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Foot mass in a simple passive walker

Ding-wei Huang and Chih-shan Shih

Department of Physics, Chung Yuan Christian University, Chung-li, Taiwan
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We study numerically the effect of foot mass in a simple passive walker. The model describes a bipedal walking driven by gravitation only. Two parameter β and γ specify the physical configuration and the environmental influence, respectively. The stable walking can be sustained in a range of gentle slopes, which can be taken as an indicator to the stability of walker. The stability decreases drastically with the increase of foot mass. The regime of chaotic walking is extended. The limping walking becomes dominant.

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I. INTRODUCTION

Human locomotion involves complicate movement of muscle and bone. Two different approaches have been developed: active dynamics and passive dynamics. The former emphasizes the sophisticate control of the neuro-muscle system. The latter focuses on the classical mechanism of the bone-joint configuration. With naive judgement, the active-dynamics approach seems to be more realistic. The importance of the passive-dynamics approach has often been overlooked. Recently a simple robot, made using a child's Tinkertoy set, has been analyzed[1, 2]. Based on passive dynamics, the two-leg toy can walk stably with no control system. Mathematical equations from classical mechanics provide a well description of the bipedal walking. The relevance of passive dynamics has been highlighted. As a good control scheme is expected to take advantage of the natural dynamics of the respective system, the knowledge of passive dynamics can be very helpful to the understanding of active dynamics[3]. Thus, to better understand the stable gaits of bipedal walking, the mechanical parameters of human body shall be further explored.

The typical passive-dynamic model can be taken as a double-pendulum[4–6]. During the bipedal walking, one foot swings freely to a forward position with the other foot remains on the ground acting like a hinge. The locomotion is described as a nature repetitive cycle between the swing leg and the stance leg. The gravitation potential is the only energy to be dissipated in the motion. In this paper, we study a typical bipedal model[7]. In this version of model, the walker is described by two physical parameters: mass and length. It is interesting to note that these two parameters are both irrelevant to the dynamics. The mass in the dynamic equations can be arbitrarily rescaled. The length can also be rescaled by combining with the gravitational acceleration into a new time scale. The only nontrivial parameter is the slope of ramp, which describes the environment. We focus on the effects

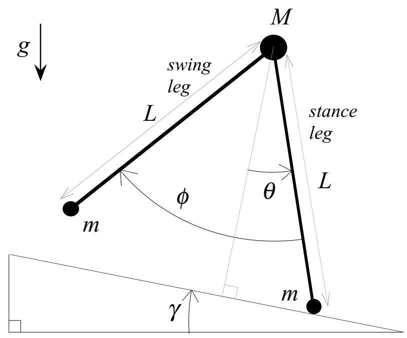

FIG. 1: Configuration of a typical passive walking model.

of foot mass. The finite mass assigned to the pointed foot introduces a new nontrivial parameter. Thus we will have two parameters: one for the physical body and the other for the environment. The feature of stable gaits are examined with numerical calculations. The influence of foot mass to the bifurcation of stable gaits is reported.

II. MODEL

Consider a simple 2D bipedal model walks passively down the ramp as shown in Fig. 1. The two legs are connected by a frictionless hinge at the hip. The mass is concentrated at the hip and feet, which are separated by a length L. The hip mass and foot mass are denoted by M and m, respectively. The ramp is specified by a slope γ. Basically, the physical configurations are parameterized by m, M, and L. The environmental conditions are parameterized by γ and the gravitational acceleration g. During walking, the stance foot is in contact with the ground and acts like a hinge. The swing foot acts like a free pendulum until it reaches heelstrike. The dynamics of the swing phase is described by the following coupled differential equations:[7]

$$\ddot{\theta}[ML + 2mL(1-\cos\phi)] - mL\ddot{\phi}(1-\cos\phi) - mL(\dot{\phi}^2 - 2\dot{\theta}\dot{\phi})\sin\phi$$
$$+ mg\sin(\theta-\phi-\gamma) - (M+m)g\sin(\theta-\gamma) = 0 \quad , \tag{1}$$
$$\ddot{\theta}L(1-\cos\phi) - \ddot{\phi}L + \dot{\theta}^2 L\sin\phi + g\sin(\theta-\phi-\gamma) = 0 \quad , \tag{2}$$

where θ and ϕ denote the stance angle and the swing angle, respectively. After rescaling the time by $\sqrt{L/g}$, the above equations have only two parameters: γ and β, where $\beta = m/M$ is the mass ratio between foot and hip. In this simple model, the variation of the physical configuration is described by a single parameter β; the influence of environment is also described by a single parameter γ. In previous studies, much emphasis was put on γ; in this work, we shall focus on β.

