Density of States of Hodge Laplacians: Decomposition effects and Sparsification of SC

Tony Savostianov [1]
[1]RWTH, Aachen , email: a.s.savostyanov@gmail.com

Abstract:
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I Sparsification of Simplicial Complex

Let $\mathcal{K}$ be a simplicial complex with the unit weights of $\mathcal{V}_1(\mathcal{K})$, $W_1 = I$. Then $L_1^{\uparrow} = B_2 W_2^2 B_2^\top$ and the **generalized effective resistance** is given by

$$\mathbf{r} = \operatorname{diag}\left(B_2^\top \left(L_1^{\uparrow} \right)^+ B_2 \right) = \operatorname{diag}\left(B_2^\top \left(B_2 W_2^2 B_2^\top \right)^+ B_2 \right) \qquad \text{(Eqn. 1)}$$

then the sparsifying measure is defined as $\mathbf{p} \sim \operatorname{diag}\left(W_2^2 \mathbf{r} \right)$ (let us temporary believe that this is correct and we do not need to touch it).

Rem .1 **(on the weird-weird-weird matrix inside r)** Let us take a further look at GER above. Let SVD $B_2 W_2 = USV^\top$; then

$$\left(B_2 W_2^2 B_2^\top \right)^+ = US^{+2}U^\top \qquad \text{(Eqn. 2)}$$

and $B_2 = USV^\top W_2^{-1}$. As a result,

$$B_2^\top \left(B_2 W_2^2 B_2^\top \right)^+ B_2 = W_2^{-\top} VSS^{+2}SV^\top W_2^{-1} \qquad \text{(Eqn. 3)}$$

Since S and S^+ are diagonal, any permutations of $SS^{+2}S$ are allowed. Then $SS^{+2}S = SS^+$. As a result for GER:

$$\begin{gathered} \mathbf{r} = \operatorname{diag}(XX^\top) \\ \text{where } X = W_2^{-1} VSS^+ = W_2^{-1} V \Pi = W_2^{-1} V_1 \end{gathered} \qquad \text{(Eqn. 4)}$$

where V_1 is the orthonormal basis of $\operatorname{im} B_2^\top$.

I References

[KS16] Rasmus Kyng and Sushant Sachdeva. Approximate gaussian elimination for laplacians: Fast, sparse, and simple. May 2016.