Solutions exam FYSA14 11ᵗʰ January 2021
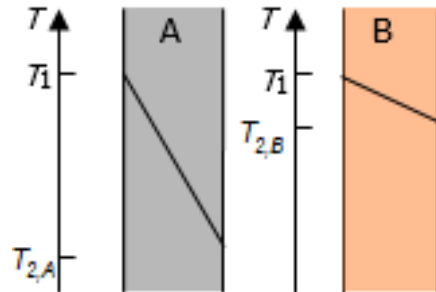
Every problem is worth 3 points. The maximum score is 24 points. G=12p, VG=19p.

1. a) The heat flow H is the same through both materials **A** and **B**. Heat flow by conduction is given by

$$H = k_A A \frac{\Delta T_A}{L} = k_B A \frac{\Delta T_B}{L}$$

Since the surface area A and thickness L are the same for both materials, it follows that

$$k_A \Delta T_A = k_B \Delta T_B$$

The temperature difference $\Delta T_A > \Delta T_B$ (because, according to the figure, $T_{2,A} < T_{2,B}$), so we can see that thermal conductivity $k_A < k_B$. Since a <u>low</u> thermal conductivity means the material is a good <u>insulator</u>, the best insulating material is material **A**.

<u>Answer:</u> Material A is the best insulating material.

b) The molar heat capacity for an ideal gas is $C_V = \frac{f}{2}R$, where f is the degrees of freedom for the gas. A monoatomic gas (like He) has no internal rotation or vibration, so $f = 3$ (translational movement gives three degrees of freedom). A diatomic gas (like N₂) has $f = 5$ (we get 2 rotational degrees of freedom, but no vibrational degrees are active at room temperature). For molecules with more than two atoms (like CO₂) the theory for degrees of freedom is more complicated, but generally larger molecules have more degrees of freedom and therefore higher C_V (CO₂ has apparent degrees of freedom $f = 6.67$).

The temperature increase when heat (Q) is added during a constant-volume process is given by

$$Q = nC_V \Delta T \qquad \Leftrightarrow \qquad \Delta T = \frac{Q}{nC_V}$$

Since the number of moles n is the same, the gas with the lowest heat capacity will see the highest temperature increase. According to the reasoning above, He has the lowest heat capacity.

<u>Answer:</u> The temperature will increase the most for gas (ii), He

c) The pV-diagram to the right shows a schematic of an isothermal expansion (i) and isobaric expansion (ii) from the same starting point (p₁, V₁, T₁) to a state where the volume V₂= 2V₁. The work $W = \int_{V_1}^{V_2} p\,dV$ is given by the area under the curve. The area under curve (ii) is larger, so the work performed during that process is greater.
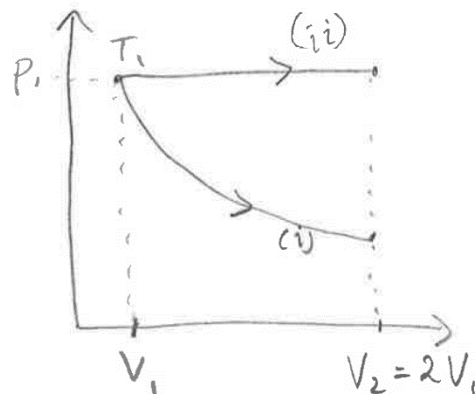
<u>Answer:</u> The work carried out by the gas will be largest for the isobaric expansion, case (ii)

2. a) The parameters the different substances (mass, starting temperature and specific heat capacity) used in this calculation are:

Water: $m_w = 0.15$ kg, $T_{0,w} = 10°C$, $c_w = 4190$ J/kg K
Saft (juice): $m_j = 0.05 \cdot 1.2 = 0.06$ kg, $T_{0,j} = 20°C$, $c_j = c_w = 4190$ J/kg K
Ice: $m_i = 0.02$ kg, $T_{0,i} = -15°C$, $c_i = 2000$ J/kg K, $L_f = 333 \cdot 10^3$ J/kg