When the swing foot hits the ground at heelstrike, a plastic collision is prescribed. The velocity of the swing foot goes to zero. The swing leg becomes the stance leg, and vice versa. The energy is not conserved; yet the angular momentum is conserved. The following transition rules are applied at $\phi = 2\theta$,

$$\{\theta, \dot{\theta}\} \;\rightarrow\; \left\{-\theta, \dot{\theta}\,\frac{\cos 2\theta}{1+\beta\sin^2 2\theta}\right\} \quad , \tag{3}$$
$$\{\phi, \dot{\phi}\} \;\rightarrow\; \left\{-\phi, \dot{\phi}\,\frac{\cos\phi}{1+\beta\sin^2\phi}\right\} \quad . \tag{4}$$

With the differential equations for the swing and the transition rules at the heelstrike, a complete description to the gait cycles can be obtained.

In the limit of massless feet, $\beta = 0$, the gaits exhibit period doubling and apparently chaotic walking as the ground slope γ increases. At small enough γ, the period-one gait cycles are stable. A scaling relation, $\theta^* \sim \gamma^{1/3}$, can be observed between the slope and the amplitude of stance angle θ^*. The period-one motion becomes unstable at $\gamma \sim 0.015$, but a stable period-two gait appears, followed by a stable period-four gait, and so on. The walker continues to have stable persistent walking solution up to $\gamma \sim 0.019$. At higher γ, we could not find persistent walking motion. As the mass shifts from hip to feet, i.e., as β increases, the whole pattern shifts toward the lower slope region. The results are summarized in Fig. 2. At $\beta = 0.07$, the onset of stable period-two gait appears at $\gamma \sim 0.005$; and the persistent walking disappears at $\gamma \sim 0.013$. The regime of chaotic motion is significantly enlarged. The characteristic period-three gait can be easily observed, which did not show up in the case of $\beta = 0$. The details are shown in Fig 3. As β continues to increase, the stable period-two gait becomes dominant. The stable period-one gait can only be observed in the limit $\gamma \rightarrow 0$. The regime of chaotic motion also shrinks. At $\beta = 0.15$, the stable period-four gait appears at $\gamma \sim 0.005$; while the chaotic motion disappears at $\gamma \sim 0.006$.

As β further increases, the feature of the stable period-two gait changes drastically. The two branches cross each other at $\beta = 0.22$. When the two branches merge, the period-two gait appears just like a period-one gait as shown in Fig. 2. However, it is still the period-two solution. Although the stance angles share the same amplitude, the two can

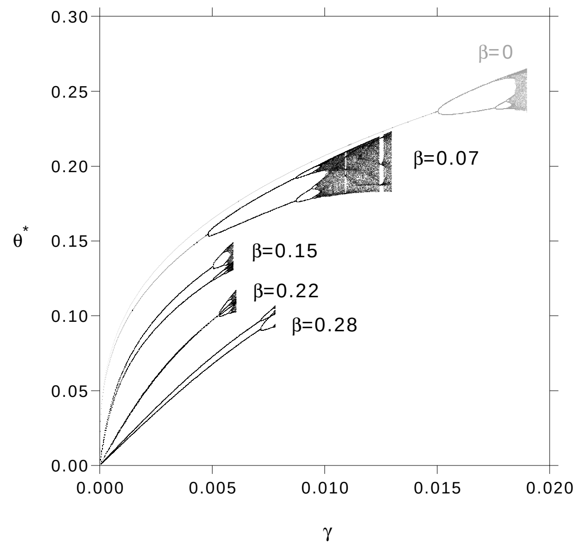

FIG. 2: Relation between stance angle θ^* and ground slope γ for various β. The case of $\beta = 0$ is shown in gray. The plotted variable θ^* denotes the amplitude of oscillations $\theta(t)$.

