# Parallel Programing

## With MATLAB Examples

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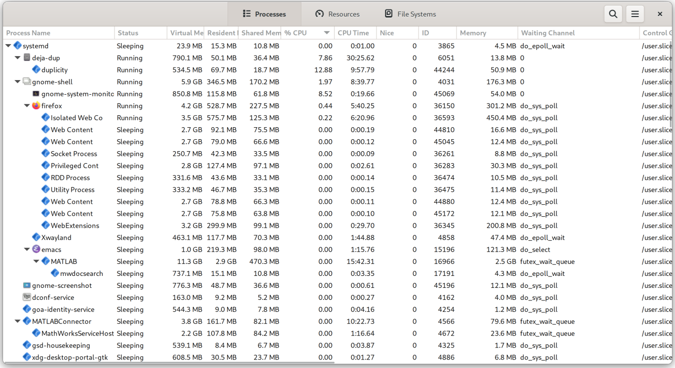
# Parallelism in the OS

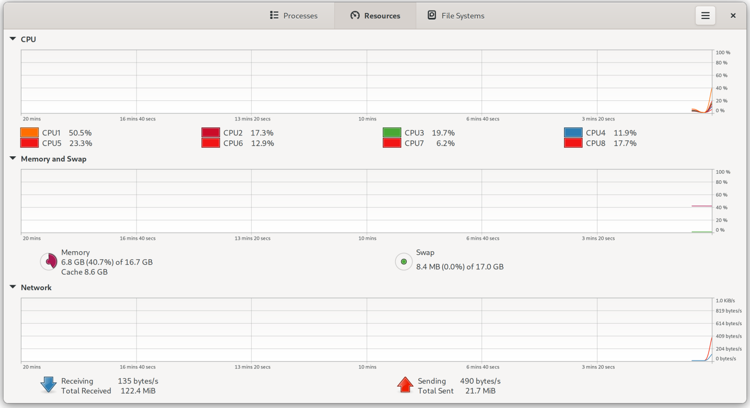
- A modern OS has a multitude of processes running, as shown by a system monitor
- OS creates an illusion of parallelism even if it runs on a single CPU not capable of multi-threading in hardware.




| Process Name | Status | Virtual Me | Resident I | Shared Mem | % CPU | CPU Time | Nice | ID | Memory | Waiting Channel | Control G |
|---|---|---|---|---|---|---|---|---|---|---|---|
| systemd | Sleeping | 23.9 MB | 15.3 MB | 10.8 MB | 0.00 | 0:01.00 | 0 | 3865 | 4.5 MB | do_epoll_wait | /user.slice |
| deja-dup | Running | 790.1 MB | 50.1 MB | 36.4 MB | 7.86 | 30:25.62 | 0 | 6051 | 13.8 MB | 0 | /user.slice |
| duplicity | Running | 534.5 MB | 69.7 MB | 18.7 MB | 12.88 | 9:57.79 | 0 | 44244 | 50.9 MB | 0 | /user.slice |
| gnome-shell | Running | 5.9 GB | 346.5 MB | 170.2 MB | 1.97 | 8:39.77 | 0 | 4031 | 176.3 MB | 0 | /user.slice |
| gnome-system-monitc | Running | 850.8 MB | 115.8 MB | 61.8 MB | 8.52 | 0:19.66 | 0 | 45069 | 54.0 MB | 0 | /user.slice |
| firefox | Running | 4.2 GB | 528.7 MB | 227.5 MB | 0.44 | 5:40.25 | 0 | 36150 | 301.2 MB | do_sys_poll | /user.slice |
| Isolated Web Co | Running | 3.5 GB | 575.7 MB | 125.3 MB | 0.22 | 6:20.96 | 0 | 36593 | 450.4 MB | do_sys_poll | /user.slice |
| Web Content | Sleeping | 2.7 GB | 92.1 MB | 75.5 MB | 0.00 | 0:00.19 | 0 | 44810 | 16.6 MB | do_sys_poll | /user.slice |
| Web Content | Sleeping | 2.7 GB | 79.0 MB | 66.6 MB | 0.00 | 0:00.12 | 0 | 45045 | 12.4 MB | do_sys_poll | /user.slice |
| Socket Process | Sleeping | 250.7 MB | 42.3 MB | 33.5 MB | 0.00 | 0:00.09 | 0 | 36261 | 8.8 MB | do_sys_poll | /user.slice |
| Privileged Cont | Sleeping | 2.8 GB | 127.4 MB | 97.1 MB | 0.00 | 0:02.61 | 0 | 36283 | 30.3 MB | do_sys_poll | /user.slice |
| RDD Process | Sleeping | 331.6 MB | 43.6 MB | 33.1 MB | 0.00 | 0:00.14 | 0 | 36474 | 10.5 MB | do_sys_poll | /user.slice |
| Utility Process | Sleeping | 333.2 MB | 46.7 MB | 35.3 MB | 0.00 | 0:00.15 | 0 | 36475 | 11.4 MB | do_sys_poll | /user.slice |
| Web Content | Sleeping | 2.7 GB | 78.8 MB | 66.3 MB | 0.00 | 0:00.11 | 0 | 44880 | 12.4 MB | do_sys_poll | /user.slice |
| Web Content | Sleeping | 2.7 GB | 75.8 MB | 63.8 MB | 0.00 | 0:00.10 | 0 | 45172 | 12.1 MB | do_sys_poll | /user.slice |
| WebExtensions | Sleeping | 3.2 GB | 299.9 MB | 99.1 MB | 0.00 | 0:29.70 | 0 | 36345 | 200.8 MB | do_sys_poll | /user.slice |
| Xwayland | Sleeping | 463.1 MB | 117.7 MB | 70.3 MB | 0.00 | 1:44.88 | 0 | 4858 | 47.4 MB | do_epoll_wait | /user.slice |
| emacs | Sleeping | 1.0 GB | 219.3 MB | 98.0 MB | 0.00 | 1:15.76 | 0 | 15196 | 121.3 MB | do_select | /user.slice |
| MATLAB | Sleeping | 11.3 GB | 2.9 GB | 470.3 MB | 0.00 | 15:42.31 | 0 | 16966 | 2.5 GB | futex_wait_queue | /user.slice |
| mwdocsearch | Sleeping | 737.1 MB | 15.1 MB | 10.8 MB | 0.00 | 0:03.35 | 0 | 17191 | 4.3 MB | do_epoll_wait | /user.slice |
| gnome-screenshot | Sleeping | 776.3 MB | 48.7 MB | 36.6 MB | 0.00 | 0:00.61 | 0 | 45196 | 12.1 MB | do_sys_poll | /user.slice |
| dconf-service | Sleeping | 163.0 MB | 9.2 MB | 5.2 MB | 0.00 | 0:00.27 | 0 | 4162 | 4.0 MB | do_sys_poll | /user.slice |
| goa-identity-service | Sleeping | 544.3 MB | 9.0 MB | 7.8 MB | 0.00 | 0:04.16 | 0 | 4254 | 1.2 MB | do_sys_poll | /user.slice |
| MATLABConnector | Sleeping | 3.8 GB | 161.7 MB | 82.1 MB | 0.00 | 10:22.73 | 0 | 4566 | 79.6 MB | futex_wait_queue | /user.slice |
| MathWorksServiceHost | Sleeping | 2.2 GB | 107.8 MB | 84.2 MB | 0.00 | 1:16.64 | 0 | 4672 | 23.6 MB | futex_wait_queue | /user.slice |
| gsd-housekeeping | Sleeping | 539.1 MB | 8.4 MB | 6.7 MB | 0.00 | 0:03.87 | 0 | 4325 | 1.7 MB | do_sys_poll | /user.slice |
| xdg-desktop-portal-gtk | Sleeping | 608.5 MB | 30.5 MB | 23.7 MB | 0.00 | 0:01.27 | 0 | 4886 | 6.8 MB | do_sys_poll | /user.slice |

