

Exercise

Optimizing Sink Position in a Wireless Sensor Network

- A. Find the lifetime of the system** when the sink is placed at the fixed position $(x_s, y_s) = (20, 20)$. Lifetime is defined as the time until the first sensor's battery dies, based on the energy consumption of the sensors.

Each sensor transmits a status update every 10 minutes. The energy consumption per transmission is determined by:

1. Energy for Transmission Circuitry (TX/RX)

$$E_c = 50 \text{ nJ/bit}$$

2. Energy for Transmission Over Distance

$$E_{tx}(d) = k \cdot d^2 \text{ nJ/bit}$$

3. Total Energy Per Transmission (Per Sensor)

$$E_{\text{total}}(d) = b \cdot (E_c + k \cdot d^2)$$

In the following, we calculated the distance of each sensor to sink. The Euclidean distance formula:

$$d = \sqrt{(x_s - x_i)^2 + (y_s - y_i)^2}$$

Sensor	Coordinates (x_i, y_i)	Distance d_i (m)
1	(1,2)	$\sqrt{(20 - 1)^2 + (20 - 2)^2} = \sqrt{361 + 324} = 26.92$
2	(10,3)	$\sqrt{(20 - 10)^2 + (20 - 3)^2} = \sqrt{100 + 289} = 19.72$
3	(4,8)	$\sqrt{(20 - 4)^2 + (20 - 8)^2} = \sqrt{256 + 144} = 20.00$
4	(15,7)	$\sqrt{(20 - 15)^2 + (20 - 7)^2} = \sqrt{25 + 169} = 13.00$
5	(6,1)	$\sqrt{(20 - 6)^2 + (20 - 1)^2} = \sqrt{196 + 361} = 23.60$
6	(9,12)	$\sqrt{(20 - 9)^2 + (20 - 12)^2} = \sqrt{121 + 64} = 13.60$
7	(14,4)	$\sqrt{(20 - 14)^2 + (20 - 4)^2} = \sqrt{36 + 256} = 16.49$
8	(3,10)	$\sqrt{(20 - 3)^2 + (20 - 10)^2} = \sqrt{289 + 100} = 19.24$
9	(7,7)	$\sqrt{(20 - 7)^2 + (20 - 7)^2} = \sqrt{169 + 169} = 18.38$

10	(12,14)	$\sqrt{(20-12)^2 + (20-14)^2} = \sqrt{64+36} = 10.00$
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Then, we computed energy consumption for each sensor.

$$E_{\text{total}}(d) = 2000 \times (50 + d^2) \text{ nJ}$$

Sensor	Distance d_i (m)	d_i^2	Energy per transmission $E_{tx}(d_i)$ (nJ)
1	26.92	724.89	$(50 + 724.89) \times 2000 = 1549780$
2	19.72	389.06	$(50 + 389.06) \times 2000 = 878120$
3	20.00	400.00	$(50 + 400) \times 2000 = 900000$
4	13.00	169.00	$(50 + 169) \times 2000 = 438000$
5	23.60	556.96	$(50 + 556.96) \times 2000 = 1213920$
6	13.60	184.96	$(50 + 184.96) \times 2000 = 469920$
7	16.49	272.06	$(50 + 272.06) \times 2000 = 644120$
8	19.24	370.24	$(50 + 370.24) \times 2000 = 840480$
9	18.38	337.91	$(50 + 337.91) \times 2000 = 775820$
10	10.00	100.00	$(50 + 100) \times 2000 = 300000$

Finally, we computed the lifetime of all sensors. As we know, each sensor starts with $E_b = 5 \text{ mJ} = 5 \times 10^6 \text{ nJ}$.

$$\text{Lifetime} = \frac{E_b}{E_{tx}(d_i)}$$

Sensor	Energy per transmission $E_{tx}(d_i)$ (nJ)	Lifetime (transmissions)	Lifetime (hours)
1	1549780	3.23	0.54
2	878120	5.69	0.95
3	900000	5.56	0.93
4	438000	11.42	1.90
5	1213920	4.12	0.69
6	469920	10.64	1.77
7	644120	7.76	1.29
8	840480	5.95	0.99
9	775820	6.44	1.07
10	300000	16.67	2.78

The lifetime of the system is determined by the sensor that dies first, which is sensor 1 with a lifetime of 0.54 hours (32.4 minutes).

For better understanding the result, we show our numerical result in Fig. 1.

