──────────────── MODULE *GameOfLife* ────────────────

EXTENDS *Integers*, *Sequences*

CONSTANT N
VARIABLE *board*

ASSUME $N \in Nat$

$vars \triangleq board$

$Pos \triangleq \{\langle x, y \rangle : x, y \in 1 .. N\}$

$TypeOK \triangleq board \in [Pos \to \text{BOOLEAN}]$

RECURSIVE $Sum(\_)$
$Sum(S) \triangleq$ IF $S = \langle \rangle$ THEN 0
$\qquad\qquad\qquad$ ELSE $Head(S) + Sum(Tail(S))$

$score(p) \triangleq$ LET $sc(a) \triangleq$ LET $x \triangleq a[1]$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad y \triangleq a[2]$
$\qquad\qquad\qquad\qquad\qquad$ IN $\quad$ CASE $\lor x = 0 \ \lor y = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \lor x > N \lor y > N$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \lor \neg board[a] \to 0$
$\qquad\qquad\qquad\qquad\qquad\qquad \Box \quad \text{OTHER} \ \to 1$
$\qquad\qquad\qquad nbrs \triangleq \langle\langle -1, \ -1 \rangle, \langle -1, \ 0 \rangle, \langle -1, \ 1 \rangle,$
$\qquad\qquad\qquad\qquad\qquad \langle 0, \qquad -1 \rangle, \qquad\qquad\quad \langle 0, \ 1 \rangle,$
$\qquad\qquad\qquad\qquad\qquad \langle 1, \qquad -1 \rangle, \langle 1, \ 0 \rangle, \langle 1, \ 1 \rangle\rangle$
$\qquad\qquad\qquad points \triangleq [n \in \text{DOMAIN } nbrs \mapsto sc(\langle p[1] + nbrs[n][1],$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad p[2] + nbrs[n][2] \rangle)]$
$\qquad\qquad$ IN $\quad Sum(points)$

$Init \quad \triangleq board \in [Pos \to \text{BOOLEAN}]$
$Next \quad \triangleq board' = [p \in Pos \mapsto$ IF $\ \lor \ (board[p] \land score(p) \in \{2, 3\})$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \lor (\neg board[p] \land score(p) = 3)$
$\qquad\qquad\qquad\qquad\qquad\qquad$ THEN TRUE
$\qquad\qquad\qquad\qquad\qquad\qquad$ ELSE $\ $ FALSE$]$

$Spec \ \triangleq \ Init \land \Box [Next]_{vars}$

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