Radar Imaging

Homework #1

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1. System Geometry

In this section, the angular position of a target is estimated using a uniform linear array (ULA). In the assignment, we are asked to consider the number of antennas as 9. In the implementation and analysis of the results, we consider the number of antennas as N. We are also given the sinusoidal signal used and its frequency.

2. Direction of Arrival Estimation

First, we will explain some of the required mathematical relationships along with the implemented code. All the code in MATLAB is commented for better understanding.

a. System Model

According to the points mentioned in the assignment, we must consider that the signal at the receiver is completely demodulated and that the phase term of the received signal is linearized around the central element. Also, using the given hints, the mathematical concepts of signal demodulation and linearization of a generic function are extracted and implemented in MATLAB. According to the given formulas, we compute:

$$lambda = \frac{C}{f_0}$$

$$dx = \frac{\lambda}{2}$$

We fix true angular position as an example, 20 degrees, to evaluate our prediction of angular position. Then we compute the length of the array:

$$L = \frac{lambda}{\Delta\theta}$$

Also, we use the number of antennas computed according to the below equations:

$$dx \cdot (1 - N) = L$$

$$N = \ round\left(\frac{L}{dx} + 1\right)$$

By calculations, the number of antennas required for the best design is 58 antennas.

We do linearization for the phase term with respect to x:

Phase term: $e^{-j2\pi\frac{f_0}{c}r_0*2}$, $t_0 = \frac{2r_0}{c}$ (The equation between distance and time)

According to the definition of angular frequency:

$$f(t\ \ ) = 2\pi f_0 t$$

Then $f'(t_0)$= $2\pi f_0 \cong\ 2\pi f_0 d_x$ (the first derivative of “f” with respect to x)

$$f_0 = \frac{\sin\ (theta\_t)}{lambda}$$

$$fx = \frac{2\sin(theta\_t)}{lambda}$$

b. Array Design and DoA Estimation

What should be the spacing between antennas?

The appropriate distance between the antennas is one of the factors affecting their performance. We fix the distance between antennas.

$$d_x = \frac{lambda}{2}$$

As mentioned, we used lambda/2, but lambda/4 is better for avoiding aliasing and also minimizing the grating lobes. In our implementation, when we set true angular lower than 30 degrees, fx remains within the unambiguous range, but for true angular >= 30 degrees, fx is placed out of the unambiguous range when the distance between antennas is equal to Lambda / 2. So, we set the distance between antennas to be equal to Lambda / 4 to avoid aliasing.

Can you propose a way to detect the direction of arrival?

In this section we use the Fast Fourier Transform (FFT) of the received demodulated signal. First, we did some preprocessing for FFT. We fix the sampling frequency and the number of FFT. Also, the spacing between frequency samples is considered in the frequency axis. Next, the fft of the received signal is calculated and the angle is estimated using f_peak. For example, when we fix True angle = 25, we get estimated angle = 25.07, and when we fix True angle = 40, we get estimated angle = 40.10, and this is a good result for our estimation. In Figure 2, true and estimated angles are marked in the FFT Spectrum.

```
Command Window
True Angle (degrees): 40.00
Estimated Angle (degrees): 40.10
Spatial Frequency Resolution (degrees): 1.12
```