Assume all the ice will melt, i.e. the final temperature $T_f \geq 0°C$. Then the ice will first warm to the melting point $T_{melt} = 0°C$, then the ice will melt, and finally the melted ice (now water) will increase its temperature from T_{melt} to T_f. The juice and the water will change their temperatures from the initial temperature to the final temperature T_f but will not go through a phase change. The sum of all heat exchanges involved is zero, i.e.

$$Q_w + Q_j + Q_i + Q_{i,m} + Q_{i,w} = 0$$

The heats in this equation are given by:

- Heat exchange for the water and juice: $Q_w = m_w c_w (T_f - T_{0,w})$, $Q_j = m_j c_w (T_f - T_{0,j})$
- Heating the ice to the melting point: $Q_i = m_i c_i (T_{melt} - T_{0,i})$
- Melting the ice: $Q_{i,m} = m_i L_f$, where L_f is the latent heat of fusion ($Q_{i,m} > 0$ since heat is transferred *into* the ice during melting)
- Heating the melted ice (OBS: the melted ice is now water and therefore has the specific heat capacity $c_w = 4190$ J/kg K): $Q_{i,w} = m_i c_w (T_f - T_{melt})$.

Putting these together gives us:

$$m_w c_w (T_f - T_{0,w}) + m_j c_w (T_f - T_{0,j}) + m_i c_i (T_{melt} - T_{0,i}) + m_i L_f + m_i c_w (T_f - T_{melt}) = 0$$

$$T_f (m_w c_w + m_j c_w + m_i c_w) = m_w c_w T_{0,w} + m_j c_w T_{0,j} + m_i c_w T_{melt} - m_i L_f - m_i c_i (T_{melt} - T_{0,i})$$

Solving the equation gives

$$T_f = \frac{m_w c_w T_{0,w} + m_j c_w T_{0,j} + m_i c_w T_{melt} - m_i L_f - m_i c_i (T_{melt} - T_{0,i})}{m_w c_w + m_j c_w + m_i c_w} = 4.21°C$$

The final temperature is $T_f = 4.21°C > 0$, so our assumption that $T_f \geq 0$ was correct.

Answer: The final temperature is 4.21°C

b) The original inner volume of the cylindrical cup (at 20°C) is $V_0 = \pi r^2 h = \pi \cdot 5^2 \cdot 12 = 942.478$ cm³. When the temperature changes, the volume will change according to

$$\Delta V = V_0 \beta \Delta T$$

Initial temperature is 20°C, final temperature is 4.21°C, and the coefficient of volume expansion for steel is $\beta = 36 \cdot 10^{-6}$ K^{-1}. This gives the

$$\Delta V = V_0 \beta (4.21 - 20) = -0.536 \text{ cm}^3$$

Answer: The volume has *decreased* by 0.536 cm³.

3. a) The dimensions of the freezer (30×40×50cm³) are illustrated in the figure to the right. Heat will flow out through all the walls of the freezer, i.e. the total surface area A for heat conduction is

$$A = 2lb + 2bh + 2lh$$

$$= 2 \cdot 0.3 \cdot 0.4 + 2 \cdot 0.4 \cdot 0.5 + 2 \cdot 0.3 \cdot 0.5$$

$$= 0.94 \text{ m}^2$$
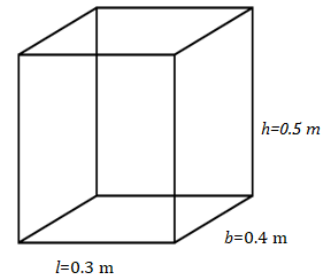
The walls of the freezer are 6 cm thick, i.e. the length of the freezer walls where the heat flows through is L=6 cm. The temperature difference between inside and outside of the freezer is $\Delta T = 20\text{-}(\text{-}18)=38°C$. The heat flow H through the walls is given by

$$H = kA\frac{\Delta T}{L} = 13.7 \text{ W}$$
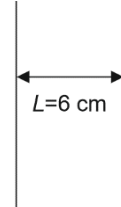
Answer: The heat flow out of the freezer through all the walls is 13.7 W