Figure: Explanations

# How many CPUs/Hardware threads do I have?



# Forking in Bash (&) — a minimal variant

```
(sleep 1e-2; echo -n "Hello, ") & \
    (sleep 1e-2; echo -n "World!")
```

# Forking in Bash (&) I

```
#!/bin/bash
# EXAMPLE: Print 'Hello, ' and 'World!'
# in random order w/o a random number generator.
# HINT: We deliberately create a race condition.
if (($#)) ;then ntimes=$1 ;else ntimes=10; fi
function hello {
    echo -n "Hello, "
}
function world {
    echo -n "World!"
}
dlay=1e-2  # Change to 5 to see processes
for (( j=0; $j<$ntimes; j=$j+1 ))
do
    # Fork with '&'
    (sleep $dlay; hello) & (sleep $dlay; world)
    echo " --Done with iteration: $j"
done
```

# Forking in Bash (&) II

```
[marek@cannonball]$ ./forkme.sh
Hello, World! --Done with iteration: 1
Hello, World! --Done with iteration: 2
World!Hello,  --Done with iteration: 3
World!Hello,  --Done with iteration: 4
Hello, World! --Done with iteration: 5
World!Hello,  --Done with iteration: 6
Hello, World! --Done with iteration: 7
Hello, World! --Done with iteration: 8
Hello, World! --Done with iteration: 9
```

# A remarkable, more rare output

```
[marek@cannonball matlabmpi]$ ./forkme.sh
Hello, World! --Done with iteration: 1
Hello, World! --Done with iteration: 2
Hello, World! --Done with iteration: 3
Hello, World! --Done with iteration: 4
Hello, World! --Done with iteration: 5
Hello, World! --Done with iteration: 6
World! --Done with iteration: 7
Hello, World!Hello,  --Done with iteration: 8
Hello, World! --Done with iteration: 9
```

# A Glossary of Terms I

program counter
: The location (address) of the instruction currently being executed; a place in a program

process
: A running program with all necessary resources (program counter, open file descriptors, memory state)

fork, forking, clone
: The UNIX/Linux system call which allows one process to create another one

IPC, inter-process communication
: The protocol by which two distinct processes can exchange information

thread (of execution)
: Formerly known as a light-weight process directly shares the state of memory (variables) with other threads; threads have separate program counters; a modern process is a collection of threads

# A Glossary of Terms II

**process/thread synchronization** Mechanisms by which one process tells another not to mess with some sensitive parts of its state; IPC can be used for proces synchronization; threads are synchronized by mutexes

**mutex, futex** A mutually exclusive lock, which a simple integer (logical) variable which is set/unset (=acquired/released) by a thread. What is important is the interpretation by another thread. A thread agrees not to do certain things when mutex is acquired by another, until it is released. Semaphores generalize mutexes to arbitrary integer values. Futex is a fast mutex, introduced by the Linux OS.

# A Glossary of Terms III

**atomicity** Some operations need to be atomic, such as changing the value of a mutex/semaphore. Atomicity means that a thread that reads the value of a mutex does not get an inconsistent value while another thread is in the process of changing it. Normal variables cannot be used as mutexes because reading and writing to them is not atomic. Atomicity is implemented using hardware (special instructions) and compiler (awarness that some variables must be changed atomically).

# Mandelbrot set definition

## Definition (The Mandelbrot set)

The set of complex numbers $c$ for which the function $f_c(z) = z^2 + c$ does not diverge to $\infty$ when iterated from $z = 0$. In other words

$$\mathcal{M} = \left\{ c \in \mathbb{C} \,:\, \sup_n \left| \underbrace{f_c(f_c(\ldots(}_{n \text{ times}} 0)\ldots) \right| < \infty \right\}$$

# The Mandelbrot set on GPU

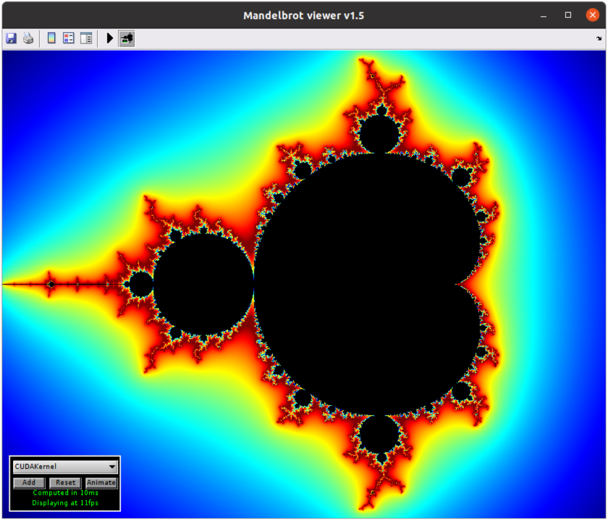



Figure: Example page: Mandelbrot Set

# The Julia definition

## Definition (The Julia set)

For fixed $c$, the set of complex numbers $z$ for which the function $f_c(z) = z^2 + c$ does not diverge to $\infty$ when iterated from $z$. In other words

$$\mathcal{J}_c = \left\{ z \in \mathbb{C} \,:\, \sup_n \left| \underbrace{f_c(f_c(\ldots(}_{n \text{ times}} z)\ldots) \right| < \infty \right\}$$