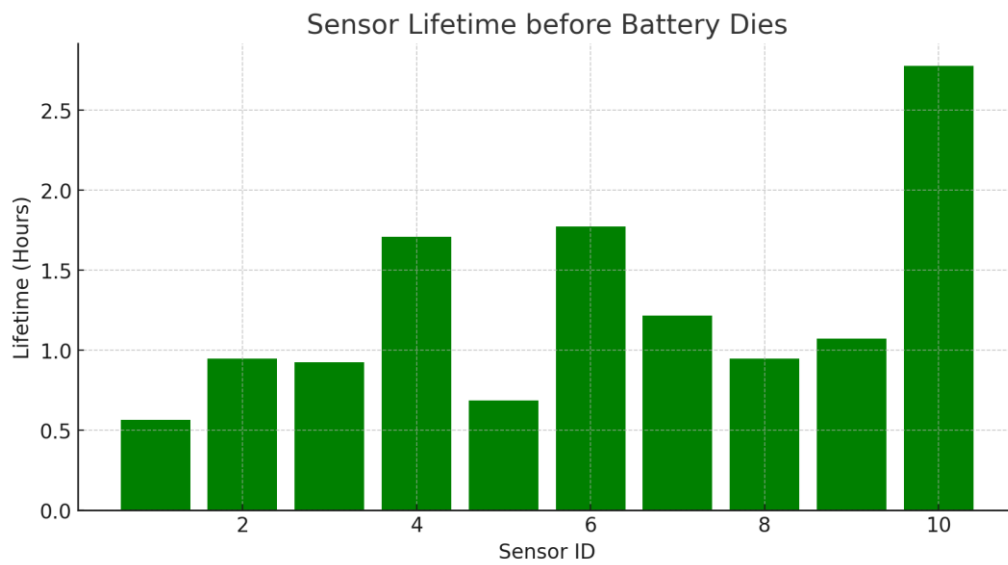


Fig. 1. Sensor lifetime when battery dies.

B. Find the optimal position of the sink that maximizes the system lifetime. Provide the coordinates (xs, ys) of the sink that minimizes the energy consumption of the worst-case sensor (the sensor that consumes the most energy).

We want to minimize:

$$\max_i (E_{\text{total}}(x_s, y_s))$$

First, we should calculate the TX/RX circuit energy consumption:

$$E_{\text{circuit}} = E_c \cdot b = 50 \times 2000 = 100000 \text{ nJ} = 0.100 \text{ mJ}$$

Then, Total energy per transmission is:

$$E_{\text{total}} = E_{\text{tx}} + E_{\text{circuit}}$$

In part A, we calculated distances from each sensor to sink and Energy Consumption for Each Sensor. We found an optimal sink position using mathematical calculations.

A reasonable initial approximation for optimal sink placement is the centroid of all sensor positions:

$$(x_c, y_c) = \left(\frac{1 + 10 + 4 + 15 + 6 + 9 + 14 + 3 + 7 + 12}{10}, \frac{2 + 3 + 8 + 7 + 1 + 12 + 4 + 10 + 7 + 14}{10} \right)$$

$$(x_c, y_c) = \left(\frac{81}{10}, \frac{68}{10} \right) = (8.1, 6.8)$$

To refine the optimal position, we evaluate different sink positions by computing the worst-case energy for each location and selecting the one that minimizes the maximum energy consumption. We select four possible candidate positions near the centroid of the sensor locations:

1. (8, 7)
2. (7, 7)
3. (8, 6)
4. (7, 6)

For each candidate position, we compute the Euclidean distance from each sensor and the total energy consumption for each sensor. The best sink position is the one that minimizes the worst-case energy.

For Sink at (8,7):

	Distance d_i (m)
1	$\sqrt{(8-1)^2 + (7-2)^2} = \sqrt{49 + 25} = \sqrt{74} = 8.6$
2	$\sqrt{(8-10)^2 + (7-3)^2} = \sqrt{4 + 16} = \sqrt{20} = 4.47$
3	$\sqrt{(8-4)^2 + (7-8)^2} = \sqrt{16 + 1} = \sqrt{17} = 4.12$
4	$\sqrt{(8-15)^2 + (7-7)^2} = \sqrt{49 + 0} = \sqrt{49} = 7.0$

5	$\sqrt{(8-6)^2 + (7-1)^2} = \sqrt{4+36} = \sqrt{40} = 6.32$
6	$\sqrt{(8-9)^2 + (7-12)^2} = \sqrt{1+25} = \sqrt{26} = 5.1$
7	$\sqrt{(8-14)^2 + (7-4)^2} = \sqrt{36+9} = \sqrt{45} = 6.7$
8	$\sqrt{(8-3)^2 + (7-10)^2} = \sqrt{25+9} = \sqrt{34} = 5.83$
9	$\sqrt{(8-7)^2 + (7-7)^2} = \sqrt{1+0} = \sqrt{1} = 1.0$
10	$\sqrt{(8-12)^2 + (7-14)^2} = \sqrt{16+49} = \sqrt{65} = 8.06$

	Total Energy Computation E_i
1	$E_1 = 2000 \times (50 + 74) = 248000$
2	$E_2 = 2000 \times (50 + 20) = 140000$
3	$E_3 = 2000 \times (50 + 17) = 134000$
4	$E_4 = 2000 \times (50 + 49) = 198000$
5	$E_5 = 2000 \times (50 + 40) = 180000$
6	$E_6 = 2000 \times (50 + 26) = 152000$
7	$E_7 = 2000 \times (50 + 45) = 190000$
8	$E_8 = 2000 \times (50 + 34) = 168000$
9	$E_9 = 2000 \times (50 + 1) = 102000$
10	$E_{10} = 2000 \times (50 + 65) = 230000$

For Sink at (7,7):