(Since asking for the "heat flow through the walls" could be interpreted in different ways, full credits are also given to students who calculated the heat flow through only one wall, or through only the 4 walls but not through the top and bottom)

b) The mass of the ice in the freezer is m_i=55 kg (calculated in c). If the temperature stays intact in the freezer through 30 hours, that means the heat flow H into the freezer will be constantly H=13.7 W during these 30 hours, i.e. the heat that flows into the freezer will be 49.3 kJ/hour or 1.48 MJ in 30 hours. The heat Q=1.48 MJ is enough to increase the temperature of the ice by

$$\Delta T = \frac{Q}{m_i c_i} = \frac{1.48 \cdot 10^6}{55 \cdot 2000} = 13.5 \text{ K}$$

This shows that the temperature in the freezer will not be constant for 30 hours.

(Since the heat flow depends on the temperature difference, and the temperature is changing, the heat flow into the freezer will not be constant 13.7 W during the 30 hours, and the actual temperature increase will be less than this)

Answer: No, there is a heat flow into the freezer which is enough to increase the temperature in the freezer. (You do not need this calculation to support the motivation that the temperature will increase)

c) We have assumed (unrealistically) that the freezer is completely filled with ice, i.e. that the volume of ice inside the freezer is the same as the volume of the freezer itself: $V = blh = 0.06$ m³. This means the mass of ice is $m_i = \rho V = 917 \cdot 0.06 = 55$ kg. The heat required to heat the ice from -18°C to the melting point (0°C) and then melt all of the ice into water is

$$Q = m_i c_i \Delta T + m_i L_f = 2.03 \cdot 10^7 \text{ J/kg}$$

Answer: The heat required for all the ice (at starting temperature -18°C) to melt into water is $2.03 \cdot 10^7$ J/kg

4. a) The lantern has the inner volume $V = \pi r^2 h = \pi (0.245)^2 \cdot 1.15 = 0.217$ m³. The gas is at atmospheric pressure (101325 Pa) and the temperature inside the lantern is 170°C=443.15 K. The ideal gas law gives us that the number of moles inside the lantern is

$$n = \frac{pV}{RT} = 5.96 \text{ mol}$$

The mass of the gas $m = n \cdot M = 173$ g, calculated from the number of moles and the molar mass M=29 g/mol.

<u>Answer</u>: The mass of all the air inside the lantern is 173 g

b) The temperature of the gas inside the lantern is 170°C=443.15 K. The average translational kinetic energy of a molecule in an ideal gas only depends on the temperature.

<u>Answer</u>: **Kinetic energy for <u>one</u> molecule:** $K_{molecule} = \frac{3}{2} kT = 9.18 \cdot 10^{-21}$ J

Kinetic energy for all the molecules in the gas combined: $K_{gas} = \frac{3}{2} nRT = 32.9$ kJ

c) The relative humidity is given by $R_{LF} = \frac{p_{H2O}(actual)}{p_{H2O}(saturated)}$, where p_{H2O} stands for the partial pressure of H_2O in the air. At 25°C, the saturation vapour pressure is 3.17 kPa. At a relative humidity of 50%, the actual partial pressure of H_2O in the air is

$$p_{H2O}(actual) = R_{LF} \cdot p_{H2O}(saturation) = 0.5 \cdot 3170 = 1585 \text{ Pa}$$