# The Julia set on GPU

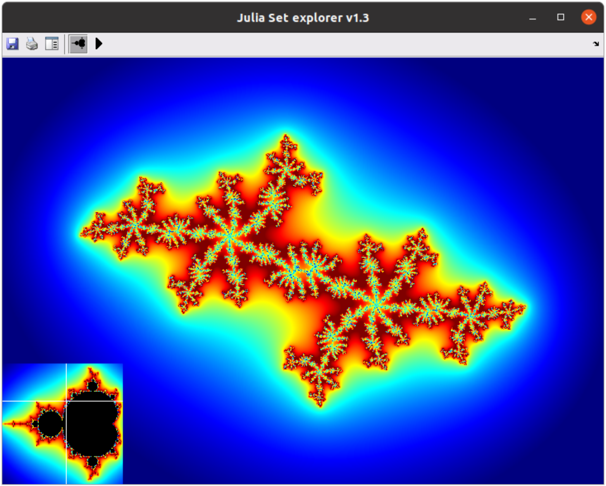



Figure: Example page: Look for a demo on MATLAB File Exchange

# MATLAB 'parfor' (parallel for) I

```
% FILE: parforEx.m
%p=gcp();
%mpiInit;
n=10;
% Evaluate x^2 for 1:n asynchronously and print results
parfor i=1:n
    x=i^2
    %pause(1);
    disp([i,x]);
end
disp('All done');
```

```
parforEx
     1     1

     5    25

     9    81

     2     4

    10   100

     3     9

     4    16

     6    36

     7    49
```

# MATLAB 'parfor' (parallel for) II

```
     8    64

All done
>>
```

## Question

Why does 'All done' print only once? Only at the end?

# Parallel pools, workers, clusters

- TIP: first install Parallel Computing Toolbox and try its GUI to configure a cluster
- Workers are a MATLAB abstraction of threads, and they should directly map to hardware (CPU, hardware threads)
- A paralel pool is a collection of workers under the management of the main thread
- A parallel pool can live on one or more CPUs, and can be distributed across many computers; these details are abstracted away
- A cluster is defined by a configuration file (a profile, eg., 'local.settings') and it specifies computers and the number of CPU used on each machine. The configuration file must be placed in one of several standard places (see 'help parcluster').

# Accumulating values, reduction variables I

```
% FILE: reductionVar.m
% This file demonstrates a useful notion of a 'Reduction Variable'
% Makes it possible to accumulate values in a parfor without using
% spmd/gop.

p=gcp('nocreate');
if isempty(p)
    p = parpool('local', 8)
end

disp(sprintf('Number of workers: %d', p.NumWorkers));

x=[];

parfor i = 1:10
    pause(rand());
    disp(i);
    x = [x, i];
end

x
```

```
>> reduction_var
Number of workers: 8

x =

     1     2     3     4     5     6     7     8     9    10
```

# Accumulating values, reduction variables II

## Fact

Deterministic: the answer is always the same.

### Reduction Variables

R2018b

MATLAB® supports an important exception, called reduction, to the rule that loop iterations must be independent. A *reduction variable* accumulates a value that depends on all the iterations together, but is independent of the iteration order. MATLAB allows reduction variables in `parfor`-loops.

Reduction variables appear on both sides of an assignment statement, such as any of the following, where `expr` is a MATLAB expression.

| | |
|---|---|
| `X = X + expr` | `X = expr + X` |
| `X = X - expr` | See Associativity in Reduction Assignments in Requirements for Reduction Assignments |
| `X = X .* expr` | `X = expr .* X` |
| `X = X * expr` | `X = expr * X` |
| `X = X & expr` | `X = expr & X` |
| `X = X \| expr` | `X = expr \| X` |
| `X = [X, expr]` | `X = [expr, X]` |
| `X = [X; expr]` | `X = [expr; X]` |
| `X = min(X, expr)` | `X = min(expr, X)` |
| `X = max(X, expr)` | `X = max(expr, X)` |
| `X = union(X, expr)` | `X = union(expr, X)` |
| `X = intersect(X, expr)` | `X = intersect(expr, X)` |

# SPMD and SIMD

SPMD Stands for “Single program, multiple data”. Multiple autonomous processors simultaneously execute the same program at independent points (program counters). Can be implemented on general purpose CPUs (Intel, AMD)

SIMD Stands for “Single-instruction, multiple data”. A vector processor processes the same instruction on different data (example: coordinatewise addition or multiplication of two vectors).

Modern CPU(s) implements both paradigms:

- SIMD uses Intel/AMD SSE instructions and vector registers;
- SPMD uses multiple threads, cores and CPUs.

# The MPI (Message Passing Interface)

- The most successful realization of SPMD; used in MATLAB; 40 years of history
- Implementations in C, C++, Fortran exist, with high-level language interfaces (e.g., Python).
- Worker becomes a lab
- Worker knows its identity, or labindex
- The main thread is now a lab with labindex==1 (recall: MATLAB has 1-based arrays)
- Labs communicate by using collective communications: labSend, labReceive, labSendReceive;
- synchronization: labBarrier, labBroadcast
- Labs can be organized as a graph with variable topology, e.g. edges of a hypercube, for the purpose of communicating with some neighbors

# Unintended blocking — a show stopper

- A blocking operation is one that stops the execution of the program (thread, process) until some condition is met
- Example: reading from a file. We wait for the data to be available (e.g., read from disk or network)
- Example: waiting for a mutex to be released
- labReceive, labBarrier are blocking operations
- A non-blocking operation does not wait for the condition to be met but immediately continues with the execution, reporting status to the caller
- Example: reading from a file in non-blockin mode reports the number of bytes successfully read. One repeatedly reads from the file, getting the file in chunks, until the end-of-file marker is found

# Deadlock (deadly embrace)

## Definition (Deadlock)

Deadlock (which is sometimes called the deadly embrace) occurs when two or more programs (threads, workers, labs) are each waiting for the others to complete a task before proceeding.