Fig. 1. Result of estimated angle

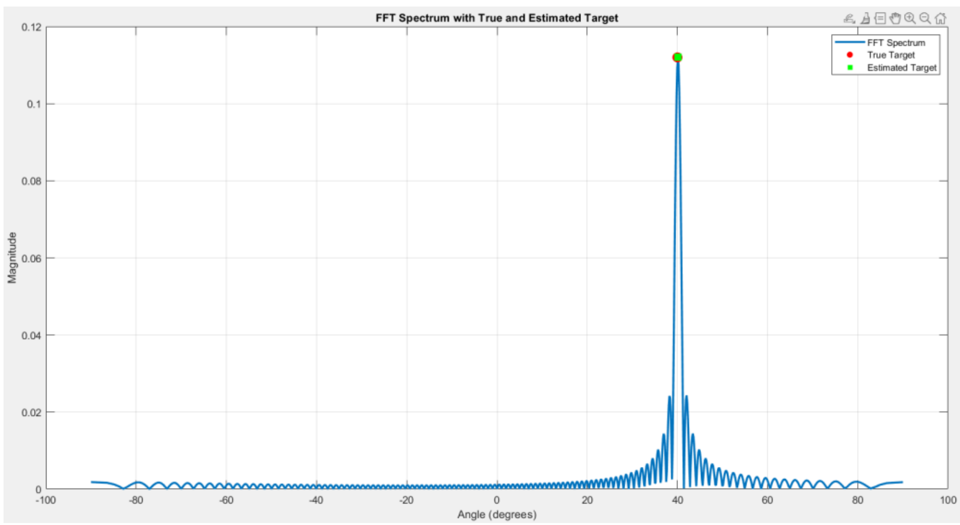

Fig. 2. FFT spectrum with marked true and estimated angular

Can you design the total array length to obtain the desired finest resolution of 2 degrees? According to the definition, Spatial frequency difference is the minimum angle that can be separated by an antenna array. In the frequency spectrum of the signal (after FFT), the minimum point is defined as the first point after the peak. This minimum value is shown by a * on the figure:

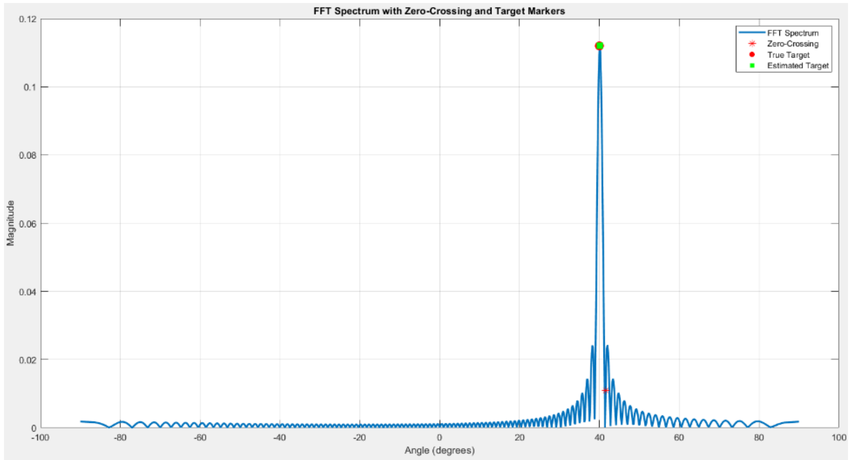

Fig.3. Display FFT spectrum with minimum point

Angular resolution using spatial frequency is obtained from the following equation

$$\Delta\theta = \arcsin\left(\frac{\Delta f \cdot \lambda}{2}\right)$$

We need to find $\Delta\theta$ value. This value for our example is and this is show good performance of our algorithm. Moreover, estimated angle is very close to our true angle. For better display of result, we show surface plot of FFT spectrum with true and estimated angle.

