Driving Scenario Design using Extended Kalman Filter (EKF)

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Abstract

This report explains the performance of the Extended Kalman Filter (EKF) in Driving Scenario Design. We filter noisy GPS measurements with a Constant Rate and Turn Velocity (CTRV) motion model. The results quantifying the accuracy and robustness of the EKF will be shown.

1 Introduction

Accurate state estimation is vital for autonomous systems, necessitating algorithms that can effectively manage sensor noise and nonlinear system dynamics. This report show the result of the EKF, which relies on linearization , on an identical vehicle tracking scenario.

2 Methodology

2.1 Simulation and Data Generation

The simulation used a time step $dt = 0.01$s. The vehicle followed a trajectory that included a significant, high-rate turn to test the filters under nonlinear conditions. Simulated GPS measurements were corrupted with Gaussian noise having a standard deviation of $\sigma = 0.75$m. The state vector $\mathbf{x}$ for the filter utilizes the CTRV model:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ v \\ \theta \\ \omega \end{bmatrix}$$

2.2 Filter Implementation

We use a nonlinear system (Extended Kalman Filter).

$$x_k = Fx_{k-1} + Bu_k + w_k, \tag{1}$$

2.2.1 EKF Formulas (Prediction Step)

The EKF prediction step estimates the prior state $\hat{\mathbf{x}}_k^-$ and the prior covariance $\mathbf{P}_k^-$ by propagating the previous estimate through the non-linear motion function f and its linearized Jacobian $\mathbf{F}_k$.

1. **State Prediction:** The predicted state is calculated directly using the non-linear CTRV motion model f on the previous state $\hat{\mathbf{x}}_{k-1}^+$:

$$\hat{\mathbf{x}}_k^- = f(\hat{\mathbf{x}}_{k-1}^+, 0)$$

2. **Covariance Prediction:** The predicted covariance is calculated by propagating the previous covariance $\mathbf{P}_{k-1}^{+}$ through the Process Jacobian $\mathbf{F}_k$ and adding the Process Noise Covariance $\mathbf{Q}$:

$$\mathbf{P}_k^{-} = \mathbf{F}_k \mathbf{P}_{k-1}^{+} \mathbf{F}_k^T + \mathbf{Q}$$

2.3 Update Step

The update step of the Extended Kalman Filter incorporates the new measurement to correct the predicted state and covariance. The equations are:

$$y_k = z_k - h(\hat{x}_k^-) \tag{2}$$
$$S_k = H_k P_k^- H_k^T + R \tag{3}$$
$$K_k = P_k^- H_k^T S_k^{-1} \tag{4}$$
$$\hat{x}_k^+ = \hat{x}_k^- + K_k y_k \tag{5}$$
$$P_k^+ = (I - K_k H_k) P_k^- \tag{6}$$

3 Results of the simulation

In Figure 1, we show our driving scenario design in MATLAB. We simulate a car in a relatively straight path and then change direction to the left.

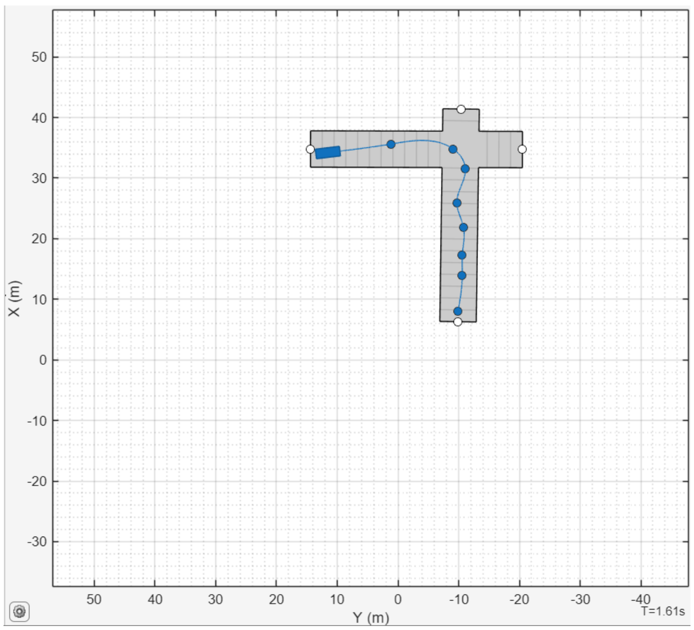

Figure 1: Our Driving Scenario Design.

