

Topological Stability and Preconditioning of Higher-Order Laplacian Operators on Simplicial Complexes

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Abstract: Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Keywords: NNs, condition number

I. Introduction

II. From Graphs to Simplicial Complex

II.1 Simplicial Complexes

Let $V = \{v_1, v_2, \dots, v_n\}$ be a set of nodes in the system; as discussed above such set may correspond to various entities and agents in the system. Let σ be a *simplex*¹ on $(k+1)$ -node subset of V ; then we refer to it as simplex of order k and all its $(k-1)$ -order subsimplices as *faces*. Then:

Def. 1

(Simplicial Complex) The collection of subsets \mathcal{K} of the nodal set $\{v_1, v_2, \dots, v_n\}$ is a *simplicial complex* if each element $\sigma \in \mathcal{K}$, referred as a *simplex*, enters \mathcal{K} with all its faces.

¹ for the purposes of the current work, a subset of V

Let $\mathcal{V}_k(\mathcal{K})$ be a set of all k -order simplices in \mathcal{K} and m_k is the cardinality of $\mathcal{V}_k(\mathcal{K})$, $m_k = |\mathcal{V}_k(\mathcal{K})|$; then $\mathcal{K} = \{\mathcal{V}_0(\mathcal{K}), \mathcal{V}_1(\mathcal{K}), \mathcal{V}_2(\mathcal{K}) \dots\}$. Note that due to the inclusion rule in Definition 1, the number of non-empty $\mathcal{V}_k(\mathcal{K})$ is finite and, moreover, uninterrupted in a sense of the order: if $\mathcal{V}_k(\mathcal{K}) = \emptyset$, then $\mathcal{V}_{k+1}(\mathcal{K})$ is also necessarily empty.

III. Matrix nearness problems

IV. Topological Stability of Simplicial Complexes

IV. References

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