SC2001: Example class 1

Theoretical Analysis

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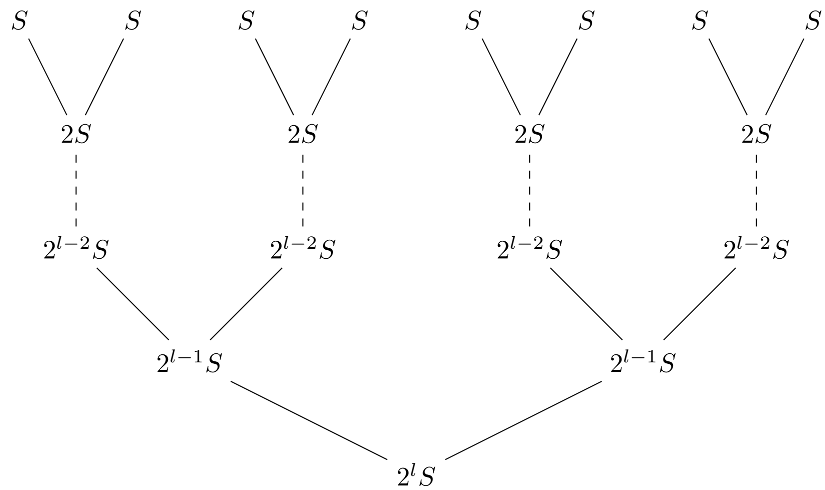
1 Hybrid Merge-Insertion Sort Analysis

1. Let n be the input size / no. of elements to be sorted.
2. Divide n elements into $\frac{n}{S}$ subarray of size S, the size of each subarray.
3. For each subarray of size S, perform insertion sort:
    - Best case: $O(S)$
    - Worst case: $O(S^2)$
    - Average case: $O(S^2)$
4. Since there are $\frac{n}{S}$ subarrays of size S, total cost of applying insertion sort as:
    - Best case: $\frac{n}{S} \cdot S = O(n)$
    - Worst case: $\frac{n}{S} \cdot S^2 = O(nS)$
    - Average case: $\frac{n}{S} \cdot S^2 = O(nS)$

5. Merge $\frac{n}{S}$ sorted subarray of size S



   - Each `merge()` doubles the subarray size. Suppose l iterations of `merge()` has to be performed to recover input size n:

$$\begin{aligned} 2^l S &= n \\ 2^l &= \frac{n}{s} \\ l \lg 2 &= \lg\left(\frac{n}{s}\right) \\ l &= \log_2\left(\frac{n}{s}\right) \end{aligned} \tag{1}$$

   - Since the cost of `merge()` is $O(n)$, the worst case total cost of merging M is:

$$\begin{aligned} M &= l \cdot n \\ &= \log_2\left(\frac{n}{s}\right) \cdot n \\ &= O(\lg\left(\frac{n}{s}\right) \cdot n) \end{aligned} \tag{2}$$

6. Combining insertion-sort & merging in Hybrid Merge-Insertion sort, we have:

   - Best Case: $O(n + \lg\left(\frac{n}{s}\right) \cdot n)$
   - Worst Case: $O(nS + \lg\left(\frac{n}{s}\right) \cdot n)$
   - Average Case: $O(nS + \lg\left(\frac{n}{s}\right) \cdot n)$

7. To derive an exact equation for total no. of keys comparisons.

   - For Insertion Sort in the average case, ith iteration's no. of key comparisons could range $1 \rightarrow i$ with uniform probability $\frac{1}{i}$.
   - Expected no. of key comparisons $= \frac{1}{i}\sum_{j=1}^{i} j$
   - Since there are $S-1$ iterations, total no. of comparisons:

$$\sum_{i=1}^{S-1} \frac{1}{i} \sum_{j=1}^{i} j = \frac{1}{2}(\frac{(S-1)(S+2)}{2}) \approx \frac{1}{4}S^2 \tag{3}$$

   - Total no. of comparisons for insertion sort $= \frac{1}{4}S^2 \cdot \frac{n}{S} = \frac{1}{4}nS$
   - For Merge Sort, the no. of comparisons $= \lg\left(\frac{n}{s}\right) \cdot n$
   - Total no. of comparisons for hybrid sort $= \frac{1}{4}nS + \lg\left(\frac{n}{s}\right) \cdot n$