3.1 Path Tracking Visualization

EKF successfully removes the noise from the raw GPS measurements. Figures 2 and 3 shows the position of our prediction on the X and Y directions over time.

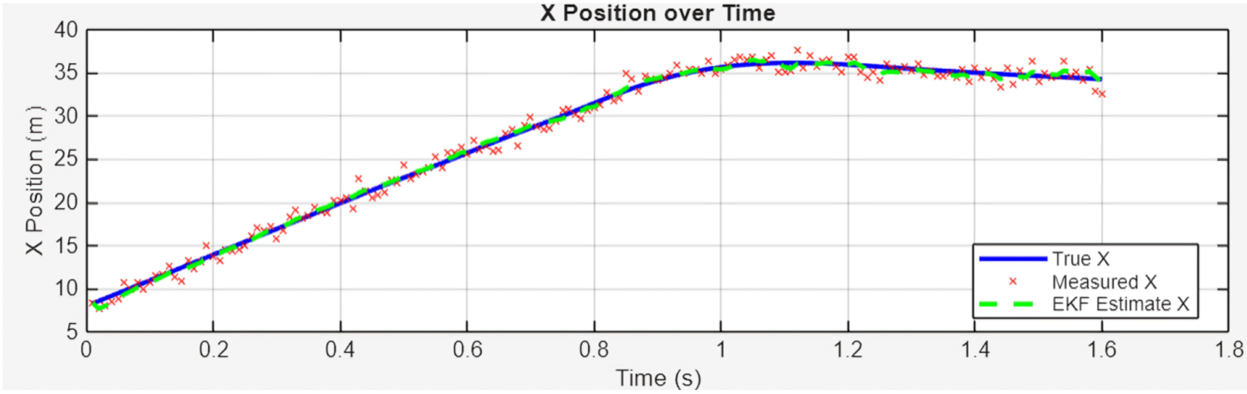

Figure 2: The position of our prediction on the X direction over time.

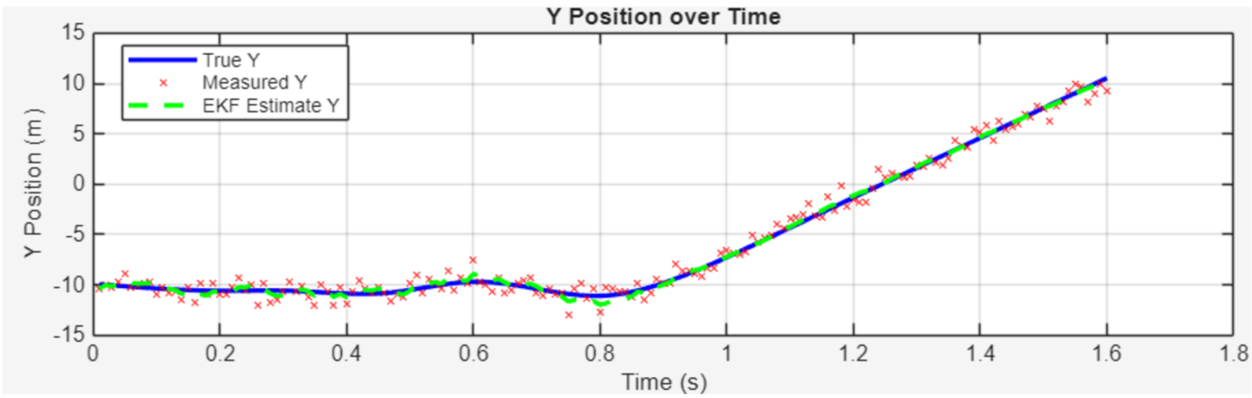

Figure 3: The position of our prediction on the Y direction over time.

The time-domain plots in Figures 2 and 3 demonstrate that the filter maintains stability and fidelity to the ground truth, even during the rapid change in the Y-coordinate. In Figure 4, we show the EKF tracking performance.