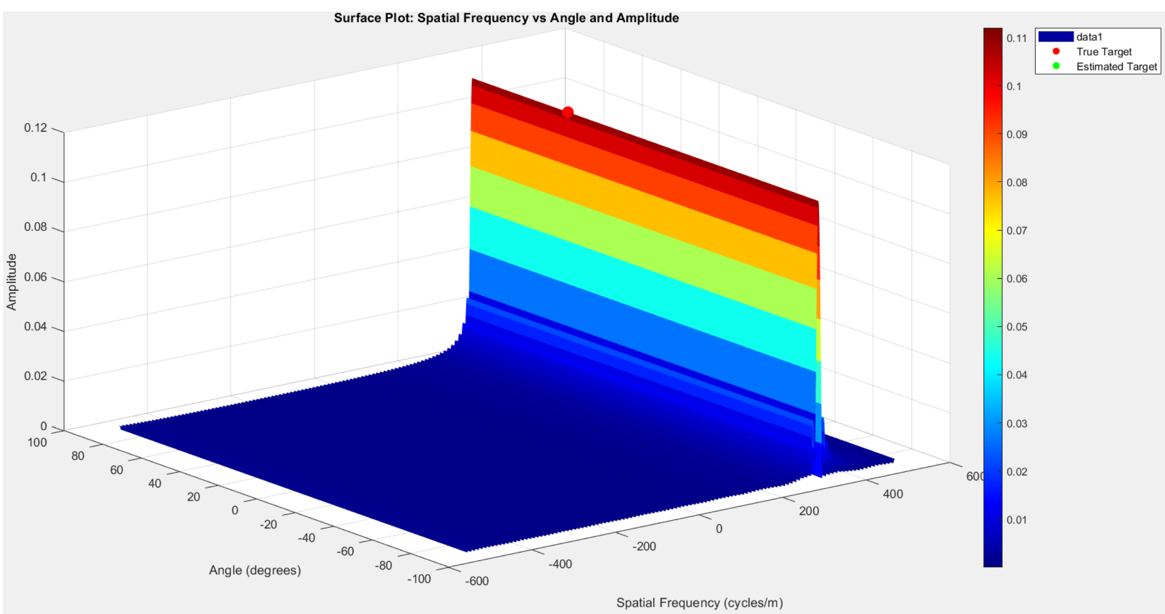

Fig.4. Surface plot of FFT spectrum with true and estimated angle

Is the resolution respected?

Yes, according to the results as we show, the resolution is respected.

Repeat the simulation with more than one target in the scene. What happens?

In this part of the task, we should consider two targets. So, we received signals from two targets. We fix the true angle for the first and second targets at 35 and 40, respectively. We received signals from targets and then summed them together. In the following, we show true and estimated angles marked in the FFT spectrum for two targets.

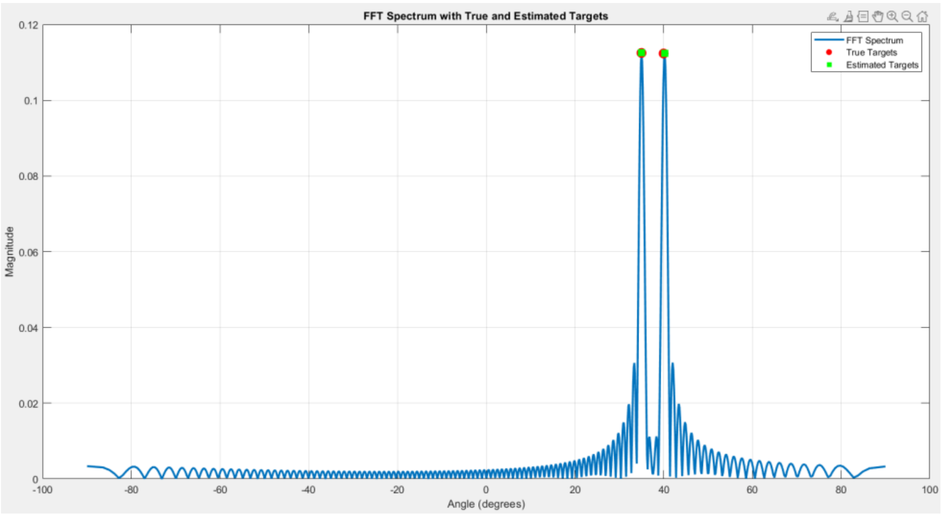

Fig.5. FFT spectrum with marked true and estimated angular for two targets

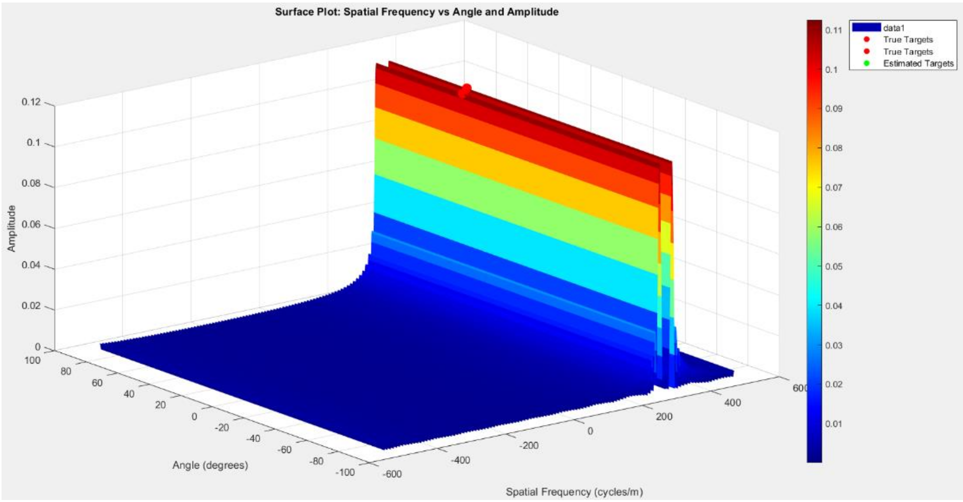

Fig.6. Surface plot of FFT spectrum with true and estimated angle for two targets

Figure 8 show the estimated angels for two targets that we obtained.

```
Command Window
True Angles (degrees): 35.00, 40.00
Estimated Angles (degrees): 35.02, 40.25
```

Fig.7. Result of estimated angle for two targets

What happens if you increase the spacing between antennas (dx)?