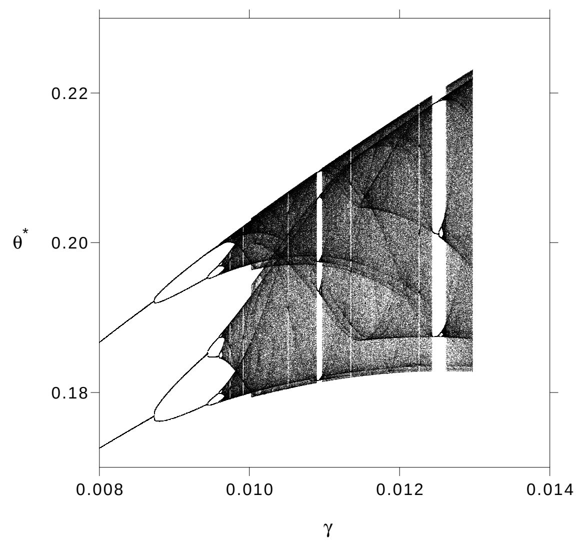

FIG. 3: Details of Fig. 2 for the case of $\beta = 0.07$.

still be distinguished by their different time dependence. The results are shown in Fig. 4(b). The gaits reveal clearly two alternating periods. Before the crossing, $\beta < 0.22$, the longer period is accompanied by a larger amplitude, see Fig. 4(a). After the crossing, $\beta > 0.22$, the shorter period has the larger amplitude, see Fig. 4(c). The crossing of the two branches also signifies a transition in the scaling relation between θ^* and γ. For small β, a $\gamma^{1/3}$-dependence is observed; for large β, a linear γ-dependence becomes apparent.

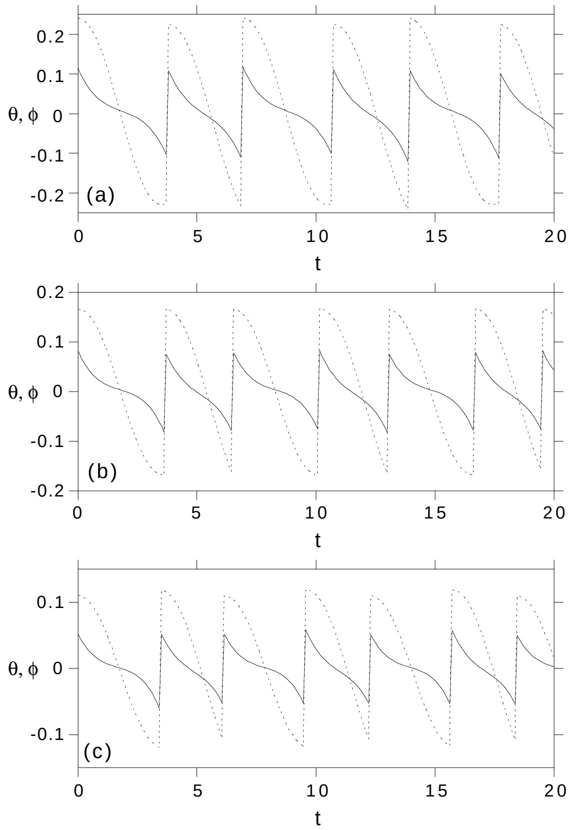

FIG. 4: Time evolution of stance angle $\theta(t)$ (solid lines) and swing angle $\phi(t)$ (dashed lines) at $\gamma = 0.004$: (a) $\beta = 0.15$; (b) $\beta = 0.22$; (c) $\beta = 0.28$. All the plots are started with the long gaits. For $\beta = 0.15$, the larger angles are associated with the long gaits; for $\beta = 0.28$, the smaller angles are associated with the long gaits.

At $\beta = 0.18$, the chaotic motion disappears at the smallest slope $\gamma \sim 0.0057$. As β increases from 0.18, the termination point of chaotic motion increases slowly. However, the chaotic regime also shrinks rapidly. For $\beta > 0.26$, the bifurcation stops abruptly before the chaotic motion can be fully developed. For $\beta > 0.3$, we cannot observe persistent motion anymore, not even the period-two solution. To summarize, the onset of the period-two gait and the stop of the chaotic motion are shown in Fig. 5, which can be taken as a phase diagram to characterize the stable solutions. For small β and γ, we find the period-one solutions. As β and/or γ increase, the period-two solutions become dominate. For large β and γ, there is no stable solution. Right on the boundary, we have the chaotic regime, which is most prominent around $(\beta, \gamma) \sim (0.07, 0.01)$.