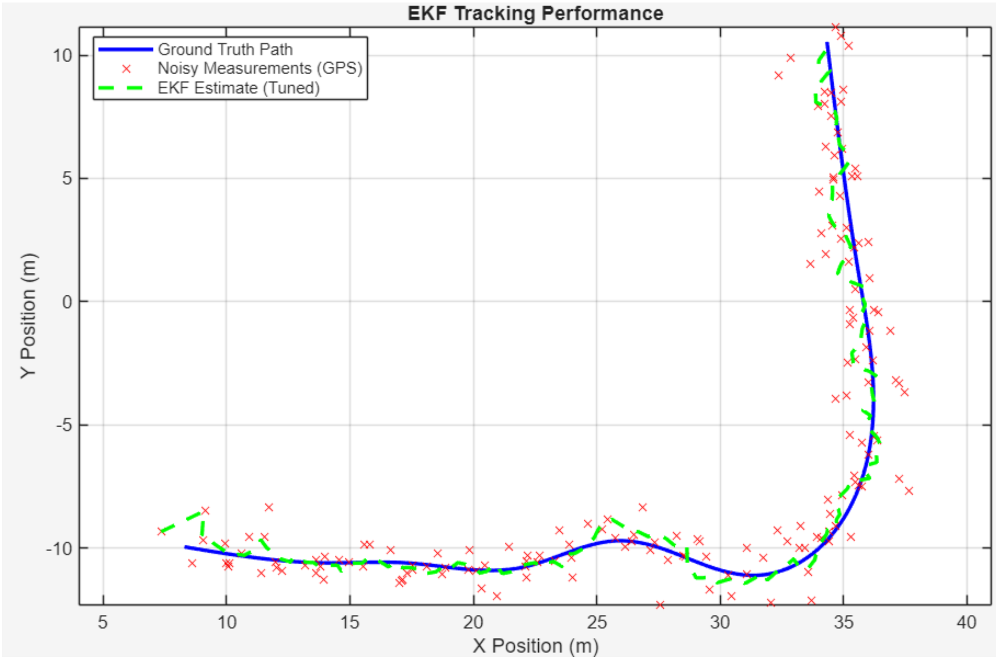

Figure 4: Performance Metrics.

The blue curve shows the actual path of the motion. The red dots are the output from the GPS sensor, which has high noise and irregular dispersion. The green dashed line is the output of the EKF, which attempts to correct the noisy data and approximate the true path.

3.2 Quantitative Performance Metrics

We use three metrics to evaluate the performance of our filter. RMSE, MAE, and MAX are our metrics for assessment. In Figure 5 we show the values of metrics in the noisy state, and after that we implement our filter.

```
Simulation complete. Generated 160 data points.
Running Extended Kalman Filter ...
EKF Complete.

 Performance Metrics
Metric:          Noisy Measurment Error.   Filtered Estimation Error.
RMSE:            1.0743 m              0.4245 m
MAE :            0.9442 m              0.3783 m
MAX :            2.4724 m              0.9612 m

Filter Bias (X, Y):
  Noisy:       [-0.065 m, -0.113 m]
  Filtered:    [-0.071 m, -0.167 m]

Overall RMSE Improvement: 60.48 %
```

Figure 5: Performance Metrics.

The results show that EKF significantly improved the accuracy of position estimation. RMSE is the best measure for examining the overall estimation quality in the presence of noise. The reduction in RMSE from

1.0743 m to 0.4245 m shows that our filter is able to remove a significant amount of sensor noise and recover the true path. A 60.48 percent improvement in RMSE for a positioning filter is considered a very desirable result.

MAE is less sensitive to outliers than RMSE. The reduction from 0.9442 to 0.3783 m emphasizes that our filter provides a consistent and stable estimate across most of the path and is not limited to improvements at a few points.

The maximum error has been reduced by almost one-third. This shows that EKF not only reduced the average error but also prevented sharp jumps in error. Bias increases slightly after filtering, but this is very small. The slight increase in bias is usually due to EKF model error or linearization approximations.