The lantern is filled with the same air as the surrounding air, which means the pressure is the same inside the lantern, and the partial pressure of H_2O is also the same, i.e.
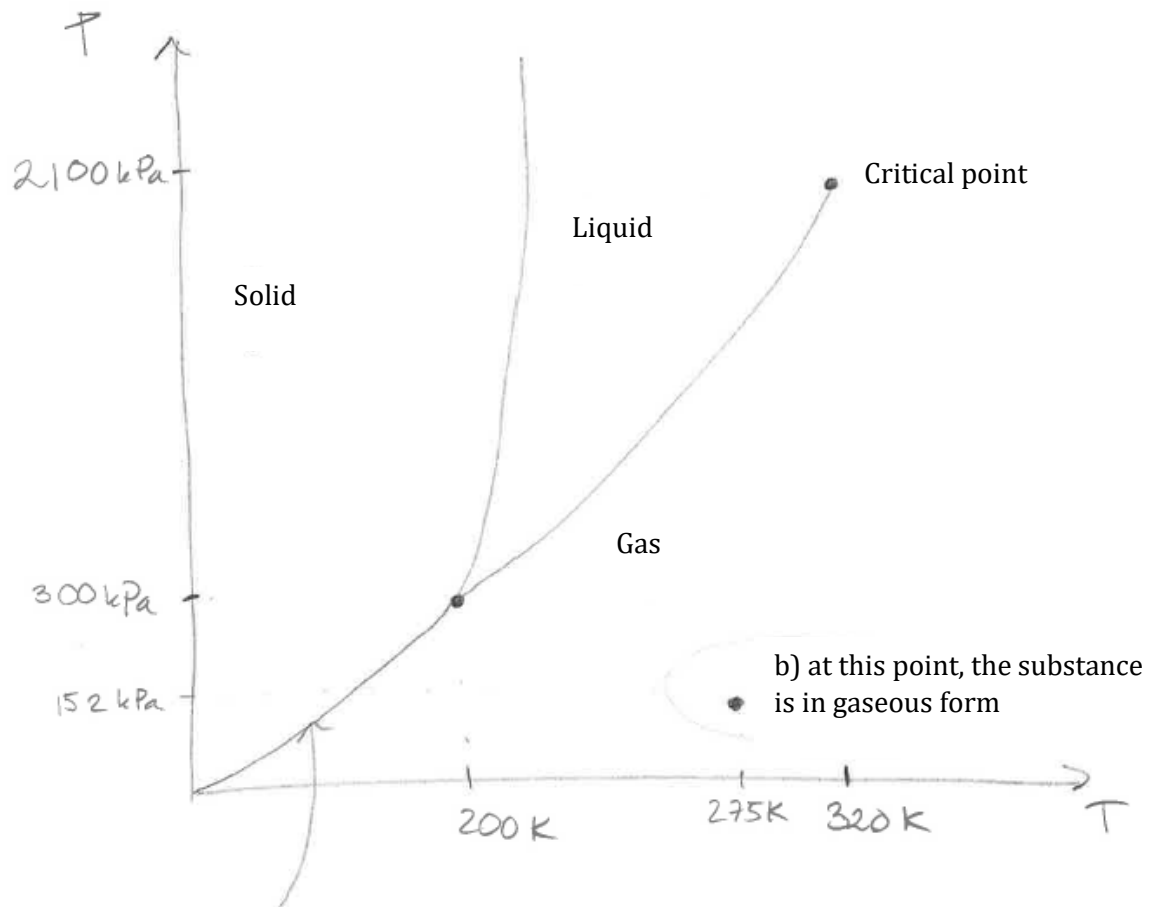
$$p_{H2O}(\ lantern) = p_{H2O}(actual) = 1585 \text{ Pa}$$

However, the saturated vapour pressure of H_2O at 100°C is $p_{H2O}(sat, 100°C) = 101.3$ kPa. This means the relative humidity inside the lantern is given by

$$R_{LF}(lantern) = \frac{p_{H2O}(actual)}{p_{H2O}(sat, 100°C)} = \frac{1585}{101300} = 1.56\%$$

<u>Answer</u>: **The relative humidity inside the lantern is 1.56%**

**5.** The figure below shows a schematic pT-diagram for the substance.

c) This line shows where the
sublimation phase transitions
take place

d) Normal air pressure, i.e. atmospheric pressure (101.3 kPa) is less than the triple point pressure (300 kPa). In the diagram above, we can see that the substance only exists in liquid form for pressures higher than the triple point pressure. Therefore, the substance will not be found in liquid form at atmospheric pressure.

