain of \mathbb{P} is 2^{Ω} .
 - \bigcirc The domain of \mathbb{P} is \mathcal{F} .
 - \bigcirc $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ for $A, B \in \mathcal{F}$.

 - $(E) \mathbb{P}(A^c) = 1 \mathbb{P}(A)$
 - $(F) \mathbb{P}(\emptyset) = 0.$
 - G If $\mathbb{P}(A) = 0$ then $A = \emptyset$.
 - (H) If $A \subseteq B$ then $\mathbb{P}(A) \leq \mathbb{P}(B)$ for $A, B \in \mathcal{F}$.

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Q 3.	(Durret 1.1.2) Let S_d =	= the empty set	plus all sets	of the form
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$$(a_1,b_1]\times\ldots(a_d,b_d]\subset\mathbb{R}^d$$

where $-\infty \le a_i < b_i \le \infty$. Show that $\sigma(S_d) = \mathbb{R}^d$, the Borel subsets of \mathbb{R}^d .

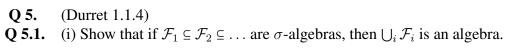
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he Proof		
		QE

(Durret 1.1.3) A σ -field $\mathcal F$ is said to be countably generated if there is a countable collection $\mathcal C\subseteq\mathcal F$

Q 4.

Q 5.



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	QED

(ii) Give an example to show that $\bigcup_i \mathcal{F}_i$ need not be a σ -algebra. THE SCRATCH AREA THE ANSWER

Q 5.2.

Q 6. (Durrett 1.1.5) A set $A \subseteq \{1, 2, ...\}$ is said to have asymptotic density θ if

$$\lim_{n\to\infty}|A\cap\{1,2,\dots,n\}|/n=\theta$$

- Let A be the collection of sets for which the asymptotic density exists.
- **Q 6.1.** Is \mathcal{A} a σ -algebra?
 - (A) Yes
 - (B) No
- **Q 6.2.** an algebra?
 - (A) Yes
 - (B) No

Q 6.3. Justify the answers in previous parts. THE PROOF QED