Name:	Student ID:	
Week2-template		Math 563, Fall 2022

- **Q 1.** (Durrett 1.2.1) Suppose X and Y are random variables on  $(\Omega, \mathcal{F}, \mathbb{P})$  and let  $A \in \mathcal{F}$ . Show that if we let  $Z(\omega) = X(\omega)$  for  $\omega \in A$  and  $Z(\omega) = Y(\omega)$  for  $\omega \in A^c$ , then Z is a random variable.
- **Q 1.1.** Complete the equation by replacing  $\square$  with the correct formula:

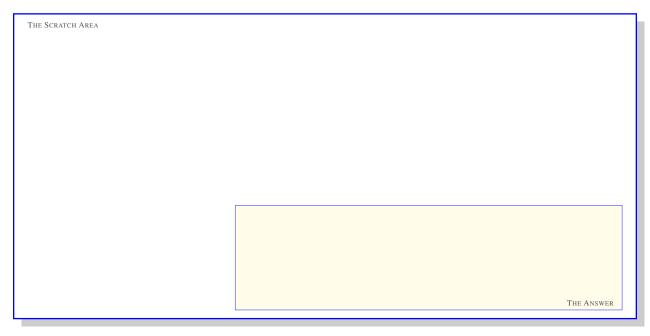
$$Z^{-1}(B) = (A \cap (\square)) \cup (A^c \cap (\square))$$

THE SCRATCH AREA	
	THE ANSWER

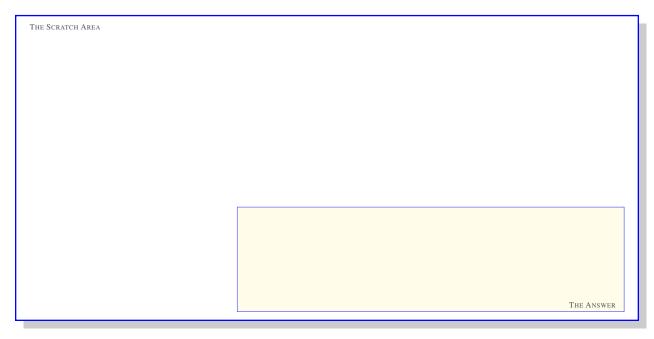
**Q 1.2.** Complete the proof.

THE PROOF	
	OFF
	QED

- **Q 2.** (Durrett 1.2.2) Let  $\chi$  have the standard normal distribution. Use Theorem 1.2.6 to get upper and lower bounds on  $\mathbb{P}(\chi \geq 4)$ .
- **Q 2.1.** What is the upper bound?



**Q 2.2.** What is the lower bound?



HE PROOF			
			QE

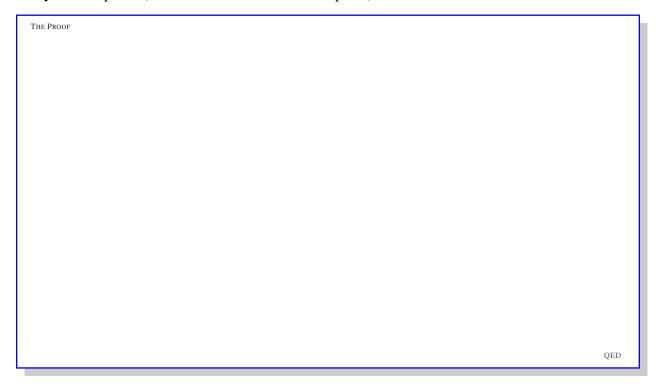
Q 3.

Q 4.	(Durrett 1.2.4) Show that if $F(x) = \mathbb{P}(X \leq x)$ is continuous then $Y = F(X)$ has a uniform
	distribution on $(0,1)$ , that is, if $y \in [0,1]$ , $P(Y \le y) = y$ .

				-	- `	- /		
O 4.1.	Prove the co	onclusion v	with an	additional	assumption	on that	F is strictly	increasing.

THE PROOF	
	QED

 ${\bf Q}$  4.2. Carry out the proof (without this additional assumption).



Q 5.	(Durrett 1.2.5) Suppose $X$ has continuous density $f$ , $\mathbb{P}(\alpha \leq X \leq \beta) = 1$ and $g$ is a function that
	is strictly increasing and differentiable on $(\alpha, \beta)$ . Then

**Q 5.1.** g(X) has density  $f(g^{-1}(y))/g'(g^{-1}(y))$  for  $y \in (g(\alpha), g(\beta))$  and 0 otherwise.

THE PROOF

**Q 5.2.** When g(x) = ax + b with a > 0,  $g^{-1}(y) = (y - b)/a$  so the answer is (1/a)f((y - b)/a).

THE PROOF

he Proof		

Q 6.

Q 7.	(Durrett	1.2.7	)
<b>V</b> /•	Durion	1.4.1	,

Q 7.1.	(i) Suppose $X$ has density function $f$ . Compute the distribution function of $X^2$ and then differen-
	tiate to find its density function.



**Q 7.2.** (ii) Work out the answer when X has a standard normal distribution to find the density of the chi-square distribution.



**Q 8.** (Custom Problem, generalizes Durrett 1.2.5) Suppose X has continuous density f, and g is a continuously differentiable function (not necessarily increasing) such that that the set

$$\{y \in \mathbb{R} : g'(y) = 0\}$$

has Lebesgue measure zero.

**Q 8.1.** Let Y = g(X). Prove that Y has density h given by the equation

$$h(x) = \sum_{y \in g^{-1}(x)} \frac{f(y)}{|g'(y)|}.$$

THE PROOF	
	QED

**Q 8.2.** Let  $g:(0,1]\to(0,1]$  be the *fractional part* of 1/x, i.e. function defined by

$$g(x) = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor$$

Let X be a continuous random variable whose distribution has probability density function (pdf):

$$f(x) = \begin{cases} \frac{1}{\pi} \frac{1}{1+x}, & 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that Y=g(X) is also a continuous variable with the same pdf f.

THE PROOF	
	QED