The estimated angle will decrease as the distance between antennas increases. Therefore, the system in this case has a lower ability to identify targets, and this issue reduces the ability of the radar system. For example, when I fix the true angle at 38 degrees, the estimate will be 38.08 degrees, and if I increase the spacing between antennas from lambda / 2 to lambda, the estimated angle will be 6.67.

3. 2D Position Estimation

In this part of the task, we want to find the distance between the target and the array. After doing some preprocessing for FFT, we get the signals received by an antenna in different time intervals. The transmitted signal is:

$$g(t) = \mathrm{sinc}(Bt)e^{j2\pi f_0 t}$$

And then we applied time delay to our signal. Then we use a cross-correlation algorithm to estimate range. This algorithm works by comparing the similarity between the transmitted and the received signals. We fix the true range at 35, and our estimate range is also 35. This result shows the good performance of our algorithm to estimate the range.

```
Command Window
True Range (m): 35.00
Estimated Range (m): 35.00
```

Fig.8. Result of estimated range

Finally, we plot the value of the estimated range.

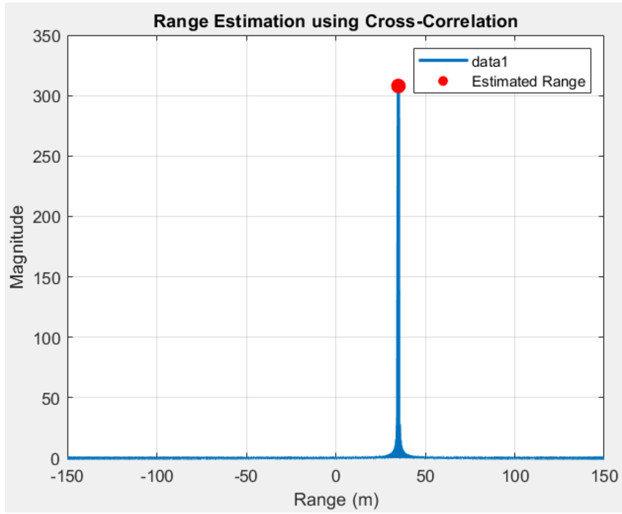

Fig.9. Display of range estimation using cross-correlation

What is the time and space resolution of such a signal?

According to the below time resolution equation, the value of time resolution in this task is 1 ns, and this shows the minimum time between two input signals from targets that radar can distinguish between them.

$$\Delta t = \frac{1}{B}$$

The equation of space resolution is

$$\Delta r = \frac{c \cdot \Delta t}{2}$$

And the value of it is 0.15 m, and this value says that the system can distinguish two targets that are at least 0.15 m from each other.

What is the expression of the received and demodulated signal at each antenna given a generic target in the scene?

For each antenna, the received antenna after reflection from the target has a delay in time and is phase-shifted and these delays depend on target position. The received signal is

$$s_{\text{received}}(t, n) = g(t - \tau_n) \cdot e^{j2\pi f_0 \tau_n}$$

The demodulated signal is

$$s_{\text{demod}}(t,n) = \text{sinc}\big(B(t-\tau_n)\big) \cdot e^{j2\pi\frac{2d_n \sin\theta}{\lambda}}$$

4. MIMO Radar

In this part of the homework, we want to estimate angle and range using MIMO radar. In the first part of the homework, we fix the number of antennas at 58, and in MIMO radar, the product between the number of transmitting antennas and the receiving antennas should be 58. So, we fix the number of transmitting antennas at 2 and the receiving antennas at 29.

```
Command Window
Number of virtual antennas: 58
Number of Tx antennas: 2
Number of Rx antennas: 29
```

