

EXTENDS *Integers*

VARIABLE *board*

$W \triangleq 4H \triangleq 5$

$Pos \triangleq (0 \dots W - 1) \times (0 \dots H - 1)$

$Piece \triangleq \text{SUBSET } Pos$

$Klotski \triangleq \{ \{ \langle 0, 0 \rangle, \langle 0, 1 \rangle \},$
 $\{ \langle 1, 0 \rangle, \langle 2, 0 \rangle, \langle 1, 1 \rangle, \langle 2, 1 \rangle \},$
 $\{ \langle 3, 0 \rangle, \langle 3, 1 \rangle \}, \{ \langle 0, 2 \rangle, \langle 0, 3 \rangle \},$
 $\{ \langle 1, 2 \rangle, \langle 2, 2 \rangle \}, \{ \langle 3, 2 \rangle, \langle 3, 3 \rangle \},$
 $\{ \langle 1, 3 \rangle \}, \{ \langle 2, 3 \rangle \}, \{ \langle 0, 4 \rangle \}, \{ \langle 3, 4 \rangle \} \}$

$KlotskiGoal \triangleq \{ \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle \} \in board$

$ChooseOne(S, P(-)) \triangleq \text{CHOOSE } x \in S : P(x) \wedge \forall y \in S : P(y) \Rightarrow y = x$

$TypeOK \triangleq board \in \text{SUBSET } Piece$

Given a position and a set of empty positions return a set of appropriately filtered von *Neumann* neighborhood points

$dir(p, es) \triangleq \text{LET } dir \triangleq \{ \langle 1, 0 \rangle, \langle 0, 1 \rangle, \langle -1, 0 \rangle, \langle 0, -1 \rangle \}$
 $\text{IN } \{ d \in dir : \wedge (p[1] + d[1] < W \wedge (p[1] + d[1]) \geq 0$
 $\wedge (p[2] + d[2] < H \wedge (p[2] + d[2]) \geq 0$
 $\wedge \langle p[1] + d[1], p[2] + d[2] \rangle \notin es \}$

Given a position and a unit translation vector return a pair of pieces, before and after translation in opposite this vector direction

$move(p, d) \triangleq \text{LET } s \triangleq \langle p[1] + d[1], p[2] + d[2] \rangle$
 $pc \triangleq ChooseOne(board, \text{LAMBDA } pc : s \in pc)$
 $\text{IN } \langle pc, \{ \langle q[1] - d[1], q[2] - d[2] \rangle : q \in pc \} \rangle$

Given specific free position and a set of all free positions return a set of boards updated by moving appropriate pieces to that free position

$update(e, es) \triangleq \text{LET } dirs \triangleq dir(e, es)$
 $moved \triangleq \{ move(e, d) : d \in dirs \}$
 $free \triangleq \{ \langle pc, m \rangle \in moved : m \cap (\text{UNION } (board \setminus \{ pc \})) = \{ \} \}$
 $\text{IN } \{ (board \setminus \{ pc \}) \cup \{ m \} : \langle pc, m \rangle \in free \}$

$Init \triangleq board = Klotski$

$Next \triangleq \text{LET } empty \triangleq (0 \dots W - 1) \times (0 \dots H - 1) \setminus \text{UNION } board$
 $\text{IN } \exists e \in empty : board' \in update(e, empty)$