> *The programs act like the overly congenial gophers in some Looney Tunes cartoons:*
> *"Oh please, you first," says one.*
> *"No no, I insist, you first," says the other.*
> *And nothing goes anywhere.*
>
> *Michael Meehan, Computerworld, Oct 29, 2001*

# An 'spmd' example (WRONG!) I

```
% FILE: race1.m
% This file demonstrates a simple race condition
% when trying to share a value between all workers
p=gcp('nocreate');
if isempty(p)
    p = parpool('local', 8)
end

disp(sprintf('Number of workers: %d', p.NumWorkers));

value = Composite();

% An incorrect way to broadcast a value and
% receive it in all workers

spmd
    pause(rand()./10);
    if labindex == 1
        for w=1:numlabs
            display(sprintf('%d sending 7 to %d',labindex,w));
            labSend(7,w);
        end
    end
    value = labReceive;
    display(sprintf('%d received %d from 1',labindex,value));
end

for w=1:p.NumWorkers
    disp(value{w});
end
```

# An ’spmd’ example (WRONG!) II

```
>> race1
Number of workers: 8
Worker 1:
  1 sending 7 to 1
Error using race1
Error detected on worker 1.

Caused by:
    Error using race1
    Destination (1) is same as source, would cause deadlock.

>>
```

# An 'spmd' example (CORRECT!) I

```
% FILE: race1Fixed.m
% This file demonstrates a simple race condition
% when trying to share a value between all workers.
% NOTE: We avoid sending to ourselves. This
% avoids the race condition.

p=gcp('nocreate');
if isempty(p)
    p = parpool('local', 8)
end

disp(sprintf('Number of workers: %d', p.NumWorkers));

value = Composite();

% A correct way to broadcast a value and
% receive it in all workers. However, lab 1 runs in O(n) time,
% so it is not an efficient way to broadcast data to others.

spmd
    pause(rand()./10);

    if labindex == 1
        for w=2:numlabs
            display(sprintf('%d sending 7 to %d',labindex,w));
            labSend(7,w)
        end
        value = 7;
```

# An 'spmd' example (CORRECT!) II

```
    else
        value = labReceive;
        display(sprintf('%d received %d from 1',labindex,value));
    end

end

for w=1:p.NumWorkers
    disp([w,value{w}]);
end
```

```
>>
Number of workers: 8
Worker 1:
  1 sending 7 to 2
  1 sending 7 to 3
  1 sending 7 to 4
  1 sending 7 to 5
  1 sending 7 to 6
  1 sending 7 to 7
  1 sending 7 to 8
Worker 2:
  2 received 7 from 1
Worker 3:
  3 received 7 from 1
Worker 4:
  4 received 7 from 1
Worker 5:
```

# An 'spmd' example (CORRECT!) III

```
  5 received 7 from 1
Worker 6:
  6 received 7 from 1
Worker 7:
  7 received 7 from 1
Worker 8:
  8 received 7 from 1
     1     7

     2     7

     3     7

     4     7

     5     7

     6     7

     7     7

     8     7

>>
```

# Broadcasting (another fix) I

```
% FILE: race1FixedBroadcast.m
% This file demonstrates a simple race condition
% when trying to share a value between all workers
% NOTE: In this version, we avoid the race condition
% by using labBroadcast.

p=gcp('nocreate');
if isempty(p)
    p = parpool('local', 8)
end

disp(sprintf('Number of workers: %d', p.NumWorkers));

value = Composite();

% An incorrect way to broadcast a value and
% receive it in all workers

root=1;
spmd
    pause(rand()./10);
    if labindex == root
        value = 7;
        value = labBroadcast(root, value);
        display(sprintf('Root==%d broadcast %d',labindex,value));
    else
        value = labBroadcast(root, 666); % Second inut ignored on root
        display(sprintf('%d received %d from root==%d',labindex,value,root));
    end
end
```

# Broadcasting (another fix) II

```
for w=1:p.NumWorkers
    disp([w,value{w}]);
end
```

```
>>
Number of workers: 8
Worker 1:
  Root==1 broadcast 7
Worker 2:
  2 received 7 from root==1
Worker 3:
  3 received 7 from root==1
Worker 4:
  4 received 7 from root==1
Worker 5:
  5 received 7 from root==1
Worker 6:
  6 received 7 from root==1
Worker 7:
  7 received 7 from root==1
Worker 8:
  8 received 7 from root==1
     1     7

     2     7

     3     7

     4     7

     5     7
```

# Broadcasting (another fix) III

```
        6        7

        7        7

        8        7

>>
```

# Linear Troop Topology

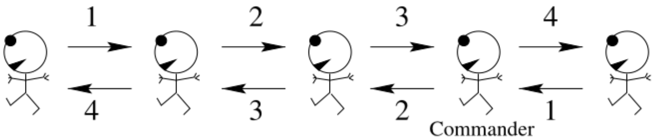



Figure: A line of soldiers counting themselves using message-passing rule-set A. The commander can add “3” from the soldier in front, “1” from the soldier behind, and “1” for himself, and deduce that there are 5 soldiers in total.

# Message-passing rule-set A (parallel pseudo-code).

1. If you are the front soldier in the line, say the number **one** to the soldier behind you.
2. If you are the rearmost soldier in the line, say the number **one** to the soldier in front of you.
3. If a soldier ahead of or behind you says a number to you, add one to it, and say the new number to the soldier on the other side.

