Density of States of Hodge Laplacians: Decomposition effects and Sparsification of SC

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Abstract:
Keywords:

I Sparsification of Simplicial Complex

Let $\mathcal{K}$ be a simplicial complex with the unit weights of $\mathcal{V}_1(\mathcal{K})$, $W_1 = I$. Then $L_1^{\uparrow} = B_2 W_2^2 B_2^\top$ and the **generalized effective resistance** is given by

$$\mathbf{r} = \operatorname{diag}\left(B_2^\top \left(L_1^{\uparrow} \right)^+ B_2 \right) = \operatorname{diag}\left(B_2^\top \left(B_2 W_2^2 B_2^\top \right)^+ B_2 \right) \qquad \text{(Eqn. 1)}$$

then the sparsifying measure is defined as $\mathbf{p} \sim \operatorname{diag}\left(W_2^2 \mathbf{r} \right)$ (let us temporary believe that this is correct and we do not need to touch it).

Rem .1 **(on the weird-weird-weird matrix inside r)** Let us take a further look at GER above. Let SVD $B_2 W_2 = USV^\top$; then

$$\left(B_2 W_2^2 B_2^\top \right)^+ = US^+U^\top \qquad \text{(Eqn. 2)}$$

and $B_2 = USV^\top W_2^{-1}$. As a result,

$$B_2^\top \left(B_2 W_2^2 B_2^\top \right)^+ B_2 = W_2^{-\top} VSS^+SV^\top W_2^{-1} = W_2^{-1} VSV^\top W_2^{-1} \qquad \text{(Eqn. 3)}$$

Note that $VSV^\top = W_2 B_2^\top B_2 W_2 = L_3^{\downarrow}$. Hence,

$$\mathbf{r} = \operatorname{diag}\left(B_2^\top B_2 \right) = 3\mathbf{1} \qquad \text{(Eqn. 4)}$$

Okay, something is definitely broken...

I References

[KS16] Rasmus Kyng and Sushant Sachdeva. Approximate gaussian elimination for laplacians: Fast, sparse, and simple. May 2016.