6.

A: The figure shows the schematic cyclic process of a refrigerator

<u>True</u>. The cycle in the *pV*-diagram is counter-clockwise, which means the net work of the cycle is less than zero. Hence, this cycle requires input work in order to move heat around, i.e. a refrigerator (or a heat pump).
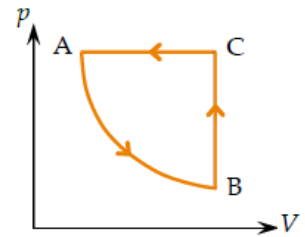
B: The figure shows the schematic cyclic process of a heat engine

<u>False</u>. A heat engine produces work, which means the net work for the cycle of a heat engine is >0.

C: $T_A > T_C$

<u>False</u>. The process C→A is an isobaric (constant pressure) process, which means $T \propto V$ (ideal gas law for constant pressure). Since $V_C > V_A$, the temperatures will be $T_C > T_A$

D: $T_C > T_A$

<u>True</u>. See motivation above

E: $\Delta U_{CA} = -\Delta U_{BC}$

<u>True</u>. $\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$ since the net change in internal energy in a cyclic process is zero. Process A→B is isothermal, so $\Delta U_{AB} = 0$ since the temperature does not change. This means that $\Delta U_{BC} + \Delta U_{CA} = 0$

F: $W_{netto} = Q_{netto}$

<u>True</u>. In a cyclic process, the change in internal energy in a cycle is equal to zero. According to the 1st law of thermodynamics, $Q = \Delta U + W$, the net heat exchange and the net work must then be equal, $W_{netto} = Q_{netto}$

7. a) Process 1→2 is adiabatic. The relation between temperature and volume in adiabatic processes is given by Poissons equations

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

For an ideal diatomic gas, the molar heat capacity at constant volume is $C_V = \frac{5}{2}R$, and the heat capacity at constant pressure is $C_p = C_V + R = \frac{7}{2}R$. This gives the heat capacity ratio $\gamma = \frac{C_p}{C_V} = 1.4$

The compression ratio for this cycle is $r = \frac{V_1}{V_2} = 11$. This gives us

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293 \cdot 11^{0.4} = 765 \text{ K}$$
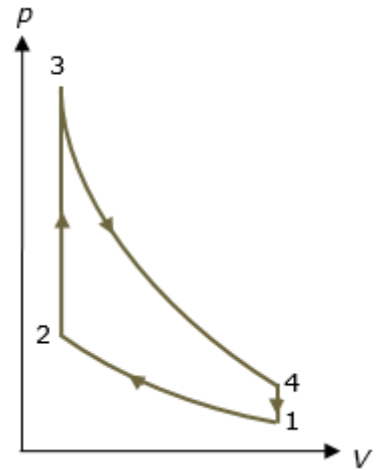
<u>Answer</u>: The temperature at point 2 is 765 K

b) Processes 1→2 and 3→4 are adiabatic, i.e. the heat exchange in these processes is zero. The heat is therefore exchanged during the isochoric processes 2→3 and 4→1. In one of these processes, the gas will absorb heat, and in the other, the gas will release heat. The amount of energy absorbed in each cycle, Q_{in}, is the heat that is *absorbed* by the gas.