# Implementation I

```
% FILE:     soldiers.m
% Mackay, algorithm 16.1: to count soldiers
% marching in line
%
% NOTE: If you are running the program on a
% processor with n=4 cores and 2*n
% hyperthreads, the setting for the 'local'
% cluster is used, and the number of workers
% in a parpool is set automatically to n,
% ignoring hyperthreading.  You can modify
% the number of worker threads using Matlab
% GUI, using Home > Parallel > Manage Cluster
% Profiles > Edit.  So, if you request 8
% workers, make sure to first edit the local
% profile and increate the number of allowed
% workers to >=8. I changed it to 64.
%
p = gcp('nocreate');
numSoldiers=5;
% Must have at least numSoldiers workers
if ~isempty(p) && p.NumWorkers < numSoldiers
    delete(p);
    p=[];
end
if isempty(p)
    % Create a local parpool with num. workers == num. soldiers
    p = parpool('local',numSoldiers);
end
mpiInit;
commander=2;
% Must have at least commander+1 workers
```

# Implementation II

```
assert(p.NumWorkers > commander);
spmd
    me=labindex;
    value=0;
    if me==commander
        [value1,source1,tag1]=labReceive;
        fprintf('%d=commander got %d from %d\n',me,value1,source1);
        [value2,source2,tag2]=labReceive;
        fprintf('%d=commander got %d from %d\n',me,value2,source2);
        value=value1+value2+1;
        fprintf('%d=commander says: count is %d\n',me,value);
    elseif me==1
        value=1;
        dest=2;
        fprintf('%d sending %d to %d\n',me,value,dest);
        labSend(value,dest);
    elseif me==numlabs
        value=1;
        dest=me-1;
        fprintf('%d sending %d to %d\n',me,value,dest);
        labSend(value,dest);
    else
        [value,source,~]=labReceive;
        value=value+1;
        if source==me-1
            dest=me+1;
        elseif source==me+1
            dest=me-1;
        end
        fprintf('%d sending %d to %d\n',me,value,dest);
        labSend(value,dest);
```

# Implementation III

```
        end
    end
```

# General topology of the troop

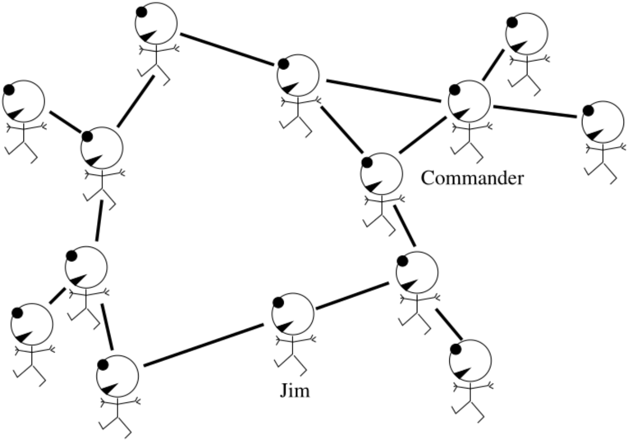



Figure: A swarm of guerillas.

# Arranging workers/labs into a graph I

```
% FILE:    buildTroop.m

% Efficient graph encoding (often used in MPI programs)
% Adjacency matrix; passed as 1-d aray, in which each vertex is followed
% Row format: node followed by neighbors, sentinel 0.
% A final zero is added to terminate the structure.
adj=[1,2,0,...
     2,1,3,11,0,...
     3,2,4,5,0,...
     4,3,0,...
     5,3,0,...
     6,8,0,...
     7,8,0,...
     8,6,7,9,0,...
     9,8,10,12,0,...
     10,9,11,0,...
     11,2,10,0,...
     12,9,13,14,0,...
     13,12,0,...
     14,12,0,...
     0];

% Automatically determine te number of soldiers (nodes)
numSoldiers = numel(find(adj==0))-1;
commander = 9;                      % Designate the commander

% Start the parpool (thread pool)
p = gcp('nocreate');
```

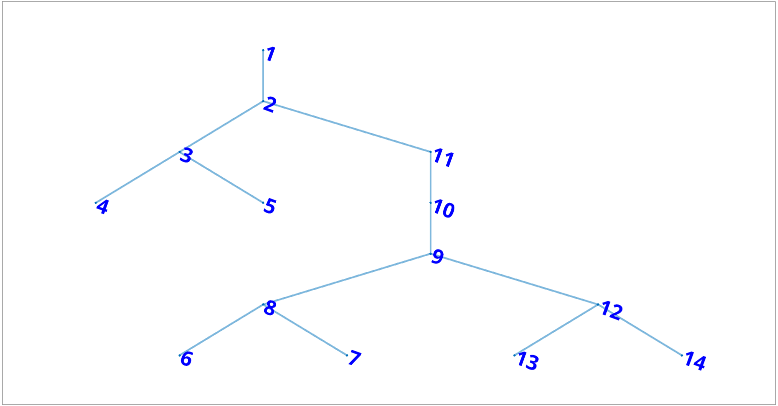
# Arranging workers/labs into a graph II

```
if ~isempty(p) && p.NumWorkers ~= numSoldiers
    delete(p);
    p=[];
end
if isempty(p)
    p = parpool('local',numSoldiers);
end


mpiInit;

% Convert nb to cell array
nb=Composite();
start=1;
for s=1:numSoldiers;
    me=adj(start);
    neighbors=[];
    n=start+1;
    % Make a list of neigbors
    while adj(n)~=0
        neighbors=[neighbors,adj(n)];
        n=n+1;
    end
    nb{me}=neighbors;
    start=n+1;
end
assert(adj(start)==0);
```

# Arranging workers/labs into a graph III

```
% A normal function call to let soldiers report the neighbors
report(nb);

% A consistency check for graph data
I=adjacency_matrix(nb);
% Check symmetry of the adjacency relation
assert(all(all(I==I')));
% Check for 'no loops' (loop=connection of edge to itself)
assert(all(diag(I)==0));
% Check for 'no cycles'; upper triangular portion of I should be nilpotent
assert(all(all( triu(I)^numSoldiers==0)));

%g = graph(I);
%plot(g,'LineWidth',4,'NodeFontSize',44,'MarkerSize',5,'NodeLabelColor','blue','
      NodeFontWeight','bold');
```

