MODULE *GameOfLife*

EXTENDS *Integers*

CONSTANT N
VARIABLE *grid*

ASSUME $N \in Nat$

$vars \triangleq grid$

RECURSIVE $Sum(\_, \_)$
$Sum(f, S) \triangleq$ IF $S = \{\}$ THEN 0
ELSE LET $x \triangleq$ CHOOSE $x \in S$: TRUE
IN $f[x] + Sum(f, S \setminus \{x\})$

$Pos \triangleq \{\langle x, y\rangle : x, y \in 1 \,..\, N\}$
$TypeOK \triangleq grid \in [Pos \to \text{BOOLEAN}\ ]$

$sc[\langle x, y\rangle \in (0 \,..\, N+1) \times (0 \,..\, N+1)] \triangleq$ CASE $\lor\ x = 0\ \lor\ y = 0$
$\lor\ x > N \lor y > N$
$\lor\ \neg grid[\langle x, y\rangle] \to 0$
$\square$ OTHER $\to 1$

$score(p) \triangleq$ LET $nbrs \triangleq \{x \in \{-1, 0, 1\} \times \{-1, 0, 1\} : x \neq \langle 0, 0\rangle\}$
$points \triangleq \{\langle p[1] + x, p[2] + y\rangle : \langle x, y\rangle \in nbrs\}$
IN $Sum(sc, points)$

$Init \triangleq grid \in [Pos \to \text{BOOLEAN}\ ]$
$Next \triangleq grid' = [p \in Pos \mapsto$ IF $\lor\ (grid[p] \land score(p) \in \{2, 3\})$
$\lor\ (\neg grid[p] \land score(p) = 3)$
THEN TRUE
ELSE FALSE$]$

$Spec \triangleq Init \land \square[Next]_{vars}$