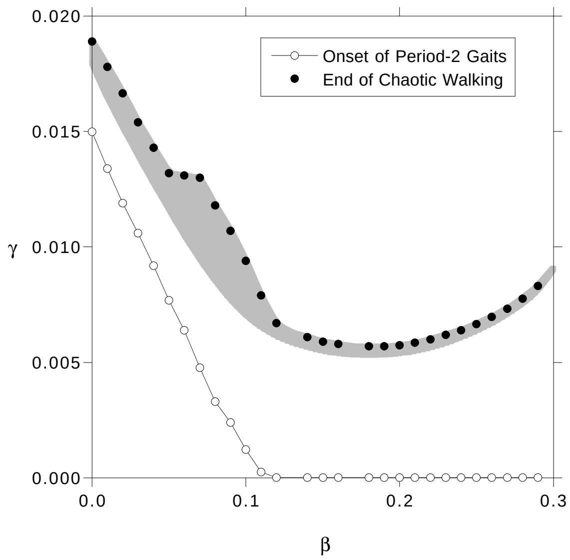

FIG. 5: Phase diagram in parameter space (β, γ) to characterize the solutions. The chaotic regime is marked by shade.

III. DISCUSSIONS

In this paper, we study the effect of foot mass in a simple 2D walker. With naive intuition, increasing the foot mass shall lower the center of gravity, and thus increase the stability of the walker. However, we find that such expectations can only be realized marginally within the range $0.2 < \beta < 0.3$, where the increase of foot mass will enable the walker to walk stably on a slightly steeper slope.

The passive dynamics of walking is described fully by deterministic mechanics. As the numerical simulations are employed, the obtained solutions are mathematically stable. All the unstable solutions revealed by some other analytic analysis are automatically excluded[7]. As the passive walker is powered by gravity only, the mechanical stability and energy efficiency can be simply measured by the maximum and minimum walking slopes, respectively. For $\beta < 0.1$, the maximum slope decreases dramatically and almost linearly with the increase of foot mass. A short pause to break the linear decreasing can be found around $\beta \sim 0.07$. After the rapid decrease, the maximum slope displays a saturation and then a slow rebound before losing all the stability for $\beta > 0.3$, see Fig. 5. If the walker can walk on level ground ($\gamma = 0$), it would be perfectly energy efficient, which implies that locomotion costs zero energy. For the straight-leg point-foot walker studied, the stable gait cycles persist as $\gamma \to 0$. The increase of foot mass does not change such a feature.

We observe that the foot mass can extend the regime of chaotic walking, which is most prominent around $\beta \sim 0.07$. With naive expectation, the periodic walking seems to be more desirable. However, if the primary objective of walking is to move efficiently without falling down, the chaotic walking can be more robust. Within the periodic regime, the stable walking can only be achieved by very specific configurations; within the chaotic regime, the stable walking can tolerate a much larger variation. It might not be surprised to find that the extended chaotic regime coincides with the pause in the rapid decreasing of stability. To keep the system in a chaotic regime might increase its stability.

The two legs of the simple walker are physically symmetric. However, we observe that the limping gaits (the period-two cycles) become dominant as the foot mass is considered. In the period-two gaits, a linear relation between the stance angle θ^* and the ground slope γ is observed. For each gait, the gravitational potential provides an amount of energy proportional to $\gamma\ \theta^*$. At each heelstrike, the loss of kinetic energy is proportional to the fourth power of θ^*, i.e., $(\theta^*)^4$. When these two energies are balanced as in the period-one gaits, a scaling relation of $\theta^* \propto \gamma^{1/3}$ is expected. In the period-two (limping) gaits, these two energies are imbalanced. At the heelstrike ending a long gait, the kinetic loss is less than the potential gain; at the heelstrike ending a short gait, the kinetic loss is larger than the potential gain. Thus some of the energy has been transferred from the long gaits to the short gaits. When the limping gaits first develop as the foot mass increases, the short gaits have smaller stance angle. When the foot mass increases, the stance angle of the short gaits becomes the larger one as more energy being transferred from the long gaits to the short gaits. A crossing at $\beta = 0.22$ has been noticed. Consequently, the kinetic loss in the heelstrikes ending the short gaits becomes the dominate dissipation of energy. The dominance of period-two gaits seems to suggest that limping walking is much more natural. The uneven erosion between one's left and right shoes seems to indicate that the limping walking can be a readily available mode of our symmetric legs. It would be interesting to further investigate the benefits of limping walking from the viewpoint of broken symmetry.

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