# A tree topology of the troop



# Message-passing rule-set B. I

**procedure** MESSAGEPASSING(*Graph*)
    $N \leftarrow$ the count your neighbours in *Graph*
    $m \leftarrow 0$ ▷ count of messages received from neighbours
    **for** $j$ from 1 to $N$ **do**
        $v_j \leftarrow -1$ ▷ initial value of message is invalid
    **end for**
    $V \leftarrow 0$ ▷ running total of messages you have received
    **if** $m == N - 1$ **then**
        Find neighbor $j$ such that $v_j == -1$ ▷ the only one who has not sent you a message
        Tell them the number $V + 1$
    **end if**

# Message-passing rule-set B. II

    **if** $V == N$ **then**<br>
        the number $V + 1$ is the required total.<br>
        **for** each neighbour $n$ **do**<br>
            say to neighbour $n$ the number $V + 1 - v_n$<br>
        **end for**<br>
    **end if**<br>
**end procedure**

# Implementation I

```
% FILE: soldiers2.m
% NOTE: Make sure to modify the parpool to allow 14 workers.

buildTroop;

% Main course: count the soldiers by message passing
spmd
    me=labindex;
    N=length(nb);                  % Neighbor count
    m=0;                           % Message count
    v=-ones(N,1);                  % Message values
    V=0;                           % Running total of messages

    labBarrier;                    % Not needed, harmless, for demo purposes

    % Receive first N-2 messages
    while m < N-1
        [isDataAvail,source]=labProbe;
        if isDataAvail             % If available, get the data
            n=find(source==nb,1);  % Find which neighbor sent the msssage
            assert(~isempty(n));   % Otherwise it is not a neighbor
            fprintf('%d sees data available from %d...\n ',me,source);
            value=labReceive(source);
            fprintf('%d received value %d from %d.\n',me,value,source);
            m=m+1;
            v(n)=value;
            V=V+value;
        end
    end

    %labBarrier;                    % Will break the code!!!
```

# Implementation II

```
    assert(m==N-1);                     % Check number of messages

    % Send the message to who has not send us a message
    n=find(v==-1,1);          % Identify who has not send us a message
    dest=nb(n);
    value_to_send=V+1;
    fprintf('%d sending %d to %d...\n',me,value_to_send,dest);
    labSend(value_to_send,dest);
    %labSendReceive(value_to_send,dest);
    fprintf('%d completed sending %d to %d.\n',me,value_to_send,dest);
    fprintf('%d waiting for message from %d...\n',me,dest);
    [value,last_source]=labReceive(dest);
    fprintf('%d received %d from %d.\n',me,value,last_source);
    assert(last_source==dest);      % Last message source
    v(n)=value;
    V=V+value;
    m=m+1;

    %labBarrier;                          % Will break the code!!!

    assert(m==N);                       % Check message count

    % Send message to everyone except the one who was the last to send us
    % a message.
    for l=1:N
        if l==n
            continue;                   % Do not send again
        end
        value_to_send=V+1-v(l);
        fprintf('%d sending %d to %d...\n',me,value_to_send,nb(l));
```

# Implementation III

```
            labSend(value_to_send,nb(l));
            fprintf('%d completed sending %d to %d.\n',me,value_to_send,nb(l));
        end
        if me==commander
            fprintf('COMMANDER %d reporting count of %d.\n', me, V+1);
        end
        fprintf('%d is done.\n',me);
    end

    % A demonstration that a Composite is a kind of cell array
    % Print the totals of all soldiers.
    for n=1:numSoldiers
        fprintf('Running total of %d is %d.\n', n, V{n});
    end
```

# A modified implementation I

```
% FILE: soldiers3.m
% This is like soldiers2, but uses labSendReceive instead
% of 2 calls labSend/labReceive. It is preferred to do it this way,
% as there is a smaller chance of programming error causing a race
% condition (labReceive when there is noone sending, or labSend when
% there is noone waiting to labReceive).

buildTroop;

% Main course: count the soldiers by message passing
spmd
    me=labindex;
    N=length(nb);                    % Neighbor count
    m=0;                             % Message count
    v=-ones(N,1);                    % Message values
    V=0;                             % Running total of messages

    fprintf('%d reached barrier.\n', me);
    labBarrier;                      % Not needed, harmless, for demo purposes
    fprintf('%d crossed barrier.\n', me);

    % Receive first N-2 messages
    while m < N-1
        [isDataAvail,source]=labProbe;
        if isDataAvail% If available, get the data
            n=find(source==nb,1);   % Find which neighbor sent the msssage
            assert(~isempty(n));    % Otherwise it is not a neighbor
            fprintf('%d sees data available from %d...\n ',me,source);
            value=labReceive(source);
            fprintf('%d received value %d from %d.\n',me,value,source);
            m=m+1;
```

# A modified implementation II

```
            v(n)=value;
            V=V+value;
        end
    end

    %labBarrier;                        % Will break the code!!!

    assert(m==N-1);                     % Check number of messages

    % Send the message to who has not send us a message
    n=find(v==-1,1);          % Identify who has not send us a message
    dest=nb(n);
    fprintf('%d noticed not receiving from %d', dest);
    value_to_send=V+1;
    fprintf('%d sending %d to %d...\n',me,value_to_send,dest);
    value = labSendReceive(dest, dest, value_to_send);
    fprintf('%d received %d from %d.\n',me,value,dest);
    v(n)=value;
    V=V+value;
    m=m+1;

    %labBarrier;                        % Will break the code!!!

    assert(m==N);                       % Check message count

    % Send message to everyone except the one who was the last to send us
    % a message.
    for l=1:N
        if l==n
            continue;                   % Do not send again
        end
```

# A modified implementation III

```
            value_to_send=V+1-v(l);
            fprintf('%d sending %d to %d...\n',me,value_to_send,nb(l));
            labSend(value_to_send,nb(l));
            fprintf('%d completed sending %d to %d.\n',me,value_to_send,nb(l));
        end
        if me==commander
            fprintf('COMMANDER %d reporting count of %d.\n', me, V+1);
        end
        fprintf('%d is done.\n',me);
    end


    % A demonstration that a Composite is a kind of cell array
    % Print the totals of all soldiers.
    for n=1:numSoldiers
        fprintf('Running total of %d is %d.\n', n, V{n});
    end
```