	Distance d_i (m)
1	7.81
2	5
3	3.16
4	8
5	6.08
6	5.38
7	7.61
8	5
9	0
10	8.60

	Total Energy Computation E_i
1	222000
2	150000
3	120000
4	228000
5	174000
6	158000
7	216000
8	150000
9	100000
10	248000

For Sink at (8,6):

	Distance d_i (m)
1	8.06
2	3.61
3	4.47
4	7.07
5	5.39
6	6.08
7	6.32
8	6.4
9	1.41
10	8.94

	Total Energy Computation E_i
1	230000
2	126000
3	140000
4	200000
5	158000
6	174000

7	180000
8	182000
9	104000
10	260000

For Sink at (7,6):

	Distance d_i (m)
1	7.21
2	4.24
3	3.61
4	8.06
5	5.1
6	6.32
7	7.28
8	5.66
9	1
10	9.43

	Total Energy Computation E_i
1	204000
2	136000
3	126000
4	230000
5	152000
6	180000
7	206000
8	164000
9	102000
10	278000

Sink Position	Worst-case Energy nJ
(8, 7)	248000
(7, 7)	230000
(8, 6)	238000
(7, 6)	228000

The best position is the one where the maximum energy consumption across all sensors is minimized. The lowest worst-case energy consumption (228000 nJ) occurs at (7,6), where the highest energy-consuming sensor. This position ensures that the sensor with the highest energy usage has the lowest possible energy consumption, thereby improving the overall system lifetime.

Also, we use python code to find the optimal position of the sink. We use Powell algorithm For optimizing in our code.

```

import numpy as np
from scipy.optimize import minimize

sensor_positions = np.array([
    (1, 2), (10, 3), (4, 8), (15, 7), (6, 1),
    (9, 12), (14, 4), (3, 10), (7, 7), (12, 14)
])

packet_size = 2000
Ec = 50
k = 1

def worst_case_energy(sink_position):
    xs, ys = sink_position
    distances = np.linalg.norm(sensor_positions - np.array([xs, ys]), axis=1)
    energies = packet_size * (Ec + k * distances**2)
    return np.max(energies) # We minimize the worst-case sensor energy

initial_guess = np.mean(sensor_positions, axis=0)

result = minimize(worst_case_energy, initial_guess, method='Powell')
optimal_sink_position = result.x
optimal_sink_position

```

array([7.80849019, 6.80055066])

Thus, the optimal sink position is (7.81,6.80) that we found from python code, minimizing energy consumption for the most energy-demanding sensor.

C. Discuss the trade-offs involved in choosing a fixed sink position versus dynamically moving the sink. Consider the impact on system lifetime and energy consumption of each sensor.

There are two main approaches for placing the sink:

1. Fixed Sink Position: The sink is placed at a single optimal location throughout the network's operation.
2. Dynamically Moving Sink: The sink moves to different positions over time to optimize energy consumption.

1. Fixed Sink Position Approach

Advantages:

Simple Implementation:

- Once the optimal sink position is determined (as we calculated in the previous task), it remains there for the entire operation.
- The sensors always send data to a fixed known location, making routing and communication straightforward.

Lower Infrastructure Cost:

- No need for additional mechanisms to move the sink, reducing complexity and hardware costs.

Predictable Energy Consumption:

- The energy required for each sensor is static and predictable, making power budgeting easier.

Suitable for Small Networks:

- If the parking lot is small, a fixed sink positioned at the optimized location can still provide a long network lifetime.

Disadvantages:

High Energy Consumption for Distant Sensors:

- Sensors that are far from the sink will have to use more transmission energy due to the squared distance dependence in energy consumption
- This causes the farthest sensor to drain its battery quickly, limiting the system's lifetime.

Unbalanced Energy Drain:

- Sensors closer to the sink consume much less energy than farther ones.
- The system's lifetime is determined by the worst-case sensor.

Not Adaptable to Changing Conditions:

- In dynamic environments where sensor conditions change (e.g., different parking congestion patterns), a fixed sink is not optimal.

2. Dynamically Moving Sink Approach

Advantages:

Balanced Energy Consumption Across Sensors:

- Since the sink moves to different positions, distant sensors at different times become closer to the sink, reducing their energy consumption.

Maximizes Network Lifetime:

- By dynamically placing the sink closer to the highest-energy-consuming sensor, we can extend the lifetime of the entire system.

Adaptability to Environmental Changes:

- If parking lot usage patterns change throughout the day, a dynamic sink can reposition itself to reduce congestion effects and minimize energy wastage.

Disadvantages:

Increased Complexity:

- Requires a movement algorithm and additional hardware (e.g., automated mobility mechanisms like drones, robotic vehicles, or movable base stations).

Higher Cost:

- Moving the sink frequently demands extra power for movement, increasing overall energy consumption.
- Deployment costs are significantly higher compared to a static sink.

Synchronization Issues:

- Sensors must always be aware of the sink's new position, requiring frequent updates and additional computational effort.
- If movement is not optimally planned, it may result in increased energy drain instead of savings.

If maximizing lifetime is the priority, a dynamically moving sink is better. If minimizing cost and complexity is more important, then a fixed optimized sink is the best choice.