For an isochoric process, we have $Q = nC_V\Delta T$. We know that $T_2 = 765 \text{ K} < T_3 = 1180 \text{ K}$. From the ideal gas law, we can see that for a constant volume-change, $p \propto T$, i.e. higher pressure means higher temperature. From this, we can deduce that $T_4 > T_1$, which means that $Q_{23} > 0$ and $Q_{41} < 0$. This in turn means that $Q_{in} = Q_{23}$ is the heat absorbed by the gas and $Q_{out} = Q_{41}$ is the heat released by the gas in each cycle.

The number of moles in the gas is given by

$$n = \frac{p_1 V_1}{RT_1} = \frac{101325 \cdot 0.5 \cdot 10^{-3}}{8.314 \cdot 293} = 0.0208 \text{ mol}$$

The heat absorbed in each cycle is

$$Q_{in} = Q_{23} = nC_V(T_3 - T_2) = 180 \text{ J}$$

<u>Answer</u>: 180 J of heat is absorbed in each cycle

c) The work can be calculated in 2 ways. The temperature in point 4 is $T_4 = T_3 \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 452 \text{ K}$

1) Processes 1→2 and 3→4 are adiabatic. $W_{net} = W_{12} + W_{34} = nC_V(T_1 - T_2) + nC_V(T_3 - T_4) = -204 + 315 = 111 \text{ J}$.

2) $W_{net} = Q_{netto} = Q_{in} + Q_{out} = Q_{23} + Q_{41} = nC_V(T_3 - T_2) + nC_V(T_4 - T_1) = 180 - 69 = 111 \text{ J}$
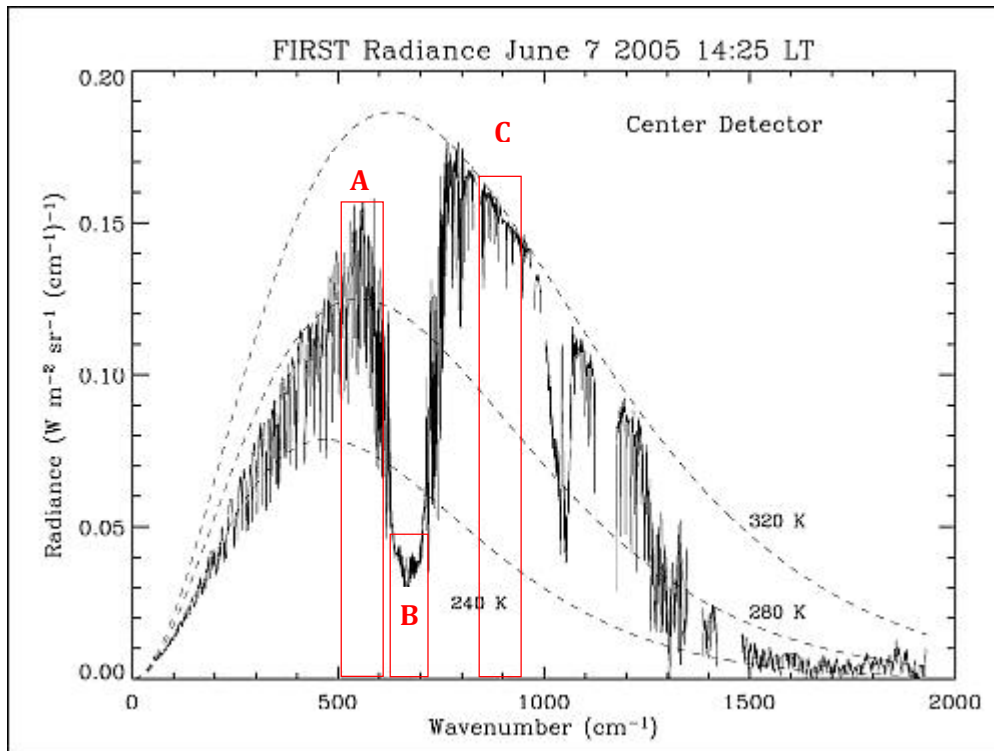
<u>Answer</u>: The net work performed by the gas during one full cycle is 111 J