# Sample output I

```
>> soldiers3
Worker  1:
  Soldier 1 reporting, sir! My neighbors are 2, sir!
Worker  2:
  Soldier 2 reporting, sir! My neighbors are 1,  3, 11, sir!
Worker  3:
  Soldier 3 reporting, sir! My neighbors are 2, 4, 5, sir!
Worker  4:
  Soldier 4 reporting, sir! My neighbors are 3, sir!
Worker  5:
  Soldier 5 reporting, sir! My neighbors are 3, sir!
Worker  6:
  Soldier 6 reporting, sir! My neighbors are 8, sir!
Worker  7:
  Soldier 7 reporting, sir! My neighbors are 8, sir!
Worker  8:
  Soldier 8 reporting, sir! My neighbors are 6, 7, 9, sir!
Worker  9:
  Soldier 9 reporting, sir! My neighbors are 8, 10, 12, sir!
Worker 10:
  Soldier 10 reporting, sir! My neighbors are 9, 11, sir!
Worker 11:
  Soldier 11 reporting, sir! My neighbors are 2, 10, sir!
Worker 12:
  Soldier 12 reporting, sir! My neighbors are 9, 13, 14, sir!
Worker 13:
  Soldier 13 reporting, sir! My neighbors are 12, sir!
Worker 14:
  Soldier 14 reporting, sir! My neighbors are 12, sir!
Worker  1:
  1 reached barrier.
```

# Sample output II

```
  1 crossed barrier.
  2 noticed not receiving from 1 sending 1 to 2...
  1 received 13 from 2.
  1 is done.
Worker  2:
  2 reached barrier.
  2 crossed barrier.
  2 sees data available from 3...
   2 received value 3 from 3.
  2 sees data available from 1...
   2 received value 1 from 1.
  11 noticed not receiving from 2 sending 5 to 11...
  2 received 9 from 11.
  2 sending 13 to 1...
  2 completed sending 13 to 1.
  2 sending 11 to 3...
  2 completed sending 11 to 3.
  2 is done.
Worker  3:
  3 reached barrier.
  3 crossed barrier.
  3 sees data available from 4...
   3 received value 1 from 4.
  3 sees data available from 5...
   3 received value 1 from 5.
  2 noticed not receiving from 3 sending 3 to 2...
  3 received 11 from 2.
  3 sending 13 to 4...
  3 completed sending 13 to 4.
  3 sending 13 to 5...
  3 completed sending 13 to 5.
```

# Sample output III

```
  3 is done.
Worker  4:
  4 reached barrier.
  4 crossed barrier.
  3 noticed not receiving from 4 sending 1 to 3...
  4 received 13 from 3.
  4 is done.
Worker  5:
  5 reached barrier.
  5 crossed barrier.
  3 noticed not receiving from 5 sending 1 to 3...
  5 received 13 from 3.
  5 is done.
Worker  6:
  6 reached barrier.
  6 crossed barrier.
  8 noticed not receiving from 6 sending 1 to 8...
  6 received 13 from 8.
  6 is done.
Worker  7:
  7 reached barrier.
  7 crossed barrier.
  8 noticed not receiving from 7 sending 1 to 8...
  7 received 13 from 8.
  7 is done.
Worker  8:
  8 reached barrier.
  8 crossed barrier.
  8 sees data available from 7...
   8 received value 1 from 7.
  8 sees data available from 6...
```

# Sample output IV

```
    8 received value 1 from 6.
   9 noticed not receiving from 8 sending 3 to 9...
   8 received 11 from 9.
   8 sending 13 to 6...
   8 completed sending 13 to 6.
   8 sending 13 to 7...
   8 completed sending 13 to 7.
   8 is done.
Worker  9:
   9 reached barrier.
   9 crossed barrier.
   9 sees data available from 12...
    9 received value 3 from 12.
   9 sees data available from 8...
    9 received value 3 from 8.
   10 noticed not receiving from 9 sending 7 to 10...
   9 received 7 from 10.
   9 sending 11 to 8...
   9 completed sending 11 to 8.
   9 sending 11 to 12...
   9 completed sending 11 to 12.
   COMMANDER 9 reporting count of 14.
   9 is done.
Worker 10:
   10 reached barrier.
   10 crossed barrier.
   10 sees data available from 9...
    10 received value 7 from 9.
   11 noticed not receiving from 10 sending 8 to 11...
   10 received 6 from 11.
   10 sending 7 to 9...
```

# Sample output V

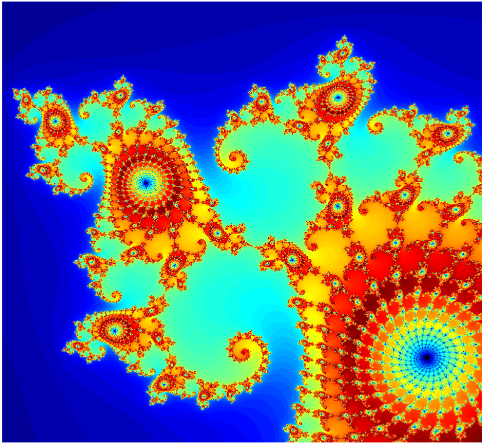
```
  10 completed sending 7 to 9.
  10 is done.
Worker 11:
  11 reached barrier.
  11 crossed barrier.
  11 sees data available from 2...
   11 received value 5 from 2.
  10 noticed not receiving from 11 sending 6 to 10...
  11 received 8 from 10.
  11 sending 9 to 2...
  11 completed sending 9 to 2.
  11 is done.
Worker 12:
  12 reached barrier.
  12 crossed barrier.
  12 sees data available from 14...
   12 received value 1 from 14.
  12 sees data available from 13...
   12 received value 1 from 13.
  9 noticed not receiving from 12 sending 3 to 9...
  12 received 11 from 9.
  12 sending 13 to 13...
  12 completed sending 13 to 13.
  12 sending 13 to 14...
  12 completed sending 13 to 14.
  12 is done.
Worker 13:
  13 reached barrier.
  13 crossed barrier.
  12 noticed not receiving from 13 sending 1 to 12...
  13 received 13 from 12.
```