Fig.10. Number of antennas in mimo radar

Then we find the positions of the virtual antennas. In the following we show the result of angle and range estimation and plot them.

```
True Angle: 20
Estimated Angle: 20.7836
```

Fig.11. Result of estimated angle for two targets

```
True Range: 14 m
Estimated Range: 14.0002 m
```

Fig.14. Result of estimated range for two

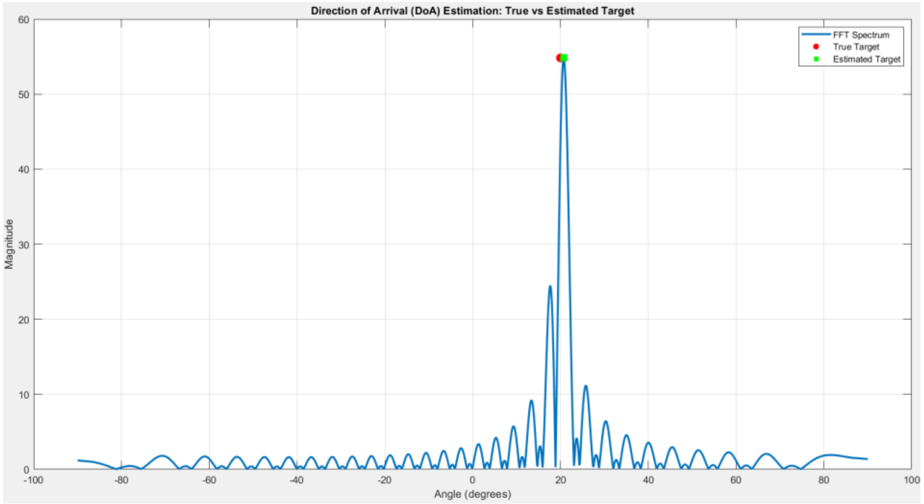

Fig.13. FFT spectrum with marked true and estimated angular in mimo radar

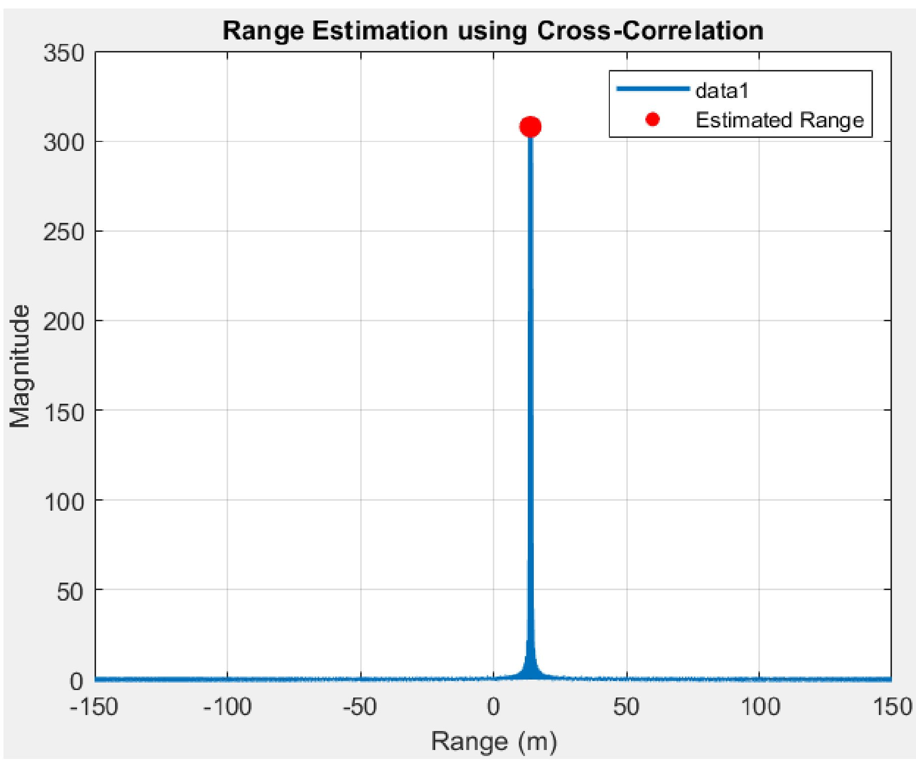

Fig.14. Display of range estimation in mimo radar