d) The efficiency $e = \frac{W_{net}}{Q_{in}} = \frac{111}{180} = 61.7\%$

Or: $e = 1 - \frac{1}{r^{(\gamma-1)}} = 1 - \frac{1}{11^{0.4}} = 61.7\%$

<u>Answer</u>: The efficiency is 61.7%

8. Blackbody radiation and climate

FIRST Radiance June 7 2005 14:25 LT

a) Estimate the average temperature in degrees Celcius for the blackbody radiation in regions **A**, **B** and **C** of the spectrum, respectively. That is, what temperature would you estimate from this radiation if you only measured the light in each specific wavenumber region?

There is not a high-precision answer to this question. You are supposed to read out the approximate temperature by comparing the intensity of the light to the perfect blackbody-spectra in the figure.

- **A: in this region, the intensity of the radiation matches the curve for blackbody radiation at 280 K, so the temperature estimated from this light would be approximately 280 K or around 10°C**
- **In region B, the light intensity matches (on average) the blackbody spectrum for 240 K, so the temperature is approximately -30°C (the light intensity is lower than the blackbody spectrum, so it is also possible to interpret this as something in the interval -70 to -30 °C)**
- **In region C, the light matches well with the blackbody spectrum for 320 K, so the temperature we would estimate here would be around 50°C**

b) Explain shortly why the blackbody radiation in regions **A**, **B** and **C** of the emission spectrum represents different temperatures?

Answers: This question is related to the content on p.5 in the 'climate compendium'. The IR radiation we measure can be emitted either by the Earth's surface or by IR-active molecules in the atmosphere (greenhouse gases). The temperature in the lower 10-15 km of the atmosphere generally decreases with altitude, so the detected radiation is emitted from: different greenhouse gases representing different atmospheric altitudes (A+B, which are radiating at lower temperatures) or the Earth's surface with the radiation being transmitted through the atmospheric IR-window (C, since that radiation represents a higher temperature more consistent with the earth's surface).

8. c) Which components (species) in the atmosphere are likely to play a role for the spectrum in region A and B, respectively. Provide a short motivation for your answer.

Answers: This question is related to the content on p.5 in the 'climate compendium'. The maximum absolute concentration of $H_2O(g)$ in the atmosphere decreases 'fast' with decreasing temperature and increasing altitude, because of the much lower saturated vapour pressure at lower temperatures. Hence, IR-emissions from $H_2O(g)$ are likely to be responsible for the A-region, which is at higher temperature and therefore comes from a lower altitude. The IR-radiation in region B is emitted at very cold temperatures (i.e. very high altitudes), so it most likely comes from a well-mixed (long-lived) and important greenhouse gas which exists in high concentrations even at high altitudes, which makes CO_2 the most qualified guess. (If you remember the absorption of H_2O and CO_2 at different wavelengths from Fig. 4 in the climate-document, then that would also be a good base for the argumentation.)

d) If we imagine the unlikely scenario, that the population on Earth wanted a significantly warmer climate on Earth. And that you were asked to design the perfect greenhouse gas to significantly warm global climate – which properties and behaviour of that gas would be ideal? Mention at least 3 aspects to consider, in order to make a gas efficient as a greenhouse gas.

Answers: This question is related to the content about global warming potential (GWP) on p.6 in the 'climate compendium'. The should have a high GWP, which is ensured by the following properties. The gas should (i) efficiently absorb IR-radiation, (ii) in a range where Earth's surface emits with a significant intensity, (iii) absorb where other Greenhouse gases do not already absorb, (iv) have a long atmospheric lifetime, so that it will be well-mixed and present at altitudes of 10-15 km at low temperatures – as well as having an impact over longer timescales. In addition, it would be advantageous if the gas was (v) easy and cheap to produce in large amounts and not toxic/reactive.