Name:	Student ID:	
Week2-template		Math 563, Fall 2022

- **Q 1.** (Durrett 1.2.1) Suppose X and Y are random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ and let $A \in \mathcal{F}$. Show that if we let $Z(\omega) = X(\omega)$ for $\omega \in A$ and $Z(\omega) = Y(\omega)$ for $\omega \in A^c$, then Z is a random variable.
- **Q 1.1.** Complete the equation by replacing \square with the correct formula:

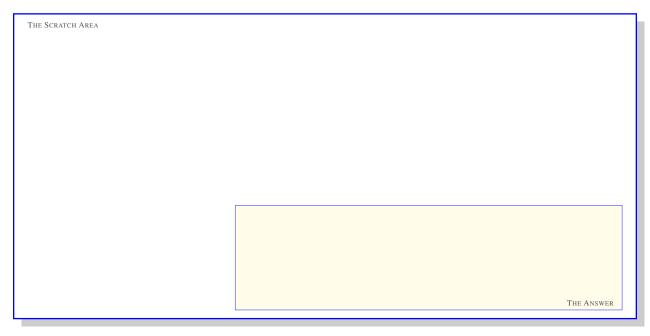
$$Z^{-1}(B) = (A \cap (\square)) \cup (A^c \cap (\square))$$

THE SCRATCH AREA	
	THE ANSWER

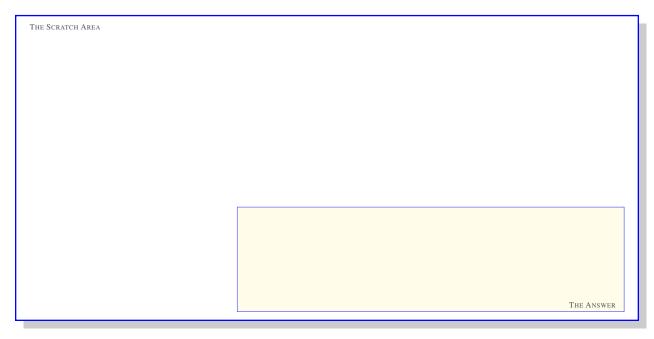
Q 1.2. Complete the proof.

THE PROOF	
	OFF
	QED

- **Q 2.** (Durrett 1.2.2) Let χ have the standard normal distribution. Use Theorem 1.2.6 to get upper and lower bounds on $\mathbb{P}(\chi \geq 4)$.
- **Q 2.1.** What is the upper bound?



Q 2.2. What is the lower bound?



HE PROOF			
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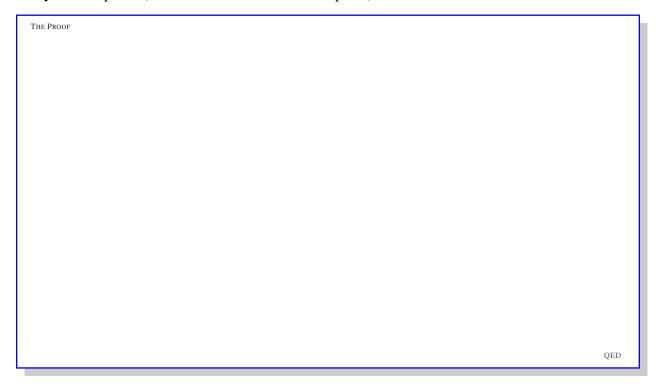
Q 3.

Q 4.	(Durrett 1.2.4) Show that if $F(x) = \mathbb{P}(X \leq x)$ is continuous then $Y = F(X)$ has a uniform
	distribution on $(0,1)$, that is, if $y \in [0,1]$, $P(Y \le y) = y$.

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O 4.1.	Prove the co	onclusion v	with an	additional	assumption	on that	F is strictly	increasing.

THE PROOF	
	QED

 ${\bf Q}$ 4.2. Carry out the proof (without this additional assumption).



Q 5.	(Durrett 1.2.5) Suppose X has continuous density f , $\mathbb{P}(\alpha \leq X \leq \beta) = 1$ and g is a function that
	is strictly increasing and differentiable on (α, β) . Then

Q 5.1. g(X) has density $f(g^{-1}(y))/g'(g^{-1}(y))$ for $y \in (g(\alpha), g(\beta))$ and 0 otherwise.

THE PROOF

Q 5.2. When g(x) = ax + b with a > 0, $g^{-1}(y) = (y - b)/a$ so the answer is (1/a)f((y - b)/a).

THE PROOF

he Proof		

Q 6.

Q 7.	(Durrett	1.2.7)
V /•	Durion	1.4.1	,

Q 7.1.	(i) Suppose X has density function f . Compute the distribution function of X^2 and then differen-
	tiate to find its density function.



Q 7.2. (ii) Work out the answer when X has a standard normal distribution to find the density of the chi-square distribution.



Q 8. (Custom Problem, generalizes Durrett 1.2.5) Suppose X has continuous, density f, and g is a continuously differentiable function (not necessarily increasing) such that that the set

$$\{y \in \mathbb{R} : g'(y) = 0\}$$

has Lebesgue measure zero.

Q 8.1. Let Y = g(X). Prove that Y has density h given by the equation

$$h(x) = \sum_{y \in g^{-1}(x)} \frac{f(y)}{|g'(y)|}.$$

THE PROOF	
	QED

Q 8.2. Let $g:(0,1]\to(0,1]$ be the *fractional part* of 1/x, i.e. function defined by

$$g(x) = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor$$

Let X be a continuous random variable whose distribution has probability density function (pdf):

$$f(x) = \begin{cases} \frac{1}{\pi} \frac{1}{1+x}, & 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that Y=g(X) is also a continuous variable with the same pdf f.

THE PROOF	
	QED