MODULE *SlidingPuzzles*

EXTENDS *Integers*

VARIABLE *board*

$W \triangleq 4 \quad H \triangleq 5$

$Pos \triangleq (0 \,..\, W-1) \times (0 \,..\, H-1)$

$Piece \triangleq \text{SUBSET } Pos$

$$\begin{aligned} Klotski \triangleq \{&\{\langle 0, 0\rangle, \langle 0, 1\rangle\},\\ &\{\langle 1, 0\rangle, \langle 2, 0\rangle, \langle 1, 1\rangle, \langle 2, 1\rangle\},\\ &\{\langle 3, 0\rangle, \langle 3, 1\rangle\}, \{\langle 0, 2\rangle, \langle 0, 3\rangle\},\\ &\{\langle 1, 2\rangle, \langle 2, 2\rangle\}, \{\langle 3, 2\rangle, \langle 3, 3\rangle\},\\ &\{\langle 1, 3\rangle\}, \{\langle 2, 3\rangle\}, \{\langle 0, 4\rangle\}, \{\langle 3, 4\rangle\}\} \end{aligned}$$

$KlotskiGoal \triangleq \{\langle 1, 3\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle\} \in board$

$ChooseOne(S, P(\_)) \triangleq \text{CHOOSE } x \in S : P(x) \land \forall y \in S : P(y) \Rightarrow y = x$

$TypeOK \triangleq board \in \text{SUBSET } Piece$

Given a position and a set of empty positions return a set of appropriately filtered von *Neumann* neighborhood points

$$\begin{aligned} dir(p, es) \triangleq\ &\text{LET } dir \triangleq \{\langle 1, 0\rangle, \langle 0, 1\rangle, \langle -1, 0\rangle, \langle 0, -1\rangle\}\\ &\text{IN } \{d \in dir : \land \langle p[1] + d[1], p[2] + d[2]\rangle \in Pos\\ &\qquad\qquad\qquad \land \langle p[1] + d[1], p[2] + d[2]\rangle \notin es\} \end{aligned}$$

Given a position and a unit translation vector return a pair of pieces, before and after translation in opposite this vector direction

$$\begin{aligned} move(p, d) \triangleq\ &\text{LET } s \triangleq \langle p[1] + d[1], p[2] + d[2]\rangle\\ &\qquad pc \triangleq ChooseOne(board, \text{LAMBDA } pc : s \in pc)\\ &\text{IN } \langle pc, \{\langle q[1] - d[1], q[2] - d[2]\rangle : q \in pc\}\rangle \end{aligned}$$

Given specific free position and a set of all free positions return a set of boards updated by moving appropriate pieces to that free position

$$\begin{aligned} update(e, es) \triangleq\ &\text{LET } dirs \triangleq dir(e, es)\\ &\qquad moved \triangleq \{move(e, d) : d \in dirs\}\\ &\qquad free \triangleq \{\langle pc, m\rangle \in moved :\\ &\qquad\qquad \land m \cap (\text{UNION } (board \setminus \{pc\})) = \{\}\\ &\qquad\qquad \land \forall p \in m : p \in Pos\}\\ &\text{IN } \{(board \setminus \{pc\}) \cup \{m\} : \langle pc, m\rangle \in free\} \end{aligned}$$

$Init \triangleq board = Klotski$

$$\begin{aligned} Next \triangleq\ &\text{LET } empty \triangleq Pos \setminus \text{UNION } board\\ &\text{IN } \exists e \in empty : board' \in update(e, empty) \end{aligned}$$