# Sample output VI

```
  13 is done.
Worker 14:
  14 reached barrier.
  14 crossed barrier.
  12 noticed not receiving from 14 sending 1 to 12...
  14 received 13 from 12.
  14 is done.
Running total of 1 is 13.
Running total of 2 is 13.
Running total of 3 is 13.
Running total of 4 is 13.
Running total of 5 is 13.
Running total of 6 is 13.
Running total of 7 is 13.
Running total of 8 is 13.
Running total of 9 is 13.
Running total of 10 is 13.
Running total of 11 is 13.
Running total of 12 is 13.
Running total of 13 is 13.
Running total of 14 is 13.
>>
```

# Basic CPU Mandelbrot I

10.24secs (without GPU)



# Basic CPU Mandelbrot II

```
%-------------------------------------------------------------
% File:     mandelbrot.m
%-------------------------------------------------------------
%
% Author:   Marek Rychlik (rychlik@arizona.edu)
% Date:     Mon Feb 27 12:27:35 2023
% Copying:  (C) Marek Rychlik, 2020. All rights reserved.
%
%-------------------------------------------------------------
% Mandelbrot without GPU or parallelization (MATLAB stock example)
maxIterations = 500;
gridSize = 1000;
xlim = [-0.748766713922161, -0.748766707771757];
ylim = [ 0.123640844894862,  0.123640851045266];

% Setup
t = tic();
x = linspace( xlim(1), xlim(2), gridSize );
y = linspace( ylim(1), ylim(2), gridSize );
[xGrid,yGrid] = meshgrid( x, y );
z0 = xGrid + 1i*yGrid;
count = ones( size(z0) );

% Calculate
z = z0;
for n = 0:maxIterations
    z = z.*z + z0;
    inside = abs( z )<=2;
    count = count + inside;
end
count = log( count );
```

# Basic CPU Mandelbrot III

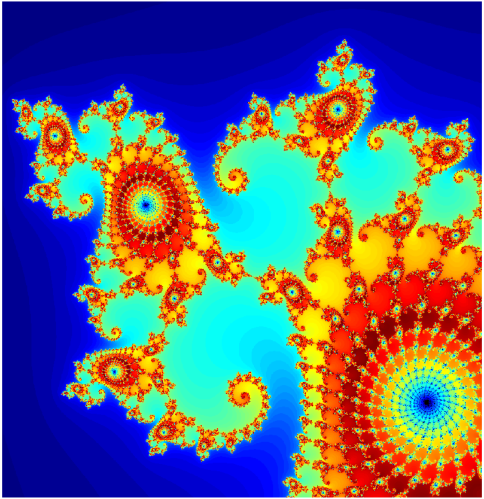
```

% Show
cpuTime = toc( t );
fig = gcf;
fig.Position = [200 200 600 600];
imagesc( x, y, count );
colormap( [jet();flipud( jet() );0 0 0] );
axis off
title( sprintf( '%1.2fsecs (without GPU)', cpuTime ) );
```

# Parallelized CPU Mandelbrot (8 workers) I

NOTE: Scaled up, higher resolution, larger radius (non-parallel time: 35 sec).

# Parallelized CPU Mandelbrot (8 workers) II

20.56 secs (without GPU, num. workers: 8, radius: 4.00, radius)



# Parallelized CPU Mandelbrot (8 workers) — code I

```
function count = mandel(x1,x2,y1,y2,gridSize,maxIterations,radius)
    x = linspace( x1, x2, gridSize(1) );
    y = linspace( y1, y2, gridSize(2) );
    [xGrid,yGrid] = meshgrid( x, y );
    z0 = xGrid + 1i*yGrid;
    count = ones( size(z0) );

    % Calculate
    z = z0;
    for n = 0:maxIterations
        z = z.*z + z0;
        inside = abs( z )<=radius;
        count = count + inside;
    end
    count = log( count );
end
```

# Parallelized CPU Mandelbrot (8 workers) — code II

```
%-----------------------------------------------------------------
% File:      mandelbrotParallel.m
%-----------------------------------------------------------------
%
% Author:    Marek Rychlik (rychlik@arizona.edu)
% Date:      Mon Feb 27 12:32:56 2023
% Copying:   (C) Marek Rychlik, 2020. All rights reserved.
%
%-----------------------------------------------------------------
% Mandelbrot without GPU, parfor (MATLAB stock example, modified)
p=gcp('nocreate');
if isempty(p)
    p = parpool('local', 8)
end
disp(sprintf('Number of workers: %d', p.NumWorkers));

maxIterations = 500;
gridSize = [2048,2048];                    % Must be divisible by 8
radius=4;
xlim = [-0.748766713922161, -0.748766707771757];
ylim = [ 0.123640844894862,  0.123640851045266];

% Setup
x1 = xlim(1); x2=xlim(2); y1=ylim(1); y2=ylim(2);

% Non-parallel calculation
tic;count0 = mandel(x1, x2, y1, y2,gridSize,maxIterations,radius);disp('Non-
    parallel time');toc
```

# Parallelized CPU Mandelbrot (8 workers) — code III

```
numSlices = p.NumWorkers*32;
dx = (x2-x1)./numSlices;
gridSizeParallel = gridSize./[numSlices,1];
count = [];

q = Par(numSlices);                    % Par is a utility class for benchmarking
    parallel loops
parfor j=1:numSlices
    Par.tic;
    countLocal = mandel(x1 + (j-1).*dx, x1 + j.*dx, y1, y2, gridSizeParallel,
        maxIterations,radius);
    count = [count, countLocal];
    q(j)=Par.toc;
end
stop(q);plot(q);

% Show
cpuTime = q.StopTime;
fig = gcf;
fig.Position = [200 200 1024 1024];
imagesc( x, y, count );
colormap( [jet();flipud( jet() );0 0 0] );
axis off
title( sprintf( '%1.2f secs (without GPU, num. workers: %d, radius: %1.2f,
    radius)', cpuTime, p.NumWorkers, radius ) );
```

# What has been left out?

- OpenMP (e.g., GOMP=GNU OpenMP); a high level interface to threads (SPMD) and vectorization (SIMD); realized as C/Fortran compiler pragmas (annotations)
- POSIX threads ("pthreads"); a C library available on most OS which allows direct access to multi-threading and thread synchronization
- Extensive C++ language constructs supporting parallelism
- Building hardware; hardware is inherently parallel; the most straightforward hardware to build is FPGA (Field-Programmable Gate Arrays); programming languages Verilog and VHDL
- University of Arizona HPC facilities