Topological Stability and Preconditioning of Higher-Order Laplacian Operators on Simplicial Complexes

Anton Savostianov[1]

[1]*Gran Sasso Science Institute, viale F.Crispi 7, L'Aquila, Italy, 67100, email:* anton.savostianov@gssi.it

Abstract: Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Keywords: NNs, condition number

I. Introduction

II. From Graphs to Simplicial Complex

II.I Simplicial Complexes

Let $V = \{v_1, v_2, \dots, v_n\}$ be a set of nodes; as discussed above, such set may refer to various interacting entities and agents in the system, e.g. neurons, genes, traffic stops, online actors, publication authors, etc. Let σ be a *simplex*[1] on $(k+1)$-node subset of V; then we refer to it as simplex of order k and all its $(k-1)$-order subsimplices as *faces*. Then:

[1] for the purposes of the current work, a subset of V

Def. 1 **(Simplicial Complex)** The collection of subsets $\mathcal{K}$ of the nodal set $\{v_1, v_2, \dots, v_n\}$ is a *simplicial complex* if each element $\sigma \in \mathcal{K}$, referred as a *simplex*, enters $\mathcal{K}$ with all its faces.

Let $\mathcal{V}_k(\mathcal{K})$ be a set of all k-order simplices in $\mathcal{K}$ and m_k is the caridnality of $\mathcal{V}_k(\mathcal{K})$, $m_k = |\mathcal{V}_k(\mathcal{K})|$; then $\mathcal{V}_0(\mathcal{K})$ is the set of nodes in the simplicial complex $\mathcal{K}$, $\mathcal{V}_1(\mathcal{K})$ — the set of edges, $\mathcal{V}_2(\mathcal{K})$ — the set of triangles, or 3-cliques, and so on, with $\mathcal{K} = \{\mathcal{V}_0(\mathcal{K}), \mathcal{V}_1(\mathcal{K}), \mathcal{V}_2(\mathcal{K}) \dots\}$. Note that due to the inclusion rule in Definition 1, the number of non-empty $\mathcal{V}_k(\mathcal{K})$ is finite and, moreover, uninterupted in a sense of the order: if $\mathcal{V}_k(\mathcal{K}) = \varnothing$, then $\mathcal{V}_{k+1}(\mathcal{K})$ is also necessarily empty.

Example (Simplicial Complex) 123

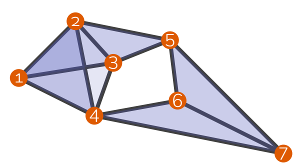

Figure 1: Example of a simplicial complex

III. Matrix nearness problems

IV. Topological Stability of Simplicial Complexes

IV. References

[KS16] Rasmus Kyng and Sushant Sachdeva. Approximate gaussian elimination for laplacians: Fast, sparse, and simple. May 2016.